

Taking advantage of $B^0 - \bar{B}^0$ entanglement to measure the direct CP violation ϕ_3/γ phase

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PROMETEO
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Introduction on phases I

- The value of the quark charged current couplings are parametrized by the Cabibbo-Kobayashi-Maskawa (CKM) matrix V

$$\begin{aligned}\mathcal{L}_{CC} &= \frac{g}{\sqrt{2}} (\bar{u} \quad \bar{c} \quad \bar{t})_L \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \gamma_\mu \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L W^{\mu+} \\ &= \frac{g}{\sqrt{2}} \bar{U}_L V \gamma_\mu D_L W^{\mu+}\end{aligned}$$

- We can eliminate 5 phases out of 9, ($q_L \rightarrow e^{i\phi_q} q_L$):

$$V = \begin{pmatrix} |V_{ud}| & |V_{us}| e^{i\chi'} & |V_{ub}| e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| e^{-i\beta} & -|V_{ts}| e^{i\beta_s} & |V_{tb}| \end{pmatrix}$$

Introduction on phases II

These are the four Aleksan, Kayser and London phases: β , γ and β_s and χ' (they even can be used as a CKM parametrization)

$$\beta = \phi_1 = \arg(-V_{cd} V_{cb}^* V_{td}^* V_{tb}) \text{ (in } B_d^0 - \bar{B}_d^0 \text{ mixing)}$$

$$\gamma = \phi_3 = \arg(-V_{ud} V_{ub}^* V_{cd}^* V_{cb}) \text{ (in tree level } B^{\pm,0} \text{ decays)}$$

$$\beta_s = \arg(-V_{ts} V_{tb}^* V_{cs}^* V_{cb}) \text{ (in } B_s^0 - \bar{B}_s^0 \text{ mixing)}$$

$$\chi' = \arg(-V_{cd} V_{cs}^* V_{ud}^* V_{us}) \text{ (in direct CPV } D \rightarrow h^+ h^-)$$

- Because $\text{Im}(V_{ij} V_{il}^* V_{kj}^* V_{kl})$ for $i \neq k, j \neq l$ is universal (Chau, Keung, Jarlskog,..) we have

$$\beta, \gamma \sim \mathcal{O}(1) ; \beta_s \sim \lambda^2 ; \chi' \sim \lambda^4$$

The phase gamma I

- It appears in V_{ub} , in fact a minimal ingredient is to measure the interference among $b \rightarrow u\bar{c}s$ transition and $b \rightarrow c\bar{u}s$ transitions.

$$A(b \rightarrow u\bar{c}s) = A(B^- \rightarrow \bar{D}^0 K^-) \propto V_{ub} V_{cs}^*$$

$$A(b \rightarrow c\bar{u}s) = A(B^- \rightarrow D^0 K^-) \propto V_{cb} V_{us}^*$$

only possible if both \bar{D}^0 and D^0 decay to a common final state f

$$A^- \equiv A\left(B^- \rightarrow \underset{\hookrightarrow f}{D} + K^-\right) = A\left(B^- \rightarrow D_{\nrightarrow f}^\perp + K^-\right) =$$
$$A(D^0 \rightarrow f) A(B^- \rightarrow D^0 K^-) + A(\bar{D}^0 \rightarrow f) A(B^- \rightarrow \bar{D}^0 K^-)$$

