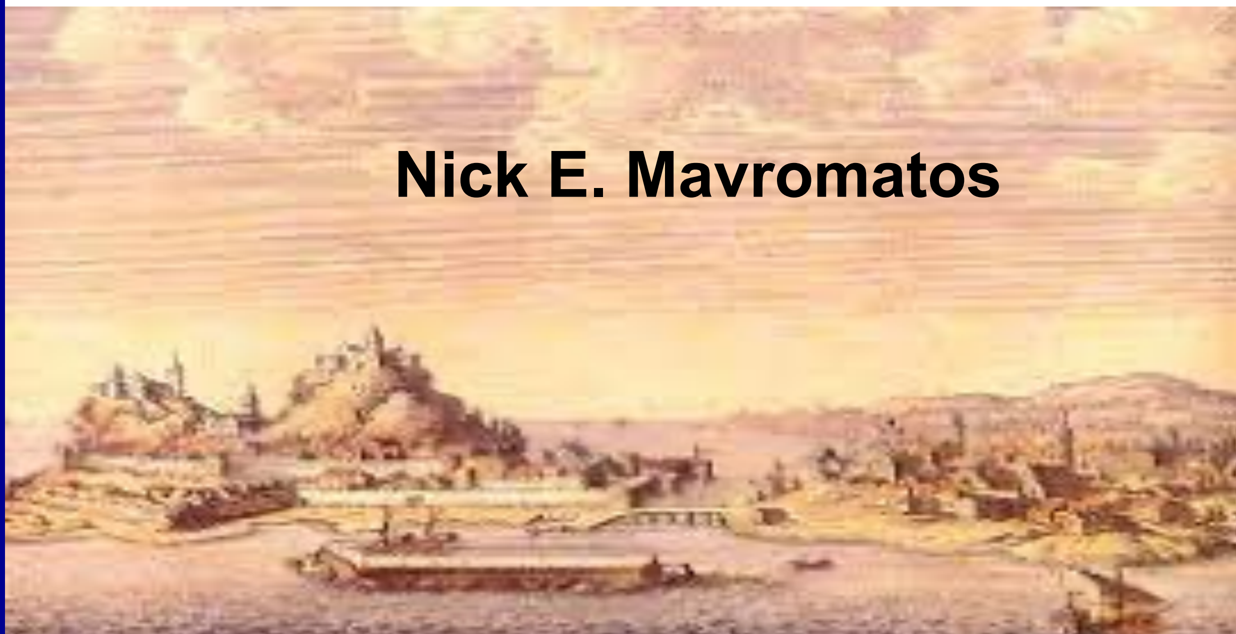
Torsion-induced Axions in String Theory, Quantum Gravity & the Cosmological Tensions



Nick E. Mavromatos

King's College London

COST – European Cooperation in Science & Technology

Quantum · Gravity · Multi · Messenger

CA18108 - Quantum gravity phenomenology in the multi-messenger approach

Workshop on the Standard Model and Beyond
August 27 – September 7, 2023

EISA
European Institute for Sciences and Their Applications

UKRI Science and Technology Facilities Council

UKRI Engineering and Physical Sciences Research Council

0. Outline

1. **Motivation: puzzles in modern cosmology, axions as dark matter (DM)**

2. **Gravitational origin of (axion) DM: axions from torsion in geometry ?**

3. **String-Inspired Gravitational Theory with Torsion & Grav. Anomalies:**

   (i) Axions in strings (``torsion"- & compactification- induced) and anomalies

   (ii) Primordial Gravitational Waves (GW) & induced Condensates of Grav. Anomalies,

   (iii) Running Vacuum Cosmology (RVM) with inflation without external inflatons

4. **Post-inflationary eras: Spontaneous Lorentz and CPT-Violation by axion backgrounds & Leptogenesis → geometric origin of Matter-antimatter asymmetry; (Meta) Stability of the (leptogenesis) Vacuum**

5. **Modern-era: cosmological tensions and stringy RVM**

6. **Conclusions & Outlook**

1. Motivation: puzzles in modern cosmology, axions as dark matter (DM)

2. Gravitational origin of (axion) DM: axions from torsion in geometry ?

3. String-Inspired Gravitational Theory with Torsion & Grav. Anom

   Quantum Gravity (QG) induced

   (i) Axions in strings (``torsion''- & compactification- induced) and anomalies

   cf. Solà 's talk

   ordial Gravitational Waves (GW) & induced Condensates of Grav. Anomalies,

   (iii) Running Vacuum Cosmology (RVM) with inflation without external inflatons

4. Post-inflationary eras: Spontaneous Lorentz and CPT-Violation by axion backgrounds & Leptogenesis → geometric origin of Matter-antimatter asymmetry; (Meta) Stability of the (leptogenesis) Vacuum

   cf. Sarkar's talk

5. Modern-era: cosmological tensions and stringy RVM

6. Conclusions & Outlook

1. Motivation: puzzles in modern cosmology, axions as dark matter (DM)

2. Gravitational origin of (axion) DM: axions from torsion in geometry ?

3. String-Inspired Gravitational Theory with Torsion & Grav. Anom

Quantum Gravity (QG) induced

   (i) Axions in strings (``torsion”- & compactification- induced) and anomalies

   (ii) Primordial Gravitational Waves (GW) & induced Condensates of Grav. Anomalies,

   (iii) Running Vacuum Cosmology (RVM) with inflation without external inflatons

4. Post-inflationary eras: Spontaneous Lorentz and CPT-Violation by axion backgrounds & Leptogenesis → geometric origin of Matter-antimatter asymmetry; (Meta) Stability of the (leptogenesis) Vacuum

5. Modern-era: cosmological tensions and string RVM

6. Conclusions & Outlook

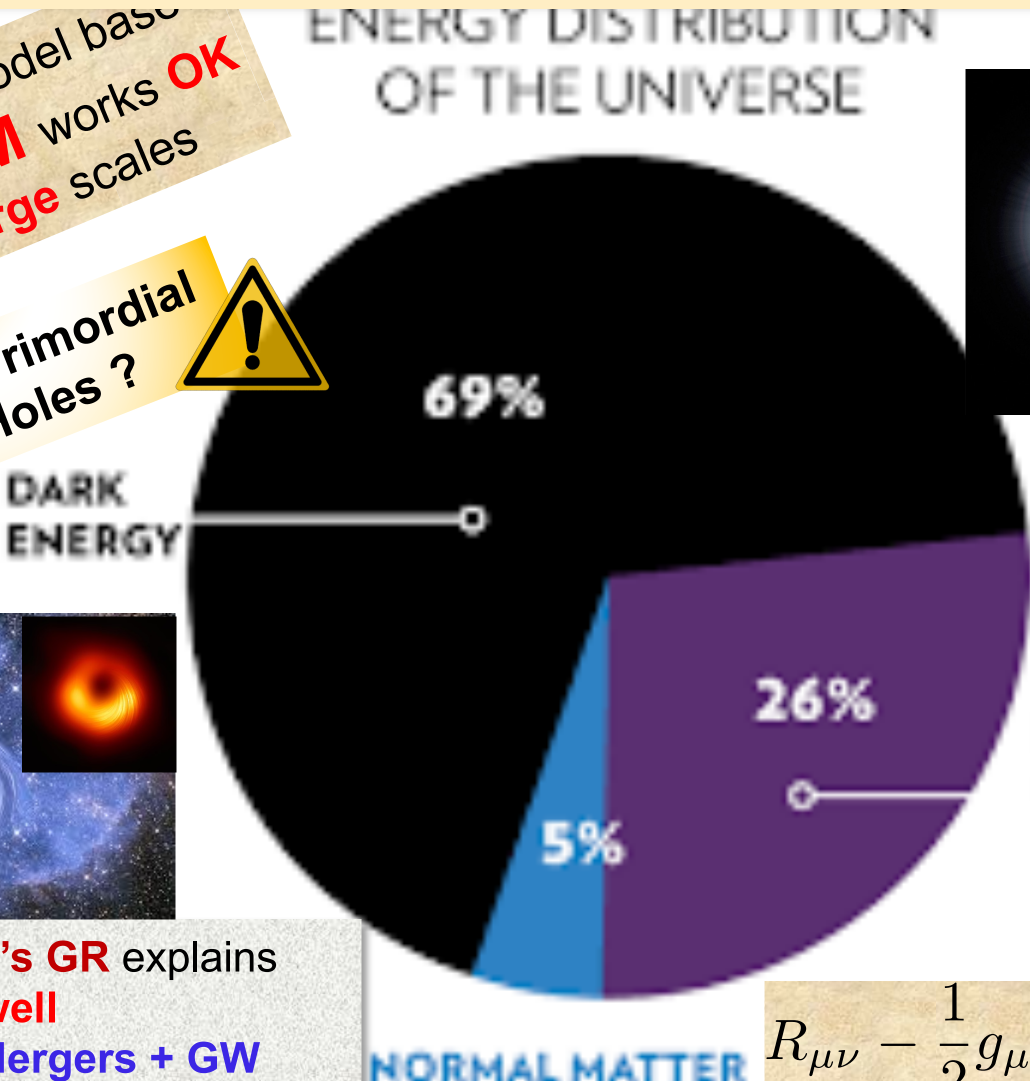
1. Motivation

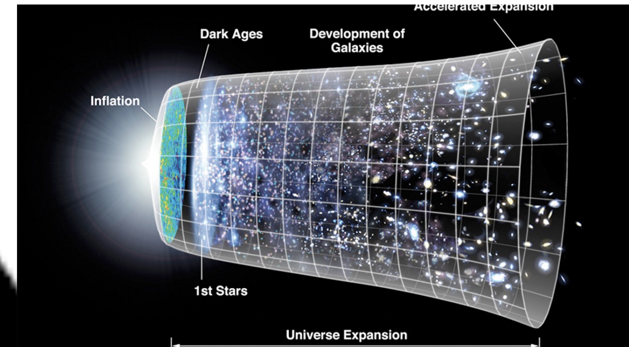
Important (> last 20 yrs) Discoveries in Cosmology/Astronomy

:k2018 data

Simplest model base
on **ΛCDM** works **OK**
for **large** scales

Are there Primordial Black Holes ?

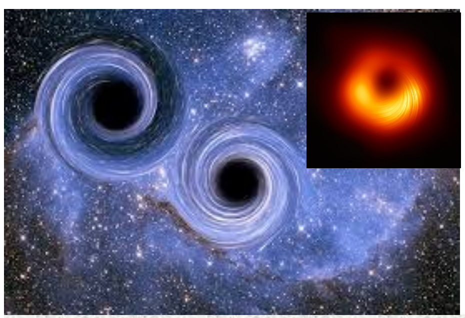




+ SnIa, BaO, Lensing

NANOGrav

29/6/23
15 year data Release.
Origin of GW background?



Also **Einstein's GR** explains **sufficiently well**
Black-Hole Mergers + GW (since 2015 LIGO),
Black-Hole `photographs' (EHT),...

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - g_{\mu\nu} \Lambda = 8\pi G \, T_{\mu\nu}$$

$$T_{\mu\nu} \ni \text{Cold Dark Matter}$$

Important (> last 20 yrs) Discoveries in Cosmology/Astronomy

3 data

Lensing

Sim
on

But....

Need to go Beyond....

Are there Primordial Black Holes (of DM type?)

What still we do not know/did not observe:

Nature of Dark Energy

Nature of Dark matter

Primordial Gravitational Waves

(through detection of B-mode polarisation in CMB from very early Universe)

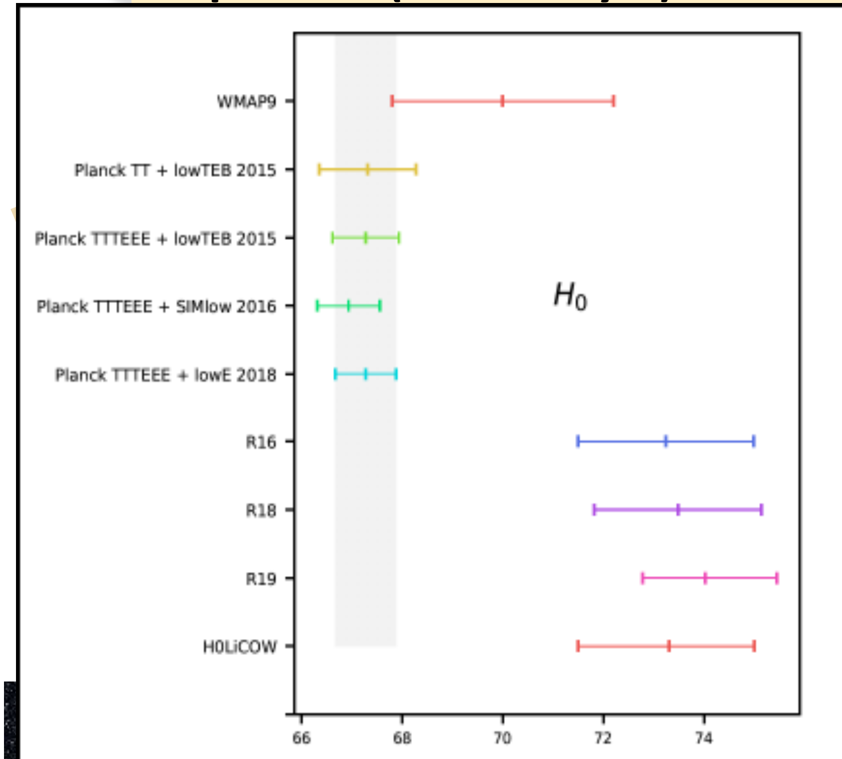
Microscopic models of Inflation

(Is it due to fundamental inflatons or dynamical e.g. Starobinsky type? ....)

Also
suffic
Black
(since
Black

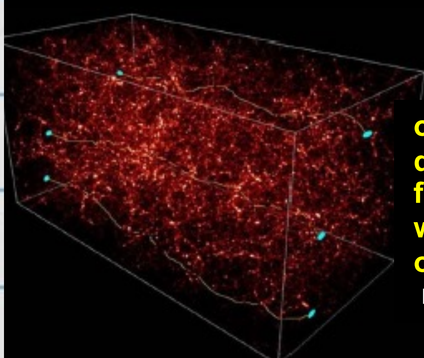
$8\pi G\, T_{\mu\nu}$

Important (> last 20 yrs) Discover…



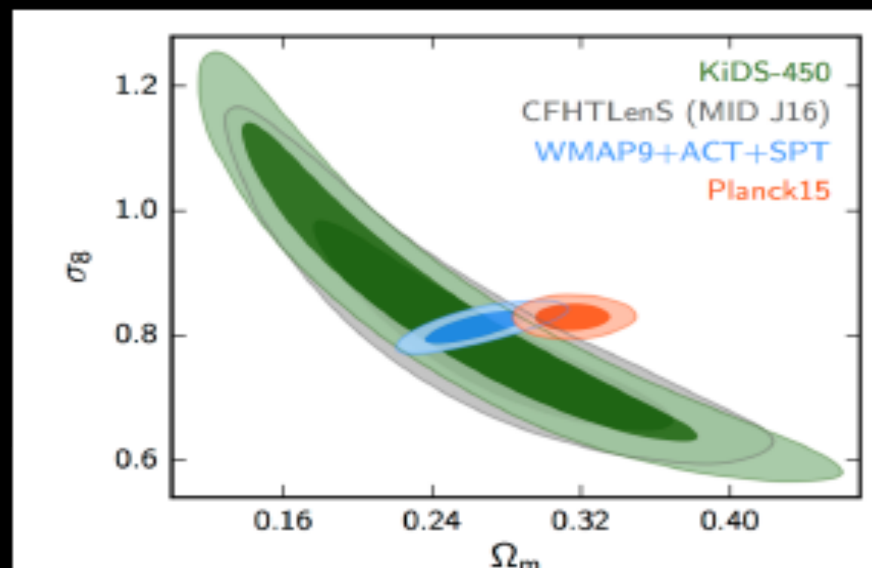
ΛCDM appears to be in tension with local measurements of present-era H_0 & also galaxy-growth data ?

Also I
suffic
Black
(since
Black

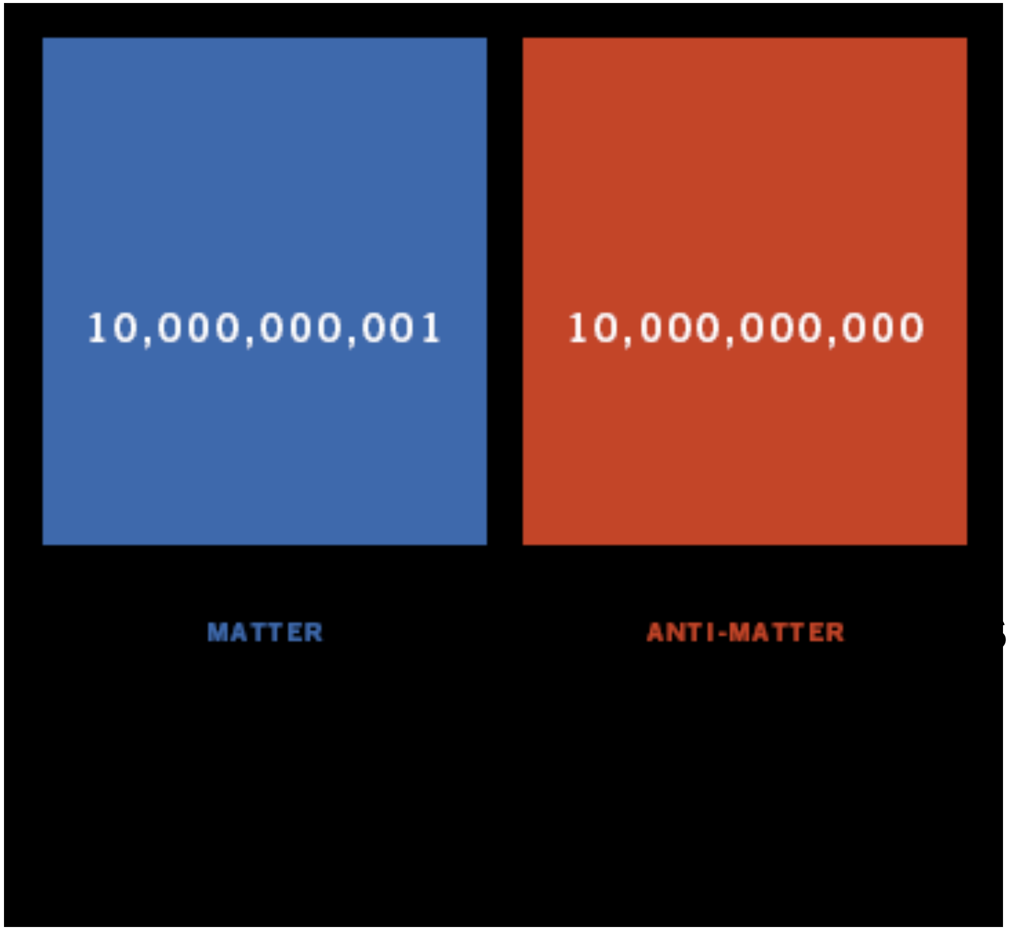


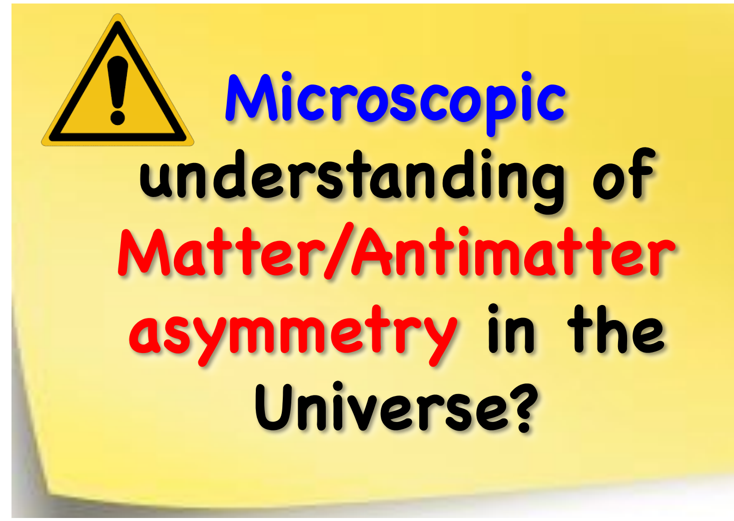
σ_8 = current matter density rms fluctuations within spheres of radius $8h^{-1}$ ($h = H_0/100$ = reduced Hubble constant)

$$S_8 \equiv \sigma_8 \sqrt{\Omega_m / 0.3}$$



Hildebrandt et al., arXiv:1606.05338.





The Baryon Asymmetry

$$\frac{n_B - \overline{n}_B}{n_B + \overline{n}_B} \sim \frac{n_B - \overline{n}_B}{s} = (8.4 - 8.9) \times 10^{-11}$$

T > 1 GeV

s = entropy density of Universe

Attempts at Explanation of Baryon Asymmetry – Sakharov 's Conditions

Baryon number violation

C-violation

and CP violation

CPT conservation assumed

Departure from thermodynamic equilibrium (non-stationary system)

CP |particle> = |anti-particle>

Need new physics beyond the SM → new sources of CP violation?

Attempts at Explanation of Baryon Asymmetry – Sakharov 's Conditions

Baryon number violation

C-violation

and CP

modynamic

tationary

What if CPTV geometries in the early Universe ? Geometric origin of Matter-Antimatter Asymmetry

CP |particle> = |anti-particle>

Need new physics beyond the SM → new sources of CP violation?

Need to go Beyond....

Axions & Axion-like Particles (ALPs)

Coupled to anomalies : **Shift symmetric interaction $a \to a + c$**
Since terms of S_a in (….) = total derivative

$$\mathcal{S}_a \ni \int d^4x \, \frac{1}{f_a} \, a(x) \left(\frac{1}{192\pi^2} \, R_{\mu\nu\rho\sigma} \, \widetilde{R}^{\mu\nu\rho\sigma} - \frac{e^2}{8\pi^2} \, F_{\mu\nu} \, \widetilde{F}^{\mu\nu} - \frac{\alpha_s}{8\pi} \, \mathcal{G}^a_{\mu\nu} \, \widetilde{\mathcal{G}}^{a\mu\nu} \right)$$

$$\mathcal{G}^a_{\mu\nu} = 2\partial_{[\mu}\mathcal{A}^a_{\nu]} + g_s \, f^{abc} \, \mathcal{A}^b_\mu \, \mathcal{A}^c_\nu \, , \quad \alpha_s = g_s^2/(4\pi) \, ,$$

$$a = 1, \ldots 8, \ \text{gluon or non} - \text{Abelian gauge group index} \, ,$$

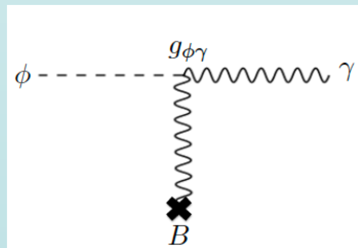
$$\widetilde{R}_{\rho\sigma\mu\nu} = \frac{1}{2} \sqrt{-g} \, \epsilon_{\mu\nu\alpha\beta} \, R_{\rho\sigma}{}^{\alpha\beta}$$

$$\widetilde{F}_{\mu\nu} = \frac{1}{2} \sqrt{-g} \, \epsilon_{\mu\nu\alpha\beta} \, F^{\alpha\beta}$$

f_a = axion coupling
[f_a] = mass dim +1

Axions & Axion-like Particles (ALPs)

Prmakoff-process:
Axion-photon conversion in the presence of magnetic field



a → γγ

$$\propto \frac{1}{f_a}\, a(x)\, \vec{E}\cdot\vec{B}$$

$$\mathcal{S}_a \ni \int d^4x\, \frac{1}{f_a}\, a(x) \left(\frac{1}{192\pi^2}\, R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} - \frac{e^2}{8\pi^2}\, F_{\mu\nu}\, \widetilde{F}^{\mu\nu} - \frac{\alpha_s}{8\pi}\, \mathcal{G}^a_{\mu\nu}\, \widetilde{\mathcal{G}}^{a\mu\nu} \right)$$

$$\mathcal{G}^a_{\mu\nu} = 2\partial_{[\mu}\mathcal{A}^a_{\nu]} + g_s\, f^{abc}\, \mathcal{A}^b_\mu\, \mathcal{A}^c_\nu\,, \quad \alpha_s = g_s^2/(4\pi)\,,$$

$$a = 1, \ldots 8, \;\; \text{gluon or non} - \text{Abelian gauge group index}\,,$$

$$\widetilde{R}_{\rho\sigma\mu\nu} = \frac{1}{2}\sqrt{-g}\, \epsilon_{\mu\nu\alpha\beta}\, R_{\rho\sigma}{}^{\alpha\beta}$$

$$\widetilde{F}_{\mu\nu} = \frac{1}{2}\sqrt{-g}\, \epsilon_{\mu\nu\alpha\beta}\, F^{\alpha\beta}$$

f_a = axion coupling
[f_a] = mass dim +1

Axions & Axion-like Particles (ALPs)

Coupled to anomalies : **Shift symmetric interaction** $a \rightarrow a + c$
Since terms of S_a in (….) = total derivative

$$\mathcal{S}_a \ni \int d^4x \, \frac{1}{f_a} \, a(x) \left(\frac{1}{192\pi^2} \, R_{\mu\nu\rho\sigma} \, \widetilde{R}^{\mu\nu\rho\sigma} - \frac{e^2}{8\pi^2} \, F_{\mu\nu} \, \widetilde{F}^{\mu\nu} - \frac{\alpha_{\rm s}}{8\pi} \, \mathcal{G}^a_{\mu\nu} \, \widetilde{\mathcal{G}}^{a\mu\nu} \right)$$

$$\mathcal{G}^a_{\mu\nu} = 2\partial_{[\mu}\mathcal{A}^a_{\nu]} + g_s \, f^{abc} \, \mathcal{A}^b_\mu \, \mathcal{A}^c_\nu \, , \quad \alpha_s = g_s^2/(4\pi) \, ,$$

$$a = 1, \ldots 8, \;\; \text{gluon or non} - \text{Abelian gauge group index} \, ,$$

$$\widetilde{R}_{\rho\sigma\mu\nu} = \frac{1}{2} \sqrt{-g} \, \epsilon_{\mu\nu\alpha\beta} \, R_{\rho\sigma}{}^{\alpha\beta}$$

$$\widetilde{F}_{\mu\nu} = \frac{1}{2} \sqrt{-g} \, \epsilon_{\mu\nu\alpha\beta} \, F^{\alpha\beta}$$

f_a = axion coupling
$[f_a]$ = mass dim +1

Axions & Axion-like Particles (ALPs)

Shift symmetry breaks to Periodicity $a/f_a \to a/f_a + 2\pi$

→ potential for axions a

$$V(a) = \Lambda^4_{\rm QCD}\left(1 - \cos\left(\frac{a}{f_a}\right)\right)$$

$$\mathcal{S}_a \ni \int d^4x\, \frac{1}{f_a}\, a(x) \left(\frac{1}{192\pi^2}\, R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} - \frac{e^2}{8\pi^2}\, F_{\mu\nu}\, \widetilde{F}^{\mu\nu} - \frac{\alpha_s}{8\pi}\, \mathcal{G}^a_{\mu\nu}\, \widetilde{\mathcal{G}}^{a\mu\nu} \right)$$

gluons

$$\mathcal{G}^a_{\mu\nu} = 2\partial_{[\mu}\mathcal{A}^a_{\nu]} + g_s\, f^{abc}\, \mathcal{A}^b_\mu\, \mathcal{A}^c_\nu\,, \quad \alpha_s = g_s^2/(4\pi)\,,$$

$a = 1, \ldots 8$, gluon or non − Abelian gauge group index,

$$\widetilde{R}_{\rho\sigma\mu\nu} = \frac{1}{2}\sqrt{-g}\,\epsilon_{\mu\nu\alpha\beta}\, R_{\rho\sigma}{}^{\alpha\beta}$$

$$\widetilde{F}_{\mu\nu} = \frac{1}{2}\sqrt{-g}\,\epsilon_{\mu\nu\alpha\beta}\, F^{\alpha\beta}$$

f_a = axion coupling

$[f_a]$ = mass dim +1

Axions & Axion-like Particles (ALPs)

→ **axion mass $m_a = \Lambda^2/f_a$**

$$V(a) = \Lambda^4_{\rm QCD}\left(1 - \cos\left(\frac{a}{f_a}\right)\right)$$

$$\mathcal{S}_a \ni \int d^4x \, \frac{1}{f_a}\, a(x) \left(\frac{1}{192\pi^2}\, R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} - \frac{e^2}{8\pi^2}\, F_{\mu\nu}\, \widetilde{F}^{\mu\nu} - \frac{\alpha_{\rm s}}{8\pi}\, \mathcal{G}^a_{\mu\nu}\, \widetilde{\mathcal{G}}^{a\mu\nu} \right)$$

gluons

$$\mathcal{G}^a_{\mu\nu} = 2\partial_{[\mu}\mathcal{A}^a_{\nu]} + g_s\, f^{abc}\, \mathcal{A}^b_\mu\, \mathcal{A}^c_\nu\,, \quad \alpha_s = g_s^2/(4\pi)\,,$$

$$a = 1, \ldots 8, \ \text{gluon or non} - \text{Abelian gauge group index}\,,$$

$$\widetilde{R}_{\rho\sigma\mu\nu} = \frac{1}{2}\sqrt{-g}\,\epsilon_{\mu\nu\alpha\beta}\, R_{\rho\sigma}{}^{\alpha\beta}$$

$$\widetilde{F}_{\mu\nu} = \frac{1}{2}\sqrt{-g}\,\epsilon_{\mu\nu\alpha\beta}\, F^{\alpha\beta}$$

f_a = axion coupling

$[f_a]$ = mass dim +1

Axions & Axion-like Particles (ALPs)

→ **axion mass $m_a = \Lambda^2/f_a$**

$$V(a) = \Lambda^4_{\rm QCD}\left(1 - \cos\left(\frac{a}{f_a}\right)\right)$$

$$\mathcal{S}_a \ni \int d^4x\, \frac{1}{f_a}\, a(x) \left(\frac{1}{192\pi^2}\, R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} - \frac{e^2}{8\pi^2}\, F_{\mu\nu}\, \widetilde{F}^{\mu\nu} - \frac{\alpha_{\rm s}}{8\pi}\, \mathcal{G}^a_{\mu\nu}\, \widetilde{\mathcal{G}}^{a\mu\nu} \right)$$

gluons

$$\mathcal{G}^a_{\mu\nu} = 2\partial_{[\mu}\mathcal{A}^a_{\nu]} + g_s\, f^{abc}\, \mathcal{A}^b_\mu\, \mathcal{A}^c_\nu\,, \quad \alpha_s = g_s^2/(4\pi)\,,$$

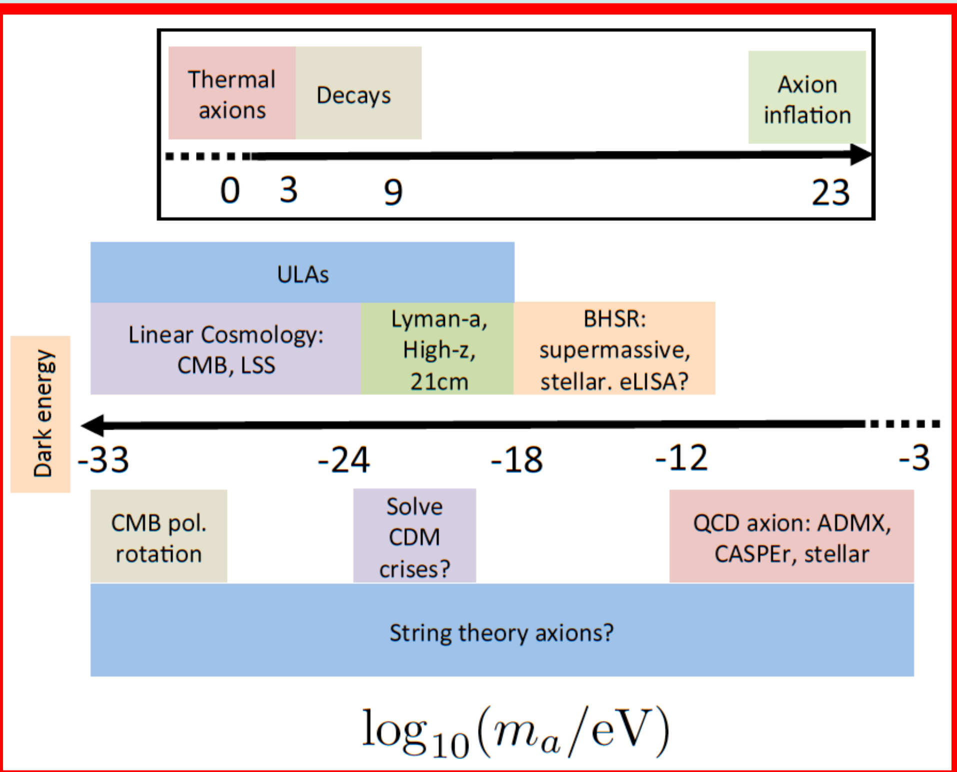
$$a = 1, \ldots 8, \;\; \text{gluon or non} - \text{Abelian gauge group index}\,,$$

$$\widetilde{R}_{\rho\sigma\mu\nu} = \frac{1}{2}\sqrt{-g}\,\epsilon_{\mu\nu\alpha\beta}\, R_{\rho\sigma}{}^{\alpha\beta}$$

$$\widetilde{F}_{\mu\nu} = \frac{1}{2}\sqrt{-g}\,\epsilon_{\mu\nu\alpha\beta}\, F^{\alpha\beta}$$

f_a = axion coupling

[f_a] = mass dim +1



Cosmological Constraints & probes of axion-like-partivcles (ALP)

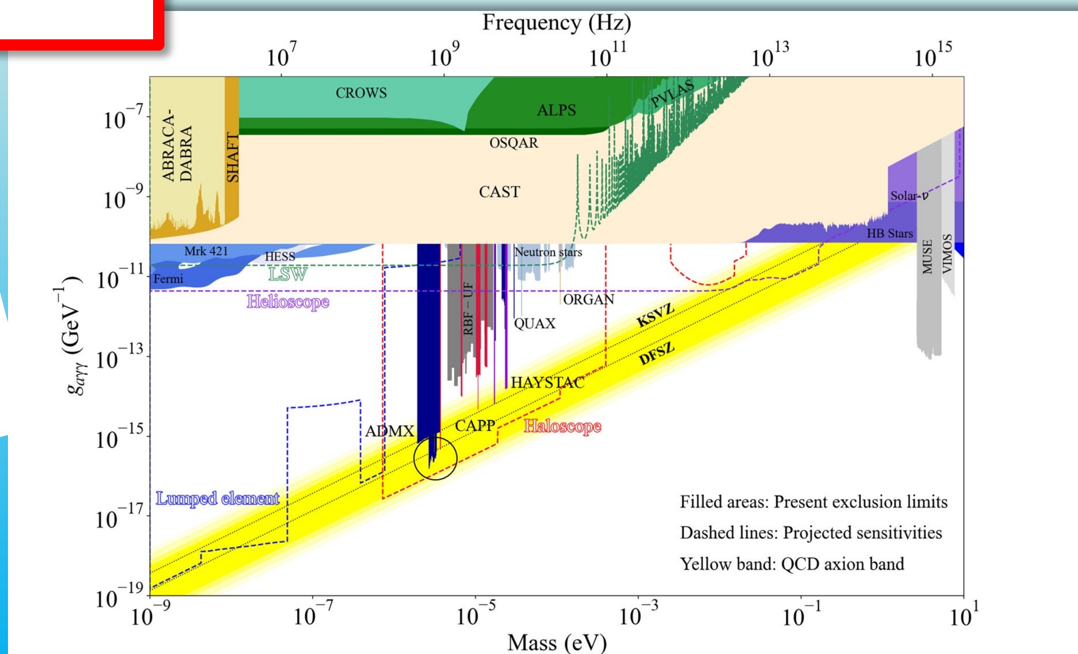
D.J.E. Marsh, Phys. Rept. 643, (2016)[arXiv:1510.07633 [astro-ph.CO]].

