Family symmetries and the origin of fermion masses and mixings

(based on my work with Graham)

Ivo de Medeiros Varzielas

CFTP, Instituto Superior Técnico, Lisbon

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Dedication

Summary of data: quark mixing

Wolfenstein parametrisation

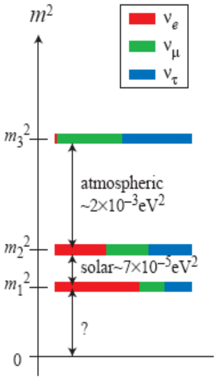
$$V_{CKM} \simeq \begin{pmatrix} 1 & \lambda & \lambda^3 \\ -\lambda & 1 & \lambda^2 \\ \lambda^3 & -\lambda^2 & 1 \end{pmatrix}$$

$\lambda \simeq 0.23$ (Sine of the Cabibbo angle)

Summary of data: lepton mixing

Tri-bi-maximal (TBM) mixing

$$V_{PMNS} \simeq \begin{pmatrix} -\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \end{pmatrix}$$



Motivation

The data

Fermion masses (heavy top, hierarchies, neutrino masses)
Fermion mixing (Cabibbo angle, near TBM, θ_{13})

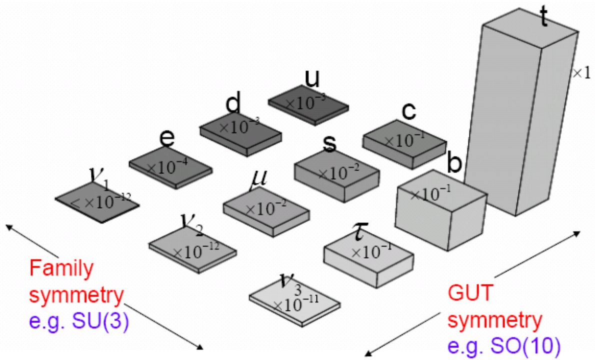
Beyond the Standard Model with family symmetries

Without $y_f H F f_R$, $\mathcal{L}_{\nu SM}$ has accidental symmetry $U(3)^6$

FS: upgrade subgroup of $U(3)^6$ to actual symmetry of $\mathcal{L}$

1. Generations charged differently under FS
2. Yukawa couplings no longer invariant
3. FS must be broken somehow...

$SO(10) \times SU(3)$?

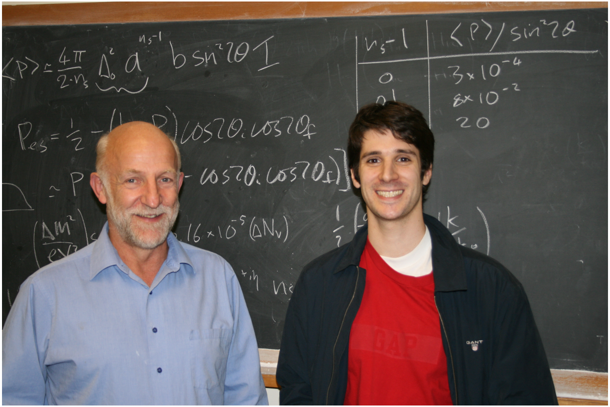


Graham Unification Theories

The challenge

ALL fermion masses and mixing (unified framework)

Improve upon existing models



Unification with family symmetry

All fermions can have the same Dirac mass structure!

P. Ramond, R.G. Roberts, G. G. Ross
`https://arxiv.org/abs/hep-ph/9303320`
G. G. Ross, M. Serna
`https://arxiv.org/abs/0704.1248`

$$\frac{M^{Dirac}}{m_3} = \begin{pmatrix} 0 & \varepsilon^3 & -\varepsilon^3 \\ \varepsilon^3 & a\varepsilon^2 + \varepsilon^3 & -a\varepsilon^2 + \varepsilon^3 \\ -\varepsilon^3 & -a\varepsilon^2 + \varepsilon^3 & 1 \end{pmatrix}$$

$$\varepsilon_d = 0.15, \quad a^d = -2/3$$
$$\varepsilon_l = 0.15, \quad a^e = -3$$
$$\varepsilon_u = 0.05, \quad a^u = 4/3$$
$$\varepsilon_\nu = 0.05, \quad a^\nu = 0$$