The phase gamma II

$$\begin{aligned} |D_{\rightarrow f}\rangle &\propto A(\bar{D}^0 \rightarrow f) |D^0\rangle - A(D^0 \rightarrow f) |\bar{D}^0\rangle \\ |D_{\rightarrow f}^\perp\rangle &\propto A(D^0 \rightarrow f)^* |D^0\rangle + A(\bar{D}^0 \rightarrow f)^* |\bar{D}^0\rangle \end{aligned}$$

in the case $f = K^+K^-$

$$A(\bar{D}^0 \rightarrow K^+K^-) \propto V_{us}^* V_{cs} ; A(D^0 \rightarrow K^+K^-) \propto V_{us} V_{cs}^*$$

giving rise to

$$A^- = aV_{cb}V_{cs}^* + bV_{ub}V_{us}^*$$

- in such a way that

$$|A^-|^2 - |A^+|^2 \propto \text{Im}(ab^*) \text{Im}(V_{cb} V_{cs}^* V_{ub}^* V_{us})$$

sensible to the interference phase

$$\arg(V_{cb} V_{cs}^* V_{ub}^* V_{us}) = \gamma + \chi' \sim \gamma$$

and needed of strong phases in $\text{Im}(ab^*)$. Gronau, London, Wyler (GLW), Atwood, Dunietz, Soni (ADS), Giri, Grossman, Soffer, Zupan (GGSZ) and many more.

Using entanglement for gamma at t=0 I

- The entangled $B^0 - \bar{B}^0$ system produced at Belle II from the decay of $\Upsilon(4s)$ is:

$$|\Psi_0\rangle = \frac{1}{\sqrt{2}} \left(|B_d^0\rangle |\bar{B}_d^0\rangle - |\bar{B}_d^0\rangle |B_d^0\rangle \right)$$

the amplitude to decay to the states f and g (simultaneously) is

$$\langle f, g | \mathcal{T} | \Psi_0 \rangle = \frac{1}{\sqrt{2}} (A_f \bar{A}_g - \bar{A}_f A_g)$$

with the usual notation $A_f = \langle f | \mathcal{T} | B^0 \rangle$; $\bar{A}_f = \langle f | \mathcal{T} | \bar{B}^0 \rangle$.

Using entanglement for gamma at t=0 II

- The rate -or the double decay rate (DDR)- at $t = 0$ can be written as

$$|\langle f, g | \mathcal{T} | \Psi_0 \rangle|^2 = \frac{1}{2} |A_f A_g|^2 \left| \frac{\bar{A}_g}{A_g} - \frac{\bar{A}_f}{A_f} \right|^2$$

and shows interference in the DDR at $t = 0$ not needed of strong phases. For example: $f = J/\psi K_{S,L} \Rightarrow \bar{A}_f/A_f = \mp 1$ and $g = (\pi\pi)_{I=2} \Rightarrow \bar{A}_g/A_g = e^{-2i\gamma}$

$$\begin{aligned} |\langle J/\psi K_S, (\pi\pi)_{I=2} | \mathcal{T} | \Psi_0 \rangle|^2 &\propto \cos^2 \gamma \\ |\langle J/\psi K_L, (\pi\pi)_{I=2} | \mathcal{T} | \Psi_0 \rangle|^2 &\propto \sin^2 \gamma \end{aligned}$$

Using entanglement for gamma I

- The use of the EPR correlation to study CP violation was proposed by Wolfenstein, Gavela et al, Falk and Petrov and Alvarez and Bernabeu among others for several decay channels in the B factories.
- To get γ from $|\bar{A}_g/A_g - \bar{A}_f/A_f|^2$ it is easy with g and f , CP eigenstates: properly chosen this DDR can be CP forbidden.
 $f = J/\psi K_S, J/\psi K_L$ (in short S or L) and $g = \pi^+\pi^-, \pi^0\pi^0, \rho_L^+\rho_L^-, \rho_L^0\rho_L^0$, will do the job from CP-conserving and CP-violating transitions:

$$\begin{array}{ccccc}
 Y(4s) & (J/\psi K_{S,L})_B & (\pi\pi)_B & \left[\begin{array}{c} (J/\psi K_S)_B \\ (\pi\pi)_B \end{array} \right]_{Y(4s)} & \left[\begin{array}{c} (J/\psi K_L)_B \\ (\pi\pi)_B \end{array} \right]_{Y(4s)} \\
 1^{--} & 0^{-+,-} & 0^{++} & 1^{++} & 1^{+-}
 \end{array}$$

