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Production of primordial black holes in single-field models of inflation

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joint work with Daniel Frolovsky and Sultan Saburov

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Plan of talk

- Modified gravity and quintessence in four spacetime dimensions
- Starobinsky model of inflation and (current/future) CMB measurements (Planck, BICEP/Keck, LiteBIRD) = *Introduction*
- Single-field extensions of the Starobinsky potential for inflation
- Production of primordial black holes (PBH) in *generalized* E- and T-models
- Production of PBH in $F(R)$ modified gravity
- PBH dark matter, induced gravitational waves (GW) and their detection
- Conclusion

Modified gravity

- Modified gravity theories are generally-covariant non-perturbative extensions of Einstein-Hilbert gravity theory by the higher-order terms. These terms are irrelevant in the Solar system but are relevant in the high-curvature regimes (inflation, black holes) or for large cosmological distances (dark energy).
- A modified gravity action has the higher-derivatives and generically suffers from Ostrogradsky instability and ghosts. However, there are exceptions. For example, in the modified gravity Largrangian quadratic in the spacetime curvature, the only ghost-free term is given by R^2 with a positive coefficient. It leads to the Starobinsky model (1980) of modified gravity with the action

$$S_{\rm Star.} = \frac{M_{\rm Pl}^2}{2} \int d^4x \sqrt{-g} \left(R + \frac{1}{6M^2} R^2 \right) \equiv \frac{M_{\rm Pl}^2}{2} \int d^4x \sqrt{-g}\, F(R) ,$$

having the only (mass) parameter M, where $M_{\rm Pl} = 1/\sqrt{8\pi G_{\rm N}} \approx 2.4 \times 10^{18}$ GeV, the spacetime signature is $(-,+,+,+,)$.

Starobinsky model of inflation

- In the high-curvature regime, the EH term can be ignored and the pure R^2-action becomes scale-invariant.
- The Starobinsky gravity has the special (attractor) solution in the FLRW universe with the Hubble function

$$H(t) \approx \left(\frac{M}{6}\right)^2 (t_{\text{end}} - t) \ ,$$

for $M(t_{\text{end}} - t) \gg 0$. This solution spontaneously breaks the scale invariance of the R^2-gravity and, hence, implies the existence of the associated Nambu-Goldstone boson called scalaron.

- Scalaron is the physical (scalar) excitation of the higher-derivative gravity. It can be revealed by rewriting the Starobinsky action into the quintessence form by the field redefinition (Legendre-Weyl transform)

$$\varphi = \sqrt{\frac{3}{2}} M_{\rm Pl} \ln F'(\chi) \quad \text{and} \quad g_{\mu\nu} \to \frac{2}{M_{\rm Pl}^2} F'(\chi) g_{\mu\nu} \, , \quad \chi = R \, ,$$

which leads to

$$S[g_{\mu\nu}, \varphi] = \frac{M_{\rm Pl}^2}{2} \int {\rm d}^4x \sqrt{-g} R - \int {\rm d}^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + V(\varphi) \right] \, ,$$

with the potential $V(\varphi) = \frac{3}{4} M_{\rm Pl}^2 M^2 \left[1 - \exp\left(-\sqrt{\frac{2}{3}} \varphi / M_{\rm Pl} \right) \right]^2 \equiv V_0 [1 - y]^2$. This potential is suitable for describing slow-roll inflation with scalaron φ as the inflaton of mass m due to the infinite plateau of the positive height $\approx V_0$ for $y \ll 1$.

- The UV cutoff of the potential is $\Lambda_{\rm UV} = M_{\rm Pl}$. The higher-order curvature terms are supposed to be suppressed by $M_{\rm Pl} \gg M$. A string theory derivation of the Starobinsky inflation is still challenging (unknown).

Starobinsky model (1980) and CMB measurements (2020)

No phenomenological input was used so far. Nevertheless, the very simple Starobinsky model of inflation is still in excellent agreement with the current CMB measurements (Planck, BICEP/Keck).

A duration of inflation is usually measured by the e-foldings number

$$N = \int_{t_*}^{t_{\text{end}}} H(t)dt \approx \frac{1}{M_{\text{Pl}}^2} \int_{\varphi_{\text{end}}}^{\varphi_*} \frac{V}{V'} d\varphi \ .$$

The standard slow roll parameters are defined by

$$\varepsilon_{\text{sr}}(\varphi) = \frac{M_{\text{Pl}}^2}{2} \left(\frac{V'}{V}\right)^2 \quad \text{and} \quad \eta_{\text{sr}}(\varphi) = M_{\text{Pl}}^2 \left(\frac{V''}{V}\right) \ .$$

The amplitude of scalar (curvature) perturbations at the horizon crossing with the pivot scale $k_* = 0.05\ \mathrm{Mpc}^{-1}$ is determined by the WMAP normalization,

$$A_s = \frac{V_*^3}{12\pi^2 M_{\mathrm{Pl}}^6 (V_*{}')^2} = \frac{3M^2}{8\pi^2 M_{\mathrm{Pl}}^2} \sinh^4\left(\frac{\varphi_*}{\sqrt{6}M_{\mathrm{Pl}}}\right) \approx 1.96 \cdot 10^{-9}$$

that implies no free parameters in the Starobinsky model,

$$M \approx 3 \cdot 10^{13}\ \mathrm{GeV} \quad \mathrm{or} \quad \frac{M}{M_{\mathrm{Pl}}} \approx 1.3 \cdot 10^{-5}\,, \quad \mathrm{and} \quad H \approx \mathcal{O}(10^{14})\ \mathrm{GeV}\,.$$

The CMB measurements give the tilt of scalar perturbations $n_s \approx 1 + 2\eta_{\mathrm{sr}} - 6\varepsilon_{\mathrm{sr}} \approx 0.9649 \pm 0.0042\ (68\%\mathrm{CL})$ and restrict the tensor-to-scalar ratio as $r \approx 16\varepsilon_{\mathrm{sr}} < 0.032\ (95\%\mathrm{CL})$. The Starobinsky inflation gives $r \approx 12/N^2 \approx 0.003$ and $n_s \approx 1 - 2/N$, with the best fit at $N \approx 55$.

Single-field extensions of Starobinsky potential

The Starobinsky inflaton potential can be generalized to the α-attractors (Kallosh, Linde, 2013) either by modifying the exponential term as (called E-models)

$$y = \exp\left(-\sqrt{\frac{2}{3\alpha}}\frac{\varphi}{M_{\mathrm{Pl}}}\right)$$

with the parameter $\alpha > 0$, or/and by using another function (called T-models)

$$V(\varphi) = V_0 \tanh^2\left(\frac{\varphi/M_{\mathrm{Pl}}}{\sqrt{6\alpha}}\right) \equiv V_0 u^2 \,, \quad u = \tanh\frac{\varphi/M_{\mathrm{Pl}}}{\sqrt{6\alpha}} \,.$$

These extensions maintain the Mukhanov-Chibisov formula for the tilt of scalar perturbations, $n_s \approx 1 - \frac{2}{N}$ but modify the tensor-to-scalar ratio as $r_\alpha \approx \frac{12\alpha}{N^2}$, so that $r_\alpha \approx 3\alpha(1-n_s)^2$.