Seesaw and Georgi-Jarlskog (GJ) factors distinguish quarks and leptons

Texture Zero for quarks

$$M_{11}^{LR} = 0 \quad (1)$$

Texture Zero for up and down quarks gives the Gatto-Sartori-Tonin (GST) relation:

$$\sin\theta_c = \left| \sqrt{\frac{m_d}{m_s}} - e^{i\delta} \sqrt{\frac{m_u}{m_c}} \right| \quad (2)$$

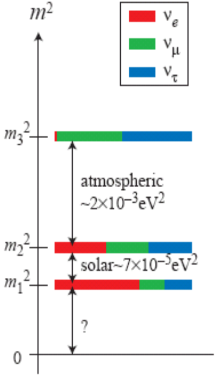
But how to get $M_{11}^{LR} = 0$... And what about the leptons?

Mass matrices from aligned VEVs

In this talk, directions:

$\langle \bar{\phi}_{\rm atm} \rangle \propto (0, 1, -1)$
$\langle \bar{\phi}_{\rm sol} \rangle \propto (1, 1, 1)$

FS invariants $(\bar{\phi}^i_{\rm atm} F_i)$, $(\bar{\phi}^i_{\rm sol} F_i)$



Mass matrices example columns (R)

Term sol L / atm R

$$+y_{\odot}(\bar{\phi}^{i}_{\mathrm{sol}}F_i)(\bar{\phi}^{j}_{\mathrm{atm}}f_{Rj})H$$

Respective mass matrix

$$+y_{\odot}\begin{pmatrix} 0 & \epsilon^3 & -\epsilon^3 \\ 0 & \epsilon^3 & -\epsilon^3 \\ 0 & \epsilon^3 & -\epsilon^3 \end{pmatrix}$$

Mass matrices example rows (L)

Term atm L / sol R

$$+y_{@}(\bar{\phi}^{i}_{\text{atm}}F_i)(\bar{\phi}^{j}_{\text{sol}}f_{Rj})H$$

Respective mass matrix

$$+y_{@}\begin{pmatrix} 0 & 0 & 0 \\ \epsilon^3 & \epsilon^3 & \epsilon^3 \\ -\epsilon^3 & -\epsilon^3 & -\epsilon^3 \end{pmatrix}$$

$SO(10) \times SU(3)$ with (near) TBM

IdMV, G.G. Ross

https://arxiv.org/abs/hep-ph/0507176

Strategy to pass the mixing to low energy after seesaw:

$$\begin{array}{c} \\ (\bar{\phi}_{\rm atm}\nu) \\ (\bar{\phi}_{\rm sol}\nu) \\ (\bar{\phi}_{\rm atm}N^c) \\ (\bar{\phi}_{\rm sol}N^c) \end{array} \begin{array}{c} \begin{array}{cccc} (\bar{\phi}_{\rm atm}\nu) & (\bar{\phi}_{\rm sol}\nu) & (\bar{\phi}_{\rm atm}N^c) & (\bar{\phi}_{\rm sol}N^c) \end{array} \\ \left(\begin{array}{cccc} 0 & 0 & 0 & \kappa^\nu \\ 0 & 0 & \kappa^\nu & 0 \\ 0 & \kappa^\nu & \kappa^M_1 & 0 \\ \kappa^\nu & 0 & 0 & \kappa^M_2 \end{array} \right) \end{array} \tag{3}$$

Gives effective LL Majorana mass terms

$$-\frac{(\kappa^\nu)^2}{\kappa^M_2}(\bar{\phi}_{\rm atm}\nu)(\bar{\phi}_{\rm atm}\nu) - \frac{(\kappa^\nu)^2}{\kappa^M_1}(\bar{\phi}_{\rm sol}\nu)(\bar{\phi}_{\rm sol}\nu) \tag{4}$$

$SO(10) \times \Delta(27)$

Directions $\langle \bar{\phi}_{\rm sol} \rangle = (1, 1, 1)$ and $\langle \bar{\phi}_{\rm atm} \rangle = (0, 1, -1)$
Easy to align in $\Delta(27)$ (discrete) family symmetry
IdMV, S. F. King, G. G. Ross
https://arxiv.org/abs/hep-ph/0607045

