Anomalous Z' extension of the Standard Model

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Based on

Pascal Anastasopoulos, Ignatios Antoniadis, K.B., Mark Goodsell, François Rondeau (in progress ...)

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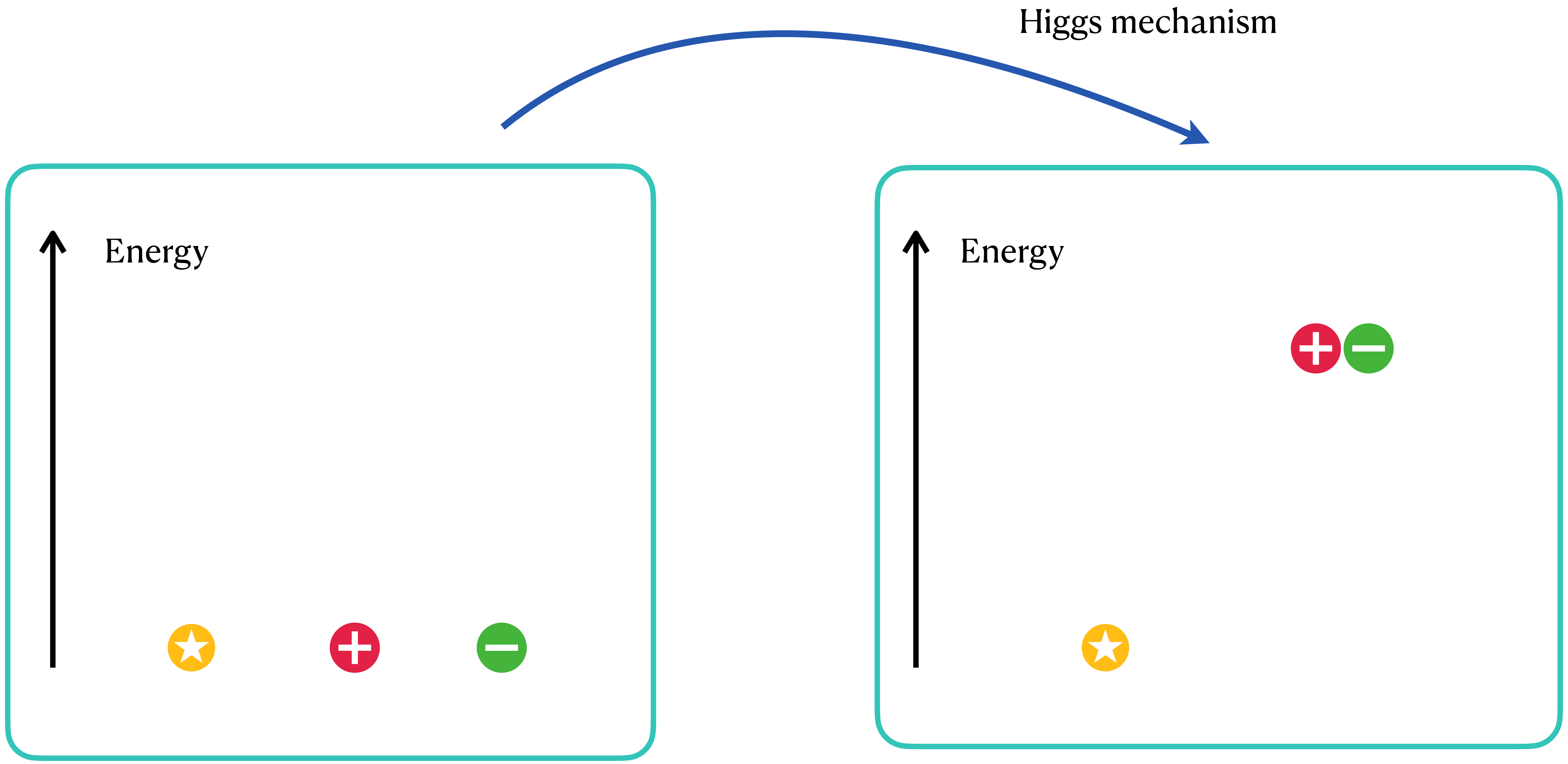
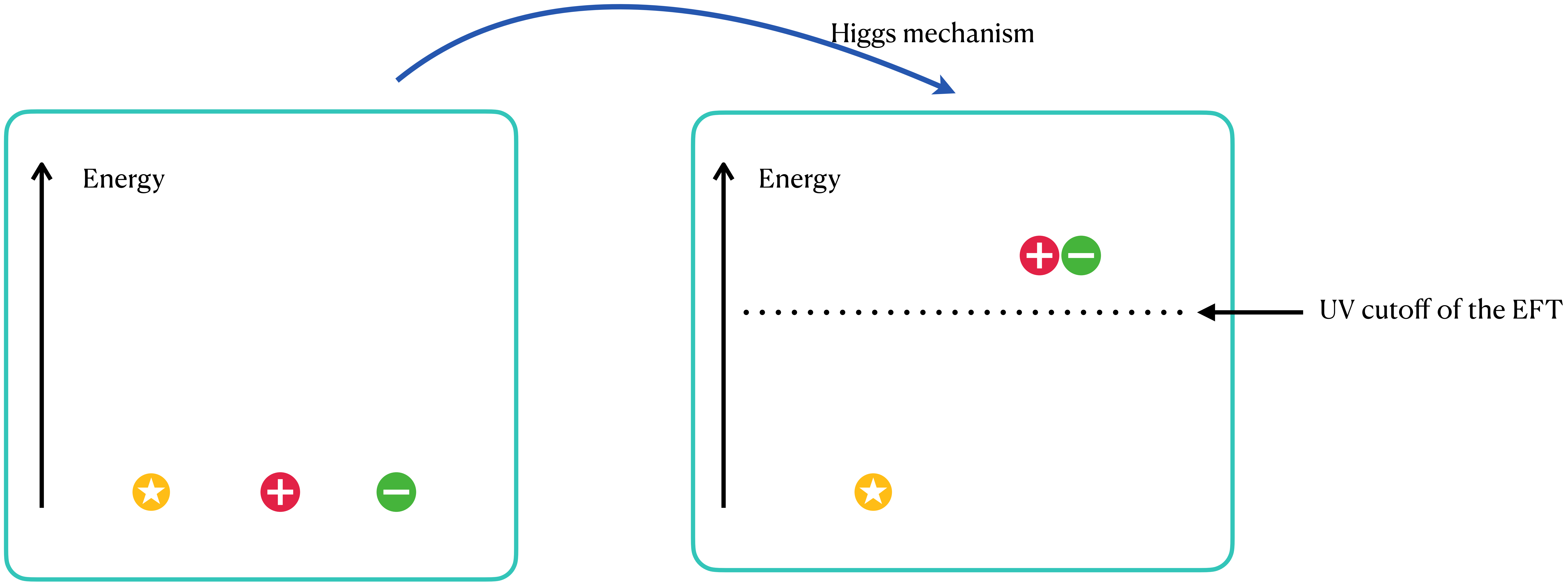
Separating the chiral fermions



Three massless chiral fermions

One massless and two massive chiral fermions

The EFT will look anomalous



Three massless chiral fermions

Anomaly cancellation among the three

One massless and two massive chiral fermions

Anomaly cancellation lost in the EFT

The Complete Model: Field Content

Our objective is:

To orchestrate a situation in which the contributions to the anomalies of the $U(1)_A$ gauge symmetry cancel out between:

- the light fields present in the effective field theory

and

- the (non-observable) heavier chiral fermions.