Further generalizations of T-models and E-models

It is possible to go further, while keeping agreement with CMB observations, by defining the generalized T-type α-attractors with the scalar potential (Kallosh, Linde, 2013)

$$V_{\mathrm{T-gen.}}(\varphi) = f^2\left(\tanh\frac{\varphi/M_{\mathrm{Pl}}}{\sqrt{6\alpha}}\right) \equiv f^2(u)\ ,$$

and the generalized E-type α-attractors (Vernov, Pozdeeva, SVK, 2021) with the potential

$$V_{\mathrm{E-gen.}}(\varphi) = \frac{3}{4}M_{\mathrm{Pl}}^2M^2\left[1-y+y^2\zeta(y)\right]^2\ ,$$

with regular functions $f(u)$ and $\zeta(y)$ that do not significantly affect the CMB tilts. The idea: *use this functional freedom to produce PBH on the scales below the inflationary scale.* (See also Dalianis, Kehagias, Tringas, 2019). The Starobinsky model is reproduced with $\alpha = 1$, $\zeta(y) = 0$ and $f(u) = \sqrt{3}M_{\mathrm{Pl}}^2M^2u/(1+u)$.

Power spectrum of perturbations

Primordial scalar perturbations (ζ) and tensor perturbations g (primordial GW) are defined by a perturbed FLRW metric,

$$ds^2 = dt^2 - a^2(t)\left(\delta_{ij} + h_{ij}(\vec{r})\right)dx^i dx^j \quad , \qquad i,j = 1,2,3 \; ,$$

where

$$h_{ij}(\vec{r}) = 2\zeta(\vec{r})\delta_{ij} + \sum_{b=1,2} g^{(b)}(\vec{r}) e_{ij}^{(b)}(\vec{r}) \quad , \quad H = \frac{da/dt}{a} \; ,$$

in terms of a local basis $e^{(b)}$ with $e_i^{i(b)} = 0$, $g_{,j}^{(b)} e_i^{j(b)} = 0$, $e_{ij}^{(b)} e^{ij(b)} = 1$.
The primordial spectrum $P_\zeta(k)$ of scalar (density) perturbations is defined by the 2-point correlation function of scalar perturbations,

$$\left\langle \frac{\delta\zeta(x)}{\zeta}\frac{\delta\zeta(y)}{\zeta} \right\rangle = \int \frac{d^3k}{k^3} e^{ik\cdot(x-y)} \frac{P_\zeta(k)}{P_0} \quad .$$

For instance, the observed CMB power spectrum is described by the Harrison-Zeldovich fit,

$$P_\zeta^{\rm HZ}(k) \approx 2.21^{+0.07}_{-0.08} \times 10^{-9} \left(\frac{k}{k_*}\right)^{n_s-1}$$

with the pivot scale $k_* = 0.05~{\rm Mpc}^{-1}$. In the slow-roll (SR) approximation, relevant for inflation, one finds

$$P_\zeta = \frac{H^2}{8M_{\rm Pl}^2\pi^2}\left(\frac{1}{\varepsilon_{\rm sr}}\right) \quad .$$

Therefore, it is possible to generate a large peak (enhancement) in the power spectrum by engineering $\epsilon_{\rm sr} \to 0$, called the ultra-slow-roll (USR) regime or the PBH production mechanism based a near-inflection point in the potential. This implies the double inflation scenario (SR $\to$ USR $\to$ SR) with two plateaus in the potential $V(\varphi)$ and in the Hubble function $H(t)$. Warning: USR is not SR !

Our generalized E-model

is defined by the potential with the dimensionless parameters $(\alpha, \beta, \gamma, \theta)$ as

$$V(\varphi) = \frac{3}{4} M_{\mathrm{Pl}}^2 M^2 \left[1 - y + \theta y^{-2} + y^2(\beta - \gamma y) \right]^2, \; y = \exp\left(-\sqrt{\frac{2}{3\alpha}} \frac{\varphi}{M_{\mathrm{Pl}}} \right).$$

Let us replace (β, γ) with the new parameters (ϕ_i, ξ) having better meaning as

$$\beta = \frac{1}{1-\xi^2} \exp\left[\sqrt{\frac{2}{3\alpha}} \frac{\phi_i}{M_{\mathrm{Pl}}} \right] , \quad \gamma = \frac{1}{3(1-\xi^2)} \exp\left[2\sqrt{\frac{2}{3\alpha}} \frac{\phi_i}{M_{\mathrm{Pl}}} \right] .$$

When $\xi = 0$, the potential has an inflection point at $\phi = \phi_i$; when $0 < \xi \ll 1$, there is also a local minimum (dip) y^-_{ext} on the r.h.s. of ϕ_i and a local maximum (bump) y^+_{ext} on the l.h.s. of ϕ_i, while both extrema are equally separated from the inflection point, $y^{\pm}_{\mathrm{ext}} = y_i\,(1 \pm \xi)$, (see also Iacconi, Assadullahi, Fasiello, Wands, 2021, for using this parametrization).

Good features of our model

(i) the existence of an attractor inflationary solution in good agreement with CMB measurements of the scalar tilt $n_s \approx 0.965$ within 1σ and the tensor-to-scalar ratio $r < 0.032$,
(ii) the two extra terms with the fine-tuned coefficients (β, γ) are needed for engineering a near-inflection point in the scalar potential and a large enhancement (peak) in the power spectrum of scalar perturbations, with the factor of 10^7 against the CMB level,
(iii) adding another term with a negative power of y and a small negative coefficient θ removes the infinite (Starobinsky) plateau, thus restricting from above the total number of e-folds for inflation, while being also needed for better (within 1σ) agreement with the observed tilt n_s of CMB.

USR regime

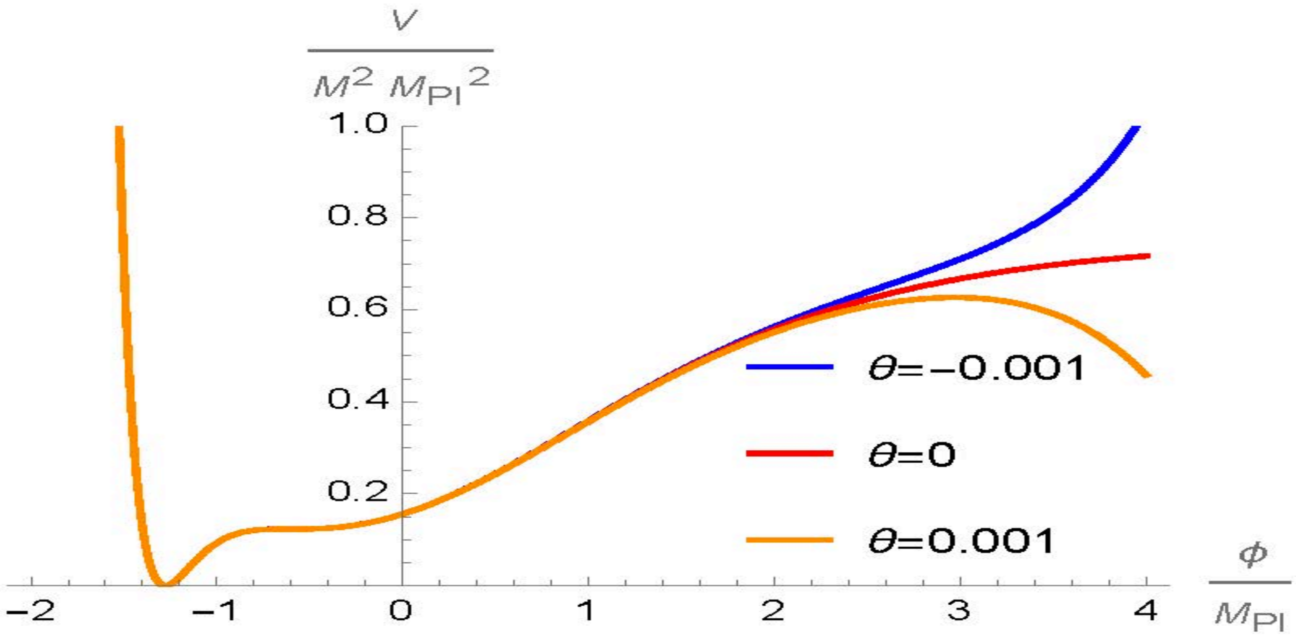
To study the USR regime, we introduce the Hubble flow functions

$$\epsilon(t) = -\frac{\dot{H}}{H^2}\,, \qquad \eta(t) = \frac{\dot{\epsilon}}{H\epsilon}\,.$$

