



PHENOMENOLOGY OF THE SUPER-WEAK $U(1)$ EXTENSION OF THE STANDARD MODEL

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based on arXiv:1812.11189 (*Symmetry*), 1911.07082 (*PRD*), 2104.11248 (*JCAP*),
2104.14571 (*PRD*), 2105.13360 with S. Iwamoto, T.J. Kärkkäinen, Z. Péli, K. Seller



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OUTLINE

1. Status of particle physics
2. Super-weak $U(1)_Z$ extension of SM
3. Neutrino masses
4. Dark matter candidate
5. Neutrino benchmarks
6. Conclusions

Status of particle physics: energy frontier

- LEP, LHC: SM describes final states of particle collisions precisely [talk by Kordas]

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- SM vacuum is metastable

[Bezrukov et al, arXiv:1205.2893; Degraasi et al, arXiv:1205.6497]

*There are some indications below discovery significance (such as lepton flavor non-universality in meson decays) [talk by Pepe-Altarelli]

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- Neutrino flavours oscillate
- Existing baryon asymmetry cannot be explained by CP asymmetry in SM
- Inflation of the early, accelerated expansion of the present Universe

[<https://pdg.lbl.gov>]

Extension of SM

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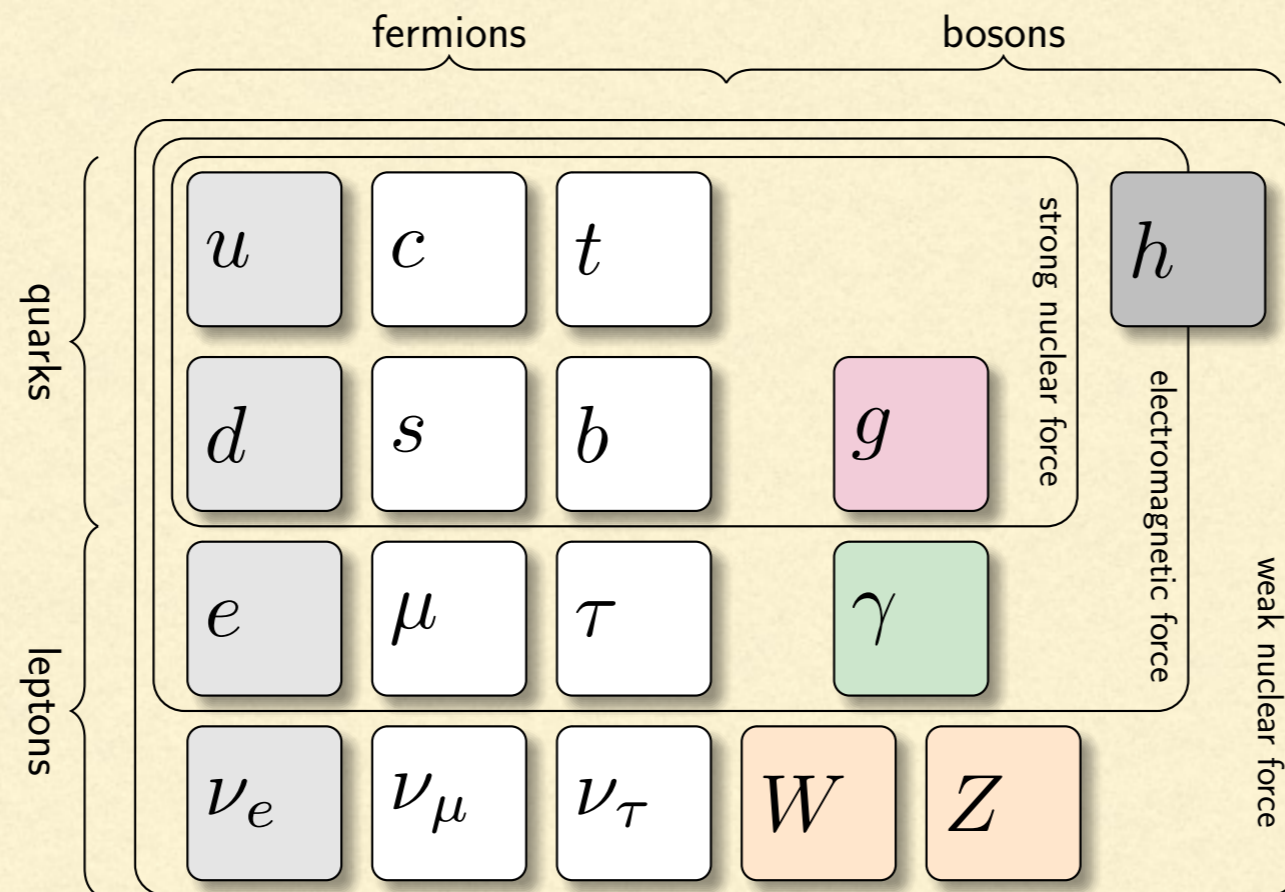
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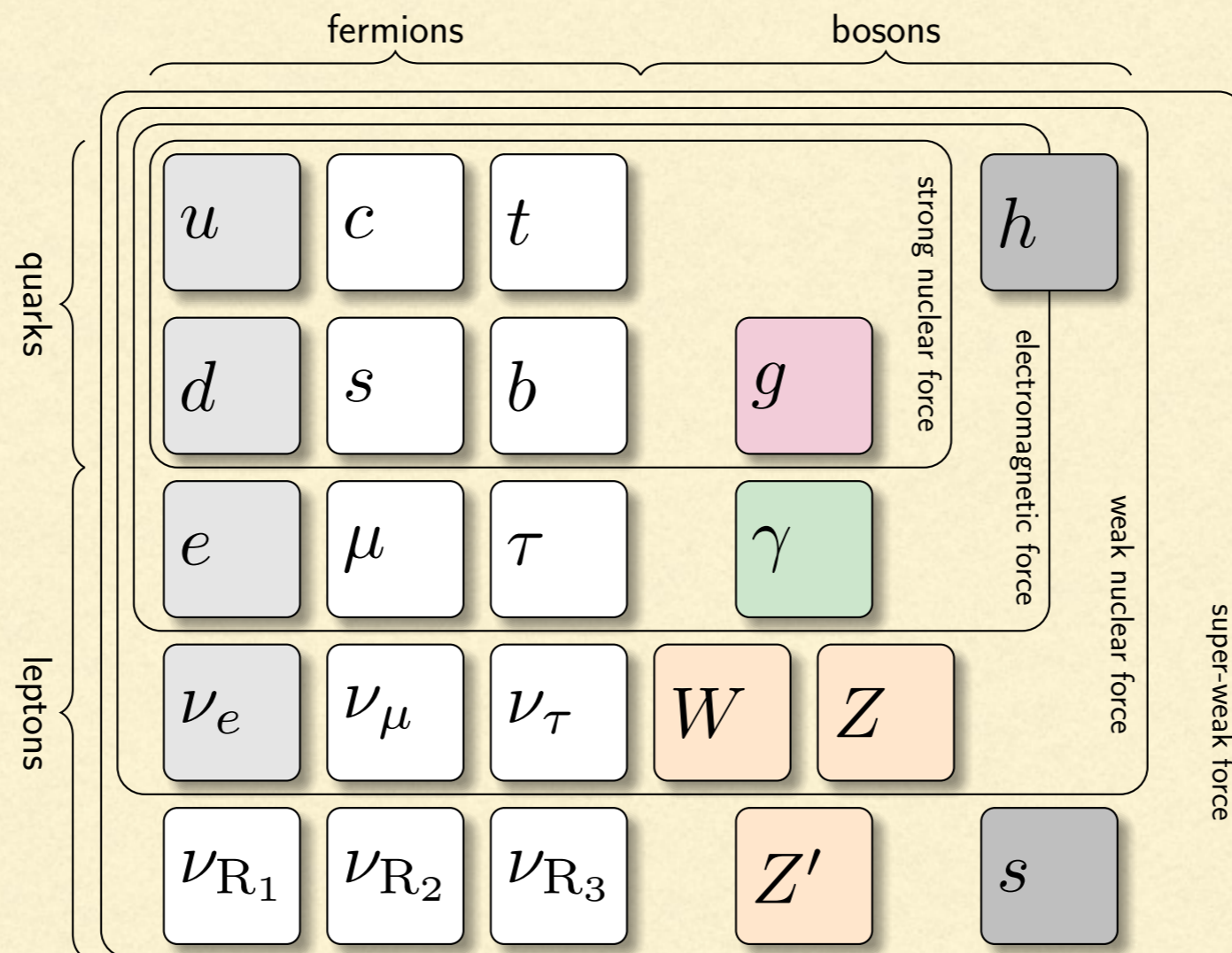
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 - gauge and gravity **anomaly cancellation** and
 - **gauge invariant Yukawa terms for neutrino** mass generation

