Istituto Nazionale di Fisica Nucleare Sezione di Napoli

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Workshop on Standard Model and Beyond

Theory of rare kaon decays

Giancarlo D'Ambrosio INFN Sezione di Napoli

Anatomy of kaon decays and prospects for lepton flavour universality violation

G. D'Ambrosio (INFN, Naples), A.M. Iyer (Indian Inst. Tech., New Delhi), F. Mahmoudi (IP2I, Lyon and CERN), S. Neshatpour (INFN, Naples) (Jun 29, 2022) Published in: JHEP 09 (2022) 148, JHEP 09 (2022) 148 • e-Print: 2206.14748 [hep-ph]

Collaboration with David Greynat and Marc Knecht On the amplitudes for the CP-conserving $K \pm (KS) \rightarrow \pi \pm (\pi 0)\ell + \ell -$ rare decay modes

> arXiv:1812.00735 JHEP, Matching long and short distances at order $O(\alpha s)$

in the form factors for $K \rightarrow \pi \ell + \ell - PLB$ arXiv:1906.03046

Collaboration with Crivellin, A., Kitahara, T and Nierste, U. e-Print: arXiv:1703.05786 PRD

Crivellin, A, GD, Hoferichter, M and Tunstall, L Phys.Rev. D 2016

Closing in on the radiative weak chiral couplings Luigi Cappiello, Oscar Cata, GD arXiv:1712.10270,,EPJC

Collaboration with Teppei Kitahara arXiv:1707.06999 PRL

Outline

- K->π νν
- K_{S,L}->μμ
- Theory
- Lepton Flavour Universality Violation in Kaons
- Conclusions

 $K \to \pi \nu \overline{\nu}$

Why we need KOTO NA62 HIKE

 $A(s \to d\nu\overline{\nu})_{\rm SM} \sim \overline{s}_L \gamma_\mu d_L \quad \overline{\nu}_L \gamma^\mu \nu_L \quad \times \left[\sum_{q=c,t} V_{qs}^* V_{qd} \ m_q^2 \right]$



 $\left[A^2\lambda^5 \left(1-\rho-i\eta\right)m_t^2+\lambda m_c^2\right]$

$$\begin{array}{l} \displaystyle \underset{\psi}{\mathsf{SM}} \quad \underbrace{V - A \otimes V - A}_{\psi} \quad \text{Littenberg} \\ \\ \displaystyle \Gamma(K_L \to \pi^0 \nu \overline{\nu}) \quad \begin{cases} \ \mathrm{CP \ violating} \\ \Rightarrow \ J = A^2 \lambda^6 \eta \\ \\ \mathrm{Only \ top} \end{cases} \end{array}$$

SM

Buchalla and Buras, hep-ph/9308272, Buras et al, 1503.02693.



History of $K^+ \rightarrow \pi^+ \nu \nu$ searches



NA62: $K^+ \to \pi^+ \nu \bar{\nu}$ RUN1=2016+2017+2018



 $BR(K^+ \to \pi^+ \nu \bar{\nu}) = (10.6^{+4.0}_{-3.4}|_{stat} \pm 0.9_{syst}) \times 10^{-11} \text{ at } 68\% \text{ CL} \qquad 3.4\sigma \text{ significance}$

Most precise determination of the decay rate to date

Prospects: Run 2 (2021-2025) with nominal beam intensity, modified beamline and additional detectors.

Aim to measure BR($K^+ \rightarrow \pi^+ \nu \nu$) at O(15%) precision by LS3

KOTO: $K^0 \rightarrow \pi^0 \nu \bar{\nu}$



Observed: 3 events in the signal box

Prospects:

J-PARC main ring upgrade: beam power to 80-100 kW KOTO DAQ upgrade: event throughput x4 KOTO will collect x 11 more data, low background Projected S.E.S. ~ O(10⁻¹¹) by 2026

Source		Number of events
K_L	$K_L \rightarrow 3\pi^0$	0.01 ± 0.01
	$K_L \rightarrow 2\gamma$ (beam halo)	$0.26\pm0.07^{\rm a}$
	Other K_L decays	0.005 ± 0.005
K^{\pm}		$0.87\pm0.25^{\rm a}$
Neutron	Hadron cluster	0.017 ± 0.002
	$CV \eta$	0.03 ± 0.01
	Upstream π^0	0.03 ± 0.03
Total	-	1.22 ± 0.26

PRL 126 (2021) 121801

Background sources studied after looking inside the blind signal region







*Approx. error on LD-subtracted rate excluding parametric contributions

- FCNC processes dominated by Z-penguin and box diagrams.
- SM rates determined by V_{CKM}, with minimal non-parametric "theory" uncertainties.
- Theory errors are being reduced [Lattice QCD, e.g. arXiv:2203.10998].
- * The current focus is on $K \rightarrow \pi \nu \nu$: uniquely clean theoretically.



Prospects: HIKE and KOTO-II

HIKE: multi-purpose high-intensity kaon decay-in-flight experiments proposed at CERN SPS

High-intensity beams at CERN North Area after LS3 with x 4-6 current NA62 nominal K^+ : 2.2 x 10¹³ decays per year K_L : 3.8 x 10¹³ decays per year

Phase 1 ($K^+ \rightarrow \pi^+ \nu \nu$ at ~5% precision), Phase 2 ($K_L \rightarrow \pi^0 l^+ l^-$ at ~20% precision) Comprehensive program of rare kaon decays, precision measurements, searches

KOTO-II: high-beam-power experiment proposed at J-PARC Hadron Hall

Increase proton beam power > 100 kW New neutral beamline at 5 degrees: larger K_L yield, momentum = 5.2 GeV/c Increase fiducial volume from 2x2m to 3x12m, new detectors

60 SM events of K_L $\rightarrow \pi^0 \nu \nu$ with S/B~1 in 3 years, ~20% precision Search for exotic particles in K_L $\rightarrow \pi^0 X$



- LHCb experiment has been designed for efficient reconstructions of b and c
- Huge production of strangeness [O(10¹³)/fb⁻¹ K⁰_S] is suppressed by its trigger efficiency [ε~1-2%@LHC Run-I, ε~18%@LHC Run-II]
- LHCb upgrade (LS2=Phase I upgrade, LS4=Phase II upgrade) could realize high efficiency for K⁰_S [ε~90%@LHC Run-III] [M. R. Pernas, HL/HE LHC meeting, Fermilab, 2018]
- In LHC Run-III and HL-LHC, we could probe the ultra rare decay Br~O(10^{-11~12})