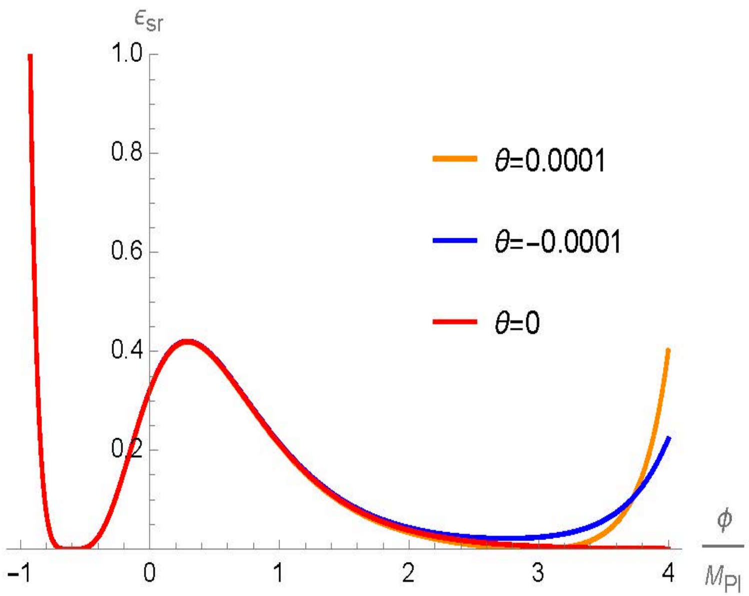
During the USR regime, the function $\epsilon(t)$ drops to very low values, whereas the function $\eta(t)$ goes from nearly zero to (-6) and back.

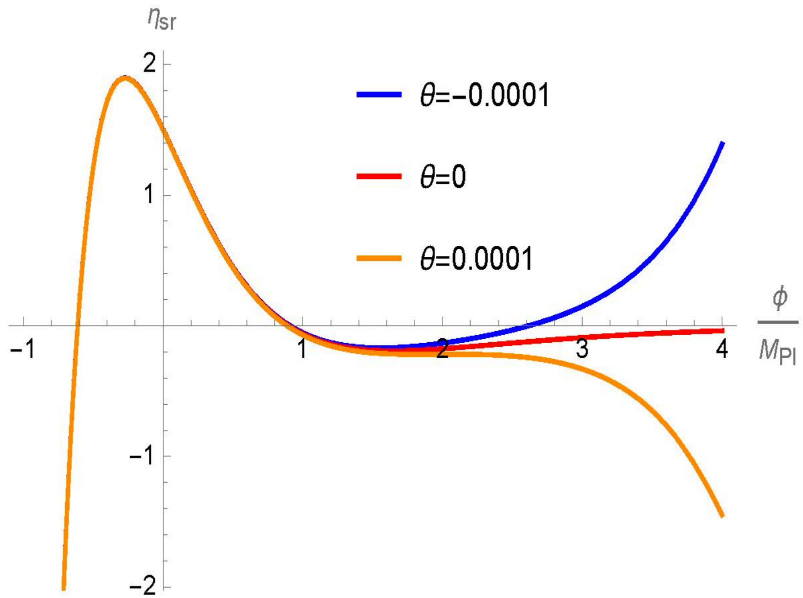
A standard procedure of (numerically) computing the power spectrum $P_R(k)$ of scalar (curvature) perturbations depending upon scale k is based on the Mukhanov-Sasaki (MS) equation. We used both approaches in our models and found that the difference between the results from numerically solving the MS equation and those derived from the SR formula is small.

