UNIVERSITY OF Southampton
School of Physics and Astronomy

HIDDeν
Hunting Invisibles: Dark sectors, Dark matter and Neutrinos

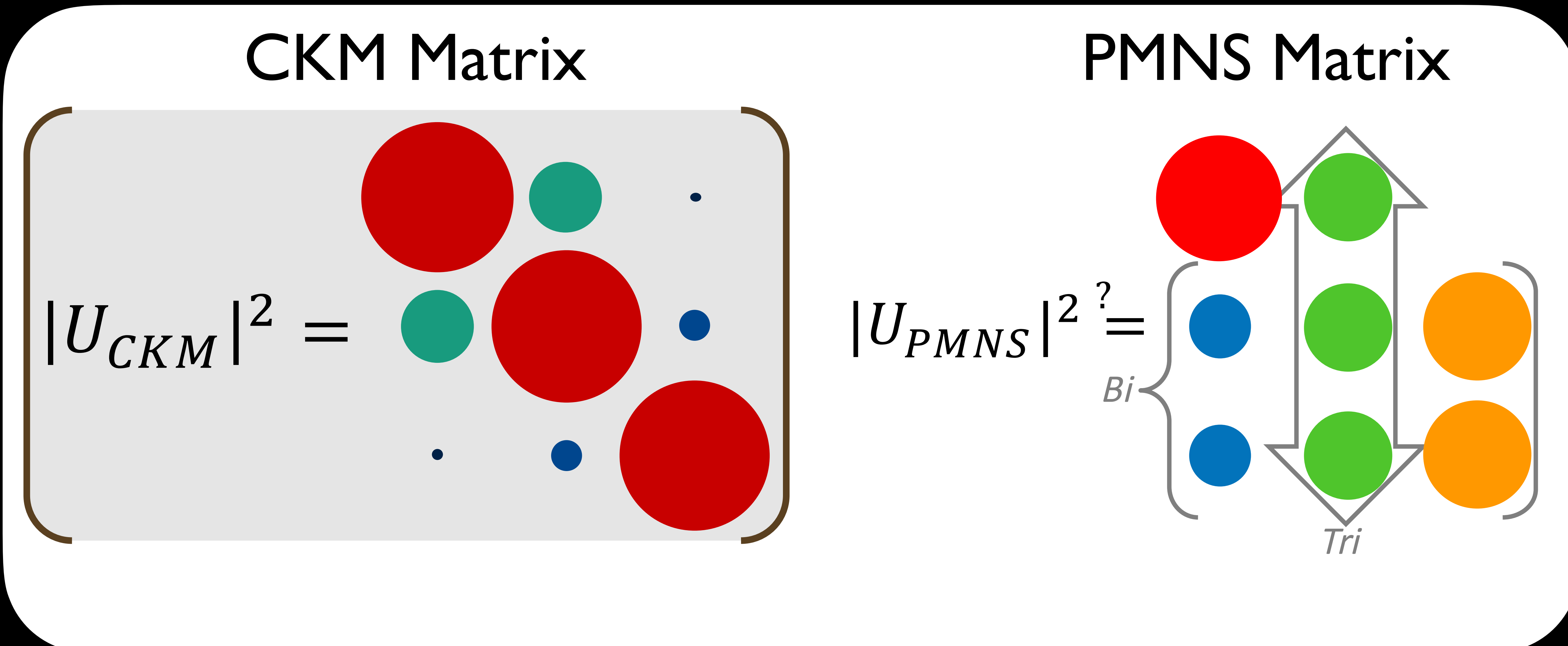
Neutrino Mixing Sum Rules and Littlest seesaw models

Steve King

2nd September

2023

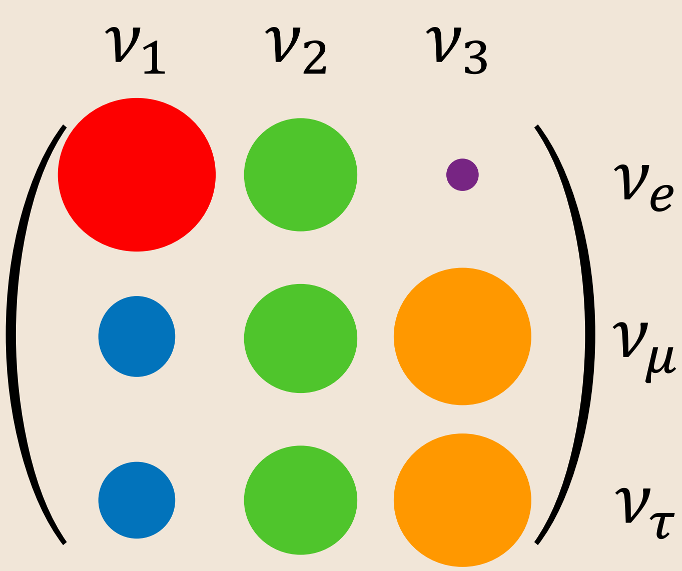
We focus on Mixing



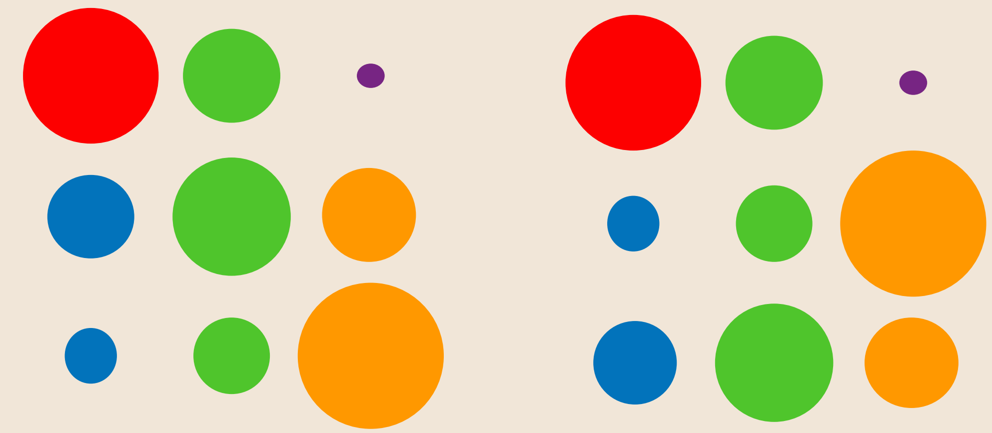
(Dirac vs Majorana with GWs at end)

Experimental open questions

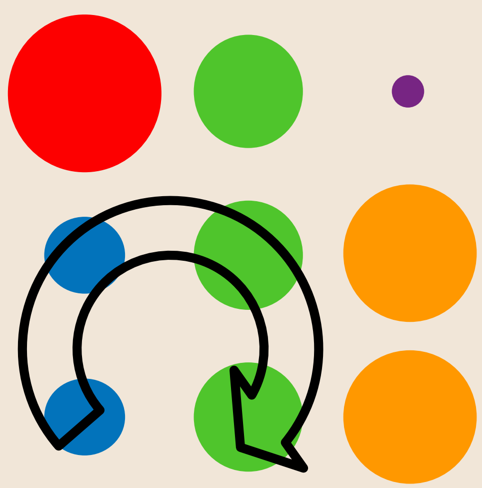
$$|U_{\text{PMNS}}|^2 \simeq$$



Octant degeneracy



Lower ($\theta_{23} < 45°$) Upper ($\theta_{23} > 45°$)



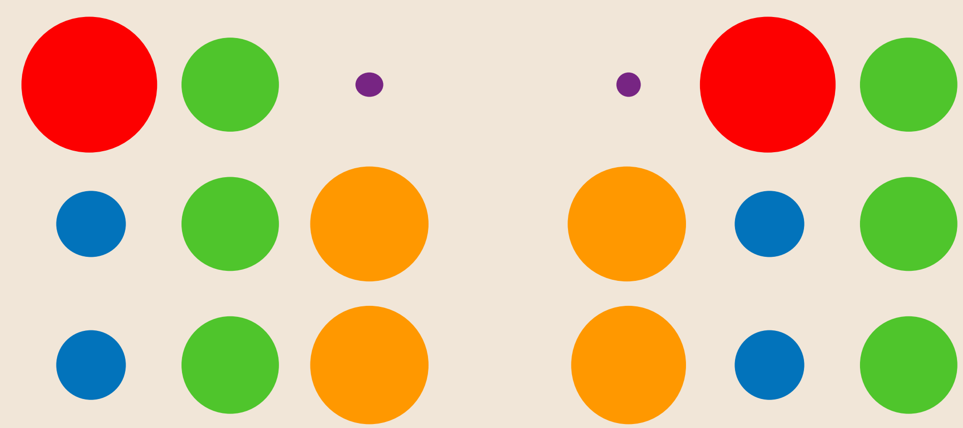
CP Violation

Complex mixing of these 4 elements causes

$$P(\nu_\alpha \to \nu_\beta) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$$

Key parameter: $\boldsymbol{\delta_{CP}}$

Mass Ordering (Hierarchy)



Normal (NO) Inverted (IO)

Precision also required (this talk)

NuFIT 5.2 (2022)

without SK atmospheric data

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.3$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.303^{+0.012}_{-0.011}$	$0.270 \to 0.341$	$0.303^{+0.012}_{-0.011}$	$0.270 \to 0.341$
$\theta_{12}/^\circ$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$
$\sin^2\theta_{23}$	$0.572^{+0.018}_{-0.023}$	$0.406 \to 0.620$	$0.578^{+0.016}_{-0.021}$	$0.412 \to 0.623$
$\theta_{23}/^\circ$	$49.1^{+1.0}_{-1.3}$	$39.6 \to 51.9$	$49.5^{+0.9}_{-1.2}$	$39.9 \to 52.1$
$\sin^2\theta_{13}$	$0.02203^{+0.00056}_{-0.00059}$	$0.02029 \to 0.02391$	$0.02219^{+0.00060}_{-0.00057}$	$0.02047 \to 0.02396$
$\theta_{13}/^\circ$	$8.54^{+0.11}_{-0.12}$	$8.19 \to 8.89$	$8.57^{+0.12}_{-0.11}$	$8.23 \to 8.90$
$\delta_{CP}/^\circ$	197^{+42}_{-25}	$108 \to 404$	286^{+27}_{-32}	$192 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.511^{+0.028}_{-0.027}$	$+2.428 \to +2.597$	$-2.498^{+0.032}_{-0.025}$	$-2.581 \to -2.408$

Second Octant

with SK atmospheric data

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 6.4$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.303^{+0.012}_{-0.012}$	$0.270 \to 0.341$	$0.303^{+0.012}_{-0.011}$	$0.270 \to 0.341$
$\theta_{12}/^\circ$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$
$\sin^2\theta_{23}$	$0.451^{+0.019}_{-0.016}$	$0.408 \to 0.603$	$0.569^{+0.016}_{-0.021}$	$0.412 \to 0.613$
$\theta_{23}/^\circ$	$42.2^{+1.1}_{-0.9}$	$39.7 \to 51.0$	$49.0^{+1.0}_{-1.2}$	$39.9 \to 51.5$
$\sin^2\theta_{13}$	$0.02225^{+0.00056}_{-0.00059}$	$0.02052 \to 0.02398$	$0.02223^{+0.00058}_{-0.00058}$	$0.02048 \to 0.02416$
$\theta_{13}/^\circ$	$8.58^{+0.11}_{-0.11}$	$8.23 \to 8.91$	$8.57^{+0.11}_{-0.11}$	$8.23 \to 8.94$
$\delta_{CP}/^\circ$	232^{+36}_{-26}	$144 \to 350$	276^{+22}_{-29}	$194 \to 344$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.507^{+0.026}_{-0.027}$	$+2.427 \to +2.590$	$-2.486^{+0.025}_{-0.028}$	$-2.570 \to -2.406$

First Octant

See Lisi talk

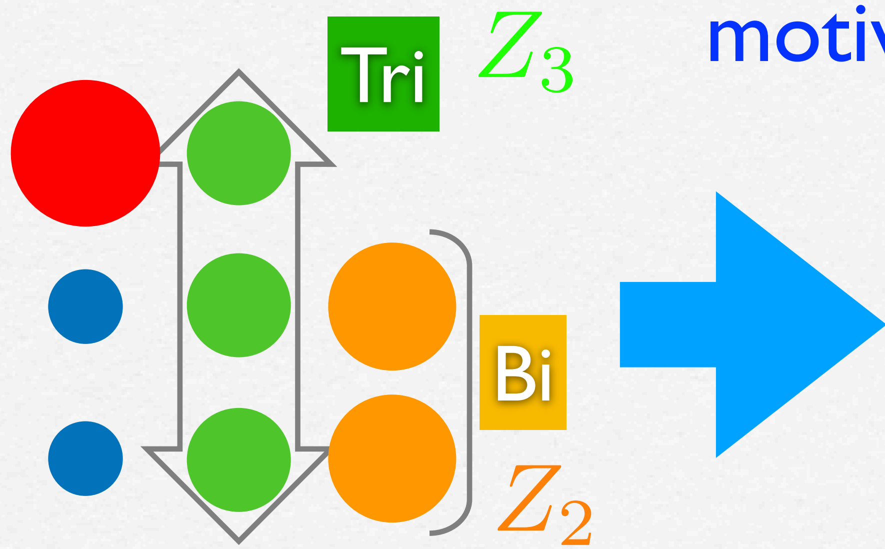
This is very impressive, but much more precise measurements of these parameters are required to match theoretical predictions based on symmetry (or maybe exclude the symmetry approach)