Particle content of SM



Particle content of SM+SW



Expected consequences

- Dirac and Majorana neutrino mass terms are generated by the SSB of the scalar fields, providing the origin of neutrino masses and oscillations
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Extensive phenomenological studies are required to confront the predictions of the model with measurements, and decide whether or not these promises are fulfilled

Particle model

- fermion fields (Weyl spinors):

$$\psi_{q,1}^f = \begin{pmatrix} U^f \\ D^f \end{pmatrix}_L \quad \psi_{q,2}^f = U_R^f, \quad \psi_{q,3}^f = D_R^f$$

$$\psi_{l,1}^f = \begin{pmatrix} \nu^f \\ \ell^f \end{pmatrix}_L \quad \psi_{l,2}^f = \nu_R^f, \quad \psi_{l,3}^f = \ell_R^f$$

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- the new U(1) kinetic term includes **kinetic mixing**:

$$\mathcal{L} \supset -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{4} F'^{\mu\nu} F'_{\mu\nu} - \frac{\epsilon}{2} F^{\mu\nu} F'_{\mu\nu}$$

Scalars

- Standard ϕ complex $SU(2)_L$ doublet and new χ complex singlet:

$$\mathcal{L}_{\phi,\chi} = [D_{\mu}^{(\phi)} \phi]^* D^{(\phi)\mu} \phi + [D_{\mu}^{(\chi)} \chi]^* D^{(\chi)\mu} \chi - V(\phi, \chi)$$

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$$V(\phi, \chi) = V_0 - \mu_{\phi}^2 |\phi|^2 - \mu_{\chi}^2 |\chi|^2 + (|\phi|^2, |\chi|^2) \begin{pmatrix} \lambda_{\phi} & \frac{\lambda}{2} \\ \frac{\lambda}{2} & \lambda_{\chi} \end{pmatrix} \begin{pmatrix} |\phi|^2 \\ |\chi|^2 \end{pmatrix}$$

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Fermion-scalar interactions

- In addition to the standard Yukawa terms we assume neutrino Yukawa terms:

$$-\mathcal{L}_{\text{SW}} \supset \frac{1}{2} \overline{\nu_R} \mathbf{Y}_N (\nu_R)^c \chi + \overline{\nu_R} \mathbf{Y}_\nu \varepsilon_{ab} L_{La} \phi_b + \text{h.c.}$$

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- These lead to Majorana and Dirac mass terms after SSB

Anomaly free charge assignment

[Dobrescu et al, hep-ph/0212073]

field	$SU(3)_c$	$SU(2)_L$	y_j	$z_j^{(a)}$	$z_j^{(b)}$	$r_j = z_j/z_\phi - y_j^{(c)}$
U_L, D_L	3	2	$\frac{1}{6}$	Z_1	$\frac{1}{6}$	0
U_R	3	1	$\frac{2}{3}$	Z_2	$\frac{7}{6}$	$\frac{1}{2}$
D_R	3	1	$-\frac{1}{3}$	$2Z_1 - Z_2$	$-\frac{5}{6}$	$-\frac{1}{2}$
ν_L, ℓ_L	1	2	$-\frac{1}{2}$	$-3Z_1$	$-\frac{1}{2}$	0
ν_R	1	1	0	$Z_2 - 4Z_1$	$\frac{1}{2}$	$\frac{1}{2}$
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gauge invariance
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After SSB neutrino mass terms appear

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Dirac and Majorana mass terms appear already at tree level by SSB (not generated radiatively)

Neutrino masses at tree level

the **weak (flavour) eigenstates**: $(\nu_e, \nu_\mu, \nu_\tau, \nu_{R,1}, \nu_{R,2}, \nu_{R,3})$

can be **transformed into the basis of $\nu_i (i = 1-6)$ mass eigenstates**
with a **6×6 unitary matrix \mathbf{U}** :

$$\mathbf{U}^T \mathbf{M}' \mathbf{U} = \mathbf{M} = \text{diag}(m_1, m_2, m_3, m_4, m_5, m_6)$$

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where \mathbf{U}_L and \mathbf{U}_R^* are **semi-unitary**: $\mathbf{U}_L \mathbf{U}_L^\dagger = \mathbf{1}_3$, $\mathbf{U}_R \mathbf{U}_R^\dagger = \mathbf{1}_3$,

but

$$\mathbf{U}_L^\dagger \mathbf{U}_L + \mathbf{U}_R^T \mathbf{U}_R^* = \mathbf{1}_6$$

useful relations collected in the appendix of our paper

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Full diagonalization is cumbersome → can use approximate diagonalization in the see-saw limit

$$\begin{pmatrix} \mathbf{M}_\nu & 0 \\ 0 & \mathbf{M}_N \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{U}_{\text{as}} \\ -\mathbf{U}_{\text{as}}^\dagger & \mathbf{1} \end{pmatrix}^T \begin{pmatrix} 0 & \mathbf{M}_D^T \\ \mathbf{M}_D & \mathbf{M}_R \end{pmatrix} \begin{pmatrix} \mathbf{1} & \mathbf{U}_{\text{as}} \\ -\mathbf{U}_{\text{as}}^\dagger & \mathbf{1} \end{pmatrix}$$

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\mathbf{M}_N is already diagonal, but \mathbf{M}_ν is not yet, can be diagonalized with U_2 unitary matrix

$$\mathbf{U}_2^T \mathbf{M}_\nu \mathbf{U}_2 = \mathbf{M}_\nu^{\text{diag}}$$

Neutrino mass matrix at one-loop order

We have experimental constraints on the upper limits the elements of M_ν^{diag} [Planck coll., arXiv:1807.06209; KATRIN coll, arXiv:1909.06048]

If at tree-level those are satisfied, loop corrections may upset those limits

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We are interested in the one-loop correction δM_L to the tree-level mass matrix of light neutrinos in

$$\delta \mathbf{M}' = \begin{pmatrix} \delta \mathbf{M}_L & \delta \mathbf{M}_D^T \\ \delta \mathbf{M}_D & \delta \mathbf{M}_R \end{pmatrix} = \mathbf{U}^* \delta \mathbf{M} \mathbf{U}^\dagger$$