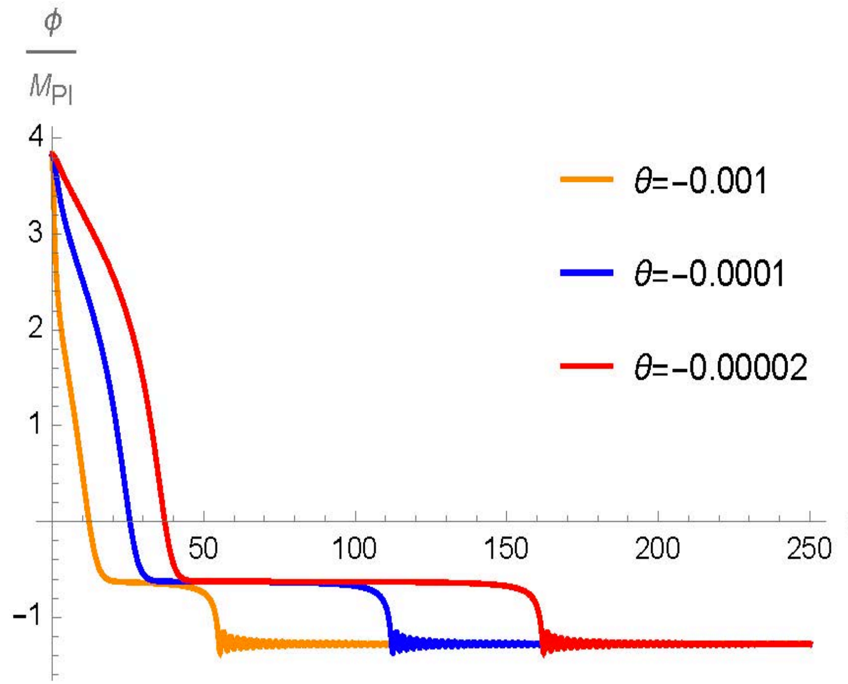
Numerical results

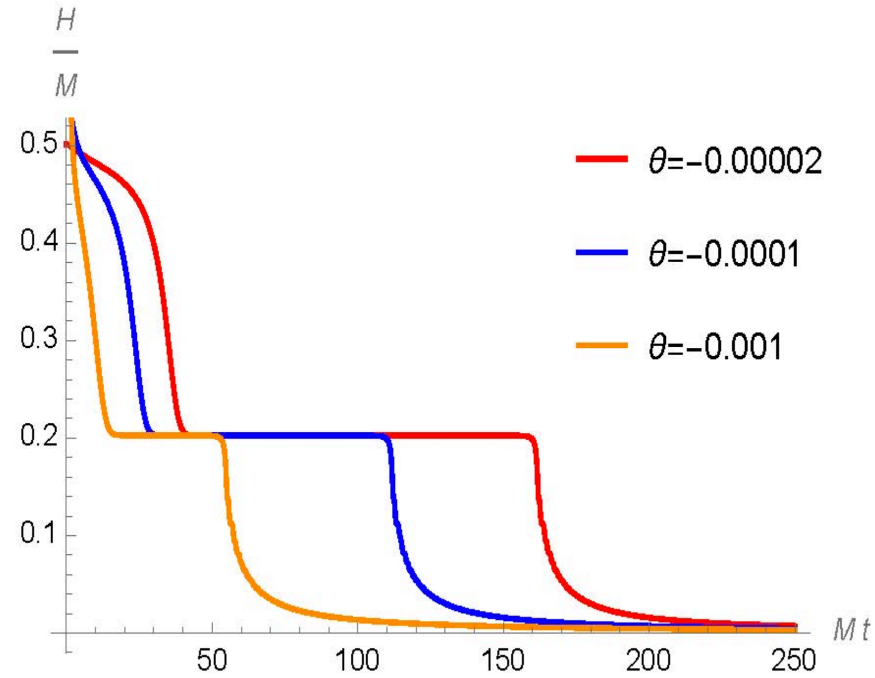


with α=0.743, ξ=0.012

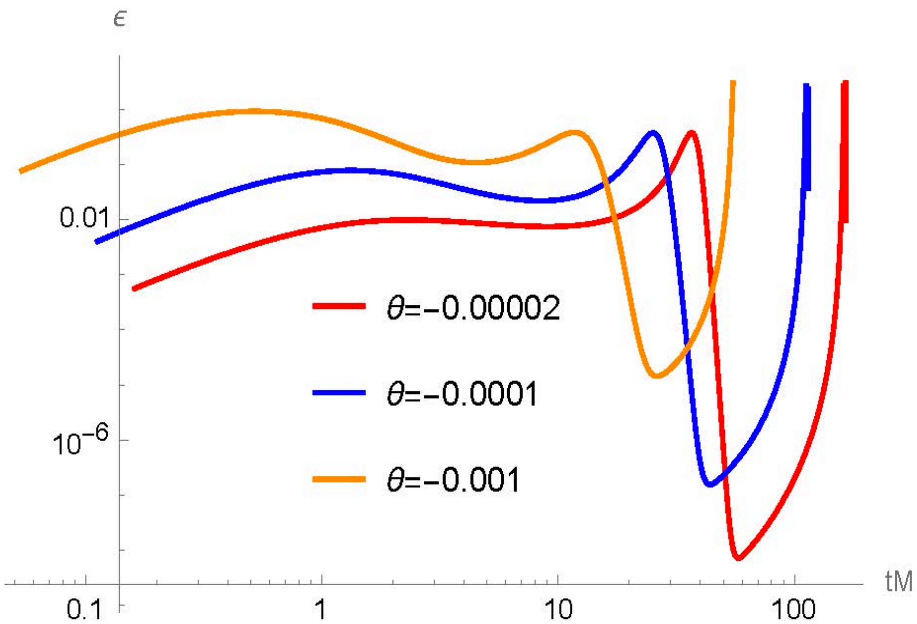


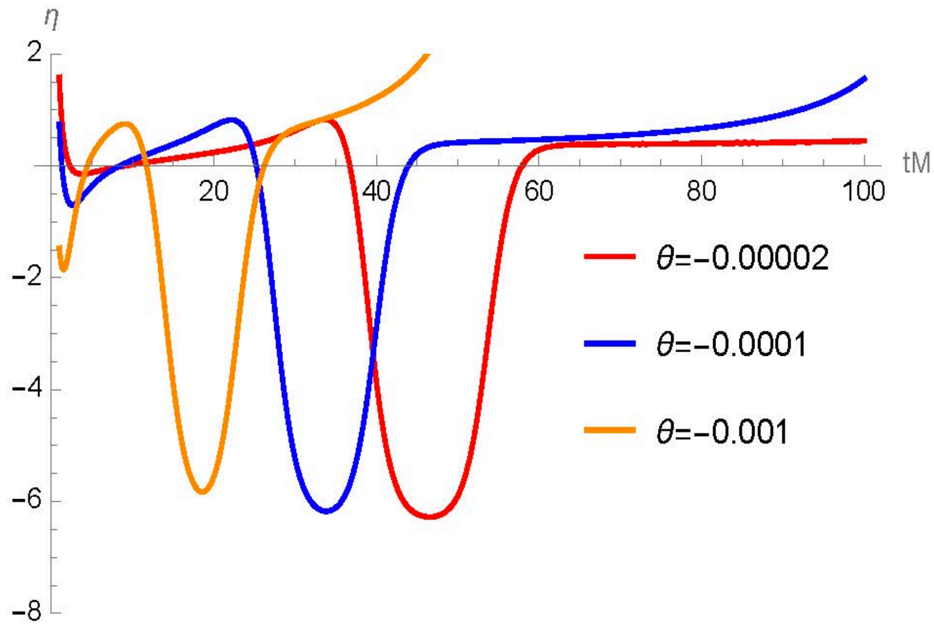




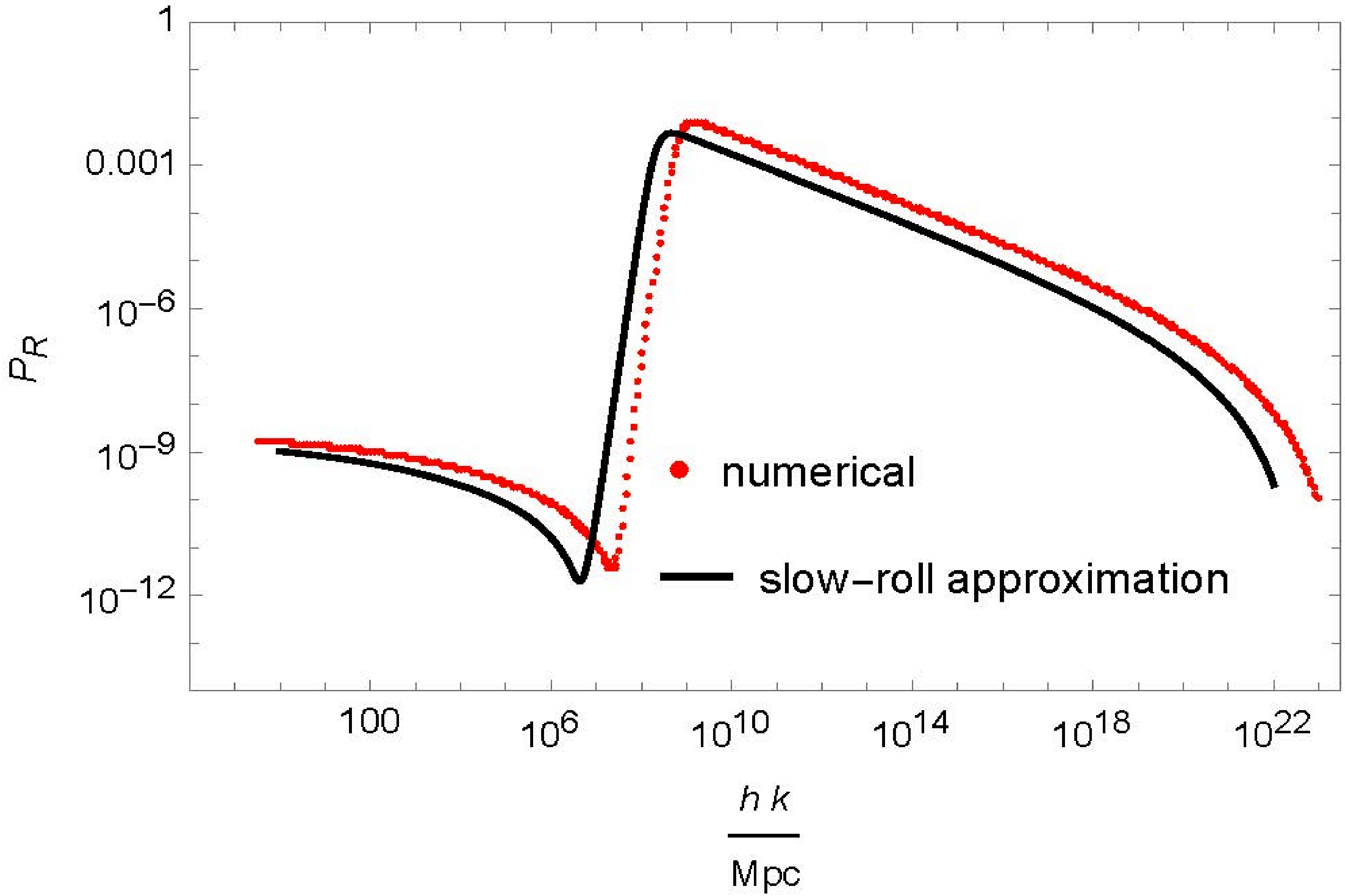


Numerical results





Comparison of our results from the Mukhanov-Sasaki equation for perturbations and from the slow-roll approximation formula



PBH masses

PBH are supposed to be formed by gravitational collapse of large (scalar) density perturbations. The masses of PBH can be estimated from given peaks (power spectrum enhancement) as follows (Pi, Sasaki, 2017):

$$M_{\rm PBH} \simeq \frac{M_{\rm Pl}^2}{H(t_{\rm peak})} \exp\left[2(N_{\rm total} - N_{\rm peak}) + \int_{t_{\rm peak}}^{t_{\rm total}} \varepsilon(t)H(t)dt\right]$$

that is very sensitive to the value of $\Delta N = N_{\rm total} - N_{\rm peak}$, while the integral gives a sub-leading correction. Increasing ΔN leads to decreasing the tilt n_s of CMB, which limits ΔN by 20 from above. On the other hand, ΔN cannot be too small when $M_{\rm PBH}$ have to exceed the Hawking (black hole) evaporation limit of 10^{15} g, which restricts ΔN from below (above 13).

After fine-tuning the parameters ξ and θ, we obtained the PBH masses in the asteroid-size range between 10^{17} g and 10^{21} g. Compare $M_{\odot} \approx 2 \cdot 10^{33}$ g.

Quantum Corrections

One-loop quantum corrections attracted a lot of attention in the recent literature. For instance, it was found that validity of the classical results is in danger because of the one-loop perturbative bound (Kristiano and Yokoyama, 2022)