C. B. Adams *et al*., in Snowmass 2021 (2022), arXive: 2203.14923

http://www.ibs.re.kr/en/

Credit: Inst. for Basic Science

$g_{a\gamma\gamma} = f_a^{-1}$ axion coupling



This Talk

I will argue that:

observed matter-antimatter asymmetry

can be linked with

Microscopic string-inspired models of Cosmology with ANOMALIES, primordial gravitational waves and induced spontaneous (through gravitational anomaly condensates) Lorentz + CPT Violation

QG

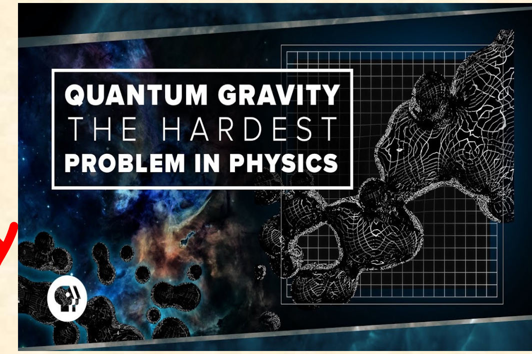
+

Range of ALPs mass in such a case?

geometric torsion interpretation of axion Dark matter

This Talk

I will argue that:



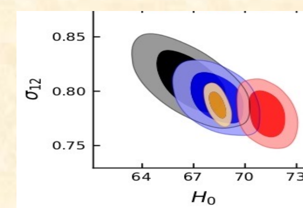
observed matter-antimatter asymmetry

can be linked with

Microscopic string-inspired models of Cosmology with ANOMALIES, primordial gravitational waves and induced spontaneous (through gravitational anomaly condensates) Lorentz + CPT Violation

QG

+

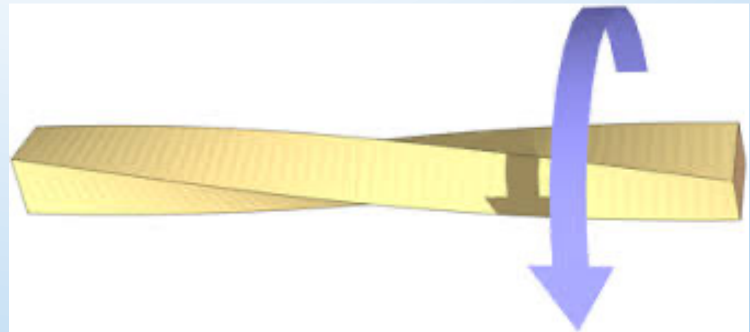


Effects of QG in alleviating cosmological tensions today!

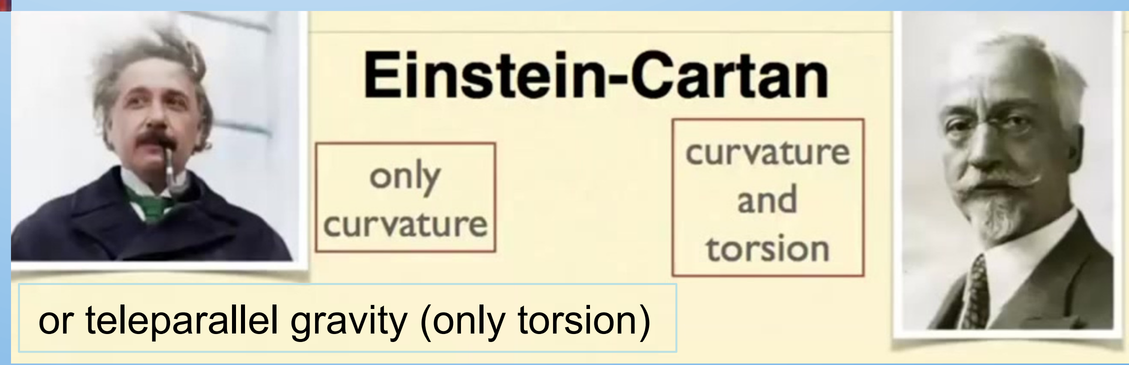
2. Geometrical origin of axion Dark matter

A Geometric Origin of (axion) Dark Matter?

A Geometric Origin of (axion) Dark Matter?



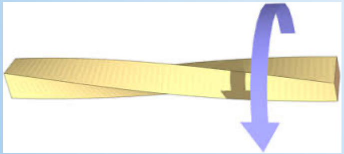
Torsion in spacetime?



Example of Einstein-Cartan theory : QED with Torsion

$$g_{\mu\nu} = e^a_{\ \mu}\, \eta_{ab}\, e^b_{\ \nu}$$

vielbein



$$\boldsymbol{T}^a = \mathbf{d}\boldsymbol{e}^a + \overline{\boldsymbol{\omega}}^a_b \wedge \boldsymbol{e}^b$$

Torsion 2-form

$$\overline{\boldsymbol{R}}^a_b = \mathbf{d}\overline{\boldsymbol{\omega}}^a_b + \overline{\boldsymbol{\omega}}^a_c \wedge \overline{\boldsymbol{\omega}}^c_b$$

Generalised curvature 2-form

Contorted
Spin connection

$$\overline{\boldsymbol{\omega}}^a_b = \boldsymbol{\omega}^a_b + \boldsymbol{K}^a_b$$

contorsion

$$\overline{\mathbf{D}}\boldsymbol{e}^a = \boldsymbol{T}^a,$$

Metricity postulate Breaks down if torsion present

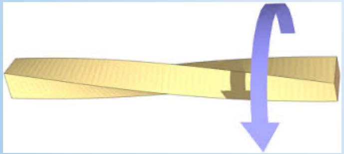
$$\overline{\nabla}_\rho g_{\mu\nu} \neq 0$$

$$\nabla_\rho g_{\mu\nu} = 0 \ \text{(torsion free)}$$

Example of Einstein-Cartan theory : QED with Torsion

$$g_{\mu\nu} = e^{a}_{\mu}\, \eta_{ab}\, e^{b}_{\nu}$$

vielbein



$$\boldsymbol{T}^a = \mathbf{d}\boldsymbol{e}^a + \bar{\boldsymbol{\omega}}^a_b \wedge \boldsymbol{e}^b$$

Torsion 2-form

$$\bar{\boldsymbol{R}}^a_b = \mathbf{d}\bar{\boldsymbol{\omega}}^a_b + \bar{\boldsymbol{\omega}}^a_c \wedge \bar{\boldsymbol{\omega}}^c_b$$

Generalised curvature 2-form

Contorted Spin connection

$$\bar{\boldsymbol{\omega}}^a_b = \boldsymbol{\omega}^a_b + \boldsymbol{K}^a_b$$

contorsion

$$\bar{\mathbf{D}}\boldsymbol{e}^a = \boldsymbol{T}^a,$$

$$\bar{\mathbf{D}}\boldsymbol{T}^a = \bar{\boldsymbol{R}}^a_b \wedge \boldsymbol{e}^b$$

$$\bar{\mathbf{D}}\bar{\boldsymbol{R}}^a_b = 0.$$

Metricity postulate Breaks down if torsion present

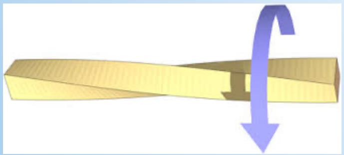
$$T^a_{\mu\nu} = e^a_\lambda\left(\Gamma^\lambda_{\nu\mu} - \Gamma^\lambda_{\mu\nu}\right) = -2 e^a_\lambda \Gamma^\lambda_{[\mu\nu]}$$

$$T^a_{bc} = -2K^a_{[bc]}, \quad K_{abc} = \tfrac{1}{2}\left(T_{cab} - T_{abc} - T_{bca}\right).$$

$$\bar{\boldsymbol{R}}^a_b = \boldsymbol{R}^a_b + \mathbf{D}\boldsymbol{K}^a_b + \boldsymbol{K}^a_c \wedge \boldsymbol{K}^c_b$$

Torsion-free

Example of Einstein-Cartan theory : QED with Torsion



fermions

$$\overline{\mathbf{D}}\psi = \mathbf{d}\psi - \frac{i}{4}\overline{\omega}_{ab}\sigma^{ab}\psi,$$

$$\overline{\boldsymbol{\omega}}^a_b = \boldsymbol{\omega}^a_b + \boldsymbol{K}^a_b$$

contorsion

$$S_\psi = \frac{i}{2}\int\left(\bar{\psi}\gamma^\mu\overline{\mathscr{D}}_\mu\psi - \left(\overline{\mathscr{D}}_\mu\bar{\psi}\right)\gamma^\mu\psi\right)\sqrt{-g}\ \mathrm{d}^4x$$

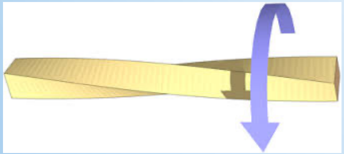
$$\overline{\mathscr{D}}_\mu = \overline{\mathrm{D}}_\mu - ieA_\mu$$

$$S_\psi = \frac{i}{2}\int\left(\bar{\psi}\gamma^\mu D_\mu\psi - \left(D_\mu\bar{\psi}\right)\gamma^\mu\psi\right)\sqrt{-g}\ \mathrm{d}^4x$$

$$+ e\int A_\mu\bar{\psi}\gamma^\mu\psi\sqrt{-g}\ \mathrm{d}^4x + \frac{1}{8}\int\bar{\psi}\{\gamma^c, \sigma^{ab}\}\psi K_{abc}\sqrt{-g}\ \mathrm{d}^4x$$

$$\{\gamma^c, \sigma^{ab}\} = 2\epsilon^{abc}{}_d\gamma^d\gamma^5$$

Example of Einstein-Cartan theory : QED with Torsion



fermions

$$\bar{\mathbf{D}}\psi = \mathbf{d}\psi - \frac{i}{4}\bar{\omega}_{ab}\sigma^{ab}\psi,$$

$$\bar{\boldsymbol{\omega}}^a_b = \boldsymbol{\omega}^a_b + K^a_b$$

contorsion

$$S_\psi = \frac{i}{2}\int\left(\bar{\psi}\gamma^\mu\bar{\mathscr{D}}_\mu\psi - \left(\bar{\mathscr{D}}_\mu\bar{\psi}\right)\gamma^\mu\psi\right)\sqrt{-g}\ \mathrm{d}^4x$$

$$\bar{\mathscr{D}}_\mu = \bar{\mathrm{D}}_\mu - ieA_\mu$$

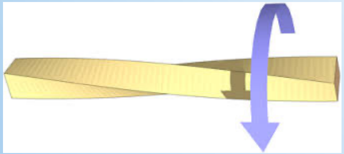
$$S_\psi = \frac{i}{2}\int\left(\bar{\psi}\gamma^\mu D_\mu\psi - \left(D_\mu\bar{\psi}\right)\gamma^\mu\psi\right)\sqrt{-g}\ \mathrm{d}^4x$$

$$T^a_{bc} = -2K^a_{[bc]}, \quad K_{abc} = \tfrac{1}{2}(T_{cab} - T_{abc} - T_{bca})$$

$$+ e\int A_\mu\bar{\psi}\gamma^\mu\psi\sqrt{-g}\ \mathrm{d}^4x + \frac{1}{8}\int\bar{\psi}\{\gamma^c, \sigma^{ab}\}\psi K_{abc}\sqrt{-g}\ \mathrm{d}^4x$$

$$\{\gamma^c, \sigma^{ab}\} = 2\epsilon^{abc}{}_d\gamma^d\gamma^5$$

Example of Einstein-Cartan theory : QED with Torsion



fermions

$$\overline{\mathbf{D}}\psi = \mathbf{d}\psi - \frac{i}{4}\overline{\omega}_{ab}\sigma^{ab}\psi,$$

$$\overline{\boldsymbol{\omega}}^a_b = \boldsymbol{\omega}^a_b + \boldsymbol{K}^a_b$$

contorsion

$$S_\psi = \frac{i}{2}\int\left(\bar{\psi}\gamma^\mu\bar{\mathscr{D}}_\mu\psi - \left(\bar{\mathscr{D}}_\mu\bar{\psi}\right)\gamma^\mu\psi\right)\sqrt{-g}\ \mathrm{d}^4x$$

$$\bar{\mathscr{D}}_\mu = \bar{\mathrm{D}}_\mu - ieA_\mu$$

$$S_\psi = \frac{i}{2}\int\left(\bar{\psi}\gamma^\mu D_\mu\psi - \left(D_\mu\bar{\psi}\right)\gamma^\mu\psi\right)\sqrt{-g}\ \mathrm{d}^4x$$

$$+ e\int A_\mu\bar{\psi}\gamma^\mu\psi\sqrt{-g}\ \mathrm{d}^4x + \frac{1}{8}\int\bar{\psi}\{\gamma^c, \sigma^{ab}\}\psi K_{abc}\sqrt{-g}\ \mathrm{d}^4x$$

$$\boldsymbol{T} = (1/3!)T_{abc}\boldsymbol{e}^a \wedge \boldsymbol{e}^b \wedge \boldsymbol{e}^c$$

$$\boldsymbol{S} = *\boldsymbol{T}$$

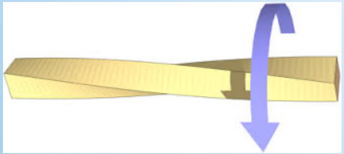
$$S_d = (1/3!)\epsilon^{abc}{}_d T_{abc}$$

4-d dual of Torsion

$$\{\gamma^c, \sigma^{ab}\} = 2\epsilon^{abc}{}_d\gamma^d\gamma^5$$

⚠

Example of Einstein-Cartan theory : QED with Torsion



fermions

$$\overline{\mathbf{D}}\psi = \mathbf{d}\psi - \frac{i}{4}\overline{\omega}_{ab}\sigma^{ab}\psi,$$

$$\overline{\boldsymbol{\omega}}^a_b = \boldsymbol{\omega}^a_b + \boldsymbol{K}^a_b$$

contorsion

$$S_\psi = \frac{i}{2}\int\left(\overline{\psi}\gamma^\mu\overline{\mathscr{D}}_\mu\psi - \left(\overline{\mathscr{D}}_\mu\overline{\psi}\right)\gamma^\mu\psi\right)\sqrt{-g}\ \mathrm{d}^4x$$

$$\overline{\mathscr{D}}_\mu = \overline{\mathrm{D}}_\mu - ieA_\mu$$

$$S_\psi \ni -\frac{3}{4}\int S_\mu\overline{\psi}\gamma^\mu\gamma^5\psi\sqrt{-g}\ \mathrm{d}^4x = -\frac{3}{4}\int \boldsymbol{S}\wedge *\,\boldsymbol{j}^5$$

$$S_d = (1/3!)\epsilon^{abc}{}_d T_{abc}$$

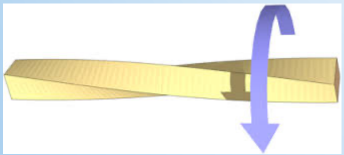
4-d dual of Torsion

$$J^\mu = \overline{\psi}\gamma^\mu\gamma^5\psi$$

Axial current

Universal, all fermion species

Example of Einstein-Cartan theory : QED with Torsion



fermions

$$\overline{\mathbf{D}}\psi = \mathbf{d}\psi - \frac{i}{4}\overline{\omega}_{ab}\sigma^{ab}\psi,$$

$$\overline{\boldsymbol{\omega}}^a_b = \boldsymbol{\omega}^a_b + \boldsymbol{K}^a_b$$

contorsion

$$S_\psi = \frac{i}{2}\int\left(\bar{\psi}\gamma^\mu\overline{\mathscr{D}}_\mu\psi - \left(\overline{\mathscr{D}}_\mu\bar{\psi}\right)\gamma^\mu\psi\right)\sqrt{-g}\ \mathrm{d}^4x$$

$$\overline{\mathscr{D}}_\mu = \overline{\mathrm{D}}_\mu - ieA_\mu$$

$$S_\psi \ni -\frac{3}{4}\int S_\mu\bar{\psi}\gamma^\mu\gamma^5\psi\sqrt{-g}\ \mathrm{d}^4x = -\frac{3}{4}\int \boldsymbol{S}\wedge *\,\boldsymbol{j}^5$$

$$S_d = (1/3!)\epsilon^{abc}{}_d T_{abc}$$

4-d dual of Torsion

Include Scalar-Curvature terms

$$S_G + S_\psi \ni \int\left[\frac{3}{4\kappa^2}\boldsymbol{S}\wedge *\,\boldsymbol{S} - \frac{3}{4}\boldsymbol{S}\wedge *\,\boldsymbol{j}^5\right]$$

Torsion & Axion-like d.o.f.

Classical torsion equation of motion $S = \frac{1}{2}\kappa^2 j^5$ → $\mathbf{d} * S = 0$

If J^5 conserved

Quantum chiral anomalies $\to \mathrm{d} * \mathbf{J}^5 \neq 0$

Add counterterms (order bny order in perturbation theory) to ensure $\mathbf{d} * S = 0$ & thus conservation of torsion charge $Q_S = \int * S$

Path integral over torsion d.o.f.

$$\int \mathscr{D}S \; \delta(\mathbf{d} * S) \; \exp\left(i \int \left[\frac{3}{4\kappa^2} S \wedge * S - \frac{3}{4} S \wedge * j^5 \right] \right)$$

Lagrange Multiplier Φ (pseudoscalar)

$$\int \mathscr{D}S \; \mathscr{D}\Phi \; \exp\left(i \int \left[\frac{3}{4\kappa^2} S \wedge * S - \frac{3}{4} S \wedge * j^5 + \Phi \mathbf{d} * S \right] \right)$$

Torsion & Axion-like d.o.f.

Classical torsion equation of motion $S = \frac{1}{2}\kappa^2 j^5$ → $\mathbf{d} * S = 0$

If J^5 conserved

Quantum chiral anomalies $\rightarrow \mathrm{d} * \mathbf{J}^5 \neq 0$

Add counterterms (order bny order in perturbation theory) to ensure $\mathbf{d} * S = 0$ **& thus conservation of torsion charge** $Q_S = \int * S$

Path integral over torsion d.o.f.

$$\int \mathscr{D} S \; \delta(\mathbf{d} * S) \; \exp\left(i \int \left[\frac{3}{4\kappa^2} S \wedge * S - \frac{3}{4} S \wedge * j^5 \right] \right)$$

Integrate out torsion S (non-propagating field)

Lagrange Multiplier Φ (pseudoscalar)

$$\Phi = (3/2\kappa^2)^{1/2} \phi$$

$$\int \mathscr{D}\phi \; \exp\left(i \int \left[-\frac{1}{2} \mathbf{d}\phi \wedge * \mathbf{d}\phi - \frac{1}{f_\phi} \mathbf{d}\phi \wedge * j^5 - \frac{1}{2 f_\phi^2} j^5 \wedge * j^5 \right] \right)$$

Axion coupling parameter $f_\phi = (3\kappa^2/8)^{-1/2}$

Torsion & Axion-like d.o.f.

$$\int \mathscr{D}\phi \, \exp\left(i \int \left[-\frac{1}{2}\mathbf{d}\phi \wedge *\mathbf{d}\phi - \frac{1}{f_\phi}\mathbf{d}\phi \wedge * \boldsymbol{j}^5 - \frac{1}{2f_\phi^2}\boldsymbol{j}^5 \wedge * \boldsymbol{j}^5 \right] \right)$$

Partially integrate

$$f_\phi = (3\kappa^2/8)^{-1/2}$$

Torsion & Axion-like d.o.f.

$$\int \mathscr{D}\phi \, \exp\left(i \int \left[-\frac{1}{2}\mathbf{d}\phi \wedge *\mathbf{d}\phi - \frac{1}{f_\phi}\mathbf{d}\phi \wedge *\boldsymbol{j}^5 - \frac{1}{2f_\phi^2}\boldsymbol{j}^5 \wedge *\boldsymbol{j}^5 \right] \right)$$

Partially integrate

Axion coupling parameter $f_\phi = (3\kappa^2/8)^{-1/2}$

$$\mathbf{d} * \boldsymbol{j}^5 = -\frac{e^2}{4\pi^2}\boldsymbol{F} \wedge \boldsymbol{F} - \frac{1}{96\pi^2}\mathrm{tr}(\overline{\boldsymbol{R}} \wedge \overline{\boldsymbol{R}}) \equiv \boldsymbol{G}(\boldsymbol{A}, \bar{\boldsymbol{\omega}})$$

$$\nabla \cdot j^5 = \frac{e^2}{8\pi^2}F^{\mu\nu *}F_{\mu\nu} - \frac{1}{192\pi^2}\overline{R}^{\alpha\beta\mu\nu *}\overline{R}_{\alpha\beta\mu\nu}$$

⚠

Can add counterterms so that only **torsion-free spin connection ω** appears in the **Anomaly**

$$\int \mathscr{D}\phi \, \exp\left(i \int \left[-\frac{1}{2}\mathbf{d}\phi \wedge *\mathbf{d}\phi + \frac{1}{f_\phi}\phi \boldsymbol{G}(\boldsymbol{A}, \boldsymbol{\omega}) - \frac{1}{2f_\phi^2}\boldsymbol{j}^5 \wedge *\boldsymbol{j}^5 \right] \right)$$

Repulsive four-fermion Characterisrtic of **Einstein-Cartan** theories

Torsion & Axion-like d.o.f.

$$\int \mathcal{D}\phi \, \exp\left(i\int\left[-\frac{1}{2}\mathbf{d}\phi\wedge *\mathbf{d}\phi - \frac{1}{f_\phi}\mathbf{d}\phi\wedge *\boldsymbol{j}^5 - \frac{1}{2f_\phi^2}\boldsymbol{j}^5\wedge *\boldsymbol{j}^5\right]\right)$$

Partially integrate

Axion coupling parameter $f_\phi = (3\kappa^2/8)^{-1/2}$

$$\mathbf{d} * \boldsymbol{j}^5 = -\frac{\alpha_s}{8\pi}\mathcal{G}^a_{\mu\nu}\widetilde{\mathcal{G}}^{a\mu\nu} - \frac{1}{96\pi^2}\mathrm{tr}(\bar{\boldsymbol{R}}\wedge\bar{\boldsymbol{R}}) \equiv \boldsymbol{G}(\boldsymbol{A}, \bar{\omega})$$

Non Abelian

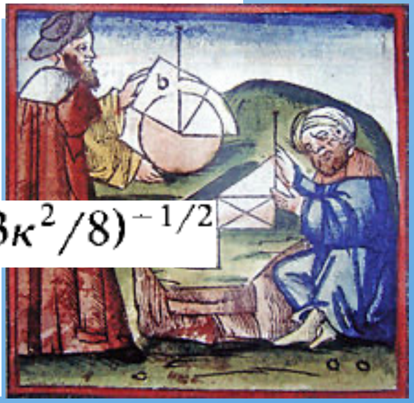
⚠

Can add counterterms so that only **torsion-free spin connection ω** appears in the **Anomaly**

$$\int \mathcal{D}\phi \, \exp\left(i\int\left[-\frac{1}{2}\mathbf{d}\phi\wedge *\mathbf{d}\phi + \frac{1}{f_\phi}\phi\boldsymbol{G}(\boldsymbol{A}, \omega) - \frac{1}{2f_\phi^2}\boldsymbol{j}^5\wedge *\boldsymbol{j}^5\right]\right)$$

Non-Abelian Gauge group Instantons can lead to potential

$$V(\phi) = \Lambda^4_{\mathrm{inst}}\left(1 - \cos(\phi/f_\phi)\right)$$

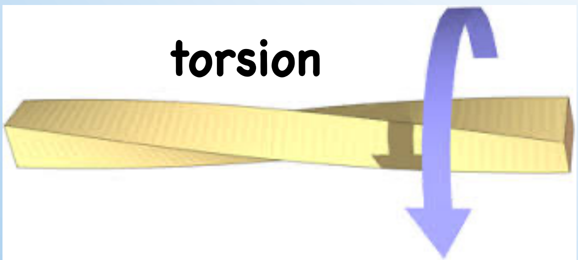


$f_\phi = (3\kappa^2/8)^{-1/2}$

→ **massive** ($m_\varphi = \Lambda_{inst}^2/f_\varphi$) **torsion-induced axion**

GEOMETRIC ORIGIN OF AXION DM?

To Recapitulate



duality

Bianchi identity

Coupling to chiral anomalies

Non-perturbative Axion mass

Geometric origin

3. String-Inspired Gravitational Theory with Torsion & Grav. Anomalies

3(i). Two kinds of Axions in String theories:

(a) String-model independent (``Torsion”- induced)

&

(b) Compactification- induced Axions

&

Anomalies

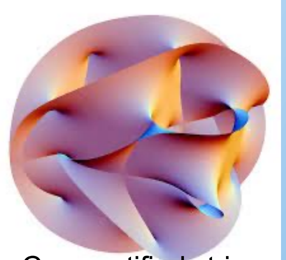
String-inspired gravitational theories with torsion and anomalies

String-Model Independent Axion

NEM,
+ Basilakos, Solà, Sarkar,

Massless gravitational (bosonic) string multiplet:

$$g_{\mu\nu} = g_{\nu\mu}\,, \quad \text{spin} = 2 \quad \text{(graviton)}$$
$$\Phi\,, \quad \text{spin} = 0 \quad \text{(dilaton)}\,,$$
$$B_{\mu\nu} = -B_{\nu\mu}\,, \quad \text{spin} = 1 \quad \text{(Kalb - Ramond (KR) field)}$$

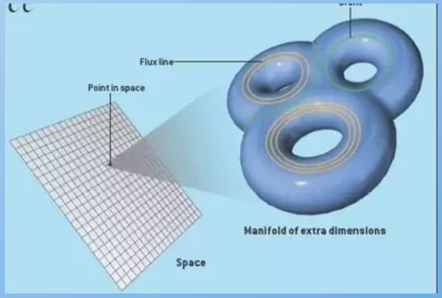

Compactified strings

Gauge symmetry in closed string sector $B_{\mu\nu} \to B_{\mu\nu} + \partial_\mu \theta_\nu - \partial_\nu \theta_\mu$

Symmetry of string σ-model vertex operators

$$\int_{\Sigma^{(2)}} d^2\sigma\, B_{\mu\nu}\epsilon^{AB}\partial_A X^\mu \partial_B X^\nu\,, \quad A, B = 1, 2$$

world sheet

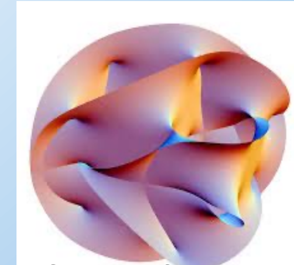


Gross and Sloan, Metsaev and Tseytlin

String-inspired gravitational theories with torsion and anomalies

Massless gravitational (bosonic) string multiplet:

$$
\begin{aligned}
&g_{\mu\nu} = g_{\nu\mu}\,, \quad \text{spin} = 2 \quad (\text{graviton}) \\
&\Phi\,, \quad \text{spin} = 0 \quad (\text{dilaton})\,, \\
&B_{\mu\nu} = -B_{\nu\mu}\,, \quad \text{spin} = 1 \quad (\text{Kalb} - \text{Ramond (KR) field})
\end{aligned}
$$

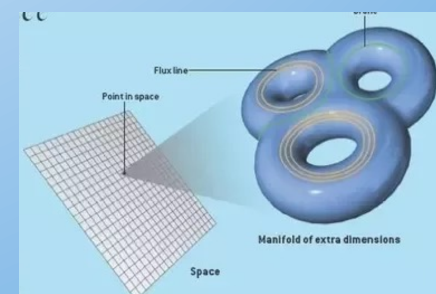


Compactified strings

Gauge symmetry in closed string sector $B_{\mu\nu} \rightarrow B_{\mu\nu} + \partial_\mu \theta_\nu - \partial_\nu \theta_\mu$

Effective target-spacetime gravitational action depends on the field strength :

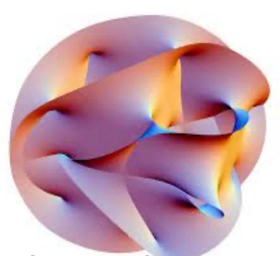
$$H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]}$$



String-inspired gravitational theories with torsion and anomalies

Massless gravitational (bosonic) string multiplet:

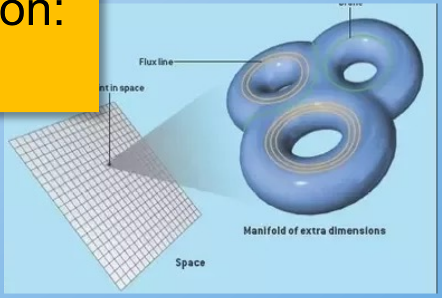
$$g_{\mu\nu} = g_{\nu\mu}\,, \quad \text{spin} = 2 \quad (\text{graviton})$$
$$\Phi\,, \quad \text{spin} = 0 \quad (\text{dilaton})\,,$$
$$B_{\mu\nu} = -B_{\nu\mu}\,, \quad \text{spin} = 1 \quad (\text{Kalb} - \text{Ramond (KR) field})$$


Compactified strings

Gauge symmetry in closed string sector $B_{\mu\nu} \rightarrow B_{\mu\nu} + \partial_\mu \theta_\nu - \partial_\nu \theta_\mu$

Effective target-spacetime gravitational action depends on the field strength :
$H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]}$

String theory: Green-Schwarz mechanism for anomaly cancellation:
$H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]} + (\alpha'/\kappa)\,(\Omega_{3L\mu\nu\rho} - \Omega_{3Y\mu\nu\rho})$



Chern-Simons terms

Gravitational **gauge**

$$\Omega_{3L} = \omega^a{}_c \wedge \mathbf{d}\omega^c{}_a + \frac{2}{3}\omega^a{}_c \wedge \omega^c{}_d \wedge \omega^d{}_a$$

$$\Omega_{3Y} = \mathbf{A} \wedge \mathbf{dA} + \mathbf{A} \wedge \mathbf{A} \wedge \mathbf{A}$$

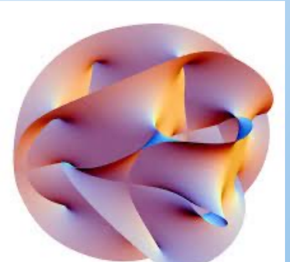
α' = Regge slope = M_s^{-2}
κ^2 = 8π G = 4d grav. constant

Massless gravitational (bosonic) string multiplet:

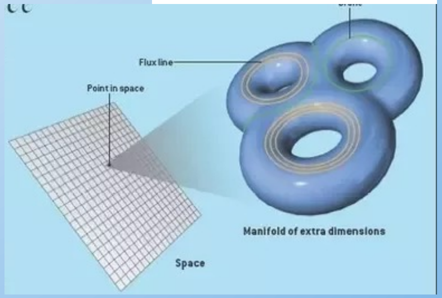
String theory: Green-Schwarz mechanism for anomaly cancellation:
$H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]} + (\alpha'/\kappa)\,(\Omega_{3L\mu\nu\rho} - \Omega_{3Y\mu\nu\rho})$

String effective action (lowest order in Regge slope)

$$S_B = -\int d^4x\sqrt{-g}\Big(\frac{1}{2\kappa^2}R + \frac{1}{6}\mathcal{H}_{\lambda\mu\nu}\mathcal{H}^{\lambda\mu\nu} + \dots\Big).$$



Compactified strings

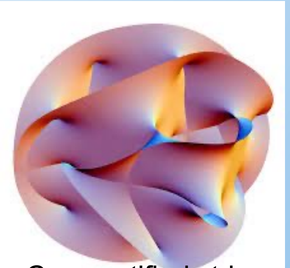


String-inspired gravitational theories with torsion and anomalies

Massless gravitational (bosonic) string multiplet:

String theory: Green-Schwarz mechanism for anomaly cancellation:

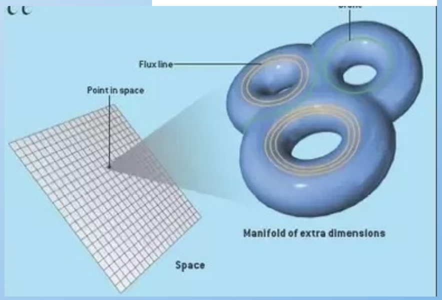
$$H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]} + (\alpha'/\kappa)\left(\Omega_{3L\mu\nu\rho} - \Omega_{3Y\mu\nu\rho}\right)$$



Compactified strings

String effective action (lowest order in Regge slope)

$$S_B = -\int d^4x\sqrt{-g}\left(\frac{1}{2\kappa^2}R + \frac{1}{6}\mathcal{H}_{\lambda\mu\nu}\mathcal{H}^{\lambda\mu\nu} + \dots\right).$$



Totally antisymmetric torsion

$$\overline{\Gamma}^{\rho}_{\mu\nu} = \Gamma^{\rho}_{\mu\nu} + \frac{\kappa}{\sqrt{3}}\mathcal{H}^{\rho}_{\mu\nu} \neq \overline{\Gamma}^{\rho}_{\nu\mu}$$

$$\overline{R}(\overline{\Gamma})$$

String-inspired gravitational theories with torsion and anomalies

Massless gravitational (bosonic) string multiplet:

String theory: Green-Schwarz mechanism for anomaly cancellation:
$H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]} + (\alpha'/\kappa)\,(\Omega_{3L\mu\nu\rho} - \Omega_{3Y\mu\nu\rho})$

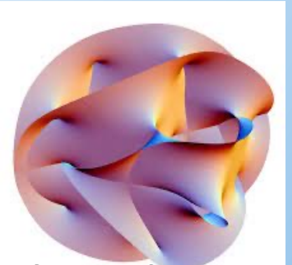
String effective action (lowest order in Regge slope)

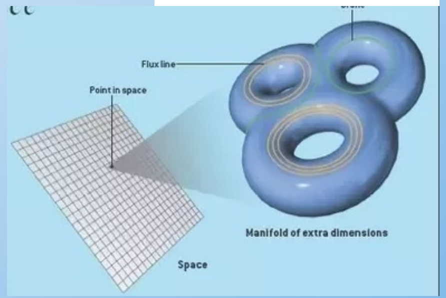
$$S_B = -\int d^4x \sqrt{-g}\left(\frac{1}{2\kappa^2} R + \frac{1}{6}\mathcal{H}_{\lambda\mu\nu}\mathcal{H}^{\lambda\mu\nu} + \dots\right).$$

Totally antisymmetric torsion

$$\overline{\Gamma}^{\rho}_{\mu\nu} = \Gamma^{\rho}_{\mu\nu} + \frac{\kappa}{\sqrt{3}}\mathcal{H}^{\rho}_{\mu\nu} \neq \overline{\Gamma}^{\rho}_{\nu\mu}$$

$$\overline{R}(\overline{\Gamma})$$


Compactified strings



Torsion → axion-like d.o.f. (as in CONTORTED QED)

String-model independent axion

Svrcek-Witten

String-inspired gravitational theories with torsion and anomalies

Massless gravitational (bosonic) string multiplet:

String theory: Green-Schwarz mechanism for anomaly cancellation:
$H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]} + (\alpha'/\kappa)\,(\Omega_{3L\mu\nu\rho} - \Omega_{3Y\mu\nu\rho})$

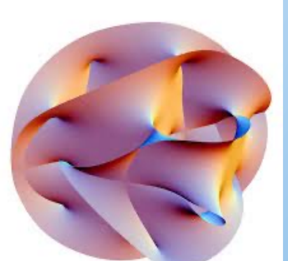
String effective action (lowest order in Regge slope)

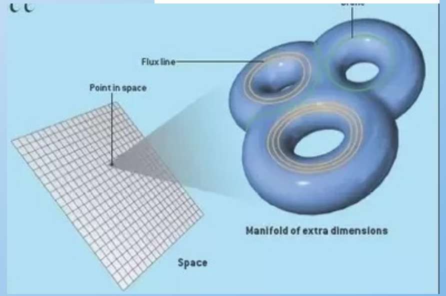
$$S_B = -\int d^4x\sqrt{-g}\Big(\frac{1}{2\kappa^2}R + \frac{1}{6}\mathcal{H}_{\lambda\mu\nu}\mathcal{H}^{\lambda\mu\nu} + \dots\Big).$$

Totally antisymmetric torsion

$$\overline{\Gamma}^{\rho}_{\mu\nu} = \Gamma^{\rho}_{\mu\nu} + \frac{\kappa}{\sqrt{3}}\mathcal{H}^{\rho}_{\mu\nu} \neq \overline{\Gamma}^{\rho}_{\nu\mu}$$

$$\overline{R}(\overline{\Gamma})$$


Compactified strings



NB: Torsion interpretation valid only up to & including $O(\alpha')$ effective action but dynamics of model-independent axion valid

Torsion → axion-like d.o.f. (as in CONTORTED QED)

String-model independent axion

Svrcek-Witten

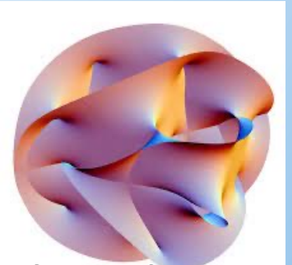
Bianchi identity constraint

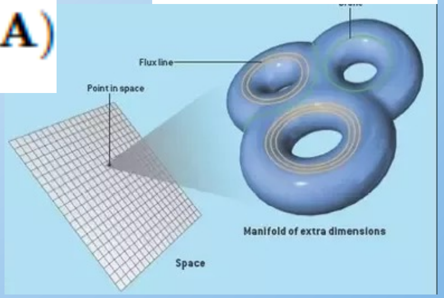
String-inspired gravitational theories with torsion and anomalies

NEM,
+ Basilakos, Solà,
Sarkar,

Massless gravitational (bosonic) string multiplet:

String theory: Green-Schwarz mechanism for anomaly cancellation:
$H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]} + (\alpha'/\kappa)\,(\Omega_{3L\mu\nu\rho} - \Omega_{3Y\mu\nu\rho})$


Compactified strings



Bianchi identity constraint

$$\varepsilon_{abc}{}^{\mu}\,\mathcal{H}^{abc}{}_{;\mu} = \frac{\alpha'}{32\,\kappa}\sqrt{-g}\left(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\right) \equiv \sqrt{-g}\,\mathcal{G}(\omega, \mathbf{A})$$

Implementation via axion-like Lagrange multiplier field b(x)

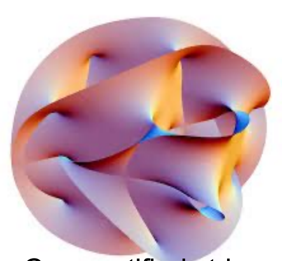
$$\Pi_x\,\delta\Big(\varepsilon^{\mu\nu\rho\sigma}\,\mathcal{H}_{\nu\rho\sigma}(x)_{;\mu} - \mathcal{G}(\omega, \mathbf{A})\Big) \Rightarrow$$
$$\int Db\,\exp\Big[i\int d^4x\sqrt{-g}\,\frac{1}{\sqrt{3}}\,b(x)\Big(\varepsilon^{\mu\nu\rho\sigma}\,\mathcal{H}_{\nu\rho\sigma}(x)_{;\mu} - \mathcal{G}(\omega, \mathbf{A})\Big)\Big]$$
$$= \int Db\,\exp\Big[-i\int d^4x\sqrt{-g}\,\Big(\partial^{\mu}b(x)\,\frac{1}{\sqrt{3}}\,\epsilon_{\mu\nu\rho\sigma}\,\mathcal{H}^{\nu\rho\sigma} + \frac{b(x)}{\sqrt{3}}\,\mathcal{G}(\omega, \mathbf{A})\Big)\Big]$$

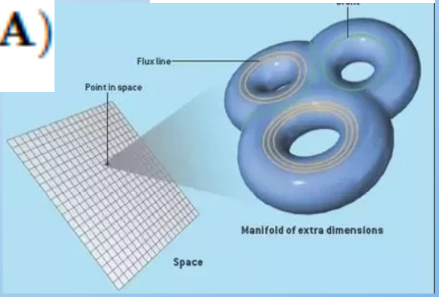
String-inspired gravitational theories with torsion and anomalies

NEM,
+ Basilakos, Solà,
Sarkar,

Massless gravitational (bosonic) string multiplet:

String theory: Green-Schwarz mechanism for anomaly cancellation:
$H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]} + (\alpha'/\kappa)\,(\Omega_{3L\mu\nu\rho} - \Omega_{3Y\mu\nu\rho})$


Compactified strings



Bianchi identity constraint

$$\varepsilon_{abc}{}^{\mu}\,\mathcal{H}^{abc}{}_{;\mu} = \frac{\alpha'}{32\,\kappa}\,\sqrt{-g}\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) \equiv \sqrt{-g}\,\mathcal{G}(\omega,\mathbf{A})$$

Implementation via axion-like Lagrange multiplier field b(x)
Integration of non-propagating H field

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

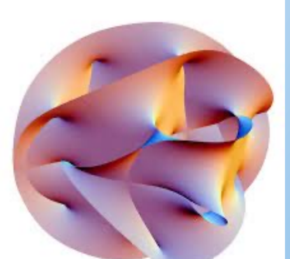
String-inspired gravitational theories with torsion and anomalies

NEM,
+ Basilakos, Solà,
Sarkar,

Massless gravitational (bosonic) string multiplet:

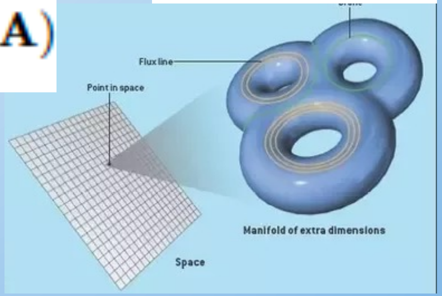
String theory: Green-Schwarz mechanism for anomaly cancellation:

$$H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]} + (\alpha'/\kappa)\,(\Omega_{3L\mu\nu\rho} - \Omega_{3Y\mu\nu\rho})$$



Bianchi identity constraint

$$\varepsilon_{abc}{}^{\mu}\, \mathcal{H}^{abc}{}_{;\mu} = \frac{\alpha'}{32\,\kappa}\sqrt{-g}\left(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\right) \equiv \sqrt{-g}\,\mathcal{G}(\omega, \mathbf{A})$$



Massive axions through
Non-Abelian gauge group
Instantons

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\frac{\alpha'}{96\,\kappa}\,b(x)\left(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\right) + \dots\Big]$$

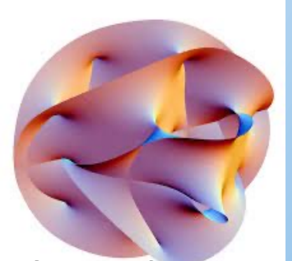
String-inspired gravitational theories with torsion and anomalies

NEM,
+ Basilakos, Solà,
Sarkar,

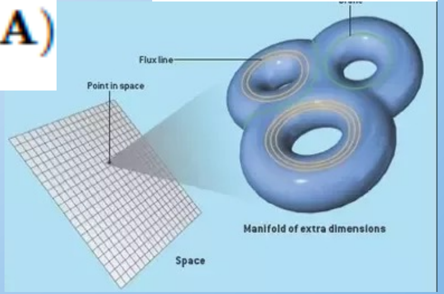
Massless gravitational (bosonic) string multiplet:

String theory: Green-Schwarz mechanism for anomaly cancellation:

$$H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]} + (\alpha'/\kappa)\,(\Omega_{3L\mu\nu\rho} - \Omega_{3Y\mu\nu\rho})$$



Compactified strings



Bianchi identity constraint

$$\varepsilon_{abc}{}^{\mu}\,\mathcal{H}^{abc}{}_{;\mu} = \frac{\alpha'}{32\,\kappa}\sqrt{-g}\left(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\right) \equiv \sqrt{-g}\,\mathcal{G}(\omega, \mathbf{A})$$

Massive axions through
Non-Abelian gauge group
Instantons

GEOMETRIC ORIGIN OF AXION DM

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\left(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\right) + \dots\Big]$$

Geometric origin of stringy axion DM

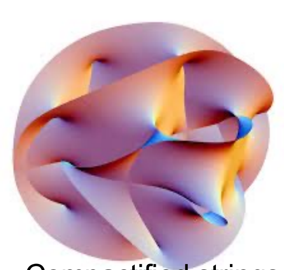
Model-dependent AXIONS IN STRINGS FROM COMPACTIFICATION

Svrcek-Witten

Co-exist with String-model independent axion

$$\int_{C_j} \beta_i = \delta_{ij}$$

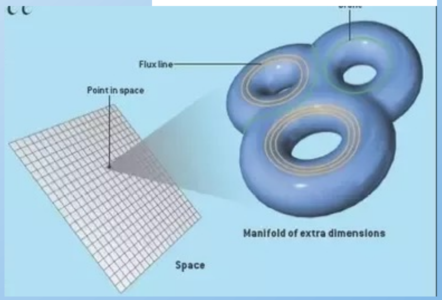
C_i = 2-cycle


Compactified strings

e.g. **zero modes β_i of KR B-field over compact manifold**

$$B = \frac{1}{2\pi}\sum_i \beta_i b_i$$

axions



1-loop Green-Schwarz anomaly-cancellation in. e.g, Heterotic strings

$$\frac{-1}{4(2\pi)^3 4!}\int B\left\{-\frac{\mathrm{Tr}F\wedge F\mathrm{tr}R\wedge R}{30}+\frac{\mathrm{Tr}F^4}{3}-\frac{(\mathrm{Tr}F\wedge F)^2}{900}\right\} \rightarrow$$

$$-\sum_i\int_Z \beta_i\wedge\frac{1}{16\pi^2}\left(\mathrm{tr}_1F\wedge F-\frac{1}{2}\mathrm{tr}R\wedge R\right)\int_M b_i\frac{\mathrm{tr}_1F\wedge F}{16\pi^2}$$

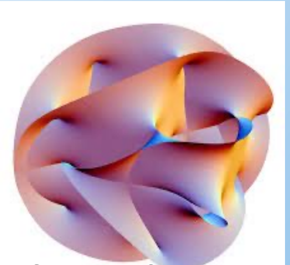
Compact manifold

String-inspired gravitational theories with torsion and anomalies

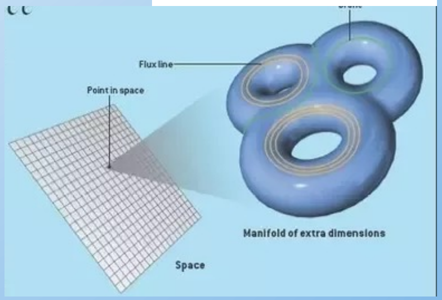
Model-dependent AXIONS IN STRINGS FROM COMPACTIFICATION

Svrcek-Witten

Co-exist with String-model independent axion



Compactified strings



$$\int_{C_j}\beta_i=\delta_{ij}$$

C_i = 2-cycle

e.g. **zero modes β_i of KR B-field over compact manifold**

$$B=\frac{1}{2\pi}\sum_i \beta_i b_i$$

axions

1-loop Green-Schwarz anomaly-cancellation in. e.g, Heterotic strings

$$\frac{-1}{4(2\pi)^3 4!}\int B\left\{-\frac{\mathrm{Tr}F\wedge F\,\mathrm{tr}R\wedge R}{30}+\frac{\mathrm{Tr}F^4}{3}-\frac{(\mathrm{Tr}F\wedge F)^2}{900}\right\}\rightarrow$$

$$-\sum_i\int_Z\beta_i\wedge\frac{1}{16\pi^2}\left(\mathrm{tr}_1F\wedge F-\frac{1}{2}\mathrm{tr}R\wedge R\right)\int_M b_i\frac{\mathrm{tr}_1F\wedge F}{16\pi^2}$$

Compact manifold

Axion coupling $F_b\sim V_Z^{1/3}/2\pi g_s^2\ell_s^4$

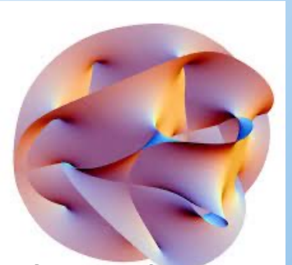
Typical values $F_b = O(10^{17})$ GeV

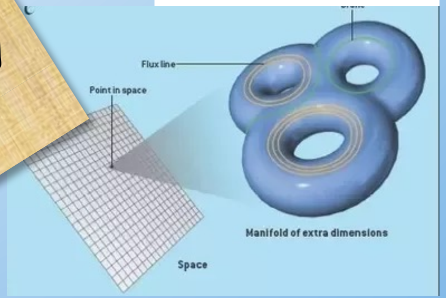
String-inspired gravitational the[...]s with torsion and anomalies

Model-dependent AXIONS IN STRINGS FROM COMPACTIFICATION

Svrcek-Witten

Co-exist with String-model independent axion


Compactified strings



$$\int_{C_j} \beta_i = \delta_{ij}$$

B

e.g. zero modes β_i of KR B-field over compact manifold

1-loop Green-Schwarz anoma[...] in. e.g, Heterotic strings

$$\frac{-1}{4(2\pi)^3 4!}\int B\left\{-\frac{\mathrm{Tr}F\wedge \ldots}{\ldots}\right.$$

$$-\sum_i \int_Z \beta_i \wedge \frac{1}{16\pi^2}\left(\mathrm{tr}_1 F \wedge \ldots \frac{\ldots F\wedge F}{16\pi^2}\right.$$

Compact manifold

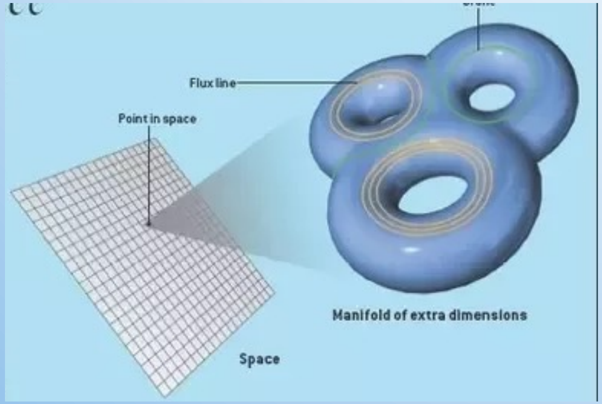
Many other types of Compactification Axions depending on the particular string theory considered

Axion coupling $F_b \sim V_Z^{1/3}/2\pi g_s^2 \ell_s^4$

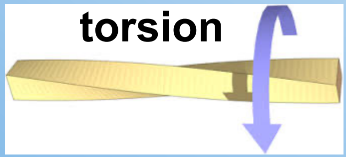
Typical values $F_b = O(10^{17})$ GeV

To Recapitulate

String-inspired gravitational theories with torsion



Compactified strings

()

Axions in Spectrum

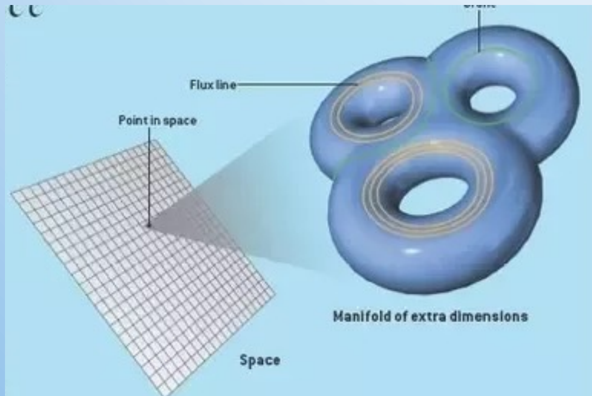
(i) From compactification

(ii) From KR field strength (4-d dual KR axion)

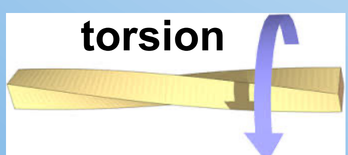
Geometric origin of (part of) stringy axion DM

To Recapitulate

String-inspired gravitational theories with torsion



Compactified strings

()

Axions in Spectrum

(i) From compactification

(ii) From KR field strength
(4-d dual KR axion)

This talk

Geometric origin of (part of) stringy axion DM

Inclusion of Fermions

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

$$+\, S_{Dirac}^{Free} + \int d^4x\sqrt{-g}\,\Big(\mathcal{F}_\mu + \frac{\kappa}{2}\sqrt{\frac{3}{2}}\,\partial_\mu b\Big)\,J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x\sqrt{-g}\,J^5_\mu J^{5\mu} + \dots\Big] + \dots$$

or Majorana

$$J^{5\mu} = \bar{\psi}_j\,\gamma^\mu\,\gamma^5\psi_j$$

Axial Current

All fermion species

torsion

cf. classically in 4 dim:
(duality relationship)

$$-3\,\sqrt{2}\partial_\sigma b \;=\; \sqrt{-g}\,\epsilon_{\mu\nu\rho\sigma}H^{\mu\nu\rho}$$

Inclusion of Fermions

$$S_B^{\rm eff} = \int d^4x \sqrt{-g}\Big[-\frac{1}{2\kappa^2} R + \frac{1}{2}\partial_\mu b\, \partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}} b(x)\left(R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\widetilde{F}^{\mu\nu}\right) + \dots \Big]$$

$$+ S_{Dirac}^{Free} + \int d^4x\sqrt{-g}\left(\mathcal{F}_\mu + \frac{\kappa}{2}\sqrt{\frac{3}{2}}\partial_\mu b\right) J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x \sqrt{-g}\, J^5_\mu J^{5\mu} + \dots \Big] + \dots$$

or Majorana

KR-axion anomalous CP-Violating interaction

$$J^{5\mu} = \bar{\psi}_j \gamma^\mu \gamma^5 \psi_j$$

Axial Current
All fermion species

cf. classically in 4 dim:
(duality relationship)

torsion

$$-3\sqrt{2}\partial_\sigma b = \sqrt{-g}\,\epsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho}$$

Inclusion of Fermions

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

$$+\,S_{Dirac}^{Free} + \int d^4x\sqrt{-g}\,\Big(\mathcal{F}_\mu + \frac{\kappa}{2}\sqrt{\frac{3}{2}}\,\partial_\mu b\Big)\,J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x\sqrt{-g}\,J^5_\mu J^{5\mu} + \dots\Big] + \dots$$

or Majorana

4-fermion contact interaction characteristic of (integrating out) torsion

$$J^{5\mu} = \bar{\psi}_j\,\gamma^\mu\,\gamma^5\psi_j$$

Axial Current

All fermion species

torsion

cf. classically in 4 dim: (duality relationship)

$$-3\,\sqrt{2}\partial_\sigma b \;=\; \sqrt{-g}\,\epsilon_{\mu\nu\rho\sigma}H^{\mu\nu\rho}$$

Inclusion of Fermions

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

$$+\, S^{Free}_{Dirac} + \int d^4x\sqrt{-g}\,\Big(\mathcal{F}_\mu + \frac{\kappa}{2}\sqrt{\frac{3}{2}}\,\partial_\mu b\Big)\,J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x\sqrt{-g}\,J^5_\mu J^{5\mu} + \dots\Big] + \dots$$

Majorana

$$\mathcal{F}^d = \varepsilon^{abcd}\,e_{b\lambda}\,\partial_a\,e^\lambda_c\,,$$

vielbeins

Vanishes for Friedmann-Lemaitre-Roberston-Walker backgrounds

torsion

cf. classically in 4 dim: (duality relationship)

$$-3\sqrt{2}\,\partial_\sigma b \;=\; \sqrt{-g}\,\epsilon_{\mu\nu\rho\sigma}\,H^{\mu\nu\rho}$$

Inclusion of Fermions

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\, R + \frac{1}{2}\,\partial_\mu b\, \partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\, b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

$$+\, S_{Dirac}^{Free} + \int d^4x\sqrt{-g}\,\Big(\quad \frac{\kappa}{2}\sqrt{\frac{3}{2}}\,\partial_\mu b\Big)\, J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x\sqrt{-g}\, J^5_\mu J^{5\mu} + \dots\Big] + \dots$$

or Majorana

$$J^{5\mu} = \bar{\psi}_j\,\gamma^\mu\,\gamma^5\psi_j$$

Axial Current (universal)

Kalb-Ramond (KR) or string-model independent ("gravitational") axion

torsion

cf. classically in 4 dim:
(duality relationship)

$$-3\,\sqrt{2}\partial_\sigma b \; = \; \sqrt{-g}\,\epsilon_{\mu\nu\rho\sigma}H^{\mu\nu\rho}$$

The Model

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

$$+\, S_{Dirac}^{Free} + \int d^4x\sqrt{-g}\,\Big(\quad \frac{\kappa}{2}\,\sqrt{\frac{3}{2}}\,\partial_\mu b\Big)\,J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x\sqrt{-g}\,J^5_\mu J^{5\mu} + \dots\Big] + \dots$$

or Majorana

$$J^{5\,\mu} = \bar{\psi}_j\,\gamma^\mu\,\gamma^5\psi_j$$

All fermion species

The Model

Anomaly terms

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \frac{2\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\left(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\right) + \dots\Big]$$

$$+\, S^{Free}_{Dirac} + \int d^4x\sqrt{-g}\left(\frac{\kappa}{2}\sqrt{\frac{3}{2}}\,\partial_\mu b\right) J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x\sqrt{-g}\,J^5_\mu J^{5\mu} + \dots\Big] + \dots$$

or Majorana

$$J^{5\mu} = \bar{\psi}_j\,\gamma^\mu\,\gamma^5\psi_j$$

All fermion species

The Model

Anomaly terms

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{2\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

$$+\, S^{Free}_{Dirac} + \int d^4x\sqrt{-g}\,\Big(\cdots \frac{\kappa}{2}\sqrt{\frac{3}{2}}\,\partial_\mu b\Big)\,J^{5\mu} - \frac{3\kappa^2}{\cdots}\int d^4x\sqrt{-g}\,J^5_\mu J^{5\mu} + \dots\Big] + \dots$$

or Majorana

$$-\int d^4x\sqrt{-g}\,\sqrt{\frac{3}{2}}\,\frac{\kappa}{2}\,b\,\nabla_\mu J^{5\,\mu}$$

$$J^{5\,\mu} = \bar{\psi}_j\,\gamma^\mu\,\gamma^5\psi_j$$

All fermion species

Non-trivial if chiral anomalies affect the conservation of axial current

NB: Anomalies: (CHIRAL)

Classically conserved current $J^{\mu 5}$

AXIAL FERMION CURRENT CEASES to be conserved @ a quantum level

$$\nabla_\mu J^{\mu 5} \propto c_1 R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu} \tilde{F}^{\mu\nu} \qquad c_1 \in \mathbb{R}$$

CHIRAL FERMIONS IN LOOP:

$J^{\mu 5} \equiv \bar{\Psi}_j \gamma^\mu \gamma^5 \Psi_j$, $j = 1 \dots N$ SPECIES

chiral fermion

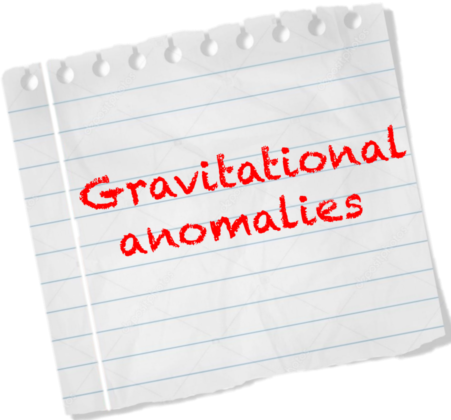
$\gamma^5 \Psi_j = \mp \Psi_j$

(LEFT OR RIGHT HANDED)

$$\tilde{F}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma},$$

$$\tilde{R}_{\mu\nu\rho\sigma} = \frac{1}{2} \varepsilon_{\mu\nu\alpha\beta} R^{\alpha\beta}{}_{\rho\sigma}$$

Gravitational Anomalies & Diffeomorphism Invariance



$$\int d^4x\, \sqrt{-g}\, b(x) \left(R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\, \widetilde{F}^{\mu\nu} \right)$$

Spoils conservation of stress tensor (diffeomorphism invariance affected in quantum theory)

Topological, does NOT contribute to stress tensor

$$\delta\Big[\int d^4x \sqrt{-g}\, b\, R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} \Big] = 4 \int d^4x \sqrt{-g}\, \mathcal{C}^{\mu\nu}\, \delta g_{\mu\nu} = -4 \int d^4x \sqrt{-g}\, \mathcal{C}_{\mu\nu}\, \delta g^{\mu\nu}$$

Cotton tensor

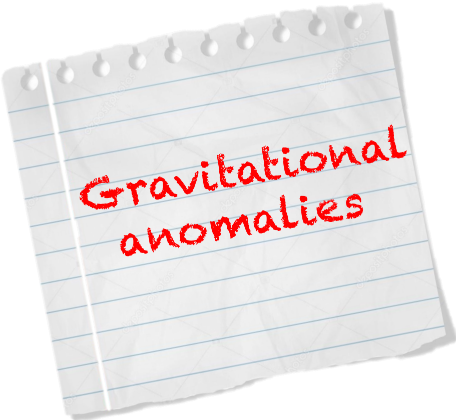
$$\mathcal{C}^{\mu\nu} = -\frac{1}{2}\Big[v_\sigma \left(\varepsilon^{\sigma\mu\alpha\beta} R^{\nu}{}_{\beta;\alpha} + \varepsilon^{\sigma\nu\alpha\beta} R^{\mu}{}_{\beta;\alpha} \right) + v_{\sigma\tau} \left(\widetilde{R}^{\tau\mu\sigma\nu} + \widetilde{R}^{\tau\nu\sigma\mu} \right) \Big] = -\frac{1}{2}\Big[\left(v_\sigma\, \widetilde{R}^{\lambda\mu\sigma\nu} \right)_{;\lambda} + (\mu \leftrightarrow \nu) \Big]$$

$$v_\sigma \equiv \partial_\sigma b = b_{;\sigma}\,, \quad v_{\sigma\tau} \equiv v_{\tau;\sigma} = b_{;\tau;\sigma}$$

Traceless $\qquad g_{\mu\nu}\, \mathcal{C}^{\mu\nu} = 0$

Jackiw, Pi (2003)

Gravitational Anomalies & Diffeomorphism Invariance



$$\int d^4x \sqrt{-g}\, b(x) \Big(R_{\mu\nu\rho\sigma} \, \widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu} \, \widetilde{F}^{\mu\nu} \Big)$$

Spoils conservation of stress tensor (diffeomorphism invariance affected in quantum theory)

Topological, does NOT contribute to stress tensor

$$\delta\Big[\int d^4x \sqrt{-g}\, b \, R_{\mu\nu\rho\sigma} \, \widetilde{R}^{\mu\nu\rho\sigma}\Big] = 4 \int d^4x \sqrt{-g}\, \mathcal{C}^{\mu\nu} \, \delta g_{\mu\nu} = -4 \int d^4x \sqrt{-g}\, \mathcal{C}_{\mu\nu} \, \delta g^{\mu\nu}$$

Cotton tensor

$$\mathcal{C}^{\mu\nu} = -\frac{1}{2}\Big[v_\sigma \Big(\varepsilon^{\sigma\mu\alpha\beta} R^{\nu}{}_{\beta;\alpha} + \varepsilon^{\sigma\nu\alpha\beta} R^{\mu}{}_{\beta;\alpha} \Big) + v_{\sigma\tau} \Big(\widetilde{R}^{\tau\mu\sigma\nu} + \widetilde{R}^{\tau\nu\sigma\mu} \Big) \Big] = -\frac{1}{2}\Big[\Big(v_\sigma \, \widetilde{R}^{\lambda\mu\sigma\nu} \Big)_{;\lambda} + (\mu \leftrightarrow \nu) \Big]$$

$$v_\sigma \equiv \partial_\sigma b = b_{;\sigma}, \quad v_{\sigma\tau} \equiv v_{\tau;\sigma} = b_{;\tau;\sigma}$$

→ not necessarily positive contributions to vacuum energy ⚠

Traceless $$g_{\mu\nu} \, \mathcal{C}^{\mu\nu} = 0$$

Gravitational Anomalies & Diffeomorphism Invariance

Einstein's equation

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R - \mathcal{C}^{\mu\nu} = \kappa^2 T^{\mu\nu}_{\text{matter}}$$

$$\mathcal{C}^{\mu\nu}{}_{;\mu} = -\frac{1}{8} v^{\nu} R^{\alpha\beta\gamma\delta} \widetilde{R}_{\alpha\beta\gamma\delta}$$

$$v_\sigma \equiv \partial_\sigma b$$

$$\kappa^2 T^{\mu\nu}_{\text{matter}\,;\mu} = -\, \mathcal{C}^{\mu\nu}{}_{;\mu} \neq 0$$

Diffeomoprphism invariance breaking by gravitational anomalies?

Gravitational Anomalies & Diffeomorphism Invariance

Einstein's equation

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R - \mathcal{C}^{\mu\nu} = \kappa^2 T^{\mu\nu}_{\rm matter}$$

$$\mathcal{C}^{\mu\nu}{}_{;\mu} = - \frac{1}{8} v^{\nu} R^{\alpha\beta\gamma\delta} \widetilde{R}_{\alpha\beta\gamma\delta}$$

$$v_\sigma \equiv \partial_\sigma b$$

$$\kappa^2 T^{\mu\nu}_{{\rm matter}\,;\mu} + \mathcal{C}^{\mu\nu}{}_{;\mu} = 0$$

No problem with diffeo

Conserved Modified stress-energy tensor

3(ii). Primordial Gravitational Waves, Anomaly condensates

The Model

Anomaly terms

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{2\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

$$+\, S_{Dirac}^{Free} + \int d^4x\sqrt{-g}\,\Big(\quad \frac{\kappa}{2}\sqrt{\frac{3}{2}}\,\partial_\mu b\Big)\,J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x\sqrt{-g}\,J^5_\mu J^{5\mu} + \dots\Big] + \dots$$

or Majorana

$$J^{5\,\mu} = \bar{\psi}_j\,\gamma^\mu\,\gamma^5\psi_j$$

All fermion species

Important Role
in early Universe
in the model → inflation

The Model

Anomaly terms

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

$$+\, S_{Dirac}^{Free} + \int d^4x\sqrt{-g}\,\Big(\frac{\kappa}{2}\sqrt{\frac{3}{2}}\,\partial_\mu b\Big)\,J^{5\mu} - \frac{3\kappa^2}{8}\int d^4x\sqrt{-g}\,J^5_\mu J^{5\mu} + \dots\Big] + \dots$$

or Majorana

$$-\int d^4x\sqrt{-g}\,\sqrt{\frac{3}{2}}\,\frac{\kappa}{2}\,b\,\nabla_\mu J^{5\,\mu}$$

$$J^{5\mu} = \bar{\psi}_j\,\gamma^\mu\,\gamma^5\psi_j$$

All fermion species

Role in Late Universe (exit from inflation Onwards) when chiral fermions are generated

The Model

Anomaly terms

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \frac{2\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

$$+ S_{Dirac}^{Free} + \int d^4x\sqrt{-g}\Big(\frac{\kappa}{2}\sqrt{\frac{3}{2}}\,\partial_\mu b\Big)\,J^{5\mu} - \frac{3\kappa^2}{\dots}\int d^4x\sqrt{\dots}\,J^5_\mu J^{5\mu} + \dots\Big] + \dots$$

or Majorana

$$-\int d^4x\sqrt{-g}\,\sqrt{\frac{3}{2}}\,\frac{\kappa}{2}\,b\,\nabla_\mu J^{5\,\mu}$$

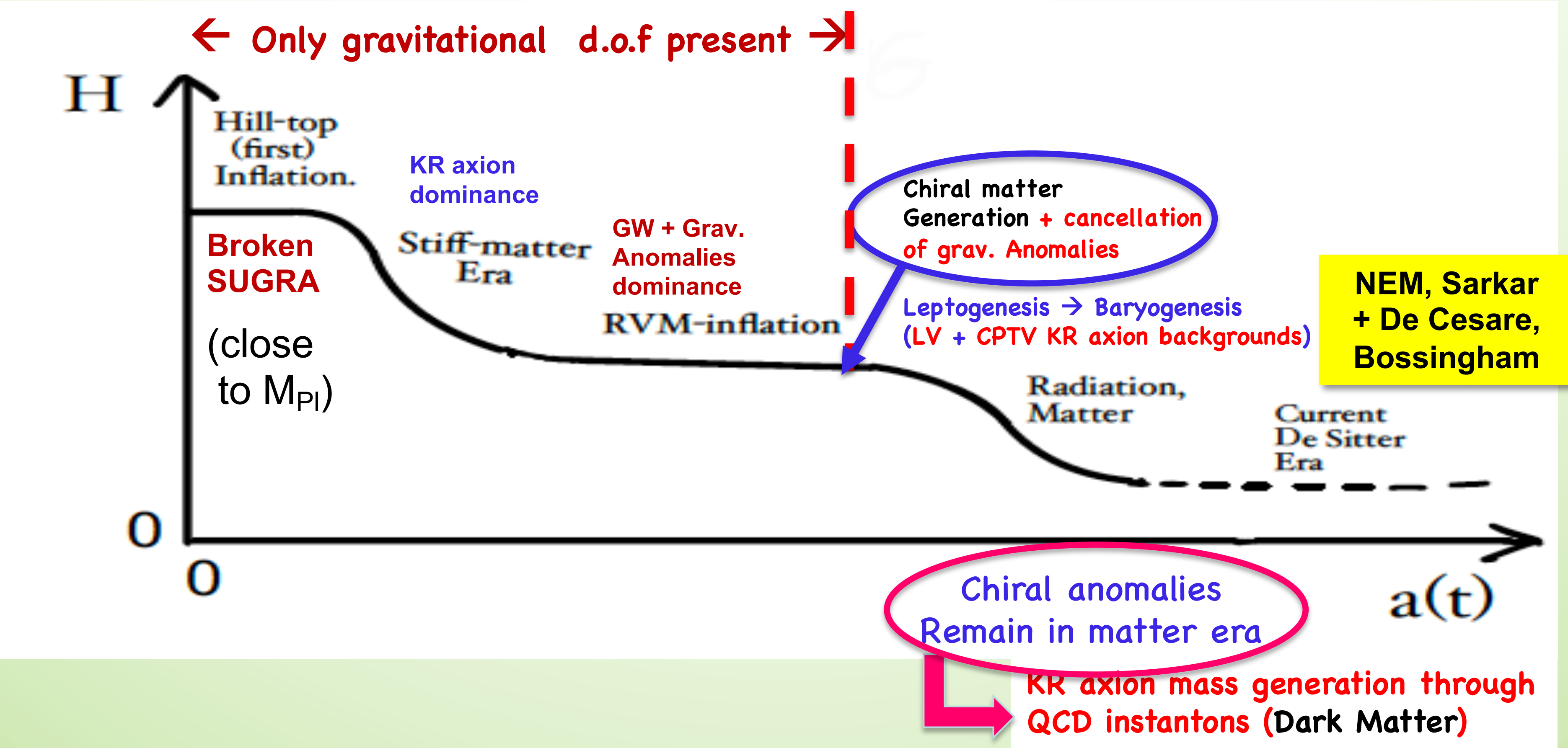
$$J^{5\,\mu} = \bar{\psi}_j\,\gamma^\mu\,\gamma^5\psi_j$$

All fermion species

Chiral-matter-induced Gravitational anomalies may cancel their primordial counterparts in post-RVM-inflationary eras

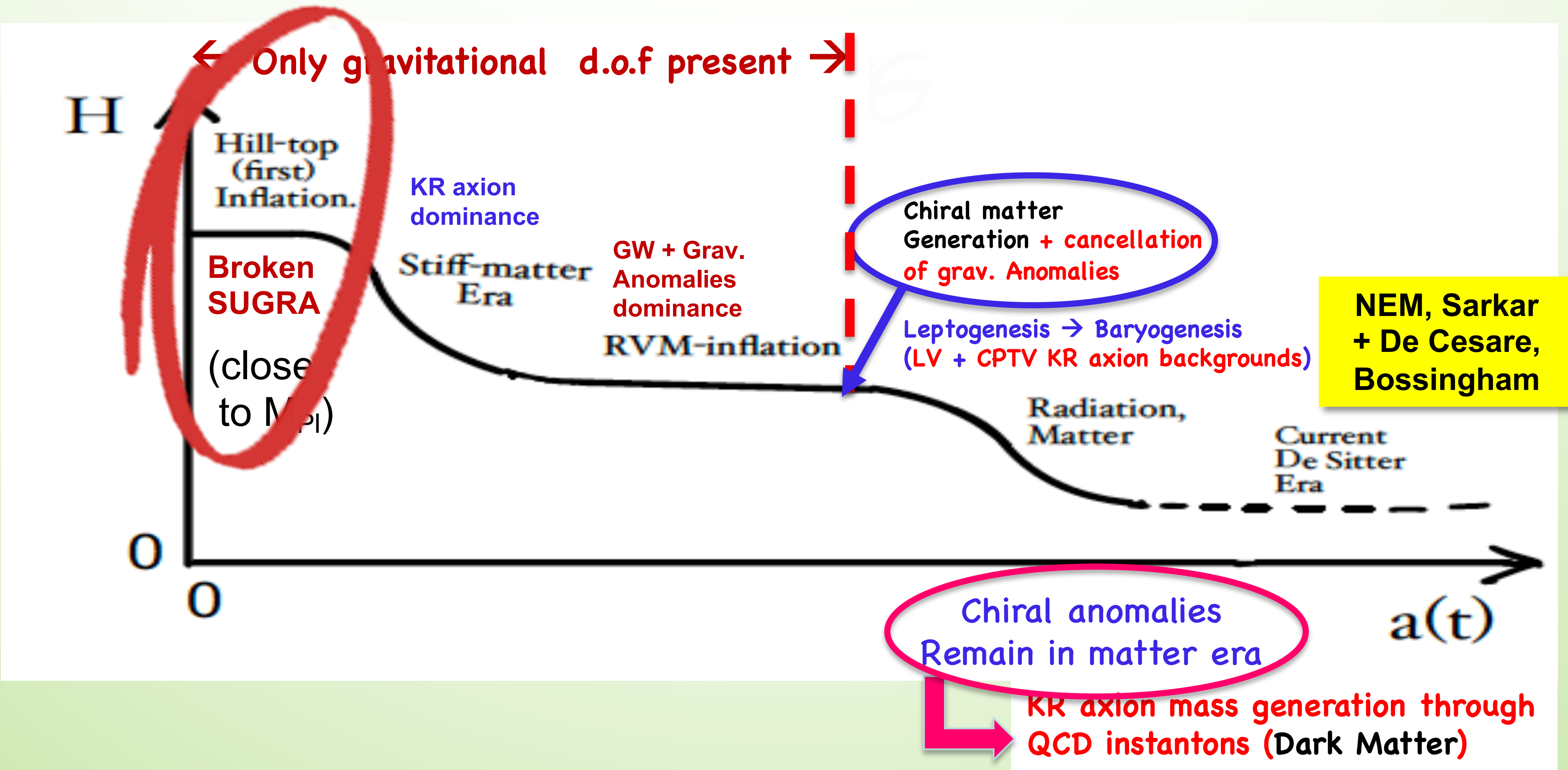
The Cosmology of the Model @ a glance

NEM,Solà
EPJ-ST
(2020)



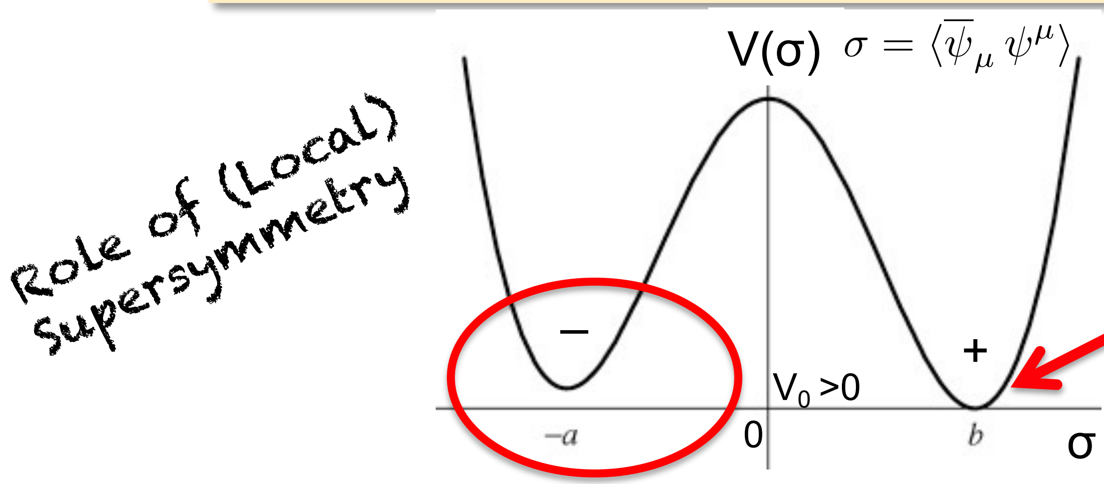
The Cosmology of the Model @ a glance

NEM, Solà
EPJ-ST
(2020)



The Model in Early Universe: only gravitational d.o.f. ($b, g_{\mu\nu}, \psi_\mu$)

Basilakos, NEM, Solà (2019-20)



SUGRA broken gravitino Condensate stabilised → RVM GW-induced Inflation

Statistical bias (percolation) in occupation probabilities of the +,- vacua

Lalak, Ovrut, Lola, G. Ross, Thomas

Primordial Gravitational Waves

Potential Origins in pre-inflationary era?