			$SU(3)$	$SU(2)$	$U(1)_Y$	$U(1)_A$
SM sector	$\mathbf{Q}_L^f$	$f = 1,2,3$	$\mathbf{3}$	$\mathbf{2}$	$1/6$	$z_{\mathbf{Q}}^f$
	$u_R^{c,f}$		$\bar{\mathbf{3}}$	$\mathbf{1}$	$-2/3$	z_u^f
	$d_R^{c,f}$		$\bar{\mathbf{3}}$	$\mathbf{1}$	$1/3$	z_d^f
	$\mathbf{L}_L^f$		$\mathbf{1}$	$\mathbf{2}$	$-1/2$	$z_{\mathbf{L}}^f$
	$e_R^{c,f}$		$\mathbf{1}$	$\mathbf{1}$	1	z_e^f
	$\nu_R^{c,f}$		$\mathbf{1}$	$\mathbf{1}$	0	z_ν^f
	H		$\mathbf{1}$	$\mathbf{2}$	$1/2$	z_H
Secluded sector	$\psi_L^{\mathbf{L}_i}$		$\mathbf{1}$	$\mathbf{2}$	$y_{\mathbf{L}}^i$	$q_{\mathbf{L}}^i$
	$(\psi_R^{\mathbf{L}_i})^c$	$i = 1,..,N_{\mathbf{L}}$	$\mathbf{1}$	$\mathbf{2}$	$-y_{\mathbf{L}}^i$	$\widetilde{q_{\mathbf{L}}}^i$
	$\psi_L^{e_j}$		$\mathbf{1}$	$\mathbf{1}$	y_e^j	q_e^j
	$(\psi_R^{e_j})^c$	$j = 1,..,N_e$	$\mathbf{1}$	$\mathbf{1}$	$-y_e^j$	$\widetilde{q_e}^j$
	$\psi_L^{d_k}$		$\mathbf{3}$	$\mathbf{1}$	y_d^k	q_d^k
	$(\psi_R^{d_k})^c$	$k = 1,..,N_d$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$-y_d^k$	$\widetilde{q_d}^k$
	$\psi_L^{\mathbf{Q}_m}$		$\mathbf{3}$	$\mathbf{2}$	$y_{\mathbf{Q}}^m$	$q_{\mathbf{Q}}^m$
	$(\psi_R^{\mathbf{Q}_m})^c$	$m = 1,..,N_{\mathbf{Q}}$	$\bar{\mathbf{3}}$	$\mathbf{2}$	$-y_{\mathbf{Q}}^m$	$\widetilde{q_{\mathbf{Q}}}^m$
	S		$\mathbf{1}$	$\mathbf{1}$	0	q_S

Table 1: The particle content of the model

Mass through Higgs

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle S \rangle = \frac{v_S}{\sqrt{2}},$$

$$v \ll v_S$$

$U(1)_A$ gauge boson mass

$$M_A \sim g_A |q_S| v_S.$$

Yukawa terms $Y_{ij} \bar{\psi}^i_L \psi^i_R \tilde{S}$, where by $\tilde{S}$ we denote S or S^*

$$M_{\psi,ij} = Y_{ij} v_S$$

$$Y_{ij} \propto \delta_{ij}$$

		$SU(3)$	$SU(2)$	$U(1)_Y$	$U(1)_A$
$\mathbf{Q}^f_L$	$f = 1, 2, 3$	$\mathbf{3}$	$\mathbf{2}$	$1/6$	$z^f_{\mathbf{Q}}$
$u^{c,f}_R$		$\bar{\mathbf{3}}$	$\mathbf{1}$	$-2/3$	z^f_u
$d^{c,f}_R$		$\bar{\mathbf{3}}$	$\mathbf{1}$	$1/3$	z^f_d
$\mathbf{L}^f_L$		$\mathbf{1}$	$\mathbf{2}$	$-1/2$	$z^f_{\mathbf{L}}$
$e^{c,f}_R$		$\mathbf{1}$	$\mathbf{1}$	1	z^f_e
$\nu^{c,f}_R$		$\mathbf{1}$	$\mathbf{1}$	0	z^f_ν
H		$\mathbf{1}$	$\mathbf{2}$	$1/2$	z_H
$\psi^{\mathbf{L}_i}_L$		$\mathbf{1}$	$\mathbf{2}$	$y^i_{\mathbf{L}}$	$q^i_{\mathbf{L}}$
$(\psi^{\mathbf{L}_i}_R)^c$	$i = 1, .., N_{\mathbf{L}}$	$\mathbf{1}$	$\mathbf{2}$	$-y^i_{\mathbf{L}}$	$\widetilde{q_{\mathbf{L}}}^i$
$\psi^{e_j}_L$		$\mathbf{1}$	$\mathbf{1}$	y^j_e	q^j_e
$(\psi^{e_j}_R)^c$	$j = 1, .., N_e$	$\mathbf{1}$	$\mathbf{1}$	$-y^j_e$	$\widetilde{q_e}^j$
$\psi^{d_k}_L$		$\mathbf{3}$	$\mathbf{1}$	y^k_d	q^k_d
$(\psi^{d_k}_R)^c$	$k = 1, .., N_d$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$-y^k_d$	$\widetilde{q_d}^k$
$\psi^{\mathbf{Q}_m}_L$		$\mathbf{3}$	$\mathbf{2}$	$y^m_{\mathbf{Q}}$	$q^m_{\mathbf{Q}}$
$(\psi^{\mathbf{Q}_m}_R)^c$	$m = 1, .., N_{\mathbf{Q}}$	$\bar{\mathbf{3}}$	$\mathbf{2}$	$-y^m_{\mathbf{Q}}$	$\widetilde{q_{\mathbf{Q}}}^m$
S		$\mathbf{1}$	$\mathbf{1}$	0	q_S

Constraints from SM fermions masses

SM Yukawa couplings

$$\bar{\mathbf{Q}}^i_L H d^j_R \rightarrow -z^i_{\mathbf{Q}} - z^j_d + z_H = 0,$$
$$\bar{\mathbf{Q}}^i_L \tilde{H} u^j_R \rightarrow -z^i_{\mathbf{Q}} - z^j_{u_R} - z_H = 0,$$
$$\bar{\mathbf{L}}^i H e^j_R, \rightarrow -z^i_{\mathbf{L}} - z^j_{e_R} + z_H = 0.$$

Dirac neutrino mass

$$\bar{\mathbf{L}}^i \tilde{H} \nu^j_R \rightarrow -z^i_{\mathbf{L}} - z^j_{\nu_R} - z_H = 0,$$

Majorana neutrino mass

$$\bar{\nu}^{c,i}_R \nu^j_R \frac{\tilde{S}^n}{\Lambda^{n-1}} \rightarrow z^i_{\nu_R} + z^j_{\nu_R} - (\varepsilon^{ij}_\nu)^n n\, q_S = 0,$$

$\varepsilon^{ij}_\nu = \pm 1$ depending if we use S or S^*

		$SU(3)$	$SU(2)$	$U(1)_Y$	$U(1)_A$
$\mathbf{Q}^f_L$	$f = 1, 2, 3$	$\mathbf{3}$	$\mathbf{2}$	$1/6$	$z^f_{\mathbf{Q}}$
$u^{c,f}_R$		$\bar{\mathbf{3}}$	$\mathbf{1}$	$-2/3$	z^f_u
$d^{c,f}_R$		$\bar{\mathbf{3}}$	$\mathbf{1}$	$1/3$	z^f_d
$\mathbf{L}^f_L$		$\mathbf{1}$	$\mathbf{2}$	$-1/2$	$z^f_{\mathbf{L}}$
$e^{c,f}_R$		$\mathbf{1}$	$\mathbf{1}$	1	z^f_e
$\nu^{c,f}_R$		$\mathbf{1}$	$\mathbf{1}$	0	z^f_ν
H		$\mathbf{1}$	$\mathbf{2}$	$1/2$	z_H
$\psi^{\mathbf{L}_i}_L$		$\mathbf{1}$	$\mathbf{2}$	$y^i_{\mathbf{L}}$	$q^i_{\mathbf{L}}$
$(\psi^{\mathbf{L}_i}_R)^c$	$i = 1, .., N_{\mathbf{L}}$	$\mathbf{1}$	$\mathbf{2}$	$-y^i_{\mathbf{L}}$	$\widetilde{q_{\mathbf{L}}}^i$
$\psi^{e_j}_L$		$\mathbf{1}$	$\mathbf{1}$	y^j_e	q^j_e
$(\psi^{e_j}_R)^c$	$j = 1, .., N_e$	$\mathbf{1}$	$\mathbf{1}$	$-y^j_e$	$\widetilde{q_e}^j$
$\psi^{d_k}_L$		$\mathbf{3}$	$\mathbf{1}$	y^k_d	q^k_d
$(\psi^{d_k}_R)^c$	$k = 1, .., N_d$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$-y^k_d$	$\widetilde{q_d}^k$
$\psi^{\mathbf{Q}_m}_L$		$\mathbf{3}$	$\mathbf{2}$	$y^m_{\mathbf{Q}}$	$q^m_{\mathbf{Q}}$
$(\psi^{\mathbf{Q}_m}_R)^c$	$m = 1, .., N_{\mathbf{Q}}$	$\bar{\mathbf{3}}$	$\mathbf{2}$	$-y^m_{\mathbf{Q}}$	$\widetilde{q_{\mathbf{Q}}}^m$
S		$\mathbf{1}$	$\mathbf{1}$	0	q_S