P.F.Harrison, D.H.Perkins and W.G.Scott, hep-ph/0202074

Tri-Bimaximal Mixing

Non-commuting Z_3 and Z_2

motivates non-abelian discrete symmetry



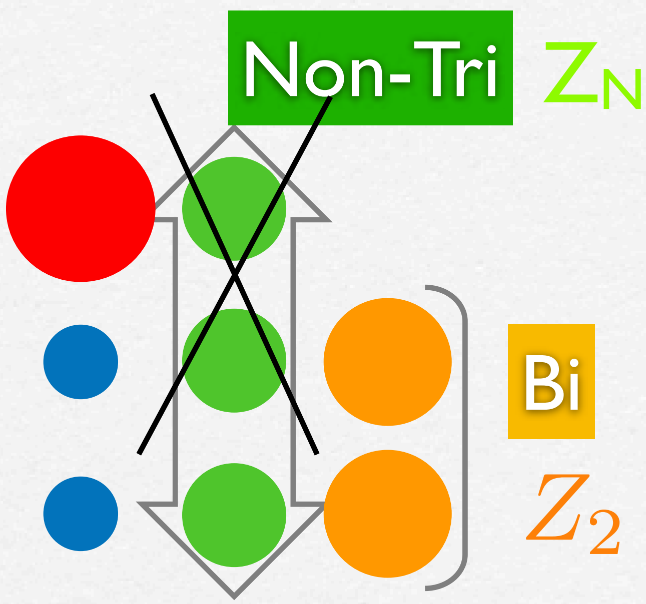
$$\begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$\sin\theta_{12} = \frac{1}{\sqrt{3}}$	$\sin\theta_{23} = \frac{1}{\sqrt{2}}$	$\sin\theta_{13} = 0$
Allowed at 3 sigma	Allowed at 3 sigma	Excluded at many sigma
$\sin\theta_{12} \equiv \frac{1+s}{\sqrt{3}}$	$\sin\theta_{23} \equiv \frac{1+a}{\sqrt{2}}$	$\sin\theta_{13} \equiv \frac{r}{\sqrt{2}}$

SFK 0710.0530

More precise measurements required to measure all the TBM deviations

Other Simple Mixing



$\phi = (1+\sqrt{5})/2$

Golden Ratio

✓ **a** $\tan\theta_{12} = 1/\phi$ $\theta_{12} = 31.7^\circ$

✓ **b** $\cos\theta_{12} = \phi/2$ $\theta_{12} = 36^\circ$

✗ **c** $\cos\theta_{12} = \phi/\sqrt{3}$ $\theta_{12} \approx 20.9^\circ$

✗ **Bimaximal** $\theta_{12} = 45^\circ$

✗ **Hexagonal** $\theta_{12} = 30^o$

Non-commuting Z_N and Z_2

motivates non-abelian discrete symmetry

$$\begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

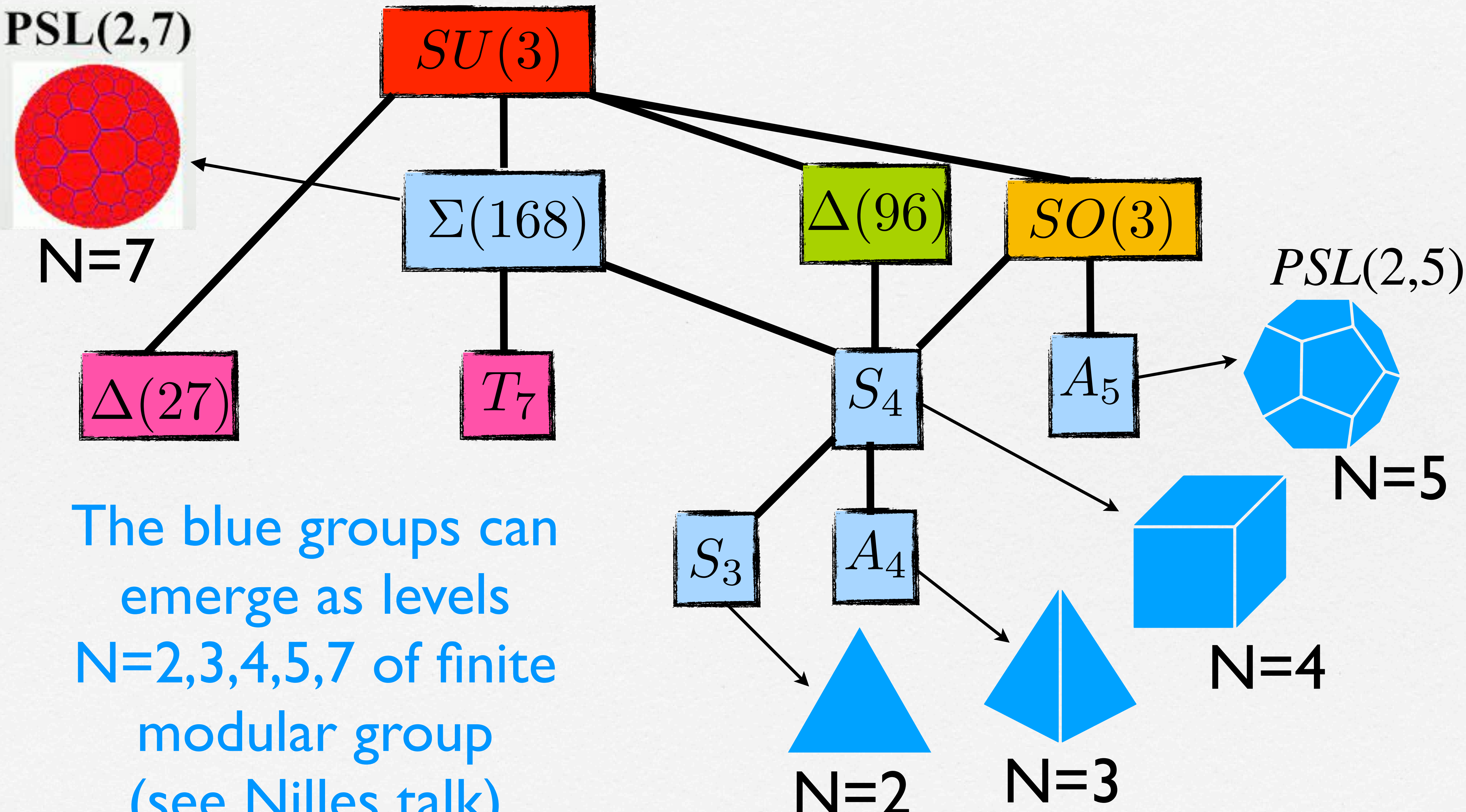
$$\sin\theta_{23} = \frac{1}{\sqrt{2}}$$ ✓

$$\sin\theta_{13} = 0$$ ✗

Allowed at 3 sigma ✓

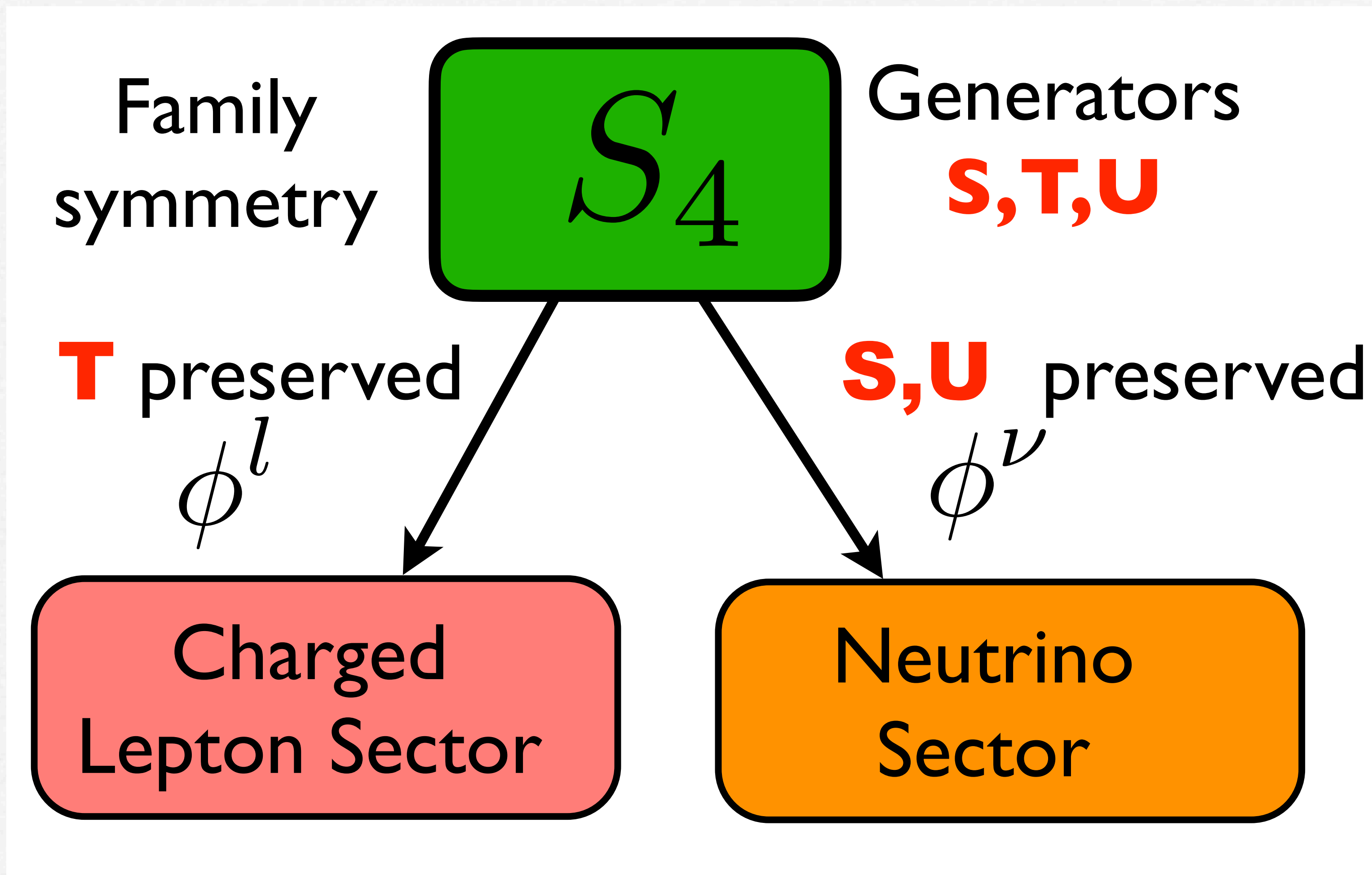
Excluded at 3 sigma ✗

Non-Abelian Discrete Symmetry



The blue groups can emerge as levels N=2,3,4,5,7 of finite modular group (see Nilles talk)