$$\frac{1}{4}(\Delta\eta)^2 \left(1.1 + \log\frac{k_e}{k_s}\right) \Delta^2_{\text{peak}} \ll 1, \text{ when } (\Delta\eta)^2 \approx 36 \text{ and } \Delta^2_{\text{peak}} \sim 10^{-2}\,,$$

where k_s correspond to the USR start and k_e corresponds to the USR end.

However, later it was found (Riotto, 2023) that the bound can be removed when the transition from the USR phase to the 2nd SR phase is mild, as is the case in our model also (a sharp transition was adopted by Kristiano and Yokoyama). Moreover, the value of Δ^2_{peak} can be lower by the one order of magnitude.

Energy density of PBH induced GW

The present-day GW density function $\Omega_{\rm GW}$ in the 2nd order with respect to perturbations is given by (Espinosa, Racco, Riotto, 2018)

$$\frac{\Omega_{\rm GW}(k)}{\Omega_r} = \frac{c_g}{72}\int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} {\rm d}d \int_{\frac{1}{\sqrt{3}}}^{\infty} {\rm d}s \left[\frac{(s^2-\frac{1}{3})(d^2-\frac{1}{3})}{s^2+d^2}\right]^2 \times P_\zeta(kx)P_\zeta(ky)\left(I_c^2+I_s^2\right) ,$$

where the constant $c_g \approx 0.4$ in the SM, and $\Omega_r = 8.6\cdot 10^{-5}$ according to the present CMB temperature.

The variables (x,y) are related to the integration variables (s,d) as

$$x = \frac{\sqrt{3}}{2}(s+d)\ , \quad y = \frac{\sqrt{3}}{2}(s-d)\ .$$

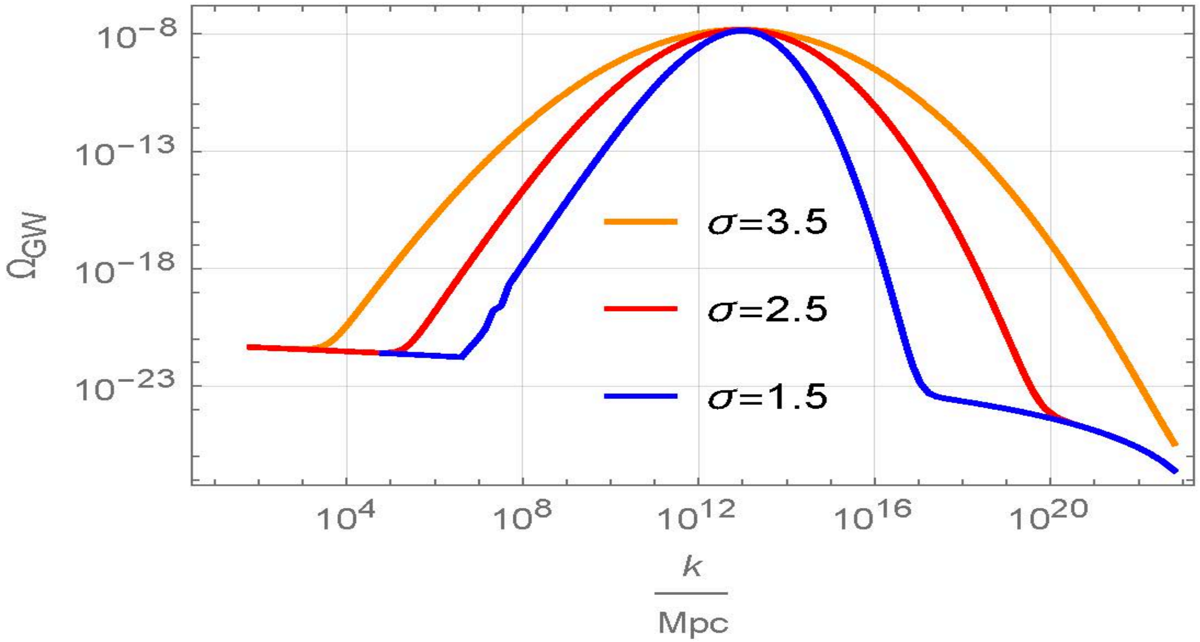
The functions I_c and I_s of $x(s,d)$ and $y(s,d)$ are (Espinosa, Racco, Riotto, 2018)

$$I_c = -36\pi \frac{(s^2+d^2-2)^2}{(s^2-d^2)^3}\theta(s-1) \, ,$$
$$I_s = -36 \frac{s^2+d^2-2}{(s^2-d^2)^2}\left[\frac{s^2+d^2-2}{s^2-d^2}\ln\left|\frac{d^2-1}{s^2-1}\right| + 2\right] \, .$$