Constraints from the extra fermions Yukawa's

Secluded sector Yukawa couplings

$$\begin{aligned}
\bar\psi_L^{\mathbf{L}_i}\psi_R^{\mathbf{L}_i}\hat S &\rightarrow -q_{\mathbf{L}}^i-\widetilde{q_{\mathbf{L}}}^i+\varepsilon_{\mathbf{L}}^i q_S &= 0\\
\bar\psi_L^{e_j}\psi_R^{e_j}\hat S &\rightarrow -q_e^j-\widetilde{q_e}^j+\varepsilon_e^j q_S &= 0\\
\bar\psi_L^{d_k}\psi_R^{d_k}\hat S &\rightarrow -q_d^k-\widetilde{q_d}^k+\varepsilon_d^k q_S &= 0\\
\bar\psi_L^{\mathbf{Q}_m}\psi_R^{\mathbf{Q}_m}\hat S &\rightarrow -q_{\mathbf{Q}}^m-\widetilde{q_{\mathbf{Q}}}^m+\varepsilon_{\mathbf{Q}}^m q_S &= 0
\end{aligned}$$

$\hat S$ denotes either S or S^*

$$\varepsilon_{\mathbf{L}}^i,\varepsilon_e^j,\varepsilon_d^k,\varepsilon_{\mathbf{Q}}^m=\pm1$$

		$SU(3)$	$SU(2)$	$U(1)_Y$	$U(1)_A$
$\mathbf{Q}_L^f$	$f=1,2,3$	$\mathbf{3}$	$\mathbf{2}$	$1/6$	$z_{\mathbf{Q}}^f$
$u_R^{c,f}$		$\bar{\mathbf{3}}$	$\mathbf{1}$	$-2/3$	z_u^f
$d_R^{c,f}$		$\bar{\mathbf{3}}$	$\mathbf{1}$	$1/3$	z_d^f
$\mathbf{L}_L^f$		$\mathbf{1}$	$\mathbf{2}$	$-1/2$	$z_{\mathbf{L}}^f$
$e_R^{c,f}$		$\mathbf{1}$	$\mathbf{1}$	1	z_e^f
$\nu_R^{c,f}$		$\mathbf{1}$	$\mathbf{1}$	0	z_ν^f
H		$\mathbf{1}$	$\mathbf{2}$	$1/2$	z_H
$\psi_L^{\mathbf{L}_i}$		$\mathbf{1}$	$\mathbf{2}$	$y_{\mathbf{L}}^i$	$q_{\mathbf{L}}^i$
$(\psi_R^{\mathbf{L}_i})^c$	$i=1,..,N_{\mathbf{L}}$	$\mathbf{1}$	$\mathbf{2}$	$-y_{\mathbf{L}}^i$	$\widetilde{q_{\mathbf{L}}}^i$
$\psi_L^{e_j}$		$\mathbf{1}$	$\mathbf{1}$	y_e^j	q_e^j
$(\psi_R^{e_j})^c$	$j=1,..,N_e$	$\mathbf{1}$	$\mathbf{1}$	$-y_e^j$	$\widetilde{q_e}^j$
$\psi_L^{d_k}$		$\mathbf{3}$	$\mathbf{1}$	y_d^k	q_d^k
$(\psi_R^{d_k})^c$	$k=1,..,N_d$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$-y_d^k$	$\widetilde{q_d}^k$
$\psi_L^{\mathbf{Q}_m}$		$\mathbf{3}$	$\mathbf{2}$	$y_{\mathbf{Q}}^m$	$q_{\mathbf{Q}}^m$
$(\psi_R^{\mathbf{Q}_m})^c$	$m=1,..,N_{\mathbf{Q}}$	$\bar{\mathbf{3}}$	$\mathbf{2}$	$-y_{\mathbf{Q}}^m$	$\widetilde{q_{\mathbf{Q}}}^m$
S		$\mathbf{1}$	$\mathbf{1}$	0	q_S

The Complete Model anomaly cancellation equations

Cancellation of the anomalies contributions

$$
\begin{aligned}
Tr[Y]_{SM} &= Tr[Y]_{secluded} &&= 0,\\
Tr[YYY]_{SM} &= Tr[YYY]_{secluded} &&= 0,\\
Tr[YT_2T_2]_{SM} &= Tr[YT_2T_2]_{secluded} &&= 0,\\
Tr[YT_3T_3]_{SM} &= Tr[YT_3T_3]_{secluded} &&= 0,\\
Tr[T_3T_3T_3]_{SM} &= Tr[T_3T_3T_3]_{secluded} &&= 0,
\end{aligned}
$$

$$
\begin{aligned}
Tr[q_A]_{SM} &= -Tr[q_A]_{secluded} &&\equiv t_A,\\
Tr[YYq_A]_{SM} &= -Tr[YYq_A]_{secluded} &&\equiv t_{YYA},\\
Tr[Yq_Aq_A]_{SM} &= -Tr[Yq_Aq_A]_{secluded} &&\equiv t_{YAA},\\
Tr[q_Aq_Aq_A]_{SM} &= -Tr[q_Aq_Aq_A]_{secluded} &&\equiv t_{AAA},\\
Tr[q_AT_2T_2]_{SM} &= -Tr[q_AT_2T_2]_{secluded} &&\equiv t_2,\\
Tr[q_AT_3T_3]_{SM} &= -Tr[q_AT_3T_3]_{secluded} &&\equiv t_3.
\end{aligned}
$$

Anomalies contributions = Triangular Feynman diagrams

		$SU(3)$	$SU(2)$	$U(1)_Y$	$U(1)_A$
$\mathbf{Q}_L^f$	$f=1,2,3$	$\mathbf{3}$	$\mathbf{2}$	$1/6$	$z_{\mathbf{Q}}^f$
$u_R^{c,f}$		$\bar{\mathbf{3}}$	$\mathbf{1}$	$-2/3$	z_u^f
$d_R^{c,f}$		$\bar{\mathbf{3}}$	$\mathbf{1}$	$1/3$	z_d^f
$\mathbf{L}_L^f$		$\mathbf{1}$	$\mathbf{2}$	$-1/2$	$z_{\mathbf{L}}^f$
$e_R^{c,f}$		$\mathbf{1}$	$\mathbf{1}$	1	z_e^f
$\nu_R^{c,f}$		$\mathbf{1}$	$\mathbf{1}$	0	z_ν^f
H		$\mathbf{1}$	$\mathbf{2}$	$1/2$	z_H
$\psi_L^{\mathbf{L}_i}$		$\mathbf{1}$	$\mathbf{2}$	$y_{\mathbf{L}}^i$	$q_{\mathbf{L}}^i$
$(\psi_R^{\mathbf{L}_i})^c$	$i=1,..,N_{\mathbf{L}}$	$\mathbf{1}$	$\mathbf{2}$	$-y_{\mathbf{L}}^i$	$\widetilde{q_{\mathbf{L}}}^i$
$\psi_L^{e_j}$		$\mathbf{1}$	$\mathbf{1}$	y_e^j	q_e^j
$(\psi_R^{e_j})^c$	$j=1,..,N_e$	$\mathbf{1}$	$\mathbf{1}$	$-y_e^j$	$\widetilde{q_e}^j$
$\psi_L^{d_k}$		$\mathbf{3}$	$\mathbf{1}$	y_d^k	q_d^k
$(\psi_R^{d_k})^c$	$k=1,..,N_d$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$-y_d^k$	$\widetilde{q_d}^k$
$\psi_L^{\mathbf{Q}_m}$		$\mathbf{3}$	$\mathbf{2}$	$y_{\mathbf{Q}}^m$	$q_{\mathbf{Q}}^m$
$(\psi_R^{\mathbf{Q}_m})^c$	$m=1,..,N_{\mathbf{Q}}$	$\bar{\mathbf{3}}$	$\mathbf{2}$	$-y_{\mathbf{Q}}^m$	$\widetilde{q_{\mathbf{Q}}}^m$
S		$\mathbf{1}$	$\mathbf{1}$	0	q_S