Ks->µµ

PHYSICAL REVIEW D

VOLUME 10, NUMBER 3

Rare decay modes of the K mesons in gauge theories

M. K. Gaillard* and Benjamin W. Lee† National Accelerator Laboratory, Batavia, Illinois 60510‡ (Received 4 March 1974)

Rare decay modes of the kaons such as $K \to \mu \overline{\mu}$, $K \to \pi \nu \overline{\nu}$, $K \to \gamma \gamma$, $K \to \pi \gamma \gamma$, and $K \to \pi e \overline{e}$ are of theoretical interest since here we are observing higher-order weak and electromagnetic interactions. Recent advances in unified gauge theories of weak and electromagnetic interactions allow in principle unambiguous and finite predictions for these processes. The above processes, which are induced $|\Delta S|=1$ transitions, are a good testing ground for the cancellation mechanism first invented by Glashow, Iliopoulos, and Maiani (GIM) in order to banish $|\Delta S| = 1$ neutral currents. The experimental suppression of $K_L \rightarrow \mu \overline{\mu}$ and nonsuppression of $K_L \rightarrow \gamma \gamma$ must find a natural explanation in the GIM mechanism which makes use of extra quark(s). The procedure we follow is the following: We deduce the effective interaction Lagrangian for $\lambda + \mathfrak{N} \rightarrow l + \overline{l}$ and $\lambda + \overline{\mathfrak{N}} \rightarrow \gamma + \gamma$ in the free-quark model; then the appropriate matrix elements of these operators between hadronic states are evaluated with the aid of the principles of conserved vector current and partially conserved axial-vector current. We focus our attention on the Weinberg-Salam model. In this model, $K \rightarrow \mu \overline{\mu}$ is suppressed due to a fortuitous cancellation. To explain the small $K_L - K_S$ mass difference and nonsuppression of $K_L \rightarrow \gamma \gamma$, it is found necessary to assume $m_{\varphi}/m_{\varphi'} << 1$, where m_{φ} is the mass of the proton quark and $m_{\phi'}$ the mass of the charmed quark, and $m_{\phi'} < 5$ GeV. We present a phenomenological argument which indicates that the average mass of charmed pseudoscalar states lies below 10 GeV. The effective interactions so constructed are then used to estimate the rates of other processes. Some of the results are the following: $K_S \rightarrow \gamma \gamma$ is suppressed; $K_S \rightarrow \pi \gamma \gamma$ proceeds at a normal rate, but $K_L \rightarrow \pi \gamma \gamma$ is suppressed; $K_L \rightarrow \pi \nu \overline{\nu}$ is very much forbidden, and $K^+ \rightarrow \pi^+ \nu \overline{\nu}$ occurs with the branching ratio of ~10⁻¹⁰; $K^+ \rightarrow \pi^+ e \overline{e}$ has the

$\Gamma(~K^0_S o \mu^+ \mu^-~)/\Gamma_{ m total}$

Test for $\Delta S = 1$ weak neutral current. Allowed by first-order weak interaction combined with electromagnetic interaction.

VALUE	CL%	DOCUMENT ID		TECN
< 2.1 $ imes$ 10 ⁻¹⁰	90	¹ AAIJ	2020AE	LHCB
•	 We do not use the following data for averages, fits, lin 	mits, etc. ● ●		
$< 8 imes 10^{-10}$	90	² AAIJ	2017BQ	LHCB
$< 9 imes 10^{-9}$	90	³ AAIJ	2013G	LHCB
$< 3.2 imes 10^{-7}$	90	GJESDAL	1973	ASPK
$< 7 imes 10^{-6}$	90	HYAMS	1969B	OSPK

 $K_{S} \rightarrow \mu \mu$



 $< 2.1 \times 10^{-10} \quad 90\% CL$

 $K_L \rightarrow \mu \mu$

- $\Gamma(K_L^0 \to \mu^+ \mu^-) / \Gamma(K_L^0 \to \pi^+ \pi^-)$



FIG. 7. Leading contributions to $\lambda + \overline{\mathfrak{A}} \rightarrow \gamma + \gamma$. To leading order in M_W^{-2} , the diagrams in (a) reduce to those of (b).

(b)

 $K_L o \gamma \gamma \mid_{\mathrm{exp}} \mathsf{known}$

Gaillard Lee

. P'

P

 $K_I \rightarrow \mathcal{U}\mathcal{U}$



 $0.98 \pm 0.55 = |ReA|^2 = (\chi_{\gamma\gamma}(M_{\rho}) + \chi_{\text{short}} - 5.12)^2$

$$|\chi_{\rm short}^{\rm SM}| = 1.96(1.11 - 0.92\bar{\rho})$$

Isidori Unterdorfer

$K_L \rightarrow \mu\mu$: our sign ignorance





CPLEAR Flavor tagging

$$p\overline{p} \rightarrow K^{-}\pi^{+}K^{0}$$

 $K^{+}\pi^{-}\overline{K}^{0}$

$$\frac{R(\tau)}{R(\tau)} \propto (1 \mp 2 \operatorname{Re}(\varepsilon_L)) (\mathrm{e}^{-\Gamma_{\mathrm{S}}\tau} + |\eta_{+-}|^2 \mathrm{e}^{-\Gamma_{\mathrm{L}}\tau} \pm 2|\eta_{+-}| \mathrm{e}^{-\frac{1}{2}(\Gamma_{\mathrm{S}}+\Gamma_{\mathrm{L}})\tau} \cos(\Delta m\tau - \phi_{+-}))$$







Can we study K^o(t)?

GD , Kitahara 1707.06999 PRL



$$\begin{aligned} |\widetilde{K}^{0}(t)\rangle &= \frac{1}{\sqrt{2}(1\pm\overline{\epsilon})} \left[e^{-iH_{S}t} \left(|K_{1}\rangle + \overline{\epsilon}|K_{2}\rangle \right) \\ &\pm e^{-iH_{L}t} \left(|K_{2}\rangle + \overline{\epsilon}|K_{1}\rangle \right) \right] \end{aligned} \qquad D = \frac{K^{0} - \overline{K}^{0}}{K^{0} + \overline{K}^{0}} \end{aligned}$$

- Short distance interfering with Large CP conserving LD contribution !
- We may be able to study the time evolution of K⁰ by tracking the associated particles (K⁻)

$$\sum_{\text{spin}} \mathcal{A}(K_1 \to \mu^+ \mu^-)^* \mathcal{A}(K_2 \to \mu^+ \mu^-)$$
$$\sim \text{Im}[\lambda_t] y_{7A}' \left\{ A_{L\gamma\gamma}^{\mu} - 2\pi \sin^2 \theta_W \left(\text{Re}[\lambda_t] y_{7A}' + \text{Re}[\lambda_c] y_c \right) \right\}$$

K_{S} -> $\mu\mu$: how to improve the THEORY error



Dispersive treatment of $K_S \rightarrow \gamma \gamma$ and $K_S \rightarrow \gamma l^+ l^-$

Gilberto Colangelo, Ramon Stucki, and Lewis C. Tunstall

LD 5×10^{-12} 20% TH err

$$K_S \to \gamma \mu \mu$$
$$K_S \to \mu \mu \mu \mu$$
$$K_S \to ee \mu \mu$$
$$K_S \to \gamma \gamma$$

$K_{L,S} \to \mu \mu$



$$Br(K_S \to \mu^+ \mu^-)_{\ell=0} = Br(K_L \to \mu^+ \mu^-) \times \frac{\tau_S}{\tau_L} \times \left(\frac{C_{int}}{C_L}\right)^2$$

[Dery et al. 2104.06427]

Short distance window GD, Kitahara 1707.06999 PRI







LHCb-upgrade Phase-II-upgrade?

Rare Kaon decay program at LHCB

PDG

Prospects

 $< 9 \times 10^{-9}$ at 90% CL $(LD)(5.0 \pm 1.5) \cdot 10^{-12}$ NP < 10^{-11} $K_S \rightarrow \mu \mu$ SM LD $\sim 2 \times 10^{-14}$ $K_S \rightarrow \mu \mu \mu \mu$ $\sim 10^{-11}$ $K_S \rightarrow ee \mu \mu$ $\sim 10^{-10}$ $K_S \rightarrow eeee$ $(2.9 \pm 1.3) \cdot 10^{-9}$ $\sim 10^{-9}$ $K_S
ightarrow \pi^0 \mu \mu$ $K_S \to \pi^+ \pi^- e^+ e^-$ (4.79 ± 0.15) · 10⁻⁵ SM LD $\sim 10^{-5}$ $K_S \rightarrow \pi^+ \pi^- \mu^+ \mu^-$ SM LD $\sim 10^{-14}$

> Rare n Strange 2017: strange physics at LHCb GD, Lewis Tunstall, Diego Martinez Santos,Veronika Chobanova, Xabier Cid Vidal, Francesco Dettori, Marc-Olivier Bettler, Teppei Kitahara,,Kei Yamamoto

Theory

	su (2), x su(2),	SU(3),	SUIZ) _L	Y
L_i	$\left(\frac{1}{2},0\right)$	1	2	- 1/2
e,	$\left(0,\frac{1}{z}\right)$	٦	4	-1
Qi	$\left(\frac{1}{z},0\right)$	3	2	76
ιί	$\left(0,\frac{1}{z}\right)$	3	7	NIM
d ز	$\left(0,\frac{1}{2}\right)$	3	4	- 1/3
Н	(o, o)	1	2	$\frac{1}{2}$



$$\begin{aligned} \mathcal{I}^{SM} &= -\frac{1}{4} \sum_{A} F^{A}_{\mu\nu} F^{A\mu\nu} - \theta \frac{9^{2}}{32\pi^{2}} G^{A}_{\mu\nu} \overline{G}^{A\mu\nu} + \sum_{f} \overline{\mathcal{V}}_{f} i D_{\mu} \mathcal{V}^{M} \mathcal{V}_{f} \\ &+ |D_{\mu}H|^{2} - V(H) - \left(\overline{\mathcal{V}}_{F}^{i} \mathcal{V}_{F}^{ii} \mathcal{V}_{F} H + h.c.\right) \end{aligned}$$



Weinberg '77 EFT SM Accidental symmetries



E.g.: Lepton Flavour Violation, unsuppressed FCNC and CP effects, B and L violation, etc..