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in detail: $\delta \mathbf{M}_L = \mathbf{U}_L^* \delta \mathbf{M} \mathbf{U}_L^\dagger$, $\delta \mathbf{M}_D = \mathbf{U}_R \delta \mathbf{M} \mathbf{U}_L^\dagger$, $\delta \mathbf{M}_R = \mathbf{U}_R \delta \mathbf{M} \mathbf{U}_R^T$

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- Calculation is non-trivial, but the **result is simple**: [Iwamoto et al, arXiv:[2104.14571](https://arxiv.org/abs/2104.14571)]

where

$$\delta\mathbf{M}_L = \frac{1}{16\pi^2} \sum_{k=1,2} \left[3(\mathbf{Z}_G)_{k1}^2 \frac{M_{V_k}^2}{v^2} \mathbf{F}(M_{V_k}^2) + (\mathbf{Z}_S)_{k1}^2 \frac{M_{S_k}^2}{v^2} \mathbf{F}(M_{S_k}^2) \right]$$

$$\mathbf{F}_{ij}(M^2) = \sum_{a=1}^6 (\mathbf{U}_L^*)_{ia} (\mathbf{U}_L^\dagger)_{aj} \frac{m_a^3}{M^2} \frac{\ln \frac{m_a^2}{M^2}}{\frac{m_a^2}{M^2} - 1}$$

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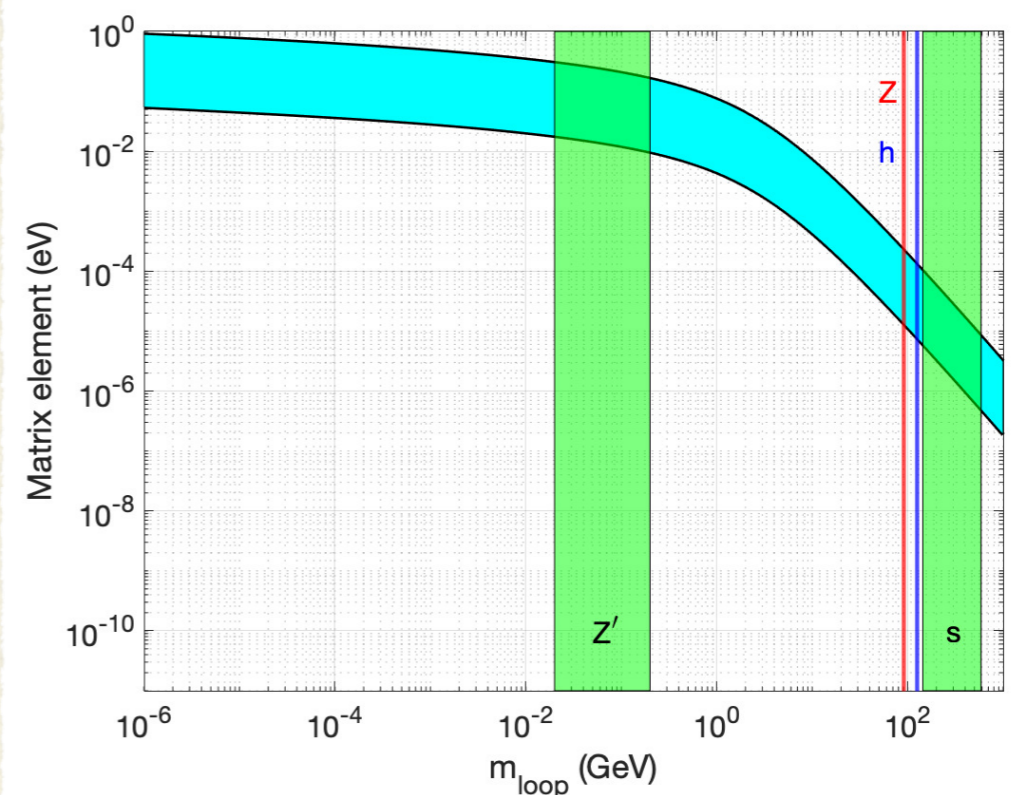
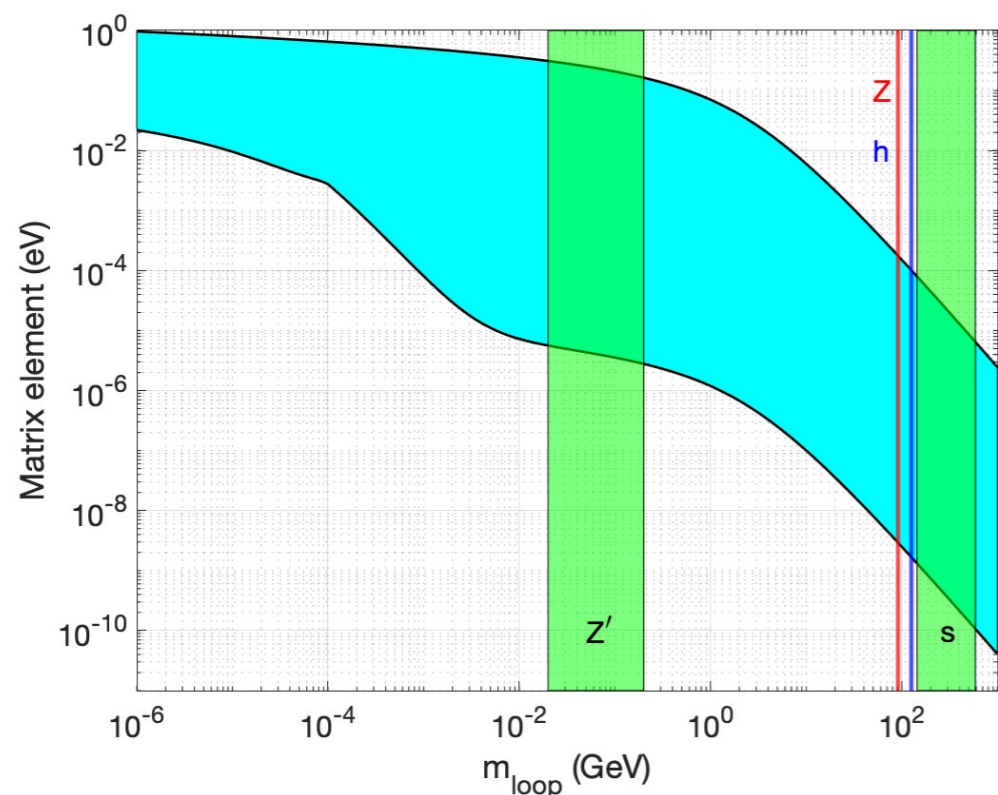
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result is **gauge independent, finite, independent of the renormalization scale**

The F_{ij} matrix



Range of the matrix elements F_{ij} represented by the blue band as a function of the mass m_{loop} of the boson in the loop.

Left: $m_1^{\text{tree}} = 0.01 \text{ eV}$, $m_4^{\text{tree}} = 30 \text{ keV}$, $m_5^{\text{tree}} \approx m_6^{\text{tree}} = 2.5 \text{ GeV}$. Right: $m_1^{\text{tree}} = 0.001 \text{ eV}$, $m_4^{\text{tree}} = 7.1 \text{ keV}$, $m_5^{\text{tree}} \approx m_6^{\text{tree}} = 3.0 \text{ GeV}$.