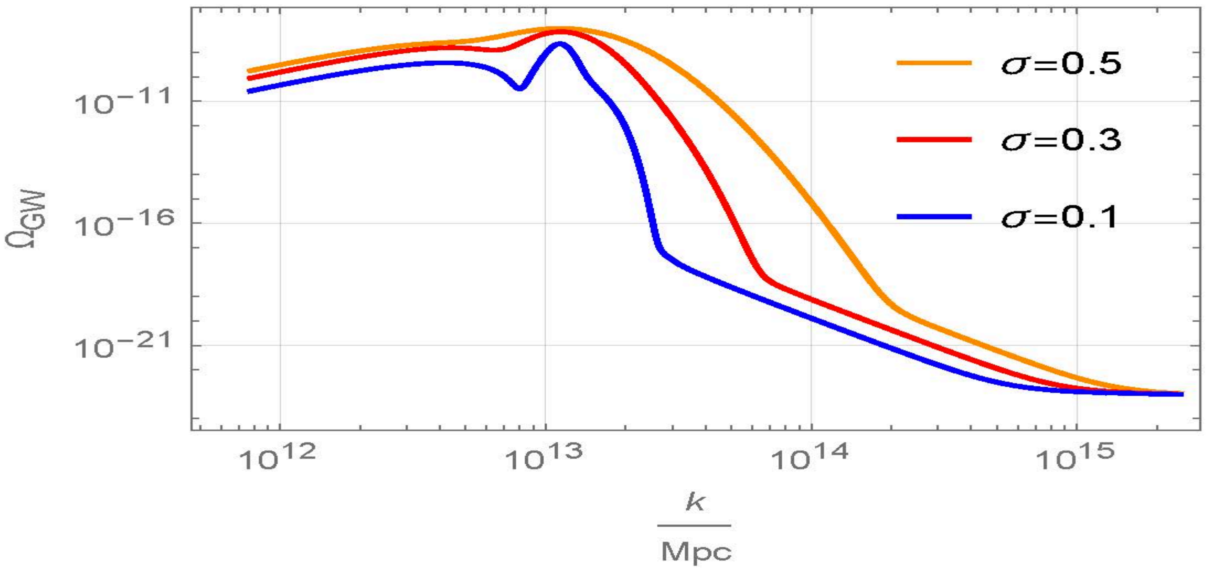
With these equations, the GW density can be numerically computed for a given power spectrum.

In our models, for broad peaks with the width $\sigma > 1$ and $\Delta^2_{\rm peak}$ of the order 10^{-3}, we obtained $\Omega^{\rm GW}(k) \sim 10^{-6} P^2_R(k)$. For sharp peaks with $\sigma < 1$ the shape of the GW spectrum is different, being far from Gaussian (see also Balaji, Domenech, Silk, 2022).

Numerical results



with the peak width σ



PBH production in modified gravity after Starobinsky inflation

We propose the modified Appleby-Battye-Starobinsky (ABS) model (2010) of $F(R)$ gravity for that purpose, defined by the smooth F-function

$$F(R) = (1 - g_1)R + gE_{AB} \ln \left[\frac{\cosh\left(\frac{R}{E_{AB}} - b\right)}{\cosh(b)} \right] + \frac{R^2}{6M^2} - \delta \frac{R^4}{48M^6} \ ,$$

where $g_1 = -g \tanh b$, $g \approx 2.25$ and $b \approx 2.89$, $0 < \delta < 4 \cdot 10^{-6}$, and

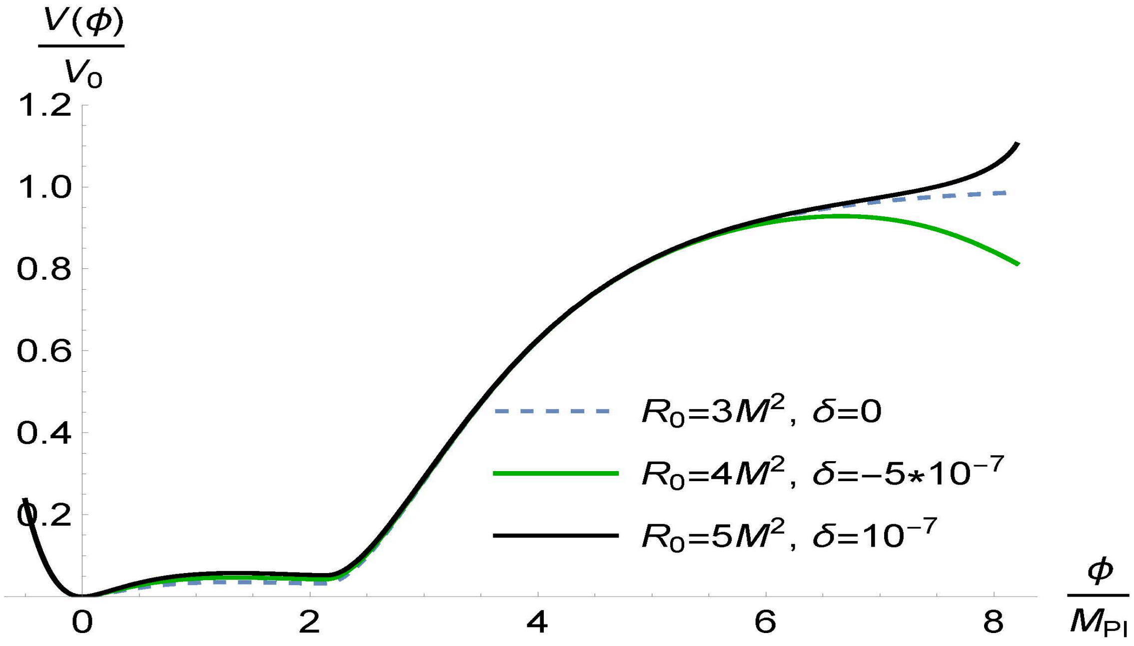
$$E_{AB} = \frac{R_0}{2g \ln(1 + e^{2b})} \quad \text{with} \quad R_0 \approx 3M^2, \ \ M \sim 10^{-5} M_{\rm Pl} \ .$$