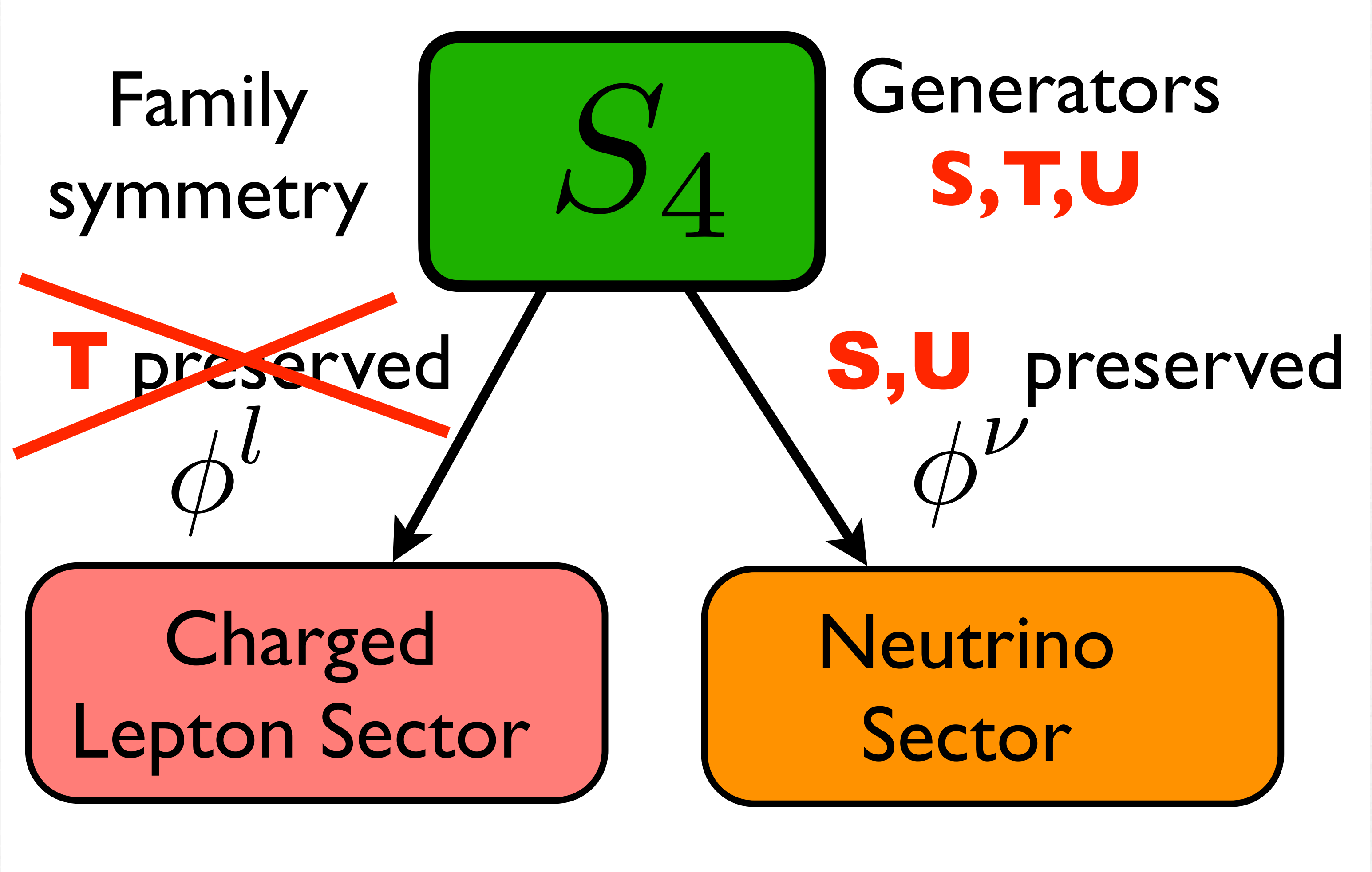
TBM from S_4



$$\Rightarrow \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

TBM excluded
so break S,T,U

Charged lepton corrections



break T

$$U_{\mathrm{PMNS}} = \begin{pmatrix} c^e_{12} & s^e_{12} e^{-i\delta^e_{12}} & 0 \\ -s^e_{12} e^{i\delta^e_{12}} & c^e_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Charged lepton rotation

Huge literature e.g. Antusch and SFK, hep-ph/0508044; I.Girardi, S.T.Petcov and A.V.Titov,1410.8056, ..

Sum rules from charged lepton corrections

Charged lepton rotation — Tri-bimaximal neutrinos

$$U_{\rm PMNS} = \begin{pmatrix} c^e_{12} & s^e_{12}e^{-i\delta^e_{12}} & 0 \\ -s^e_{12}e^{i\delta^e_{12}} & c^e_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$= \begin{pmatrix} \cdots & \cdots & \frac{s^e_{12}}{\sqrt{2}}e^{-i\delta^e_{12}} \\ \cdots & \cdots & \frac{c^e_{12}}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\rightarrow s_{13} = \frac{s^e_{12}}{\sqrt{2}}$$

Suggests $\theta^e_{12} \approx \theta_C$

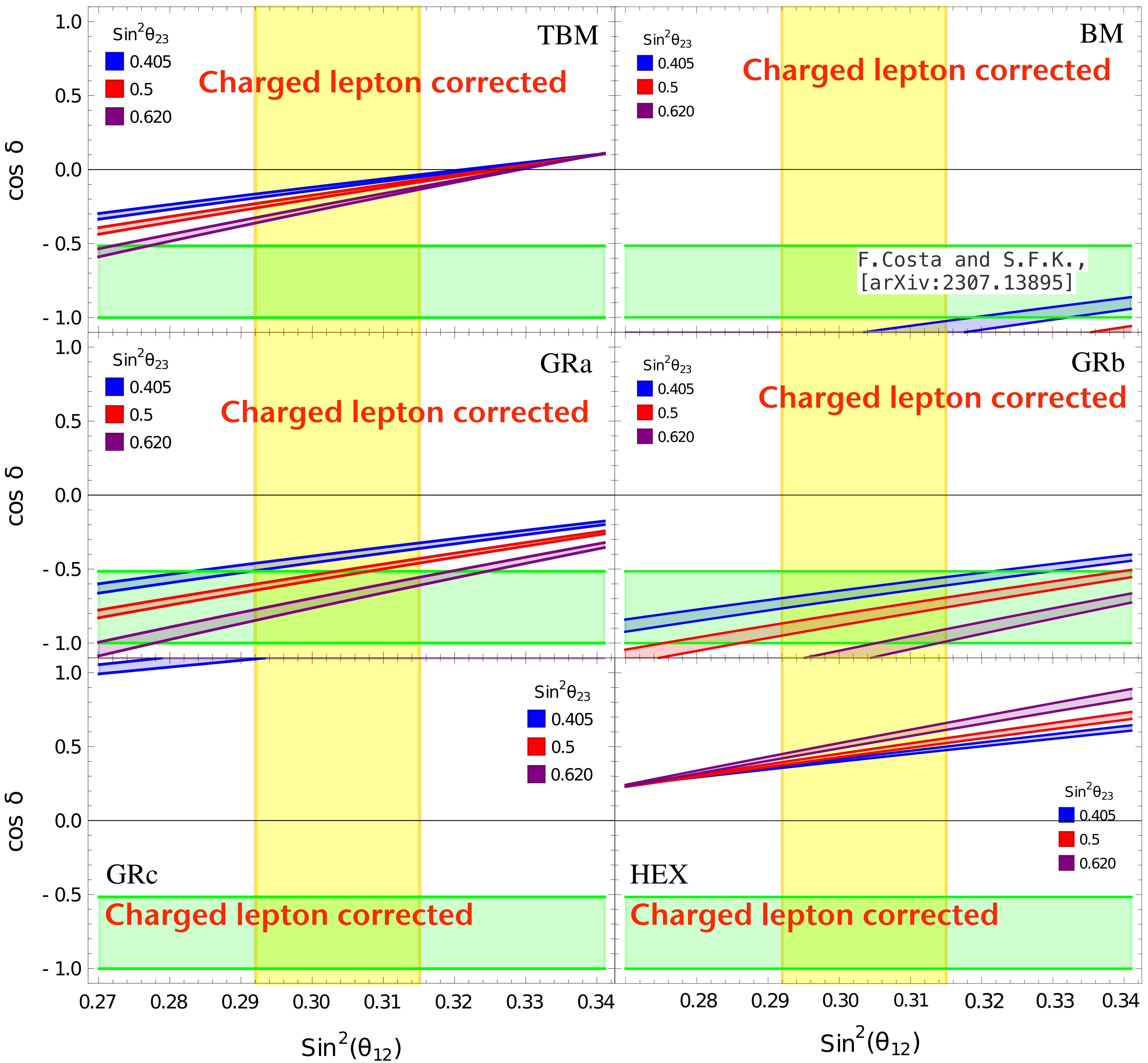
$$\rightarrow c_{23}c_{13} = \frac{1}{\sqrt{2}} \rightarrow s^2_{23} < \frac{1}{2}$$

Valid also with θ^e_{23}

Sum Rule prediction for CP phase

$$\frac{|U_{\tau 1}|}{|U_{\tau 2}|} = \frac{|s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta}|}{|-c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta}|} = \frac{1}{\sqrt{2}} \rightarrow \cos\delta = \frac{t_{23}s^2_{12} + s^2_{13}c^2_{12}/t_{23} - \frac{1}{3}(t_{23} + s^2_{13}/t_{23})}{\sin 2\theta_{12} s_{13}}$$