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e.g., assuming the active neutrino masses to be $O(10^{-3})\text{eV}$:

$$(\delta\mathbf{M}_L)_{ij} < O(10^{-7})\text{eV} + O(10^{-21}) \times \left(\frac{M_{Z'}}{100 \text{ MeV}} \right)^2 \mathbf{F}_{ij}(M_{Z'}^2)$$

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- DM particles are produced by the decay of Z' , so **we consider m_4 in $[10, 50] \text{ MeV}$, hence T_{dec} is $\mathcal{O}(1 \text{ MeV})$**
- **electrons** and **active neutrinos** are **abundant** in the cosmic soup, heavier fermions are negligible.

Evolution of comoving number density

- **Comoving number density** of DM particle a is determined by

$$\frac{d\mathcal{Y}_a}{dz} \propto \sum_{\text{particles}} \left[\begin{aligned} &(\text{rate of creation processes of particle } a) \\ &- (\text{rate of processes annihilating particle } a) \end{aligned} \right]$$

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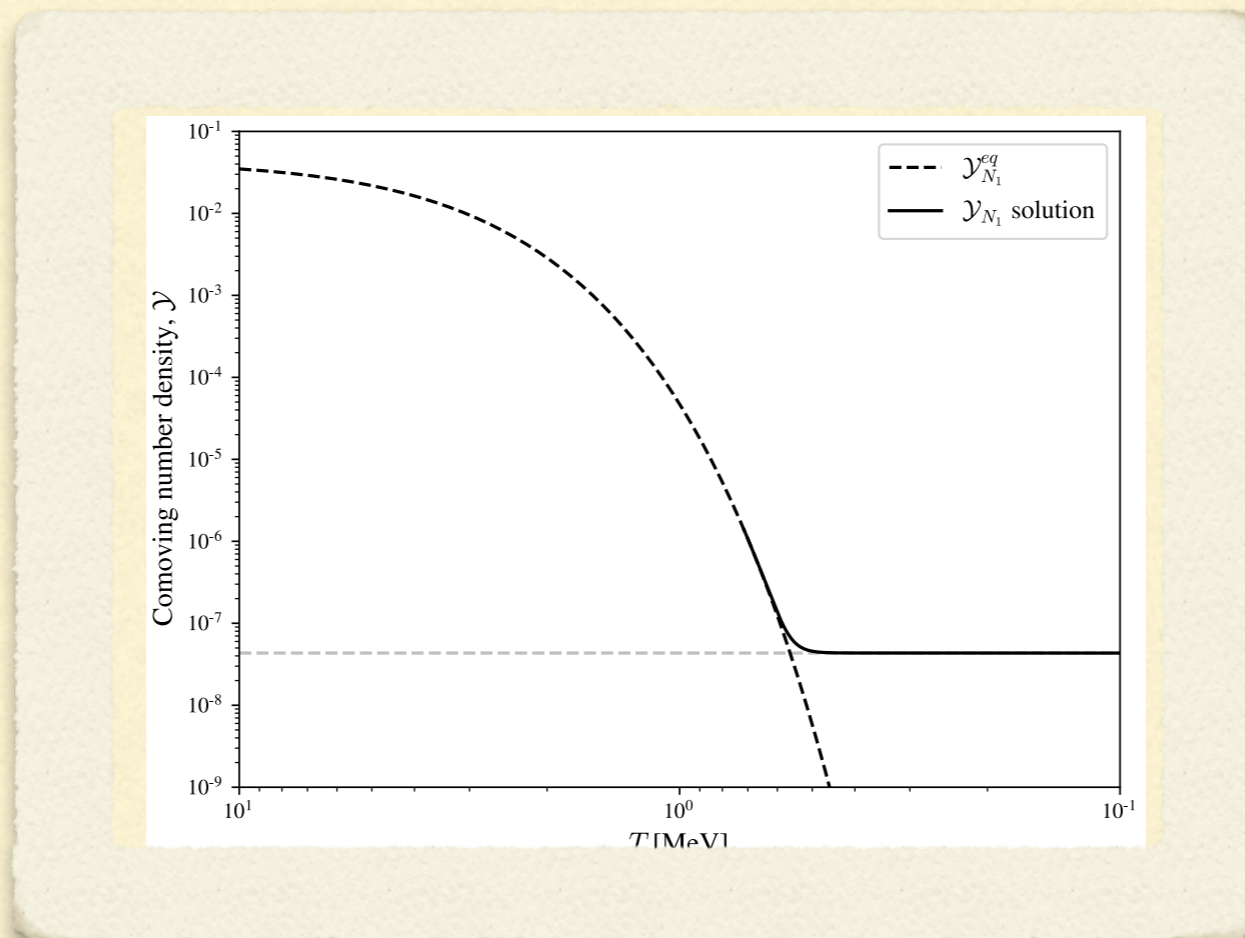
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rate = (cross section or decay rate) \times (available initial particle abundance)

$$\langle \sigma v_{M\phi 1} \rangle \propto \int_{4\mu^2}^{\infty} ds \sigma(s) (s - 4m_{\text{in}}^2) \sqrt{s} K_1 \left(\frac{\sqrt{s}}{T} \right) \quad \langle \Gamma \rangle = \Gamma \frac{K_1(z)}{K_2(z)}$$

K_i Bessel function of the 2nd kind

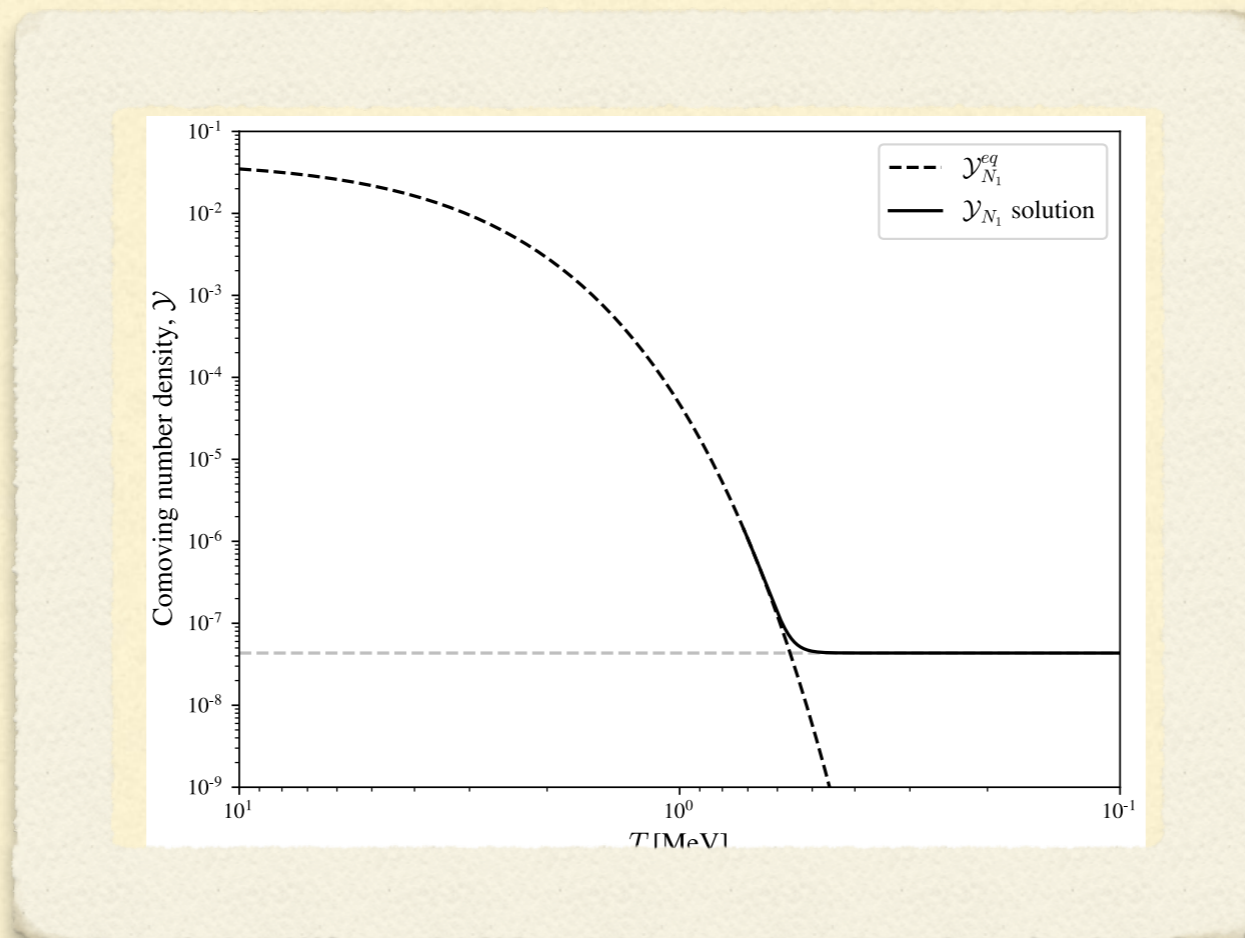
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Example solution to the Boltzmann equation in the freeze-out case. The horizontal line indicates the relic density corresponding to