Anomaly equations from SM fermions

Cancellation of the anomalies contributions

$$
\begin{aligned}
Tr[q_A]_{SM} &= \sum_f[6z^f_{\mathbf{Q}} + 3z^f_u + 3z^f_d + 2z^f_{\mathbf{L}} + z^f_e + z^f_\nu] &&= t_A, \\
Tr[YYq_A]_{SM} &= \sum_f[6(y^f_{\mathbf{Q}})^2 z^f_{\mathbf{Q}} + 3(y^f_u)^2 z^f_u + 3(y^f_d)^2 z^f_d + 2(y^f_L)^2 z^f_{\mathbf{L}} + (y^f_e)^2 z^f_e] &&= t_{YYA}, \\
Tr[Yq_Aq_A]_{SM} &= \sum_f[6y^f_{\mathbf{Q}}(z^f_{\mathbf{Q}})^2 + 3y^f_u(z^f_u)^2 + 3y^f_d(z^f_d)^2 + 2y^f_L(z^f_{\mathbf{L}})^2 + y^f_e(z^f_e)^2] &&= t_{YAA}, \\
Tr[q_Aq_Aq_A]_{SM} &= \sum_f[6(z^f_{\mathbf{Q}})^3 + 3(z^f_u)^3 + 3(z^f_d)^3 + 2(z^f_{\mathbf{L}})^3 + (z^f_e)^3 + (z^f_\nu)^3] &&= t_{AAA}, \\
Tr[q_AT_2T_2]_{SM} &= \sum_f[3z^f_{\mathbf{Q}} + z^f_{\mathbf{L}}] &&= t_2, \\
Tr[q_AT_3T_3]_{SM} &= \sum_f[2z^f_{\mathbf{Q}} + z^f_u + z^f_d] &&= t_3,
\end{aligned}
$$

Impose relations from Yukawa coupling constraints:

$$
\begin{aligned}
Tr[q_A]_{SM} &= \sum_f[2z^f_{\mathbf{L}} + z^f_e + z^f_\nu] &&= t_A \\
Tr[YYq_A]_{SM} &= -\frac{1}{2}\sum_f[3z^f_{\mathbf{Q}} + z^f_{\mathbf{L}}] &&= t_{YYA} \\
Tr[Yq_Aq_A]_{SM} &= -2\sum_f[3z^f_{\mathbf{Q}} + z^f_{\mathbf{L}}]z_H &&= t_{YAA} \\
Tr[q_Aq_Aq_A]_{SM} &= \sum_f[z^3_H + 3z_H(z^f_{\mathbf{L}})^2 + (z^f_{\mathbf{L}})^3 - 3z^2_H z^f_{\mathbf{L}} - 18z^2_H z^f_{\mathbf{Q}} + (z^f_\nu)^3] &&= t_{AAA} \\
Tr[q_AT_2T_2]_{SM} &= \sum_f[3z^f_{\mathbf{Q}} + z^f_{\mathbf{L}}] &&= t_2 \\
Tr[q_AT_3T_3]_{SM} &= 0 &&= t_3
\end{aligned}
$$

$$t_{YYA} = -\frac{1}{2}t_2\ , \quad t_{YAA} = -2z_H t_2\ , \quad t_3 = 0. \qquad t_A = 0\ , \quad t_{AAA} = -6z^2_H t_2$$

(neutrino Dirac mass constraints)

		$SU(3)$	$SU(2)$	$U(1)_Y$	$U(1)_A$
$\mathbf{Q}^f_L$	$f = 1, 2, 3$	$\mathbf{3}$	$\mathbf{2}$	$1/6$	$z^f_{\mathbf{Q}}$
$u^{c,f}_R$		$\bar{\mathbf{3}}$	$\mathbf{1}$	$-2/3$	z^f_u
$d^{c,f}_R$		$\bar{\mathbf{3}}$	$\mathbf{1}$	$1/3$	z^f_d
$\mathbf{L}^f_L$		$\mathbf{1}$	$\mathbf{2}$	$-1/2$	$z^f_{\mathbf{L}}$
$e^{c,f}_R$		$\mathbf{1}$	$\mathbf{1}$	1	z^f_e
$\nu^{c,f}_R$		$\mathbf{1}$	$\mathbf{1}$	0	z^f_ν
H		$\mathbf{1}$	$\mathbf{2}$	$1/2$	z_H
$\psi^{\mathbf{L}_i}_L$		$\mathbf{1}$	$\mathbf{2}$	$y^i_{\mathbf{L}}$	$q^i_{\mathbf{L}}$
$(\psi^{\mathbf{L}_i}_R)^c$	$i = 1, .., N_{\mathbf{L}}$	$\mathbf{1}$	$\mathbf{2}$	$-y^i_{\mathbf{L}}$	$\widetilde{q_{\mathbf{L}}}^i$
$\psi^{e_j}_L$		$\mathbf{1}$	$\mathbf{1}$	y^j_e	q^j_e
$(\psi^{e_j}_R)^c$	$j = 1, .., N_e$	$\mathbf{1}$	$\mathbf{1}$	$-y^j_e$	$\widetilde{q_e}^j$
$\psi^{d_k}_L$		$\mathbf{3}$	$\mathbf{1}$	y^k_d	q^k_d
$(\psi^{d_k}_R)^c$	$k = 1, .., N_d$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$-y^k_d$	$\widetilde{q_d}^k$
$\psi^{\mathbf{Q}_m}_L$		$\mathbf{3}$	$\mathbf{2}$	$y^m_{\mathbf{Q}}$	$q^m_{\mathbf{Q}}$
$(\psi^{\mathbf{Q}_m}_R)^c$	$m = 1, .., N_{\mathbf{Q}}$	$\bar{\mathbf{3}}$	$\mathbf{2}$	$-y^m_{\mathbf{Q}}$	$\widetilde{q_{\mathbf{Q}}}^m$
S		$\mathbf{1}$	$\mathbf{1}$	0	q_S

Universality hypothesis

Assume universality with respect to both U(1)'s

$\forall\ i=1,\dots,N_{\mathbf{L}}$	$\forall\ j=1,\dots,N_e$	$\forall\ k=1,\dots,N_d$	$\forall\ m=1,\dots,N_{\mathbf{Q}}$
$\varepsilon^i_{\mathbf{L}}=\varepsilon_{\mathbf{L}}$	$\varepsilon^j_e=\varepsilon_e$	$\varepsilon^k_d=\varepsilon_d$	$\varepsilon^m_{\mathbf{Q}}=\varepsilon_{\mathbf{Q}}$
$y^i_{\mathbf{L}}=y_{\mathbf{L}}$	$y^j_e=y_e$	$y^k_d=y_d$	$y^m_{\mathbf{Q}}=y_{\mathbf{Q}}$
$q^i_{\mathbf{L}}=q_{\mathbf{L}}$	$q^j_e=q_e$	$q^k_d=q_d$	$q^m_{\mathbf{Q}}=q_{\mathbf{Q}}$
$\widetilde{q_{\mathbf{L}}}^i=\widetilde{q_{\mathbf{L}}}$	$\widetilde{q_e}^j=\widetilde{q_e}$	$\widetilde{q_d}^k=\widetilde{q_d}$	$\widetilde{q_{\mathbf{Q}}}^m=\widetilde{q_{\mathbf{Q}}}$

		$SU(3)$	$SU(2)$	$U(1)_Y$	$U(1)_A$
$\mathbf{Q}^f_L$	$f=1,2,3$	$\mathbf{3}$	$\mathbf{2}$	$1/6$	$z^f_{\mathbf{Q}}$
$u^{c,f}_R$		$\bar{\mathbf{3}}$	$\mathbf{1}$	$-2/3$	z^f_u
$d^{c,f}_R$		$\bar{\mathbf{3}}$	$\mathbf{1}$	$1/3$	z^f_d
$\mathbf{L}^f_L$		$\mathbf{1}$	$\mathbf{2}$	$-1/2$	$z^f_{\mathbf{L}}$
$e^{c,f}_R$		$\mathbf{1}$	$\mathbf{1}$	1	z^f_e
$\nu^{c,f}_R$		$\mathbf{1}$	$\mathbf{1}$	0	z^f_ν
H		$\mathbf{1}$	$\mathbf{2}$	$1/2$	z_H
$\psi^{\mathbf{L}_i}_L$		$\mathbf{1}$	$\mathbf{2}$	$y^i_{\mathbf{L}}$	$q^i_{\mathbf{L}}$
$(\psi^{\mathbf{L}_i}_R)^c$	$i=1,..,N_{\mathbf{L}}$	$\mathbf{1}$	$\mathbf{2}$	$-y^i_{\mathbf{L}}$	$\widetilde{q_{\mathbf{L}}}^i$
$\psi^{e_j}_L$		$\mathbf{1}$	$\mathbf{1}$	y^j_e	q^j_e
$(\psi^{e_j}_R)^c$	$j=1,..,N_e$	$\mathbf{1}$	$\mathbf{1}$	$-y^j_e$	$\widetilde{q_e}^j$
$\psi^{d_k}_L$		$\mathbf{3}$	$\mathbf{1}$	y^k_d	q^k_d
$(\psi^{d_k}_R)^c$	$k=1,..,N_d$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$-y^k_d$	$\widetilde{q_d}^k$
$\psi^{\mathbf{Q}_m}_L$		$\mathbf{3}$	$\mathbf{2}$	$y^m_{\mathbf{Q}}$	$q^m_{\mathbf{Q}}$
$(\psi^{\mathbf{Q}_m}_R)^c$	$m=1,..,N_{\mathbf{Q}}$	$\bar{\mathbf{3}}$	$\mathbf{2}$	$-y^m_{\mathbf{Q}}$	$\widetilde{q_{\mathbf{Q}}}^m$
S		$\mathbf{1}$	$\mathbf{1}$	0	q_S