This derivation: P.Ballett, S.F.K., C.Luhn, S.Pascoli and M.A.Schmidt, 1410.7573

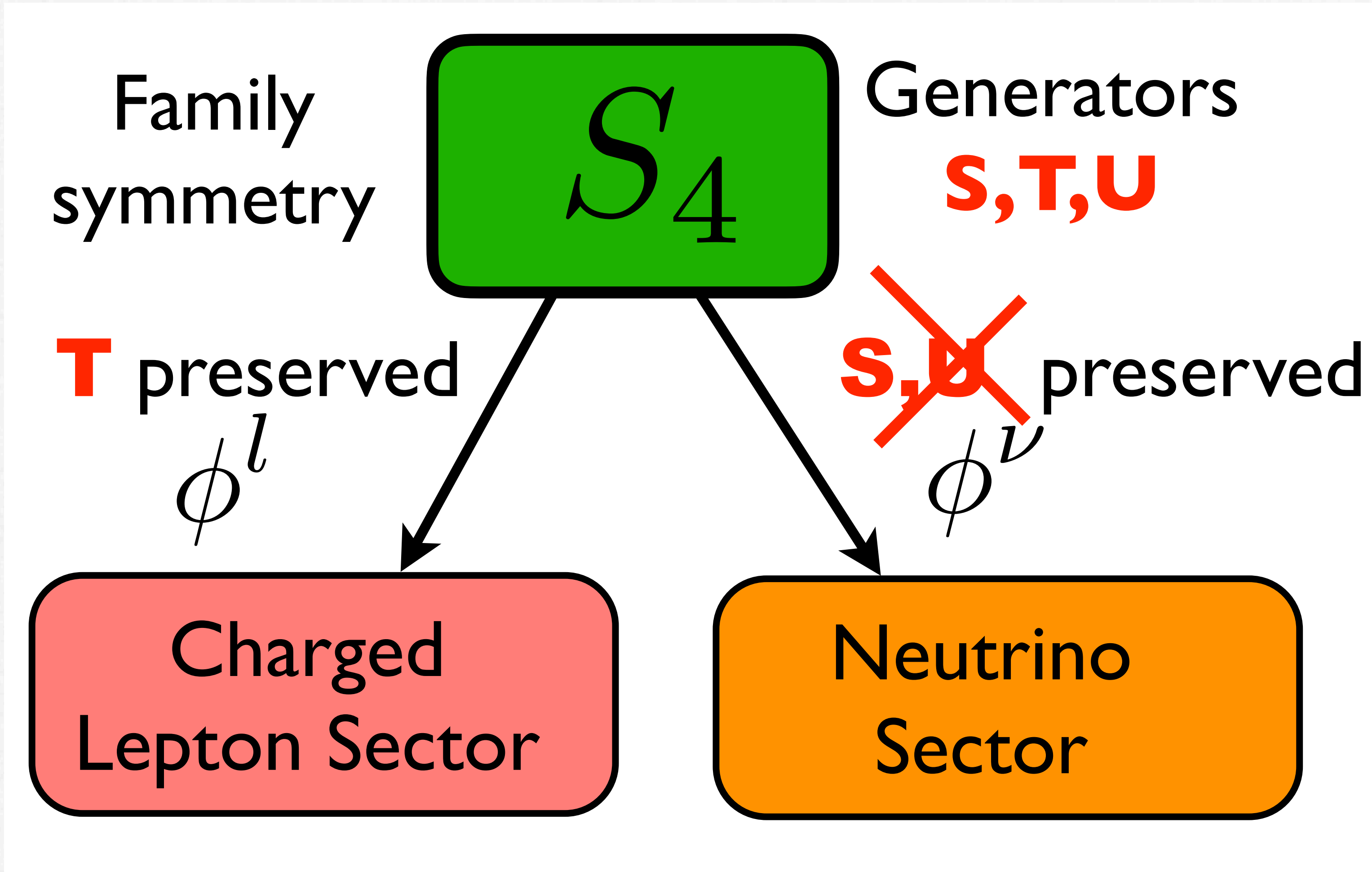


More precise measurements required to exclude these cases

Yellow and green are one sigma ranges

At 3 sigma the entire range is allowed

Preserving a column of TBM



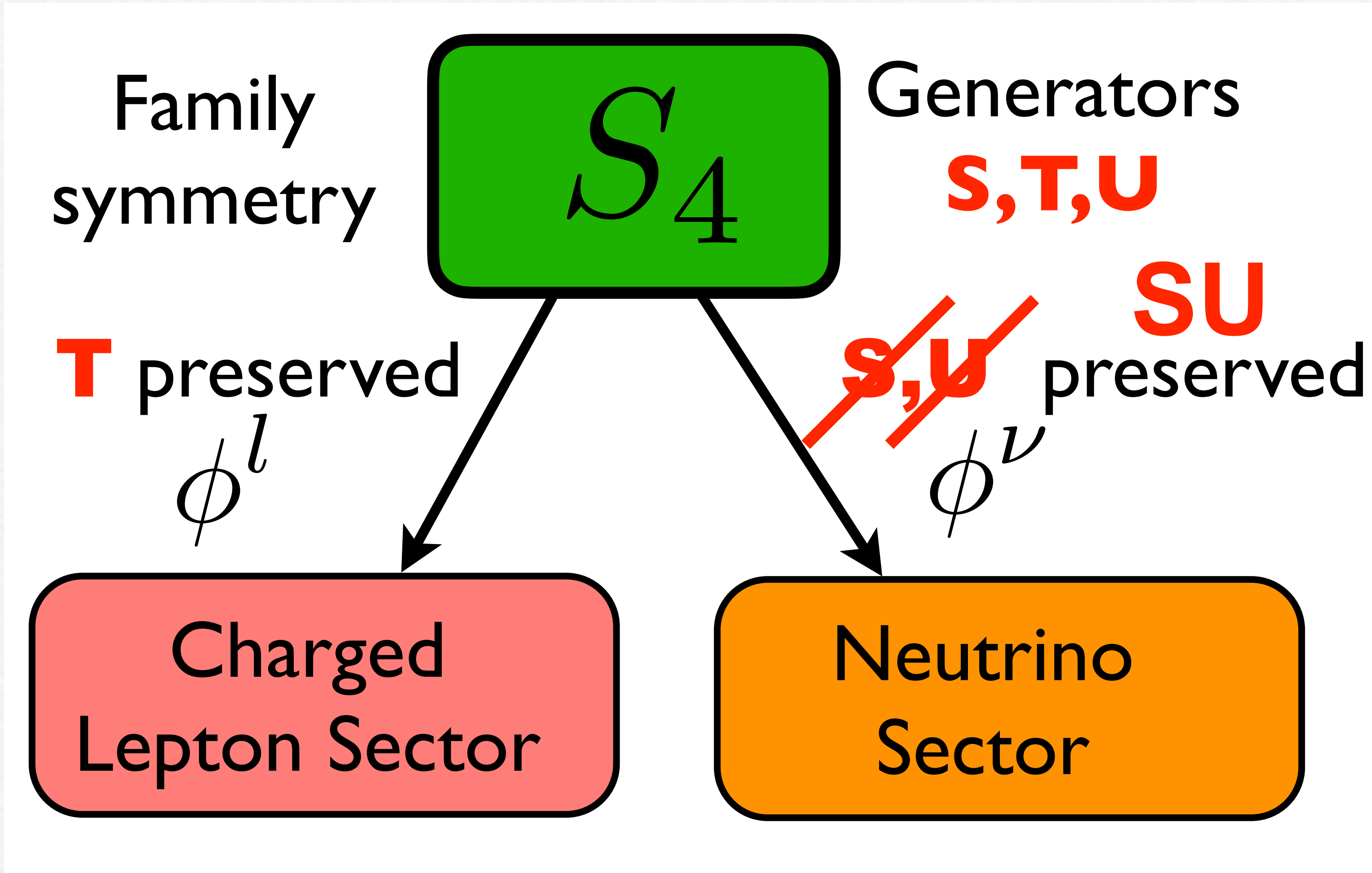
break U

Alternatively
A4 with just
S and T

$$U_{\rm TM2} \approx \begin{pmatrix} - & \frac{1}{\sqrt{3}} & - \\ - & \frac{1}{\sqrt{3}} & - \\ - & -\frac{1}{\sqrt{3}} & - \end{pmatrix}$$

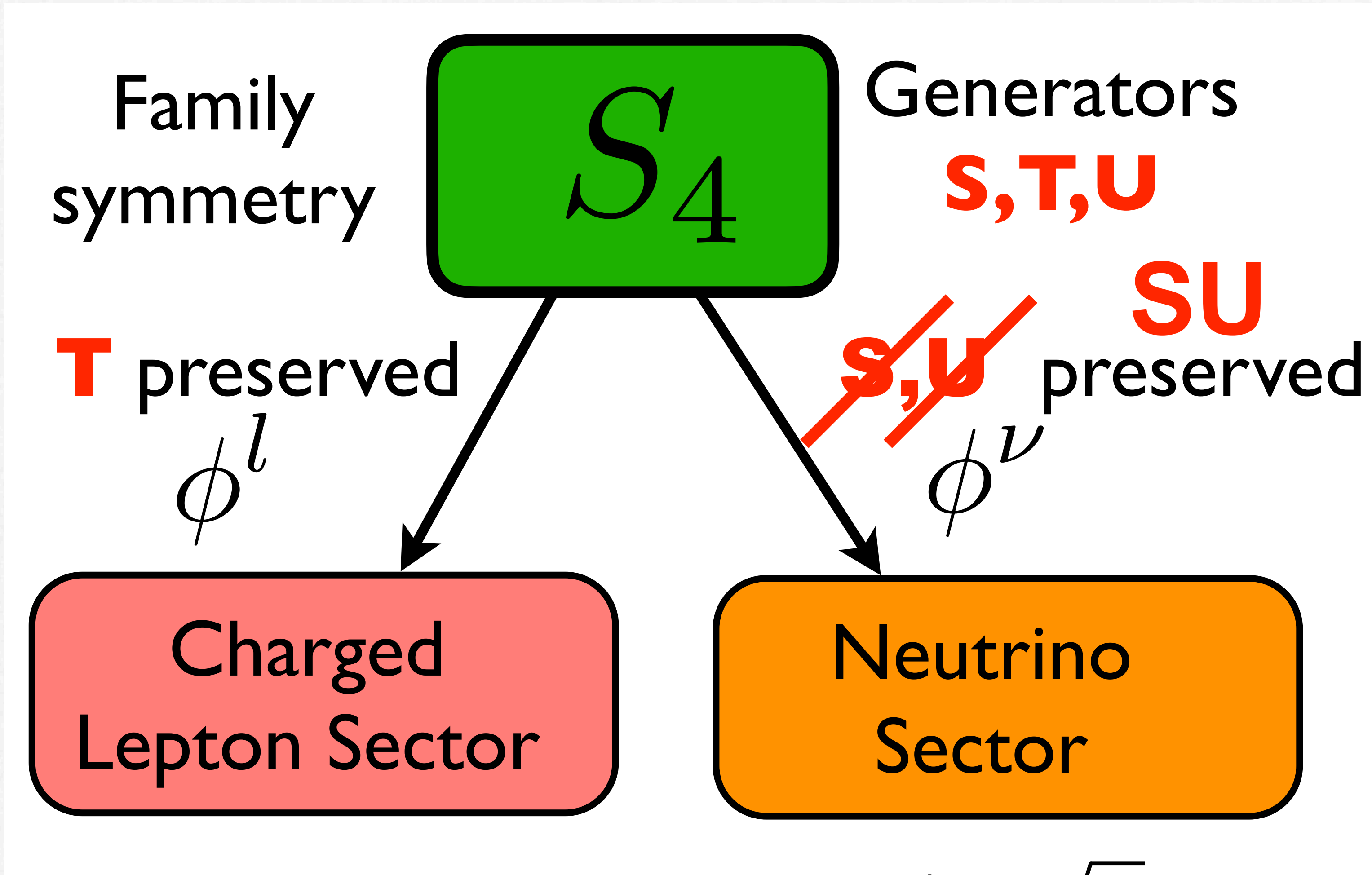
preserved 2nd column

Preserving a column of TBM



break S,U separately preserve SU

Preserving a column of TBM



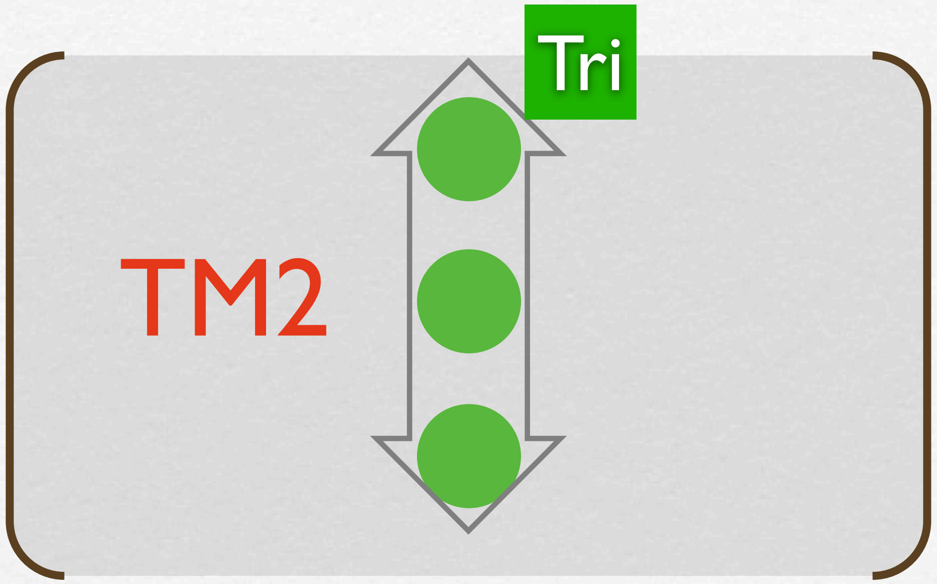
break S,U separately preserve SU

$$U_{\mathrm{TM1}} \approx \begin{pmatrix} \sqrt{\frac{2}{3}} & - & - \\ -\frac{1}{\sqrt{6}} & - & - \\ \frac{1}{\sqrt{6}} & - & - \end{pmatrix}$$

preserved 1st column

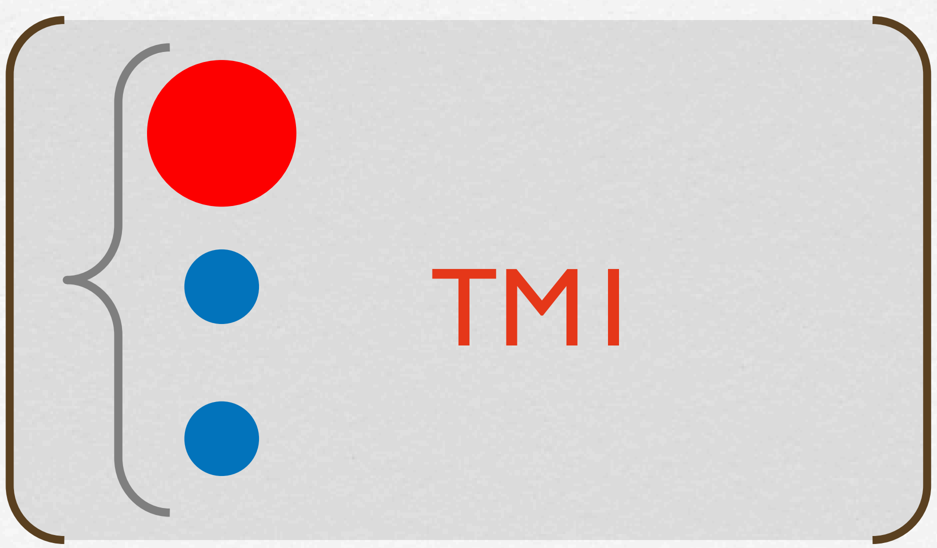
C.H.Albright and W.Rodejohann, 0812.0436; C.H.Albright, A.Dueck and W.Rodejohann, 1004.2798

Tri-maximal Mixing



Second column of TBM

$$U_{\text{TM2}} \approx \begin{pmatrix} \text{—} & \frac{1}{\sqrt{3}} & \text{—} \\ \text{—} & \frac{1}{\sqrt{3}} & \text{—} \\ \text{—} & -\frac{1}{\sqrt{3}} & \text{—} \end{pmatrix}$$



First column of TBM

$$U_{\text{TM1}} \approx \begin{pmatrix} \sqrt{\frac{2}{3}} & \text{—} & \text{—} \\ -\frac{1}{\sqrt{6}} & \text{—} & \text{—} \\ \frac{1}{\sqrt{6}} & \text{—} & \text{—} \end{pmatrix}$$

C.H.Albright and W.Rodejohann, 0812.0436; C.H.Albright, A.Dueck and W.Rodejohann, 1004.2798

Sum rules from preserved columns

$$U_{\rm TM2} \approx \begin{pmatrix} - & \frac{1}{\sqrt{3}} & - \\ - & \frac{1}{\sqrt{3}} & - \\ - & -\frac{1}{\sqrt{3}} & - \end{pmatrix}$$

Solar angle disfavoured

$$|U_{e2}| = s_{12}c_{13} = \sqrt{\frac{1}{3}} \rightarrow s_{12}^2 > \frac{1}{3}$$

$$|U_{\mu 2}| = |c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta}| = \sqrt{\frac{1}{3}}$$

$$|U_{\tau 2}| = |-c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta}| = \sqrt{\frac{1}{3}}$$

TM2 Sum Rule prediction for CP phase →

$$\cos\delta = \frac{2c_{13}\cot 2\theta_{23}\cot 2\theta_{13}}{\sqrt{2-3s_{13}^2}}$$

$$U_{\rm TM1} \approx \begin{pmatrix} \sqrt{\frac{2}{3}} & - & - \\ -\frac{1}{\sqrt{6}} & - & - \\ \frac{1}{\sqrt{6}} & - & - \end{pmatrix}$$

Solar angle favoured

$$|U_{e1}| = c_{12}c_{13} = \sqrt{\frac{2}{3}} \rightarrow s_{12}^2 < \frac{1}{3}$$

$$|U_{\mu 1}| = |-s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta}| = \sqrt{\frac{1}{6}}$$