$$\Omega_{\text{DM}} = 0.265, M_{Z'} = 30 \text{ MeV}, M_1 = 10 \text{ MeV}, g_z = 1.06 \cdot 10^{-3}.$$

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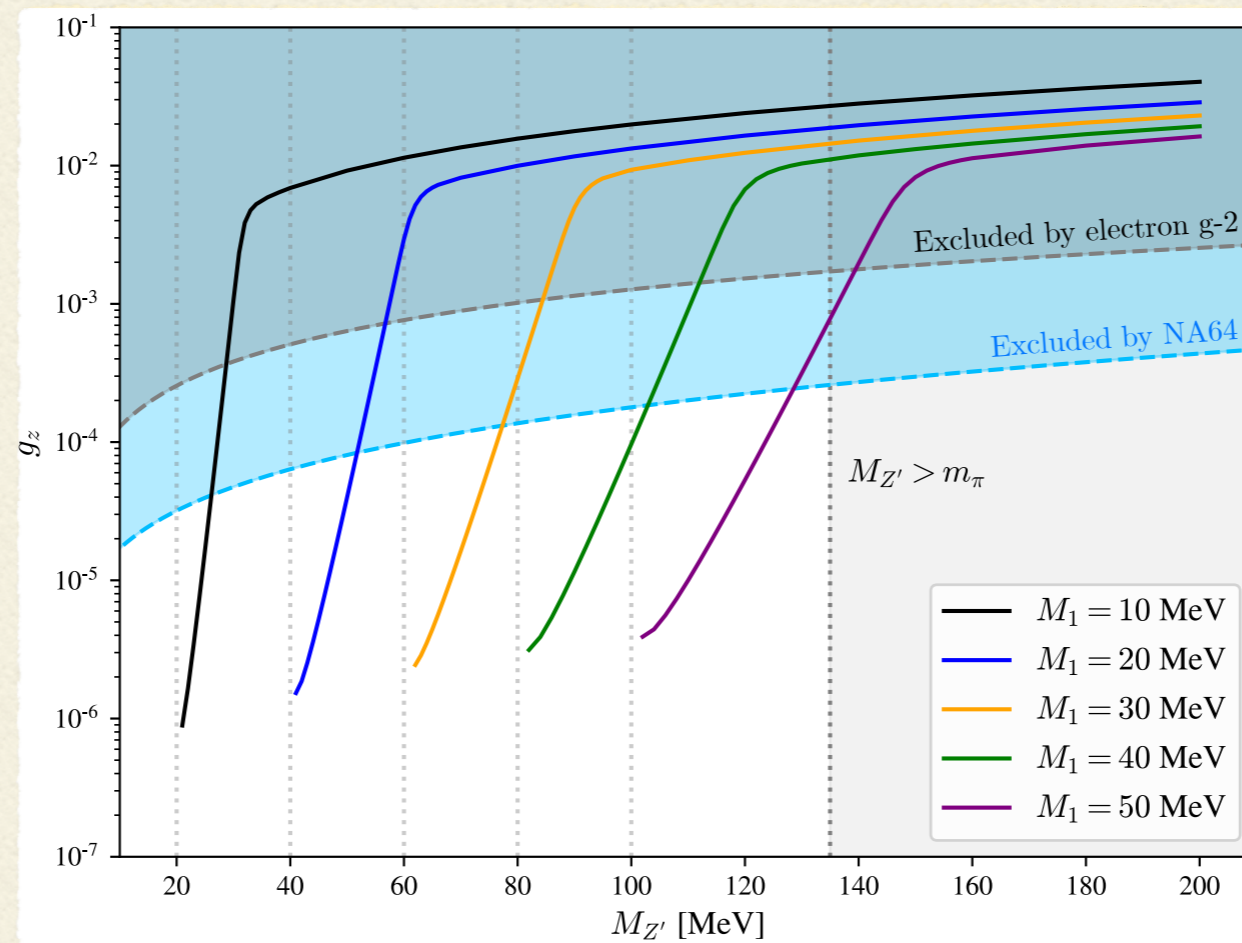
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It is essential for the superweak model DM candidate that the resonance can dominate the integral in the rate

Resonant enhancement



Parameter space for the freeze-out scenario of dark matter production in the supeweak model

Benchmark points

- Using the Casas-Ibarra parametrization the active-sterile mixing matrix $\mathbf{U}_{as} = \mathbf{M}_D^\dagger \mathbf{M}_R^{-1}$ can be written as

$$\mathbf{U}_{as} = \mathbf{U}_{PMNS} \sqrt{\mathbf{M}_\nu^{\text{diag}}} (\mathbf{iR}^\dagger) \mathbf{M}_R^{-1/2}$$

where \mathbf{R} is an orthogonal matrix

[better: see talk by Pereira]