Effective Majorana mass terms

$$-\frac{(\kappa^\nu)^2}{\kappa_2^M}(\bar{\phi}_{\rm atm}\nu)(\bar{\phi}_{\rm atm}\nu) - \frac{(\kappa^\nu)^2}{\kappa_1^M}(\bar{\phi}_{\rm sol}\nu)(\bar{\phi}_{\rm sol}\nu) \tag{5}$$

Democratic contribution fills all entries

$$M^{LR}_{11} = 0; M^{RR}_{11} \neq 0; M^{LL}_{11} \neq 0 \tag{6}$$

TBM in neutrino sector, modified slightly by charged lepton matrix which is not diagonal in this basis: θ_{13} too small!

Universal Texture Zero

IdMV, G. G. Ross, J. Talbert
`https://arxiv.org/abs/1710.01741`
Preserve the M_{11} texture zero in the Majorana mass matrix and into the effective neutrino mass matrix after seesaw:

$$M_{11}^{LR} = M_{11}^{RR} = M_{11}^{LL} = 0 \tag{7}$$

Not TBM in neutrino sector.
Large θ_{13} (correlated with other angles).

$SO(10) \times \Delta(27)$ with UTZ, seesaw

$$
\begin{array}{c} \\ (\bar{\phi}_{\rm sol}\nu) \\ (\bar{\phi}_{\rm atm}\nu) \\ (\bar{\phi}_{\rm sol}N^c) \\ (\bar{\phi}_{\rm atm}N^c) \end{array}
\begin{array}{c} \begin{array}{cccc} (\bar{\phi}_{\rm sol}\nu) & (\bar{\phi}_{\rm atm}\nu) & (\bar{\phi}_{\rm sol}N^c) & (\bar{\phi}_{\rm atm}N^c) \end{array} \\
\begin{pmatrix} 0 & 0 & 0 & \kappa_2^\nu \\ 0 & 0 & \kappa_2^\nu & \kappa_1^\nu \\ 0 & \kappa_2^\nu & 0 & \kappa_2^M \\ \kappa_2^\nu & \kappa_1^\nu & \kappa_2^M & \kappa_1^M \end{pmatrix} \end{array} \tag{8}
$$

Seesaw into effective Majorana mass terms

$$
\begin{array}{c} \\ (\bar{\phi}_{\rm sol}\nu) \\ (\bar{\phi}_{\rm atm}\nu) \end{array}
\begin{array}{c} \begin{array}{cc} (\bar{\phi}_{\rm sol}\nu) & (\bar{\phi}_{\rm atm}\nu) \end{array} \\
\begin{pmatrix} 0 & \frac{(\kappa_2^\nu)^2}{\kappa_2^M} \\ \frac{(\kappa_2^\nu)^2}{\kappa_2^M} & f\left(\kappa_{1,2}^{\nu,M}\right) \end{pmatrix} \end{array} . \tag{9}
$$

Results (summarised)

Universal Texture Zero: labour saving devices are good!

Nice family symmetry GUT model with UTZ.
Important postdictions:
The Cabibbo angle (GST),
Expected charged lepton masses (GJ),
Daya-Bay reactor angle (θ_{13}).

Higgs mediators: terms

$$P_S = M\bar{X}^i X_i + \bar{\phi}_3^i \bar{\phi}_3^i \bar{X}^i H / M_X^a$$
$$= M\bar{X}^i \left(X_i + \bar{\phi}_3^i \bar{\phi}_3^i H / M M_X^a \right)$$

with $\langle \bar{\phi}_3 \rangle \propto (0, 0, 1)$

$$H_l = X_3 - H \frac{M\, M_X^a}{\langle \bar{\phi}_3 \rangle^2} \approx X_3$$

Then if

$$P_Y = \sum_i X_i \Psi_i \Psi_i^c$$

Renormalizable Yukawa for 3rd generation, as only X_3 is light!