• Veneziano: maybe the solution is a two scale



Generic Flavor structures strongly constrained

Operator	Bounds on Λ in TeV ($c_{NP} = 1$)		Bounds on $c_{\rm NP}$ ($\Lambda = 1$ TeV)		Observables	
	Re	Im	Re	Im		
$(\bar{s}_L \gamma^{\mu} d_L)^2$	9.8×10^{2}	1.6×10^{4}	9.0×10^{-7}	3.4×10^{-9}	America	
$(\bar{s}_R d_L)(\bar{s}_L d_R)$	1.8×10^{4}	3.2×10^{8}	6.9×10^{-9}	2.6×10^{-11}	Δm_R , ϵ_R	
$(\bar{c}_L \gamma^{\mu} u_L)^2$	1.2×10^{3}	2.9×10^{3}	5.6×10^{-7}	1.0×10^{-7}	Amu: la/slu du	
$(\bar{c}_R u_L)(\bar{c}_L u_R)$	6.2×10^{3}	1.5×10^{4}	5.7×10^{-8}	1.1×10^{-8}	Δm_D , $ q/p _D$, φ_D	
$(\bar{b}_L \gamma^\mu d_L)^2$	6.6×10^{2}	9.3×10^{2}	2.3×10^{-8}	1.1×10^{-6}	Amp : $\sin(2\beta)$ from $B_1 \rightarrow \psi K$	
$(\bar{b}_R d_L)(\bar{b}_L d_R)$	2.5×10^{3}	3.6×10^{3}	3.9×10^{-7}	1.9×10^{-7}	Δm_{B_d} , $\delta m(2p)$ from $D_d \rightarrow \varphi R$	
$(b_L \gamma^\mu s_L)^2$	1.4×10^{2}	2.5×10^{2}	5.0×10^{-5}	1.7×10^{-5}	Am_{B} : $\sin(\phi)$ from $B \rightarrow ih\phi$	
$(\bar{b}_R s_L)(\bar{b}_L s_R)$	4.8×10^2	8.3×10^2	8.8×10^{-6}	2.9×10^{-6}	Δm_{B_s} , $\operatorname{sm}(\psi_s)$ from $B_s \to \psi \psi$	

Isidori Nir Perez 10

Problem already known since '86 technicolour (Chivukula Georgi) susy (Hall Randall) extra dimensions (Rattazzi Zafferoni)

Maybe there is an energy gap between the theory of flavor and the EW scale, ameliorating also a clash from the scale of the bounds in the table above and the requirement of solving the hierarchy problem



$$\begin{aligned} \chi^{SM} &= -\frac{1}{4} \sum_{A} F^{A}_{\mu\nu} F^{A\mu\nu} - \theta \frac{9^{2}}{32\pi^{2}} G^{A}_{\mu\nu} G^{A\mu\nu} + \sum_{f} \overline{\psi}_{f} i D_{\mu} \chi^{M} \psi_{f} \\ &+ |D_{\mu}H|^{2} - V(H) - (\overline{\psi}_{F}^{i} \psi_{F}^{i} \psi_{F} H + h.c.) \end{aligned}$$

$$G_F = \underbrace{\operatorname{U}(3)_Q \otimes \operatorname{U}(3)_U \otimes \operatorname{U}(3)_D \otimes \operatorname{U}(3)_L \otimes \operatorname{U}(3)_E}_{\text{global symmetry}} + \underbrace{Y_{U,D,E}}_{Y_{U,D,E}}$$

$$\mathcal{L}_{MFV}^{Y} = \mathcal{L}_{SM}^{Y} + \dim_{-}6$$

	su (2), x su(2),	SU(3),	SUIZ) _L	Y
L_i	(1,0)	1	2	- 1/2
ei	$\left(0,\frac{1}{z}\right)$	٦	4	- 1
Qi	$\left(\frac{1}{z},0\right)$	3	2	76
ι	$\left(0,\frac{1}{z}\right)$	3	7	2/m
di	$\left(0,\frac{1}{2}\right)$	3	4	- 1/3
Н	(o, o)	1	2	$\frac{1}{2}$

U(3) ->U(2)





Shift of paradigma

Dvali & Shifman '00 Panico & Pomarol '16 … Bordone *et al.* '17 Allwicher, GI, Thomsen '20 Barbieri '21 Davighi & G.I. '23

Main idea:

- Flavor non-universal interactions already at the TeV scale:
- 1st & 2nd gen. have small masses because they are coupled to NP at heavier scales




G. Isidori – Kaon Physics: the next step

Kaon 2016, Birmingham, Sept 2016

Lepton Flavor Universality

Example-II: neutral currents, $\mu^+\mu^- vs$. e^+e^-



...but a potential more promising effect could appear in our beloved $K \rightarrow \pi v v$ decays....

Anatomy of kaon decays and prospects for lepton flavour universality violation

arxiv 2206.14748 GD, A.M. Iyer, F. Mahmoudi, S. Neshatpour

Motivated by B-anomalies we study LFUV Kaon decays

$$\mathcal{H}_{ ext{eff}} = -rac{4G_F}{\sqrt{2}}\lambda_t^{sd}rac{lpha_e}{4\pi}\sum_k C_k^\ell O_k^\ell\,,$$

$$O_{9}^{\ell} = (\bar{s}\gamma_{\mu}P_{L}d) (\bar{\ell}\gamma^{\mu}\ell), \qquad O_{10}^{\ell} = (\bar{s}\gamma_{\mu}P_{L}d) (\bar{\ell}\gamma^{\mu}\gamma_{5}\ell),$$
$$O_{L}^{\ell} = (\bar{s}\gamma_{\mu}P_{L}d) (\bar{\nu}_{\ell}\gamma^{\mu}(1-\gamma_{5})\nu_{\ell}),$$

$$\delta C_L^\ell \equiv \delta C_9^\ell = -\delta C_{10}^\ell$$

$$\delta C_L^\tau = \delta C_L^\mu$$





Figure 7: The bounds from individual observables. The right panel is the zoomed version of the left panel. The coloured regions correspond to 68% CL when there is a measurement and the dashed ones to upper limits at 90% CL. $K_L \rightarrow \mu \bar{\mu}$ has been shown for both signs of the long-distance contribution. For $K_L \rightarrow \pi^0 e\bar{e}$ and $K_L \rightarrow \pi^0 \mu \bar{\mu}$, constructive interference between direct and indirect CP-violating contributions has been assumed.



Very recent development: HIKE full projections

Italy before 2023



World for several millennia





Pompei

World before VII BC





Conclusions

- Interesting new experiments and phenomenology
- Interplay with high energy experiments very important
- LHCB: K_S->µµ extraordinary result: interference effect!!!Short distance window



Conclusions

- Flavour anomalies: interplay with K-> $\pi\nu\nu$ but 10% measurement needed!
- LHCB: K_S->µµ extraordinary result: ^a/_{2×10⁻¹²}
 interference effect!!!Short distance window



- weak chiral lagrangian
- LFUV in Kaons very useful
- Rich rare kaon program

 $K \to \pi \nu \overline{\nu}$

Why we need KOTO and NA62

 $A(s \to d\nu\overline{\nu})_{\rm SM} \sim \overline{s}_L \gamma_\mu d_L \quad \overline{\nu}_L \gamma^\mu \nu_L \quad \times \left[\sum_{q=c,t} V_{qs}^* V_{qd} \ m_q^2 \right]$





SM

Buchalla and Buras, hep-ph/9308272, Buras et al, 1503.02693.