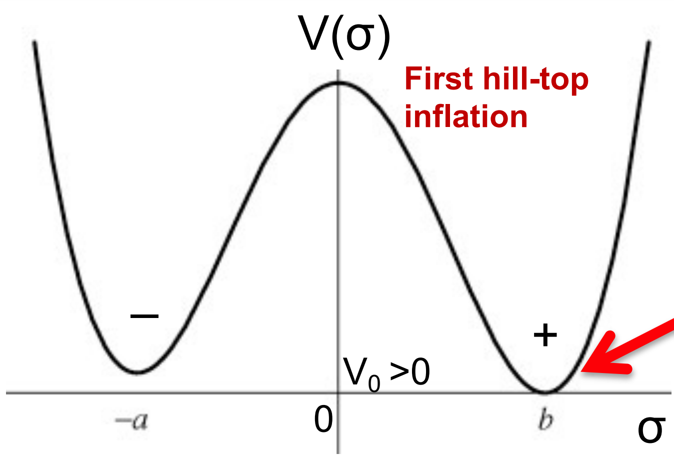
Collapse/collisions of Domain walls formed in theories with (approximate) discrete symmetry breaking, e.g. via bias in double-well potentials of some condensate (gravitino ψ_μ or gaugino)

NEM, Solà EPJ-ST (2020)

Ellis, NEM, Alexandre, Houston

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$, ψ_μ)

Basilakos, NEM, Solà (2019-20)



SUGRA broken gravitino Condensate stabilised → RVM GW-induced Inflation

Pre-RVM inflationary phase: superstring/supergravity Effective action → **Imaginary parts** → **instabilities**

First Hill-top inflation = finite life –time → System **tunnels** to **RVM inflationary vacuum (GW condense)**

NEM, Solà EPJ-ST (2020)

Ellis, NEM, Alexandre, Houston

N=1 SUGRA & QG effects

Alexandre
Houston. NEM

$$\Gamma \simeq -\frac{1}{2\kappa^2}\int d^4x\sqrt{g}\left[\left(\widehat{R}-2\Lambda_1\right)+\alpha_1\,\widehat{R}+\alpha_2\,\widehat{R}^2\right]$$

Effective action Γ in the presence of cosmol. constant Λ > 0

$$\widehat{R}_{\lambda\mu\nu\rho}=\frac{\Lambda}{3}\left(\widehat{g}_{\lambda\nu}\,\widehat{g}_{\mu\rho}-\widehat{g}_{\lambda\rho}\,\widehat{g}_{\mu\nu}\right)\qquad \Lambda_1\equiv\Lambda_0-\kappa^{-2}\,\alpha_0\qquad \alpha_0=\alpha_0^B=\kappa^4\,\Lambda_0^2\left[0.027-0.018\ln\left(-\frac{3\Lambda_0}{2\,\mu^2}\right)\right]$$

$$\alpha_1=\frac{\kappa^2}{2}\left(\alpha_1^F+\alpha_1^B\right)\ ,\qquad \alpha_2=\frac{\kappa^2}{8}\left(\alpha_2^F+\alpha_2^B\right)$$

F=integrating out gravitinos

B=integrating our gravitons **(QG)**

$$\alpha_1^F=0.067\,\tilde{\kappa}^2\sigma_c^2-0.021\,\tilde{\kappa}^2\sigma_c^2\ln\left(\frac{\Lambda}{\mu^2}\right)+0.073\,\tilde{\kappa}^2\sigma_c^2\ln\left(\frac{\tilde{\kappa}^2\sigma_c^2}{\mu^2}\right)\ ,$$

$$\alpha_2^F=0.029+0.014\ln\left(\frac{\tilde{\kappa}^2\sigma_c^2}{\mu^2}\right)-0.029\ln\left(\frac{\Lambda}{\mu^2}\right)\ ,$$

$$\alpha_1^B=-0.083\Lambda_0+0.018\,\Lambda_0\ln\left(\frac{\Lambda}{3\mu^2}\right)+0.049\,\Lambda_0\ln\left(-\frac{3\Lambda_0}{\mu^2}\right)\ ,$$

$$\alpha_2^B=0.020+0.021\ln\left(\frac{\Lambda}{3\mu^2}\right)-0.014\ln\left(-\frac{6\Lambda_0}{\mu^2}\right)\ .$$

μ = RG scale

In cosmological setting we may replace $\Lambda \sim 3H_I^{\,2}$ for inflation or
More generally $\Lambda \sim 3\,H^2(t)$ for slowly time-varying H(t)

N=1 SUGRA & QG effects

Alexandre Houston. NEM

$$\Gamma \simeq -\frac{1}{2\kappa^2}\int d^4x\sqrt{g}\left[\left(\widehat{R}-2\Lambda_1\right)+\alpha_1\,\widehat{R}+\alpha_2\,\widehat{R}^2\right]$$

Effective action Γ in the presence of cosmol. constant Λ > 0

$$\widehat{R}_{\lambda\mu\nu\rho}=\frac{\Lambda}{3}\left(\widehat{g}_{\lambda\nu}\,\widehat{g}_{\mu\rho}-\widehat{g}_{\lambda\rho}\,\widehat{g}_{\mu\nu}\right) \qquad \Lambda_1\equiv\Lambda_0-\kappa^{-2}\,\alpha_0 \qquad \alpha_0=\alpha^B \ldots 3\ln\left(-\frac{3\Lambda_0}{2\,\mu^2}\right)\Big]$$

...t gravitinos

$$\alpha_1=\frac{\kappa^2}{2}\left(\alpha_1^F+\alpha_1^B\right)\ , \quad \alpha_2=\frac{\kappa^2}{8}\left(\alpha^F \ldots\right.$$

...ntegrating our gravitons **(QG)**

$$\alpha_1^F=0.067\,\tilde{\kappa}^2\sigma^2 \ldots$$

$$\ldots 0.014\ln\left(\frac{\tilde{\kappa}^2\sigma_c^2}{\mu^2}\right)- -0.029\ln\left(\frac{\Lambda}{\mu^2}\right)\ ,$$

$$\alpha_1^B=-0.083\Lambda_0+0.018\,\Lambda_0\ln\left(\frac{\Lambda}{3\mu^2}\right)+ 0.049\,\Lambda_0\ln\left(-\frac{3\Lambda_0}{\mu^2}\right)\ ,$$

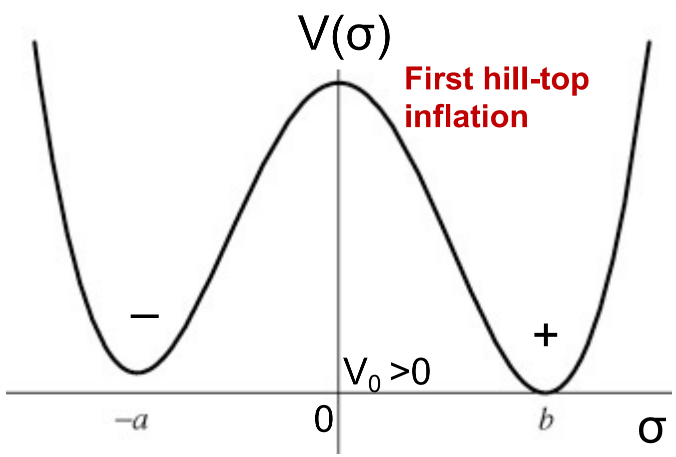
$$\alpha_2^B=0.020+0.021\ln\left(\frac{\Lambda}{3\mu^2}\right)- 0.014\ln\left(-\frac{6\Lambda_0}{\mu^2}\right)\ .$$

μ = RG scale

$$\Gamma \propto \int \left(c_1+c_2\ln H\right)H^2+\left(c_3+c_4\ln H\right)H^4$$

In cosmological setting we may replace $\Lambda \sim 3H_I^{\,2}$ for inflation or
More generally $\Lambda \sim 3H^2(t)$ for slowly time-varying H(t)

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$, ψ_μ)



$H_I^{\rm first}$ Hubble parameter of first inflation

Pre-RVM inflationary phase: superstring/supergravity Effective action → **Imaginary parts** → **instabilities**

$$\mathrm{Im}\Gamma^{(1)\,E}_{\rm eff} \simeq \kappa^2\,\pi^3\,\Big(0.4\,\frac{\Lambda_0^2}{\Lambda} - 1.2\,\Lambda_0 + 1.3\,\Lambda\Big)$$

Decay rate of false vacuum γ can be estimated

NEM, Solà

$$\gamma \simeq \pi\,(H_I^{\rm first})^4\,\Big|\frac{\mu^2}{|\Lambda_0|}\Big|^{-16\pi^2\,(H_I^{\rm first})^2\kappa^2}$$

$$\kappa^2\,(H_I^{\rm first})^2 \ll 1, \quad |\Lambda_0| \ll \mu^2$$

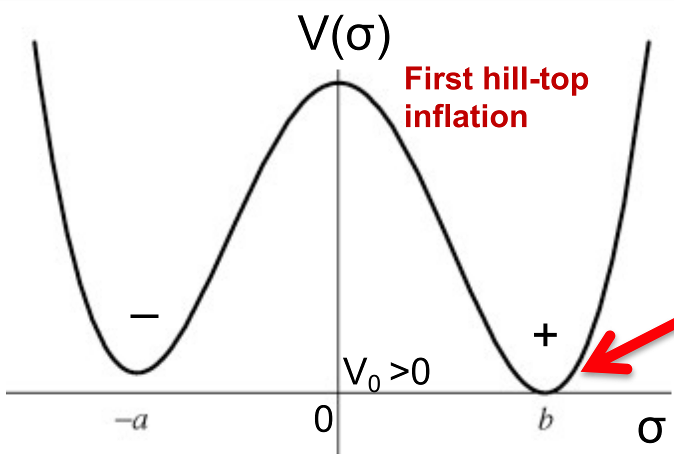
$$\mu^2\kappa^2 = \mathcal{O}(1)$$

$$\mu^{-2}|\Lambda_0| \sim \kappa^2|\Lambda_0| \ll 1$$

Decay rate of first inflation is O(H$_i^{\rm first}$)

The Model in Early Universe:
only gravitational d.o.f. (b, $g_{\mu\nu}$, ψ_μ)

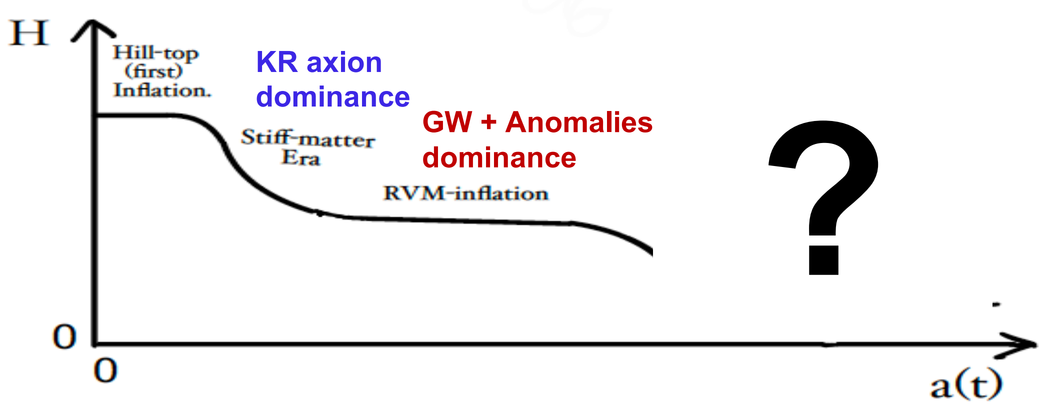
Basilakos, NEM, Solà (2019-20)



SUGRA broken gravitino Condensate stabilised → RVM GW-induced Inflation

Pre-RVM inflationary phase: superstring/supergravity Effective action → Imaginary parts → instabilities

First Hill-top inflation = finite life –time → System **tunnels** to RVM inflationary vacuum (GW condense)

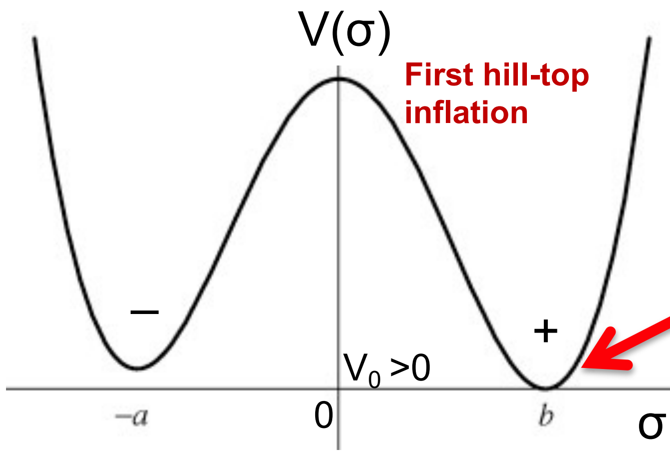


NEM, Solà EPJ-ST (2020)

Ellis, NEM, Alexandre, Houston

The Model in Early Universe:
only gravitational d.o.f. (b, $g_{\mu\nu}$, ψ_μ)

Basilakos, NEM, Solà (2019-20)

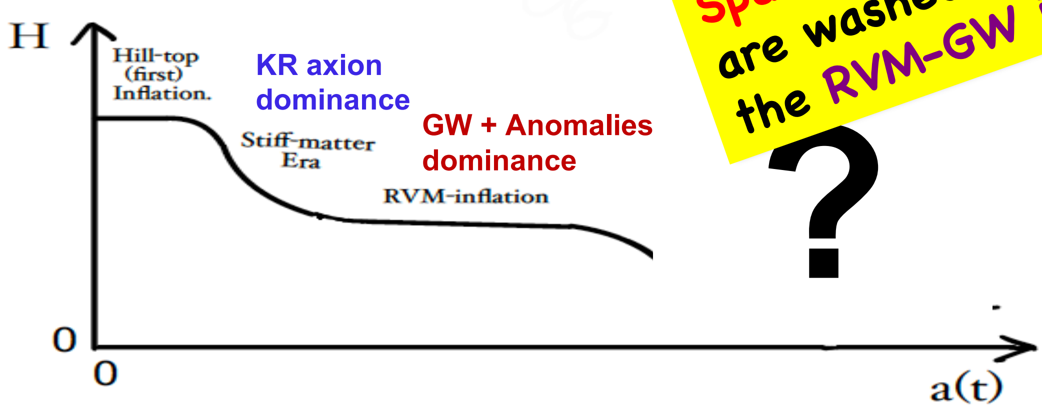


SUGRA broken gravitino Condensate stabilised → RVM GW-induced Inflation

Pre-RVM inflationary phase: superstring/supergravity Effective action → *Imaginary parts* → **instabilities**

First Hill-top inflation = finite life –ti

System **tunnels** to RVM inflationary va



First inflation ensures any Spatial inhomogeneities are washed out before the RVM-GW inflation

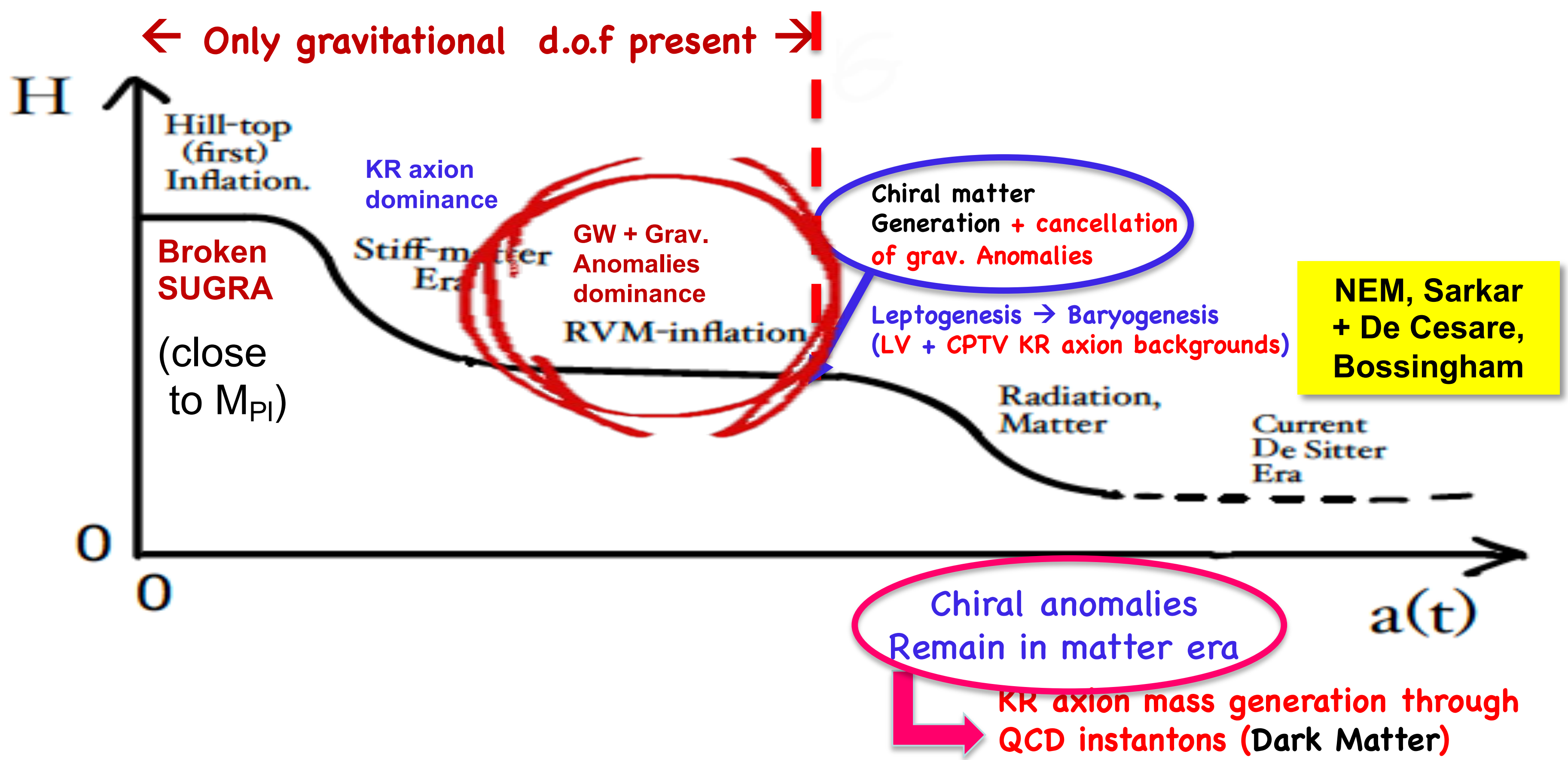
?

NEM,Solà EPJ-ST (2020)

Ellis, NEM, Alexandre, Houston

The Cosmology of the Model @ a glance

NEM,Solà
EPJ-ST
(2020)



$$\langle R_{\mu\nu\rho\sigma} \widetilde{R}^{\mu\nu\rho\sigma} \rangle = \frac{16}{a^4} \kappa^2 \int^{\widetilde{\mu}} \frac{d^3k}{(2\pi)^3} \frac{H^2}{2k^3} k^4 \Theta + O(\Theta^3)$$

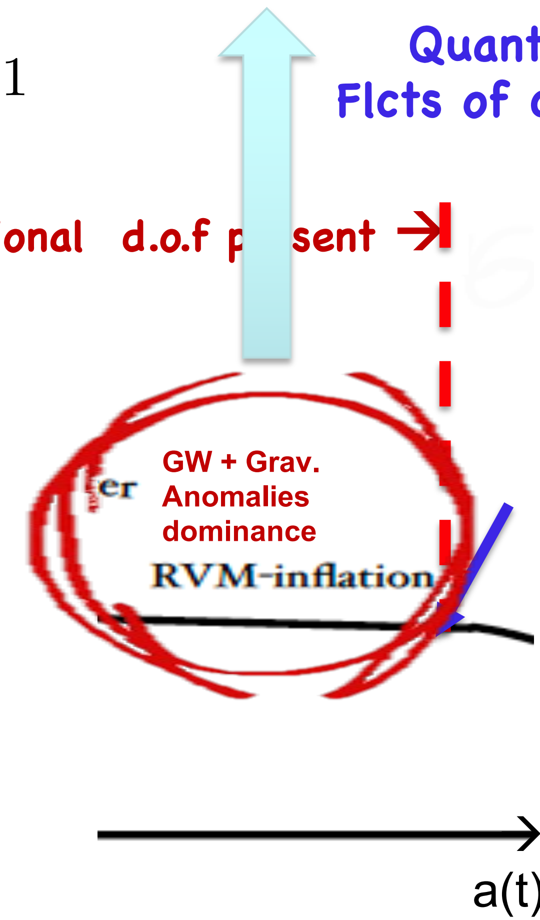
$$\kappa = M_{\rm Pl}^{-1}, \quad \dot{b} \equiv db/dt, \quad a(t) \sim e^{Ht}$$

$$\Theta = \sqrt{\frac{2}{3}} \frac{\kappa^3}{12} H \dot{b} \ll 1$$

Quantum graviton Flcts of chiral GW type

H ≈ const. (inflation)

← Only gravitational d.o.f p sent →



Alexander, Peskin, Sheikh -Jabbari

Lyth, Quimbay Rodriguez,

Can be shown (including Chern-Simons grav. anomalies)

E.o.S. of Running vacuum

$p(t) = -\rho(t) > 0$, H(t) = mild variation

UV cutoff $\tilde{\mu}$

$$\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle = \frac{16}{a^4}\kappa^2\int^{\tilde{\mu}}\frac{d^3k}{(2\pi)^3}\frac{H^2}{2k^3}k^4\Theta + O(\Theta^3)$$

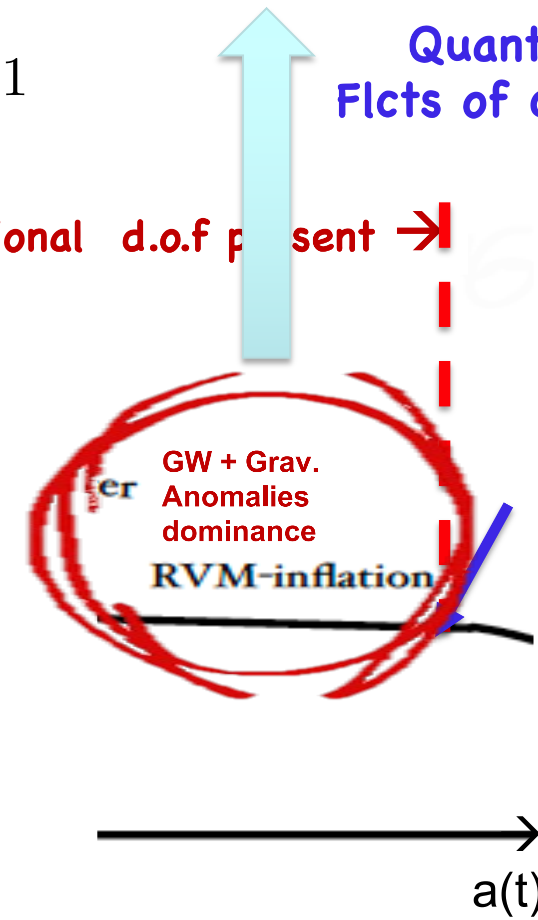
$$\kappa = M_{\rm Pl}^{-1},$$
$$\dot{b} \equiv db/dt$$
$$a(t) \sim e^{Ht}$$

$$\Theta = \sqrt{\frac{2}{3}}\frac{\kappa^3}{12}H\dot{b} \ll 1$$

Quantum graviton Flcts of chiral GW type

H ≈ const.
(inflation)

← Only gravitational d.o.f present →



Only an estimate in effective field theories as it depends on physics in UV regime. Hence, full string theory computation possibly depends on infinite towers of Massive string states…

Can be shown (including Chern-Simons grav. anomalies)

E.o.S. of Running vacuum

p(t) = - ρ(t) > 0, H(t) = mild variation

The Model in Early Universe:
only gravitational d.o.f. ($b, g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

$$b\,\mathcal{K}^{\mu}{}_{;\mu}$$

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$

$$\Big(= \int d^4x\,\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b - \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\partial_\mu b(x)\,\mathcal{K}^\mu + \dots\Big]\Big)$$

$$+\sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\int d^4x\sqrt{-g}\,\langle b(x)\,R_{\mu\mu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\rangle$$

Condensate < …> of Gravitational Anomalies

Cosmological-Constant-like

$$g\mathcal{CS} = \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\int d^4x\,\sqrt{-g}\,\Big(\langle\, b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\,\rangle + :b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}:\Big)$$

quantum ordered

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\frac{\alpha'}{96\,\kappa}\mathcal{K}^\alpha(t)\right)\right] = 0 \quad \Rightarrow \quad \dot{\bar{b}} = \sqrt{\frac{2}{3}}\frac{\alpha'}{96\,\kappa} < \mathcal{K}^0 > \sim \text{constant}$$

$$<\mathcal{K}^0> = \text{const.}$$

Spontaneous LV (+ CPTV) solution

$$\dot{\bar{b}} \propto \epsilon^{ijk} H_{ijk} = \text{constant}$$

$$\Lambda \equiv \langle b(x)\, R_{\mu\mu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma}\rangle \simeq 5.86 \times 10^7\, \epsilon \mathcal{N}\, H^4 > 0$$

Positive total energy density since Λ-term dominates

$$\rho_{\rm total} = \rho_b + \rho_{gCS} + \rho_\Lambda \simeq 3 M_{\rm Pl}^4 \left[-1.7 \times 10^{-3} \left(\frac{H}{M_{\rm Pl}}\right)^2 + \left(1.17 - 1.37\right) \times 10^7 \left(\frac{H}{M_{\rm Pl}}\right)^4 \right] > 0$$

Dark Energy ("running vacuum model (RVM) type") cf. talk by Solà

Equation of state :

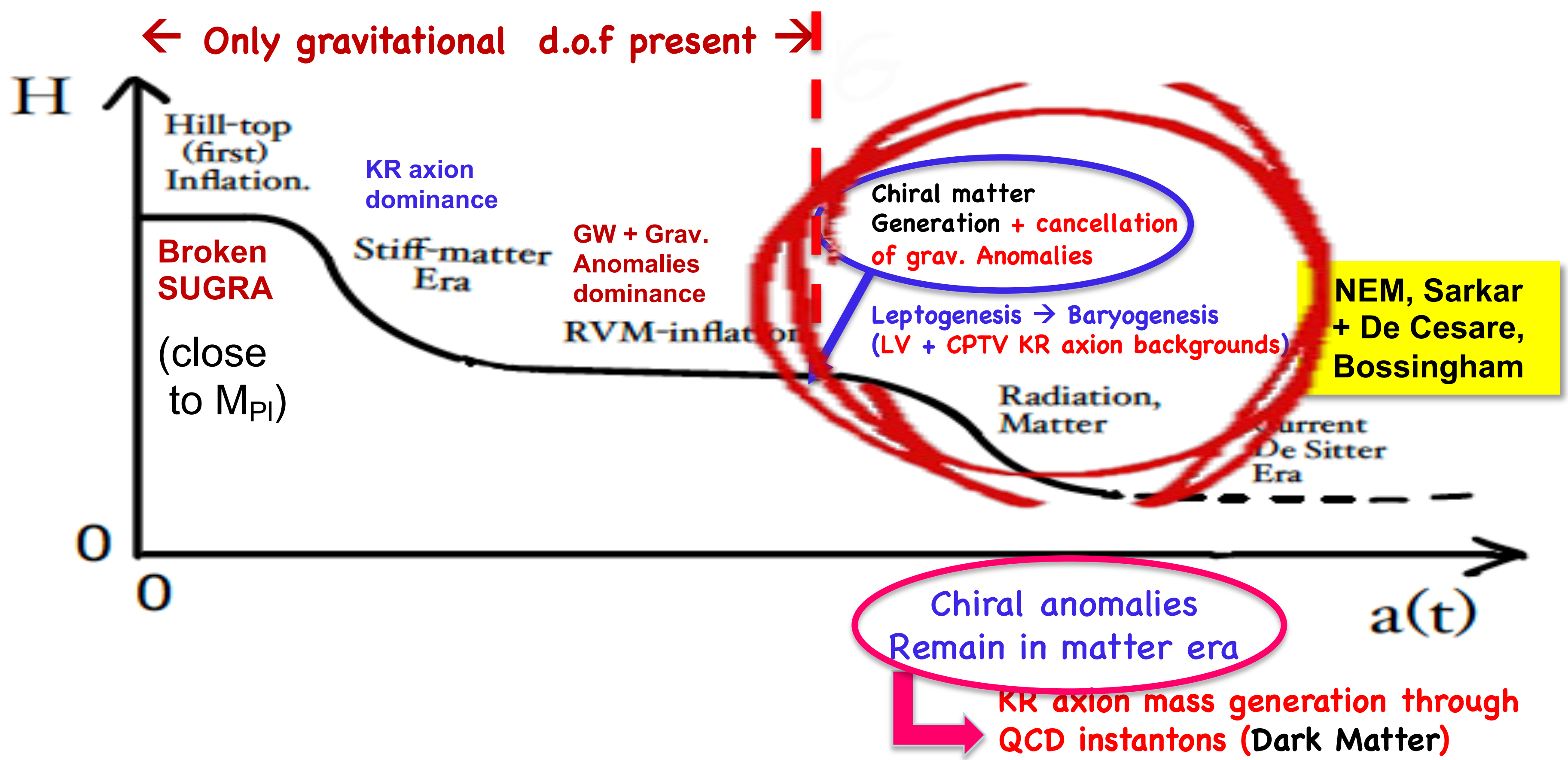
$0 > \rho_b + \rho_{gCS} = -(p_b + p_{gCS})$ cf. phantom "matter"

$0 < \rho_\Lambda = -p_\Lambda$ → dominates →

$0 < \rho_b + \rho_{gCS} + \rho_\Lambda = -(p_b + p_{gCS} + p_\Lambda)$ true RVM vacuum

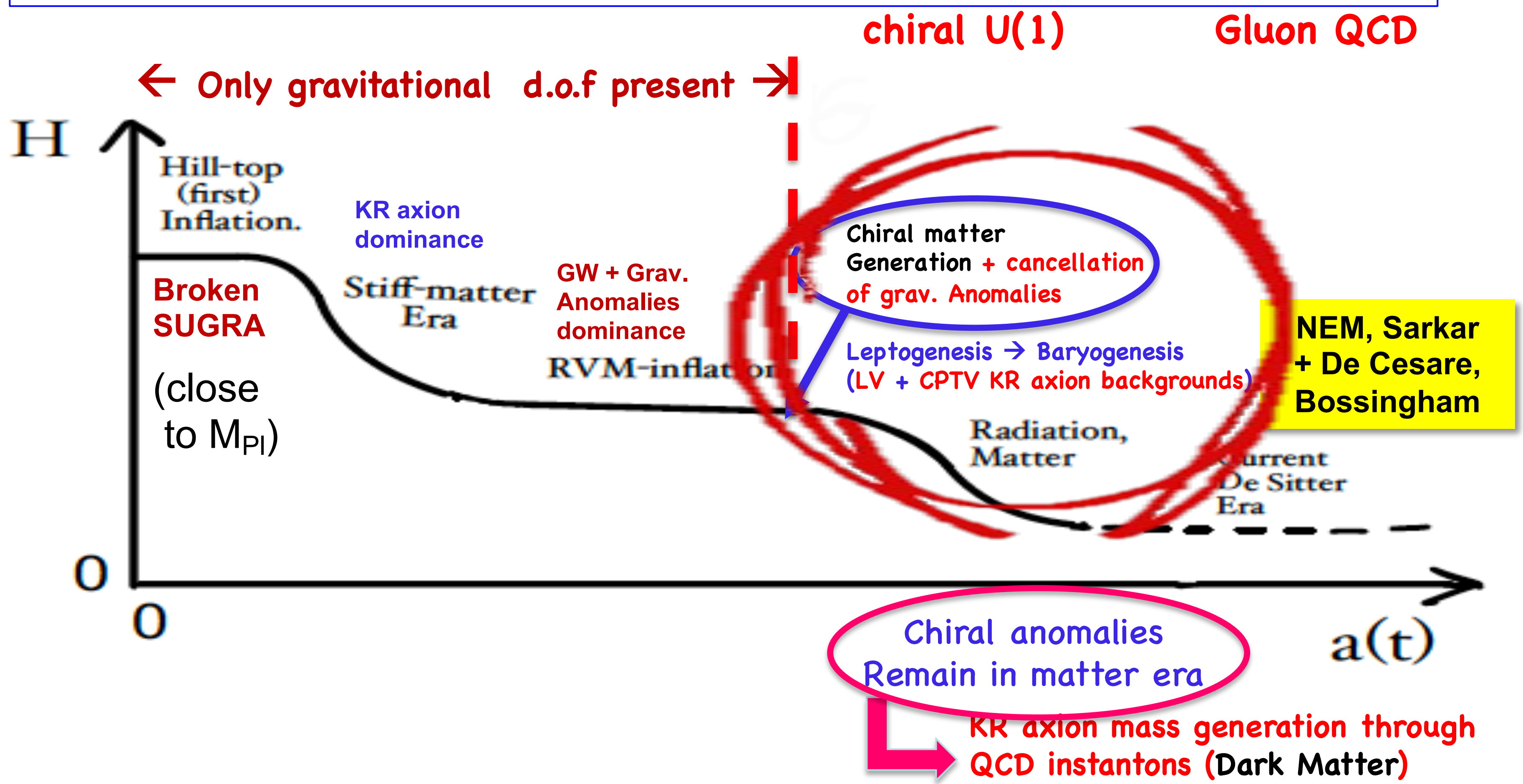
The Cosmology of the Model @ a glance

NEM, Solà
EPJ-ST
(2020)