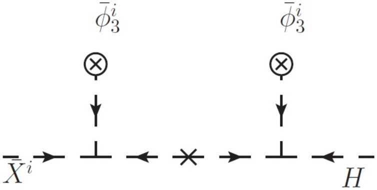
Higgs mediators: matrix

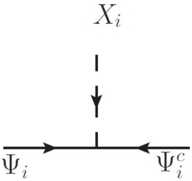
Mass matrix of $(H, \bar{X}^3, X_3)$:

$$\begin{pmatrix} 0 & \langle\bar{\phi}_3\rangle^2/M_X^a & 0 \\ \langle\bar{\phi}_3\rangle^2/M_X^a & 0 & M \\ 0 & M & 0 \end{pmatrix}$$

State $H_l = X_3 - H\frac{M\, M_X^a}{\langle\bar{\phi}_3\rangle^2}$ with mass 0

Higgs mediators: diagrams





Generalisation: the scalars

$$
\begin{aligned}
P_S = & \, M\bar{X}X + M\bar{Y}Y + \phi\bar{Z}Z \\
& + \bar{\phi}_3^i \bar{\phi}_3^i \bar{X}^i H / M_X^a \\
& + a\bar{\phi}_{23}^i \bar{\phi}_{23}^i \bar{Y}^{(a),i} H / M_X^b \\
& + b[\bar{\phi}_{23} \bar{\phi}_{23} \bar{Y}^{(b)}]_+ H / M_X^b \\
& + [\bar{\phi}_{23} \bar{\phi}_{123} \bar{Z}]_- H / M_X^c \, ,
\end{aligned}
$$

$$
\langle \bar{\phi}_{23} \rangle / M_X = (0, \epsilon_f, -\epsilon_f) \, , \quad \langle \bar{\phi}_{123} \rangle / M_X \sim (\epsilon_f^2, \epsilon_f^2, \epsilon_f^2)
$$

Higgs light state

$$H_l \approx X_3 + \left(\frac{\langle\bar{\phi}_{23}\rangle^2}{\langle\bar{\phi}_3\rangle^2}\right)\frac{M_X^a}{M_X^b}\left(a(Y_2^{(a)} + Y_3^{(a)}) - 2bY_1^{(b)}\right)$$
$$+ \left(\frac{\langle\bar{\phi}_{23}\rangle\langle\bar{\phi}_{123}\rangle}{\langle\bar{\phi}_3\rangle^2}\right)\frac{M_X^a}{M_X^c}(2Z_1 - Z_2 - Z_3) - H\frac{M\, M_X^a}{\langle\bar{\phi}_3\rangle^2}\,,$$

Generalisation: Yukawa

$$P_Y = X_i \Psi_i \Psi_i^c + \left(a' Y_i^{(a)} \Psi_i \Psi_i^c + b' [Y^{(b)} \Psi \Psi^c]_+ \right) \Sigma / M_X + [Z \Psi \Psi^c]_-$$

Fermion masses

$$\langle \bar{\phi}_{23} \rangle / M_d = (0, \epsilon_f, -\epsilon_f), \quad \langle \bar{\phi}_{123} \rangle / \langle \bar{\phi}_{23} \rangle \sim \epsilon_f$$

$$M_f \sim \begin{pmatrix} 0 & -\epsilon_f^3 & \epsilon_f^3 \\ \epsilon_f^3 & a^f \epsilon_f^2 & -2ba^f \epsilon_f^2 + 2\epsilon_f^3 \\ -\epsilon_f^3 & -2ba^f \epsilon_f^2 - 2\epsilon_f^3 & 1 \end{pmatrix}$$

Fermion masses, Σ

$$a^f \propto < \Sigma > / M_X \,, \quad a^\nu \sim 0, \; a^l \sim 3a^d \sim 3a^u/2$$

$$M_f \sim \begin{pmatrix} 0 & -\epsilon_f^3 & \epsilon_f^3 \\ \epsilon_f^3 & a^f \epsilon_f^2 & -2ba^f \epsilon_f^2 + 2\epsilon_f^3 \\ -\epsilon_f^3 & -2ba^f \epsilon_f^2 - 2\epsilon_f^3 & 1 \end{pmatrix}$$

Results (summarised)

Higgs mediators: now we are cooking with gas!

Alternative to Froggatt-Nielsen style completion:
Renormalizable (3rd generation) Yukawa couplings,
Diminute fermionic sector.

Conclusion