K+-> π+ν ν



Misiak, Urban; Buras, Buchalla; Brod, Gorbhan, Stamou`11, Straub

$$\begin{split} \lambda_{q} &= V_{qd}^{*} V_{qs} \\ \mathcal{B}(K^{+}) \sim \kappa_{+} \left[\left(\frac{\mathrm{Im}\lambda_{t}}{\lambda^{5}} X_{t} \right)^{2} + \left(\frac{\mathrm{Re}\lambda_{c}}{\lambda} \left(P_{c} + \delta P_{c,u} \right) + \frac{\mathrm{Re}\lambda_{t}}{\lambda^{5}} X_{t} \right)^{2} \right] \\ \downarrow \\ \chi_{13} & \text{LD} \\ \mathcal{B}(K^{\pm}) &= (8.82 \pm 0.8 \pm 0.3) \times 10^{-11} & \text{TH} \\ \frac{V_{cb}}{V_{cb}} & \text{nonpert QCD} \\ & \left(1.73^{+1.15}_{-1.05} \right) \times 10^{-10} & \text{E949} \\ < 11 \cdot 10^{-10} 90\% \text{ CL} & \text{NA62} \end{split}$$

UV sensitivity

$$\mathcal{L} \sim \frac{1 - 0.3 i}{(180 \text{ TeV})^2} (\overline{s}_L \gamma_\mu d_L \overline{\nu}_L \gamma^\mu \nu_L)$$

Cristina Lazzeroni, ECFA mtg 2022

 δC_L^e

Kaon Global Fit

For example, recent paper with global fits to set of kaon measurements Deviation of Wilson coefficients from SM, for NP scenarios with only lefthanded quark currents.



Bounds from individual observables. Coloured regions are 68%CL measurements Dashed lines are 90%CL upper limits



With projections: central value for existing measurements kept the same, A upper bounds extrapolated to central value consistent with SM, B central value of all observables is projected to the best-fit points obtained from fits to existing data

$$\mathcal{H}_{\text{eff}} = -\frac{1}{\sqrt{2}} \mathcal{N}_t \frac{1}{4\pi} \sum_k \mathcal{O}_k \mathcal{O}_k$$
$$\mathcal{O}_L^{\ell} = (\bar{s}\gamma_{\mu} P_L d) \left(\bar{\nu}_{\ell} \gamma^{\mu} (1 - \gamma_5) \nu_{\ell} \right)$$

 $\mathcal{U} = \frac{4G_F}{\sqrt{sd}} \alpha_e \sum C^{\ell} O^{\ell}$

 $C_k^\ell = C_{k,{\rm SM}}^\ell + \delta C_k^\ell$

Observable	SM prediction	Experimental results	Ref.	HIKE projections	
$BR(K^+ \to \pi^+ \nu \bar{\nu})$	$(7.86\pm0.61)\times10^{-11}$	$(10.6^{+4.0}_{-3.5}\pm0.9) imes10^{-11}$	[110]	5% (Phase 1)	
$BR(K_L \to \pi^0 v \bar{v})$	$(2.68 \pm 0.30) \times 10^{-11}$	$< 300 \times 10^{-11}$ @90% CL	[144]	20% (Phase 3)	
LFUV $(a^{\mu\mu}_+ - a^{ee}_+)$	0	-0.031 ± 0.017	[145, 146]	±0.007 (Phase 1)	
$BR(K_L \to \mu \mu) (+)$	$(6.82^{+0.77}_{-0.29}) imes 10^{-9}$	$(6.84 \pm 0.11) \times 10^{-9}$	[147]	1% (Phase 2)	
$BR(K_L \to \mu \mu) (-)$	$(8.04^{+1.47}_{-0.98}) imes 10^{-9}$	(0.04 ± 0.11) × 10	[147]	1 % (r hase 2)	
$BR(K_S \rightarrow \mu \mu)$	$(5.15\pm1.50)\times10^{-12}$	$< 2.1(2.4) \times 10^{-10}$ @90(95)% CL	[148]	Upper bound kept to current value	
$BR(K_L \to \pi^0 ee)(+)$	$(3.46^{+0.92}_{-0.80}) \times 10^{-11}$	$< 28 \times 10^{-11}$ @90% CI	[140]	20% (Phase 2)	
$BR(K_L \to \pi^0 ee)(-)$	$(1.55^{+0.60}_{-0.48}) \times 10^{-11}$	28×10 4 @90% CL	[149]	20% (Plase 2)	
$BR(K_L \to \pi^0 \mu \mu)(+)$	$(1.38^{+0.27}_{-0.25}) \times 10^{-11}$	$< 38 \times 10^{-11}$ @00% CI	[150]	20% (Phase 2)	
$BR(K_L \to \pi^0 \mu \mu)(-)$	$(0.94^{+0.21}_{-0.20}) \times 10^{-11}$	< 56 × 10 @90% CL	[150]	20% (Phase 2)	

Table 5: The SM predictions, current experimental status and the expected HIKE sensitivity for the different observables. The "(+)" and "(-)" signs in the first column correspond to constructive and destructive interference of the amplitudes.



Figure 17: Global fits in the $\{\delta C_L^e, \delta C_L^\mu (= \delta C_L^\tau)\}$ plane with current data (purple contours) and the projected scenarios (green regions).

Kaons and LFUV

Deviation of Wilson coefficients from SM, for NP scenarios with only left-handed quark currents. $C^{\ell} - C^{\ell}$

[arXiv:2206.14748]

 $\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} \lambda_t^{sd} \frac{\alpha_e}{4\pi} \sum_k C_k^\ell O_k^\ell$

$$O_L^{\ell} = (\bar{s}\gamma_{\mu}P_L d) \left(\bar{\nu}_{\ell}\gamma^{\mu}(1-\gamma_5)\nu_{\ell}\right)$$

 $C_k^\ell = C_{k,\mathrm{SM}}^\ell + \delta C_k^\ell \qquad \delta C_L^\ell \equiv \delta C_9^\ell = -\delta C_{10}^\ell$



Individual measurements can be combined in a fit. Projections can be used for future experiments **Projections:**

A: predicted central values for observables with only upper bound is projected to be as SM prediction; for measured ones current central values are taken.

B: central values for all observables are projected with best-fit points obtained from fits of existing data.



Very recent development: HIKE full projections

 $\begin{array}{c} & & & \\ & &$

[CERN Physics Beyond Colliders Document in preparation, and paper In preparation by D'Ambrosio, Mahmoudi, Neshatpour]

26

 $\mathcal{L}_{\text{Yukawa}}^{\text{SM}} = Y_d^{ij} \bar{Q}_L^i \phi D_R^j + Y_u^{ij} \bar{Q}_L^i \tilde{\phi} U_R^j + Y_e^{ij} \bar{L}_L^i \phi E_R^j + \text{h.c.}$

arxiv 2206.1474



Figure 4: $BR(K_L \to \mu \bar{\mu})$ as a function of $\delta C_L^{\mu} (\equiv \delta C_9^{\mu} = -\delta C_{10}^{\mu})$ assuming both possible signs for the long-distance contribution from $A_{L\gamma\gamma}^{\mu}$ on the left panel. $BR(K_S \to \mu \bar{\mu})$ as a function of NP contributions in δC_L^{μ} on the right panel. In the left (right) panel, the grey band indicates the experimental measurement (upper limit) while the coloured bands correspond to the theoretical uncertainties. The LHCb bound and prospect for $BR(K_S \to \mu \bar{\mu})$ are from Ref. [44] and Ref. [51], respectively.