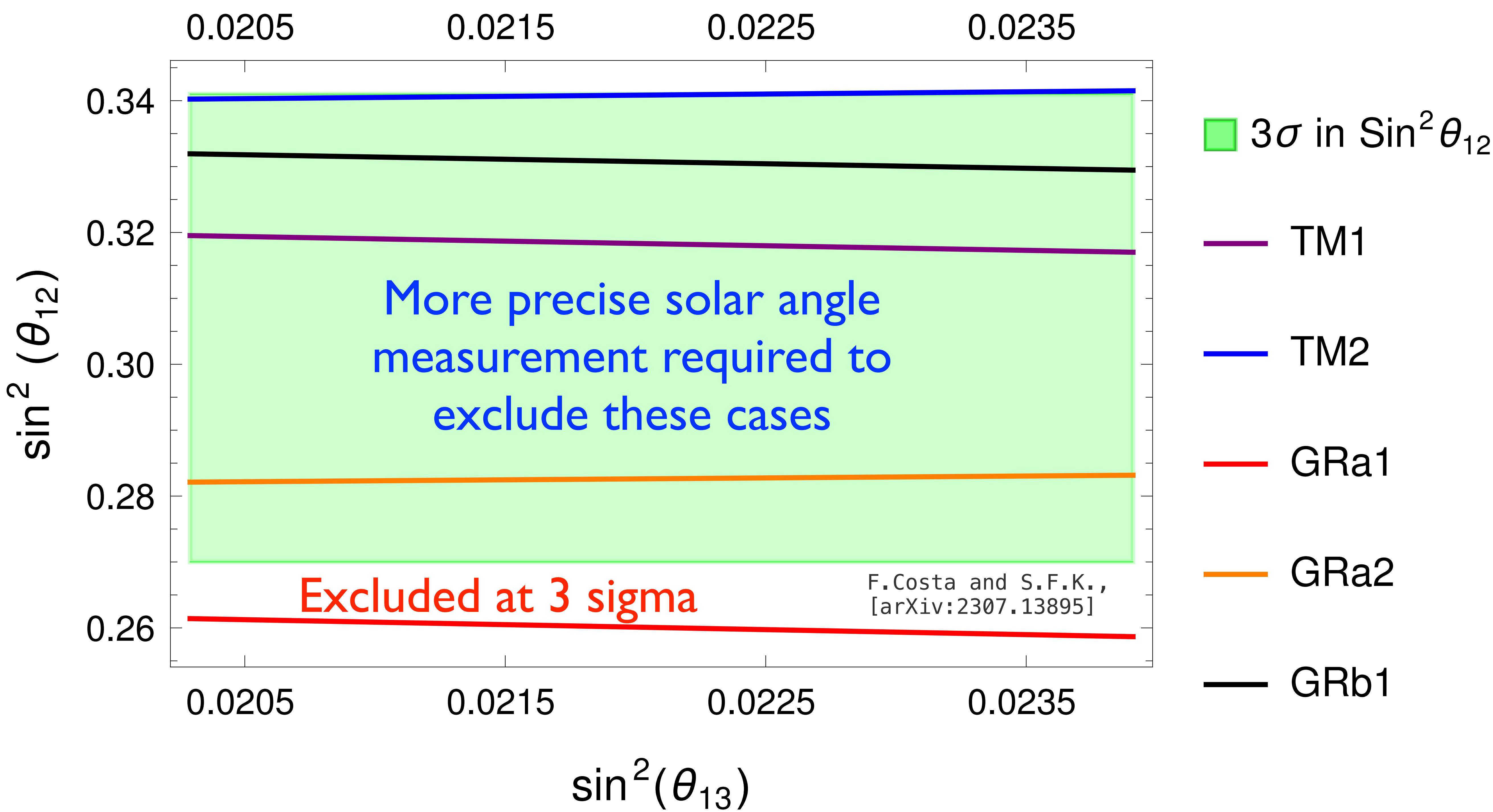
$$|U_{\tau 1}| = |s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta}| = \sqrt{\frac{1}{6}}$$

TM1 Sum Rule prediction for CP phase →

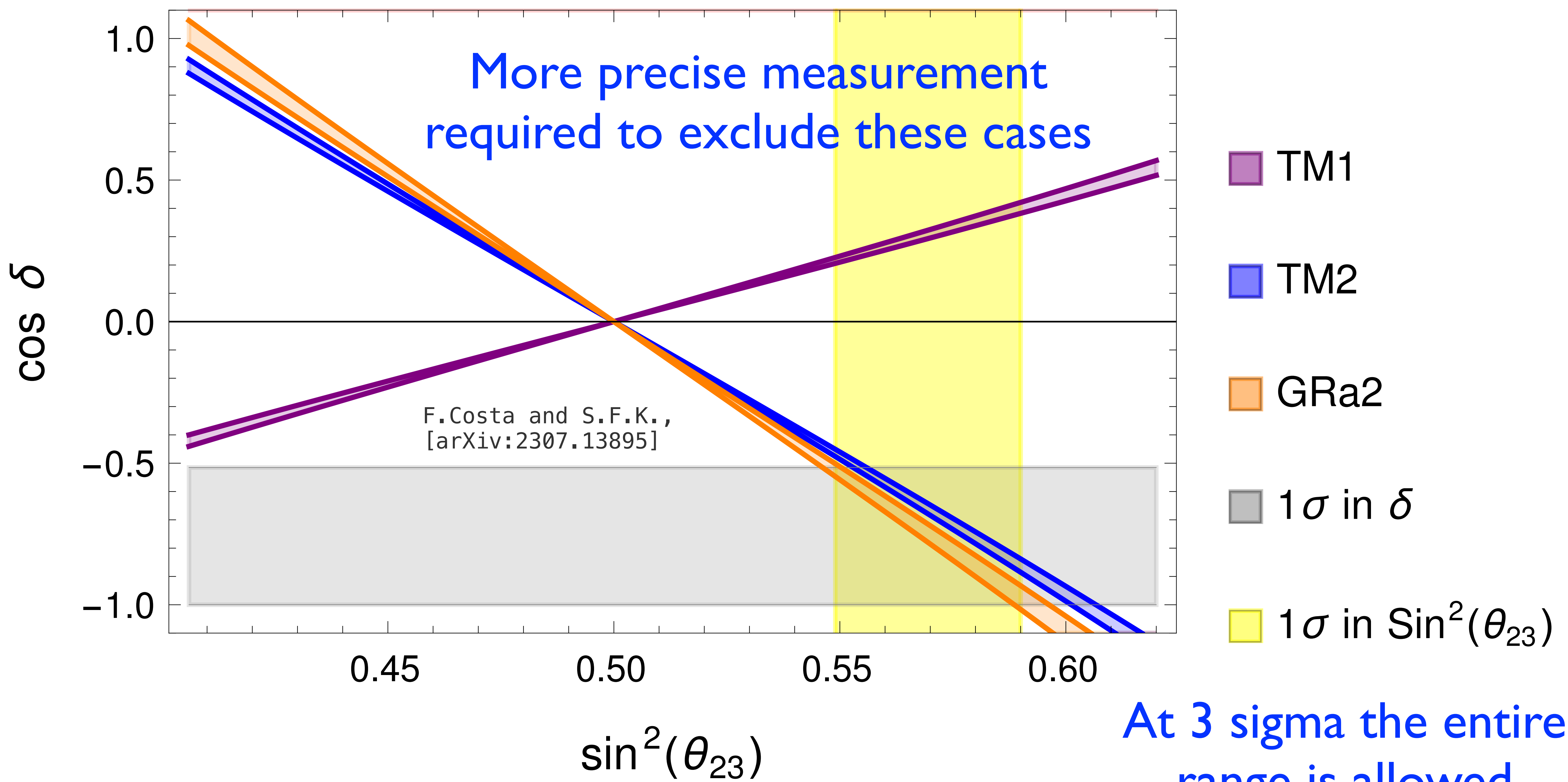
$$\cos\delta = -\frac{\cot 2\theta_{23}(1-5s_{13}^2)}{2\sqrt{2}s_{13}\sqrt{1-3s_{13}^2}}$$

Solar angle predictions from preserved columns

TM1	$\cos\theta_{12} = \sqrt{\frac{2}{3}}\frac{1}{\cos\theta_{13}}$	TM2	$\sin\theta_{12} = \frac{1}{\sqrt{3}\cos\theta_{13}}$
BM1	$\cos\theta_{12} = \frac{1}{\sqrt{2}\cos\theta_{13}}$	BM2	$\cos\theta_{12} = \frac{1}{\sqrt{2}\cos\theta_{13}}$
GRa1	$\cos\theta_{12} = \frac{\cos\theta}{\cos\theta_{13}}$	GRa2	$\cos\theta_{12} = \frac{\sin\theta}{\cos\theta_{13}}$
GRb1	$\cos\theta_{12} = \frac{1+\sqrt{5}}{4\cos\theta_{13}}$	GRb2	$\sin\theta_{12} = \frac{\sqrt{5+\sqrt{5}}}{4\cos\theta_{13}}$
GRc1	$\cos\theta_{12} = \frac{1+\sqrt{5}}{2\sqrt{3}\cos\theta_{13}}$	GRc2	$\sin\theta_{12} = \frac{1+\sqrt{5}}{2\sqrt{3}\cos\theta_{13}}$
HEX1	$\cos\theta_{12} = \frac{\sqrt{3}}{2\cos\theta_{13}}$	HEX2	$\sin\theta_{12} = \frac{1}{2\sqrt{2}\cos\theta_{13}}$

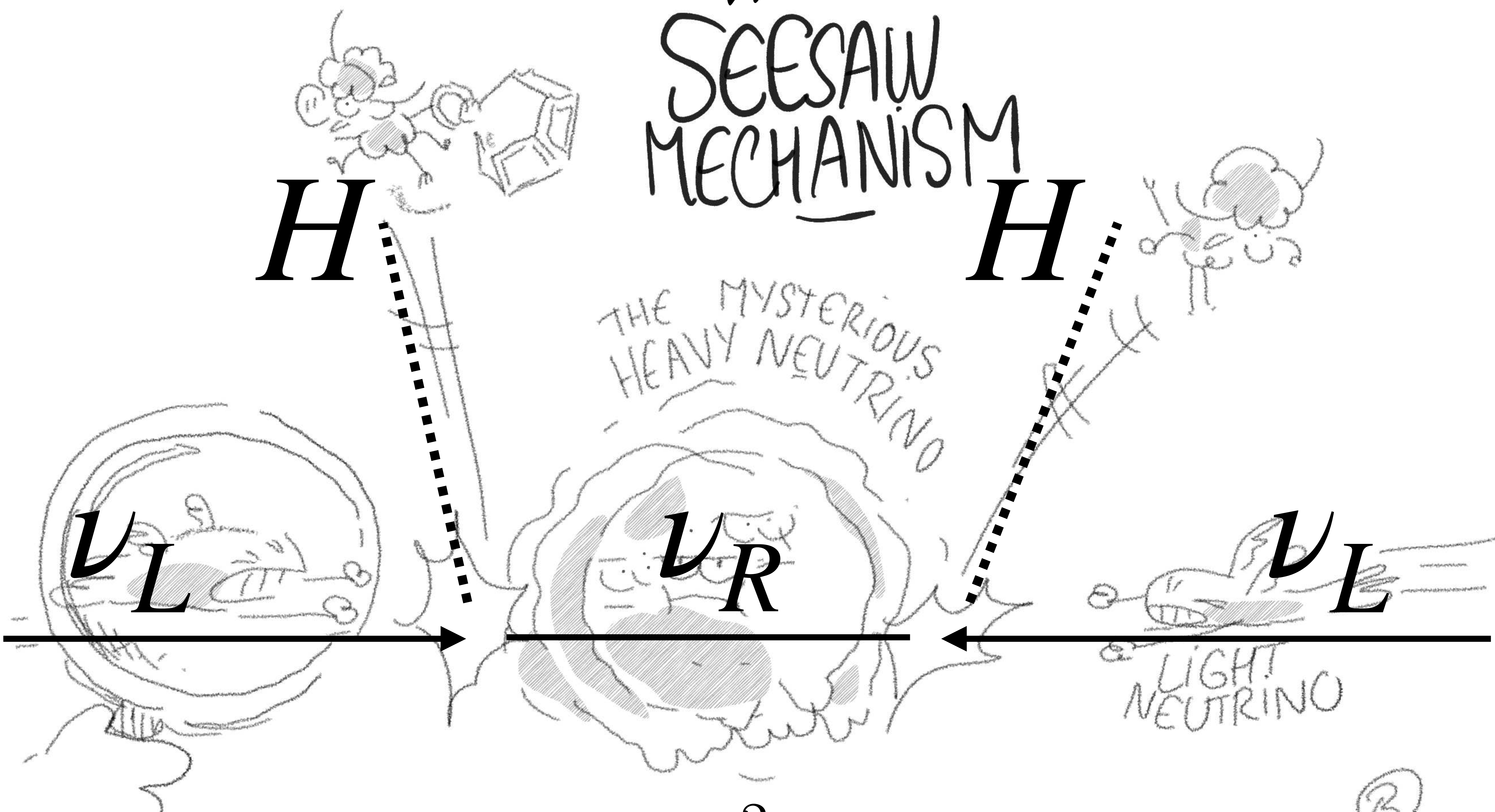


CP phase predictions from preserved columns of simple mixing patterns



At 3 sigma the entire range is allowed

Let's discuss the… *Type Ia* SEESAW MECHANISM



$$m_\nu \approx \frac{m_{\text{Dirac}}^2}{M_R}$$

SFK 9806440, 9912492, 0204360

Sequential Dominance (SD)

Motivation: naturalness and minimality

Assume red RHN dominates seesaw

Assume black RHN is subdominant

Assume primed RHN is irrelevant

Heavy Majorana

Diagonal basis

$$M_{RR} = \begin{pmatrix} Y & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & \not{X'} \end{pmatrix}$$

Dirac

$$m_{LR} = \begin{pmatrix} d & a & \not{a'} \\ e & b & \not{b'} \\ f & c & \not{c'} \end{pmatrix}$$

Predicts normal hierarchy $m_1 = 0$

$$m_3 \sim (e^2 + f^2)/Y \qquad \tan\theta_{23} \sim e/f$$

$$m_2 \sim a^2/(s_{12}^2 X) \qquad \tan\theta_{12} \sim \sqrt{2}a/(b-c)$$

More precise results depend on phases

Further assuming d=0 $\theta_{13} \lesssim m_2/m_3$

Predicted before measurement!