Therefore

$$Y(4s) \rightarrow (J/\psi K_S)_B (\pi\pi)_B \quad ; \quad \text{is CP allowed}$$

$$Y(4s) \rightarrow (J/\psi K_L)_B (\pi\pi)_B \quad ; \quad \text{is CP forbidden}$$

Using entanglement for gamma II

- With $B_H = pB^0 + q\bar{B}^0$ $B_L = pB^0 - q\bar{B}^0$ the B eigenstate

$$|\Psi_0\rangle = \frac{1}{2\sqrt{2}pq} (|B_L\rangle |B_H\rangle - |B_H\rangle |B_L\rangle)$$

$$\begin{aligned} \langle f, t_0; g, t_0 + t | \mathcal{T} | \Psi_0 \rangle &= \frac{e^{-i(\mu_L + \mu_H)t_0}}{2\sqrt{2}pq} \times \\ &\times \left(e^{-i\mu_H t} A_L^f A_H^g - e^{-i\mu_L t} A_H^f A_L^g \right) \end{aligned}$$

where $A_H^f = pA_f + q\bar{A}_f$ and $A_L^f = pA_f - q\bar{A}_f$.

Using entanglement for gamma III

- So the double decay rate to the state f at t_0 and to the state g at $t_0 + t$ integrated for t_0 is given by
 $Y(4s) \rightarrow (B^0 \bar{B}^0 - \bar{B}^0 B^0) \rightarrow (f, t_0; g, t_0 + t)$

$$\begin{aligned} I(f, g; t) &= \frac{e^{-\Gamma|t|}}{16\Gamma|pq|^2} \left| e^{-i\Delta Mt/2} A_L^f A_H^g - e^{i\Delta Mt/2} A_H^f A_L^g \right|^2 \\ &= \frac{e^{-\Gamma|t|}}{16\Gamma|pq|^2} \left| \begin{array}{l} \cos\left(\frac{\Delta Mt}{2}\right) (A_L^f A_H^g - A_H^f A_L^g) \\ -i \sin\left(\frac{\Delta Mt}{2}\right) (A_L^f A_H^g + A_H^f A_L^g) \end{array} \right|^2 \end{aligned}$$

- Our general expression for the DDR normalized to

$$\int_0^\infty dt \sum_{f \leq g} I(f, g; t) = 1 \text{ is}$$

$$\begin{aligned} \hat{I}(f, g; t) &\equiv \frac{\Gamma}{\langle \Gamma_f \rangle \langle \Gamma_g \rangle} I(f, g; t) = \\ &= e^{-\Gamma|t|} \left[I_d^{fg} \cos^2 \left(\frac{\Delta Mt}{2} \right) + I_m^{fg} \sin^2 \left(\frac{\Delta Mt}{2} \right) + I_{od}^{fg} \sin(\Delta Mt) \right] \end{aligned}$$

Important consistency properties I

- We have formally

$$\hat{I}(f, g; t) = \hat{I}(g, f; -t)$$

with the following implications

$$I_d^{fg} = I_d^{gf} ; I_m^{fg} = I_m^{gf} ; I_{od}^{fg} = -I_{od}^{gf}$$

- Because

$$\hat{I}(g, S; t) = \left| \langle B_{\rightarrow S}^\perp | B_{\rightarrow g}(t) \rangle \right|^2$$

and $|B_{\rightarrow S}^\perp\rangle$ and $|B_{\rightarrow L}^\perp\rangle$ are orthogonal

$$\hat{I}(g, S; t) + \hat{I}(g, L; t) = \langle B_{\rightarrow g}(t) | B_{\rightarrow g}(t) \rangle = e^{-\Gamma t}$$

therefore we have

$$I_d^{Sg} + I_d^{Lg} = 1; I_m^{Sg} + I_m^{Lg} = 1; I_{od}^{Sg} + I_{od}^{Lg} = 0$$

- **One can measure all three observables** $I_{d,m,od}^{L_S g}$ for all the (f, g) and (g, f) channels $f = J/\psi K_S, J/\psi K_L$ and $g = (\rho_L^+ \rho_L^-), (\rho_L^0 \rho_L^0), (\pi^+ \pi^-), (\pi^0 \pi^0)$. **Just measuring ratios** $I_{d,m}^{L_S g} / I_{od}^{L_S g}$!!!