Theory of Kaon Physics

$$K_L \rightarrow \pi^0 e^+ e^-$$
 : summary

 $Br(K_L \to \pi^0 e^+ e^-) \le 2.8 \cdot 10^{-10} \text{ at } 90\% \text{ CL}$ KTeV



CP conserving NA48

$$Br(K_L \to \pi^0 e^+ e^-) < 3 \cdot 10^{-12}$$

$$V-A \otimes V-A \Rightarrow \langle \pi^0 e^+ e^- | (\bar{s}d)_{V-A} (\bar{e}e)_{V-A} | K_L \rangle$$
 violates CP



Possible large interference: $a_S < -0.5$ or $a_S > 1$; short distance probe even for a_S large

$$|\mathbf{2}) + \mathbf{3})|^{2} = \left[15.3 \ a_{S}^{2} - 6.8 \frac{Im\lambda_{t}}{10^{-4}} \ a_{S} + 2.8 \left(\frac{Im\lambda_{t}}{10^{-4}}\right)^{2}\right] \cdot 10^{-12}$$

$$[17.7 \pm 9.5 + 4.7] \cdot 10^{-12}$$

Isidori Mescia, Smith



	$C^\ell_{ m dir}$	$C_{ m int}^\ell$	$C_{ m mix}^\ell$	$C^\ell_{\gamma\gamma}$
$\ell = e$	$(4.62 \pm 0.24)(w_{7V}^2 + w_{7A}^2)$	$(11.3 \pm 0.3) w_{7V}$	14.5 ± 0.5	≈ 0
$\ell=\mu$	$(1.09 \pm 0.05)(w_{7V}^2 + 2.32w_{7A}^2)$	$(2.63 \pm 0.06) w_{7V}$	3.36 ± 0.20	5.2 ± 1.6

arxiv 2206.14748



 $BR^{SM}(K_L \to \pi^0 e\bar{e}) = 3.46^{+0.92}_{-0.80} \left(1.55^{+0.60}_{-0.48}\right) \times 10^{-11}$ $BR^{SM}(K_L \to \pi^0 \mu \bar{\mu}) = 1.38^{+0.27}_{-0.25} \left(0.94^{+0.21}_{-0.20}\right) \times 10^{-11}$

 ${
m BR}^{
m exp}(K_L o \pi^0 e \bar{e}) < 28 imes 10^{-11}$ at 90% CL ${
m BR}^{
m exp}(K_L o \pi^0 \mu \bar{\mu}) < 38 imes 10^{-11}$ at 90% CL .

Observable	SM prediction	Exp results	Ref.	Experimental Err. Projections	
$BR(K^+ \to \pi^+ \nu \nu)$	$(7.86 \pm 0.61) \times 10^{-11}$	$(10.6^{+4.0}_{-3.5}\pm 0.9) \times 10^{-11}$	[15]	10% (@2025) 5% (CERN; long-term) [58]	
${\rm BR}(K^0_L\to\pi^0\nu\nu)$	$(2.68 \pm 0.30) \times 10^{-11}$	$< 3.0 \times 10^{-9}$ @90% CL	[17]	20%(CERN; long-term [58]) 15% (KOTO [61])	
${ m LFUV}(a_+^{\mu\mu}-a_+^{ee})$	0	-0.031 ± 0.017	[16, 42]	± 0.007 (assuming ± 0.005 for each mode)	
$BR(K_L \to \mu \mu) (+)$	$(6.82^{+0.77}_{-0.29}) \times 10^{-9}$	$(6.84 \pm 0.11) \times 10^{-9}$	[43]	experimental uncertainty kept to current value	
$BR(K_L \to \mu \mu) \ (-)$	$(8.04^{+1.47}_{-0.98}) \times 10^{-9}$	(0.01 ± 0.11) × 10			
$BR(K_S \to \mu \mu)$	$(5.15 \pm 1.50) \times 10^{-12}$	$< 2.1(2.4) \times 10^{-10}$ @90(95)% CL	[44]	$< 8 \times 10^{-12}$ @95% CL (CERN; long-term [51])	
$BR(K_L \to \pi^0 ee)(+)$	$(3.46^{+0.92}_{-0.80}) \times 10^{-11}$	$< 28 \times 10^{-11}$ @90% CL	[56]		
$BR(K_L \to \pi^0 ee)(-)$	$(1.55^{+0.60}_{-0.48}) \times 10^{-11}$		[00]	observation (CEBN: long-term 58)	
$BR(K_L \to \pi^0 \mu \mu)(+)$	$(1.38^{+0.27}_{-0.25}) \times 10^{-11}$	$< 38 \times 10^{-11}$ @90% CL	[57]	(we assume 100% error)	
$BR(K_L \to \pi^0 \mu \mu)(-)$	$(0.94^{+0.21}_{-0.20}) \times 10^{-11}$		[31]		

arxiv 2206.1474



Figure 7: The bounds from individual observables. The right panel is the zoomed version of the left panel. The coloured regions correspond to 68% CL when there is a measurement and the dashed ones to upper limits at 90% CL. $K_L \rightarrow \mu \bar{\mu}$ has been shown for both signs of the long-distance contribution. For $K_L \rightarrow \pi^0 e\bar{e}$ and $K_L \rightarrow \pi^0 \mu \bar{\mu}$, constructive interference between direct and indirect CP-violating contributions has been assumed.

Observable	SM prediction	Experimental results	Ref.	HIKE projections	
$BR(K^+ \to \pi^+ \nu \bar{\nu})$	$(7.86\pm0.61)\times10^{-11}$	$(10.6^{+4.0}_{-3.5}\pm0.9) imes10^{-11}$	[110]	5% (Phase 1)	
$BR(K_L \to \pi^0 v \bar{v})$	$(2.68 \pm 0.30) \times 10^{-11}$	$< 300 \times 10^{-11}$ @90% CL	[144]	20% (Phase 3)	
LFUV $(a_+^{\mu\mu} - a_+^{ee})$	0	-0.031 ± 0.017	[145, 146]	±0.007 (Phase 1)	
$BR(K_L \to \mu \mu) (+)$	$(6.82^{+0.77}_{-0.29}) imes 10^{-9}$	$(6.84 \pm 0.11) \times 10^{-9}$	[147]	1% (Phase 2)	
$BR(K_L \to \mu \mu) (-)$	$(8.04^{+1.47}_{-0.98}) imes 10^{-9}$	(0.04 ± 0.11) × 10	[147]	1 % (Flase 2)	
$BR(K_S \rightarrow \mu \mu)$	$(5.15\pm1.50)\times10^{-12}$	$< 2.1(2.4) \times 10^{-10}$ @90(95)% CL	[148]	Upper bound kept to current value	
$BR(K_L \to \pi^0 ee)(+)$	$(3.46^{+0.92}_{-0.80}) \times 10^{-11}$	$< 28 \times 10^{-11}$ @90% CI	[140]	20% (Phase 2)	
$BR(K_L \to \pi^0 ee)(-)$	$(1.55^{+0.60}_{-0.48}) \times 10^{-11}$	28×10 4 @90% CL	[149]	20% (Plase 2)	
$BR(K_L \to \pi^0 \mu \mu)(+)$	$(1.38^{+0.27}_{-0.25}) \times 10^{-11}$	$< 38 \times 10^{-11}$ @00% CI	[150]	20% (Phase 2)	
$BR(K_L \to \pi^0 \mu \mu)(-)$	$(0.94^{+0.21}_{-0.20}) \times 10^{-11}$	< 58×10 €90% CL	[150]	2070 (Flase 2)	

Table 5: The SM predictions, current experimental status and the expected HIKE sensitivity for the different observables. The "(+)" and "(-)" signs in the first column correspond to constructive and destructive interference of the amplitudes.



Figure 17: Global fits in the $\{\delta C_L^e, \delta C_L^\mu (= \delta C_L^\tau)\}$ plane with current data (purple contours) and the projected scenarios (green regions).