Anomalies from the extra fermions

Cancellation of the anomalies contributions

$$
\begin{aligned}
Tr[q_A]_{secluded} &= \Big(2\varepsilon_{\mathbf{L}} N_{\mathbf{L}} + \varepsilon_e N_e + 3\varepsilon_d N_d + 6\varepsilon_{\mathbf{Q}} N_{\mathbf{Q}}\Big) q_S &&= -t_A, \\
Tr[YYq_A]_{secluded} &= \Big(2\varepsilon_{\mathbf{L}} y_{\mathbf{L}}^2 N_{\mathbf{L}} + \varepsilon_e y_e^2 N_e \\
&\quad + 3\varepsilon_d y_d^2 N_d + 6\varepsilon_{\mathbf{Q}} y_{\mathbf{Q}}^2 N_{\mathbf{Q}}\Big) q_S &&= -t_{YYA}, \\
Tr[Yq_Aq_A]_{secluded} &= -q_S^2\Big(2y_{\mathbf{L}} N_{\mathbf{L}} + y_e N_e + 3y_d N_d + 6y_{\mathbf{Q}} N_{\mathbf{Q}}\Big) \\
&\quad + 2q_S\Big(2\varepsilon_{\mathbf{L}} y_{\mathbf{L}} q_{\mathbf{L}} N_{\mathbf{L}} + \varepsilon_e y_e q_e N_e \\
&\quad + 3\varepsilon_d y_d q_d N_d + 6\varepsilon_{\mathbf{Q}} y_{\mathbf{Q}} q_{\mathbf{Q}} N_{\mathbf{Q}}\Big) &&= -t_{YAA}, \\
Tr[q_Aq_Aq_A]_{secluded} &= q_S^3\Big(2\varepsilon_{\mathbf{L}} N_{\mathbf{L}} + \varepsilon_e N_e + 3\varepsilon_d N_d + 6\varepsilon_{\mathbf{Q}} N_{\mathbf{Q}}\Big) \\
&\quad - 3q_S^2\Big(2q_{\mathbf{L}} N_{\mathbf{L}} + q_e N_e + 3q_d N_d + 6q_{\mathbf{Q}} N_{\mathbf{Q}}\Big) \\
&\quad + 3q_S\Big(2\varepsilon_{\mathbf{L}} q_{\mathbf{L}}^2 N_{\mathbf{L}} + \varepsilon_e q_e^2 N_e \\
&\quad + 3\varepsilon_d q_d^2 N_d + 6\varepsilon_{\mathbf{Q}} q_{\mathbf{Q}}^2 N_{\mathbf{Q}}\Big) &&= -t_{AAA}, \\
Tr[q_AT_2T_2]_{secluded} &= (\varepsilon_{\mathbf{L}} N_{\mathbf{L}} + 3\varepsilon_{\mathbf{Q}} N_{\mathbf{Q}}) q_S &&= -t_2, \\
Tr[q_AT_3T_3]_{secluded} &= (\varepsilon_d N_d + 2\varepsilon_{\mathbf{Q}} N_{\mathbf{Q}}) q_S &&= -t_3.
\end{aligned}
$$

		$SU(3)$	$SU(2)$	$U(1)_Y$	$U(1)_A$
$\mathbf{Q}_L^f$	$f = 1,2,3$	$\mathbf{3}$	$\mathbf{2}$	$1/6$	$z_{\mathbf{Q}}^f$
$u_R^{c,f}$		$\bar{\mathbf{3}}$	$\mathbf{1}$	$-2/3$	z_u^f
$d_R^{c,f}$		$\bar{\mathbf{3}}$	$\mathbf{1}$	$1/3$	z_d^f
$\mathbf{L}_L^f$		$\mathbf{1}$	$\mathbf{2}$	$-1/2$	$z_{\mathbf{L}}^f$
$e_R^{c,f}$		$\mathbf{1}$	$\mathbf{1}$	1	z_e^f
$\nu_R^{c,f}$		$\mathbf{1}$	$\mathbf{1}$	0	z_ν^f
H		$\mathbf{1}$	$\mathbf{2}$	$1/2$	z_H
$\psi_L^{\mathbf{L}_i}$		$\mathbf{1}$	$\mathbf{2}$	$y_{\mathbf{L}}^i$	$q_{\mathbf{L}}^i$
$(\psi_R^{\mathbf{L}_i})^c$	$i = 1,..,N_{\mathbf{L}}$	$\mathbf{1}$	$\mathbf{2}$	$-y_{\mathbf{L}}^i$	$\widetilde{q_{\mathbf{L}}}^i$
$\psi_L^{e_j}$		$\mathbf{1}$	$\mathbf{1}$	y_e^j	q_e^j
$(\psi_R^{e_j})^c$	$j = 1,..,N_e$	$\mathbf{1}$	$\mathbf{1}$	$-y_e^j$	$\widetilde{q_e}^j$
$\psi_L^{d_k}$		$\mathbf{3}$	$\mathbf{1}$	y_d^k	q_d^k
$(\psi_R^{d_k})^c$	$k = 1,..,N_d$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$-y_d^k$	$\widetilde{q_d}^k$
$\psi_L^{\mathbf{Q}_m}$		$\mathbf{3}$	$\mathbf{2}$	$y_{\mathbf{Q}}^m$	$q_{\mathbf{Q}}^m$
$(\psi_R^{\mathbf{Q}_m})^c$	$m = 1,..,N_{\mathbf{Q}}$	$\bar{\mathbf{3}}$	$\mathbf{2}$	$-y_{\mathbf{Q}}^m$	$\widetilde{q_{\mathbf{Q}}}^m$
S		$\mathbf{1}$	$\mathbf{1}$	0	q_S

Solving in the case where all N's are non zero

The equations $t_3 = 0$ and $t_A = 0$ lead to:

$$2\varepsilon_{\mathbf{L}} N_{\mathbf{L}} + \varepsilon_e N_e = 0,$$
$$2\varepsilon_{\mathbf{Q}} N_{\mathbf{Q}} + \varepsilon_d N_d = 0.$$

Since the N's are positive integers, we must have $\varepsilon_{\mathbf{L}}\varepsilon_e = -1$, $\varepsilon_{\mathbf{Q}}\varepsilon_d = -1$ and thus

