The Multipolar structure of the Local Universe

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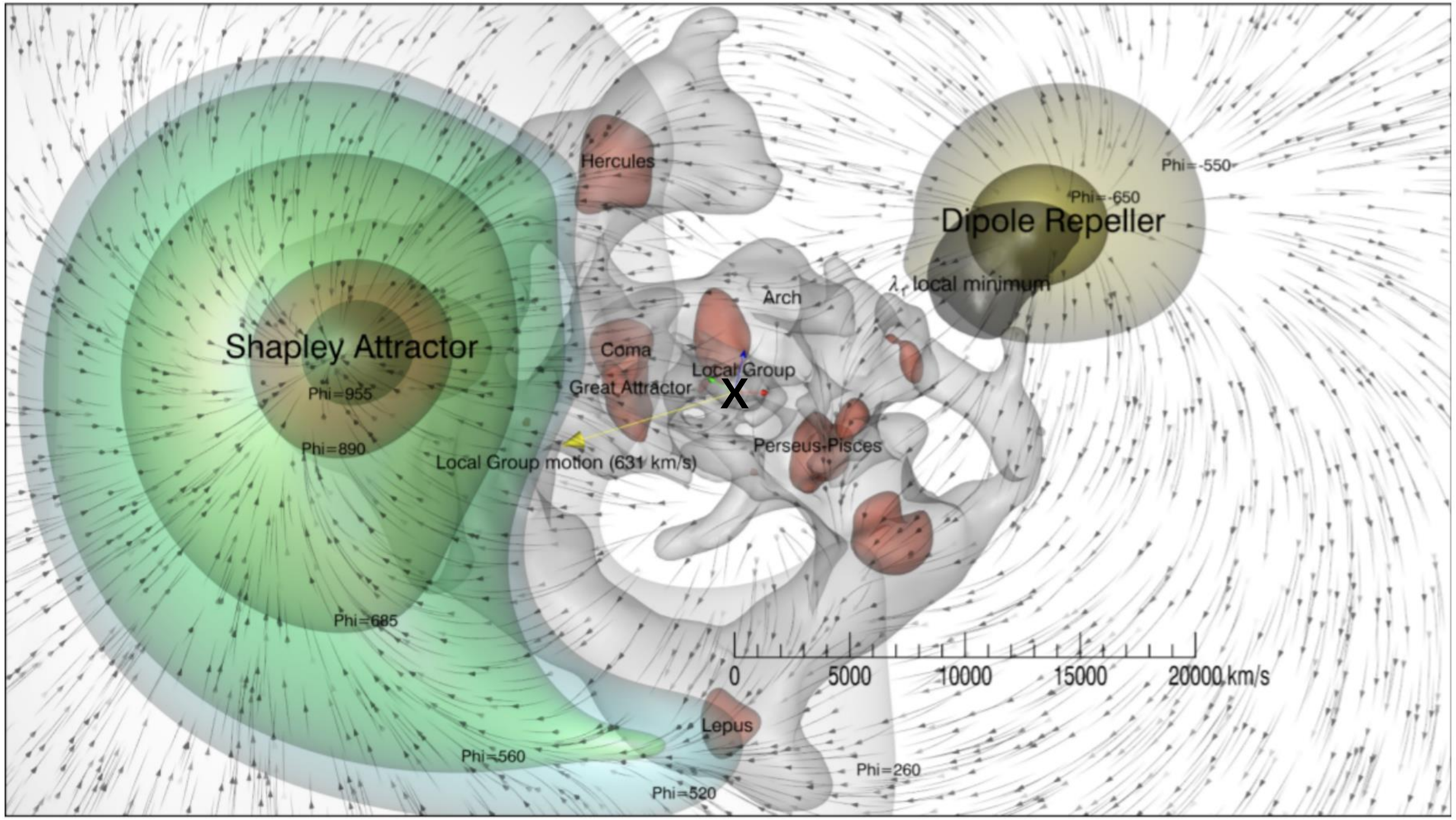
Corfu 2023: Tensions in Cosmology (10/9/2023)

Introduction

The cosmological principle is violated in the local universe.

Our interest is in exploring the local structure of spacetime by a model-independent way.

The local universe structure



Hoffman, Pomarède, Tully and Courtois 2017 (using Cosmicflows-2)

The expansion rate fluctuation

The expansion rate fluctuation **for a matter comoving observer** is defined as

$$\eta(\boldsymbol{n}, z) \equiv \log\left(\frac{z}{d_L(\boldsymbol{n}, z)}\right) - a_0$$

z is the redshift in km/s

d_L is the luminosity distance in Mpc

$\boldsymbol{n}$ is the direction of the line of sight

where

$$a_0 \equiv \int_S \log\left(\frac{z}{d_L(\boldsymbol{n}, z)}\right) \mathrm{d}\Omega$$

is the monopole.

The estimator is a random variable with Gaussian distribution and statistically unbiased.

It measures the anisotropy and their evolution.

The expansion rate fluctuation

The expansion rate fluctuation is defined as

$$\eta(\boldsymbol{n}, z) \equiv \log\left(\frac{z}{d_L(\boldsymbol{n}, z)}\right) - a_0$$

The luminosity distance can be expanded up to the third order in redshift (with respect to the matter observer) as (Heinesen 2021)

$$d_L(z, \boldsymbol{n}) = \left(\frac{1}{\mathbb{H}(\boldsymbol{n})}\right) z + \left(\frac{1 - \mathbb{Q}(\boldsymbol{n})}{2\mathbb{H}(\boldsymbol{n})}\right) z^2 + \left(\frac{\mathbb{R}(\boldsymbol{n}) + \mathbb{Q}(\boldsymbol{n}) - \mathbb{J}(\boldsymbol{n}) + 3\mathbb{Q}^2(\boldsymbol{n}) - 1}{6\mathbb{H}(\boldsymbol{n})}\right) z^3 + \mathcal{O}(z^4)$$

In Friedmann–Lemaître–Robertson–Walker spacetime these expansion coefficients are

$\mathbb{H}_o \xrightarrow{FLRW} H_o$ (Hubble parameter)

$\mathbb{R}_o \xrightarrow{FLRW} \Omega_{ko}$ (curvature parameter)

$\mathbb{Q}_o \xrightarrow{FLRW} q_0$ (deceleration parameter)

$\mathbb{J}_o \xrightarrow{FLRW} j_o$ (jerk parameter)

The expansion rate fluctuation

In any given spacetime these invariant expansion coefficients can be calculated at the event of observation **o** (now and here) by (Kristian & Sachs 1966, Clarkson & Maartens 2010, Heinesen 2021)

$$\mathbb{H} \stackrel{\circ}{=} K^\mu K^\nu \Theta_{\mu\nu}$$

$$\mathbb{Q} \stackrel{\circ}{=} -3 + \frac{K^\mu K^\nu K^\alpha \nabla_\alpha \Theta_{\mu\nu}}{\mathbb{H}^2}$$

$$\mathbb{R} \stackrel{\circ}{=} 1 + \mathbb{Q} - \frac{K^\mu K^\nu R_{\mu\nu}}{2\mathbb{H}^2}$$

$$\mathbb{J} \stackrel{\circ}{=} 10\mathbb{Q} - 15 + \frac{K^\mu K^\nu K^\alpha K^\beta \nabla_\alpha \nabla_\beta \Theta_{\mu\nu}}{2\mathbb{H}^3}$$

$$K^\mu \equiv \frac{k^\mu}{k^\nu u_\nu}$$

$$\Theta_{\mu\nu} \equiv \nabla_\mu u_\nu$$

u_ν is the 4-velocity for the matter particles

k^μ is the (past pointing) 4-momentum of the light.

The expansion rate fluctuation

The expansion rate fluctuations are related to the cosmographic parameters by

$$\eta(\boldsymbol{n}, z) \;=\; \log \mathbb{H}_o(\boldsymbol{n}) - \frac{1 - \mathbb{Q}_o(\boldsymbol{n})}{2 \ln 10} z + \frac{7 - \mathbb{Q}_o(\boldsymbol{n})(10 + 9\mathbb{Q}_o(\boldsymbol{n})) + 4(\mathbb{J}_o(\boldsymbol{n}) - \mathbb{R}_o(\boldsymbol{n}))}{24 \ln 10} z^2 - a_0$$

To find the multipoles of the cosmographic parameters, we need to decompose the expansion rate fluctuation into spherical harmonic components at different redshifts

$$\eta(\theta, \phi, z) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m}(z) Y_{\ell m}(\theta, \phi)$$

The spherical harmonic coefficients can be calculated by

$$a_{\ell m}(z) \equiv \int_0^{2\pi} \int_0^{\pi} \eta(\theta, \phi, z) Y^*_{\ell m}(\theta, \phi) \sin\theta \; d\theta d\phi$$

The expansion rate fluctuation

We chose the CMB-comoving observer (who does not see the CMB dipole) but, in general, is not comoving with matter.

If the observer is not the matter-observer (the observer who is comoving with the cosmological matter fluid element, we need to add a correction.

In general, for any other observer (with bar)

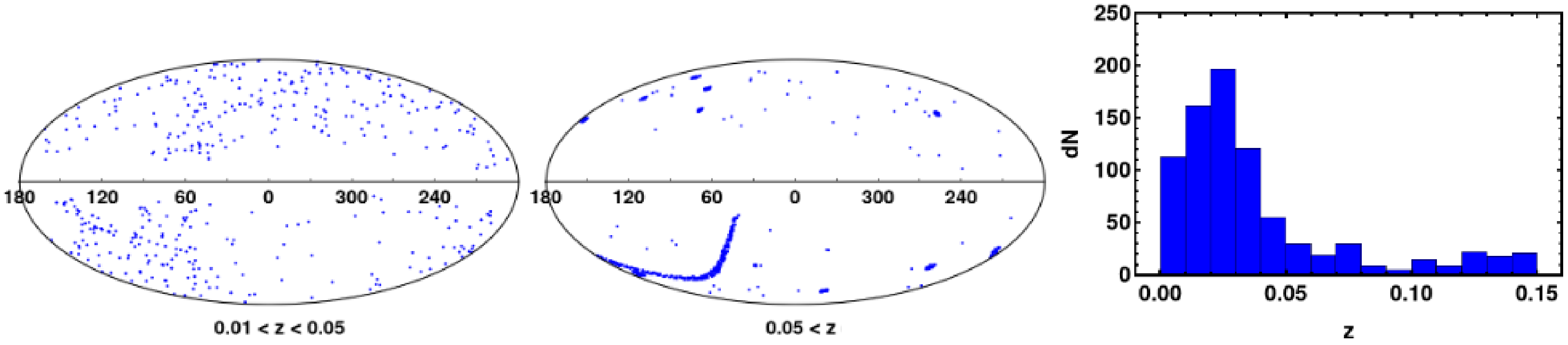
$$\bar{\eta}(\boldsymbol{n}, \bar{z}) \approx \eta(\boldsymbol{n}, \bar{z}) - \log\left(1 - \frac{\boldsymbol{v}_0 \cdot \boldsymbol{n}}{\bar{z}}(1 + \bar{z})\right) + a_0 - \bar{a}_0$$

where $\boldsymbol{v}_0$ is the velocity of the matter observer with respect to the boosted observer at time and position of observation.