π	2π	3π	N_i	
$\pi^+\gamma^*$	$\pi^+\pi^0\gamma^*$		$N_{14}^r - N_{15}^r K^+ \to \pi$	-+l+l-
$\pi^0 \gamma^* (S)$	$\pi^0 \pi^0 \gamma^* (L)$		$2N_{14}^r + N_{15}^r K_S \to \pi^0$	$l^{+}l^{-}$
$\pi^+\gamma\gamma$	$\pi^+\pi^0\gamma\gamma$		$N_{14} - N_{15} - 2N_{18}$	
	$\pi^+\pi^-\gamma\gamma~(S)$,,	
	$\pi^+\pi^0\gamma$	$\pi^+\pi^+\pi^-\gamma$	$N_{14} - N_{15} - N_{16} - N_{17}$	
	$\pi^+\pi^-\gamma~(S)$	$\pi^+\pi^0\pi^0\gamma$	"	
		$\pi^+\pi^-\pi^0\gamma (L)$	"	
		$\pi^+\pi^-\pi^0\gamma$ (S)	$7(N_{14}^r - N_{16}^r) + 5(N_{15}^r + N_{17})$	
	$\pi^+\pi^-\gamma^*(L)$		$N_{14}^r - N_{15}^r - 3(N_{16}^r - N_{17})$	
	$\pi^+\pi^-\gamma^*$ (S)		$N_{14}^r - N_{15}^r - 3(N_{16}^r + N_{17})$	
	$\pi^+\pi^0\gamma^*$		$N_{14}^r + 2N_{15}^r - 3(N_{16}^r - N_{17})$	
	$\pi^+\pi^-\gamma (L)$	$\pi^+\pi^-\pi^0\gamma$ (S)	$N_{29} + N_{31}$	
		$\pi^+\pi^+\pi^-\gamma$	"	
	$\pi^+\pi^0\gamma$	$\pi^+\pi^0\pi^0\gamma$	$3N_{29} - N_{30}$	
		$\pi^+\pi^-\pi^0\gamma~(S)$	$5N_{29} - N_{30} + 2N_{31}$	
		$\pi^+\pi^-\pi^0\gamma$ (L)	$6N_{28} + 3N_{29} - 5N_{30}$	

 $\mathcal{L}_{\Delta S=1} = \mathcal{L}_{\Delta S=1}^2 + \mathcal{L}_{\Delta S=1}^4 + \dots = G_8 F^4 \underbrace{\langle \lambda_6 D_\mu U^\dagger D^\mu U \rangle}_{K \to 2\pi/3\pi} + \underbrace{G_8 F^2 \sum_i N_i W_i}_{K^+ \to \pi^+ \gamma \gamma, K \to \pi l^+ l^-} + \dots$





electrons and μ 's in the final state

observables
$$\begin{cases}
R = \frac{\Gamma(K^+ \to \pi^+ \mu^+ \mu^-)}{\Gamma(K^+ \to \pi^+ e^+ e^-)} \\
\text{lepton spectrum} \quad z = \frac{q^2}{M_K^2}
\end{cases}$$
'97 Initial data inconsistency e and us: LEV?

General consideration ff



$$i \int d^4x e^{iqx} \langle \pi(p) | T \{ J^{\mu}_{\text{elm}}(x) \mathcal{L}_{\Delta S=1}(0) \} | K(k) \rangle = \frac{W(z)}{(4\pi)^2} \left[z(k+p)^{\mu} - (1-r_{\pi}^2)q^{\mu} \right]$$

$$W_i(z) = G_F M_K^2 W_i^{\text{pol}}(z) + W_i^{\pi\pi}(z)$$

GD,Ecker,Isidori,Portoles



 $A(K^+ \to \pi^+ \pi^+ \pi^-) = \alpha_0 + \alpha_+ Y + \gamma (Y^2 + X^2/3) + \beta_+ (Y^2 - X^2/3) ,$ $A(K_S \to \pi^+ \pi^- \pi^0) = b_2 X - d_2 X Y ,$

$$s_i = (k - p_i)^2$$
, $s_0 = \frac{1}{3}(s_1 + s_2 + s_3)$, $X = \frac{s_1 - s_2}{M_\pi^2}$, $Y = \frac{s_3 - s_0}{M_\pi^2}$,

$$W_i^{\text{pol}}(z) = a_i + b_i z \qquad (i = +, S)$$

$$W_i(z) = G_F M_K^2 W_i^{\text{pol}}(z) + W_i^{\pi\pi}(z)$$

$$\begin{split} W_{i}^{\text{pol}}(z) &= a_{i} + b_{i}z \qquad (i = +, S) \\ W_{i}^{\pi\pi}(z) &= \frac{1}{r_{\pi}^{2}} \begin{bmatrix} \alpha_{i} + \beta_{i} \frac{z - z_{0}}{r_{\pi}^{2}} \end{bmatrix} F(z) \ \chi(z) \qquad \begin{matrix} z_{0} = 1/3 + r_{\pi}^{2} \\ \chi(z) &= \frac{4}{9} - \frac{4r_{\pi}^{2}}{3z} - \frac{1}{3}(1 - \frac{4r_{\pi}^{2}}{z})G(z/r_{\pi}^{2}) \\ F(z) &= 1 + z/r_{V}^{2} \\ K \rightarrow 3\pi \text{ slopes} \end{split}$$

 $A(K^+ \to \pi^+ \pi^+ \pi^-) = \alpha_0 + \alpha_+ Y + \gamma (Y^2 + X^2/3) + \beta_+ (Y^2 - X^2/3) ,$ $A(K_S \to \pi^+ \pi^- \pi^0) = b_2 X - d_2 X Y ,$



Weak chiral couplings

$$egin{aligned} K^{\pm} &
ightarrow \pi^{\pm} \gamma^{*}: \ K_{S} &
ightarrow \pi^{0} \gamma^{*}: \ K^{\pm} &
ightarrow \pi^{\pm} \pi^{0} \gamma: \ K^{+} &
ightarrow \pi^{+} \gamma \gamma: \end{aligned}$$

$$egin{aligned} a_+ &= -0.578 \pm 0.016 \ [3, \ 4] \ a_S &= (1.06^{+0.26}_{-0.21} \pm 0.07) \ [5, \ 6] \ X_E &= (-24 \pm 4 \pm 4) \ {
m GeV^{-4}} \ [7] \ \hat{c} &= 1.56 \pm 0.23 \pm 0.11 \ [8] \ . \end{aligned}$$

$$\begin{split} \mathcal{N}_E^{(1)} &\equiv N_{14}^r - N_{15}^r = \frac{3}{64\pi^2} \left(\frac{1}{3} - \frac{G_F}{G_8} a_+ - \frac{1}{3} \log \frac{\mu^2}{m_K m_\pi} \right) - 3L_9^r \\ \mathcal{N}_S &\equiv 2N_{14}^r + N_{15}^r = \frac{3}{32\pi^2} \left(\frac{1}{3} + \frac{G_F}{G_8} a_S - \frac{1}{3} \log \frac{\mu^2}{m_K^2} \right) \; ; \\ \mathcal{N}_E^{(0)} &\equiv N_{14}^r - N_{15}^r - N_{16}^r - N_{17} = -\frac{|\mathcal{M}_K| f_\pi}{2G_8} X_E \; ; \\ \mathcal{N}_0 &\equiv N_{14}^r - N_{15}^r - 2N_{18}^r = \frac{3}{128\pi^2} \hat{c} - 3(L_9^r + L_{10}^r) \; , \end{split}$$

Decay mode	counterterm combination	expt. value
$K^\pm \to \pi^\pm \gamma^*$	$N_{14} - N_{15}$	-0.0167(13)
$K_S \rightarrow \pi^0 \gamma^*$	$2N_{14} + N_{15}$	+0.016(4)
$K^{\pm} ightarrow \pi^{\pm} \pi^{0} \gamma$	$N_{14} - N_{15} - N_{16} - N_{17}$	+0.0022(7)
$K^\pm \to \pi^\pm \gamma \gamma$	$N_{14} - N_{15} - 2N_{18}$	-0.0017(32)