The Cosmology of the Model @ a glance

$$\partial_\mu\left[\sqrt{-g}\left(\sqrt{\frac{3}{8}}\frac{\alpha'}{\kappa}J^{5\mu} - \frac{\alpha'}{\kappa}\sqrt{\frac{2}{3}}\frac{1}{96}\mathcal{K}^\mu\right)\right] = \sqrt{\frac{3}{8}}\frac{\alpha'}{\kappa}\left(\frac{\alpha_{\rm EM}}{2\pi}\sqrt{-g}\,F^{\mu\nu}\,\widetilde{F}_{\mu\nu} + \frac{\alpha_s}{8\pi}\sqrt{-g}\,G^a_{\mu\nu}\,\widetilde{G}^{a\mu\nu}\right)$$



Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

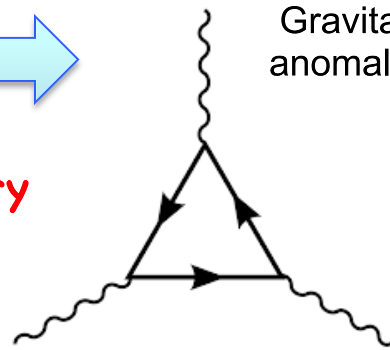
Big-Bang, pre-inflationary phase (broken Sugra)

Basilakos, NEM, Solà

forward direction

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves → Gravitational anomaly (GA) →



Undiluted constant KR axial backround

$$B_\mu = M_{\rm Pl}^{-1}\,\dot{\overline{b}}\,\delta_{\mu 0}$$

$$\dot{\overline{b}} \sim \sqrt{2\varepsilon}\,M_{\rm Pl}\,H \sim 0.14\,M_{\rm Pl}\,H$$

From a pre-inflationary era after Big-Bang

chiral matter generation @ inflation exit

Cancellation of GA

Radiation Era

$$B_0 \propto T^3$$

Leptogenesis induced by RHN (tree-level) decays

NEM, Sarkar + De Cesare, Bossingham

$$N_I \;\rightarrow\; \overline{\phi}\,\ell\,,\;\; \phi\,\overline{\ell}$$

ΔL In the (approx.) constant LV + CPTV background $B_\mu = M_{\rm Pl}^{-1}\,\dot{\overline{b}}\,\delta_{\mu 0}$

B-L conserving sphelaron processes → Baryongenesis

Matter Era

Possible potential (mass) generation for b → axion Dark matter

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

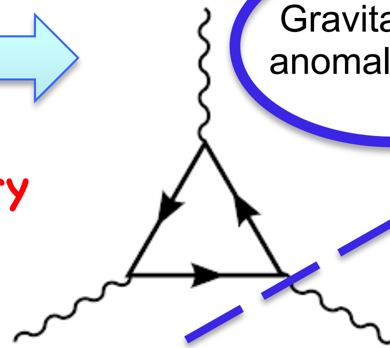
Big-Bang, pre-inflationary phase (broken Sugra)

Basilakos, NEM, Solà

forward direction

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves



Gravitational anomaly (GA)

Undiluted constant KR axial backround

$B_\mu = M_{\rm Pl}^{-1}\dot{b}\,\delta_{\mu 0}$

$\dot{b} \sim \sqrt{2\varepsilon}\,M_{\rm Pl}\,H \sim 0.14\,M_{\rm Pl}\,H$

From a pre-inflationary era after Big-Bang

chiral matter generation @ inflation exit

Radiation Era

Cancellation of GA

$B_0 \propto T^3$

Leptogenesis induced by RHN (tree-level) decays

$N_I \;\rightarrow\; \overline{\phi}\,\ell\,,\;\; \phi\,\overline{\ell}$

NEM, Sarkar + De Cesare, Bossingham

ΔL in the (approx.) constant LV + CPTV background $B_\mu = M_{\rm Pl}^{-1}\dot{b}\,\delta_{\mu 0}$

B-L conserving sphelaron processes → **Baryongenesis**

Matter Era

Possible potential (mass) generation for b → axion Dark matter

4. Lorentz- & CPT-Violating

Leptogenesis →

→ Baryogenesis

in models with Massive Right-handed Neutrinos

cf. also Sarben Sarkar's talk

Models with Right-handed Majorana Neutrinos N_I, $I=1,2,\ldots$

$$L = L_{SM} + \bar{N}_I i\partial_\mu \gamma^\mu N_I - F_{\alpha I}\, \bar{L}_\alpha N_I \tilde{\phi} - \frac{M_I}{2}\, \bar{N}_I^c N_I + \text{h.c.}$$

Light Neutrino Masses through see saw

$$m_\nu = -M^D \frac{1}{M_I} [M^D]^T .$$

$$M_D = F_{\alpha I} v$$

$$v = \langle \phi \rangle \sim 175 \text{ GeV}$$

$$M_D \ll M_I$$

Models with Right-handed Majorana Neutrinos N_I, I=1,2,...

$$L = L_{SM} + \bar{N}_I i\partial_\mu \gamma^\mu N_I - F_{\alpha I}\, \bar{L}_\alpha N_I \tilde{\phi} - \frac{M_I}{2}\, \bar{N}_I^c N_I + \text{h.c.}$$

+

$$\mathcal{L}_{\rm int} = -\overline{N}_I\, B_\mu\, \gamma^\mu\, \gamma^5\, N_I$$

Add interaction with approximately constant axial background B_μ (e.g. generated by torsion)

Isotropy & Homogeneity: B_0 = non trivial, B_i = 0, i=1,2,3

In our KR-torsion-induced axion background

$$B_\mu = M_{\rm Pl}^{-1}\, \dot{\bar{b}}\, \delta_{\mu 0}$$

$$\dot{\bar{b}} \sim \sqrt{2\,\varepsilon}\, M_{\rm Pl}\, H \sim 0.14\, M_{\rm Pl}\, H$$

CPT Violation

de Cesare, NEM, Sarkar
Eur.Phys.J. C75, 514 (2015)

Early Universe
$T >> T_{EW}$

$$\mathcal{L} = i\overline{N}\not{\partial}N - \frac{m}{2}\left(\overline{N^c}N + \overline{N}N^c\right) - \overline{N}\not{B}\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

Heavy Right-Handed-Neutrino (*N*) interact with **axial (approx.)**

constant background with only temporal component $B_0 \propto \dot{b} \neq 0$

CPT Violation

de Cesare, NEM, Sarkar
Eur.Phys.J. C75, 514 (2015)

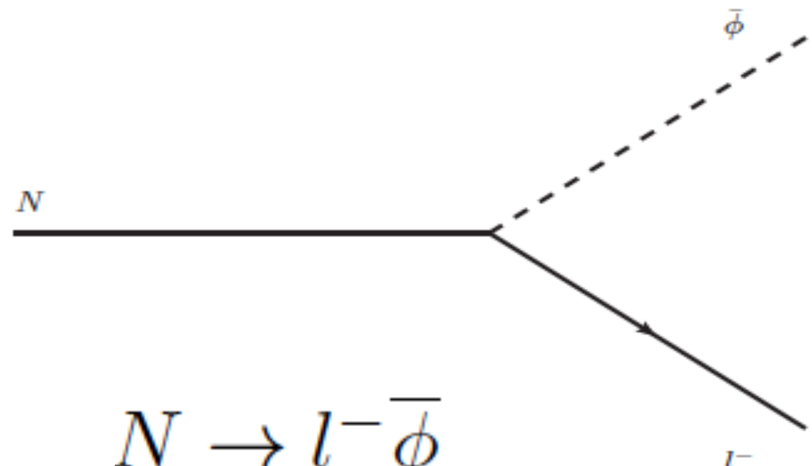
Early Universe
$T >> T_{EW}$

$$\mathcal{L} = i\overline{N}\not\partial N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}\not B\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

Heavy Right-Handed-Neutrino (*N*) interact with **axial (approx.)** **constant background** with only temporal component $B_0 \propto \dot{b} \neq 0$

Produce Lepton asymmetry

Lepton number & CP Violations
@ tree-level due to
Lorentz/CPTV Background



$$N \rightarrow l^+\phi \qquad\qquad N \rightarrow l^-\overline{\phi}$$

$$\Gamma_1 = \sum_k \frac{|Y_k|^2}{32\pi^2}\frac{m^2}{\Omega}\frac{\Omega + B_0}{\Omega - B_0} \neq \Gamma_2 = \sum_k \frac{|Y_k|^2}{32\pi^2}\frac{m^2}{\Omega}\frac{\Omega - B_0}{\Omega + B_0}$$

$B_0 \neq 0$

CPV & LV

$$\Omega = \sqrt{B_0^2 + m^2}$$

$$\mathcal{L} = i\overline{N}\partial\!\!\!/ N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}B\!\!\!/\gamma^5 N - Y_k \overline{L}_k \tilde{\phi} N + h.c.$$

(approx.) Constant B_0 Background

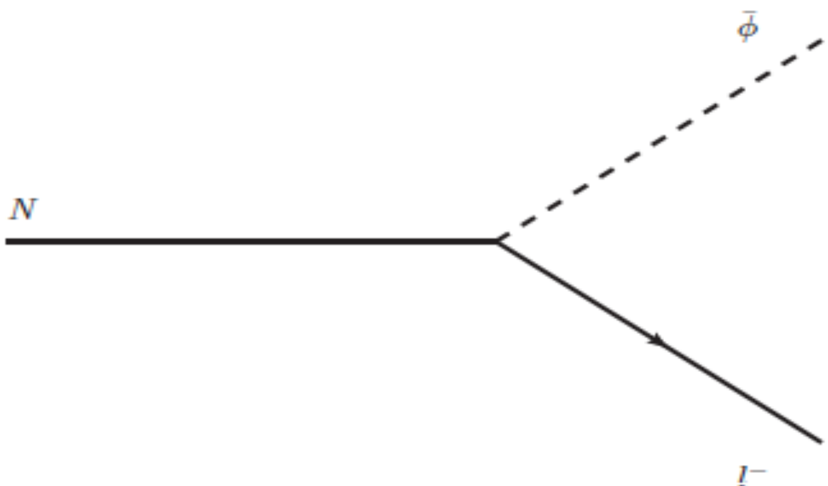
CPT Violation

Early Universe
$T \gg T_{EW}$

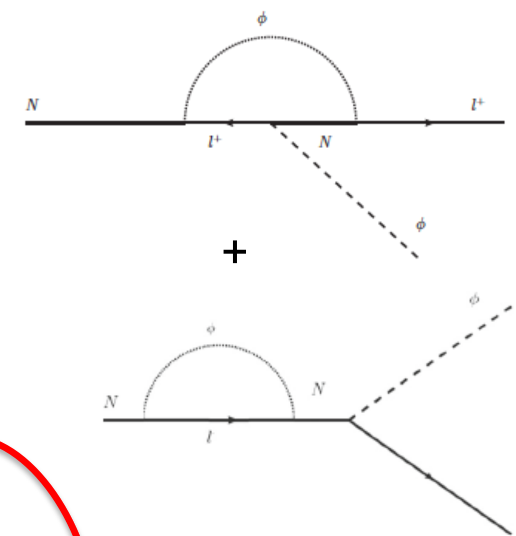
Lepton number & CP Violations @ tree-level due to Lorentz/CPTV Background

$$N_I \to \overline{\phi}\,\ell\,,\;\; \phi\,\overline{\ell}$$

Produce Lepton asymmetry



Contrast with one-loop conventional CPV Leptogenesis (in absence of H-torsion)



Fukugita, Yanagida,

CPTV Thermal

$$\mathcal{L} = i\overline{N}\partial\!\!\!/ N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}B\!\!\!/\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

CPT Violation

Early Universe
$T > 10^5$ GeV

(approx.) Constant $B^0 \neq 0$ background

Lepton number & CP Violations @ tree-level due to Lorentz/CPTV Background

$$N_I \rightarrow \overline{\phi}\,\ell\,,\ \phi\,\overline{\ell}$$

$$\frac{\Delta L}{n_\gamma} \simeq 10^{-10},$$

$$\frac{B_0}{m} \simeq 10^{-8}$$

Produce Lepton asymmetry

Solving system of Boltzmann eqs

$$\frac{\Delta L^{TOT}}{s} \simeq \frac{g_N}{7\,e\,(2\pi)^{3/2}}\frac{B_0}{m} \simeq 0.007\,\frac{B_0}{m}$$

$$Y_k \sim 10^{-5}$$

$$m \geq 100\text{TeV} \rightarrow$$

$$B^0 \sim 1\text{MeV}$$

$$T_D \simeq m \sim 100 \text{ TeV}$$

$m \leq 10^4$ TeV (Higgs mass stabilty)

Situation faced in post-RVM-inflationary eras in our models

Similar order of magnitude estimates if $B^0 \sim T^3$ during Leptogenesis era

Bossingham, NEM, Sarkar

CPTV Thermal

$$\mathcal{L} = i\overline{N}\not{\partial}N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}\not{B}\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

If the anomaly condensate ceases to exist at the end of the RVM-inflationary phase EOM implies

$$\partial_\mu\left(\sqrt{-g}\,\partial^\mu b\right) = 0$$

- For FLRW universe the radiation era scale factor $a(t) \sim 1/T$, with T the temperature
- Implies scaling of CPTV axion background B^0 with T :

$$B^0(T) \sim B(t_{\rm exit})\left(\frac{T}{T_{\rm exit}}\right)^3$$

- The suffix "exit" denotes quantities at the exit phase (end) of the RVM inflation

Situation faced in post-RVM-inflationary eras in our models

$$T_D \simeq m \sim 100 \text{ TeV}$$

Similar order of magnitude estimates if $B^0 \sim T^3$ during Leptogenesis era

Bossingham, NEM, Sarkar

CPTV Thermal

$$\mathcal{L} = i\overline{N}\partial\!\!\!/ N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}B\!\!\!/\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

If the anomaly condensate ceases to exist at the end of the RVM-inflationary phase EOM implies

$$\partial_\mu\left(\sqrt{-g}\,\partial^\mu b\right) = 0$$

- For FLRW universe the radiation era scale factor $a(t) \sim 1/T$, with T the temperature
- Implies scaling of CPTV axion background B^0 with T :

$$B^0(T) \sim B(t_{\rm exit})\left(\frac{T}{T_{\rm exit}}\right)^3$$

- The suffix "exit" denotes quantities at the exit phase (end) of the RVM inflation

$$B^0 \sim 1\,\text{MeV}$$

$$T_D \simeq m \sim 100\ \text{TeV}$$

Situation faced in post-RVM-inflationary eras in our models

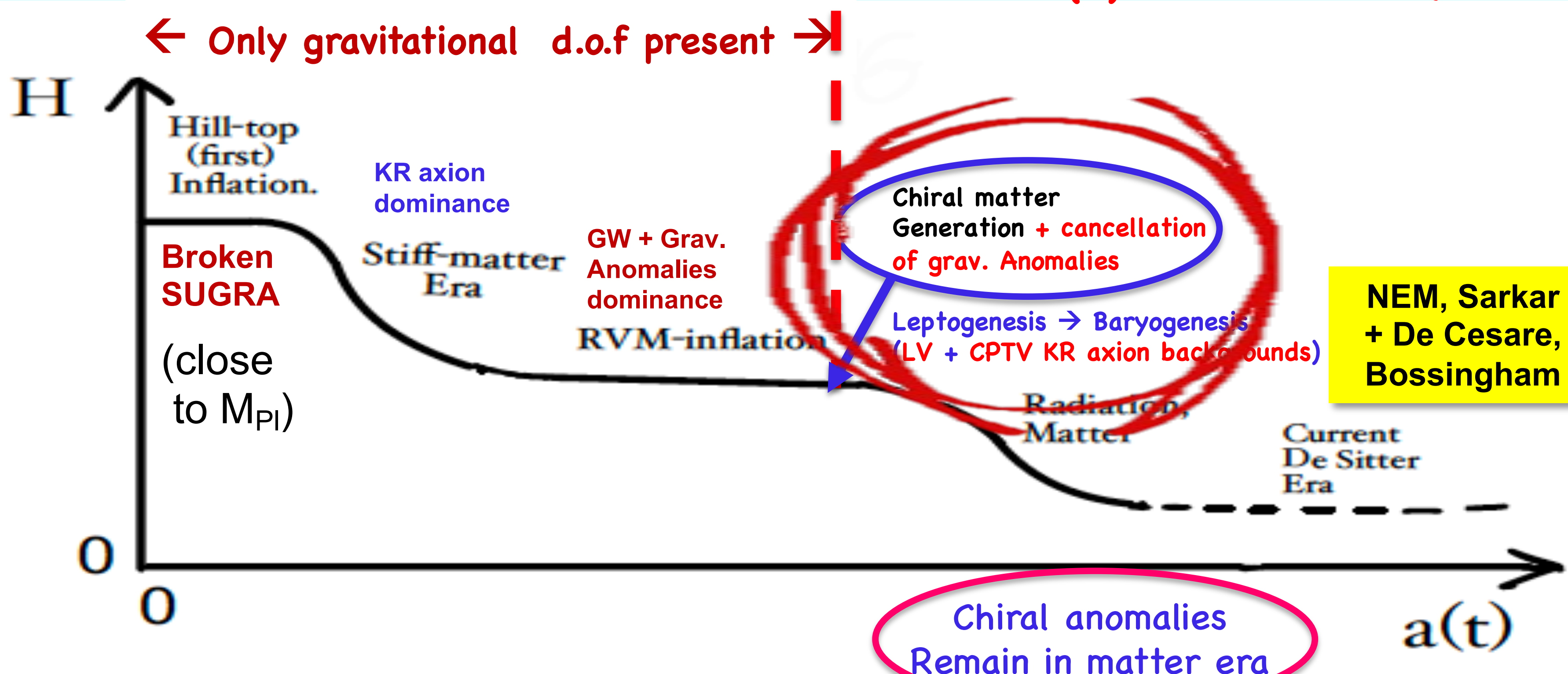
Similar order of magnitude estimates if $B^0 \sim T^3$ during Leptogenesis era

Bossingham, NEM, Sarkar

NB: The Cosmology of the Model @ a glance

$$\partial_\mu\Big[\sqrt{-g}\,\Big(\sqrt{\frac{3}{8}}\frac{\alpha'}{\kappa}\,J^{5\mu} - \frac{\alpha'}{\kappa}\sqrt{\frac{2}{3}}\,\frac{1}{96}\,\mathcal{K}^\mu\Big)\Big] = \sqrt{\frac{3}{8}}\frac{\alpha'}{\kappa}\,\Big(\frac{\alpha_{\rm EM}}{2\pi}\,\sqrt{-g}\,F^{\mu\nu}\,\widetilde{F}_{\mu\nu} + \frac{\alpha_s}{8\pi}\,\sqrt{-g}\,G^a_{\mu\nu}\,\widetilde{G}^{a\mu\nu}\Big)$$

chiral U(1) Gluon QCD



NEM, Sarkar + De Cesare, Bossingham

Chiral anomalies Remain in matter era

KR axion mass generation through QCD instantons (Dark Matter)

CPTV Thermal

$$\mathcal{L} = i\overline{N}\not{\partial}N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}\not{B}\gamma^5 N - Y_k \overline{L}_k \tilde{\phi} N + h.c.$$

If the anomaly condensate ceases to exist at the end of the RVM-inflationary phase EOM implies

$$\partial_\mu\left(\sqrt{-g}\,\partial^\mu b\right) = 0$$

- For FLRW universe the radiation era scale factor $a(t) \sim 1/T$, with T the temperature
- Implies scaling of CPTV axion background B^0 with T :

$$B^0(T) \sim B(t_{\rm exit})\left(\frac{T}{T_{\rm exit}}\right)^3$$

- The suffix "exit" denotes quantities at the exit phase (end) of the RVM inflation

Situation faced in post-RVM-inflationary eras in our models

$$B^0 \sim 1\,\mathrm{MeV}$$

$$T_D \simeq m \sim 100\ \mathrm{TeV}$$

Similar order of magnitude estimates if $B^0 \sim T^3$ during Leptogenesis era

Bossingham, NEM, Sarkar

CPTV Thermal

$$\mathcal{L} = i\overline{N}\partial\!\!\!/ N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}B\!\!\!/\gamma^5 N - Y_k \overline{L}_k \tilde{\phi} N + h.c.$$

If the anomaly condensate ceases to exist at the end of the RVM-inflationary phase EOM implies

$$\partial_\mu\left(\sqrt{-g}\,\partial^\mu b\right) = 0$$

- For FLRW universe the radiation era scale factor $a(t) \sim 1/T$, with T the temperature
- Implies scaling of CPTV axion background B^0 with T :

$$B^0(T) \sim B(t_{\rm exit})\left(\frac{T}{T_{\rm exit}}\right)^3$$

$T_{exit} \sim H/(2\pi)$ Gibbons-Hawking Temperature of de Sitter spacetime

- The suffix "exit" denotes quantities at the exit phase (end) of the RVM inflation

Situation faced in post-RVM-inflationary eras in our models

$B^0 \sim 1\,\text{MeV}$

$$T_D \simeq m \sim 10^4\ \text{TeV}$$

(Higgs mass stability OK !)

Similar order of magnitude estimates if $B^0 \sim T^3$ during Leptogenesis era

Bossingham, NEM, Sarkar

CPTV Thermal

$$\mathcal{L} = i\overline{N}\not{\partial}N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}\not{B}\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

Early Universe
$T > 10^5$ GeV

CPT Violation

(approx.) Constant $B^0 \neq 0$ background

Lepton number & CP Violations @ tree-level due to Lorentz/CPTV Background

$$N_I \rightarrow \overline{\phi}\,\ell\,,\ \phi\,\overline{\ell}$$

$$\frac{\Delta L}{n_\gamma} \simeq 10^{-10},$$

$$\frac{B_0}{m} \simeq 10^{-8}$$

Produce Lepton asymmetry

Equilibrated electroweak
B+L violating sphaleron interactions

B-L conserved

$$L = \frac{2}{M} l_L l_L \phi\phi + \text{H.c.}$$

where

$$l_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

Environmental Conditions Dependent

Observed Baryon Asymmetry In the Universe (BAU)

Fukugita, Yanagida,

$$\frac{n_B - \overline{n}_B}{n_B + \overline{n}_B} \sim \frac{n_B - \overline{n}_B}{s} = (8.4 - 8.9) \times 10^{-11}$$

T > 1 GeV

CPTV Thermal

$$\mathcal{L} = i\overline{N}\partial\!\!\!/ N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}B\!\!\!/\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

CPT Violation

Early Universe
$T > 10^5$ GeV

(approx.) Constant $B^0 \neq 0$ background

Lepton number & CP Violations @ tree-level due to Lorentz/CPTV Background

$$N_I \to \overline{\phi}\,\ell\,,\ \phi\,\overline{\ell}$$

$$\frac{\Delta L}{n_\gamma} \simeq 10^{-10}, \quad \frac{B_0}{m} \simeq 10^{-8}$$

Produce Lept... try

Equilibrated electroweak B+L violating sphaleron in...

B-L conserved

Envir...
Co... ...ndent

Kuzmin, Rubakov, Shaposhnikov
Gavela, Hernandez, Orloff, Pene, Quimbay

$$L = \frac{2}{M} l_L l_L \phi\phi + \text{H.c.}$$

where

$$l_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

Fukugita, Yanagida,

Observed Baryon Asymmetry In the Universe (BAU)

$$\frac{n_B - \overline{n}_B}{n_B + \overline{n}_B} \sim \frac{n_B - \overline{n}_B}{s} = (8.4 - 8.9) \times 10^{-11}$$

$T > 1$ GeV

Stability issues with CPTV Leptogenesis Vacuum

NEM & Sarkar (2023)

Existence of massive **right-handed neutrinos** (RHN) of mass $\boldsymbol{m_N}$

$$\mathcal{S}_{\rm int}^{\rm b-J^5} = -\frac{1}{f_b}\int d^x\,\sqrt{-g}\,\widetilde{b}(x)\,J^{5\mu}{}_{;\mu}$$

Non conserved chiral current due to RHN mass

$$\bar{N}\,\left(\not{p} - m_N - X[\tilde{b}]\right) N$$

Axion-RHN interactions

$$X\left[\tilde{b}\right] = W_0\left[\tilde{b}\right] + iW_1\left[\tilde{b}\right]\gamma^5 + V_\mu\left[\tilde{b}\right]\gamma^\mu + A_\mu\left[\tilde{b}\right]\gamma^\mu\gamma^5.$$

Our case: $W_1\left[\tilde{b}\right] = \dfrac{2m_N}{f_b}\tilde{b}$

S. Ellis,, Quevillon, Vuong, T. You, Zhang

EFFECTIVE FIELD THEORY (EFT) - integrating out heavy RHN

Validity of EFT: $f_b \geq m_N$

EFT Axion potential generated : up to, say, dim 6 operators

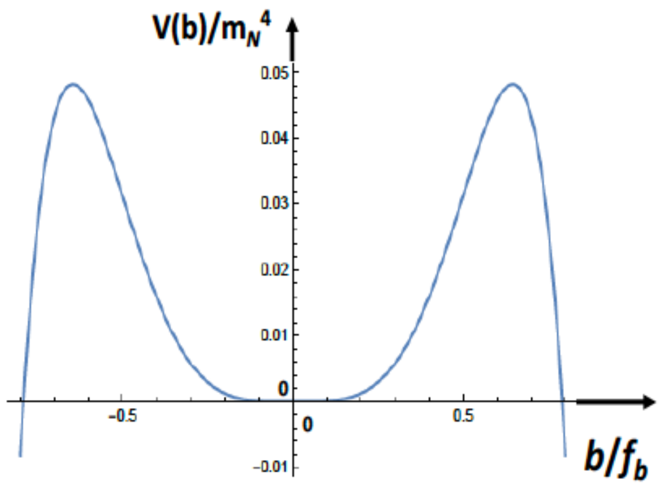
$$V_{\rm eff}[b] = a_2\left(W_1\left[\tilde{b}\right]\right)^2 + a_4\left(W_1\left[\tilde{b}\right]\right)^4 + a_6\left(W_1\left[\tilde{b}\right]\right)^6$$

$$a_2 = 4m_N^2\left(1 - \frac{1}{2}\ln\frac{m_N^2}{\mu^2}\right) \qquad a_4 = \frac{5}{6} - \ln\frac{m_N^2}{\mu^2} \qquad a_6 = -\frac{1}{3m_N^2}\,.$$

Runaway Instability?

$$W_1\left[\tilde{b}\right] = \frac{2m_N}{f_b}\tilde{b} \qquad \mu \lesssim m_N$$

RG scale



Real mass for axions: $\mu \gtrsim e^{-1}\, m_N$

Matching: $m_N \simeq \mu$

Allowed regime: $e^{-1} m_N \lesssim \mu \lesssim m_N\,.$

Coleman approach to calculating lifetime of false vacuum Using instanton (bounce) solutions in Euclidean formalism

$$\tau = T_U \min_{\mu} \mathcal{T}(\mu)\,, \quad \mathcal{T}(\mu) \sim T_U^{-4}\,\mu^{-4}\,\exp\left(\frac{8\pi^2}{3|\lambda_{\rm eff}(\eta,\mu)|}\right)$$

Lifetime of Universe

$$V_{\rm eff}[b] \simeq \frac{\lambda_{\rm eff}(b,\mu)}{4}\,b^4\,, \quad \lambda_{\rm eff}(b,\mu) \equiv \frac{2^6\,m_N^4}{f_b^4}\,\left[\frac{5}{6} - \ln(\frac{m_N^2}{\mu^2}) - \frac{4}{3\,f_b^2}\,b^2\right]$$

Do not include the mass terms

$$\frac{d\lambda_{\rm eff}}{d{\rm ln}\mu} = 2^7\,\frac{m_N^4}{f_b^4} > 0$$

Minimize τ w.r.t. *μ*

$$\frac{\tau(\mu=\mu_{\rm min})}{T_U} \sim 6.5\times 10^{-240}\left(\frac{M_{\rm Pl}}{m_N}\right)^4 e^{\frac{5\pi}{4\sqrt{3}}\,\frac{f_b^2}{m_N^2}}$$

$\mu = m_N$:

$$\frac{\tau}{T_U} \simeq 1.6\times 10^{-185}\,\exp\left[1.5\left(\frac{f_b}{m_N}\right)^4\right]$$

Vacuum metastable for $f_b \gg m_N$: $\tau \gg T_U$

Coleman approach to calculating lifetime of false vacuum Using instanton (bounce) solutions in Euclidean formalism

$$\tau = T_U \min_{\mu} \mathcal{T}(\mu)\,, \quad \mathcal{T}(\mu) \sim T_U^{-4}\,\mu^{-4}\,\exp\Big(\frac{8\pi^2}{3|\lambda_{\rm eff}(\eta,\mu)|}\Big)$$

Lifetime of Universe

$$V_{\rm eff}[b] \simeq \frac{\lambda_{\rm eff}(b,\mu)}{4}\, b^4$$

Do not include the mass terms

This is the case of the Leptogenesis model of de Cesare, NEM, Sarkar for large string mass scales $\kappa^2 \sim \alpha'$

$$f_b^{\rm str} = 96\sqrt{\frac{3}{2}}\,\frac{\kappa}{\alpha'} \simeq M_{Pl}$$

$\gg m_N \sim 10^{5-7}$ GeV (stability of Higgs mass)

Minimize τ w.r.t. μ $\quad \frac{\tau(\mu = \mu}{T_U}$

$\mu = m_N$:

$$\frac{\tau}{T_U} \simeq 1.6 \times 10^{-185} \exp\Big[1.5\Big(\frac{f_b}{m_N}\Big)^4\Big]$$

Vacuum metastable for $f_b \gg m_N$: $\quad \tau \gg T_U$

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

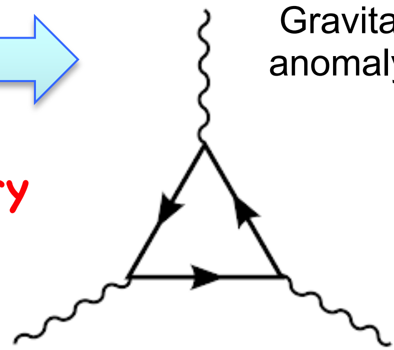
Big-Bang, pre-inflationary phase (broken Sugra)

Basilakos, NEM, Solà

forward direction

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves → Gravitational anomaly (GA) → Undiluted constant KR axial backround



$$B_\mu = M_{\rm Pl}^{-1}\,\dot{\overline{b}}\,\delta_{\mu 0}$$

$$\dot{\overline{b}} \sim \sqrt{2\varepsilon}\,M_{\rm Pl}\,H \sim 0.14\,M_{\rm Pl}\,H$$

From a pre-inflationary era after Big-Bang

chiral matter generation @ inflation exit

Cancellation of GA

Radiation Era

$$B_0 \propto T^3$$

Leptogenesis induced by RHN (tree-level) decays

$$N_I \;\rightarrow\; \overline{\phi}\,\ell\,,\;\; \phi\,\overline{\ell}$$

ΔL In the (approx.) constant LV + CPTV background $B_\mu = M_{\rm Pl}^{-1}\,\dot{\overline{b}}\,\delta_{\mu 0}$

B-L conserving sphelaron processes → Baryongenesis

Matter Era

Possible potential (mass) generation for b → axion Dark matter

Chiral anomalies @ QCD era (instantons)

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

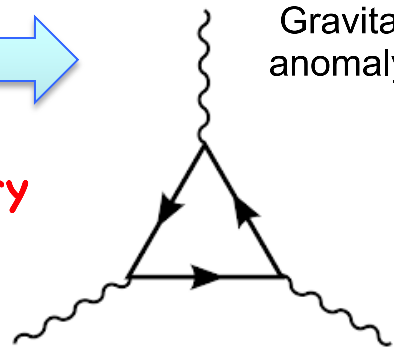
Big-Bang, pre-inflationary phase (broken Sugra)

Basilakos, NEM, Solà

forward direction

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves



Gravitational anomaly (GA)

Undiluted constant KR axial backround

$$B_\mu = M_{\rm Pl}^{-1}\,\dot{\bar{b}}\,\delta_{\mu 0}$$

$$\dot{\bar{b}} \sim \sqrt{2\varepsilon}\,M_{\rm Pl}\,H \sim 0.14\,M_{\rm Pl}\,H$$

chiral matter generation @ inflation exit

From a pre-inflationary era after Big-Bang

Cancellation of GA

Radiation Era

$$B_0 \propto T^3$$

Leptogenesis induced by RHN (tree-level) decays

$$N_I \;\rightarrow\; \overline{\phi}\,\ell\,,\;\; \phi\,\overline{\ell}$$

ΔL In the (approx.) constant LV + CPTV background $B_\mu = M_{\rm Pl}^{-1}\,\dot{\bar{b}}\,\delta_{\mu 0}$

B-L conserving sphelaron processes → Baryongenesis

Matter Era

Possible potential (mass) generation for b → axion Dark matter

Chiral anomalies @ QCD era (instantons)

Cosmic Time

forward direction

Summary of (stringy-RVM) Cosmological Evolution

Basilakos, NEM, Solà

Collider bound

10 TeV = O(10^{-14}) M_{Pl} < M_s ≤ M_{Pl}

$$V_b^{QCD} \simeq \Lambda_{QCD}^4 \Big(1 - \cos(\frac{b}{f_b})\Big)\,,$$

$$f_b = \sqrt{\frac{8}{3}}\, \frac{M_s^2}{M_{Pl}}$$

$$\Lambda_{QCD} \sim 218 \text{ MeV}$$

Remaining chiral anomalies

@ QCD Era

T~ 200 MeV

$$S_b^{eff} = \int d^4x \sqrt{-g}\Big[\frac{1}{2}\,\partial_\mu b\,\partial^\mu b - \frac{\alpha'}{\kappa}\,\sqrt{\frac{3}{8}}\,\frac{\alpha_s}{8\pi}\,b(x)\,G^a_{\mu\nu}\,\widetilde{G}^{a\mu\nu}\Big]$$

Instanton-effects-induced KR-axion potential and mass due to QCD chiral anomaly

Matter Era

Possible potential (mass) generation for b → axion Dark matter

$$2\times 10^{-11}\,\text{eV} < m_b = \frac{\Lambda_{QCD}^2}{f_b} < 2\times 10^{17}\,\text{eV}$$

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

Basilakos, NEM, Solà

forward direction

Collider bound

10 TeV = O(10^{-14}) $M_{Pl} < M_s \leq M_{Pl}$

$$V_b^{\rm QCD} \simeq \Lambda_{\rm QCD}^4\Big(1 - \cos(\frac{b}{f_b})\Big)\,,$$

$$f_b = \sqrt{\frac{8}{3}}\,\frac{M_s^2}{M_{\rm Pl}}$$

$$\Lambda_{\rm QCD} \sim 218 \;\; \rm MeV$$

Remaining chiral anomalies

@ QCD Era

T~ 200 MeV

$$S_b^{\rm eff} = \int d^4x\sqrt{-g}\Big[\frac{1}{2}\,\partial_\mu b\,\partial^\mu b - \frac{\alpha'}{\kappa}\,\sqrt{\frac{3}{8}}\,\frac{\alpha_s}{8\pi}\,b(x)\,G^a_{\mu\nu}\,\widetilde{G}^{a\mu\nu}\Big]$$

Instanton-effects-induced KR-axion potential and mass due to QCD chiral anomaly

Matter Era

Possible potential (mass) generation for b → axion Dark matter

$$2\times 10^{-11}\,{\rm eV} \; < m_b = \frac{\Lambda^2_{\rm QCD}}{f_b} \; < \; 2\times 10^{17}\,{\rm eV}$$

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

Basilakos, NEM, Solà

forward direction

Collider bound

10 TeV = O(10^{-14}) M_{Pl} < M_s ≤ M_{Pl}

$$V_b^{\rm QCD} \simeq \Lambda_{\rm QCD}^4 \Big(1 - \cos(\frac{b}{f_b})\Big) ,$$

$$f_b = \sqrt{\frac{8}{3}}\, \frac{M_s^2}{M_{\rm Pl}}$$

$$\Lambda_{\rm QCD} \sim 218 \ {\rm MeV}$$

Remaining chiral anomalies

@ QCD Era

T~ 200 MeV

$$S_b^{\rm eff} = \int d^4x \sqrt{-g}\Big[\frac{1}{2}\,\partial_\mu b\, \partial^\mu b - \frac{\alpha'}{\kappa}\, \sqrt{\frac{3}{8}}\, \frac{\alpha_s}{8\pi}\, b(x)\, G^a_{\mu\nu}\, \widetilde{G}^{a\mu\nu}\Big]$$

Mass upper bound restricted further (severely) by cosmological & other constraints

Matter Era

Possible poten

$$2 \times 10^{-11}\, {\rm eV} \ < m_b = \frac{\Lambda^2_{\rm QCD}}{f_b} < \ 2 \times 10^{17}\, {\rm eV}$$

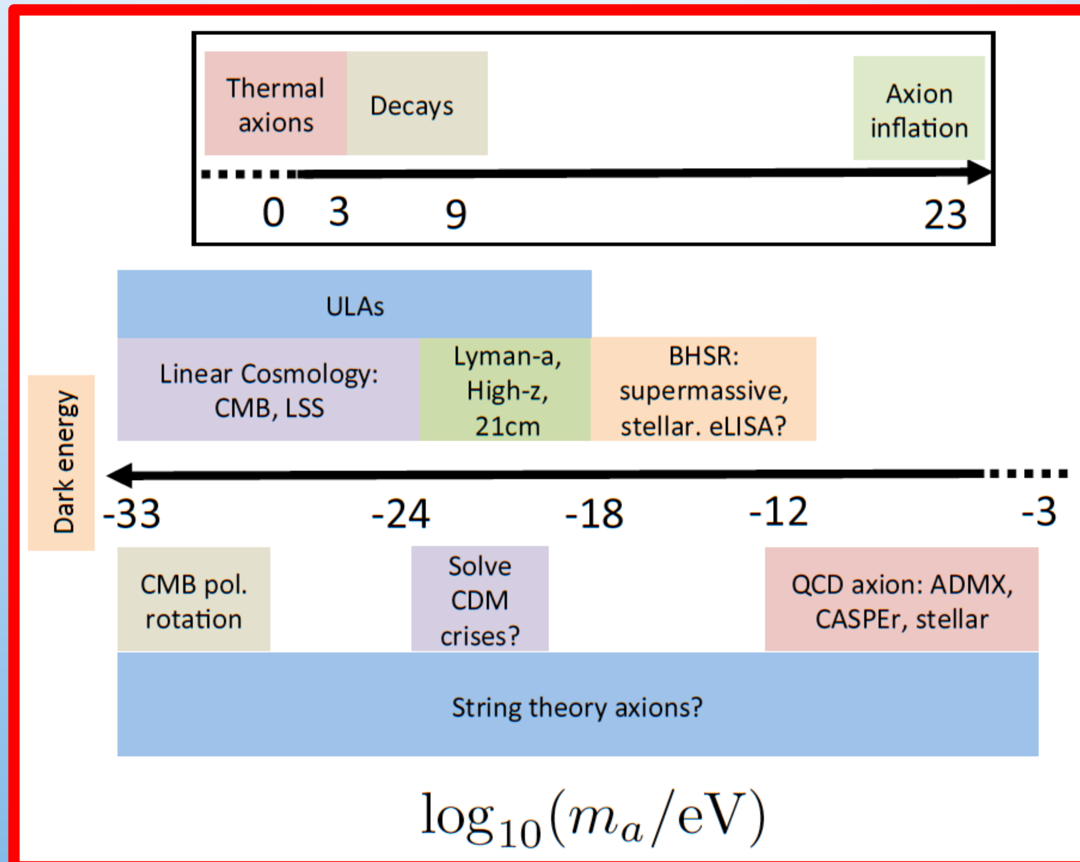
Back to Phenomenology

Axion Cosmology

D.J.E. Marsh,
Phys. Rept. 643, 1 (2016)
[arXiv:1510.07633 [astro-ph.CO]].