SFK 0506297

Constrained Sequential Dominance

Heavy Majorana → M_{RR}

Diagonal basis

$$M_{RR} = \begin{pmatrix} Y & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & \cancel{X'} \end{pmatrix}$$

Dirac → m_{LR}

$$m_{LR} = \begin{pmatrix} d & a & \cancel{a'} \\ e & b & \cancel{b'} \\ f & c & \cancel{c'} \end{pmatrix}$$

$$m_1 = 0$$

$$\frac{\Delta m_{21}^2}{\Delta m_{31}^2} = \frac{m_2^2}{m_3^2}$$

We can add further constraints to enhance predictivity

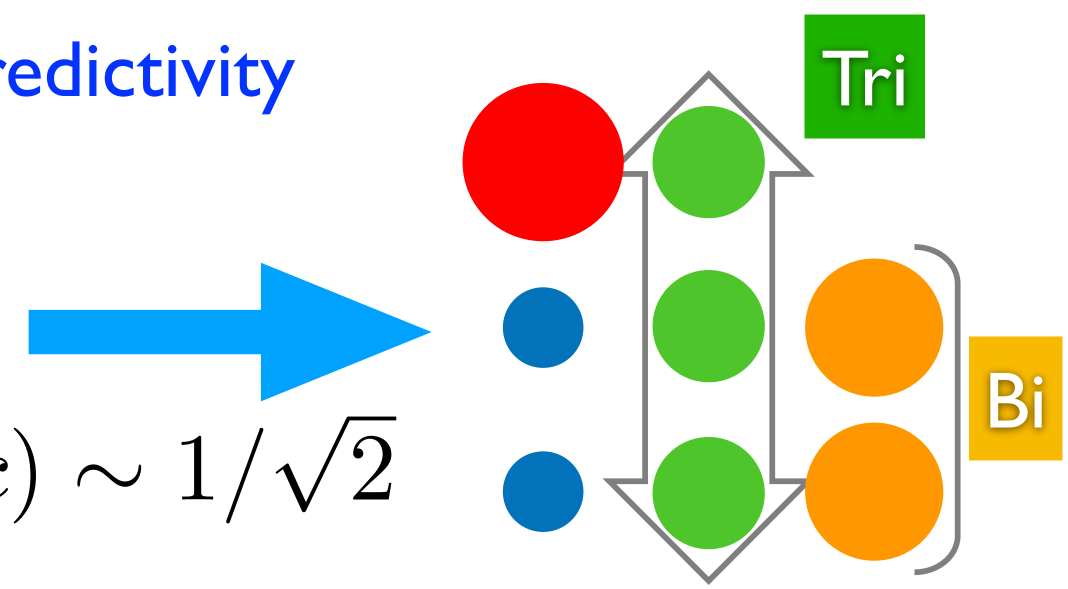
CSD

d=0 $\quad e = f$

$a = b = -c$

$$\tan\theta_{23} \sim e/f \sim 1$$

$$\tan\theta_{12} \sim \sqrt{2}a/(b-c) \sim 1/\sqrt{2}$$



It turns out that this gives exact tri-bimaximal mixing with $\theta_{13} = 0$

Accidentally occurs due to orthogonality of two columns

More general examples called CSD(n) can give approximate TBM with $\theta_{13} \neq 0$

SFK 1304.6264; 1512.07531

Constrained Sequential Dominance

$$M_{RR} = \begin{pmatrix} Y & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & \cancel{X'} \end{pmatrix} \qquad m_{LR} = \begin{pmatrix} d & a & \cancel{a'} \\ e & b & \cancel{b'} \\ f & c & \cancel{c'} \end{pmatrix}$$

More generally assume the two columns of the Dirac matrix are proportional to

$$\begin{pmatrix} d \\ e \\ f \end{pmatrix} \propto \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} a \\ b \\ c \end{pmatrix} \propto \begin{pmatrix} 1 \\ n \\ n-2 \end{pmatrix}$$

CSD(n)

(n=real number)

(can be enforced by symmetry - see later)

$$\tan\theta_{23} \sim e/f \sim 1 \qquad \theta_{13} \neq 0$$

$$\tan\theta_{12} \sim \sqrt{2}a/(b-c) \sim 1/\sqrt{2}$$

Approximate TBM independently of n (which cancels) but depends on phases

The case n=1 corresponds to the exact TBM case previously

But precise results for general n depend on relative phase of columns

CSD(n) possibilities

Two possibilities: Normal Flipped

$$\begin{pmatrix} d \\ e \\ f \end{pmatrix} \propto \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} a \\ b \\ c \end{pmatrix} \propto \begin{pmatrix} 1 \\ n \\ n-2 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} a \\ b \\ c \end{pmatrix} \propto \begin{pmatrix} 1 \\ n-2 \\ n \end{pmatrix}$$

The two predictions only differ in atmospheric angle and CP phase (solar angle, reactor angle and neutrino mass unchanged) (n= real number)

Octant flipped $\tan\theta_{23} \to \cot\theta_{23}$ $\delta \to \delta + \pi$

Alternatively we could use the following (only differs by unphysical phases):

$$\begin{pmatrix} d \\ e \\ f \end{pmatrix} \propto \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \qquad \begin{pmatrix} a \\ b \\ c \end{pmatrix} \propto \begin{pmatrix} 1 \\ n \\ 2-n \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} a \\ b \\ c \end{pmatrix} \propto \begin{pmatrix} 1 \\ 2-n \\ n \end{pmatrix}$$

CSD(n) results

Original case:

$$\begin{pmatrix} d \\ e \\ f \end{pmatrix} \propto \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} a \\ b \\ c \end{pmatrix} \propto \begin{pmatrix} 1 \\ n \\ n-2 \end{pmatrix}$$

$$m^{\nu}_{(n)} = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_b e^{i\eta} \begin{pmatrix} 1 & n & n-2 \\ n & n^2 & n(n-2) \\ n-2 & n(n-2) & (n-2)^2 \end{pmatrix}$$

Shows n~3 is viable! Bjorkeroth, SFK 1412.6996

$$\theta_{13} \sim (n-1)\frac{\sqrt{2}}{3}\frac{m_2}{m_3}$$

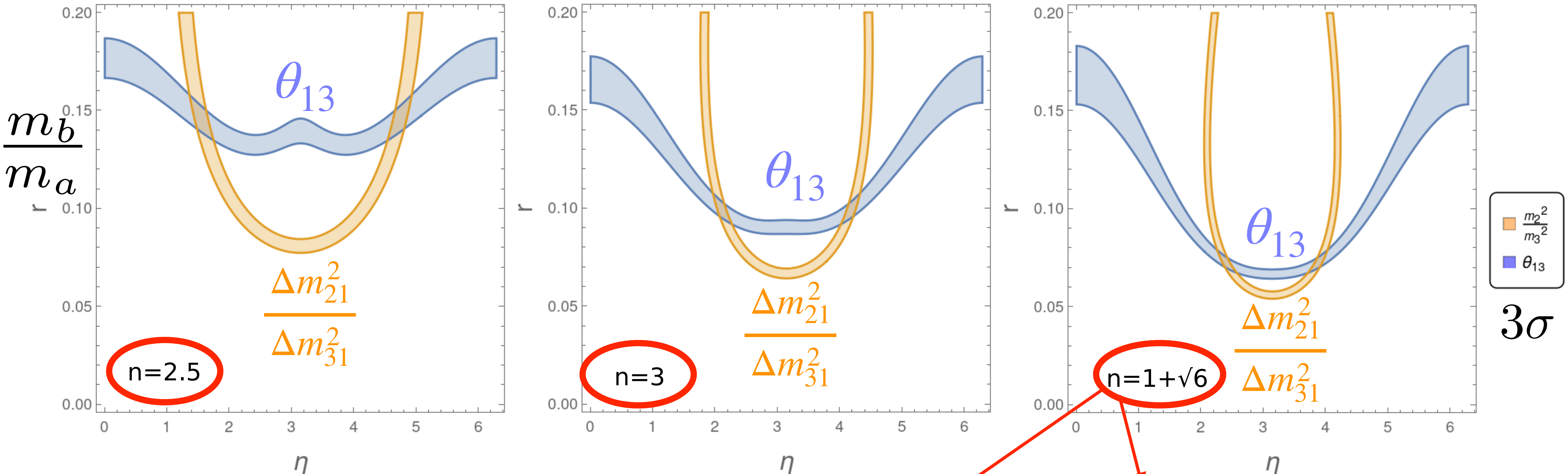
$m_1 = 0$

n	m_a (meV)	m_b (meV)	η (rad)	θ_{12} (°)	θ_{13} (°)	θ_{23} (°)	$\lvert\delta_{CP}\rvert$ (°)	m_2 (meV)	m_3 (meV)	χ^2	
1	24.8	2.89	3.14	35.3	0	45.0	0	8.66	49.6	485	CSD(1)=TBM
2	19.7	3.66	0	34.5	7.65	56.0	0	8.85	48.8	95.1	CSD(2) Antusch 1108.4278
3	27.3	2.62	2.17	34.4	8.39	44.5	92.2	8.69	49.5	3.98	CSD(3) SFK 1304.6264
4	36.6	1.95	2.63	34.3	8.72	38.4	120	8.61	49.8	8.82	CSD(4) SFK 1305.4846
5	45.9	1.55	2.88	34.2	9.03	34.4	142	8.53	50.0	33.8	

SFK 1512.07531 F.Costa, S.F.K., 2307.13895 Ding, SFK

CSD(~3) =Littlest Seesaw

Highly predictive - 3 inputs for 9 observables (6 "measured")



Use θ_{13} and $\frac{\Delta m^2_{21}}{\Delta m^2_{31}}$ to fit η and $\frac{m_b}{m_a}$.

Then θ_{12} , θ_{23} and δ are all predicted!