Luigi Cappiello, Oscar Cata,GD arXiv:1712.10270,,EPJC

LFUV in Kaons

$$\frac{\Gamma(K^+ \to \pi^+ \mu^+ \mu^-)}{\Gamma(K^+ \to \pi^+ e^+ e^-)}$$

SD << LD



Collaboration with Crivellin, A Hoferichter, M and Tunstall, Phys.Rev. D 2016

LFUV: Kaons

Channel	a_+	b_+	Reference	
ee	-0.587 ± 0.010	-0.655 ± 0.044	E865	·
ee	-0.578 ± 0.016	-0.779 ± 0.066	NA48/2	$a_{\perp}^{\rm NP} = \frac{2\pi\sqrt{2}}{2} V_{ud} V_{ud}^* * C_{7V}^{\rm NP}$
$\mu\mu$	-0.575 ± 0.039	-0.813 ± 0.145	NA48/2	$\alpha + \alpha +$

$$C_{7V}^{\mu\mu} - C_{7V}^{ee} = \alpha \frac{a_{+}^{\mu\mu} - a_{+}^{ee}}{2\pi\sqrt{2}V_{ud}V_{us}^{*}} \qquad \stackrel{MFV}{\Longrightarrow} C_{9V}^{B,\mu\mu} - C_{9V}^{B,ee} = \alpha \frac{a_{+}^{\mu\mu} - a_{+}^{ee}}{2\pi\sqrt{2}V_{td}V_{ts}^{*}} = -19 \pm 79$$
LHCB-NA62 PLEASE!!

High statistics: nominal # of decays 50 times greater than NA48/2



$$i \int d^4x e^{iqx} \langle \pi(p) | T \{ J^{\mu}_{\text{elm}}(x) \mathcal{L}_{\Delta S=1}(0) \} | K(k) \rangle = \frac{W(z)}{(4\pi)^2} \left[z(k+p)^{\mu} - (1-r_{\pi}^2)q^{\mu} \right]$$

General structure of the amplitude

$$\begin{split} \mathcal{A}^{K \to \pi \ell^+ \ell^-}(s) &= -e^2 \times \bar{\mathbf{u}}(p_-) \gamma_{\rho} \mathbf{v}(p_+) \times \frac{1}{s} \times i \int d^4 x \, \langle \pi(p) | \mathbf{T}\{j^{\rho}(0) \mathcal{L}_{\text{non-lept}}^{\Delta S=1}(x)\} | K(k) \rangle \\ &- e^2 \times \bar{\mathbf{u}}(p_-) \gamma_{\rho} \mathbf{v}(p_+) \times \left(-\frac{G_{\rm F}}{\sqrt{2}} V_{us} V_{ud} \right) \times \frac{C_{7V}(\boldsymbol{\nu})}{4\pi \alpha} \, \langle \pi(p) | (\bar{s} \gamma^{\rho} d)(0) | K(k) \rangle \\ &= -e^2 \times \bar{\mathbf{u}}(p_-) \gamma^{\rho} \mathbf{v}(p_+) (k+p)_{\rho} \times \frac{W_{K\pi}(z)}{16\pi^2 M_K^2} \qquad [z \equiv s/M_K^2] \end{split}$$

 $W_{K\pi}(z) = W^{\mathrm{LD}}_{K\pi}(z; \boldsymbol{\nu}) + W^{\mathrm{SD}}_{K\pi}(z; \boldsymbol{\nu})$

$$j^{
ho}(x) = \sum_{q=u,d,s} e_q(ar q \gamma^{
ho} q)(x) \qquad \mathcal{L}^{\Delta S=1}_{ ext{non-lept}}(x) = \left(-rac{G_{ ext{F}}}{\sqrt{2}}V_{us}V_{ud}
ight) imes \sum_{I=1}^6 C_I(
u)Q_I(x;
u)$$

$$u rac{dC_{7V}(
u)}{d
u} = rac{lpha}{lpha_s(
u)} \sum_{J=1}^6 \gamma_{J,7V}(lpha_s) C_J(
u)$$

E. Witten, Nucl. Phys. B 122, 109 (1977) F. J. Gilman, M. B. Wise, Phys Rev D 21, 3150 (1980) C. Dib et al., Phys Lett B 218, 487 (1989); Phys Rev D 39, 2639 (1989) J. Flynn, L. Randall, Nucl Phys B 326, 31 (1989) [Nucl Phys B 334, 580 (1990)] A. J. Buras et al., Nucl Phys B 423, 349 (1994)

GD Greynat Knecht

Predicting a_+ , b_+ ? Going beyond the low-energy expansion

requires an unsubtracted dispersion relation

$$W(z)|_{\pi\pi} = \frac{1}{\pi} \int_0^\infty dx \frac{\operatorname{Abs} W(x/M_K^2)|_{\pi\pi}}{x - zM_K^2 - i0}$$

with

$$\frac{\operatorname{Abs} W(s/M_K^2)|_{\pi\pi}}{16\pi^2 M_K^2} = \theta(s - 4M_\pi^2) \times \frac{s - 4M_\pi^2}{s} \lambda_{K\pi}^{-1/2}(s) \times F_V^{\pi*}(s) \times f_1^{\pi^+\pi^- \to K^+\pi^-}(s)$$

Then a_+ and b_+ are given by spectral sum rules

$$G_F M_K^2 a_+|_{\pi\pi} = W(0)|_{\pi\pi} = \frac{1}{\pi} \int_0^\infty \frac{dx}{x} \operatorname{Abs} W(x/M_K^2)|_{\pi\pi}$$

and

$$G_F M_K^2 b_+|_{\pi\pi} = W'(0)|_{\pi\pi} - \frac{1}{60} \left(\frac{M_K^2}{M_\pi^2}\right)^2 \left(\alpha_+ - \beta_+ \frac{s_0}{M_\pi^2}\right)$$
$$= \frac{M_K^2}{\pi} \int_0^\infty \frac{dx}{x^2} \operatorname{Abs} W(x/M_K^2)|_{\pi\pi} - \frac{1}{60} \left(\frac{M_K^2}{M_\pi^2}\right)^2 \left(\alpha_+ - \beta_+ \frac{s_0}{M_\pi^2}\right)$$

requires $F_V^{\pi*}(s)$ and $f_1^{\pi^+\pi^-\to K^+\pi^-}(s)$ beyond low-energy expansion

GD Greynat Knecht

Combined fit (e^+e^-)

$\beta_+ \cdot 10^8$	a_+	b_+	$\chi^2/d.o.f$
_3.06	+0.483	+1.632	86.7/39
-3.30	-0.598	-0.678	48.8/39
-2.88	+0.489	+1.630	60.4/39
-2.00	-0.592	-0.680	45.4/39
_1.80	+0.495	+1.629	74.8/39
-1.00	-0.585	-0.682	42.8/39



NA48/2 + E865

GD Greynat Knecht

Fit to NA48/2 data ($\mu^+\mu^-$)

$\beta_+ \cdot 10^8$	a_+	b_+	$\chi^2/d.o.f$
_3.06	+0.372	+2.102	11.9/15
-3.30	-0.611	-0.746	15.9/15
_2 88	+0.384	+2.081	12.1/15
-2.00	-0.598	-0.768	15.2/15
_1.80	+0.397	+2.060	12.4/15
-1.00	-0.585	-0.790	14.5/15



NA48/2

As in the NA48/2 data for the e^+e^- channel, the data show a slight preference for the positive solution
Robustness of determinations of a_+ and b_+

Impact of remaining two-loop contributions, not contained in $W_{\mbox{\tiny BOL}}(z)$

Predictions for a_+ and b_+ ?