Data

Pantheon+ sample (Scolnic et al. 2022)

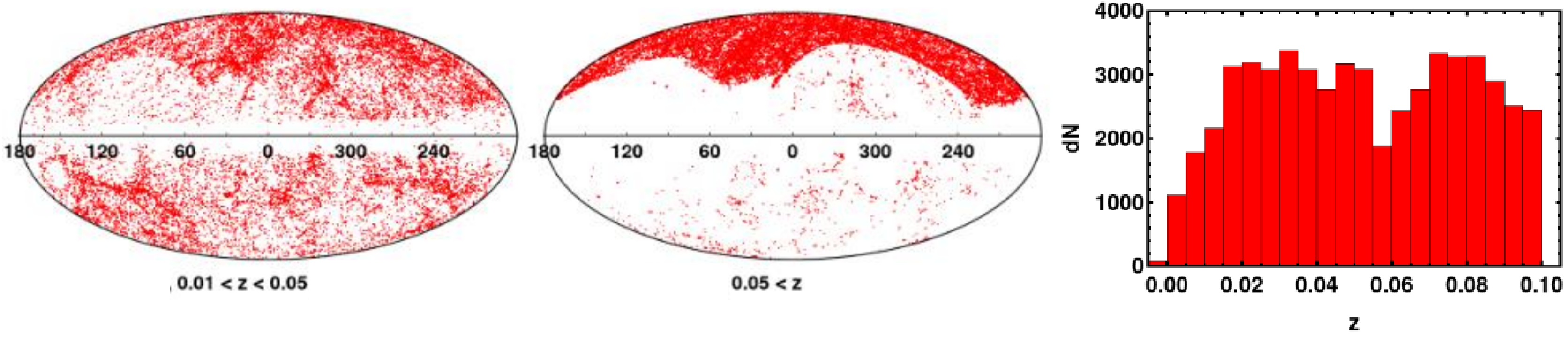
- Contains 1701 SNIa.
- We use objects at (0.01 < z < 0.09), we have 623.



Mollweide projection in galactic coordinates

Cosmicflows-4 (CF4) (Tully et al. 2023)

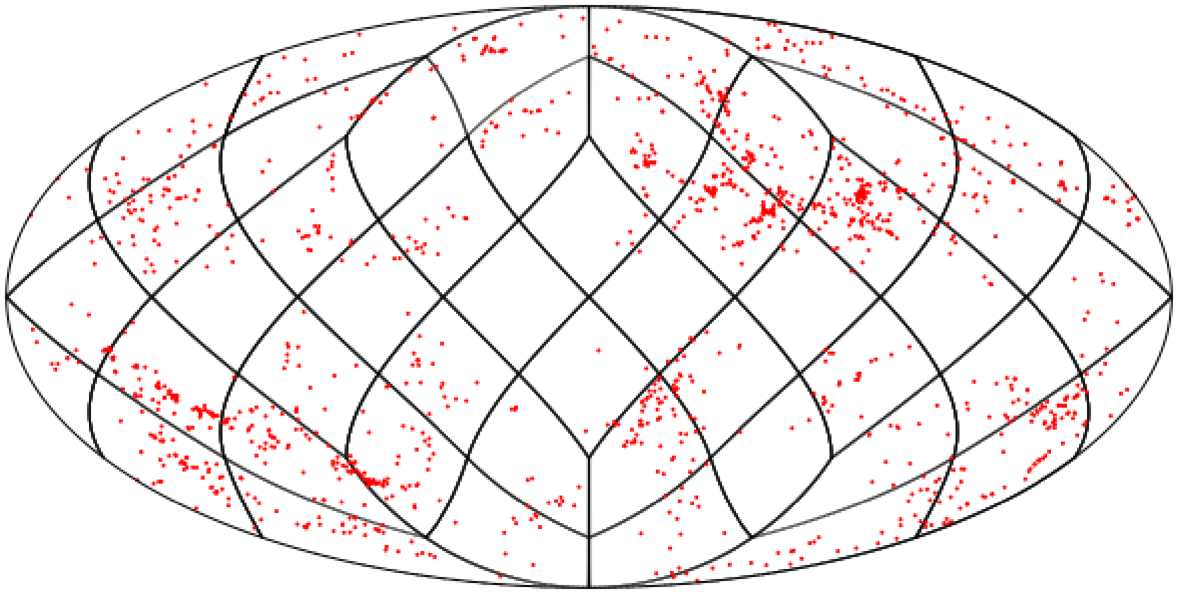
- Contains around 55800 objects.
- Around 47800 objects in (0.01 < z < 0.09)



Mollweide projection in galactic coordinates

The expansion rate fluctuation reconstruction

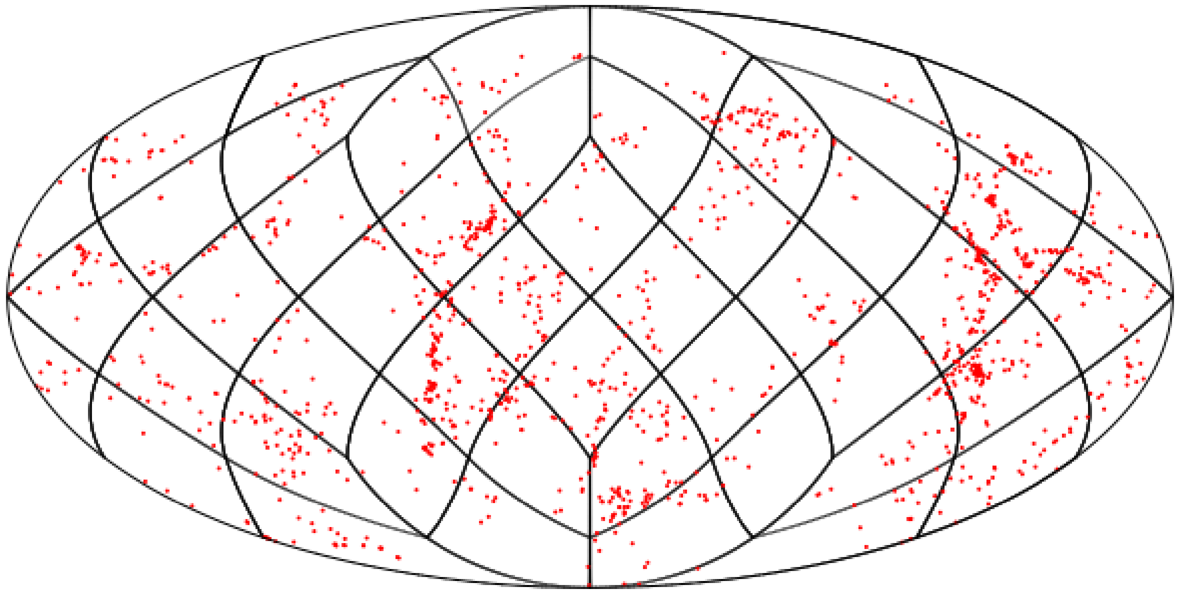
HEALPix tessellation



The expansion rate fluctuation reconstruction

HEALPix tessellation

Rotation to handle angular incompleteness



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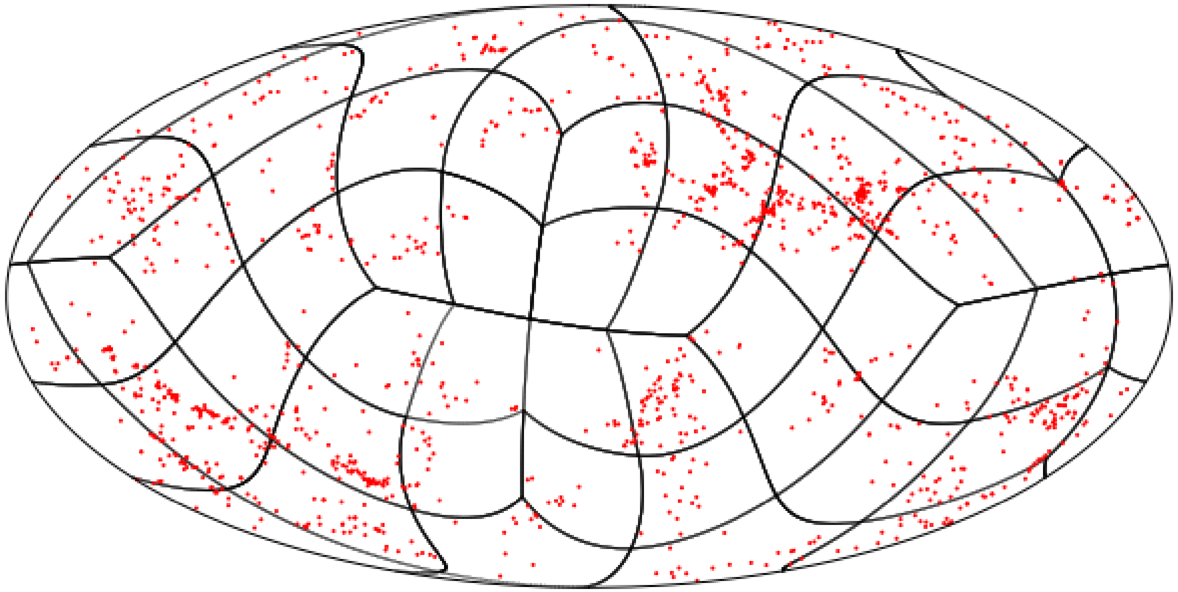