C. B. Adams *et al*.,
in Snowmass 2021 (2022),
arXive: 2203.14923

Cosmological Constraints & probes of axions



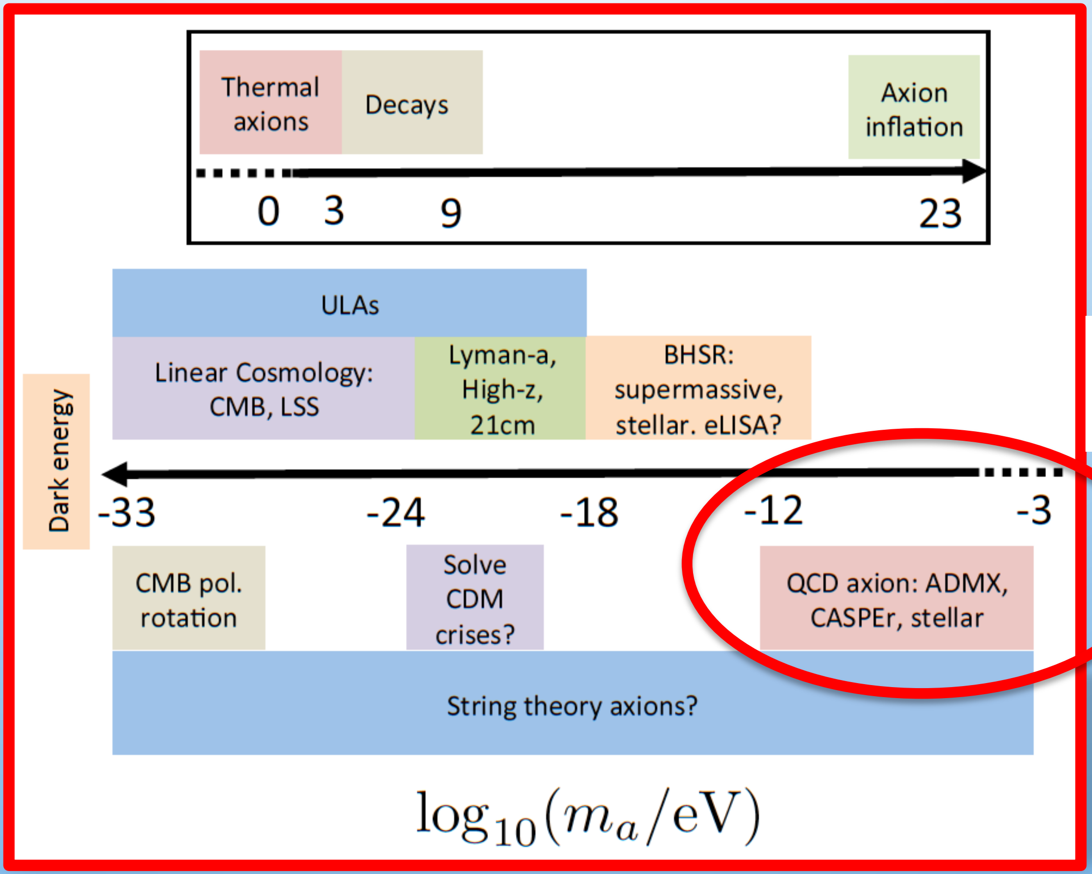
Back to Phenomenology

Axion Cosmology

D.J.E. Marsh,
Phys. Rept. 643, 1 (2016)
[arXiv:1510.07633 [astro-ph.CO]].

C. B. Adams *et al.*,
in Snowmass 2021 (2022),
arXive: 2203.14923

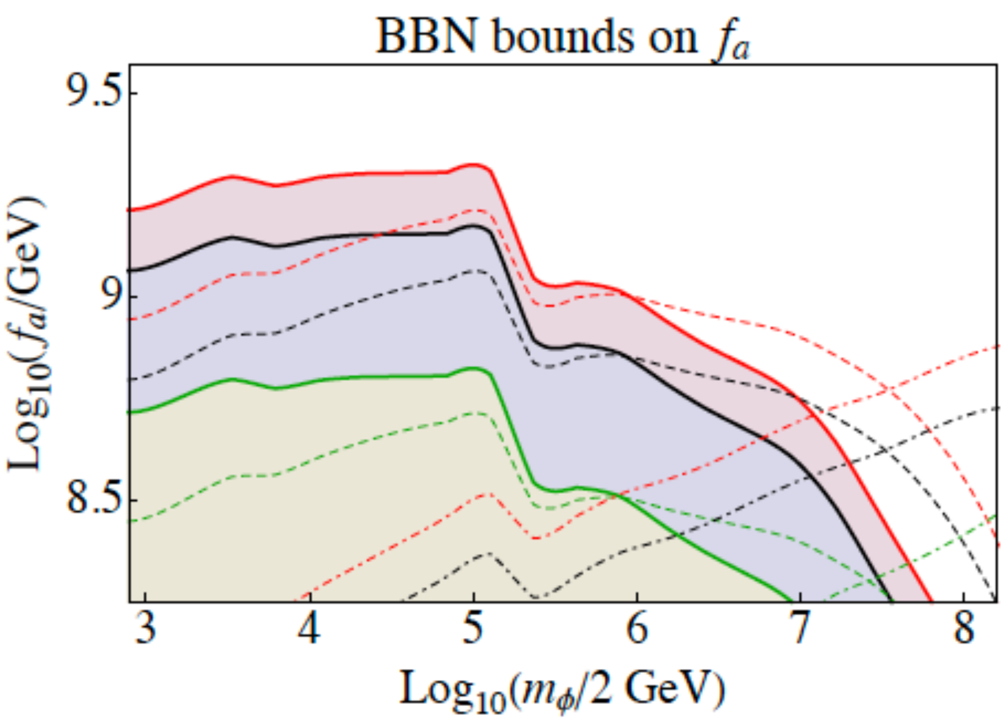
Cosmological Constraints & probes of axions



Stringy RVM KR b axion

BBN Constraints

$c_f = 1 \quad \mathcal{L}_f = c_f m_f \phi \bar{\psi} \gamma^5 \psi / f_a$



$\Delta N_{\text{eff}} = 0.1, 0.5, 1$ (green, black, red)

J. P. Conlon and M. C. D. Marsh, JHEP**10**, 214 (2013), 1304.1804.

BBN constraints rule out $f_a \leq 10^9$ GeV for a wide range of Masses m_ϕ

For KR axion coupling

$$f_b = \sqrt{\frac{8}{3}} \frac{M_s^2}{M_{\text{Pl}}} < 10^9\ \text{GeV}$$

Excludes
$m_b = \Lambda^2_{QCD} / f_b > 4 \times 10^{-2}$ eV

Allowed:
2×10^{-11} eV $< m_b <$ 0.04 eV

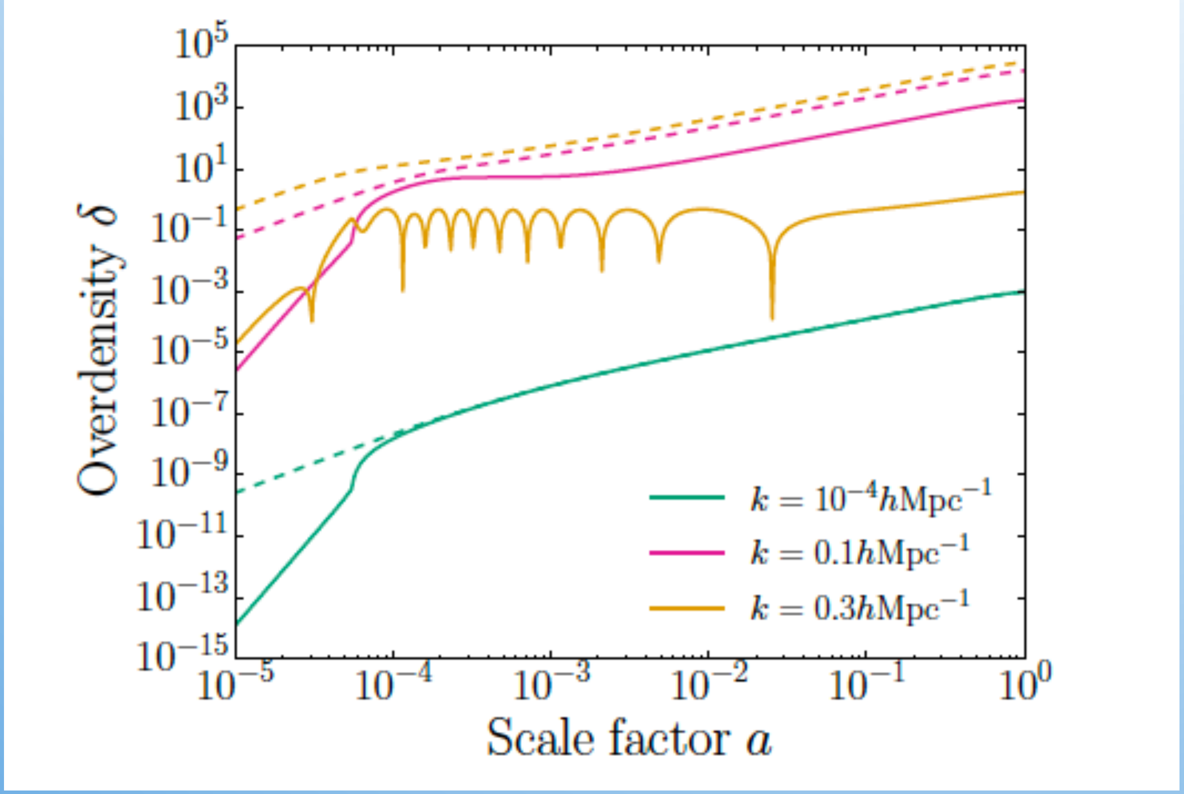
D.J.E. Marsh,
Phys. Rept. 643, 1 (2016) [arXiv:1510.07633 [astro-ph.CO]].

NB Ultra Light Axion (ULA) DM (allowed in string theory)

Compactification actions, NOT KR b in stringy RVM

D.J.E. Marsh, Phys. Rept. 643, 1 (2016) [arXiv:1510.07633 [astro-ph.CO]].

Contribution to galactic growth if dominant DM species

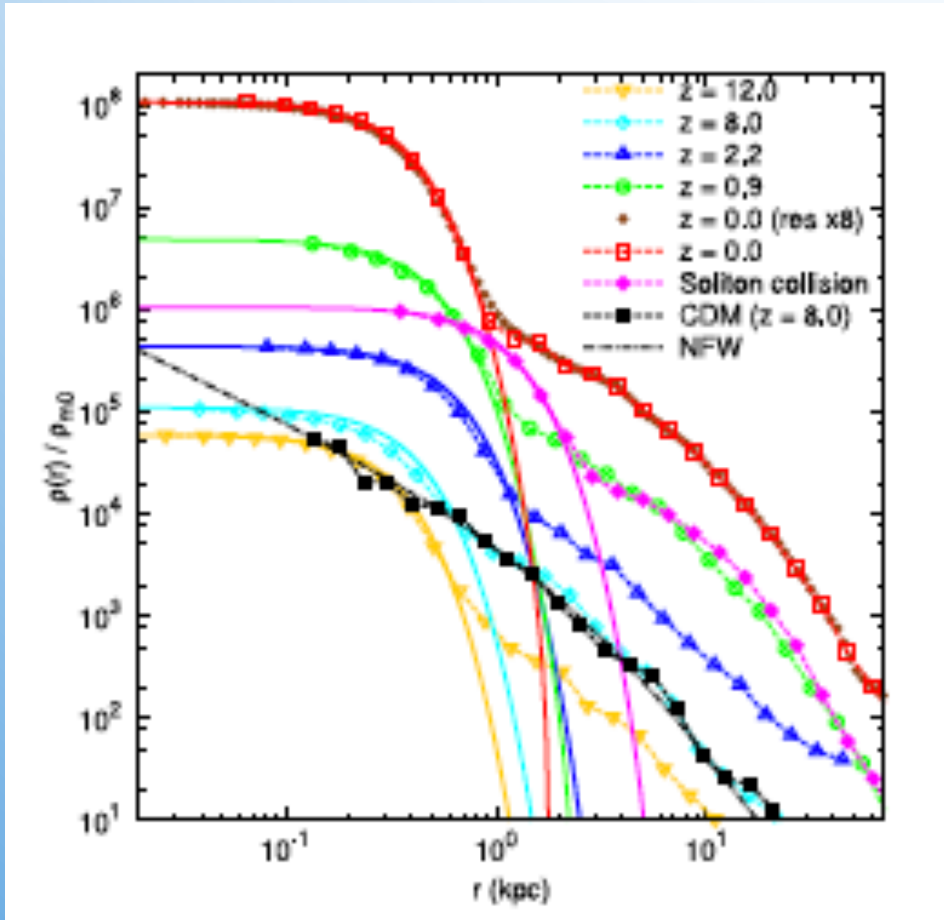


$$m_a = 10^{-26}\ \mathrm{eV}$$

R. Hlozek, D. Grin, D. J. E. Marsh, and P. G. Ferreira, Phys. Rev. D91, 103512 (2015), 1410.2896.

NB **Compactification actions, NOT KR b in stringy RVM**

Halo Density Profiles and ULA

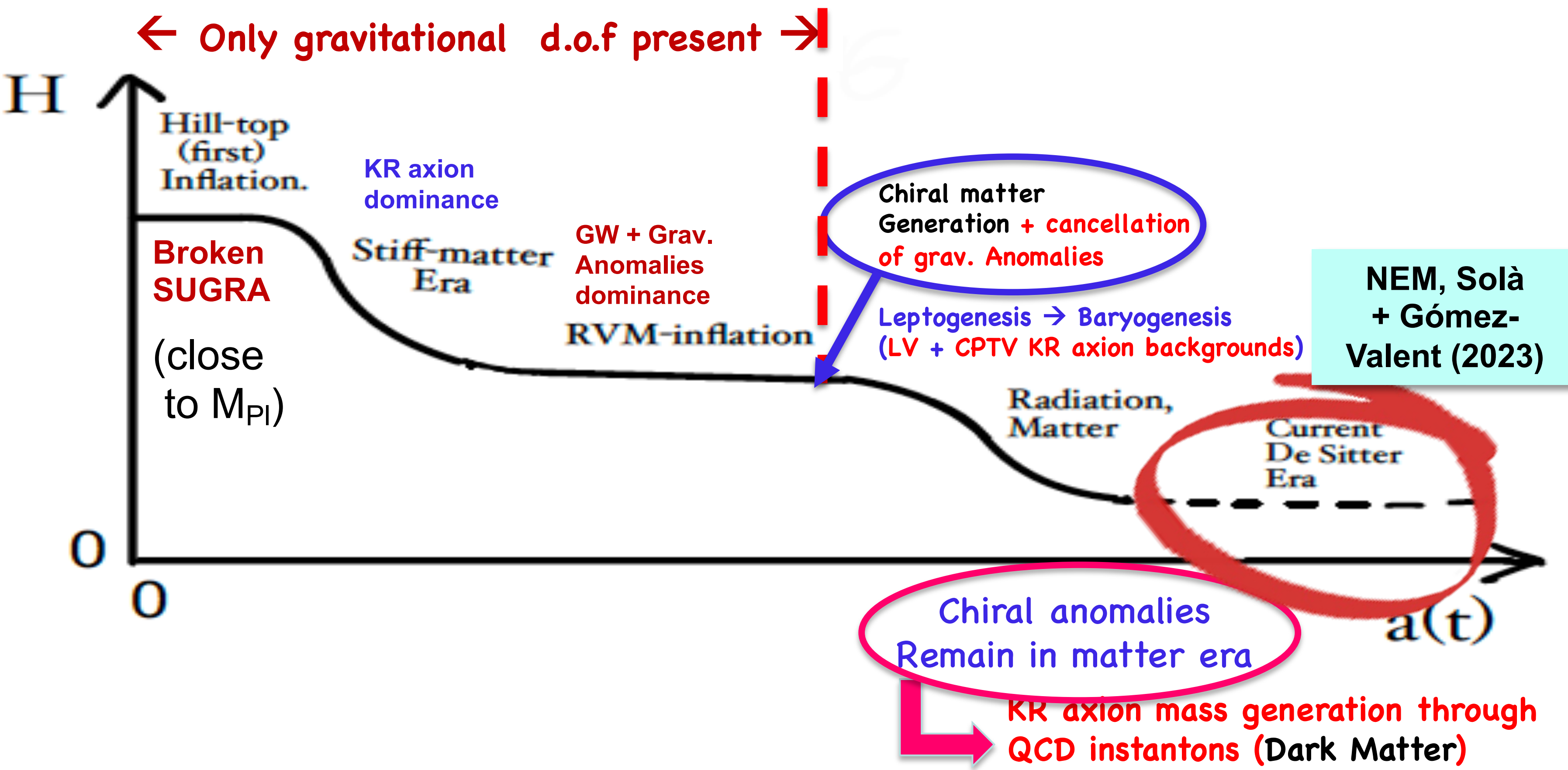


$$m_a = 8.1 \times 10^{-23} \text{ eV}$$

D.J.E. Marsh,
Phys. Rept. 643, 1 (2016) [arXiv:1510.07633 [astro-ph.CO]].

The Cosmology of the Model @ a glance

NEM,Solà
EPJ-ST
(2020)



5. Modern Era: Cosmological Tensions & stringy RVM

Cosmic Time

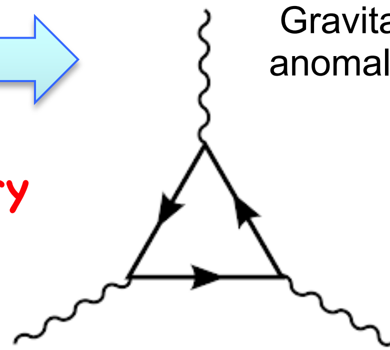
Summary of (stringy-RVM) Cosmological Evolution

Big-Bang, pre-inflationary phase (broken Sugra)

Basilakos, NEM, Solà

forward direction

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves →  Gravitational anomaly (GA) →

Undiluted constant KR axial backround

$$B_\mu = M_{\rm Pl}^{-1}\,\dot{\overline{b}}\,\delta_{\mu 0}$$

$$\dot{\overline{b}} \sim \sqrt{2\,\varepsilon}\,M_{\rm Pl}\,H \sim 0.14\,M_{\rm Pl}\,H$$

chiral matter generation @ inflation exit

From a pre-inflationary era after Big-Bang

Radiation Era

Cancellation of GA

$$B_0 \propto T^3$$

Leptogenesis induced by RHN (tree-level) decays

$$N_I \;\rightarrow\; \overline{\phi}\,\ell\,,\;\; \phi\,\overline{\ell}$$

B-L conserving sphelaron processes → Baryongenesis

Consistent with current bounds on LV & CPTV
$B_0 < 10^{-2}$ eV,
$B_i < 10^{-22}$ eV

Matter Era Possible potential (mass) generation f... b – → axion Dark matter

Modern de-Sitter Era GA resurfacing

$$\dot{b}_{\rm today} \sim \sqrt{2\,\varepsilon'}\,M_{\rm Pl}\,H_0$$

$H_0 \sim 10^{-42}$ GeV $\approx 10^{-60}\, M_{\rm Pl} \approx 10^{-33}$ eV

$$\varepsilon' \sim \varepsilon = \mathcal{O}(10^{-2})$$ Phenomenology

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

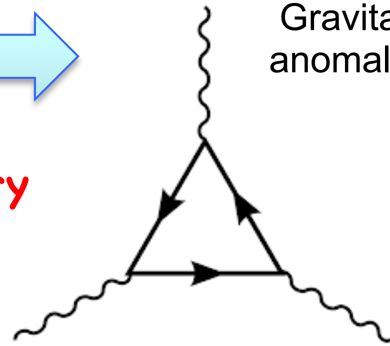
Big-Bang, pre-inflationary phase (broken Sugra)

Basilakos, NEM, Solà

forward direction

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves → Gravitational anomaly (GA) →



From a pre-inflationary era after Big-Bang

Undiluted constant KR axial backround

$$B_\mu = M_{\rm Pl}^{-1}\,\dot{\overline{b}}\,\delta_{\mu 0}$$

$$\dot{\overline{b}} \sim \sqrt{2\varepsilon}\,M_{\rm Pl}\,H \sim 0.14\,M_{\rm Pl}\,H$$

chiral matter generation @ inflation exit

Radiation Era

$$B_0 \propto \dot{\overline{b}} \propto T^3 + \text{subleading } (\sim T^2) \text{ chiral U(1) anomaly terms}$$

$$B_0\Big|_{\rm today} \sim 2.435 \times 10^{-34}\ {\rm eV}$$

Consistent with current bounds on LV & CPTV
$B_0 < 10^{-2}$ eV,
$B_i < 10^{-22}$ eV

Matter Era

Possible potential (mass) generation f… b – → axion Dark matter

Modern de-Sitter Era GA resurfacing

$$\dot{b}_{\rm today} \sim \sqrt{2\,\varepsilon'}\,M_{\rm Pl}\,H_0$$

$H_0 \sim 10^{-42}$ GeV $\approx 10^{-60}\,M_{\rm Pl} \approx 10^{-33}$ eV

$$\varepsilon' \sim \varepsilon = \mathcal{O}(10^{-2})$$ **Phenomenology**

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

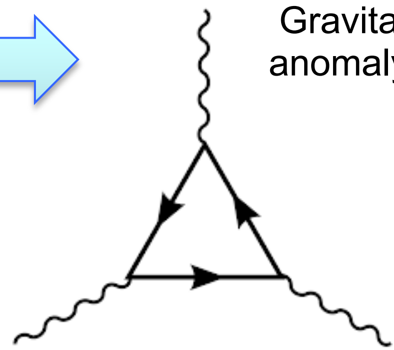
forward direction

Big-Bang, pre-inflationary phase (broken Sugra)

Basilakos, NEM, Solà

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves → Gravitational anomaly (GA) →



Undiluted constant KR axial backround

$$B_\mu = M_{\rm Pl}^{-1}\,\dot{\overline{b}}\,\delta_{\mu 0}$$

$$\dot{\overline{b}} \sim \sqrt{2\varepsilon}\,M_{\rm Pl}\,H \sim 0.14\,M_{\rm Pl}\,H$$

chiral matter generation @ inflation exit

Cancellation of GA

Radiation Era

$$B_0 \propto T^3$$

Leptogenesis induced by RHN (tree-level) decays

$$N_I \to \overline{\phi}\,\ell\,,\ \ \phi\,\overline{\ell}$$

B-L conserving sphe

Consistent with current bounds on LV & CPTV
$B_0 < 10^{-2}$ eV,
$B_i < 10^{-22}$ eV

Need to understand Modern Era better

Matter Era

P

Dark matter

Modern de-Sitter Era

GA resurfacing

$$\dot{b}_{\rm today} \sim \sqrt{2\,\varepsilon'}\,M_{\rm Pl}\,H_0$$

$$\varepsilon' \sim \varepsilon = \mathcal{O}(10^{-2})$$

$H_0 \sim 10^{-42}$ GeV
$\approx 10^{-60}\,M_{\rm Pl} \approx 10^{-33}$ eV

Phenomenology

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

Big-Bang, pre-inflationary phase (broken Sugra)

Basilakos, NEM, Solà

forward direction

RVM Inflationary (de Sitter) Phase

Primordial
Gr
Wa

Gravitational

Undiluted constant
KR axial backround

$$B_\mu = M_{\rm Pl}^{-1}\,\dot{\bar b}\,\delta_{\mu 0}$$

$$\dot{\bar b} \sim \sqrt{2\varepsilon}\, M_{\rm Pl} H \sim 0.14\, M_{\rm Pl} H$$

chiral matter
generation
[illegible]lation exit

Distinguishing feature from ΛCDM
Fit data & Alleviate data tensions

$$\rho_{\rm RVM}^{\rm today}(H) = 3M_{\rm Pl}^4\left(c_0 + \nu_0\left(\frac{H_0}{M_{\rm Pl}}\right)^2\right)$$

$$0 < \nu_0 = \mathcal{O}(10^{-3})$$

$$\frac{3}{\kappa^2}\,c_0 \simeq 10^{-122}\, M_{\rm Pl}^4$$

Gómez-Valent
Solà, Moreno-
Pulido, de Cruz-
Perez

Ra

B_0

Lep
RH

N_I

B-L

Ma

[illegible]xion [illegible] matter

$$\ldots \sqrt{2\varepsilon'}\, M_{\rm Pl} H_0$$

$$\varepsilon' \sim \varepsilon = \mathcal{O}(10^{-2})$$

Modern de-Sitter Era

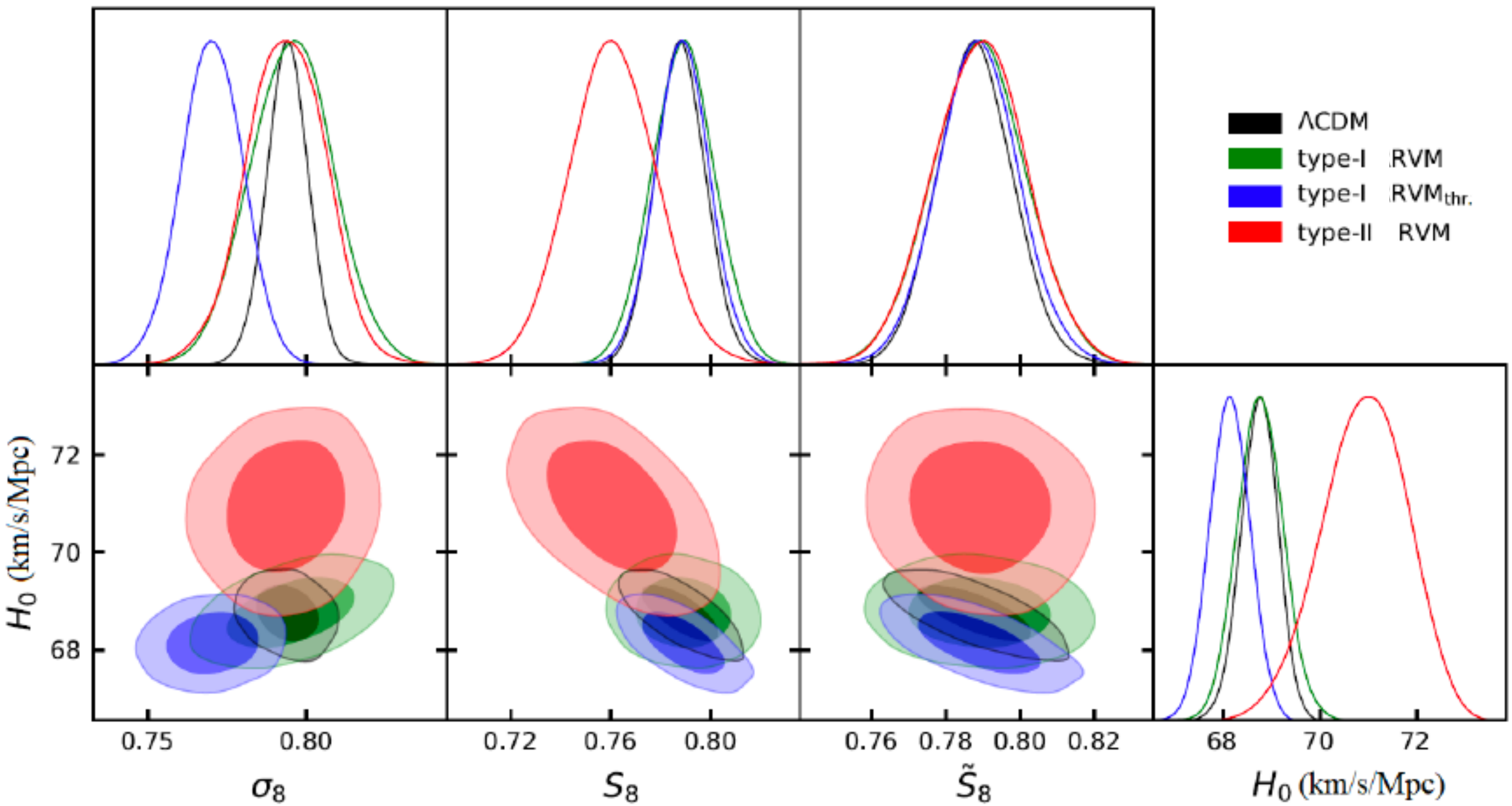
GA resurfacing

RVM-type
Running Dark Energy

If tensions are not due to statistics

Solà, Gómez-Valent, De Cruz Perez, Moreno-Pulido, (Planck 2018 data)

Alleviation of the H_0, σ_8 tension by RVM model

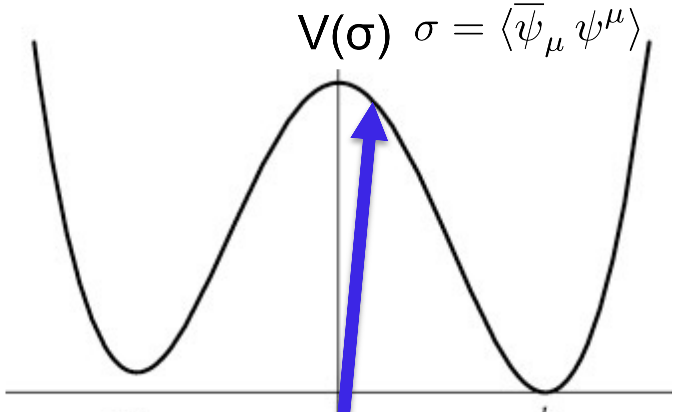


Stringy RVM

Integrating out graviton in (primordial) SUGRA

Gómez-Valent, NEM, Solà (2023)

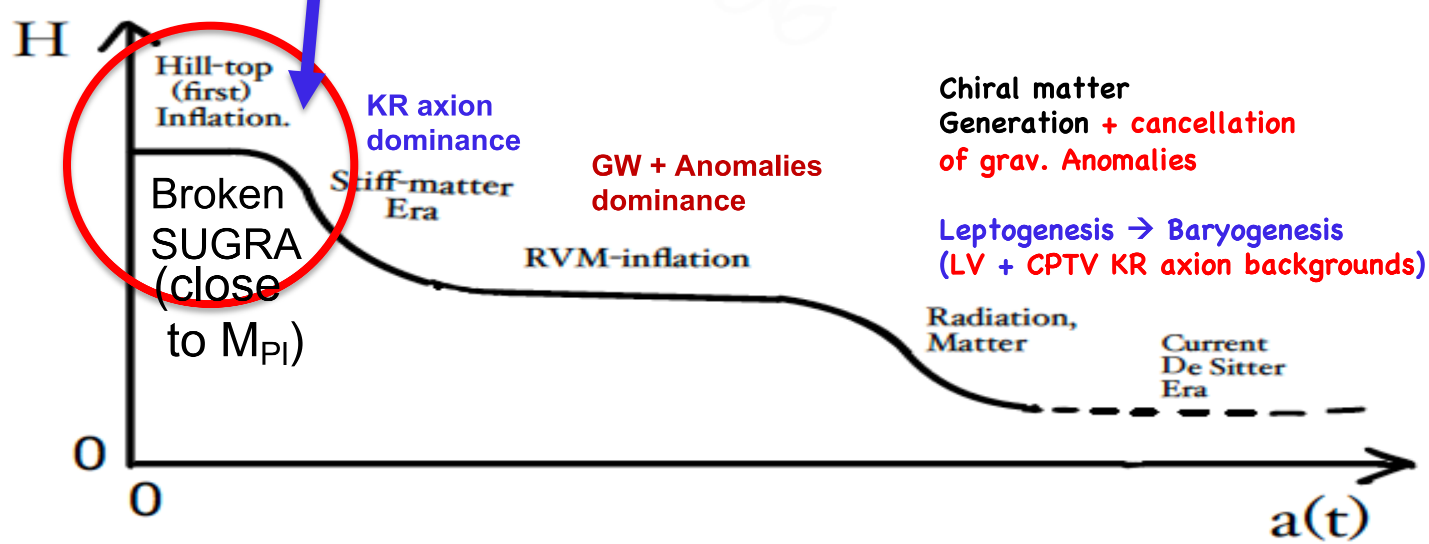
$$\rho \propto (c_1 + c_2 \ln H)\, H^2 + (c_3 + c_4 \ln H)\, H^4 + \Lambda$$



SUGRA broken dynamically by gravitino ψ_μ condensate stabilised →

RVM GW-induced Inflation

NEM, Solà EPJ-ST (2020)



NB:

N=1 SUGRA & QG effects

Alexandre Houston. NEM

$$\Gamma \simeq -\frac{1}{2\kappa^2}\int d^4x\sqrt{g}\left[\left(\widehat{R}-2\Lambda_1\right)+\alpha_1\,\widehat{R}+\alpha_2\,\widehat{R}^2\right]$$

Effective action Γ in the presence of cosmol. constant Λ > 0

$$\widehat{R}_{\lambda\mu\nu\rho}=\frac{\Lambda}{3}\left(\widehat{g}_{\lambda\nu}\,\widehat{g}_{\mu\rho}-\widehat{g}_{\lambda\rho}\,\widehat{g}_{\mu\nu}\right)\qquad \Lambda_1\equiv\Lambda_0-\kappa^{-2}\,\alpha_0\qquad \alpha_0=\alpha_0^B=\kappa^4\,\Lambda_0^2\left[0.027-0.018\ln\left(-\frac{3\Lambda_0}{2\,\mu^2}\right)\right]$$

$$\alpha_1=\frac{\kappa^2}{2}\left(\alpha_1^F+\alpha_1^B\right)\;,\qquad \alpha_2=\frac{\kappa^2}{8}\left(\alpha_2^F+\alpha_2^B\right)$$