GD Greynat Knecht

Comparing $W_{2\mathrm{loop}}(z)$ and $W_{\scriptscriptstyle \mathrm{BOL}}(z)$



solid lines: $|W_{2\text{loop}}(z)|^2$ full two loops dash-dotted lines $|W_{\text{BOL}}(z)|^2$ red curves: $a_+ = -0.585$, $b_+ = -0.779$, $\beta_+ = -2.88 \cdot 10^{-8}$ blue curves: $a_+ = -0575$, $b_+ = -0.779$, $\beta_+ = -0.99 \cdot 10^{-8}$



*Approx. error on LD-subtracted rate excluding parametric contributions

- FCNC processes dominated by Z-penguin and box diagrams.
- SM rates determined by V_{CKM}, with minimal non-parametric "theory" uncertainties.
- Theory errors are being reduced [Lattice QCD, e.g. arXiv:2203.10998].
- * The current focus is on $K \rightarrow \pi \nu \nu$: uniquely clean theoretically.



Kaons and LFUV

Deviation of Wilson coefficients from SM, for NP scenarios with only left-handed quark currents. $C_k^\ell = C_{k,\text{SM}}^\ell + \delta C_k^\ell \qquad \delta C_l^\ell \equiv \delta C_9^\ell = -\delta C_{10}^\ell$





[arXiv:2206.14748]

0

Projections:

A: predicted central values for observables with only upper bound is projected to be as SM prediction; for measured ones current central values are taken.

B: central values for all observables are projected with best-fit points obtained from fits of existing data.



Very recent development: HIKE full projections

[CERN Physics Beyond Colliders Document in preparation, and paper In preparation by D'Ambrosio, Mahmoudi, Neshatpour]

HIKE Phase 2: sensitivity

SM signal yields and backgrounds in 5 years of operation:

Mode	N_S	N_B	$N_S/\sqrt{N_S+N_B}$
$K_L \rightarrow \pi^0 e^+ e^-$	70	83	5.7
$K_L \rightarrow \pi^0 \mu^+ \mu^-$	100	53	8.1

Sensitivity to the CKM parameters (assuming an improved $|a_{S}|$ measurement with $K_{S} \rightarrow \pi^{0}\ell^{+}\ell^{-}$ at LHCb):

$$\frac{\delta(\mathrm{Im}\lambda_t)}{\mathrm{Im}\lambda_t}\bigg|_{K_L\to\pi^0 e^+e^-} = 0.33, \quad \frac{\delta(\mathrm{Im}\lambda_t)}{\mathrm{Im}\lambda_t}\bigg|_{K_L\to\pi^0\mu^+\mu^-} = 0.28.$$

GD Greynat Knecht

Predicting a_+ , b_+ ? Going beyond the low-energy expansion

requires an unsubtracted dispersion relation

$$W(z)|_{\pi\pi} = \frac{1}{\pi} \int_0^\infty dx \frac{\operatorname{Abs} W(x/M_K^2)|_{\pi\pi}}{x - zM_K^2 - i0}$$

with

$$\frac{\operatorname{Abs} W(s/M_K^2)|_{\pi\pi}}{16\pi^2 M_K^2} = \theta(s - 4M_\pi^2) \times \frac{s - 4M_\pi^2}{s} \lambda_{K\pi}^{-1/2}(s) \times F_V^{\pi*}(s) \times f_1^{\pi^+\pi^- \to K^+\pi^-}(s)$$

Then a_+ and b_+ are given by spectral sum rules

$$G_F M_K^2 a_+|_{\pi\pi} = W(0)|_{\pi\pi} = \frac{1}{\pi} \int_0^\infty \frac{dx}{x} \operatorname{Abs} W(x/M_K^2)|_{\pi\pi}$$

and

$$G_F M_K^2 b_+|_{\pi\pi} = W'(0)|_{\pi\pi} - \frac{1}{60} \left(\frac{M_K^2}{M_\pi^2}\right)^2 \left(\alpha_+ - \beta_+ \frac{s_0}{M_\pi^2}\right)$$
$$= \frac{M_K^2}{\pi} \int_0^\infty \frac{dx}{x^2} \operatorname{Abs} W(x/M_K^2)|_{\pi\pi} - \frac{1}{60} \left(\frac{M_K^2}{M_\pi^2}\right)^2 \left(\alpha_+ - \beta_+ \frac{s_0}{M_\pi^2}\right)$$

requires $F_V^{\pi*}(s)$ and $f_1^{\pi^+\pi^-\to K^+\pi^-}(s)$ beyond low-energy expansion

Predicting a_+ , b_+ ? Going beyond the low-energy expansion

Simple approach: unitarize both using the inverse amplitude method

T. N. Truong, Phys Rev Lett 61, 2526 (1988)

A. Dobado et al, Phys Lett B 235, 134 (1990)

T. Hannah, Phys Rev D 55, 5613 (1997)

A. Dobado, J. R. Pelaez, Phys Rev D 56, 3057 (1997)

J. Nieves et al., Phys Rev D 65, 036002 (2002)

$$a_{+}|_{\pi\pi} = -(1.574^{+0.003}_{-0.020})$$
 $b_{+}|_{\pi\pi} = -(0.622^{+0.012}_{-0.017})$ for $\beta_{+} = -0.85 \cdot 19^{-8}$

note: position of the ρ resonance much too low for $\beta_+=-2.88...$ (phase goes through $\pi/2$ at $s\sim M_\rho^2/2!$)

$$a_{+} = -1.58 + \begin{cases} -0.10 \div +0.03 \text{ NDR} \\ -0.14 \div +0.07 \text{ HV} \end{cases}$$
$$b_{+} = -0.76 + \begin{cases} -0.04 \div +0.03 \text{ NDR} \\ -0.07 \div +0.03 \text{ HV} \end{cases}$$

GD Greynat Knecht

Matching LD and SD at NLO





$$BR(K^+ \to \pi^+ \nu \overline{\nu}) < 11 \times 10^{-10} @ 90\% CL$$

 $BR(K^+ \to \pi^+ \nu \overline{\nu}) < 14 \times 10^{-10} @ 95\% CL$

- One event observed in Region 2
- Full exploitation of the CLs method in progress
- The results are compatible with the Standard Model
- For comparison: $BR(K^+ \to \pi^+ \nu \overline{\nu}) = 28^{+44}_{-23} \times 10^{-11} @ 68\% CL$

$$BR(K^+ \to \pi^+ \nu \overline{\nu})_{SM} = (8.4 \pm 1.0) \times 10^{-11}$$

 $BR(K^+ \to \pi^+ \nu \overline{\nu})_{exp} = (17.3^{+11.5}_{-10.5}) \times 10^{-11} \text{ (BNL, "kaon decays at rest")}$

Prospects



Processing of 2017 data on-going

- \star ~ 20 times more data than the presented statistics
- Expected reduction of upstream background
- ★ Methods to improve the reconstruction efficiency under study

2018 data taking under way

- * Further mitigation of the upstream background is expected
- ✤ Processing in parallel with data taking
- ☆ Final 2018 reprocessing expected beginning 2019
- Expect ~ 20 SM events from the 2017+2018 data sample. The analysis of this sample should provide:
 - ☆ Input to the European Strategy for Particle Physics
 - Solid extrapolation to the ultimate sensitivity of NA62 achievable after LS2

 $K_{L} \rightarrow \pi^{0} \nu \bar{\nu}$

$$B(K_L) = (3.14 \pm 0.17 \pm 0.06) \times 10^{-11}$$
 TH
 $B(K_L) < 2.6 \times 10^{-8}$ at 90% C.L. E391a

Model-independent bound, based on SU(2) properties dim-6 operators for $\overline{s}d\overline{v}v$ Grossman Nir

$$B(K_L) \leq \frac{\tau_L}{\tau_+} \times B(K^{\pm})_{E949} \leq 1.4 \times 10^{-9}$$
 at 90%C.L.

UV sensitivity

$$\mathcal{L} \sim \frac{1 - 0.3 i}{(180 \text{ TeV})^2} (\overline{s}_L \gamma_\mu d_L \overline{\nu}_L \gamma^\mu \nu_L)$$