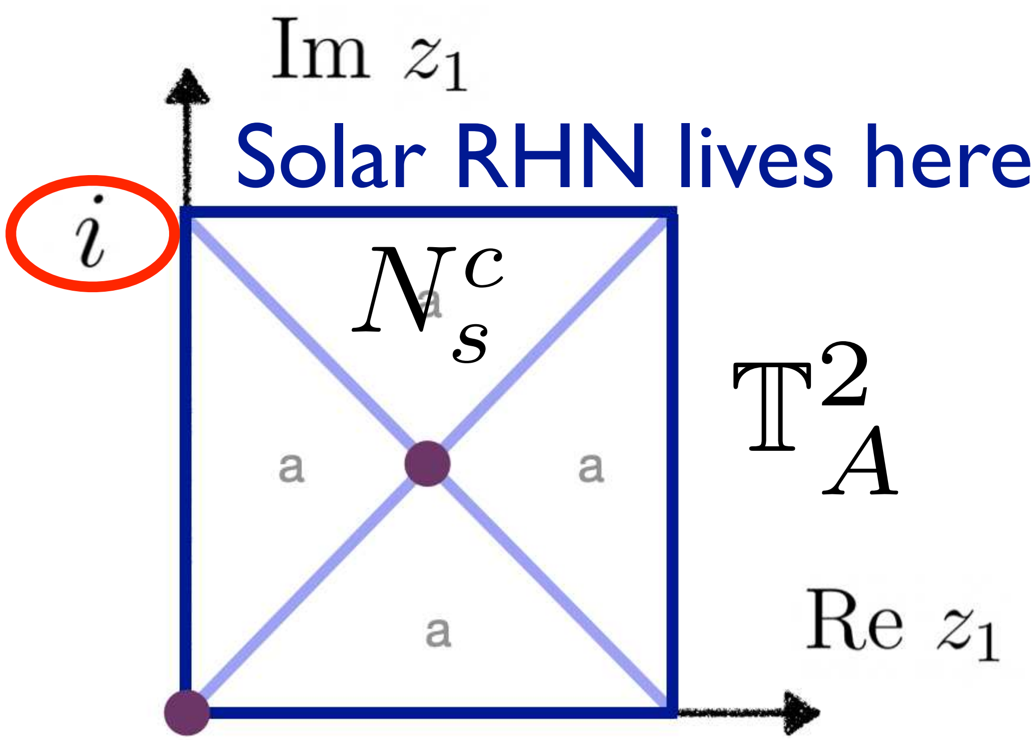
Precision required

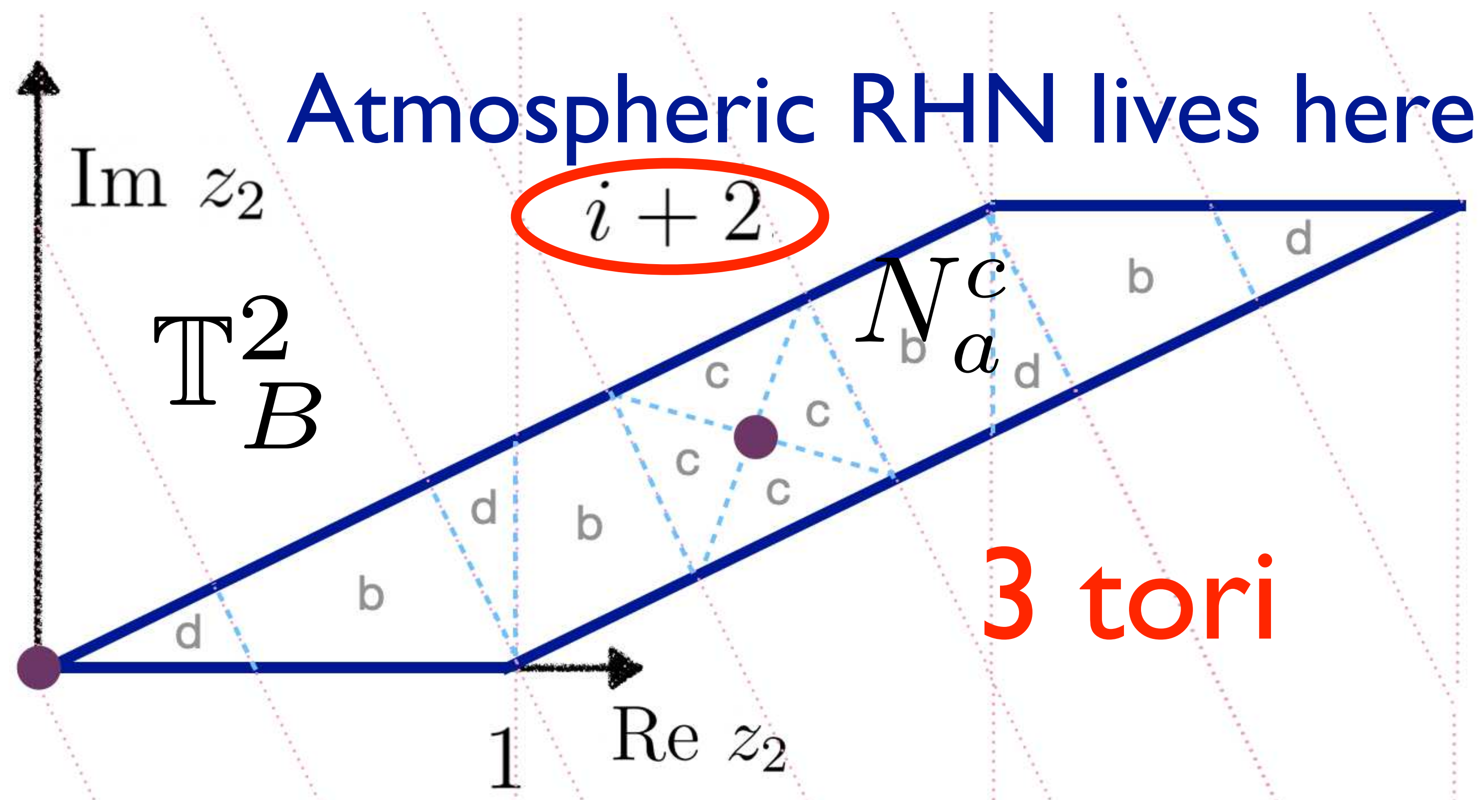
Modular Littlest seesaw			Flipped modular Littlest seesaw		
	bf	allowed ranges		bf	allowed ranges
η/π	1.240	$[1.197, 1.276]$	η/π	0.742	$[0.725, 0.806]$
r	0.0734	$[0.0684, 0.0786]$	r	0.0758	$[0.0683, 0.0786]$
$\sin^2\theta_{13}$	0.0223	$[0.0205, 0.0240]$	$\sin^2\theta_{13}$	0.0231	$[0.0205, 0.0240]$
$\sin^2\theta_{12}$	0.318	$[0.317, 0.319]$	$\sin^2\theta_{12}$	0.318	$[0.317, 0.319]$
$\sin^2\theta_{23}$	0.447	$[0.408, 0.483]$	$\sin^2\theta_{23}$	0.535	$[0.517, 0.595]$
δ_{CP}/π	-0.575	$[-0.640, -0.522]$	δ_{CP}/π	-0.452	$[-0.478, -0.354]$
β/π	0.474	$[0.408, 0.555]$	β/π	-0.441	$[-0.562, -0.409]$
m_2^2/m_3^2	0.0297	$[0.0270, 0.0321]$	m_2^2/m_3^2	0.0283	$[0.0270, 0.0321]$

De Anda, SFK 2304.05958

Littlest Modular Seesaw $n = 1 + \sqrt{6}$

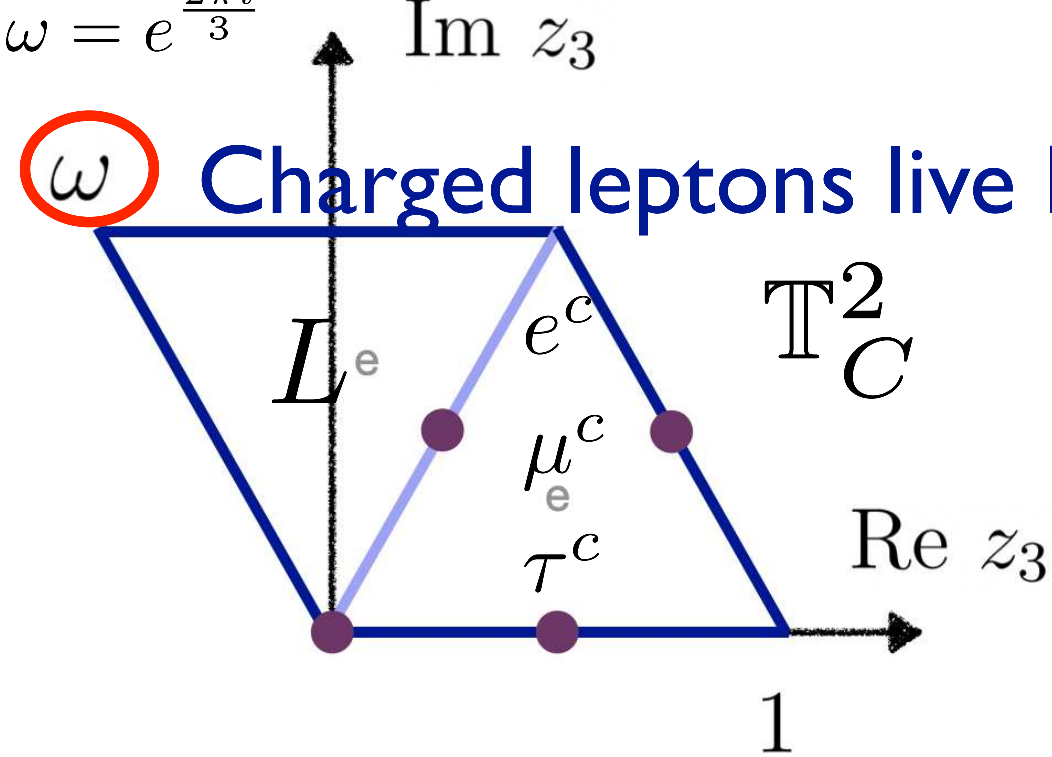
10d model with orbifold $(\mathbb{T}^2)^3/(\mathbb{Z}_4 \times \mathbb{Z}_2)$





3 tori

$\omega = e^{\frac{2\pi i}{3}}$



Lattice vectors for each torus are $(1, \tau_i)$

$$\tau_1 = i, \ \ \tau_2 = i + 2, \ \ \tau_3 = \omega$$

which define 3 fixed moduli

I.de Medeiros Varzielas, S.F.K. and M.Levy, 2211.00654   De Anda, SFK 2304.05958

Littlest Modular Seesaw $n = 1+\sqrt{6}$

Field	S_4^A	S_4^B	S_4^C	$2k_A$	$2k_B$	$2k_C$	Loc
L	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	0	0	0	$\mathbb{T}_C^2$
e^c	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	0	-6	$\mathbb{T}_C^2$
μ^c	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	0	-4	$\mathbb{T}_C^2$
τ^c	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	0	-2	$\mathbb{T}_C^2$
N_a^c	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	-4	0	$\mathbb{T}_B^2$
N_s^c	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	-2	0	0	$\mathbb{T}_A^2$
Φ_{BC}	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	0	0	0	Bulk
Φ_{AC}	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3}$	0	0	0	Bulk

Yuk/Mass	S_4^A	S_4^B	S_4^C	$2k_A$	$2k_B$	$2k_C$
$Y_e(\tau_3)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	0	0	6
$Y_\mu(\tau_3)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	0	0	4
$Y_\tau(\tau_3)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	0	0	2
$Y_a(\tau_2)$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	0	4	0
$Y_s(\tau_1)$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	2	0	0
$M_a(\tau_2)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	8	0
$M_s(\tau_1)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	4	0	0

Yukawa couplings are modular forms evaluated at the fixed points of the moduli fields (the lattice vectors)

G.J.Ding, S.F.K, X.G.Liu and J.N.Lu,1910.03460
G.J.Ding, S.F.K. and C.Y.Yao, 2103.16311

	τ	$Y_{\mathbf{3}}^{(2)}(\tau),\ Y_{\mathbf{3,I}}^{(6)}(\tau)$	$Y_{\mathbf{3}}^{(4)}(\tau),\ Y_{\mathbf{3'}}^{(6)}(\tau)$
τ_1	i	$(1, 1+\sqrt{6}, 1-\sqrt{6})$	$(1, -\frac{1}{2}, -\frac{1}{2})$
	$i+1$	$(1, -\frac{\omega}{3}(1+i\sqrt{2}), -\frac{\omega^2}{3}(1+i\sqrt{2}))$	$(0, 1, -\omega)$
τ_2	$i+2$	$(1, \frac{1}{3}(-1+i\sqrt{2}), \frac{1}{3}(-1+i\sqrt{2}))$	$(0, 1, -1)$
	$i+3$	$(1, \omega(1+\sqrt{6}), \omega(1-\sqrt{6}))$	$(1, -\frac{\omega}{2}, -\frac{\omega^2}{2})$

	τ	$Y_{\mathbf{3}}^{(2)}(\tau)$	$Y_{\mathbf{3}}^{(4)}(\tau), Y_{\mathbf{3'}}^{(4)}(\tau)$	$Y_{\mathbf{3,II}}^{(6)}(\tau), Y_{\mathbf{3'}}^{(6)}(\tau)$
τ_3	ω	$(0, 1, 0)$	$(0, 0, 1)$	$(1, 0, 0)$
	$\omega+1$	$(1, 1, -\frac{1}{2})$	$(1, -\frac{1}{2}, 1)$	$(1, -2, -2)$
	$\omega+2$	$(1, -\frac{\omega^2}{2}, \omega)$	$(1, \omega^2, -\frac{\omega}{2})$	$(1, -2\omega^2, -2\omega)$
	$\omega+3$	$(1, \omega, -\frac{\omega^2}{2})$	$(1, -\frac{\omega}{2}, \omega^2)$	$(1, -2\omega, -2\omega^2)$

$\omega = e^{\frac{2\pi i}{3}}$

$$\frac{1}{\Lambda}\left[L\Phi_{BC}Y_aN_a^c + L\Phi_{AC}Y_sN_s^c\right]H_u + \left[LY_ee^c + LY_\mu\mu^c + LY_\tau\tau^c\right]H_d + \frac{1}{2}M_aN_a^cN_a^c + \frac{1}{2}M_sN_s^cN_s^c\,.$$

Flipped case (2nd octant)

$$\begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} \qquad \begin{pmatrix} 0 & b \\ a & b\left(1-\sqrt{6}\right) \\ -a & b\left(1+\sqrt{6}\right) \end{pmatrix}$$

Charged leptons   Dirac neutrinos

Conclusions

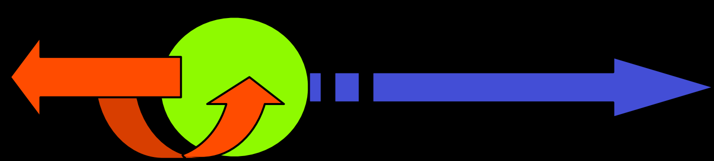
- Mixing sum rules are relics of simple mixing patterns enforced by symmetry and predict $\cos\delta$ (not δ)
- Discussed minimal predictive Type Ia seesaw CSD(n)
- "Littlest Seesaw" $n \approx 3$ predicts θ_{12} , θ_{23} and δ
- n=$1+\sqrt{6} \approx$ 3.45 enforced by modular symmetry
- Such theories motivate precision measurements

Dirac or Majorana?