The observables I

- Our observables will be $I_{d,m,od}^{L_S g}$ for each channel
 $g = (\rho_L^+ \rho_L^-), (\rho_L^0 \rho_L^0), (\pi^+ \pi^-), (\pi^0 \pi^0)$ in both time ordering making a total of 16 channels. We use

$$\left(\frac{q}{p}\right)_B = e^{-2i\phi_M} ; \frac{\bar{A}_g}{A_g} = \rho_g e^{-2i\phi_g} ; (\lambda = q\bar{A}_f / pA_f)$$

$$\frac{\bar{A}_S}{A_S} = \left(\frac{p}{q}\right)_K \frac{V_{cs} V_{cb}^*}{V_{cs}^* V_{cb}} = - \left(\frac{V_{cd} V_{cb}^*}{V_{cd}^* V_{cb}}\right) e^{+2i\chi'} = -1$$

$$\frac{\bar{A}_S}{A_S} = -\frac{\bar{A}_L}{A_L} = -1 ; \lambda_L = -\lambda_S = -e^{-2i\phi_M} ; \phi_M = \beta$$

The observables II

for the observable we get

$$\begin{aligned} I_d^{Lg} &= \frac{\left| \left(\frac{\bar{A}_g}{A_{g^-}} - \frac{\bar{A}_f}{A_{f^-}} \right) \right|^2}{\left(1 + |\lambda_f|^2 \right) \left(1 + |\lambda_g|^2 \right)} \\ &= \frac{\left| \rho_g e^{-2i\phi_g} \mp 1 \right|^2}{2 \left(1 + \rho_g^2 \right)} = \frac{1}{2} \left[1 \mp \frac{2\rho_g \cos(2\phi_g)}{\left(1 + \rho_g^2 \right)} \right] \end{aligned}$$

it is present at $t = 0$ it is CP forbidden for $f = J/\psi K_L$ and CP allowed for $f = J/\psi K_S$. It is sensible to the phases in g and f . If

there were not penguin pollution ($\rho_g = 1$) in the decays $g = (\rho_L^+ \rho_L^-), (\rho_L^0 \rho_L^0), (\pi^+ \pi^-), (\pi^0 \pi^0)$ all ϕ_g would be $\phi_g = \gamma$ and we would have

$$I_d^{Lg} = \sin^2 \gamma \text{ and } I_d^{Sg} = \cos^2 \gamma$$

The observables III

It is clear why we have named I_d for **direct** CP, because mixing is not needed.

- For the other observables we get

$$I_m^{L_S g} = \frac{|(1 - \lambda_g \lambda_f)|^2}{(1 + |\lambda_f|^2)(1 + |\lambda_g|^2)} = \frac{1}{2} \left[1 \mp \frac{2\rho_g \cos(4\phi_M + 2\phi_g)}{(1 + \rho_g^2)} \right]$$

$$I_{od}^{L_S g} = \frac{2 \operatorname{Im} [(\lambda_g^* - \lambda_f^*) (1 - \lambda_g \lambda_f)]}{(1 + |\lambda_f|^2)(1 + |\lambda_g|^2)} = (\mp) \frac{(1 - \rho_g^2)}{(1 + \rho_g^2)} \sin(2\phi_M)$$

depending also on the mixing phase. The quantities to be extracted from $I_{d,m,od}^{L_S g}$, for each channel g , are ϕ_g , ρ_g and ϕ_M . The one we are mainly interested is ϕ_g that if they were not penguin pollution in all the g channels we will have $\phi_g = \gamma$ (and also $\rho_g = 1$). **The way of getting γ from ϕ_g is by the Gronau and London isospin analysis.**