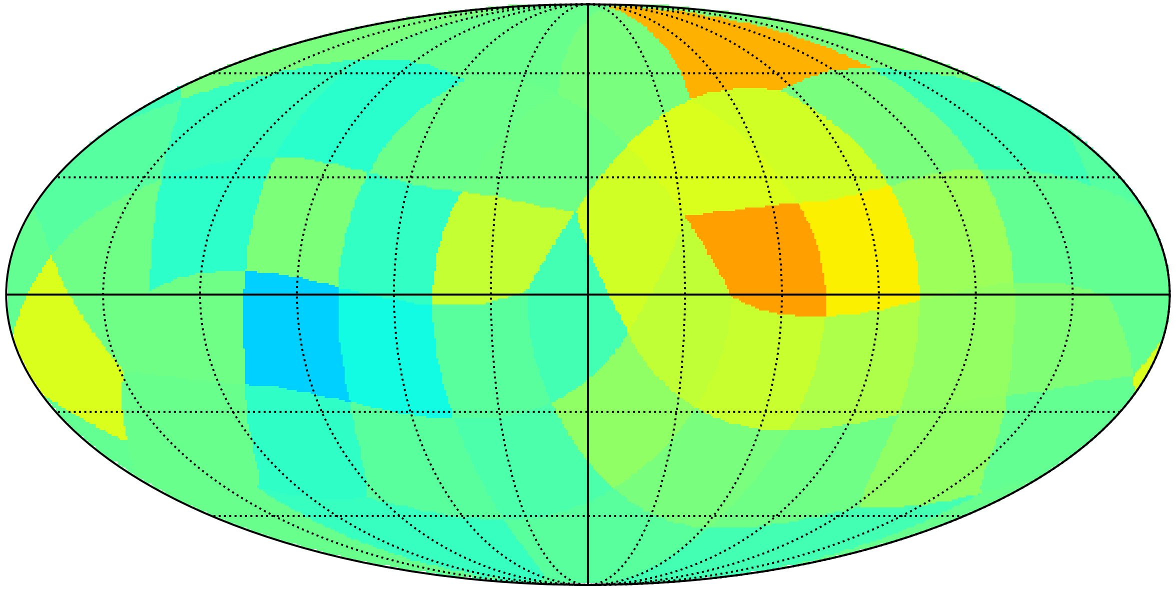
The expansion rate fluctuation reconstruction

HEALPix tessellation

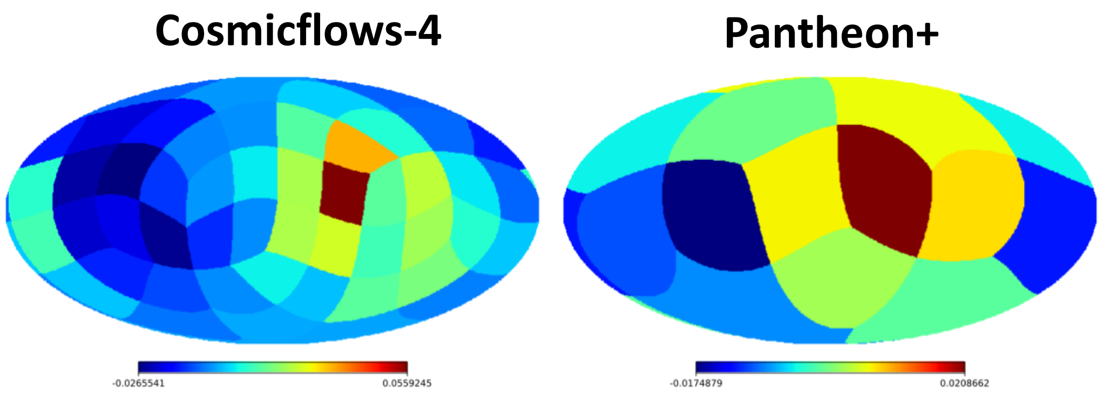
Rotation to handle angular incompleteness

Signal is then decomposed on the SH basis

$$\eta = \sum_{lm} a_{lm} Y_{lm}\,(\theta, \phi)$$



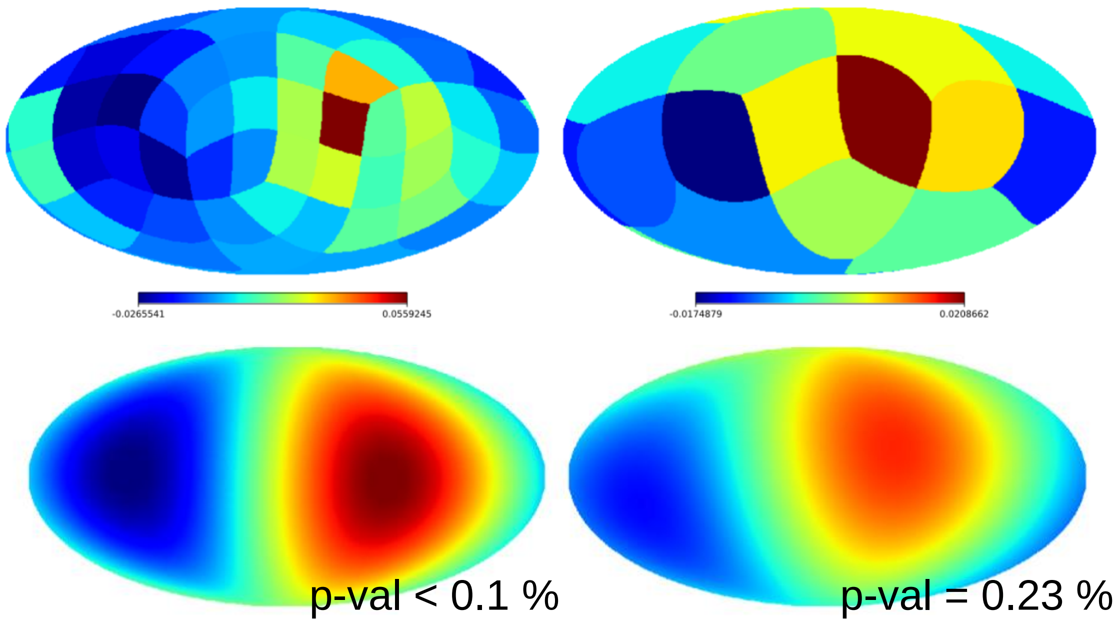
Multipoles of the expansion rate fluctuation by HEALpix (Kalbouneh et al. 2023)



η map (for $0.01 < z < 0.05$) using 48 pixels for Cosmicflows-4, and 12 pixels for Pantheon+.

Multipoles of the expansion rate fluctuation

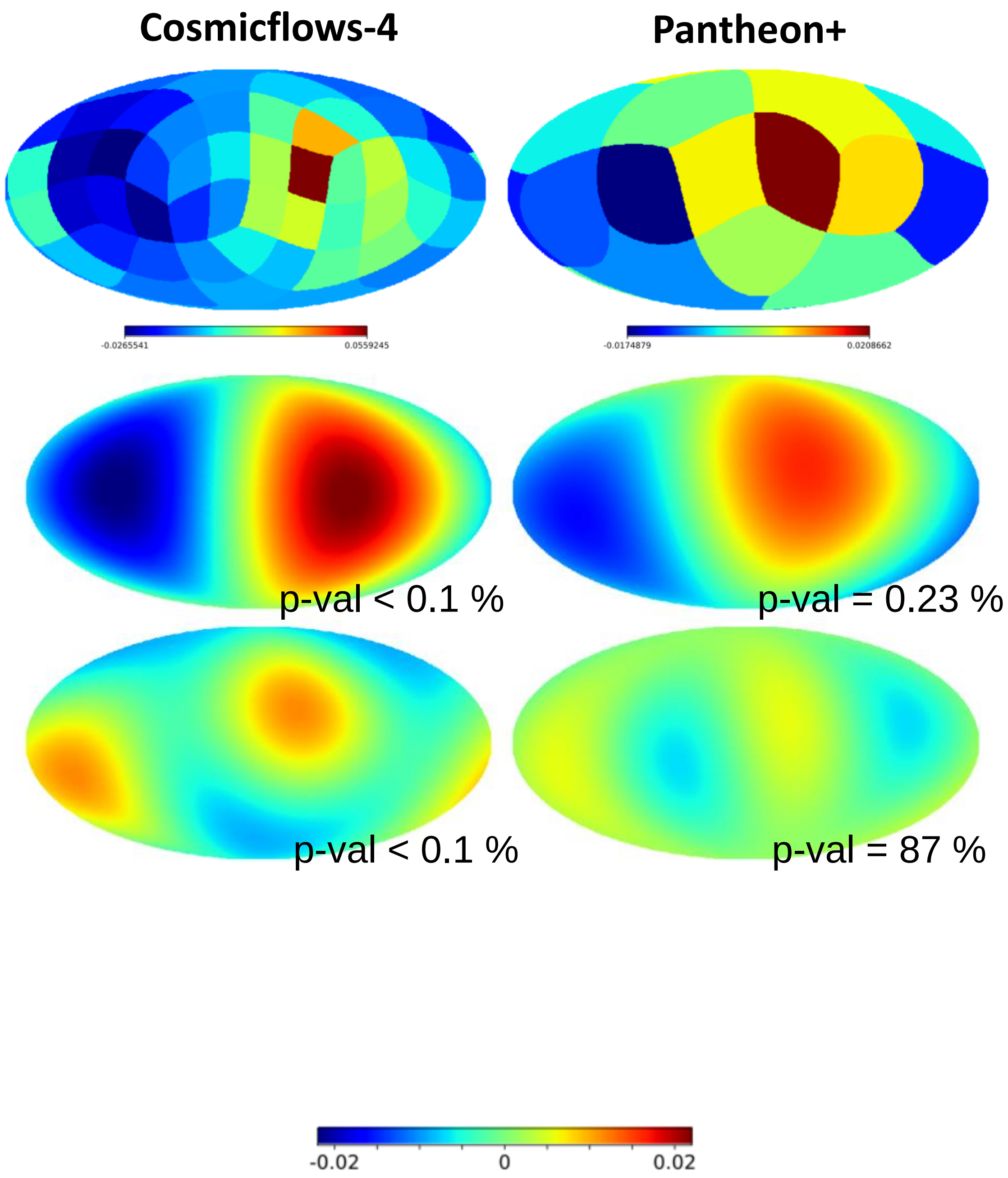
Cosmicflows-4 **Pantheon+**



η map (for $0.01 < z < 0.05$) using 48 pixels for Cosmicflows-4, and 12 pixels for Pantheon+.

Dipole $\approx$ 0.02 at $(l, b) \approx$ (290±3,-4±3) for CF4 and $\approx$ (314±21,9±16) for pantheon+.