F=integrating out gravitinos

B=integrating our gravitons **(QG)**

$$\alpha_1^F=0.067\,\tilde{\kappa}^2\sigma_c^2-0.021\,\tilde{\kappa}^2\sigma_c^2\ln\left(\frac{\Lambda}{\mu^2}\right)+0.073\,\tilde{\kappa}^2\sigma_c^2\ln\left(\frac{\tilde{\kappa}^2\sigma_c^2}{\mu^2}\right)\,,$$

$$\alpha_1^B=-0.083\Lambda_0+0.018\,\Lambda_0\ln\left(\frac{\Lambda}{3\mu^2}\right)+0.049\,\Lambda_0\ln\left(-\frac{3\Lambda_0}{\mu^2}\right)\,,$$

$$\alpha_2^F=0.029+0.014\ln\left(\frac{\tilde{\kappa}^2\sigma_c^2}{\mu^2}\right)-0.029\ln\left(\frac{\Lambda}{\mu^2}\right)\,,$$

$$\alpha_2^B=0.020+0.021\ln\left(\frac{\Lambda}{3\mu^2}\right)-0.014\ln\left(-\frac{6\Lambda_0}{\mu^2}\right)\,.$$

μ = RG scale

In cosmological setting we may replace $\Lambda \sim 3H_I^{\,2}$ for inflation or
More generally $\Lambda \sim 3\,H^2(t)$ for slowly time-varying H(t)

NB: N=1 SUGRA & QG effects

Alexandre Houston. NEM

Effective action Γ in the presence of cosmol. constant $\Lambda > 0$

$$\Gamma \simeq -\frac{1}{2\kappa^2}\int d^4x\sqrt{g}\left[\left(\widehat{R}-2\Lambda_1\right)+\alpha_1\,\widehat{R}+\alpha_2\,\widehat{R}^2\right]$$

$$\widehat{R}_{\lambda\mu\nu\rho}=\frac{\Lambda}{3}\left(\widehat{g}_{\lambda\nu}\,\widehat{g}_{\mu\rho}-\widehat{g}_{\lambda\rho}\,\widehat{g}_{\mu\nu}\right) \qquad \Lambda_1\equiv\Lambda_0-\kappa^{-2}\,\alpha_0 \qquad \alpha_0 = \alpha^B \ldots \ln\left(-\frac{3\Lambda_0}{2\,\mu^2}\right)\Big]$$

$$\alpha_1=\frac{\kappa^2}{2}\left(\alpha_1^F+\alpha_1^B\right)\;,\quad \alpha_2=\frac{\kappa^2}{8}\left(\alpha^F \ldots\right.$$

... gravitinos

... integrating our gravitons **(QG)**

$$\alpha_1^F = 0.067\,\tilde{\kappa}^2\sigma^2 \ldots$$

$$\ldots 0.014\ln\left(\frac{\tilde{\kappa}^2\sigma_c^2}{\mu^2}\right)-$$

$$-0.029\ln\left(\frac{\Lambda}{\mu^2}\right)\;,$$

$$\alpha_1^B=-0.083\Lambda_0+0.018\,\Lambda_0\ln\left(\frac{\Lambda}{3\mu^2}\right)+$$

$$0.049\,\Lambda_0\ln\left(-\frac{3\Lambda_0}{\mu^2}\right)\;,$$

$$\alpha_2^B=0.020+0.021\ln\left(\frac{\Lambda}{3\mu^2}\right)-$$

$$0.014\ln\left(-\frac{6\Lambda_0}{\mu^2}\right)\;.$$

μ = *RG scale*

$$\Gamma \propto \int \left(c_1+c_2\ln H\right)H^2+\left(c_3+c_4\ln H\right)H^4$$

In cosmological setting we may replace $\Lambda \sim 3H_I^{\,2}$ for inflation or
More generally $\Lambda \sim 3\,H^2(t)$ for slowly time-varying $H(t)$

Stringy RVM

Gómez-Valent, NEM, Solà (2023)

Integrating out graviton in SUGRA

$$\rho \propto (c_1 + c_2 \ln H)\, H^2 + (\times + \times \ln H)\, H^4 + \Lambda$$

Not dominant today

Demanding alleviation of tensions can **constrain supergravity model** in pre-RVM-inflationary phase of StRVM, assuming **ln*H*** originate from this

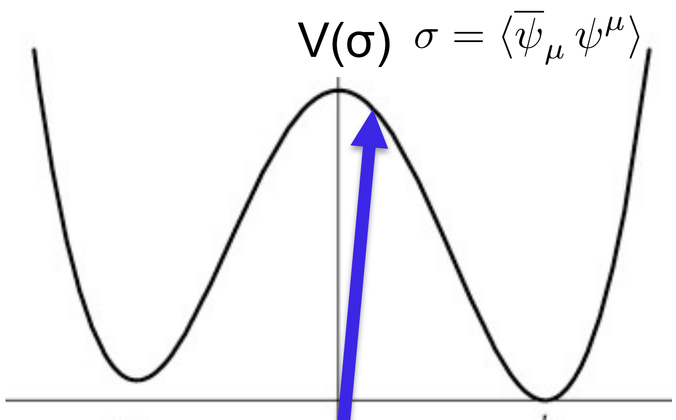
$$S = -\int d^4x \sqrt{-g} \left[c_0 + R \left(c_1 + c_2 \ln \left(\frac{R}{R_0} \right) \right) \right] + S_m$$

Stringy RVM

Integrating out graviton in (primordial) SUGRA

Gómez-Valent, NEM, Solà (2023)

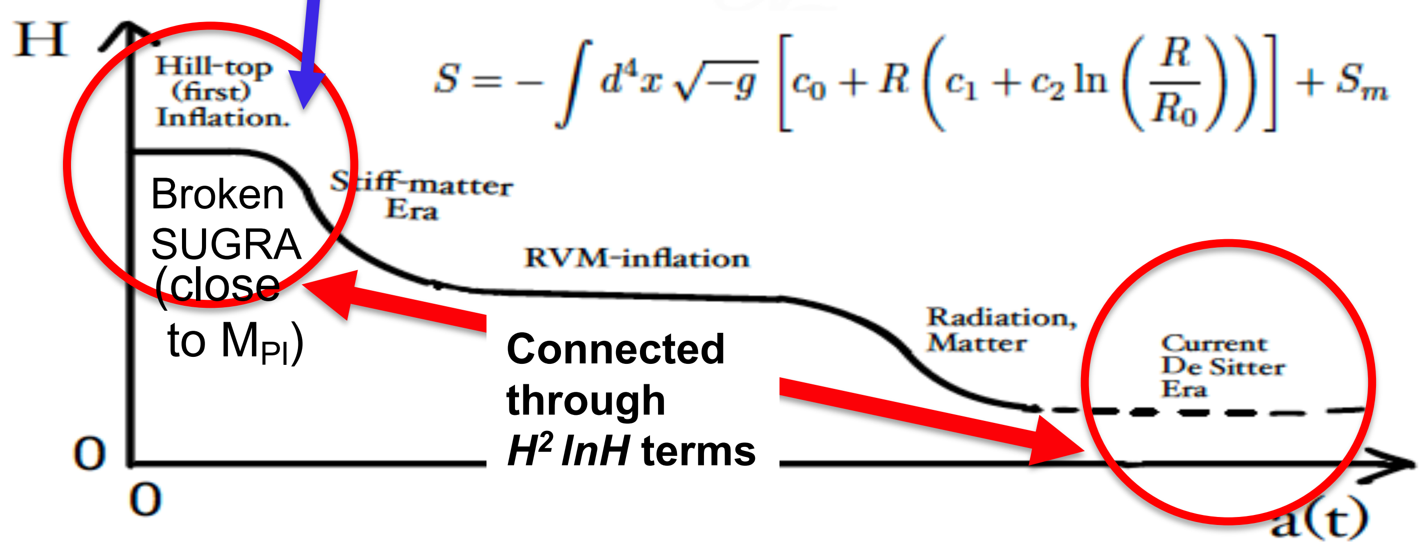
$$\rho \propto (c_1 + c_2 \ln H)\, H^2 + (c_3 + c_4 \ln H)\, H^4 + \Lambda$$



SUGRA broken dynamically by gravitino ψ_μ condensate stabilised →
RVM GW-induced Inflation

R =12 H^2

$$S = -\int d^4x \sqrt{-g}\left[c_0 + R\left(c_1 + c_2 \ln\left(\frac{R}{R_0}\right)\right)\right] + S_m$$



Stringy RVM

Gómez-Valent, NEM, Solà (2023)

Integrating out graviton in SUGRA

$$\rho \propto (c_1 + c_2 \ln H)\, H^2 + (\times + \times \ln H)\, H^4 + \Lambda$$

Not dominant today

Demanding alleviation of tensions can **constrain supergravity model** in pre-RVM-inflationary phase of StRVM, assuming $\ln H$ originate from this

$$S = -\int d^4x \sqrt{-g}\left[c_0 + R\left(c_1 + c_2 \ln\left(\frac{R}{R_0}\right)\right)\right] + S_m$$

Alleviation of H_0 & σ_{12} growth tensions

$$|\epsilon| \equiv \left|\frac{c_2}{c_1 + c_2}\right| \lesssim 10^{-7} \ll (aH/k^2)$$

CMB, large-scale structures OK

Primordial SUGRA model:

$$c_1 - c_2 \ln\left(\kappa^2 H_0^2\right) = \frac{1}{2\kappa^2}\left[1 + \frac{1}{2}\kappa^4 f^2 \left(0.083 - 0.049 \ln\left(3\kappa^4 f^2\right)\right)\right]$$

$$c_2 = -0.0045\, \kappa^2 f^2 < 0$$

f = scale of primordial **SUSY dynamical breaking**

$$\sqrt{|f|} \gtrsim 10^{-5/4}\, \kappa^{-1} \sim 10^{17}\ \mathrm{GeV}$$

Natural !!!

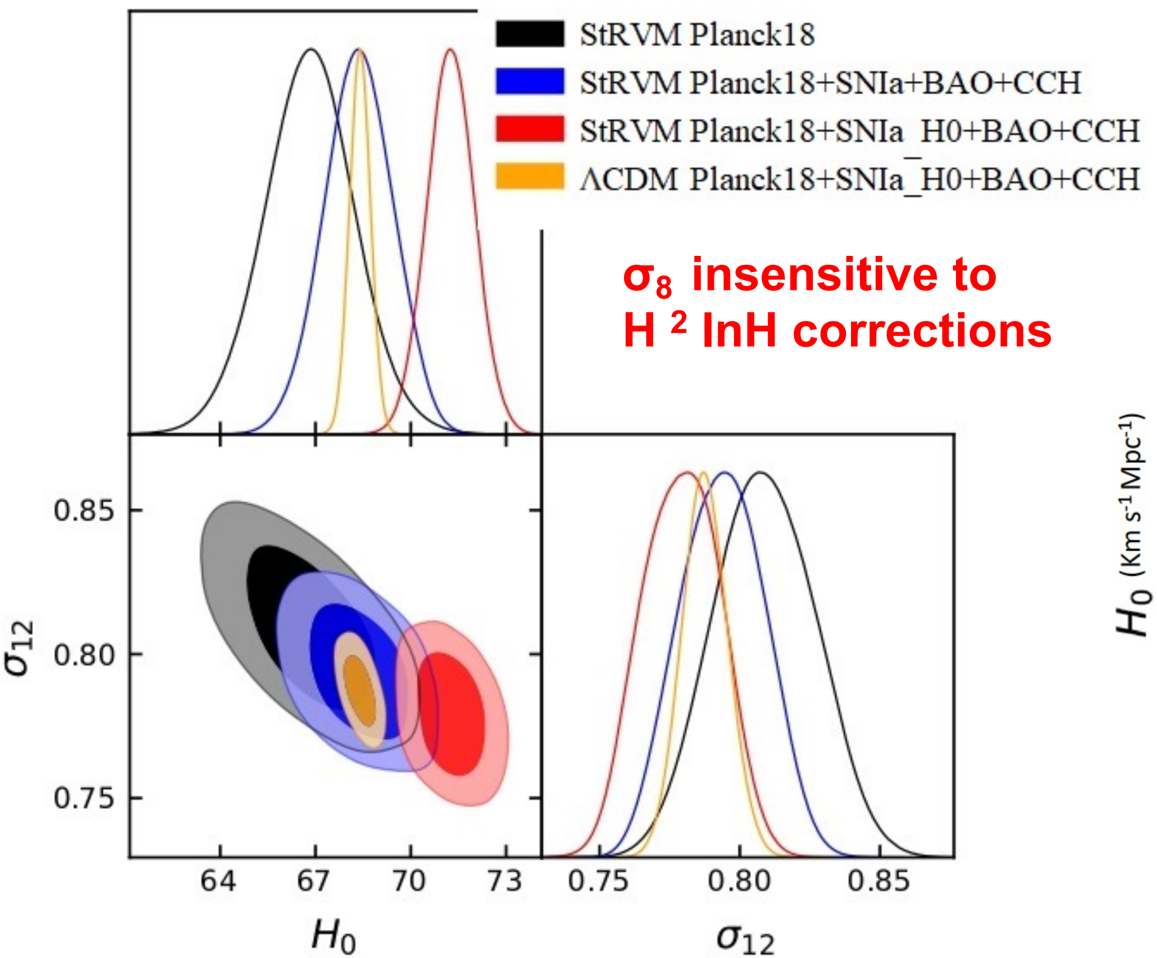
Stringy RVM

Gómez-Valent, NEM, Solà (2023)

Integrating out graviton in SUGRA

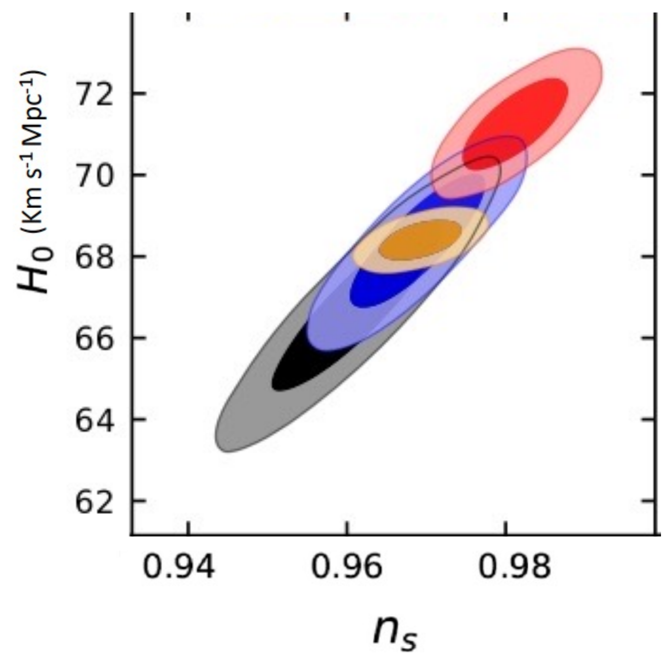
$$\rho \propto (c_1 + c_2 \ln H)\, H^2 + (\times + \times \ln H)\, H^4 + \Lambda$$

Not dominant today



σ_8 insensitive to $H^2 \ln H$ corrections

Alleviation of H_0 & σ_{12} growth tensions



NB:

Integrating out massive matter in QFT

Moreno Pulido, Solà, Cheraghchi (2020-23)

$$\rho \propto (c_1 + c_2 \ln H)\, H^2 + (\times + \times \ln H)\, H^4 + \Lambda$$

Not dominant today

Terms of the form
$(H^2 - H_0^2)\ln(H)$
also arise in ***QFT***
by integrating out
massive matter
(fermionic & bosonic)
fields

NB: suppressed
today compared to
SUGRA effects,
if latter present

Alleviation of H_0 & $\sigma_{12,\,8}$ growth tensions ?

6. Conclusions & Outlook

Cosmological (stringy RVM) Evolution: the Whole & its Parts

Cosmic Time

forward direction

Pre RVM-Inflationary era

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves

Gravitational anomaly (GA)

Undiluted constant KR axial backround

We exist because of Anomalies !

Paraphrasing C. Sagan: we are anomalously made of star stuff !

Leptogenesis induced by RHN (tree-level) decays

Spontaneous Lorentz and CPT Violation

Matter Era

axion Dark matter

Modern de-Sitter Era

RVM-type Running Dark Energy

Cosmological (stringy RVM) Evolution: the Whole & its Parts

Cosmic Time

forward direction

Pre RVM-Inflationary era

RVM Inflationary (de Sitter) Phase

Undiluted constant KR axial backround

Primordial Gravitational Waves

Gravitational anomaly (GA)

We exist because of Anomalies !

OUTLOOK: (i) Incorporate other model-dependent stringy axions → Axiverse
Interesting Cosmology (*eg* Marsh 2015)
could be ultralight → AION etc

Leptogenesis induced by RHN (tree-level) decays

Spontaneous

Matter Era

axion Dark matter

Modern de-Sitter Era

RVM-type Running Dark Energy

Cosmological (stringy RVM) Evolution: the Whole & its Parts

Cosmic Time

forward direction

Pre RVM-Inflationary era

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves

Gravitational anomaly (GA)

Undiluted constant KR axial backround

xist because
malies !

OUTLOOK: (ii) Look for imprints of the LV & CPTV KR axial background in CMB in early eras.

Leptogenesis induced by RHN (tree-level) decays

Spontaneous Lorentz and CPT Violation

Matter Era

axion Dark matter

Modern de-Sitter Era

RVM-type Running Dark Energy

Cosmological (stringy RVM) Evolution: the Whole & its Parts

Cosmic Time

forward direction

Pre RVM-Inflationary era

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves

Gravitational anomaly (GA)

Undiluted constant KR axial backround

OUTLOOK: (iii) Can we also get evidence of ν < 0 coefficient of H^2 during RVM inflation?

$$\rho_{\rm RVM}^{\rm string} \simeq 3\, M_{\rm Pl}^4 \Big[-1.7 \times 10^{-3} \Big(\frac{H}{M_{\rm Pl}}\Big)^2 + \mathcal{O}(10^7) \Big(\frac{H}{M_{\rm Pl}}\Big)^4 \Big]$$

Leptogenesis induced by RHN (tree-level) decays

Spontaneous Lorentz and CPT Violation

Matter Era

axion Dark matter

Modern de-Sitter Era

RVM-type Running Dark Energy

Cosmological (stringy RVM) Evolution: the Whole & its Parts

Cosmic Time

forward direction

Pre RVM-Inflationary era

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves

Gravitational anomaly (GA)

Undiluted constant KR axial backround

We exist because of Anomalies !

Leptogenesis induced by RHN (tree-level) decays

Spontaneous Lorentz and CPT Violation

OUTLOOK: (iv) Understand origin and nature of Modern de-Sitter era and its effects (vs cosmological data)

Matter Era

Modern de-Sitter Era

RVM-type Running Dark Energy

References:

Thank you!

a microscopic (string-inspired) model for RVM Universe....

Links with :
spontaneous Lorentz violation (via (gravitational axion) backgrounds)
and
Matter-Antimatter Asymmetry
in theories with
Right-Handed Neutrinos

Basilakos, NEM, Solà
(i) JCAP 12 (2019) 025
(ii) IJMD28 (2019) 1944002
(iii) Phys.Rev.D 101 (2020) 045001
(iv) Phys.Lett.B 803 (2020) 135342
(v) Universe 2020, 6(11), 218

NEM, Solà
(vi) EPJST 230 (2020), 2077
(vii) EPJPlus 136 (2021), 1152

NEM
(viii) 2205.07044 (Book ch. Springer)
(ix) Universe 7 (2021), 480
(x) Phil. Trans. A380 (2022) 2222

NEM, Spanos, Stamou,
(xi) Phys. Rev. D106 (2022), 063532

Gómez-Valent, NEM, Solà,
(xii) 2305.15774

(i) NEM & Sarben Sarkar, EPJC 73 (2013), 2359
(ii) John Ellis, NEM & Sarkar, PLB 725 (2013), 407
(iii) De Cesare, NEM & Sarkar, EPJC 75 (2015), 514
(iv) Bossingham, NEM & Sarkar, EPJC 78 (2018), 113; 79 (2019), 50
(v) NEM & Sarben Sarkar, EPJC 80 (2020), 558
(vi) NEM & Sarben Sarkar, 2306.02122 [hep-th]

SPARES

3(iii). Spontaneous Lorentz & CPT Violation by axion backgrounds and Running-Vacuum-Model Inflation without inflatons

Effective action contains **CP violating axion-like coupling**

$$\sqrt{-g}\,\mathcal{K}^{\mu}(\omega)_{;\mu}$$

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} \qquad '\Big) + \dots\Big]$$

(i) Assume de Sitter era, first, to discuss anomaly condensate in the presence of GW perturbation

(ii) deduce Running Vacuum Model (RVM) vacuum behaviour

and

(iii) Inflation is obtained self consistently from RVM evolution

Basilakos, Lima, Solà

Effective action contains **CP violating axion-like coupling**

$$\partial_\mu\left(\sqrt{-g}\,\mathcal{K}^\mu(\omega)\right)$$

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\qquad '\Big) + \dots\Big]$$

$$ds^2 = dt^2 - a^2(t)\Big[(1-h_+(t,z))\,dx^2 + (1+h_+(t,z))\,dy^2 + 2h_\times(t,z)\,dx\,dy + dz^2\Big]$$

Average over inflationary space time in the presence of **primordial Gravitational waves**

$$n_\star \equiv \frac{\mathcal{N}(t)}{\sqrt{-g}}$$ Proper density of sources

$b(x)=b(t)$

Alexander, Peskin, Sheikh -Jabbari

μ = UV k-momentum Cut-off

$$\frac{d}{dt}\left(\sqrt{-g}\,\mathcal{K}^0(t)\right) = \langle R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\rangle = \frac{16}{a^4}\,\kappa^2\, n^* \int^{\mu} \frac{d^3k}{(2\pi)^3}\,\frac{H^2}{2k^3}\,k^4\,\Theta + O(\Theta^3)$$

Homogeneity & Isotropy

$$\Theta = \sqrt{\frac{2}{3}}\,\frac{\kappa^3}{12}\,H\dot{b} \ll 1$$

H ≈ const. (inflation)

$$\kappa = M_{\rm Pl}^{-1},$$

$$\dot{b} \equiv db/dt$$

$$a(t) \sim e^{Ht}$$

Effective action contains **CP violating axion-like coupling**

$$\partial_\mu\left(\sqrt{-g}\,\mathcal{K}^\mu(\omega)\right)$$

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\left(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} \qquad '\right) + \dots\Big]$$

$$n_\star \equiv \frac{\mathcal{N}(t)}{\sqrt{-g}}$$ Proper density of sources

b(x)=b(t)

<….> of CS term calculable using **weak canonical quantum gravity** via creation and annihilation coefficients of **graviton modes**

non-zero only for chiral GW situations

$$\frac{d}{dt}\left(\sqrt{-g}\,\mathcal{K}^0(t)\right) = \langle R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\rangle = \frac{16}{a^4}\,\kappa^2\, n^* \int^{\mu} \frac{d^3k}{(2\pi)^3}\,\frac{H^2}{2k^3}\,k^4\,\Theta + O(\Theta^3)$$

Homogeneity & Isotropy

$$\Theta = \sqrt{\frac{2}{3}}\,\frac{\kappa^3}{12}\,H\dot{b} \ll 1$$

$$\kappa = M_{\rm Pl}^{-1},$$

$$\dot{b} \equiv db/dt$$

$$a(t) \sim e^{Ht}$$

H ≈ const. (inflation)

$$\widehat{h}_{ij}(\mathbf{x},\eta) = \frac{\sqrt{2}}{M_{\rm Pl}} \int \frac{d^3k}{(2\pi)^{3/2}} \sum_{p=L,R} \left(e^{i\mathbf{k}\cdot\mathbf{x}}\, \epsilon^{p}_{ij}(\mathbf{k})\, \widehat{h}_p(\mathbf{k},\eta) \right)$$

$$\widehat{h}_p(\mathbf{k},\eta) = h_p(\mathbf{k},\eta)\, \widehat{a}_p(\mathbf{k}) + h^{\star}_p(-\mathbf{k},\eta)\, \widehat{a}^{\dagger}_p(-\mathbf{k}),$$

$$\partial_\mu\left(\sqrt{-g}\,\mathcal{K}^\mu(\omega)\right)$$

$$n_\star \equiv \frac{\mathcal{N}(t)}{\sqrt{-g}}$$ Proper density of sources

b(x)=b(t)

<….> of CS term calculable using **weak canonical quantum gravity** via creation and annihilation coefficients of **graviton modes**

non-zero only for chiral GW situations

$$\frac{d}{dt}\left(\sqrt{-g}\,\mathcal{K}^0(t)\right) = \langle R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\rangle = \frac{16}{a^4}\,\kappa^2 n^* \int^{\mu} \frac{d^3k}{(2\pi)^3}\,\frac{H^2}{2k^3}\,k^4\,\Theta + O(\Theta^3)$$

$$\langle 0|R_{\mu\nu\rho\sigma}\,\widehat{{}^*R^{\mu\nu\rho\sigma}}|0\rangle = \frac{16}{a(\eta)^4 M_{\rm Pl}^2} \int \frac{d^3k}{(2\pi)^3}\Big[k^2 h^{\star}_L(k,\eta)\, h'_L(k,\eta) - k^2 h^{\star}_R(k,\eta \ldots k,\eta)$$
$$- h^{\star\prime}_L(k,\eta)\, h''_L(k,\eta) + h^{\star\prime}_R(k,\eta)\, h''_R(k,\eta)\Big], \quad (42)$$

Solutions (backgrounds) to the Eqs of Motion

$\alpha' = M_s^{-2}$

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^\alpha(t)\right)\right] = 0 \quad \Rightarrow \quad \dot{\bar{b}} = \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^0$$

$n_\star \equiv \frac{\mathcal{N}(t)}{\sqrt{-g}}$ Proper density of sources

$$\frac{d}{dt}\left(\sqrt{-g}\,\mathcal{K}^0(t)\right) = \langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle = \frac{16}{a^4}\kappa^2 n^\star \int^\mu \frac{d^3k}{(2\pi)^3}\frac{H^2}{2k^3}k^4\Theta + O(\Theta^3)$$

$$\Theta = \sqrt{\frac{2}{3}}\frac{\kappa^3}{12}H\dot{b} \propto \mathcal{K}^0$$

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\Big[\sqrt{-g}\Big(\partial^\alpha\bar{b}-\sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^\alpha(t)\Big)\Big]=0 \quad\Rightarrow\quad \dot{\bar{b}}=\sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}<\mathcal{K}^0>\sim \text{constant}$$

$$\frac{d}{dt}\Big(\sqrt{-g}\,\mathcal{K}^0(t)\Big)=\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle=\frac{16}{a^4}\kappa^2 n^*\int^{\mu}\frac{d^3k}{(2\pi)^3}\frac{H^2}{2k^3}k^4\Theta+O(\Theta^3)$$

$$\Theta=\sqrt{\frac{2}{3}}\frac{\kappa^3}{12}H\dot{b}\propto\mathcal{K}^0$$

time evolution of Anomaly

μ = UV k-momentum Cut-off

$$\mathcal{K}^0(t)\simeq\mathcal{K}^0_{\rm begin}(0)\exp\Big[-3Ht\Big(1-3\times10^{-4}\,n_\star\Big(\frac{H}{M_{\rm Pl}}\Big)^2\Big(\frac{\mu}{M_s}\Big)^4\Big)\Big]$$

$$n_\star^{1/4}\frac{\mu}{M_s}\sim 7.6\times\Big(\frac{M_{\rm Pl}}{H}\Big)^{1/2}$$

$$\Rightarrow <\mathcal{K}^0>=\text{const.}$$

Planck Data $H/M_{\rm Pl}<10^{-4}$ $\Rightarrow$ $n_\star\gtrsim 3.3\times10^{13}$

$\mu/M_s=1$

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^\alpha(t)\right)\right] = 0 \quad \Rightarrow \quad \dot{\bar{b}} = \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}<\mathcal{K}^0> \sim \text{constant}$$

$$\dot{\bar{b}} \propto \epsilon^{ijk} H_{ijk} = \text{constant}$$

$$\frac{d}{dt}\left(\sqrt{-g}\,\mathcal{K}^0(t)\right) = \langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle = \frac{16}{a^4}\kappa^2 n^* \int^\mu \frac{d^3k}{(2\pi)^3}\frac{H^2}{2k^3}k^4\Theta + O(\Theta^3)$$

time evolution of Anomaly

$$\Theta = \sqrt{\frac{2}{3}}\frac{\kappa^3}{12}H\dot{b} \propto \mathcal{K}^0$$

μ = UV k-momentum Cut-off

$$\mathcal{K}^0(t) \simeq \mathcal{K}^0_{\rm begin}(0)\exp\left[-3Ht\left(1 - 3\times 10^{-4}\, n_\star \left(\frac{H}{M_{\rm Pl}}\right)^2\left(\frac{\mu}{M_s}\right)^4\right)\right]$$

$$n_\star^{1/4}\frac{\mu}{M_s} \sim 7.6\times\left(\frac{M_{\rm Pl}}{H}\right)^{1/2} \quad \Rightarrow \quad <\mathcal{K}^0> = \text{const.}$$

Spontaneous LV (+ CPTV) solution

Planck Data $\quad H/M_{\rm Pl} < 10^{-4} \quad \Rightarrow \quad n_\star \gtrsim 3.3\times 10^{13}$

$\mu / M_s = 1$

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^\alpha(t)\right)\right] = 0 \quad \Rightarrow \quad \dot{\bar{b}} = \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}<\mathcal{K}^0> \sim \text{constant}$$

$$\dot{\bar{b}} \sim \varepsilon_{ijk} H^{ijk}$$

≈ constant torsion

Using **slow-roll assumption** *b*

$$\varepsilon = \frac{1}{2}\frac{1}{(HM_{\rm Pl})^2}\dot{\bar{b}}^2 \sim 10^{-2}$$

Planck Data

$$\dot{\bar{b}} \sim \sqrt{2\varepsilon}\, M_{\rm Pl}\, H \sim 0.14\, M_{\rm Pl}\, H$$

$$H = H_{\rm infl} \simeq \text{const.}$$

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^\alpha(t)\right)\right] = 0 \quad \Rightarrow \quad \dot{\bar{b}} = \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}<\mathcal{K}^0> \sim \text{constant}$$

Using **slow-roll assumption** *b*

$$\varepsilon = \frac{1}{2}\frac{1}{(HM_{\rm Pl})^2}\dot{\bar{b}}^2 \sim 10^{-2}$$

Planck Data

$$\dot{\bar{b}} \sim \sqrt{2\,\varepsilon}\,M_{\rm Pl}\,H \sim 0.14\,M_{\rm Pl}\,H$$

$$H = H_{\rm infl} \simeq \text{const.}$$

@ end of Inflationary era

$$b_{\rm end} \sim b_{\rm initial} + 0.14\,M_{\rm Pl}\,H_{\rm infl}\,t_{\rm end},$$

$$t_{\rm end}H_{\rm infl} \sim \mathcal{N} = \text{e} - \text{foldings}$$

~ 55-70

Fix $b_{initial}$ to arrange approx. constant condensate during appropriate time period (inflation)

Gravitational Anomaly Condensates → Dynamical Inflation

Basilakos, NEM, Solà

$$\Lambda \equiv \langle b(x)\, R_{\mu\mu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} \rangle \simeq 5.86 \times 10^7 \, \epsilon \mathcal{N} \, H^4 > 0$$

e-foldings

Positive Cosmological Constant-like

Positive total energy density since Λ-term dominates

$$\rho_{\rm total} = \rho_b + \rho_{gCS} + \rho_\Lambda \simeq 3M_{\rm Pl}^4 \Big[-1.7 \times 10^{-3} \Big(\frac{H}{M_{\rm Pl}}\Big)^2 + \Big(1.17 - 1.37\Big) \times 10^7 \, (\frac{H}{M_{\rm Pl}})^4 \Big] > 0$$

Gravitational Anomaly Condensates → Dynamical Inflation

Basilakos, NEM, Solà

$$\Lambda \equiv \langle b(x)\, R_{\mu\mu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} \rangle \simeq 5.86 \times 10^7\, \epsilon \mathcal{N}\, H^4 > 0$$

Positive
Cosmological
Constant-like

Positive total energy density since Λ-term dominates

$$\rho_{\rm total} = \rho_b + \rho_{gCS} + \rho_\Lambda \simeq 3M_{\rm Pl}^4 \Big[-1.7 \times 10^{-3} \Big(\frac{H}{M_{\rm Pl}}\Big)^2 + \Big(1.17 - 1.37\Big) \times 10^7 \,(\frac{H}{M_{\rm Pl}})^4 \Big] > 0$$

Dark Energy
("running vacuum model (RVM) type")
cf. talk by Solà

Gravitational Anomaly Condensates → Dynamical Inflation

Basilakos, NEM, Solà

$$\Lambda \equiv \langle b(x)\, R_{\mu\mu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma}\rangle \simeq 5.86 \times 10^7\, \epsilon \mathcal{N}\, H^4 > 0$$

Positive
Cosmological
Constant-like

Positive total energy density since Λ-term dominates

$$\rho_{\rm total} = \rho_b + \rho_{gCS} + \rho_\Lambda \simeq 3M_{\rm Pl}^4 \Big[-1.7 \times 10^{-3} \Big(\frac{H}{M_{\rm Pl}}\Big)^2 + \Big(1.17 - 1.37\Big) \times 10^7 \, (\frac{H}{M_{\rm Pl}})^4 \Big] > 0$$

Self-consistent derivation of early inflation from RVM evolution

$$\dot{H} + \frac{3}{2}(1+\omega)H^2 \left(1 - \nu - \frac{c_0}{H^2} - \alpha \frac{H^2}{H_I^2}\right) = 0$$

Basilakos, Lima, Solà, Perico

Solution

$$H(a) = \left(\frac{1-\nu}{\alpha}\right)^{1/2} \frac{H_I}{\sqrt{D\, a^{3(1-\nu)(1+\omega_m)} + 1}} \qquad D > 0$$

Early de Sitter (unstable) $Da^{4(1-\nu)} \ll 1$ ⇒ $H^2 = (1-\nu)H_I^2/\alpha$

Gravitational Anomaly Condensates → Dynamical Inflation

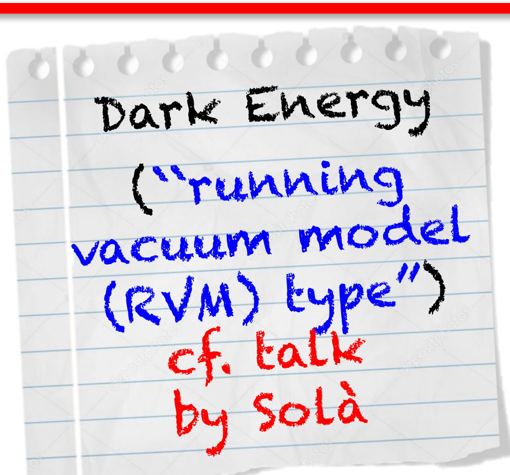
NEM, Solà

$$\Lambda \equiv \langle b(x)\, R_{\mu\mu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} \rangle \simeq 5.86 \times 10^7\, \epsilon \mathcal{N}\, H^4 > 0$$

Positive
Cosmological
Constant-like

Positive total energy density since Λ-term dominates

$$\rho_{\rm total} = \rho_b + \rho_{gCS} + \rho_\Lambda \simeq 3M_{\rm Pl}^4 \Big[-1.7 \times 10^{-3} \Big(\frac{H}{M_{\rm Pl}}\Big)^2 + \Big(1.17 - 1.37\Big) \times 10^7 \,(\frac{H}{M_{\rm Pl}})^4 \Big] > 0$$



Equation of state :

$0 > \rho_b + \rho_{gCS} = -\,(p_b + p_{gCS})$ cf. phantom "matter"

$0 < \rho_\Lambda = -p_\Lambda$ → dominates →

$0 < \rho_b + \rho_{gCS} + \rho_\Lambda = -\,(p_b + p_{gCS} + p_\Lambda)$ true RVM vacuum

Gravitational Anomaly Condensates → Dynamical Inflation

Basilakos, NEM, Solà

$$\Lambda \equiv \langle b(x)\, R_{\mu\mu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} \rangle \simeq 5.86 \times 10^{7}\, \epsilon \mathcal{N}\, H^4 > 0$$

Positive Cosmological Constant-like

Positive total energy density since Λ-term dominates

$$\rho_{\rm total} = \rho_b + \rho_{gCS} + \rho_\Lambda \simeq 3M_{\rm Pl}^4 \Big[-1.7 \times 10^{-3} \Big(\frac{H}{M_{\rm Pl}}\Big)^2 + \Big(1.17 - 1.37\Big) \times 10^{7} \Big(\frac{H}{M_{\rm Pl}}\Big)^4 \Big] > 0$$

Dark Energy
("running vacuum model (RVM) type")
cf. talk by Solà

RVM-like terms drive inflation contain scalar d.o.f. from the anomaly condensate

But slow roll is due to the KR axion field $\epsilon \sim \frac{1}{2}\frac{1}{(H M_{\rm Pl})^2}\dot{\bar{b}}^2 \sim 10^{-2}$