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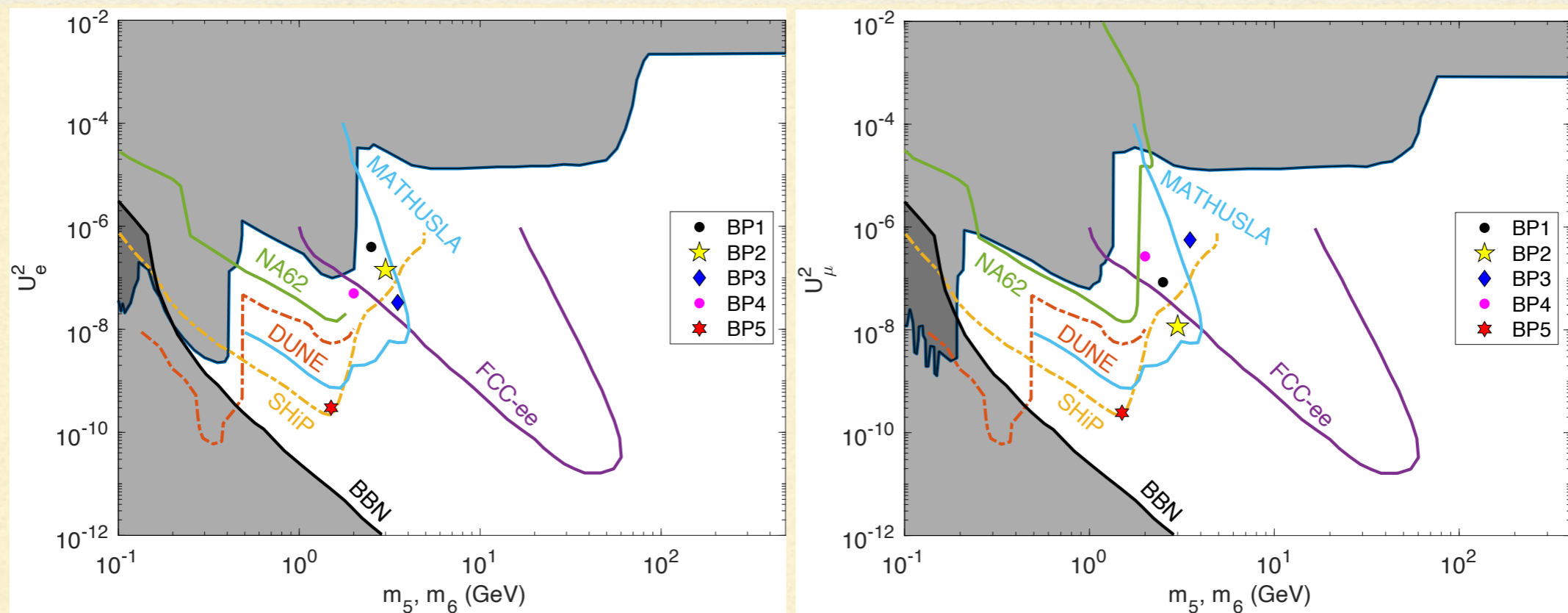
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knowing the PMNS matrix experimentally and assuming values for the masses of the neutrinos, we have to scan over the full parameter space of the \mathbf{R} matrix to find the possible \mathbf{U}_{as} matrix elements.

Benchmark points



Constraints in logarithmic ($U_X^2 = \sum_{i=4}^6 |U_{Xi}|^2, m_j$) plane ($j = 5,6$) from above are given by several experiments (shaded area). Experimental sensitivities of future experiments are given by colored lines. Left plot: $X=e$. Right plot: $X=\mu$

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- **Neutrino masses are generated by SSB at tree level**
- **One-loop corrections to the tree-level neutrino mass matrix computed and found to be small (below 1‰) in the parameter space relevant in the super-weak model**

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- Valid benchmark points are found that will be testable in SHIP and MATHUSLA experiments: motivation for systematic exploration of the parameter space
- Cosmological and particle physics consequences of the scalar sector is to be explored [Péli et al, arXiv:[1911.07082](https://arxiv.org/abs/1911.07082)]

the end

Appendix

Kinetic mixing

- **New fields:** 3 right-handed neutrinos ν_{Rf} , a new scalar χ , and new $U(1)_z$ gauge boson B'

- **kinetic mixing:** $\mathcal{L} \supset -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}F'^{\mu\nu}F'_{\mu\nu} - \frac{\epsilon}{2}F^{\mu\nu}F'_{\mu\nu}$

- **covariant derivative:** $\mathcal{D}_\mu^{U(1)} = -i(yg_y B_\mu + zg_z B'_\mu)$

- **or equivalently can choose basis s. t.:** $D_\mu^{U(1)} = -i \begin{pmatrix} y & z \end{pmatrix} \begin{pmatrix} \hat{g}_{yy} & \hat{g}_{yz} \\ \hat{g}_{zy} & \hat{g}_{zz} \end{pmatrix} \begin{pmatrix} \hat{B}_\mu \\ \hat{B}'_\mu \end{pmatrix}$

and can parametrize the coupling matrix s.t.:

$$\hat{g} = \begin{pmatrix} \hat{g}_{yy} & \hat{g}_{yz} \\ \hat{g}_{zy} & \hat{g}_{zz} \end{pmatrix} = \begin{pmatrix} g_y & -\eta g'_z \\ 0 & g'_z \end{pmatrix} \begin{pmatrix} \cos \epsilon' & \sin \epsilon' \\ -\sin \epsilon' & \cos \epsilon' \end{pmatrix} \text{ with } \begin{aligned} g'_z &= g_z / \sqrt{1 - \epsilon^2} \\ \eta &= \epsilon g_y / g_z. \end{aligned}$$

Mixing in the neutral gauge sector

$$\begin{pmatrix} \hat{B}^\mu \\ W^{3\mu} \\ \hat{B}'^\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_W & -\cos \theta_Z \sin \theta_W & -\sin \theta_Z \sin \theta_W \\ \sin \theta_W & \cos \theta_Z \cos \theta_W & \cos \theta_W \sin \theta_Z \\ 0 & -\sin \theta_Z & \cos \theta_Z \end{pmatrix} \begin{pmatrix} A^\mu \\ Z^\mu \\ Z'^\mu \end{pmatrix}$$

where θ_W is the Weinberg angle & θ_Z is the Z — Z' mixing, implicitly: $\tan(2\theta_Z) = 2\kappa/(1 - \kappa^2 - \tau^2)$, with

$$\kappa = \cos \theta_W (\gamma'_y - 2\gamma'_z)$$

$$\tau = 2 \cos \theta_W \gamma'_z \tan \beta$$

$$\gamma'_y = (\epsilon/\sqrt{1 - \epsilon^2})(g_y/g_L), \quad \gamma'_z = g'_z/g_L$$

$$\tan \beta = w/v$$

$$\left(\sin \theta_Z = \operatorname{sgn}(\kappa) \left[\frac{1}{2} \left(1 - \frac{1 - \kappa^2 - \tau^2}{\sqrt{(1 + \kappa^2 + \tau^2)^2 - 4\tau^2}} \right) \right]^{1/2}, \quad \cos \theta_Z = \left[\frac{1}{2} \left(1 + \frac{1 - \kappa^2 - \tau^2}{\sqrt{(1 + \kappa^2 + \tau^2)^2 - 4\tau^2}} \right) \right]^{1/2} \right)$$

Masses of the neutral gauge bosons

$$M_Z^2 = \left(\frac{M_W}{\cos \theta_W} \right)^2 \left[(\cos \theta_Z - \kappa \sin \theta_Z)^2 + (\tau \sin \theta_Z)^2 \right]$$

$$M_{Z'}^2 = \left(\frac{M_W}{\cos \theta_W} \right)^2 \left[(\sin \theta_Z + \kappa \cos \theta_Z)^2 + (\tau \cos \theta_Z)^2 \right]$$

obeying

$$(Z \rightarrow Z') \Rightarrow (\cos \theta_Z, \sin \theta_Z) \rightarrow (\sin \theta_Z, -\cos \theta_Z)$$

Scalar and Goldstone mixing

$$\begin{pmatrix} h \\ s \end{pmatrix} = \mathbf{Z}_S \begin{pmatrix} h' \\ s' \end{pmatrix} \equiv \begin{pmatrix} \cos \theta_S & -\sin \theta_S \\ \sin \theta_S & \cos \theta_S \end{pmatrix} \begin{pmatrix} h' \\ s' \end{pmatrix} \quad \begin{pmatrix} \sigma_Z \\ \sigma_{Z'} \end{pmatrix} = \mathbf{Z}_G \begin{pmatrix} \sigma_\phi \\ \sigma_\chi \end{pmatrix}$$