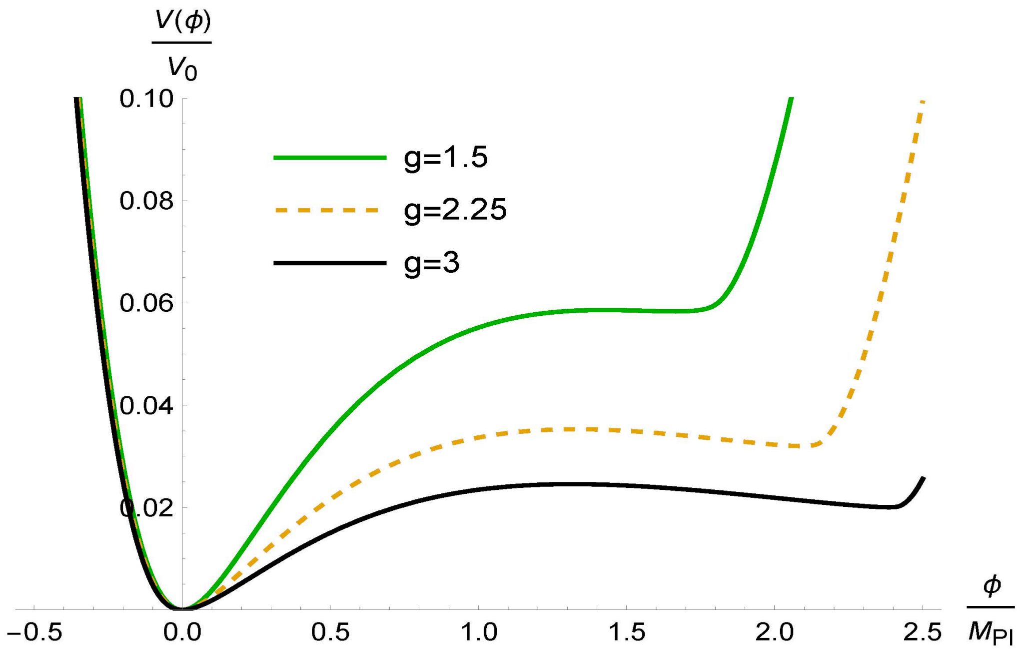
It is consistent with Starobinsky inflation and CMB measurements, has no ghosts ($F'(R) > 0$, $F''(R) > 0$), and the corresponding inflaton potential has two plateaus, leading to a large peak in the power spectrum. The last term can be interpreted as a quantum correction.

Consistency with CMB, and PBH masses

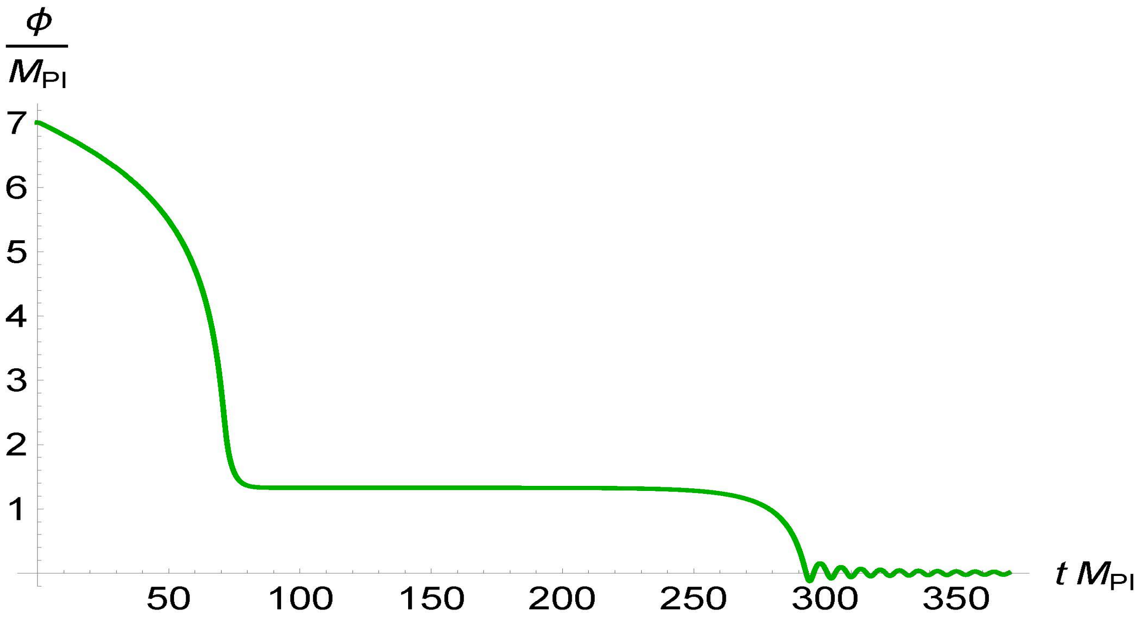
Demanding:
(i) a large enhancement (peak) in the power spectrum by the factor of 10^7 against the CMB level of 10^{-9},
(ii) consistency with the latest CMB measurements,
$n_s = 0.9649 \pm 0.0042$ (within 1σ) and $r < 0.032$, and
(iii) PBH masses beyond 10^{15} g,
we found $\triangle N$ must be restricted between 17 and 22 e-folds, while the total duration of inflation is between 54 and 66 e-folds.
The possible range of the parameter δ is between $1.02 \cdot 10^{-8}$ and $8.74 \cdot 10^{-8}$.
The PBH masses found are between 10^{16} g and 10^{20} g, i.e. of the asteroid-size again.