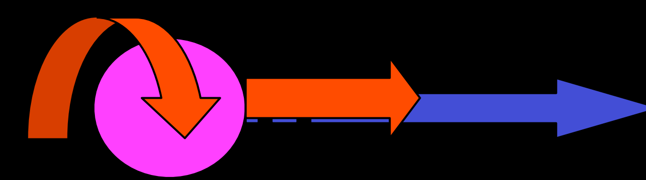
ν_L

Dirac

ν_R



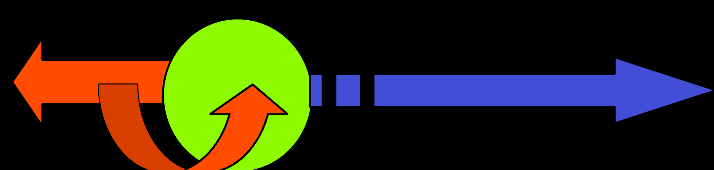
$m_{\mathrm{D}}\bar{\nu}_L \nu_R$



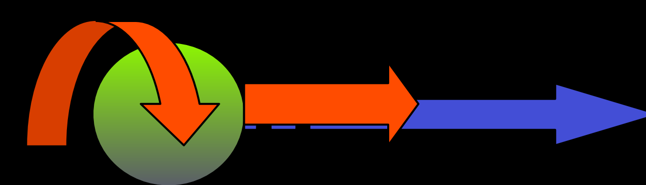
ν_L

Majorana

ν_L^c



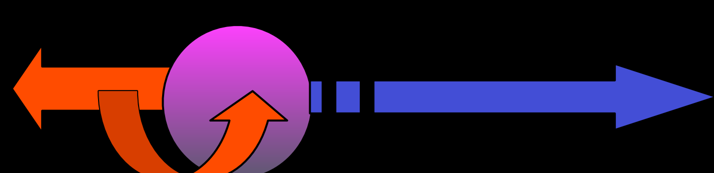
$m_{\mathrm{L}}\bar{\nu}_L \nu_L^c$



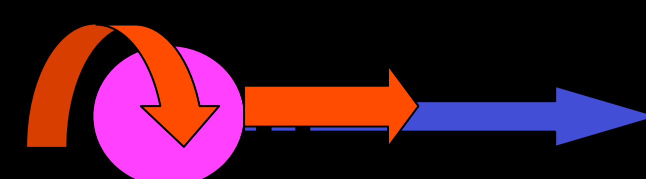
ν_R^c

Majorana

ν_R



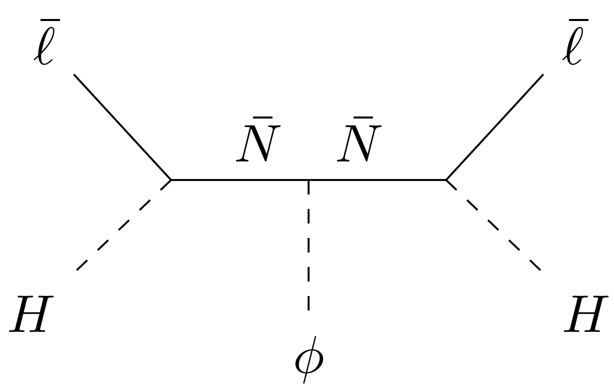
$M_{\mathrm{R}}\bar{\nu}_R^c \nu_R$



SFK, Marfatia, Rahat 2306.05389

Majorana vs. Dirac Seesaw

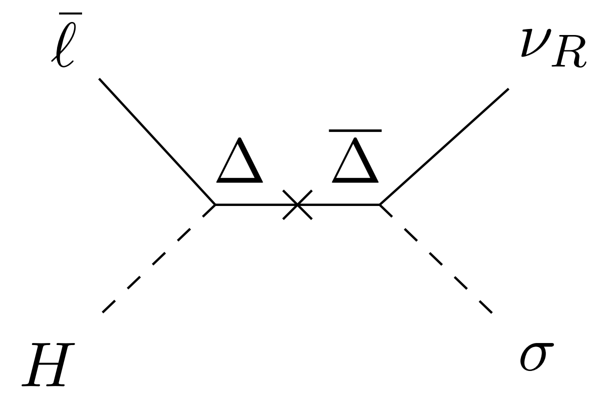
$$-\mathcal{L}_M \supset \mathcal{Y}\ \bar{\ell} H \bar{N} + \bar{N}\bar{N}^T\phi$$



$$\mathcal{M}_M = \frac{1}{\sqrt{2}} v^2\ \mathcal{Y}\ \mathcal{M}_N^{-1}\ \mathcal{Y}^T$$

U(1)$_{B-L}$ broken
Cosmic string
GWs

$$-\mathcal{L}_D \supset \mathcal{Y}_L \bar{\ell} H \Delta_R + \mathcal{Y}_R \overline{\Delta}_L \sigma \nu_R + \mathcal{M}_\Delta \overline{\Delta}\Delta$$



odd
Z_2
odd

$$\mathcal{M}_D = \frac{1}{\sqrt{2}}\ v\ u\ \mathcal{Y}_L \mathcal{M}_\Delta^{-1} \mathcal{Y}_R$$

U(1)$_L$ preserved
Z_2 broken
Domain Walls
GWs

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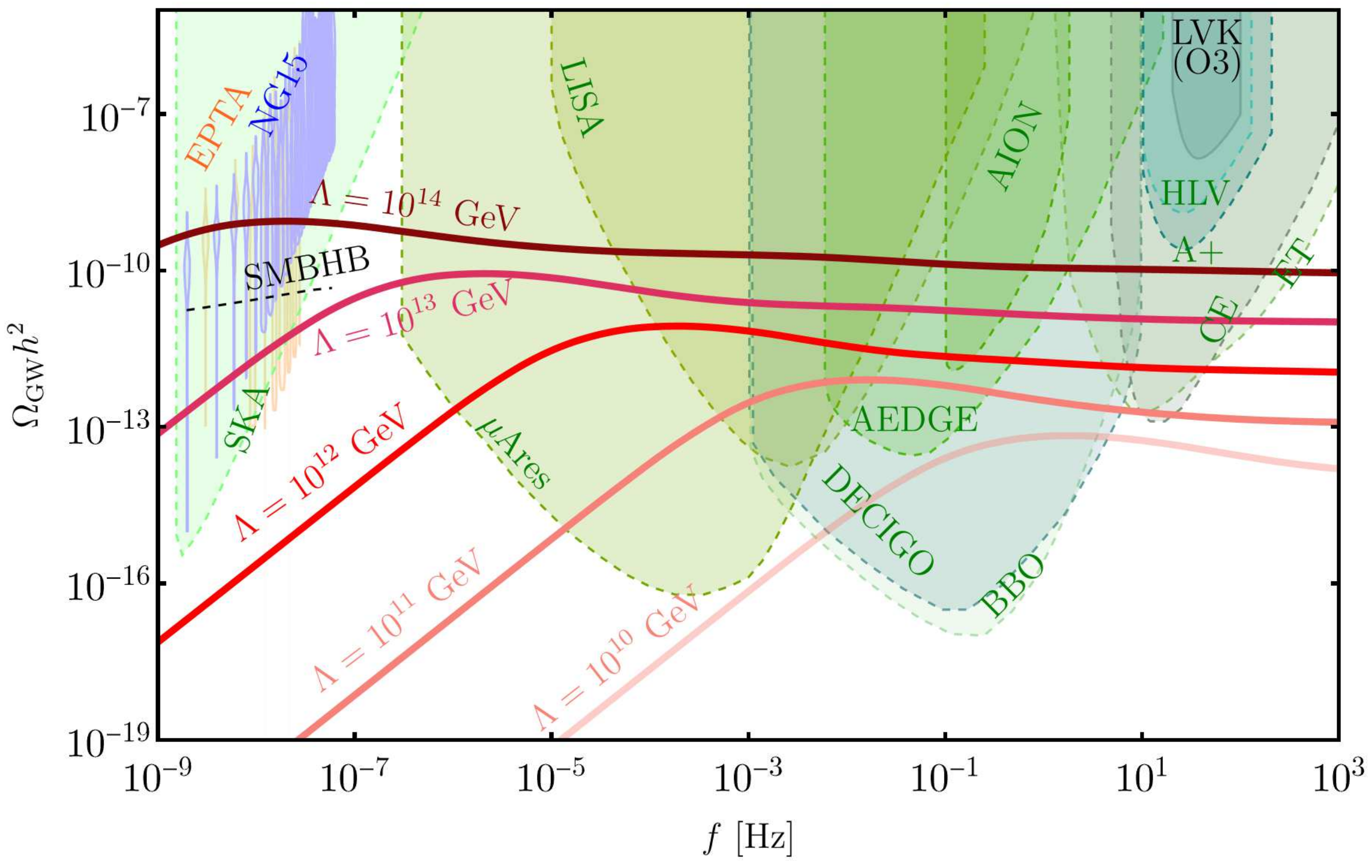
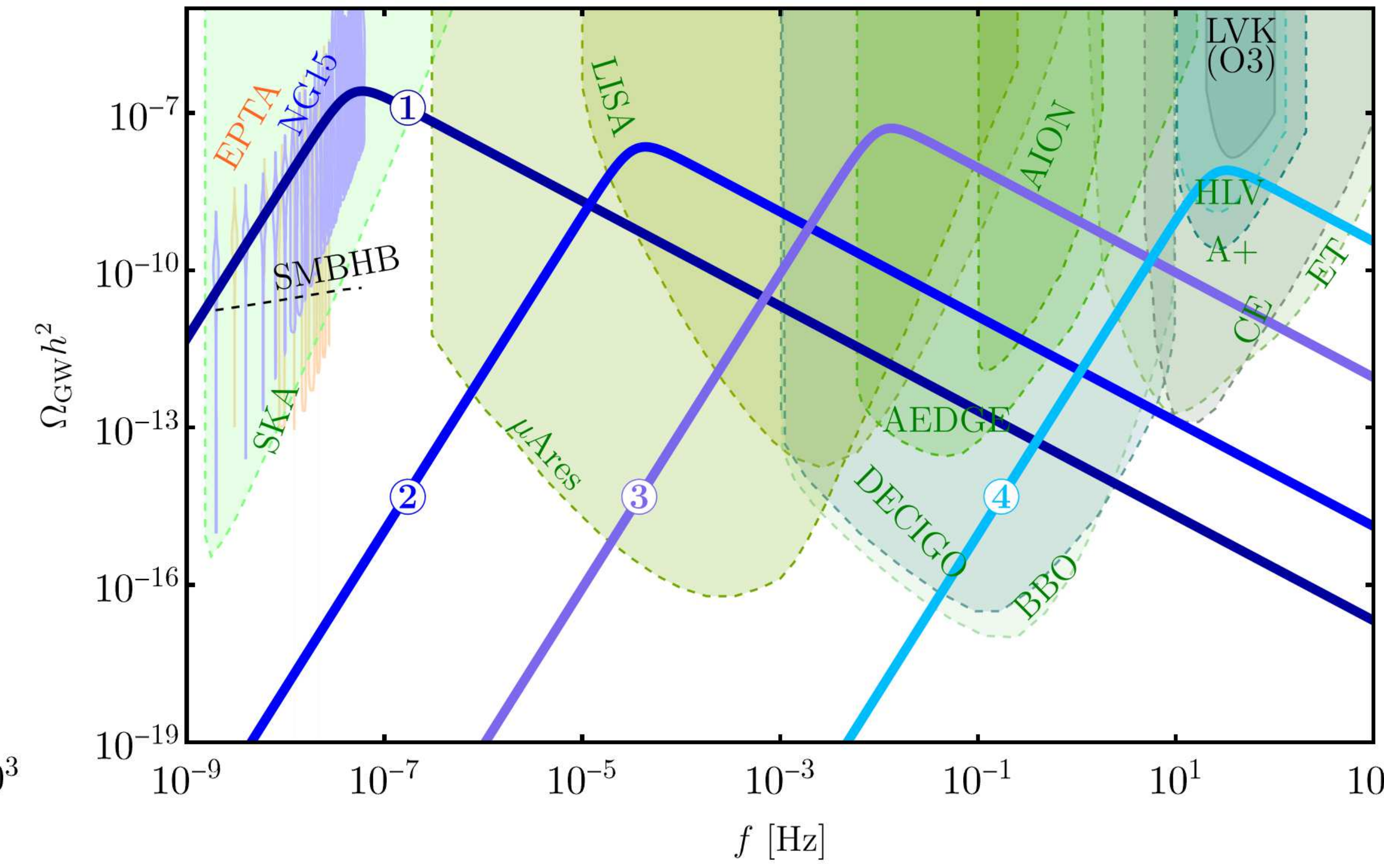
Majorana vs. Dirac





Flat GW spectrum from cosmic strings

Peaked GW spectrum from domain walls

Benchmark Point	u [GeV]	$V_{\rm bias}$ [GeV^4]	$y_{\max}(M_\Delta < M_{\rm Pl})$
①	0.97×10^6	0.86	2.70
②	5.2×10^7	7.14×10^{10}	0.37
③	2.7×10^9	9.3×10^{20}	0.051
④	3.63×10^{11}	1.38×10^{34}	0.004

Majorana vs Dirac can be distinguished from shape of GW spectrum

$$V(\sigma) = \frac{\lambda}{4}(\sigma^2 - u^2)^2 \qquad \Delta V(\sigma) = \epsilon u \sigma \left(\frac{\sigma^2}{3} - u^2\right)$$

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NANOGrav 15-year data