Multipoles of the expansion rate fluctuation

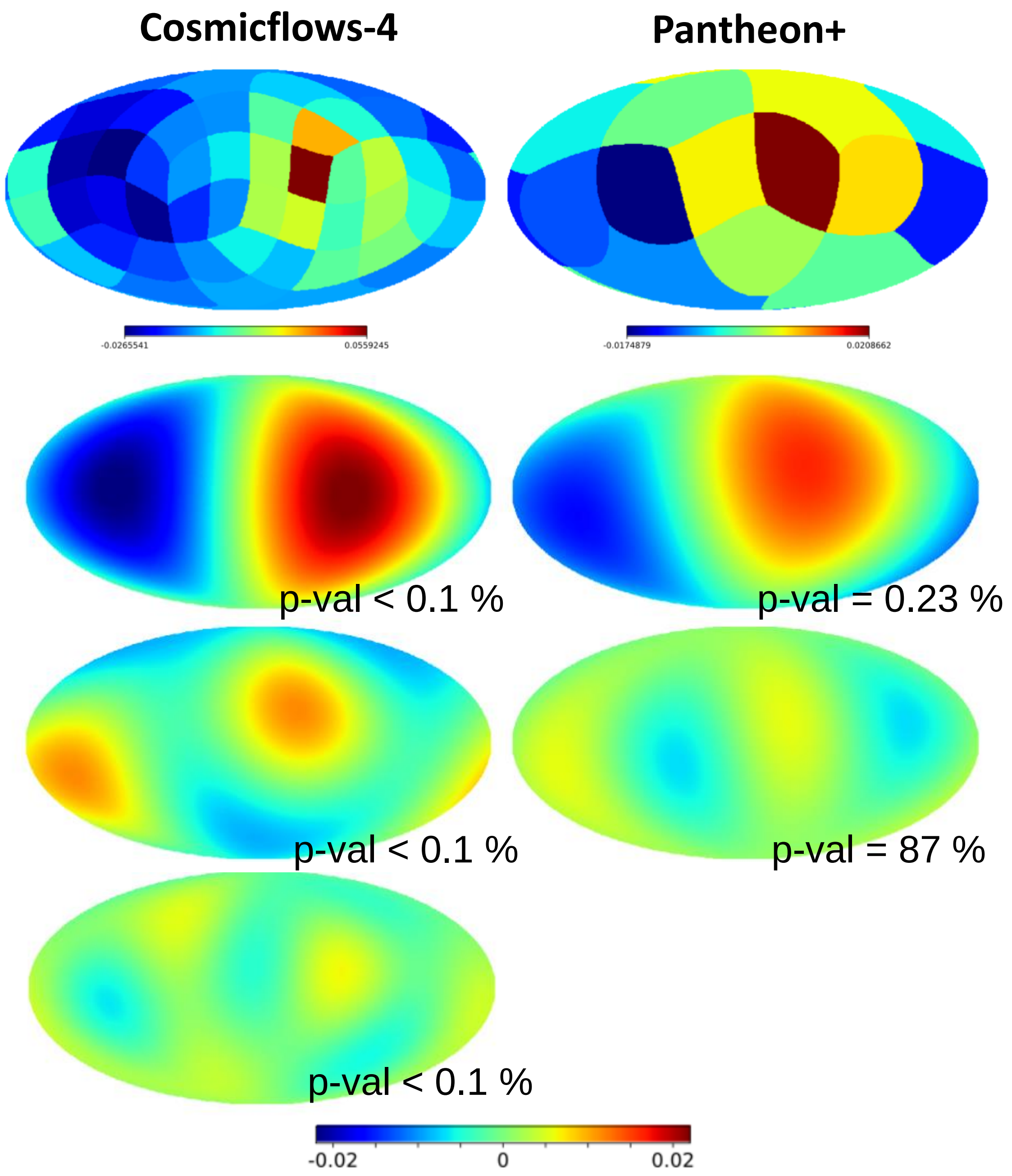


η map (for $0.01 < z < 0.05$) using 48 pixels for Cosmicflows-4, and 12 pixels for Pantheon+.

Dipole ≈ 0.02 at $(l, b) \approx (290\pm3, -4\pm3)$ for CF4 and $\approx (314\pm21, 9\pm16)$ for pantheon+.

Qudrupole ≈ 0.01
With a maximum at $(l, b) \approx (320,20)$
The quadrupole of Pantheon+ is not significant.

Multipoles of the expansion rate fluctuation



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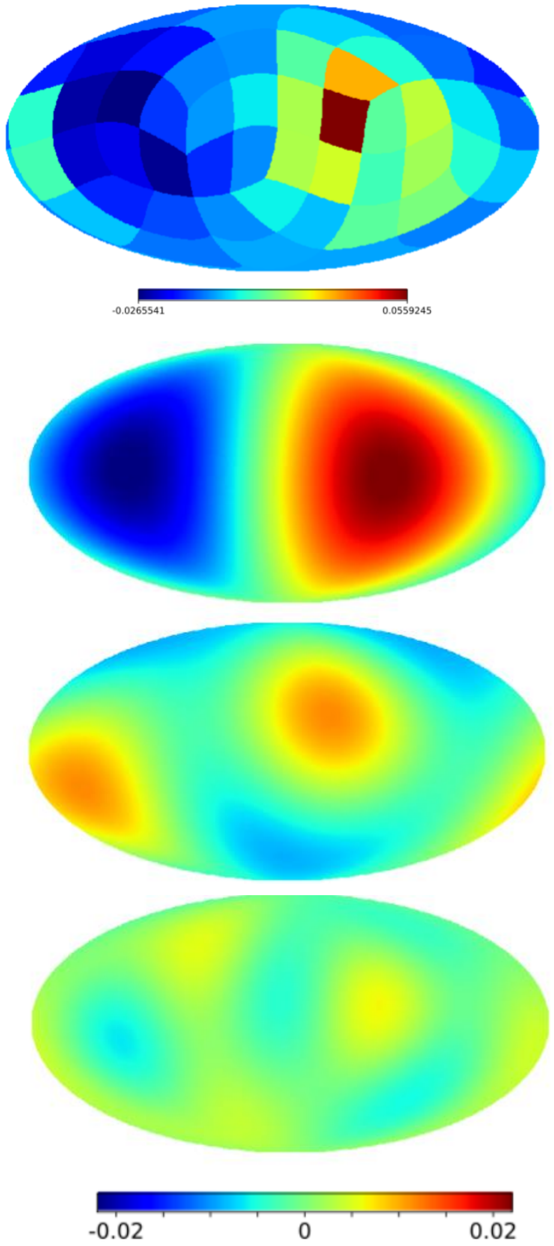
Qudrupole ≈ 0.01
With a maximum at $(l, b) \approx (320, 20)$
The quadrupole of Pantheon+ has p-value ≈ 87%

Octupole ≈ 0.006
Only can be measured by cosmicflows-4.
It has a maximum at $(l, b) \approx (306, 12)$

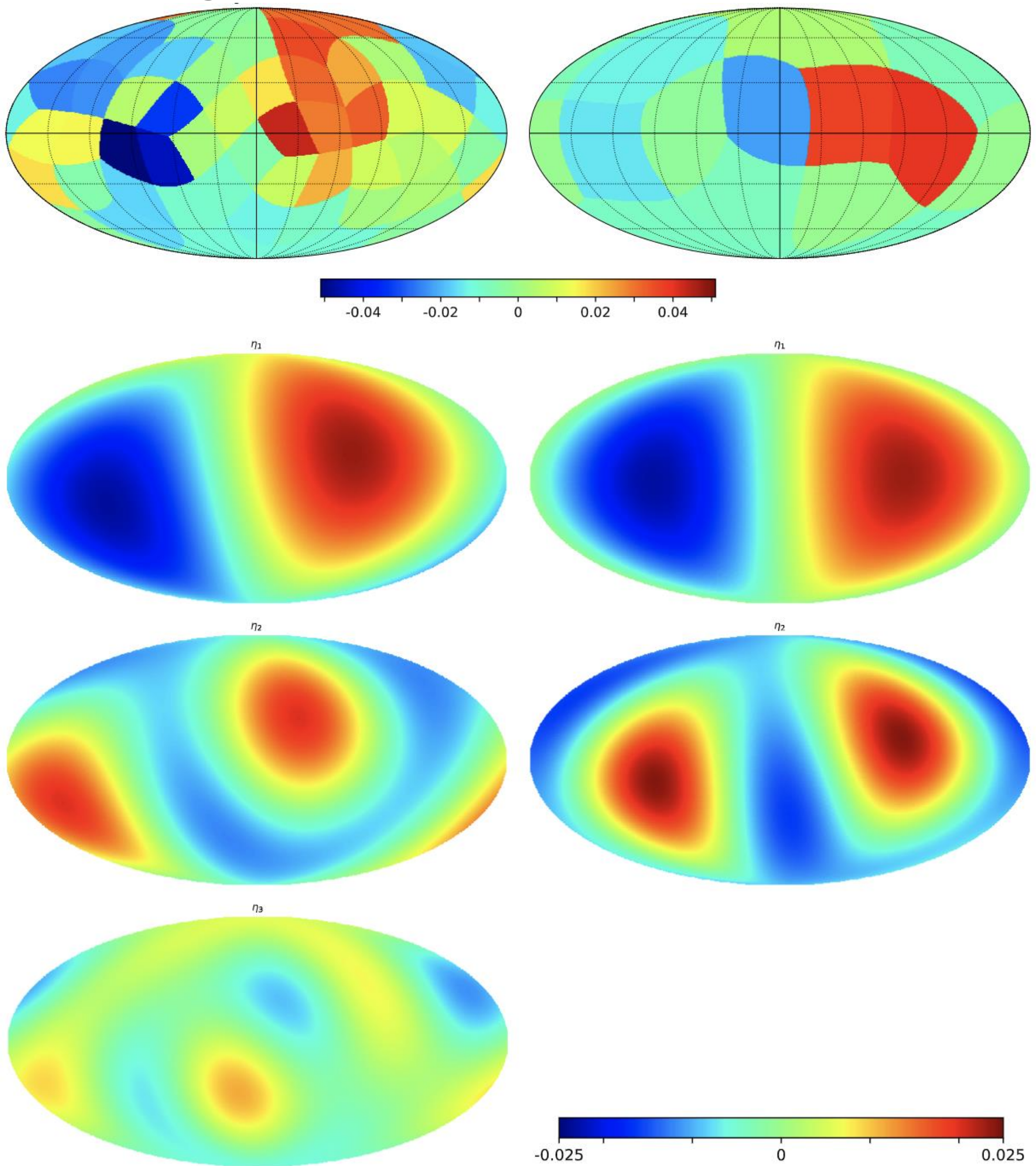
Multipoles of the expansion rate fluctuation