$$N_e = 2N_{\mathbf{L}}, \quad N_d = 2N_{\mathbf{Q}}.$$

		$SU(3)$	$SU(2)$	$U(1)_Y$	$U(1)_A$
$\mathbf{Q}_L^f$	$f = 1,2,3$	$\mathbf{3}$	$\mathbf{2}$	$1/6$	$z_{\mathbf{Q}}^f$
$u_R^{c,f}$		$\bar{\mathbf{3}}$	$\mathbf{1}$	$-2/3$	z_u^f
$d_R^{c,f}$		$\bar{\mathbf{3}}$	$\mathbf{1}$	$1/3$	z_d^f
$\mathbf{L}_L^f$		$\mathbf{1}$	$\mathbf{2}$	$-1/2$	$z_{\mathbf{L}}^f$
$e_R^{c,f}$		$\mathbf{1}$	$\mathbf{1}$	1	z_e^f
$\nu_R^{c,f}$		$\mathbf{1}$	$\mathbf{1}$	0	z_ν^f
H		$\mathbf{1}$	$\mathbf{2}$	$1/2$	z_H
$\psi_L^{\mathbf{L}_i}$		$\mathbf{1}$	$\mathbf{2}$	$y_{\mathbf{L}}^i$	$q_{\mathbf{L}}^i$
$(\psi_R^{\mathbf{L}_i})^c$	$i = 1, .., N_{\mathbf{L}}$	$\mathbf{1}$	$\mathbf{2}$	$-y_{\mathbf{L}}^i$	$\widetilde{q_{\mathbf{L}}}^i$
$\psi_L^{e_j}$		$\mathbf{1}$	$\mathbf{1}$	y_e^j	q_e^j
$(\psi_R^{e_j})^c$	$j = 1, .., N_e$	$\mathbf{1}$	$\mathbf{1}$	$-y_e^j$	$\widetilde{q_e}^j$
$\psi_L^{d_k}$		$\mathbf{3}$	$\mathbf{1}$	y_d^k	q_d^k
$(\psi_R^{d_k})^c$	$k = 1, .., N_d$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$-y_d^k$	$\widetilde{q_d}^k$
$\psi_L^{\mathbf{Q}_m}$		$\mathbf{3}$	$\mathbf{2}$	$y_{\mathbf{Q}}^m$	$q_{\mathbf{Q}}^m$
$(\psi_R^{\mathbf{Q}_m})^c$	$m = 1, .., N_{\mathbf{Q}}$	$\bar{\mathbf{3}}$	$\mathbf{2}$	$-y_{\mathbf{Q}}^m$	$\widetilde{q_{\mathbf{Q}}}^m$
S		$\mathbf{1}$	$\mathbf{1}$	0	q_S

Solving in the case where all N's are non zero

12 parameters: $y_{\mathbf{L}}$, y_e, $y_{\mathbf{Q}}$, y_d, $q_{\mathbf{L}}$, q_e, $q_{\mathbf{Q}}$, q_d, q_S, z_H, $N_{\mathbf{L}}$ and $N_{\mathbf{Q}}$

We demand:

- All charges are rational numbers.
- The lepton-like extra fermions $\psi^{\mathbf{L}}$ have electric charges 0 or ± 1, and ψ^e electric charge ± 1.
- The quark-like extra fermions $\psi^{\mathbf{Q}}$ and ψ^d have electric charges $\pm 1/3$ or $\pm 2/3$. Indeed, this condition ensures that when the color forces confine, the resulting bound states can all carry integer charges.
- We will consider $\varepsilon_{\mathbf{L}} = 1$, $\varepsilon_e = -1$, $\varepsilon_d = 1$ and $\varepsilon_{\mathbf{Q}} = -1$

We get:

$$y_{\mathbf{L}} = \pm\frac{1}{2}, \quad y_{\mathbf{Q}} = \pm\frac{1}{6}, \quad y_d = \pm\frac{2}{3}, \quad y_e = \pm 1, \qquad N_{\mathbf{L}} = N_{\mathbf{Q}}.$$

		$SU(3)$	$SU(2)$	$U(1)_Y$	$U(1)_A$
$\mathbf{Q}_L^f$	$f = 1,2,3$	$\mathbf{3}$	$\mathbf{2}$	$1/6$	$z_{\mathbf{Q}}^f$
$u_R^{c,f}$		$\bar{\mathbf{3}}$	$\mathbf{1}$	$-2/3$	z_u^f
$d_R^{c,f}$		$\bar{\mathbf{3}}$	$\mathbf{1}$	$1/3$	z_d^f
$\mathbf{L}_L^f$		$\mathbf{1}$	$\mathbf{2}$	$-1/2$	$z_{\mathbf{L}}^f$
$e_R^{c,f}$		$\mathbf{1}$	$\mathbf{1}$	1	z_e^f
$\nu_R^{c,f}$		$\mathbf{1}$	$\mathbf{1}$	0	z_ν^f
H		$\mathbf{1}$	$\mathbf{2}$	$1/2$	z_H
$\psi_L^{\mathbf{L}_i}$		$\mathbf{1}$	$\mathbf{2}$	$y_{\mathbf{L}}^i$	$q_{\mathbf{L}}^i$
$(\psi_R^{\mathbf{L}_i})^c$	$i = 1,..,N_{\mathbf{L}}$	$\mathbf{1}$	$\mathbf{2}$	$-y_{\mathbf{L}}^i$	$\widetilde{q_{\mathbf{L}}}^i$
$\psi_L^{e_j}$		$\mathbf{1}$	$\mathbf{1}$	y_e^j	q_e^j
$(\psi_R^{e_j})^c$	$j = 1,..,N_e$	$\mathbf{1}$	$\mathbf{1}$	$-y_e^j$	$\widetilde{q_e}^j$
$\psi_L^{d_k}$		$\mathbf{3}$	$\mathbf{1}$	y_d^k	q_d^k
$(\psi_R^{d_k})^c$	$k = 1,..,N_d$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$-y_d^k$	$\widetilde{q_d}^k$
$\psi_L^{\mathbf{Q}_m}$		$\mathbf{3}$	$\mathbf{2}$	$y_{\mathbf{Q}}^m$	$q_{\mathbf{Q}}^m$
$(\psi_R^{\mathbf{Q}_m})^c$	$m = 1,..,N_{\mathbf{Q}}$	$\bar{\mathbf{3}}$	$\mathbf{2}$	$-y_{\mathbf{Q}}^m$	$\widetilde{q_{\mathbf{Q}}}^m$
S		$\mathbf{1}$	$\mathbf{1}$	0	q_S

Simple solutions when all N's are non zero

3 cases:

Case 1:
$$\begin{cases} q_{\mathbf{L}} = q_e = \dfrac{2q_S z_H + q_S^2\left[y_{\mathbf{L}} + y_e + 3(y_d + y_{\mathbf{Q}})\right] - 2z_H^2(y_{\mathbf{Q}} - y_d)}{2q_S(y_{\mathbf{L}} - y_e + y_{\mathbf{Q}} - y_d)}, \\ q_{\mathbf{Q}} = q_d = \dfrac{2q_S z_H + q_S^2\left[y_{\mathbf{L}} + y_e + 3(y_d + y_{\mathbf{Q}})\right] + 2z_H^2(y_{\mathbf{L}} - y_e)}{-6q_S(y_{\mathbf{L}} - y_e + y_{\mathbf{Q}} - y_d)}. \end{cases}$$

Case 2:
$$\begin{cases} q_{\mathbf{L}} = -q_e = \dfrac{2q_S z_H + q_S^2\left[y_{\mathbf{L}} + y_e + 3(y_d + y_{\mathbf{Q}})\right] - 2z_H^2(y_{\mathbf{Q}} - y_d)}{2q_S(y_{\mathbf{L}} + y_e)}, \\ q_{\mathbf{Q}} = q_d = -\dfrac{z_H^2}{3q_S}. \end{cases}$$

Case 3:
$$\begin{cases} q_{\mathbf{L}} = q_e = -\dfrac{z_H^2}{q_S}, \\ q_{\mathbf{Q}} = -q_d = \dfrac{2q_S z_H + q_S^2\left[y_{\mathbf{L}} + y_e + 3(y_d + y_{\mathbf{Q}})\right] + 2z_H^2(y_{\mathbf{L}} - y_e)}{-6q_S(y_{\mathbf{Q}} + y_d)}. \end{cases}$$

		$SU(3)$	$SU(2)$	$U(1)_Y$	$U(1)_A$
$\mathbf{Q}_L^f$	$f = 1, 2, 3$	$\mathbf{3}$	$\mathbf{2}$	$1/6$	$z_{\mathbf{Q}}^f$
$u_R^{c,f}$		$\bar{\mathbf{3}}$	$\mathbf{1}$	$-2/3$	z_u^f
$d_R^{c,f}$		$\bar{\mathbf{3}}$	$\mathbf{1}$	$1/3$	z_d^f
$\mathbf{L}_L^f$		$\mathbf{1}$	$\mathbf{2}$	$-1/2$	$z_{\mathbf{L}}^f$
$e_R^{c,f}$		$\mathbf{1}$	$\mathbf{1}$	1	z_e^f
$\nu_R^{c,f}$		$\mathbf{1}$	$\mathbf{1}$	0	z_ν^f
H		$\mathbf{1}$	$\mathbf{2}$	$1/2$	z_H
$\psi_L^{\mathbf{L}_i}$		$\mathbf{1}$	$\mathbf{2}$	$y_{\mathbf{L}}^i$	$q_{\mathbf{L}}^i$
$(\psi_R^{\mathbf{L}_i})^c$	$i = 1, .., N_{\mathbf{L}}$	$\mathbf{1}$	$\mathbf{2}$	$-y_{\mathbf{L}}^i$	$\widetilde{q_{\mathbf{L}}}^i$
$\psi_L^{e_j}$		$\mathbf{1}$	$\mathbf{1}$	y_e^j	q_e^j
$(\psi_R^{e_j})^c$	$j = 1, .., N_e$	$\mathbf{1}$	$\mathbf{1}$	$-y_e^j$	$\widetilde{q_e}^j$
$\psi_L^{d_k}$		$\mathbf{3}$	$\mathbf{1}$	y_d^k	q_d^k
$(\psi_R^{d_k})^c$	$k = 1, .., N_d$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$-y_d^k$	$\widetilde{q_d}^k$
$\psi_L^{\mathbf{Q}_m}$		$\mathbf{3}$	$\mathbf{2}$	$y_{\mathbf{Q}}^m$	$q_{\mathbf{Q}}^m$
$(\psi_R^{\mathbf{Q}_m})^c$	$m = 1, .., N_{\mathbf{Q}}$	$\bar{\mathbf{3}}$	$\mathbf{2}$	$-y_{\mathbf{Q}}^m$	$\widetilde{q_{\mathbf{Q}}}^m$
S		$\mathbf{1}$	$\mathbf{1}$	0	q_S