- In general we will have for each g channel a departure from the universal γ value that we call

$$\epsilon_g = \gamma - \phi_g$$

The neutral and charged B meson decays differ in the presence versus absence, respectively, of the penguin contribution to the amplitudes for each final $h = \pi, \rho_L$ system. The charged decay amplitudes $A_{+0} = A(B^+ \rightarrow h^+ h^0)$ and $\bar{A}_{+0} = A(B^- \rightarrow h^- h^0)$ have a final $(h^\pm h^0)$ isospin 2 state and, therefore, only the $\Delta I = 3/2$ tree-level amplitude contributes with the weak phase γ .

- It is convenient to define, with the same notation for both neutral decay channels $\pi\pi$ and $\rho_L\rho_L$ and using $g = \pm$ or 00 for the corresponding decay charges, the quantities

$$a_g = \frac{A_g}{A_{+0}} ; \bar{a}_g = \frac{\bar{A}_g}{\bar{A}_{+0}}$$

in such a way that the double ratio fixes the penguin pollution parameters ρ_g and ϵ_g

$$\frac{\bar{a}_g}{a_g} = \rho_g e^{2i\epsilon_g}$$

- The isospin triangular relations with these complex ratios are

$$\frac{1}{\sqrt{2}}a_{+-} = 1 - a_{00} ; \frac{1}{\sqrt{2}}\bar{a}_{+-} = 1 - \bar{a}_{00}$$

that allows to obtain $\text{Re} \left(\begin{smallmatrix} (-) \\ a \\ +- \end{smallmatrix} \right)$ and $\text{Re} \left(\begin{smallmatrix} (-) \\ a \\ 00 \end{smallmatrix} \right)$ in terms of

$\left| \begin{smallmatrix} (-) \\ a \\ +- \end{smallmatrix} \right|^2$ and $\left| \begin{smallmatrix} (-) \\ a \\ 00 \end{smallmatrix} \right|^2$ and of course also $\text{Im} \left(\begin{smallmatrix} (-) \\ a \\ +- \end{smallmatrix} \right)$ and

$\text{Im} \left(\begin{smallmatrix} (-) \\ a \\ 00 \end{smallmatrix} \right)$. Therefore we can get a_g and \bar{a}_g from the branching

ratios of the processes $B^\pm \rightarrow h^\pm h^0 ; B^0, \bar{B}^0 \rightarrow h^+ h^-, h^0 h^0$ fixing ϵ_g

Isospin analysis IV

and ρ_g . The summary of our isospin analysis with the present PDG data is

g	ρ_g	ϵ_g
$\rho_L^+ \rho_L^-$	1.007 ± 0.076	$0.008 \pm \mathbf{0.091}$
$\rho_L^0 \rho_L^0$	0.972 ± 0.241	0.007 ± 0.345
$\pi^+ \pi^-$	1.392 ± 0.062	$\pm (0.307 \pm 0.170)$
$\pi^0 \pi^0$	1.306 ± 0.206	$\pm (0.427 \pm 0.172)$

- Because the $\rho_L^+ \rho_L^-$ is the one with largest branching ratio, $\delta\epsilon_{\rho_L^+ \rho_L^-} = \mathbf{0.091} = \mathbf{5.2^\circ}$ gives us an estimate of the uncertainty, due to the present knowledge of the penguin pollution, in the determination of γ/ϕ_3 . (Important improvements are expected from Belle II and LHCb).

Potential estimate of the method I

The intrinsic accuracy of the proposed method is controlled by the ability to extract ϕ_g .

We generate events according to the double decay rate time distributions fixing all values as $\rho_g, \phi_g, \phi_M = \beta$ and the isopin analysis parameters.