Cosmicflows-4

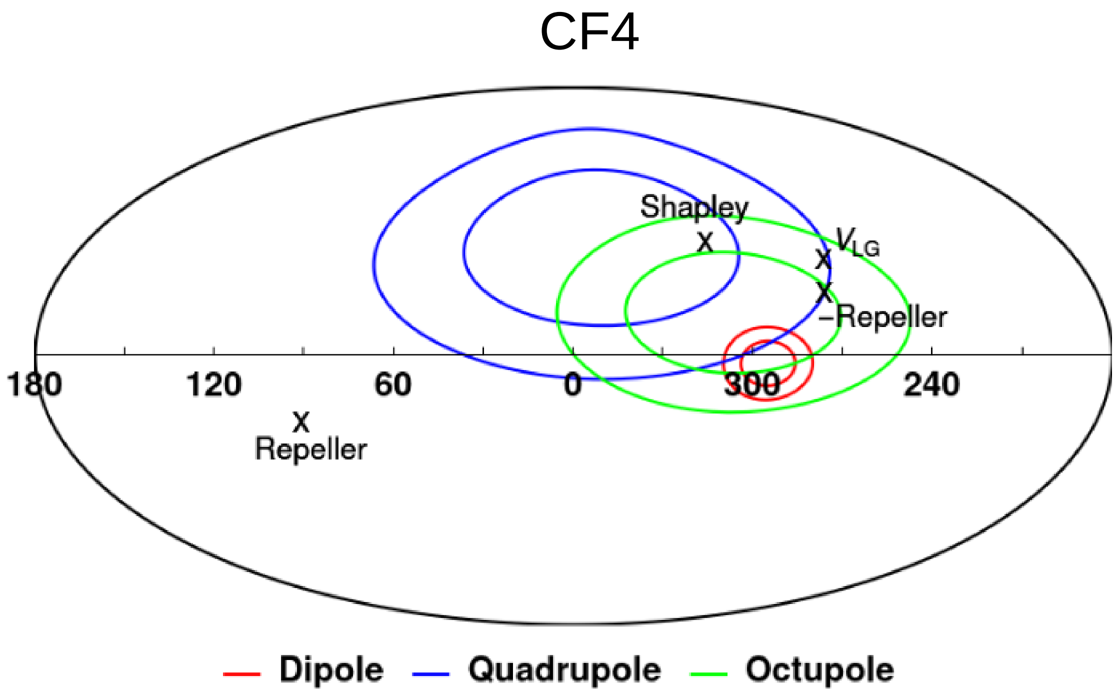


That confirms what we found in Cosmicflows-3 **(Kalbouneh, Marinoni & Bel , 2023)**

Cosmicflows-3 galaxies | **Cosmicflows-3 SNIa**



Multipoles of the expansion rate fluctuation

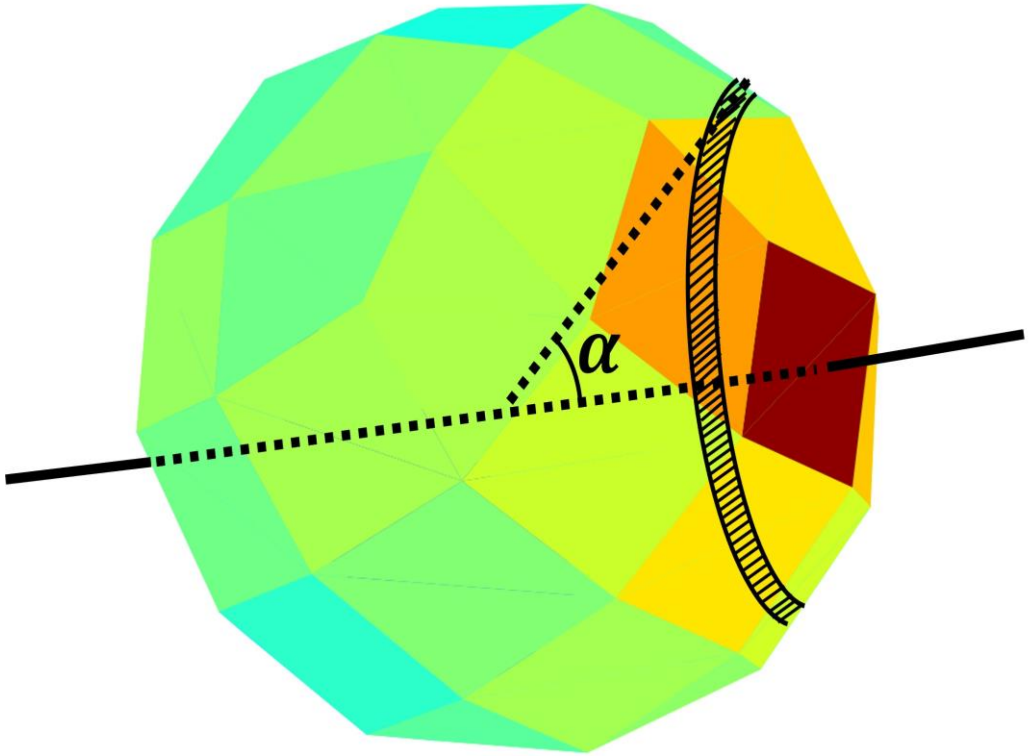


The dipole is aligned with the maximum of the quadrupole.

The octupole also has a maximum in that direction.

Axial symmetry of the expansion rate fluctuation

For CF4 at 0.01< z < 0.05, using the multipoles (up to l=3 with 16 d.o.f) almost has the same effect of using use axial symmetric multipoles (up to l=3 with only 6 d.o.f) on χ^2_{red}.



For the axial symmetric field on a sphere, it is sufficient to expand it using the Legendre basis.

$$\eta(\alpha, z) = \sum_{\ell=0}^{\infty} a_\ell(z) P_\ell(\cos\alpha)$$

and

$$a_\ell(z) = \frac{2\ell + 1}{2} \int_{-1}^{1} \eta(\cos\alpha, z) P_\ell(\cos\alpha)\, \mathrm{d}(\cos\alpha)$$

Axial symmetry of the expansion rate fluctuation

The relation between the multipoles of η and the multipoles of the cosmographic coefficients (up to the octupole) at linear order of z

$$\bar{a}_0(\bar{z}) \approx \log a_0^{\mathbb{H}} - \frac{1 - a_0^{\mathbb{Q}}}{2 \ln 10} \bar{z}$$

monopole

$$\bar{a}_1(\bar{z}) \approx \frac{a_1^{\mathbb{Q}}}{2 \ln 10} \bar{z} + \frac{v_0(1 + \bar{z})}{\ln 10 \bar{z}}$$

dipole

$$\bar{a}_2(\bar{z}) \approx \frac{a_2^{\mathbb{H}}}{a_0^{\mathbb{H}} \ln 10} + \frac{a_2^{\mathbb{Q}}}{2 \ln 10} \bar{z}$$

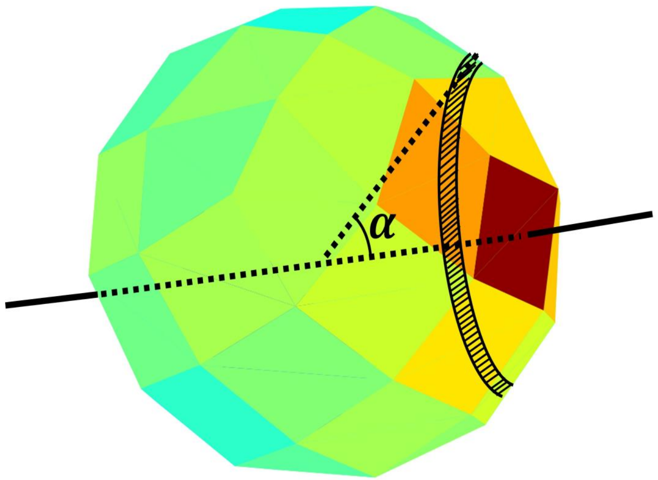
quadrupole

$$\bar{a}_3(\bar{z}) \approx \frac{a_3^{\mathbb{Q}}}{2 \ln 10} \bar{z}$$

octupole

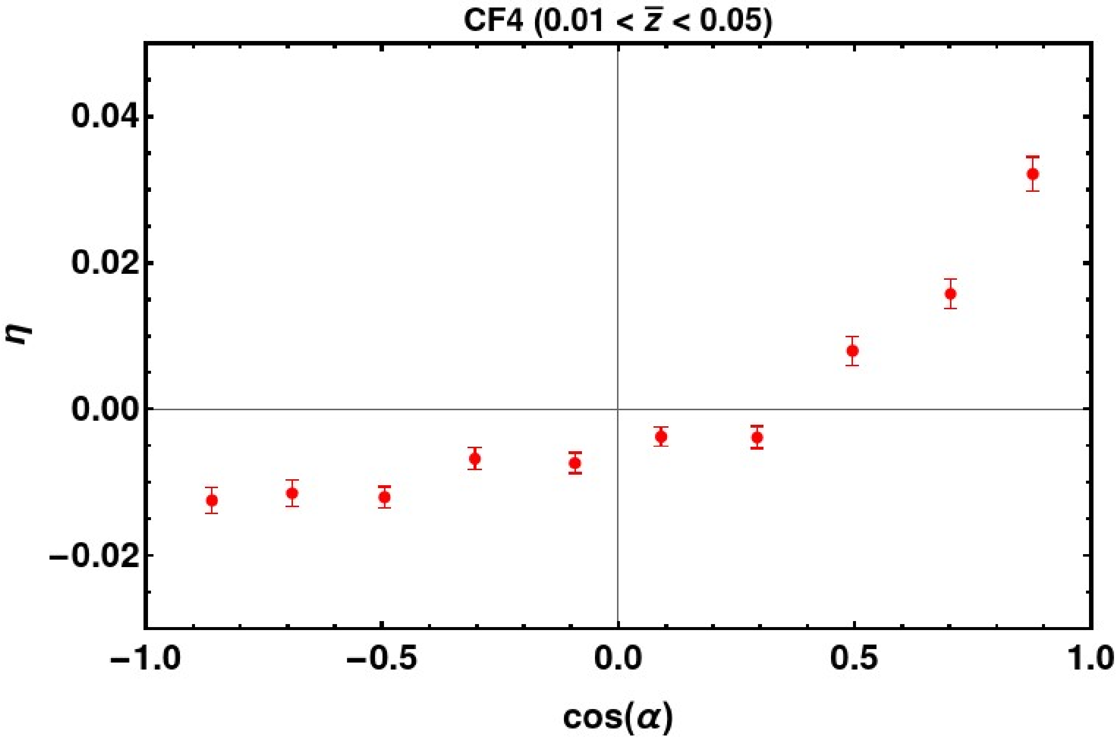
Axial symmetry of the expansion rate fluctuation

We fix the axis of symmetry to at (l , b) = **(292,2)**, which minimizes χ^2 .