Gravitational Anomaly Condensates → Dynamical Inflation

Cannot obtain such H^4 terms in **ordinary Quantum Field Theories by integrating out matter fields**

There you obtain H^6 and higher…

NEM Solà

Moreno Pulido, Solà + Cheraghchi

Positive total energy density since Λ-term dominates

$$\rho_{\rm total} = \rho_b + \rho_{gCS} + \rho_\Lambda \simeq 3M_{\rm Pl}^4 \Big[-1.7 \times 10^{-3} \Big(\frac{H}{M_{\rm Pl}}\Big)^2 + \Big(1.17 - 1.37\Big) \times 10^7 \Big(\frac{H}{M_{\rm Pl}}\Big)^4 \Big] > 0$$

Dark Energy ("running vacuum model (RVM) type") cf. talk by Solà

RVM-like terms drive inflation contain scalar d.o.f. from the anomaly condensate

But slow roll is due to the KR axion field $\epsilon \sim \frac{1}{2}\frac{1}{(H M_{\rm Pl})^2}\dot{b}^2 \sim 10^{-2}$

Gravitational Anomaly Condensates → Dynamical Inflation

NEM Solà

Cannot obtain such H^4 terms
in **ordinary Quantum Field Theories**
You need the **condensate of the gravitational anomalies**
which have **CP-violating couplings**
with the **gravitational axions**

Another important role of CP-violation in Early Universe

$$\rho_{\rm total} = \rho_b + \rho_{gCS} + \rho_\Lambda \simeq 3M_{\rm Pl}^4 \Big[-1.7\times 10^{-3}\Big(\frac{H}{M_{\rm Pl}}\Big)^2 + \Big(1.17 - 1.37\Big)\times 10^7\,\big(\frac{H}{M_{\rm Pl}}\big)^4\Big] > 0$$

RVM-like terms drive inflation contain scalar d.o.f. from the anomaly condensate

Dark Energy ("running vacuum model (RVM) type") cf. talk by Solà

But slow roll is due to the KR axion field $\epsilon \sim \frac{1}{2}\frac{1}{(HM_{\rm Pl})^2}\dot{b}^2 \sim 10^{-2}$

Gravitational Anomaly Condensates → Dynamical Inflation

Basilakos, NEM, Solà

$$\Lambda \equiv \langle b(x)\, R_{\mu\mu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} \rangle \simeq 5.86 \times 10^7\, \epsilon \mathcal{N}\, H^4 > 0$$

Positive Cosmological Constant-like

Positive total energy density since Λ-term dominates

$$\rho_{\rm total} = \rho_b + \rho_{gCS} + \rho_\Lambda \simeq 3M_{\rm Pl}^4 \left[-1.7 \times 10^{-3} \Big(\frac{H}{M_{\rm Pl}}\Big)^2 + \Big(1.17 - 1.37\Big) \times 10^7 \Big(\frac{H}{M_{\rm Pl}}\Big)^4 \right] > 0$$

Negative coefficient ν < 0
due to CS anomaly
in early Universe, unlike
late-era RVM

RVM-like terms drive inflation contain scalar d.o.f. from the anomaly condensate

But slow roll is due to the KR axion field $\epsilon \sim \frac{1}{2} \frac{1}{(H M_{\rm Pl})^2} \dot{b}^2 \sim 10^{-2}$

Cosmological Evolution of RVM

Basilakos, Lima, Solà + Gomez Valent + ... (2013 - 2018)

$\omega = \rho_m / p_m$ *m* = matter, radiation

$$\nabla^\mu T_{\mu\nu} = 0 \quad \Rightarrow \quad \dot{\rho}_m + 3(1+\omega)H\rho_m = -\dot{\rho}^{\Lambda}_{\rm RVM}$$

$$\dot{H} + \frac{3}{2}(1+\omega)H^2\left(1 - \nu - \frac{c_0}{H^2} - \alpha\frac{H^2}{H_I^2}\right) = 0$$

Solution

$$H(a) = \left(\frac{1-\nu}{\alpha}\right)^{1/2} \frac{H_I}{\sqrt{D\, a^{3(1-\nu)(1+\omega_m)} + 1}}$$

$D > 0$

Early de Sitter (unstable) $\quad Da^{4(1-\nu)} \ll 1 \quad \Rightarrow \quad H^2 = (1-\nu)H_I^2/\alpha$

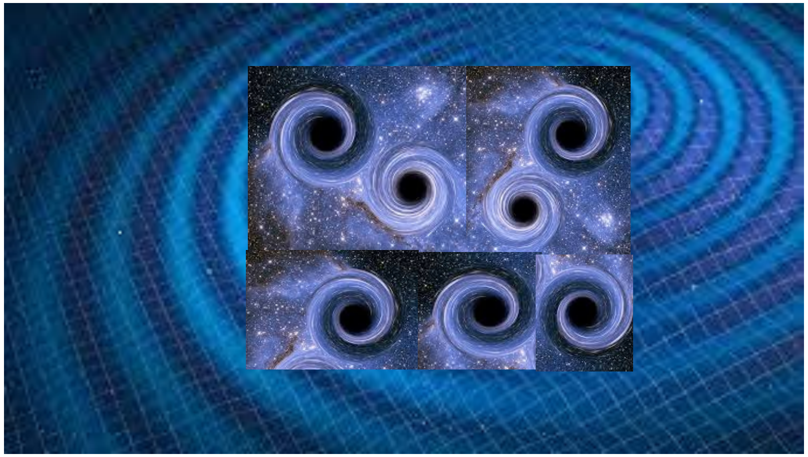
Radiation $\quad Da^{4(1-\nu)} \gg 1 \quad \Rightarrow \quad H^2 \sim a^{3(1-\nu)(1+\omega_m)} \sim a^{-4}$

$\omega = 1/3$

Late dark-Energy dominated era $\quad H^2(a) = H_0^2\left[\tilde{\Omega}_{m0}\, a^{-3(1-\nu)} + \tilde{\Omega}_{\Lambda 0}\right]$ $\tilde{\Omega}_{\Lambda 0}$ dominant

The combined important role of string-model independent and Compactification axions

3(iv). Enhanced cosmic perturbations and densities of primordial black holes during RVM inflation and Gravitational Wave profiles



World-Sheet Instantons, Axion Monodromy like potentials & deviations from scale invariance

NEM, Universe 7 (2021) 12, 480, e-Print: 2111.05675 [hep-th]

Anomaly condensate → linear axion potential

$$V_{\text{eff}} \ni \langle R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} \rangle\, b(x)$$

approximately de Sitter provided we have, during the duration of inflation:

$$b(t) = \overline{b}(0) + 0.14 M_{\rm Pl}\, H\, t_{end} \simeq \overline{b}(0)$$

order of magnitude

< 0

N=e-folds

beginning of inflation

$$|\overline{b}(0)| \gtrsim \mathcal{O}(10)\, M_{\rm Pl}$$

Distance-swampland conjectures?

World-Sheet Instantons, Axion Monodromy like potentials & deviations from scale invariance

NEM, Universe 7 (2021) 12, 480, e-Print: 2111.05675 [hep-th]

Anomaly condensate → linear axion potential

$$V_{\rm eff} \ni \langle R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma}\rangle\, b(x)$$

$$V(b) \simeq b\, \widetilde{\Lambda}_0^4 \sqrt{\frac{2}{3}}\, \frac{M_{\rm Pl}}{96\, M_s^2} \equiv b\, \frac{\widetilde{\Lambda}_0^4}{f_b} \equiv b\, \Lambda_0^3$$

Such a potential can **also** arise in **appropriate brane compactifications** (eg type IIB strings)

L. McAllister, E. Silverstein and A. Westphal, Phys. Rev. D 82 (2010), 046003 [arXiv:0808.0706 [hep-th]].

World-Sheet Instantons, Axion Monodromy like potentials & deviations from scale invariance

NEM, Universe 7 (2021) 12, 480, e-Print: 2111.05675 [hep-th]

Anomaly condensate → linear axion potential

$$V_{\rm eff} \ni \langle R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma}\rangle\, b(x)$$

$$V(b) \simeq b\, \widetilde{\Lambda}_0^4 \sqrt{\frac{2}{3}}\, \frac{M_{\rm Pl}}{96\, M_s^2} \equiv b\, \frac{\widetilde{\Lambda}_0^4}{f_b} \equiv b\, \Lambda_0^3$$

Such a potential can **also** arise in **appropriate brane compactifications** (eg type IIB strings)

L. McAllister, E. Silverstein and A. Westphal, Phys. Rev. D 82 (2010), 046003 [arXiv:0808.0706 [hep-th]].

We may extend the model to include other **stringy axions** arising from **compactification**

$$V^{\rm lin}_{a_I} = a_I(x)\, \frac{f_b}{f_a}\, \Lambda_0^3$$

$$\Lambda_0 = 8.4 \times 10^{-4}\, M_{\rm Pl}$$

f_a = *axion coupling*

canonical kinetic terms for a-axions

$$f_b \equiv \left(\sqrt{\frac{2}{3}}\, \frac{M_{\rm Pl}}{96\, M_s^2}\right)^{-1}$$

World-Sheet Instantons, Axion Monodromy like potentials & deviations from scale invariance

NEM, Universe 7 (2021) 12, 480, e-Print: 2111.05675 [hep-th]

NEM, Spanos, Stamou PRD106 (2022), 063532

Anomaly condensate → linear axion potential

$$V_{\rm eff} \ni \langle R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma}\rangle\, b(x)$$

world-sheet (non-perturbative) instantons → periodic potential perturbations

$$V^{b}_{\rm wsinst} \simeq \Lambda_b^4 \cos\Big(\frac{b}{f_b}\Big) \qquad \Lambda_b^4 \sim M_s^4\, e^{-S_{\rm wsinst}} \quad \rightarrow \quad \Lambda_b \ll \Lambda_0\,.$$

$$V^{a_I}_{\rm wsinst} \simeq \Lambda_I^4 \cos\Big(\frac{a_I}{f_{a_I}}\Big) \qquad \Lambda_0 \gg \Lambda_I \ne \Lambda_b,$$

Restrict to $I = 1:\ a_1 \equiv a$

$$V_{brane-compact.-effects}(a) \ni \Lambda_2^4\, \frac{1}{f_a}\, a + \Lambda_1^4 \Big(1 + \xi_a\, \frac{a}{f_a}\Big) \cos\Big(\frac{a}{f_a}\Big)$$

warp factor

$$\frac{\Lambda_2^4}{f_a} \sim \frac{\epsilon}{L} \sqrt{\frac{3}{(2\pi)^3}}\, M_s^3$$

L. McAllister, E. Silverstein and A. Westphal, Phys. Rev. D 82 (2010), 046003 [arXiv:0808.0706 [hep-th]].

World-Sheet Instantons, Axion Monodromy like potentials & deviations from scale invariance

Anomaly condensate → linear axion potential $V_{\rm eff} \ni \langle R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} \rangle\, b(x)$

world-sheet (non-perturbative) instantons → periodic potential perturbations

$$V(a,\,b) = {\Lambda_1}^4 \left(1 + f_a^{-1}\, \tilde{\xi}_1\, a(x)\right) \cos({f_a}^{-1} a(x)) + \frac{1}{f_a}\left(f_b\, {\Lambda_0}^3 + \Lambda_2^4\right) a(x) + {\Lambda_0}^3\, b(x)$$

Case I

$$\left(\frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a\, \Lambda_0^3}\right)^{1/3} \Lambda_0 < \Lambda_1 \ll \Lambda_0$$

NEM, Solà + Basilakos

NEM, Spanos, Stamou
PRD106 (2022), 063532

Case II

$$\Lambda_0 \ll \left(\frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a\, \Lambda_0^3}\right)^{1/3} \Lambda_0 < \Lambda_1$$

Zhou, Jiang, Cai, Sasaki, Pi,
Phys. Rev. D 102 (2020) no.10, 103527

World-Sheet Instantons, Axion Monodromy like potentials & deviations from scale invariance

Anomaly condensate → linear axion potential

$$V_{\rm eff} \ni \langle R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} \rangle\, b(x)$$

world-sheet (non-perturbative) instantons → periodic potential perturbations

$$V(a,\,b) = \Lambda_1{}^4 \Big(1 + f_a^{-1}\, \tilde{\xi}_1\, a(x)\Big)\, \cos(f_a{}^{-1} a(x)) + \frac{1}{f_a}\Big(f_b\, \Lambda_0{}^3 + \Lambda_2^4\Big)\, a(x) + \Lambda_0{}^3\, b(x)$$

Case I

$$\Big(\frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a\, \Lambda_0^3}\Big)^{1/3} \Lambda_0 < \Lambda_1 \ll \Lambda_0$$

Enhancement of cosmic perturbations

Case

$$\Lambda_0 \ll \Big(\frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a\, \Lambda_0^3}\Big)^{1/3} \Lambda_0 < \Lambda_1$$

NEM, Solà + Basilakos

NEM, Spanos, Stamou PRD106 (2022), 063532

Zhou, Jiang, Cai, Sasaki, Pi, Phys. Rev. D 102 (2020) no.10, 103527

World-Sheet Instantons, Axion Monodromy like potentials & deviations from scale invariance

Anomaly condensate → linear axion potential

$$V_{\rm eff} \ni \langle R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} \rangle \, b(x)$$

world-sheet (non-perturbative) instantons → periodic potential perturbations

$$V(a,\,b) = {\Lambda_1}^4 \Big(1 + f_a^{-1}\,\tilde{\xi}_1\, a(x)\Big)\, \cos({f_a}^{-1} a(x)) + \frac{1}{f_a}\Big(f_b\,{\Lambda_0}^3 + \Lambda_2^4\Big)\, a(x) + {\Lambda_0}^3\, b(x)$$

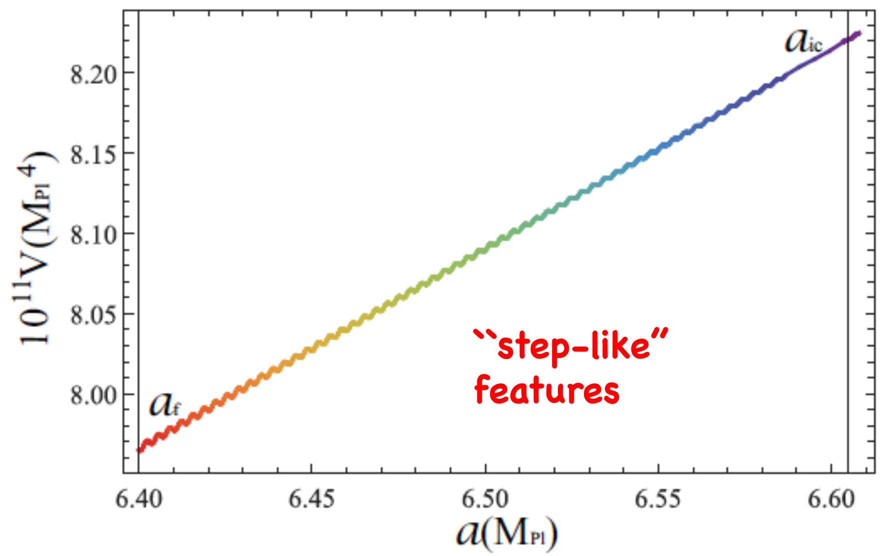
Case I $$\Big(\frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a\,\Lambda_0^3}\Big)^{1/3}\Lambda_0 < \Lambda_1 \ll \Lambda_0$$

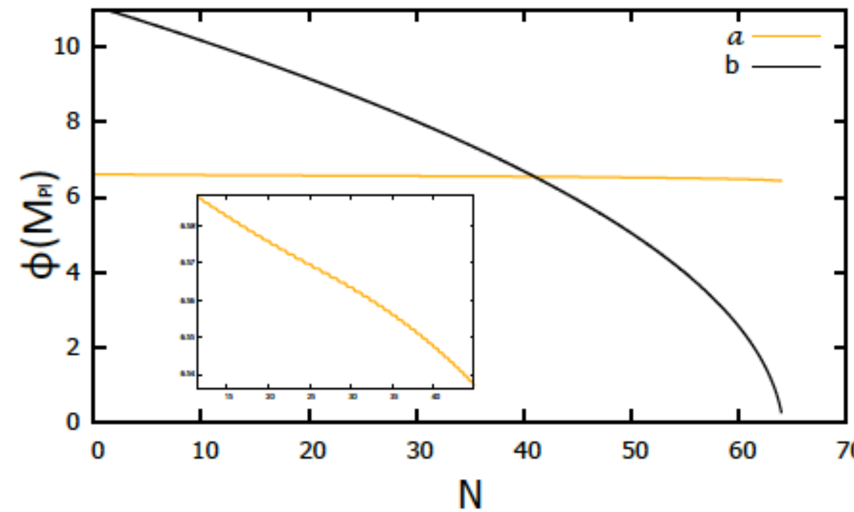
NEM, Spanos, Stamou
PRD106 (2022), 063532

The enhancement of cosmic perturbations mechanism

NEM, Spanos, Stamou
PRD106 (2022), 063532

b-field + condensate drive inflation, **a-axion ends inflation**





$$V(a,\,b) = \Lambda_1{}^4 \left(1 + f_a^{-1}\,\tilde{\xi}_1\, a(x)\right)\, \cos(f_a{}^{-1} a(x)) + \frac{1}{f_a}\left(f_b\,\Lambda_0{}^3 + \Lambda_2^4\right) a(x) + \Lambda_0{}^3\, b(x)$$

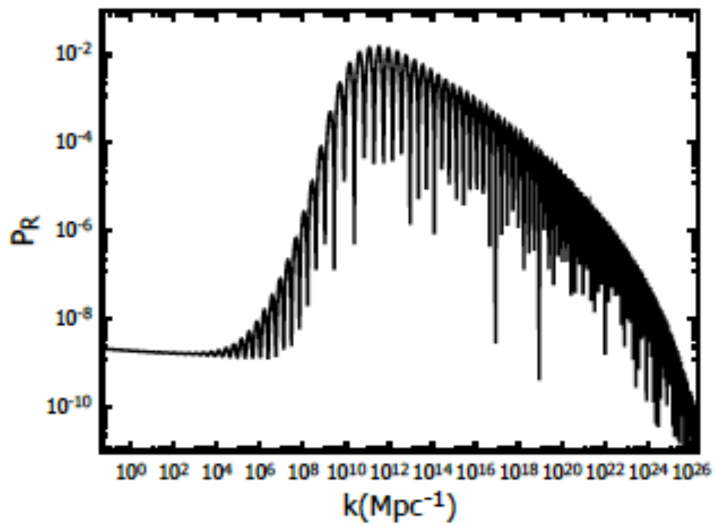
$$n_s = 1 + \frac{d\ln P_R}{d\ln k} \qquad r = \frac{P_T}{P_R} \qquad P_T = \frac{2}{\pi^2}H^2$$

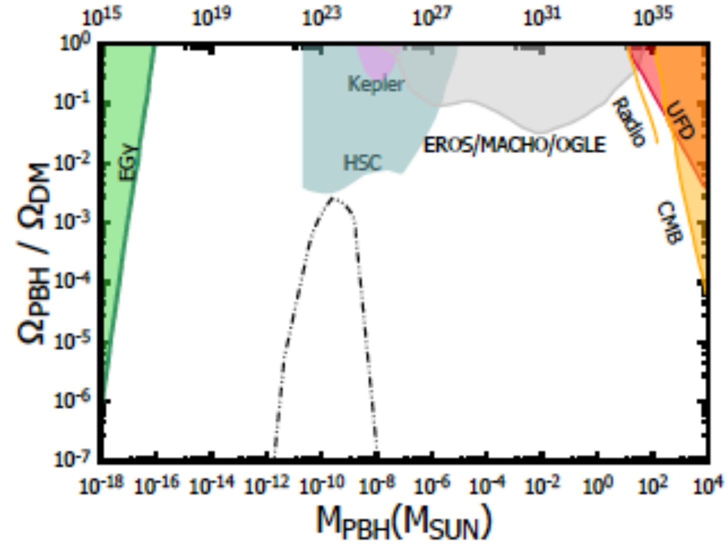
SET	g_1	g_2	ξ	$f(M_{Pl})$	$\Lambda_0(M_{Pl})$	$\Lambda_1(M_{Pl})$	$\Lambda_3(M_{Pl})$
1	0.021	0.904	−0.15	2.5×10^{-4}	8.4×10^{-4}	8.19×10^{-4}	2.32×10^{-4}
2	0.026	0.774	−0.20	2.5×10^{-4}	8.4×10^{-4}	7.89×10^{-4}	2.49×10^{-4}

SET	a_{ic}	b_{ic}	n_s	r
1	6.605	11.1	0.9638	0.062
2	4.932	11.4	0.9619	0.060

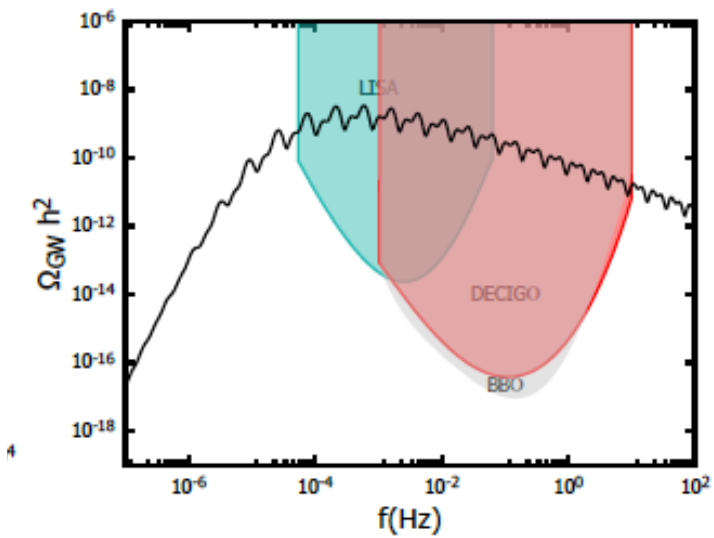
Primordial Black Hole (PBH) and GW enhanced production during inflation

NEM, Spanos, Stamou
PRD106 (2022), 063532





SET 1



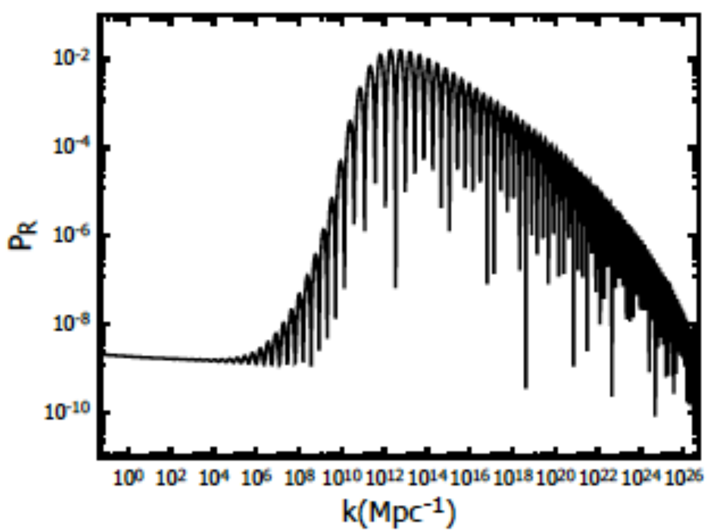
fractional PBH abundance

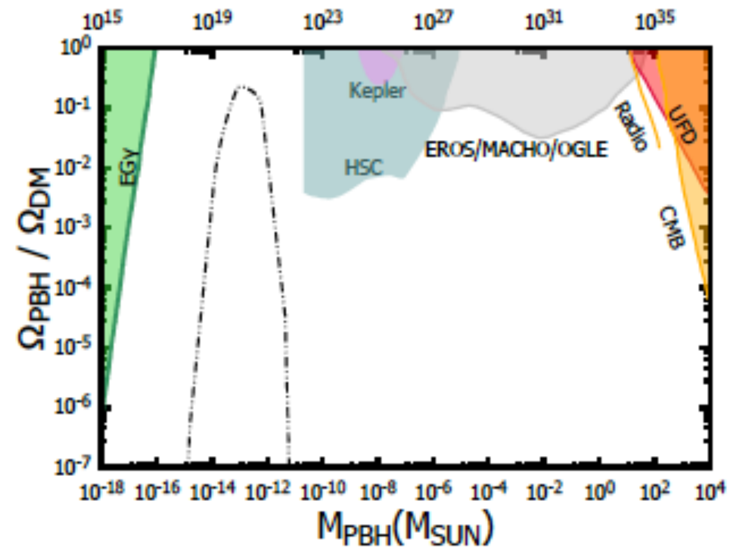
$$f_{PBH} = \int_k dM_{\rm PBH}(k)\,\frac{1}{M_{\rm PBH}(k)}\,\frac{\Omega_{PBH}}{\Omega_{DM}}(M_{\rm PBH}(k))$$

$$f_{PBH} = 0.01$$

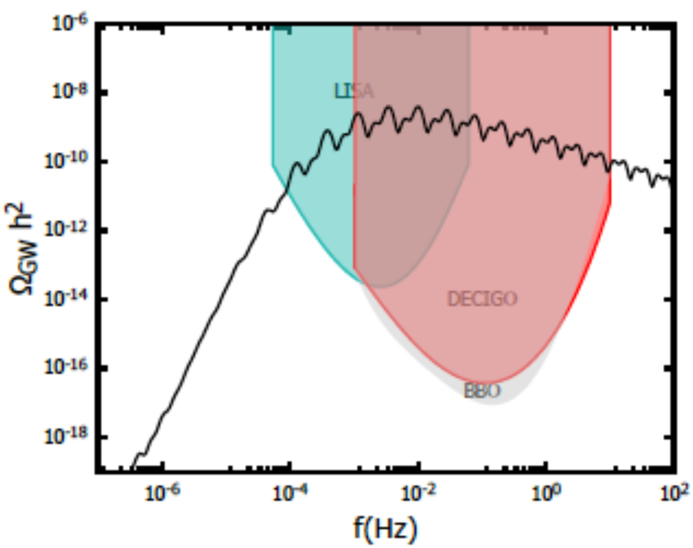
Primordial Black Hole (PBH) and GW enhanced production during inflation

NEM, Spanos, Stamou
PRD106 (2022), 063532





SET 2



fractional PBH abundance

$$f_{PBH} = \int_k dM_{\rm PBH}(k)\,\frac{1}{M_{\rm PBH}(k)}\,\frac{\Omega_{PBH}}{\Omega_{DM}}(M_{\rm PBH}(k))$$

$$f_{PBH} = 0.80.$$

World-Sheet Instantons, Axion Monodromy like potentials & deviations from scale invariance

NEM, Spanos, Stamou
PRD106 (2022), 063532

Anomaly condensate → **linear axion potential**

$$V_{\rm eff} \ni \langle R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} \rangle\, b(x)$$

world-sheet (non-perturbative) instantons → periodic potential perturbations

$$V(a,\, b) = {\Lambda_1}^4 \Big(1 + f_a^{-1}\, \tilde{\xi}_1\, a(x)\Big)\, \cos({f_a}^{-1} a(x)) + \frac{1}{f_a}\Big(f_b\, {\Lambda_0}^3 + \Lambda_2^4\Big)\, a(x) + {\Lambda_0}^3\, b(x)$$

Case II

$$\Lambda_0 \ll \Big(\frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a\,\Lambda_0^3}\Big)^{1/3} \Lambda_0 < \Lambda_1$$

specific set of parameters
enhancement due to **inflection points** in the potential →
different enhancement mechanism than in

Zhou, Jiang, Cai, Sasaki, Pi,
Phys. Rev. D 102 (2020) no.10, 103527

World-Sheet Instantons, Axion Monodromy like potentials & deviations from scale invariance

NEM, Spanos, Stamou
PRD106 (2022), 063532

Anomaly condensate → linear axion potential

$$V_{\rm eff} \ni \langle R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma}\rangle\, b(x)$$

world-sheet (non-perturbative) instantons → periodic potential perturbations

$$V(a,\,b) = {\Lambda_1}^4 \left(1 + f_a^{-1}\,\tilde{\xi}_1\, a(x)\right) \cos({f_a}^{-1} a(x)) + \frac{1}{f_a}\left(f_b\, {\Lambda_0}^3 + \Lambda_2^4\right) a(x) + {\Lambda_0}^3\, b(x)$$

$$\Lambda_0 = 8.4\times 10^{-4} M_{\rm Pl}, \quad g_1 = 110, \quad g_2 = 1.779\times 10^4, \quad \xi = -0.09, \quad f = 0.09\, M_{\rm Pl}\,.$$

SET 3 $(a_{ic}, b_{ic}) = 7.5622, 0.522$

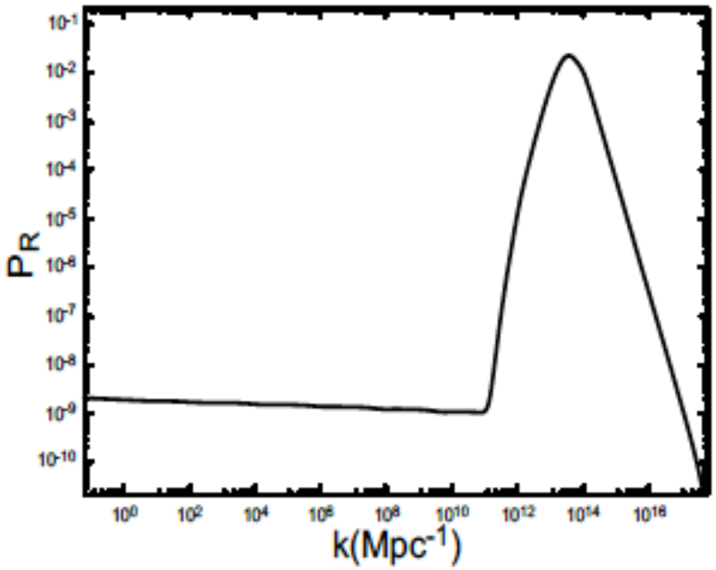
Case II $\quad \Lambda_0 \ll \left(\frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a\,\Lambda_0^3}\right)^{1/3} \Lambda_0 < \Lambda_1$

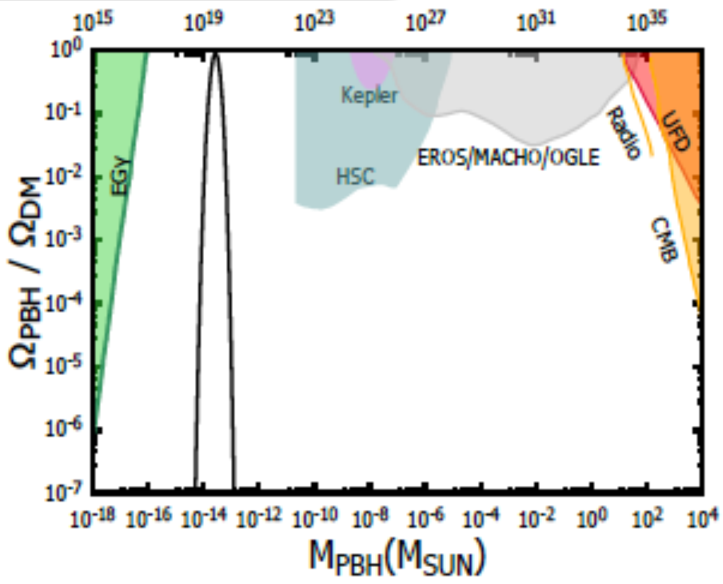
⚠ **specific set of parameters**
enhancement due to **inflection points** in the potential →
different enhancement mechanism than in

Zhou, Jiang, Cai, Sasaki, Pi,
Phys. Rev. D 102 (2020) no.10, 103527

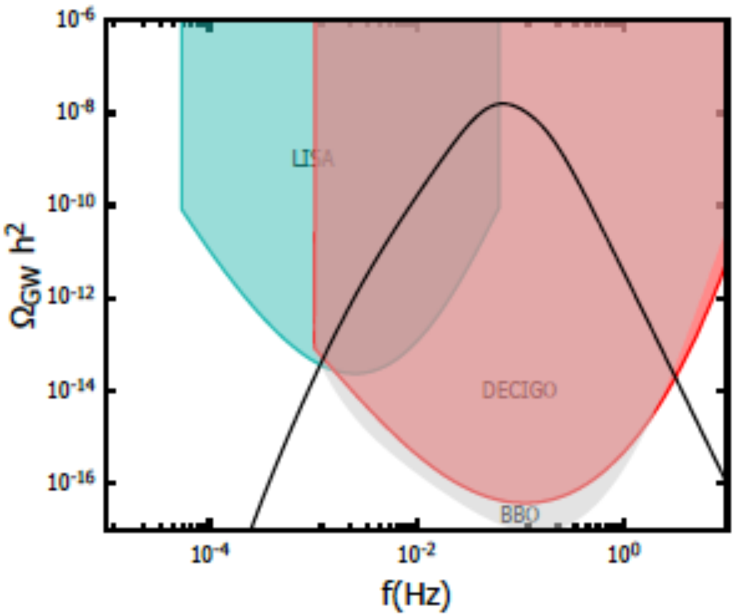
Primordial Black Hole (PBH) and GW enhanced production during inflation in Case 2

NEM, Spanos, Stamou PRD106 (2022), 063532





SET 3



fractional PBH abundance

$$f_{PBH} = \int_k dM_{\rm PBH}(k) \frac{1}{M_{\rm PBH}(k)} \frac{\Omega_{PBH}}{\Omega_{DM}}(M_{\rm PBH}(k))$$

$$f_{PBH} = 0.762$$

SUMMARY: Primordial Black Hole (PBH) and GW enhanced production during inflation in Cases 1 + 2

NEM, Spanos, Stamou
PRD106 (2022), 063532

SET	P_R^{peak}	$M_{PBH}^{peak}(M_\odot)$	f_{PBH}
1	1.466×10^{-2}	2.394×10^{-10}	0.009
2	1.365×10^{-2}	8.313×10^{-14}	0.799
3	2.24×10^{-2}	1.791×10^{-14}	0.762

NANOGrav

29/6/23
15 year data
Release.
Origin of
GW background?

Hence in both hierarchies of scales :

1: $\left(\frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a\,\Lambda_0^3}\right)^{1/3}\Lambda_0 < \Lambda_1 \ll \Lambda_0$, **2:** $\Lambda_0 \ll \left(\frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a\,\Lambda_0^3}\right)^{1/3}\Lambda_0 < \Lambda_1$

one may get **significant enhancement** of cosmic perturbations, ands PBH production, and thus a **significant portion** of PBH could play **the role of DM**, also, as a result, **profiles of GW** could **change** during radiation, in principle **falsifiable predictions** at **interferometers, distinguishing 1 from 2.**

Cancellation of Gravitational Anomalies in Radiation Era by:

Chiral Fermionic Matter generation @ end of Inflation

Basilakos, NEM,Solà (2019-20)

Required by consistency of quantum theory of matter and radiation (**diffeomorphism invariance**)

$$S^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \frac{\alpha'}{\kappa}b(x)\nabla_\mu\Big(\sqrt{\frac{2}{3}}\frac{1}{96}\mathcal{K}^\mu - \sqrt{\frac{3}{8}}J^{5\mu}\Big)\Big] + \dots$$

$$J^{5\mu} = \sum_j \bar{\psi}_j\gamma^\mu\gamma^5\psi_j,$$

Chiral current, including RHN

$$\partial_\mu\Big[\sqrt{-g}\Big(\sqrt{\frac{3}{8}}\frac{\alpha'}{\kappa}J^{5\mu} - \frac{\alpha'}{\kappa}\sqrt{\frac{2}{3}}\frac{1}{96}K^\mu\Big)\Big] = \sqrt{\frac{3}{8}}\frac{\alpha'}{\kappa}\Big(\frac{\alpha_{\rm EM}}{2\pi}\sqrt{-g}\,F^{\mu\nu}\widetilde{F}_{\mu\nu} + \frac{\alpha_s}{8\pi}\sqrt{-g}\,G^a_{\mu\nu}\widetilde{G}^{a\mu\nu}\Big)$$

chiral U(1) Gluon QCD

instanton generated potential for KR axion b-field during matter dominance → axion Dark Matter

Cancellation of Gravitational Anomalies in Radiation Era by:

Chiral Fermionic Matter generation @ end of Inflation

Basilakos, NEM, Solà (2019-20)

Required by consistency of quantum theory of matter and radiation (**diffeomorphism invariance**)

Scale factor ***a(t) ~ T^{-1}***

Possibly also QCD

$$\dot{\bar{b}} \propto T^3 + \text{subleading } (\sim T^2) \text{ chiral U(1) anomaly terms}$$

sufficiently slowly varying during leptogenesis (brief) epoch → qualitatively similar to approximately const. background

Bossingham, NEM, Sarkar