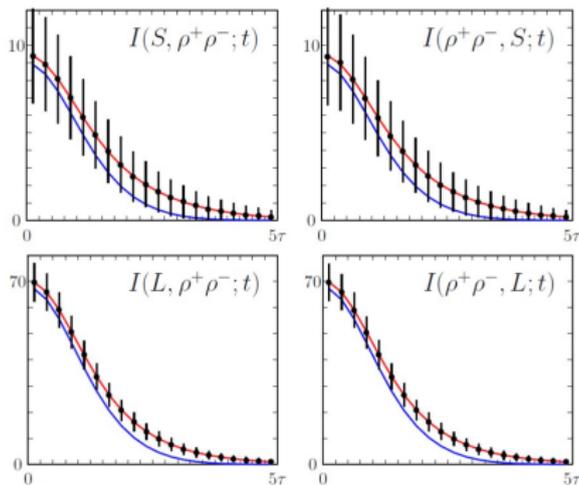
Under the assumption that Belle II can collect 1000 $\rho_L^+ \rho_L^-$ events in the categories $(L, \rho_L^+ \rho_L^-)$, $(S, \rho_L^+ \rho_L^-)$, $(\rho_L^+ \rho_L^-, L)$, $(\rho_L^+ \rho_L^-, S)$, 50 $\rho_L^0 \rho_L^0$, 200 $\pi^+ \pi^-$ and 50 $\pi^0 \pi^0$

For each g , we generate values of t , the events, distributed according to the four double-decay intensities.

In order to incorporate the effect of experimental time resolution, each t is randomly displaced following a normal distribution with zero mean and $\sigma = 1ps$. Additional experimental effects such as efficiencies are not included. Generation proceeds until the chosen number of events. Events are binned.

Potential estimate of the method II

The procedure is repeated in order to obtain mean values and standard deviations in each bin: these constitute our simulated data, as we illustrate with 20 bins. There are no significant differences if one considers, for example, 15 or 10 bins.



Potential estimate of the method III

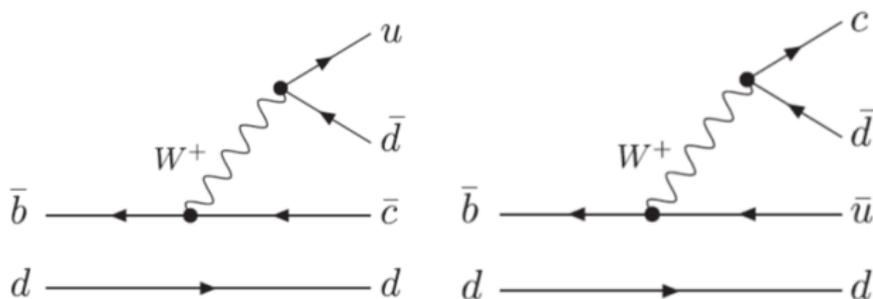
The fit to these simulations gives $I_d^{S\rho_L^+\rho_L^-} = 0.1170 \pm 0.0138$,
 $I_m^{S\rho_L^+\rho_L^-} = 0.1658 \pm 0.0456$ and $I_{od}^{S\rho_L^+\rho_L^-} = 0.0000 \pm 0.0198$. Together with
the other fits it give rise to

g	ϕ_g	ρ_g
$\rho_L^+\rho_L^-$	$1.222 \pm \mathbf{0.020}$	1.00 ± 0.06
$\rho_L^0\rho_L^0$	1.22 ± 0.09	1.00 ± 0.24
$\pi^+\pi$	1.57 ± 0.12	1.35 ± 0.12
$\pi^0\pi^0$	1.57 ± 0.18	1.35 ± 0.24
	$\phi_M = 0.384 \pm 0.031$	

We conclude that, since $\gamma = \phi_g + \epsilon_g$, the error $\delta\phi_{\rho_L^+\rho_L^-} = \mathbf{0.020} = \mathbf{1.1^\circ}$
gives an idea of the intrinsic statistical limiting error we would expect in
the determination of γ for the assumed number of events.