For each shell in redshift, we calculate the Legendre coefficients by

$$a_\ell = \frac{2\ell + 1}{N_{bins}} \sum_{i=1}^{N_{bins}} \eta(\alpha_i) P_\ell(\cos \alpha_i)$$



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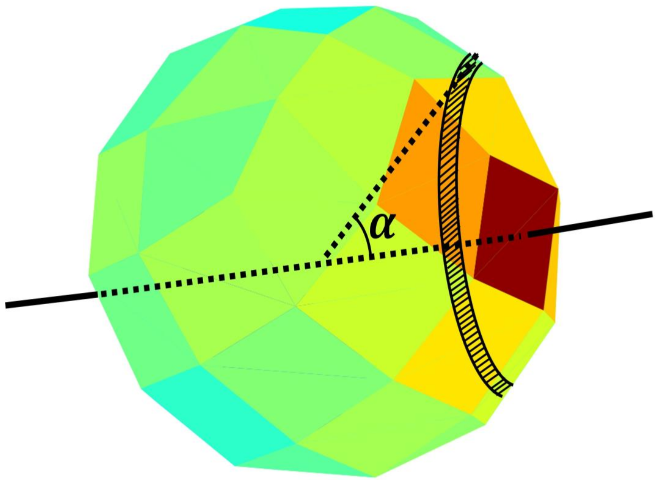

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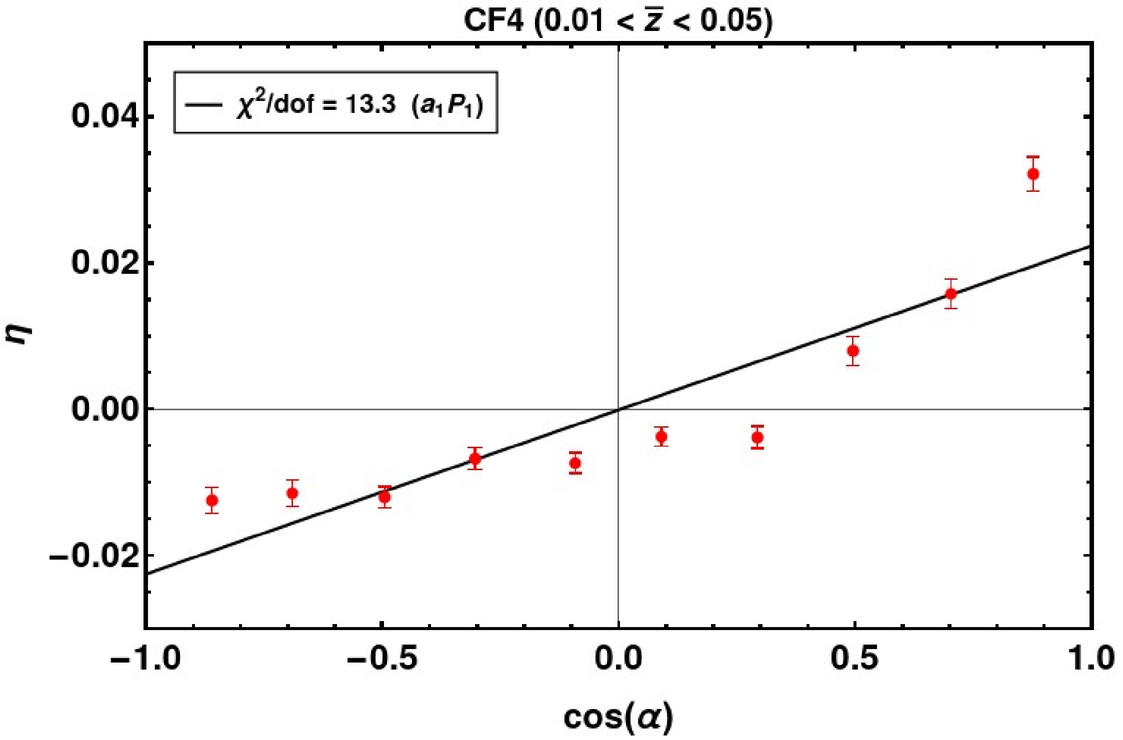

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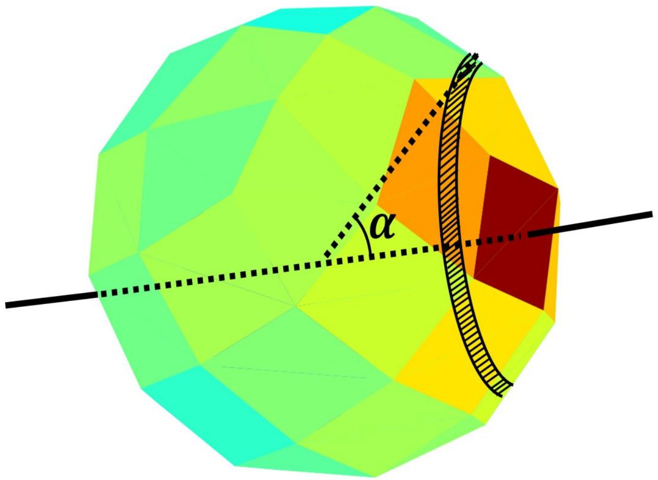

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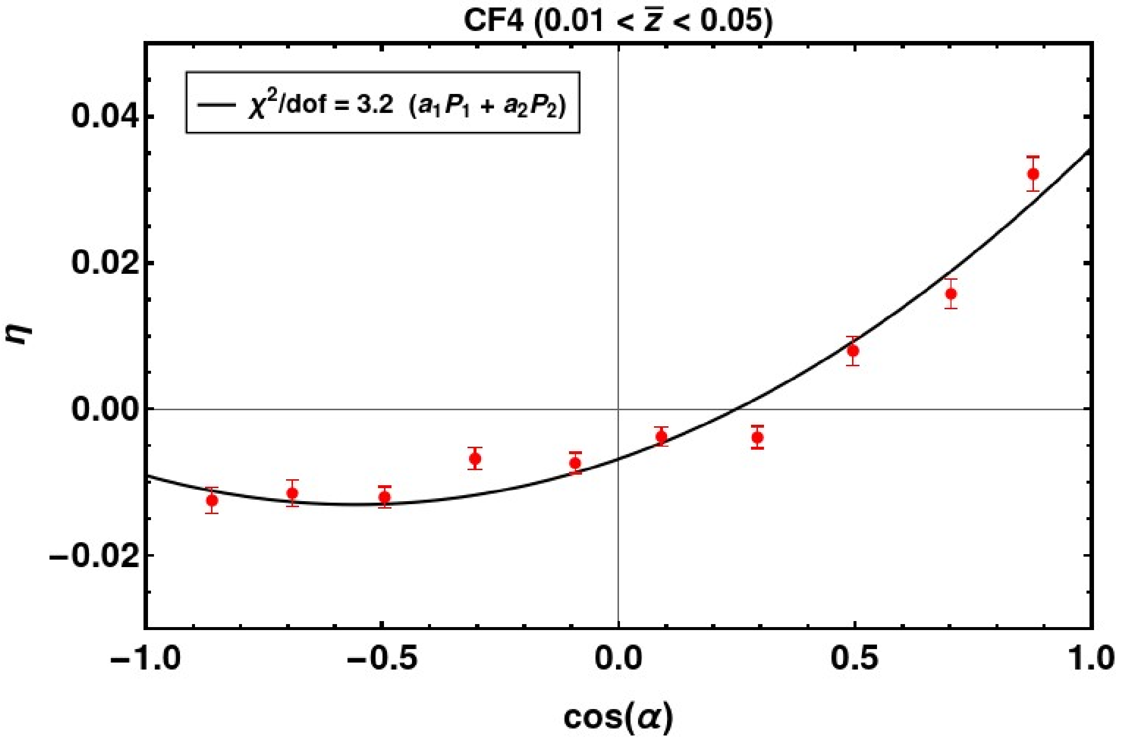

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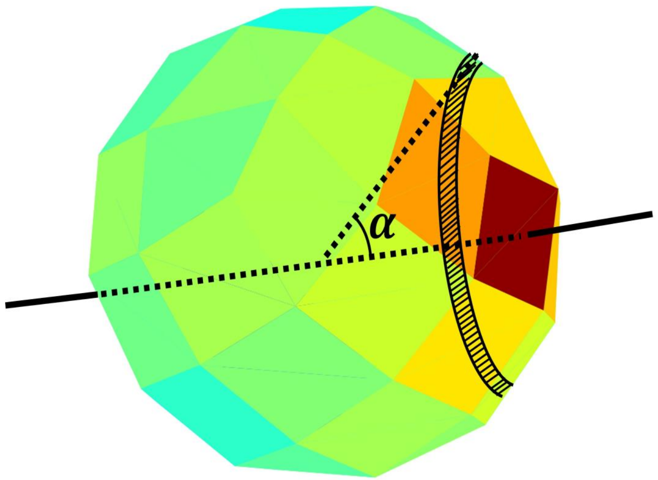