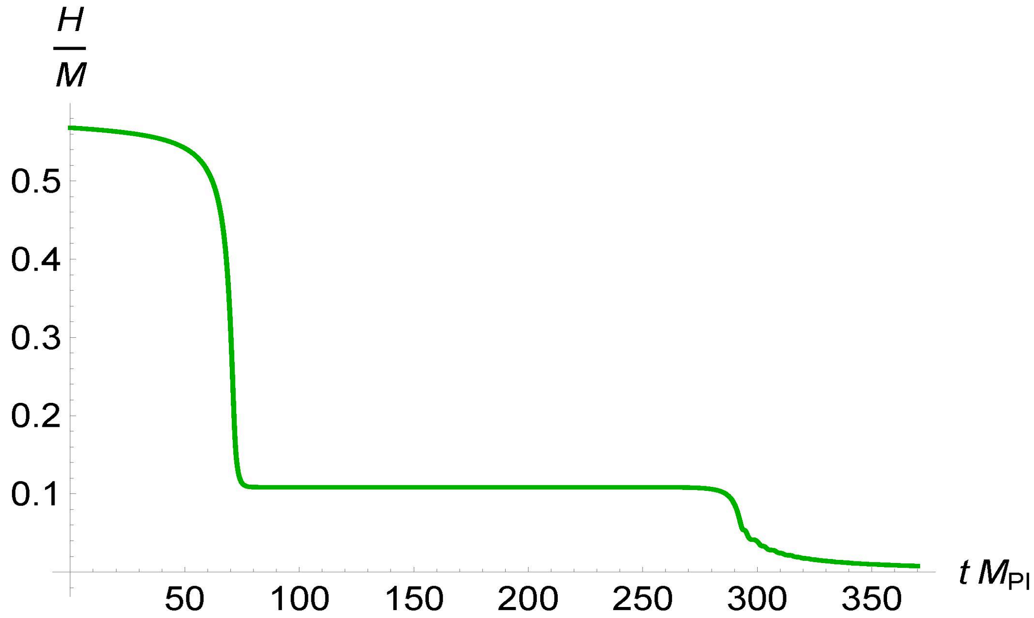
Numerical results



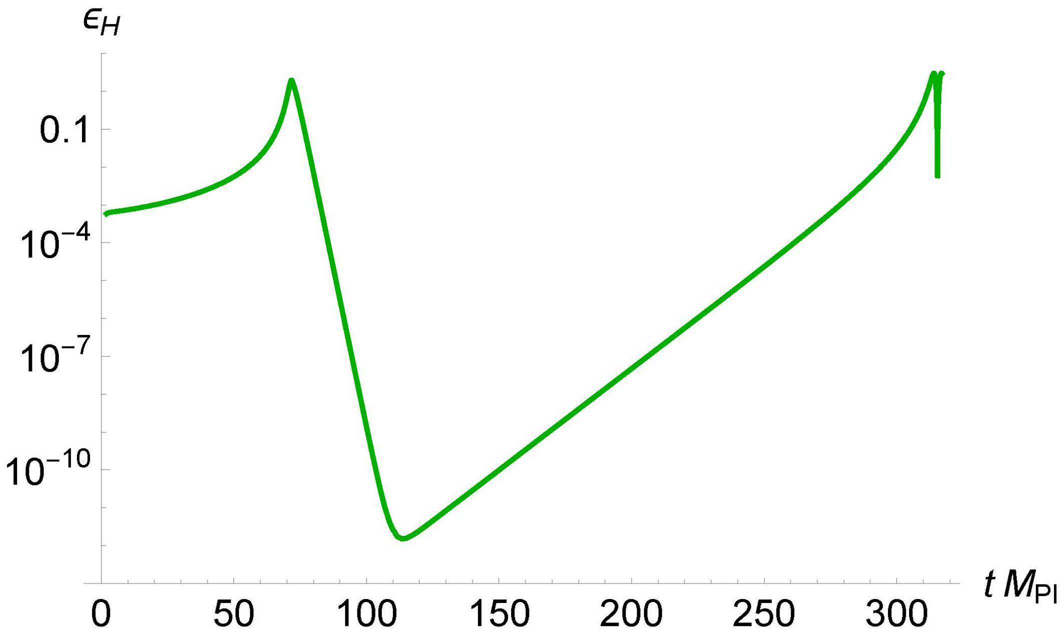


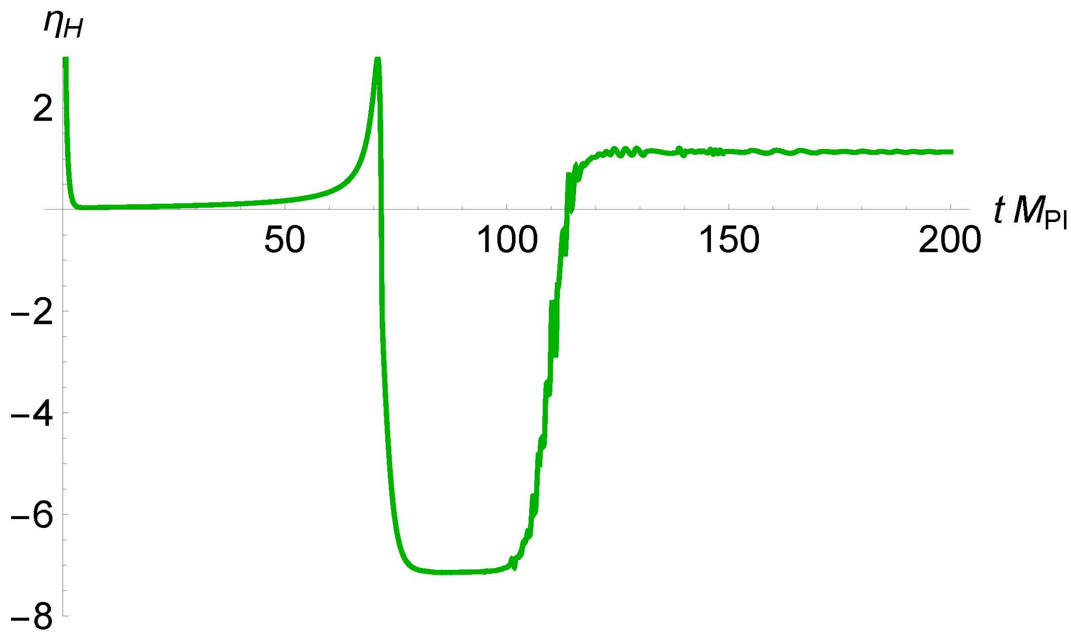
Potential and dynamics



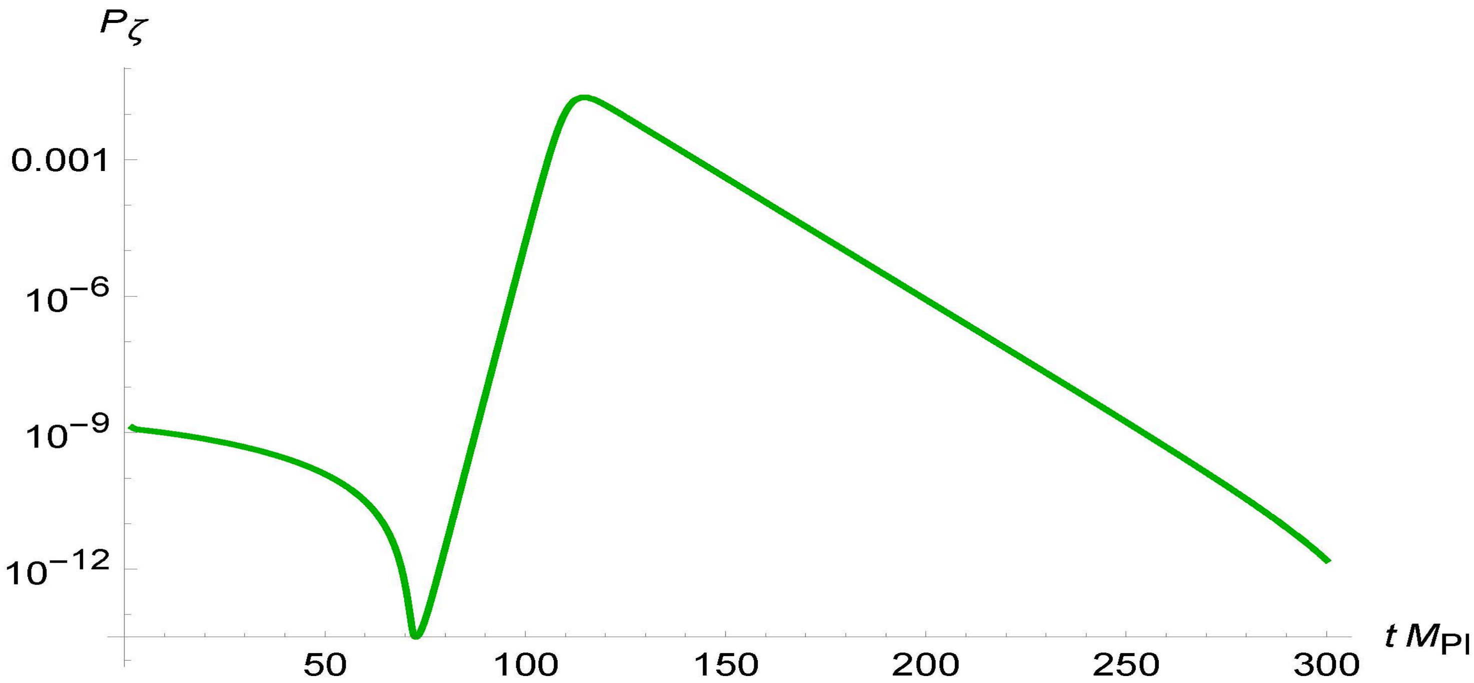


Numerical results





Hubble flow parameters and power spectrum



Conclusion

- Our approach is phenomenological: from viable inflation to efficient PBH production included on smaller scales, and the induced GW.

- The PBH masses are possible in the window between 10^{17} g and 10^{21} g, where they can form (the whole or part of) current dark matter.

- It is necessary to fine-tune some of the parameters in order to get that.

- The modified gravity origin of inflation and PBH formation is possible.

- The PBH-induced GW may be detectable by the future space-based gravitational interferometers (LISA, DECIGO, TianQin, Taiji).

Thank you for your attention!