Other Channels (preliminary) I

- $f = J/\psi K_{S,L}$. $g = \pi^+ D^-, \rho^+ D^-, \pi^+ D^* (2010)^-, \rho^+ D^* (2010)^-$.
 $g \neq \bar{g}$. In general we will have



$$\frac{\bar{A}_g}{A_g} = -\rho_g e^{i(\gamma + \Delta_g)} ; \frac{\bar{A}_{\bar{g}}}{A_{\bar{g}}} = -\frac{1}{\rho_g} e^{i(\gamma - \Delta_g)}$$

with $\rho_g \sim \lambda^2$ and Δ_g a strong phase, for $\pi^+ D^-$ we have
 $\rho_{\pi^+ D^-} = 0.018$.

Other Channels (preliminary) II

- By measuring (f, g) and (f, \bar{g}) one can get γ without isospin analysis
- The estimated precision extracted from this table for $\Delta_g = 0$

g	ρ_g	$B_r(B^0 \rightarrow g)$	$B_r(B^0 \rightarrow \bar{g})$	$\delta\gamma$
$D^- \pi^+$	0.0178	2.5×10^{-3}	7.3×10^{-7}	8°
$D^{*-} \pi^+$	0.0181	2.7×10^{-3}	$5.9 \times 10^{-7} (cal)$	7°
$D^- \rho^+$	0.0071	7.6×10^{-3}	$3.8 \times 10^{-7} (cal)$	11°
$D^{*-} \rho^+$	0.0145	6.8×10^{-3}	$14 \times 10^{-7} (cal)$	
$D^- a_1^+$		6.0×10^{-3}		
$D^{*-} a_1^+$		13.0×10^{-3}		

On the Upsilon (5S) (preliminary) I

- About 90% of $B_s \bar{B}_s$ pairs produced at the $\Psi(5s)$ are $CP = -$ eigenstates (Atwood and Soni) and therefore entangled states of the form

$$|\Psi_0\rangle = \frac{1}{\sqrt{2}} \left(|B_s^0\rangle |\bar{B}_s^0\rangle - |\bar{B}_s^0\rangle |B_s^0\rangle \right)$$

- Differences among the $|\Psi_0\rangle$ in B_d or B_s system including $\Delta\Gamma_s$ should be included.
- Analogous channel to $B_d^0 \rightarrow D^- \pi^+$ is $B_s^0 \rightarrow D_s^- K^+$. In this case both paths are of the same order (Fleischer and Malami)

$$\rho_g = \left| \frac{\bar{A}_g}{A_g} \right| = 0.40 \pm 0.13$$

Much more similar to the $\rho\rho$ channel. With $f = J/\psi\phi$ and $g = D_s^- K^+$ a 4° precision in γ is not excluded.

Conclusions I

- $B^0 - \bar{B}^0$ entanglement affords a simple way to have interferences in the decays provided $B^0, \bar{B}^0 \rightarrow f$ and $B^0, \bar{B}^0 \rightarrow g$:
 $|A_f \bar{A}_g - \bar{A}_f A_g|^2 \neq 0$
- No strong phases appear as essential ingredients.
- The eight (f, g) CP channels $f = J/\psi K_S, /\psi K_S$ and $g = (\pi\pi)^0, (\rho_L \rho_L)^0$ have a tree level common γ phase. $\rho_L^+ \rho_L^-$ is the benchmark channel.
- Several constraining consistencies among the different intensities appear.
- We find that an intrinsic accuracy of the order of $\pm 1^\circ$ degree could be achievable for the relative phase ϕ_g . From the isospin analysis of γ , there is a limitation of $\pm 5^\circ$, to be improved by the existing experimental facilities.
- Other channels -without isospin analysis- and $B_s \bar{B}_s$ entangled states are under consideration