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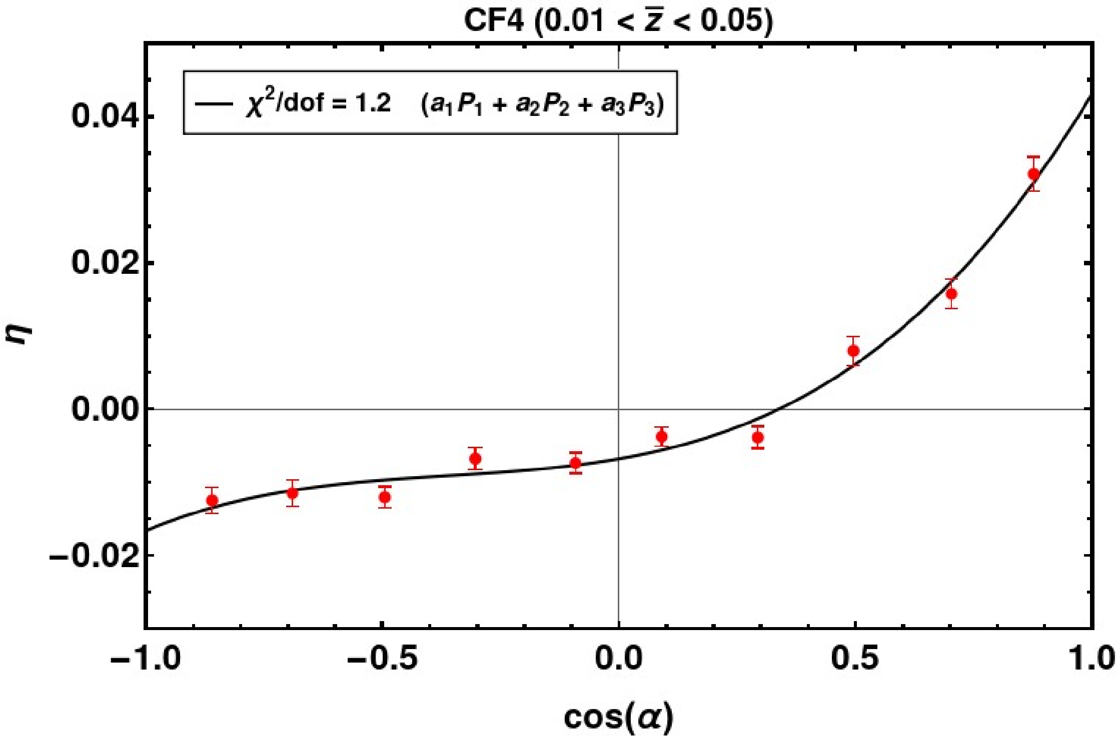
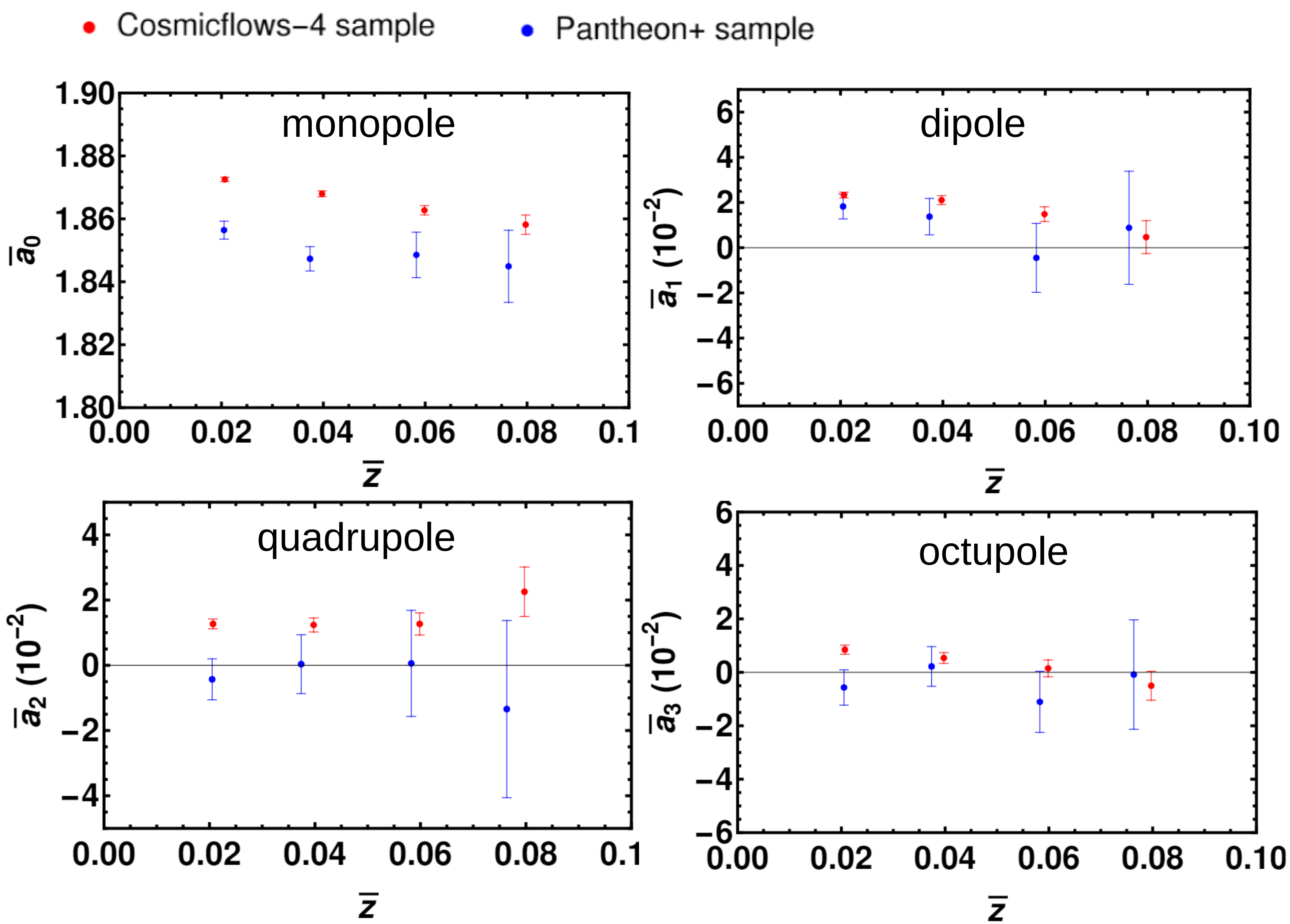
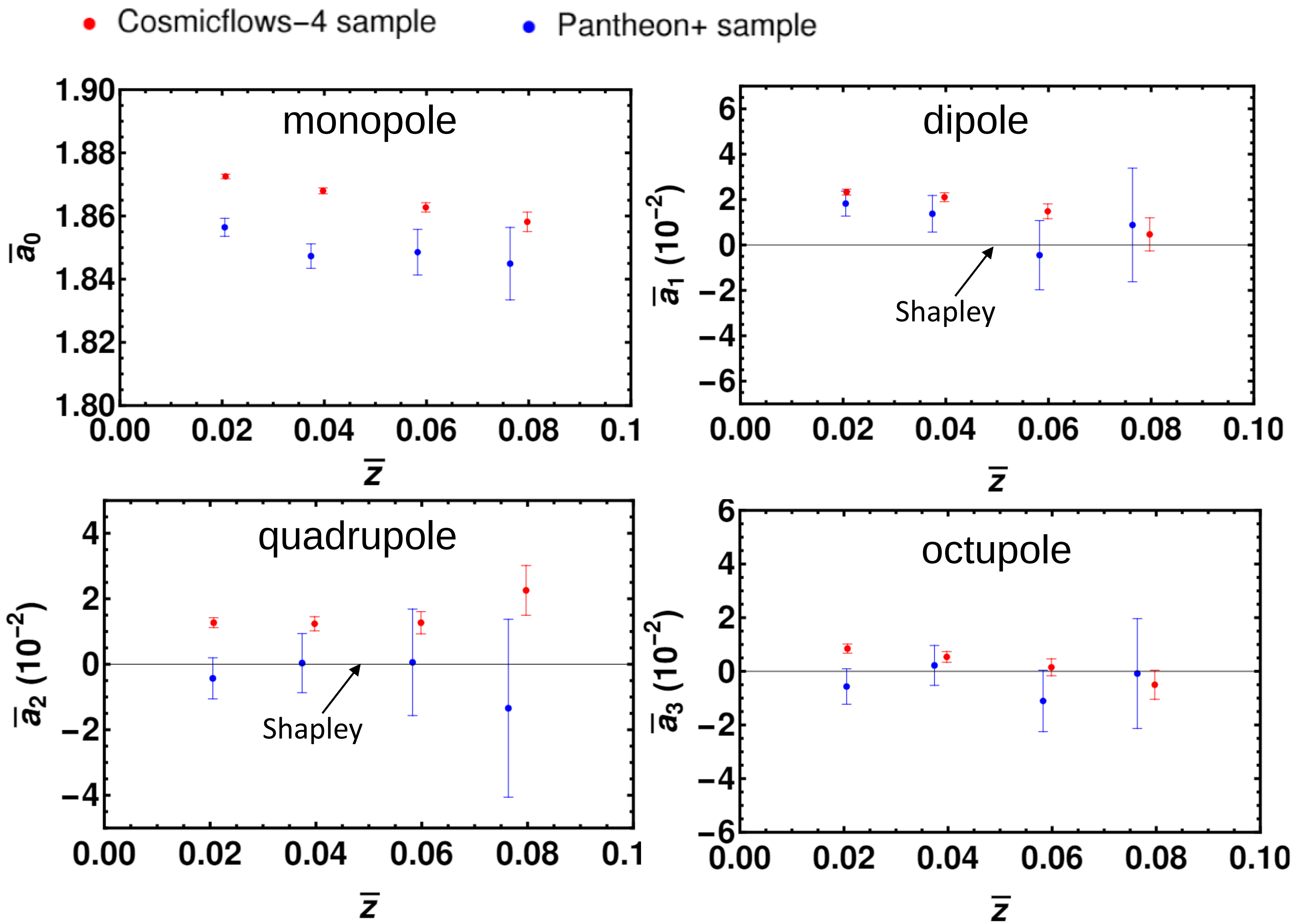


Results (Measured multipoles of $\bar{\eta}$)



Up to redshift 0.09.