Solution: *Example 1*

			$SU(3)$	$SU(2)$	$U(1)_Y$
SM sector	$\mathbf{Q}_L^f$	$f = 1,2,3$	$\mathbf{3}$	$\mathbf{2}$	$1/6$
	$u_R^{c,f}$		$\bar{\mathbf{3}}$	$\mathbf{1}$	$-2/3$
	$d_R^{c,f}$		$\bar{\mathbf{3}}$	$\mathbf{1}$	$1/3$
	$\mathbf{L}_L^f$		$\mathbf{1}$	$\mathbf{2}$	$-1/2$
	$e_R^{c,f}$		$\mathbf{1}$	$\mathbf{1}$	1
	$\nu_R^{c,f}$		$\mathbf{1}$	$\mathbf{1}$	0
	H		$\mathbf{1}$	$\mathbf{2}$	$1/2$
Secluded sector	$\psi_L^{\mathbf{L}_i}$		$\mathbf{1}$	$\mathbf{2}$	$-1/2$
	$(\psi_R^{\mathbf{L}_i})^c$	$i = 1,..,N_{\mathbf{L}}$	$\mathbf{1}$	$\mathbf{2}$	$+1/2$
	$\psi_L^{e_j}$		$\mathbf{1}$	$\mathbf{1}$	-1
	$(\psi_R^{e_j})^c$	$j = 1,..,2N_{\mathbf{L}}$	$\mathbf{1}$	$\mathbf{1}$	$+1$
	$\psi_L^{d_k}$		$\mathbf{3}$	$\mathbf{1}$	$-2/3$
	$(\psi_R^{d_k})^c$	$k = 1,..,2N_{\mathbf{L}}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$+2/3$
	$\psi_L^{\mathbf{Q}_m}$		$\mathbf{3}$	$\mathbf{2}$	$+1/6$
	$(\psi_R^{\mathbf{Q}_m})^c$	$m = 1,..,N_{\mathbf{L}}$	$\bar{\mathbf{3}}$	$\mathbf{2}$	$-1/6$
	S		$\mathbf{1}$	$\mathbf{1}$	0

Table 3: Anomaly-free solution with $q_{\mathbf{L}} = q_e, q_d = q_{\mathbf{Q}}$, and $N_{\mathbf{L}} = 3$, $z_L = 1$. The $U(1)_A^3$ anomaly is $t_{AAA} = -36$. With the Majorana mass term for the RH neutrino, we can take $n = 2$, $\varepsilon_\nu = +1$, $z_L = -2$, $z_{\mathbf{Q}} = 4/3$, which implies $z_u = -7/3$, $z_d = -1/3$, $z_e = 1$, $z_\nu = 1$

Solution: *Example 2*

			$SU(3)$	$SU(2)$	$U(1)_Y$	$U(1)_A$
SM sector	$\mathbf{Q}_L^f$	$f = 1, 2, 3$	$\mathbf{3}$	$\mathbf{2}$	$1/6$	$2/3$
	$u_R^{c,f}$		$\bar{\mathbf{3}}$	$\mathbf{1}$	$-2/3$	$-8/3$
	$d_R^{c,f}$		$\bar{\mathbf{3}}$	$\mathbf{1}$	$1/3$	$4/3$
	$\mathbf{L}_L^f$		$\mathbf{1}$	$\mathbf{2}$	$-1/2$	2
	$e_R^{c,f}$		$\mathbf{1}$	$\mathbf{1}$	1	0
	$\nu_R^{c,f}$		$\mathbf{1}$	$\mathbf{1}$	0	-4
	H		$\mathbf{1}$	$\mathbf{2}$	$1/2$	2
Secluded sector	$\psi_L^{\mathbf{L}_i}$		$\mathbf{1}$	$\mathbf{2}$	$-1/2$	-3
	$(\psi_R^{\mathbf{L}_i})^c$	$i = 1, .., N_{\mathbf{L}}$	$\mathbf{1}$	$\mathbf{2}$	$+1/2$	5
	$\psi_L^{e_j}$		$\mathbf{1}$	$\mathbf{1}$	$+1$	-3
	$(\psi_R^{e_j})^c$	$j = 1, .., 2N_{\mathbf{L}}$	$\mathbf{1}$	$\mathbf{1}$	-1	1
	$\psi_L^{d_k}$		$\mathbf{3}$	$\mathbf{1}$	$+2/3$	$1/3$
	$(\psi_R^{d_k})^c$	$k = 1, .., 2N_{\mathbf{L}}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$-2/3$	$5/3$
	$\psi_L^{\mathbf{Q}_m}$		$\mathbf{3}$	$\mathbf{2}$	$+1/6$	$1/3$
	$(\psi_R^{\mathbf{Q}_m})^c$	$m = 1, .., N_{\mathbf{L}}$	$\bar{\mathbf{3}}$	$\mathbf{2}$	$-1/6$	$-7/3$
	S		$\mathbf{1}$	$\mathbf{1}$	0	2

Table 4: Anomaly-free solution with $q_{\mathbf{L}} = q_e, q_d = q_{\mathbf{Q}}$, and $N_{\mathbf{L}} = 3$, $z_L = 2$. The $U(1)_A^3$ anomaly is $t_{AAA} = -288$. With the Majorana mass term for the RH neutrino, we can take $n = 1$, $\varepsilon_\nu = -1$, $z_L = -1$, $z_{\mathbf{Q}} = 5/3$, which implies $z_u = -11/3$, $z_d = 1/3$, $z_e = 3$, $z_\nu = -1$

Solution: Example 3

			$SU(3)$	$SU(2)$	$U(1)_Y$	$U(1)_A$
SM sector	$\mathbf{Q}_L^f$	$f=1,2,3$	$\mathbf{3}$	$\mathbf{2}$	$1/6$	$1/3$
	$u_R^{c,f}$		$\bar{\mathbf{3}}$	$\mathbf{1}$	$-2/3$	$-10/3$
	$d_R^{c,f}$		$\bar{\mathbf{3}}$	$\mathbf{1}$	$1/3$	$8/3$
	$\mathbf{L}_L^f$		$\mathbf{1}$	$\mathbf{2}$	$-1/2$	1
	$e_R^{c,f}$		$\mathbf{1}$	$\mathbf{1}$	1	2
	$\nu_R^{c,f}$		$\mathbf{1}$	$\mathbf{1}$	0	-4
	H		$\mathbf{1}$	$\mathbf{2}$	$1/2$	3
Secluded sector	$\psi_L^{\mathbf{L}_i}$		$\mathbf{1}$	$\mathbf{2}$	$-1/2$	-3
	$(\psi_R^{\mathbf{L}_i})^c$	$i=1,..,N_{\mathbf{L}}$	$\mathbf{1}$	$\mathbf{2}$	$+1/2$	6
	$\psi_L^{e_j}$		$\mathbf{1}$	$\mathbf{1}$	-1	-3
	$(\psi_R^{e_j})^c$	$j=1,..,2N_{\mathbf{L}}$	$\mathbf{1}$	$\mathbf{1}$	$+1$	0
	$\psi_L^{d_k}$		$\mathbf{3}$	$\mathbf{1}$	$-2/3$	0
	$(\psi_R^{d_k})^c$	$k=1,..,2N_{\mathbf{L}}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$+2/3$	3
	$\psi_L^{\mathbf{Q}_m}$		$\mathbf{3}$	$\mathbf{2}$	$+1/6$	0
	$(\psi_R^{\mathbf{Q}_m})^c$	$m=1,..,N_{\mathbf{L}}$	$\bar{\mathbf{3}}$	$\mathbf{2}$	$-1/6$	-3
	S		$\mathbf{1}$	$\mathbf{1}$	0	3

Table 5: Anomaly-free solution with $q_{\mathbf{L}} = q_e, q_d = q_{\mathbf{Q}}$, and $N_{\mathbf{L}} = 1$, $z_L = 1$. The $U(1)_A^3$ anomaly is $t_{AAA} = -324$.