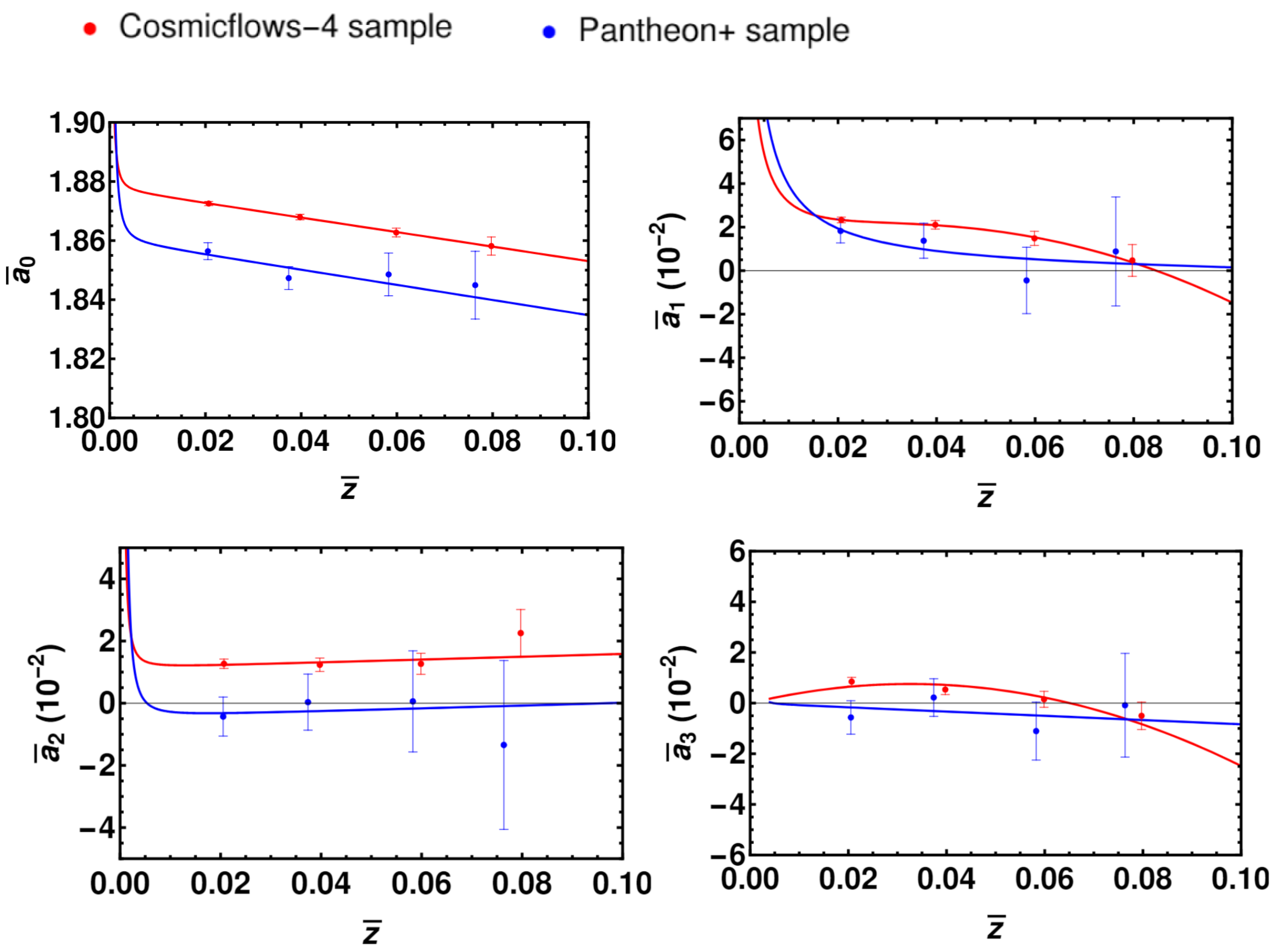
Results (Measured multipoles of $\bar{\eta}$)



Up to redshift 0.09.

Note the dipole and the quadrupole after Shapley supercluster.

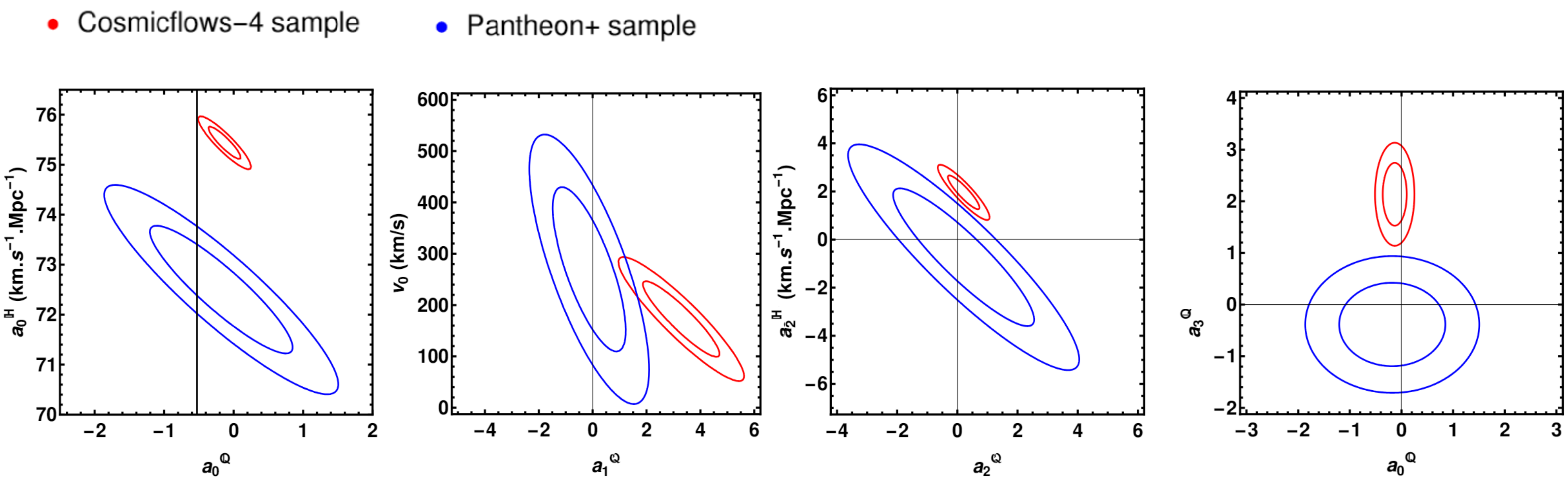
Results (Measured multipoles of $\bar{\eta}$ vs theoretical prediction from cosmography)



For Pantheon+, fit up to linear order of redshift is enough for all the multipoles, and that gives $\frac{\chi^2_{min}}{dof} = 0.5$.

For Cosmicflows-4, we have $\frac{\chi^2_{min}}{dof} = 0.6$.

Results (The multipoles of the cosmographic parameters)

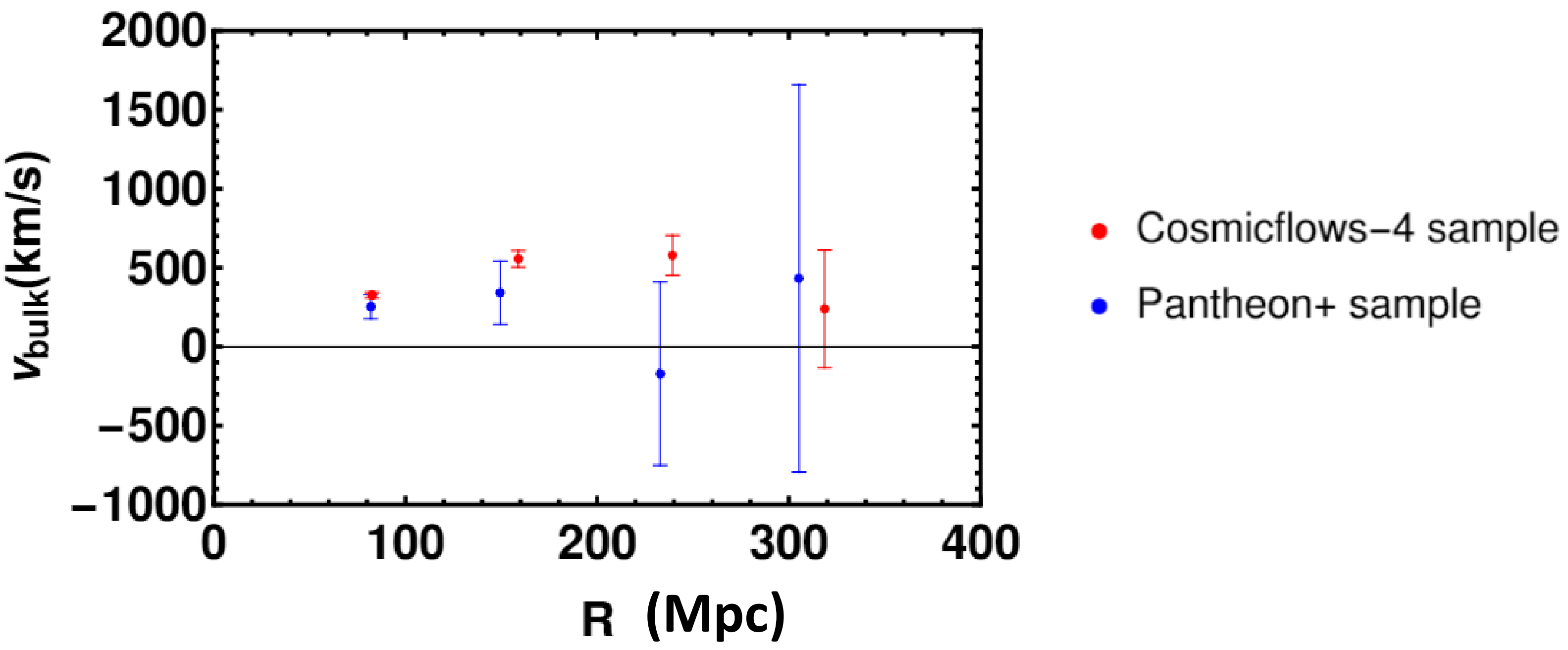


- The monopole of the Hubble parameter is different for both samples.
- The monopole $\mathbb{Q}_0$ is higher than the q_0 in the standard model.
- Both samples agree on the value of v_0.
- There is a dipole $\mathbb{Q}_0$ and an octupole in CF4, but not a quadrupole.
- The Hubble parameter has a quadrupole in CF4.
- Pantheon+ has no signal for the multipoles of the deceleration and Hubble parameter.

Results : The bulk velocity in shells (a model dependent analysis)

In the standard model the dipole of η presents the bulk velocity as

$$\bar{a}_1 \approx \frac{v_{bulk}(1+\bar{z})}{\bar{z}\ln 10}$$



Conclusions

We measured the multipoles of the expansion rate fluctuation (η) and their redshift evolution in the CF4 and Pantheon+ catalogues.

We confirm the axially symmetric nature of local anisotropies (already found in CF3 an Pantheon) (Kalbouneh et al. 2023).

We find evidence for a nonvanishing (and still aligned) dipole and a quadrupole even beyond 200 Mpc (the Shapley supercluster) in CF4 sample.

We extract from the multipoles of η the multipoles of Hubble and the deceleration parameters, in addition to the relative velocity between the CMB observer and the dust matter observer.

Future work, apply multipolar analysis of eta at redshift $z > 0.1$.