Solution: some N's vanish

			$SU(3)$	$SU(2)$	$U(1)_Y$	$U(1)_A$
SM sector	$\mathbf{Q}_L^f$	$f = 1, 2, 3$	$\mathbf{3}$	$\mathbf{2}$	$1/6$	$1/3$
	$u_R^{c,f}$		$\bar{\mathbf{3}}$	$\mathbf{1}$	$-2/3$	$-4/3$
	$d_R^{c,f}$		$\bar{\mathbf{3}}$	$\mathbf{1}$	$1/3$	$2/3$
	$\mathbf{L}_L^f$		$\mathbf{1}$	$\mathbf{2}$	$-1/2$	-2
	$e_R^{c,f}$		$\mathbf{1}$	$\mathbf{1}$	1	3
	$\nu_R^{c,f}$		$\mathbf{1}$	$\mathbf{1}$	0	1
	H		$\mathbf{1}$	$\mathbf{2}$	$1/2$	1
Secluded sector	$\psi_L^{\mathbf{L}_i}$		$\mathbf{1}$	$\mathbf{2}$	0	$1/2$
	$(\psi_R^{\mathbf{L}_i})^c$	$i = 1, .., N_{\mathbf{L}}$	$\mathbf{1}$	$\mathbf{2}$	0	$1/2$
	$\psi_L^{e_j}$		$\mathbf{1}$	$\mathbf{1}$	$+1/2$	$1/2$
	$(\psi_R^{e_j})^c$	$j = 1, .., 2N_{\mathbf{L}}$	$\mathbf{1}$	$\mathbf{1}$	$-1/2$	$-3/2$
	S		$\mathbf{1}$	$\mathbf{1}$	0	1

Table 9: Anomaly-free solution with $(N_d, N_{\mathbf{Q}}) = (0,0)$, $(N_{\mathbf{L}}, N_e) \neq (0,0)$, written with the choice of the free parameters $q_S = 1$, $z_H = 1$, $z_{\mathbf{Q}} = 1/3$ and $N_{\mathbf{L}} = 3$. The $U(1)_A^3$ anomaly is $t_{AAA} = 18$.

The EFT UV Cutoff: Preskill bound

Often quoted is the Preskill cutoff

$$\Lambda_{eff} \sim \left| \frac{64\pi^3 M_A}{[g_A^3 t_{AAA}^{(light)} + 2g_A^2 g_Y t_{YAA}^{(light)} + g_A g_Y^2 t_{AYY}^{(light)} + g_A g_2^2 t_2^{(light)} + g_A g_3^2 t_3^{(light)}]} \right|$$

More precisely, the Preskill bound is

$$\Lambda_{eff} \lesssim \left| \frac{64\pi^3 M_A}{[g_A^3 t_{AAA}^{(light)} + 2g_A^2 g_Y t_{YAA}^{(light)} + g_A g_Y^2 t_{AYY}^{(light)} + g_A g_2^2 t_2^{(light)} + g_A g_3^2 t_3^{(light)}]} \right|$$

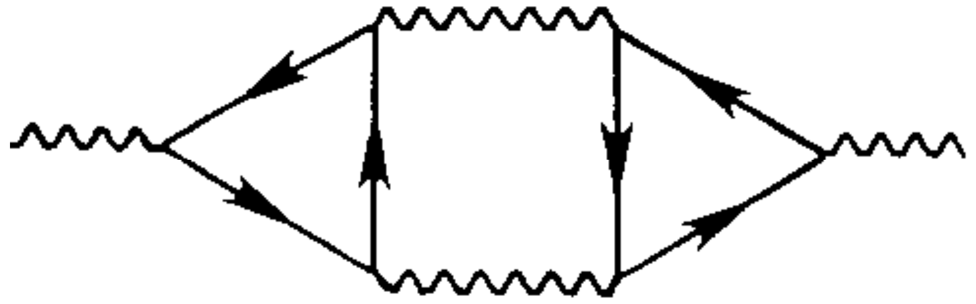
The EFT UV Cutoff

The tree-level mass for the gauge boson Z'_A

$$M_A^{(0)} = g_A|q_S|v_S$$

The predominant radiative contribution to the $U(1)_A$ gauge boson Z'_A mass

$$M_A{}^{(1)} \simeq \left| \frac{[g_A^3 t_{AAA}^{(light)} + 2g_A^2 g_Y t_{YAA}^{(light)} + g_A g_Y^2 t_{AYY}^{(light)} + g_A g_2^2 t_2^{(light)} + g_A g_3^2 t_3^{(light)}]\Lambda_{eff}}{64\pi^3} \right|$$



The effective theory cut-off scale Λ_{eff} will be approximately equal to the mass scale of the heavy fermions, i.e.,

$$\Lambda_{eff} \simeq M_f$$

the heavy secluded fermion mass originates from the Yukawa coupling

$$M_f \simeq Y_{ij} v_S \simeq v_S$$

The EFT UV Cutoff

The ratio of the loop-induced mass with respect of the tree-level one is now of order:

$$\frac{{M_A}^{(1)}}{{M_A}^{(0)}} \simeq \frac{g_A^2 \left|t^{(h)}_{AAA}\right|}{64\pi^3 q_S} \quad \xrightarrow[\text{all extra fermions heavy}]{} \quad \frac{{M_A}^{(1)}}{{M_A}^{(0)}} \simeq \frac{3 g_A^2 z_H^2 N_{\mathbf{L}}}{16\pi^3}$$

Indeed, the dominance of the anomaly loop-induced mass for the Z'_A requires that $g_A {z_H}^2 N_{\mathbf{Q}} \sim 10^3$. To achieve a light Z'_A with a mass $M_A \ll M_f$, it necessitates a coupling $g_A \ll 1$. This, in turn, implies significantly large charges and/or a large number of fields $N_{\mathbf{Q}}$, especially if we assume $q_S = 1$.

The EFT UV Cutoff

One potential resolution to this issue is that only some of the fermions are heavy enough

fermions $\psi_L^{\mathbf{L}i}$ with large charges $q\mathbf{L} \gg q_S$ are heavy and inaccessible.

$$\frac{{M_A}^{(1)}}{{M_A}^{(0)}} \simeq \frac{g_A^2 \left| t_{AAA}^{(h)} \right|}{64\pi^3 q_S} \qquad \xrightarrow[N_{\mathbf{L}} \psi_L^{\mathbf{L}} \text{ heavy}]{} \qquad \frac{{M_A}^{(1)}}{{M_A}^{(0)}} \simeq \frac{3 g_A^2 q_{\mathbf{L}}^2 N_{\mathbf{L}}}{32\pi^3}$$

$g_A^2 q_{\mathbf{L}}^2 N_{\mathbf{L}}$ that needs to be of order $\sim 10^3$.

The EFT UV Cutoff

$$\Lambda_{eff} \simeq M_f \qquad M_f \simeq Y_{ij} v_S \simeq v_S \qquad M_A^{(0)} = g_A |q_S| v_S$$

$$\Longrightarrow \qquad \Lambda_{eff} \simeq \frac{M_A}{g_A q_S} \qquad (q_S > 0)$$

In the case where g_A is hierarchically the smallest coupling in the theory, the magnetic Swampland Conjecture can be used to put a bound as:

$$\Lambda_{eff} \lesssim \Lambda_{QG} \simeq g_A M_P \Rightarrow M_A \lesssim g_A^2 M_p$$

Conclusions

In summary, we have explored a scenario where the gauge symmetry appears to be anomalous at low energies but is completed in the ultraviolet (UV) such that there are no anomalies.

We have defined a set of models that achieve this, with the Standard Model (SM) being charged under this U(1) extension.

We provided explicit examples of such models and discussed the location of the cutoff scale for the effective field theory (EFT).

Work in progress …