- where the scalar mixing angle is related to the potential parameters:

$$\tan(2\theta_S) = -\frac{\lambda v w}{\lambda_\phi v^2 - \lambda_\chi w^2}$$

- and for the Goldstone mixing angle is related to the neutral gauge boson mixing angle:

$$\tan \theta_G = \tan \theta_Z \frac{M_{Z'}}{M_Z}$$

Neutral current couplings

$$\Gamma_{V\bar{f}f}^\mu = -ie\gamma^\mu(C_{V\bar{f}f}^R P_R + C_{V\bar{f}f}^L P_L)$$

for neutrinos

$$eC_{Z\nu\nu}^L = \frac{g_L}{2\cos\theta_W} \left[\cos\theta_Z - (\gamma'_y - \gamma'_z) \sin\theta_Z \cos\theta_W \right], \quad eC_{Z\nu\nu}^R = -\frac{g_L}{2}\gamma'_z \sin\theta_Z,$$
$$eC_{Z'\nu\nu}^L = \frac{g_L}{2\cos\theta_W} \left[\sin\theta_Z + (\gamma'_y - \gamma'_z) \cos\theta_Z \cos\theta_W \right], \quad eC_{Z'\nu\nu}^R = \frac{g_L}{2}\gamma'_z \cos\theta_Z,$$

obeying

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Masses of the neutral gauge bosons again

can also be expressed with chiral couplings:

$$M_Z^2 = \frac{v^2 e^2}{\cos^2 \theta_G} \left(C_{Z\nu\nu}^L - C_{Z\nu\nu}^R \right)^2$$

$$M_{Z'}^2 = \frac{v^2 e^2}{\sin^2 \theta_G} \left(C_{Z'\nu\nu}^L - C_{Z'\nu\nu}^R \right)^2$$

which are crucial for checking gauge independence

Neutral current couplings on mass basis

recall: $\Gamma_{V\bar{f}f}^\mu = -ie\gamma^\mu (C_{V\bar{f}f}^R P_R + C_{V\bar{f}f}^L P_L)$

which reads **on the basis of propagating mass eigenstates as**

$$\Gamma_{V\nu_i\nu_j}^\mu = -ie\gamma^\mu \left(\Gamma_{V\nu\nu}^L P_L + \Gamma_{V\nu\nu}^R P_R \right)_{ij}$$

where

$$\Gamma_{V\nu\nu}^L = C_{V\nu\nu}^L \mathbf{U}_L^\dagger \mathbf{U}_L - C_{V\nu\nu}^R \mathbf{U}_R^T \mathbf{U}_R^*$$

$$\Gamma_{V\nu\nu}^R = -C_{V\nu\nu}^L \mathbf{U}_L^T \mathbf{U}_L^* + C_{V\nu\nu}^R \mathbf{U}_R^\dagger \mathbf{U}_R = -\left(\Gamma_{V\nu\nu}^L \right)^*$$

and also: $\Gamma_{S_k/\sigma_k\nu_i\nu_j} = \left(\Gamma_{S_k/\sigma_k\nu\nu}^L P_L + \Gamma_{S_k/\sigma_k\nu\nu}^R P_R \right)_{ij}$

$$\Gamma_{S_k\nu\nu}^L = -i \left[\left(\mathbf{M} \mathbf{U}_L^\dagger \mathbf{U}_L + \mathbf{U}_L^T \mathbf{U}_L^* \mathbf{M} \right) \frac{(\mathbf{Z}_S)_{k1}}{v} + \mathbf{U}_R^\dagger \mathbf{M}_N \mathbf{U}_R^* \frac{(\mathbf{Z}_S)_{k2}}{w} \right] \Gamma_{S_k/\sigma_k\nu\nu}^R = -\left(\Gamma_{S_k/\sigma_k\nu\nu}^L \right)^*$$

$$\Gamma_{\sigma_k\nu\nu}^L = - \left[\left(\mathbf{M} \mathbf{U}_L^\dagger \mathbf{U}_L + \mathbf{U}_L^T \mathbf{U}_L^* \mathbf{M} \right) \frac{(\mathbf{Z}_G)_{k1}}{v} + \mathbf{U}_R^\dagger \mathbf{M}_N \mathbf{U}_R^* \frac{(\mathbf{Z}_G)_{k2}}{w} \right]$$

Neutrino mass matrix at one-loop order

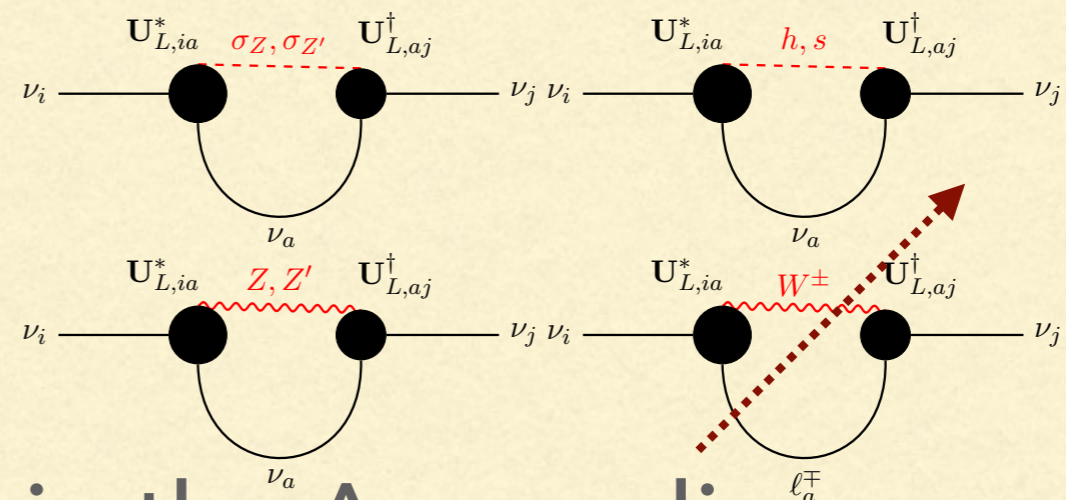
calculation is simple conceptually
self energy can be decomposed as

$$i\Sigma(p) = \mathbf{A}_L(p^2)\not{p}P_L + \mathbf{A}_R(p^2)\not{p}P_R + \mathbf{B}_L(p^2)P_L + \mathbf{B}_R(p^2)P_R$$

and

$$\delta\mathbf{M}_L = \mathbf{U}_L^* \mathbf{B}_L(0) \mathbf{U}_L^\dagger$$

takes contributions from



with Feynman rules given in the Appendix

Neutrino mass matrix at one-loop order

calculation involves "miracles" technically

neutral vectors – with notation $\mathbf{m}_\ell^{(n)} = \text{diag} \left(\frac{m_1^n}{\ell^2 - m_1^2}, \dots, \frac{m_6^n}{\ell^2 - m_6^2} \right) :$

$$\delta \mathbf{M}_L^V = ie^2 \left(C_{V\nu\nu}^L - C_{V\nu\nu}^R \right)^2 \int \frac{d^d \ell}{(2\pi)^d} \mathbf{U}_L^* \left[\frac{d \mathbf{m}_\ell^{(1)}}{\ell^2 - M_V^2} + \frac{\mathbf{m}_\ell^{(3)}}{M_V^2} \left(\frac{1}{\ell^2 - \xi_V M_V^2} - \frac{1}{\ell^2 - M_V^2} \right) \right] \mathbf{U}_L^\dagger$$

scalars:

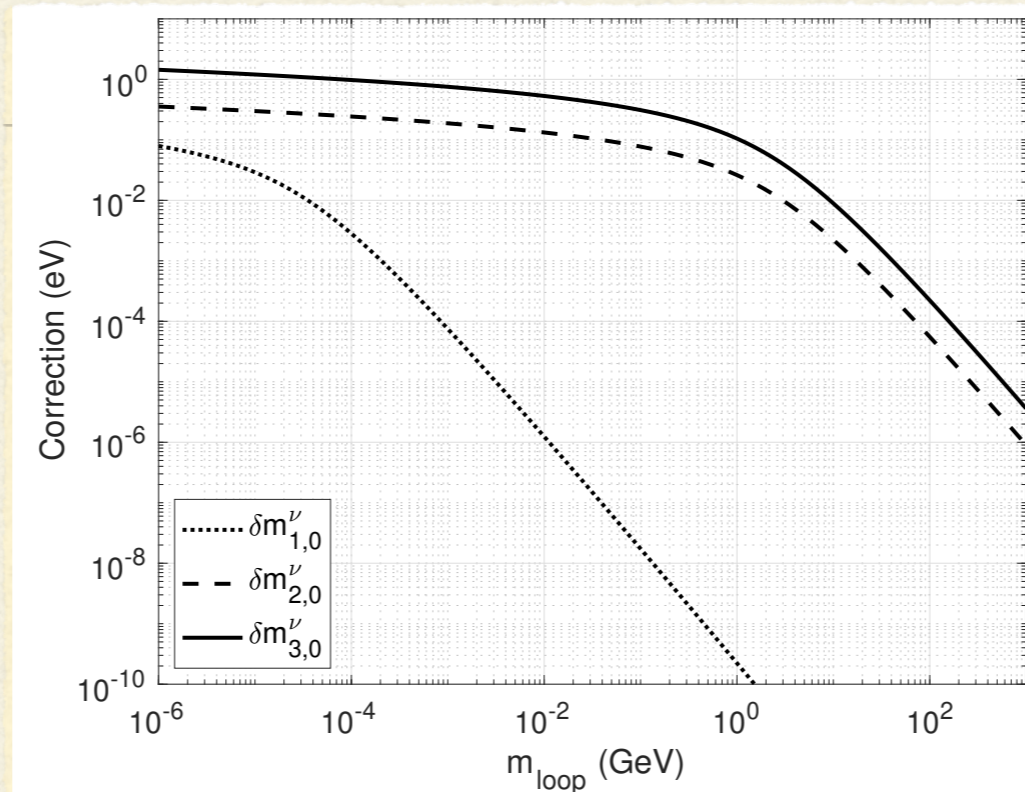
$$\delta \mathbf{M}_L^{S_k} = i \int \frac{d^d \ell}{(2\pi)^d} \mathbf{U}_L^* \mathbf{M} \mathbf{m}_\ell^{(1)} \mathbf{M} \mathbf{U}_L^\dagger \left(\frac{(\mathbf{Z}_S)_{k1}}{v} \right)^2 \frac{1}{\ell^2 - M_{S_k}^2}$$

Goldstones:

$$\delta \mathbf{M}_L^{\sigma_V} = -ie^2 \left(C_{V\nu\nu}^L - C_{V\nu\nu}^R \right)^2 \int \frac{d^d \ell}{(2\pi)^d} \mathbf{U}_L^* \frac{\mathbf{m}_\ell^{(3)}}{M_V^2} \mathbf{U}_L^\dagger \frac{1}{\ell^2 - \xi_V M_V^2}$$

gauge terms cancel

Numerical estimates

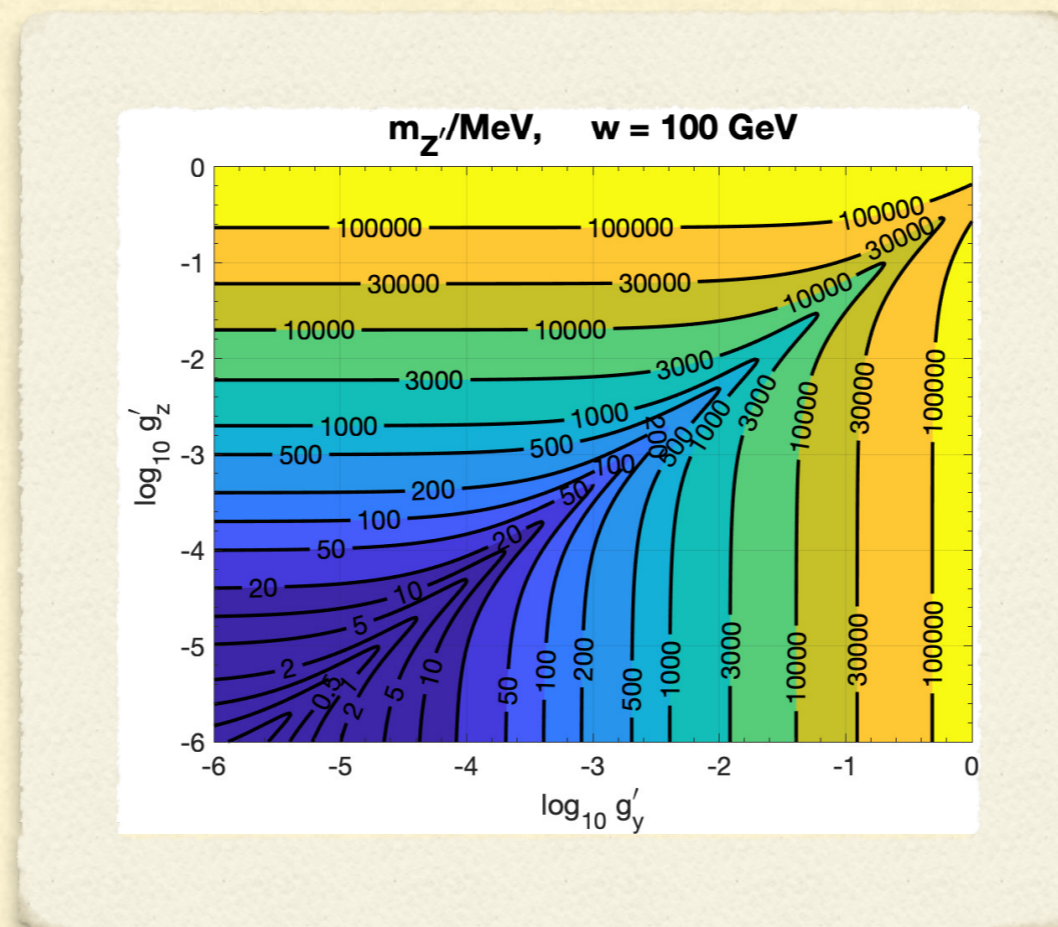


Eigenvalues of the matrix F as a function of the mass of the boson in the loop m_{loop} , assuming $m_1^{\text{tree}} = 0.01$ eV, $m_4^{\text{tree}} = 30$ keV, $m_5^{\text{tree}} \approx m_6^{\text{tree}} = 2.5$ GeV, and normal neutrino mass hierarchy

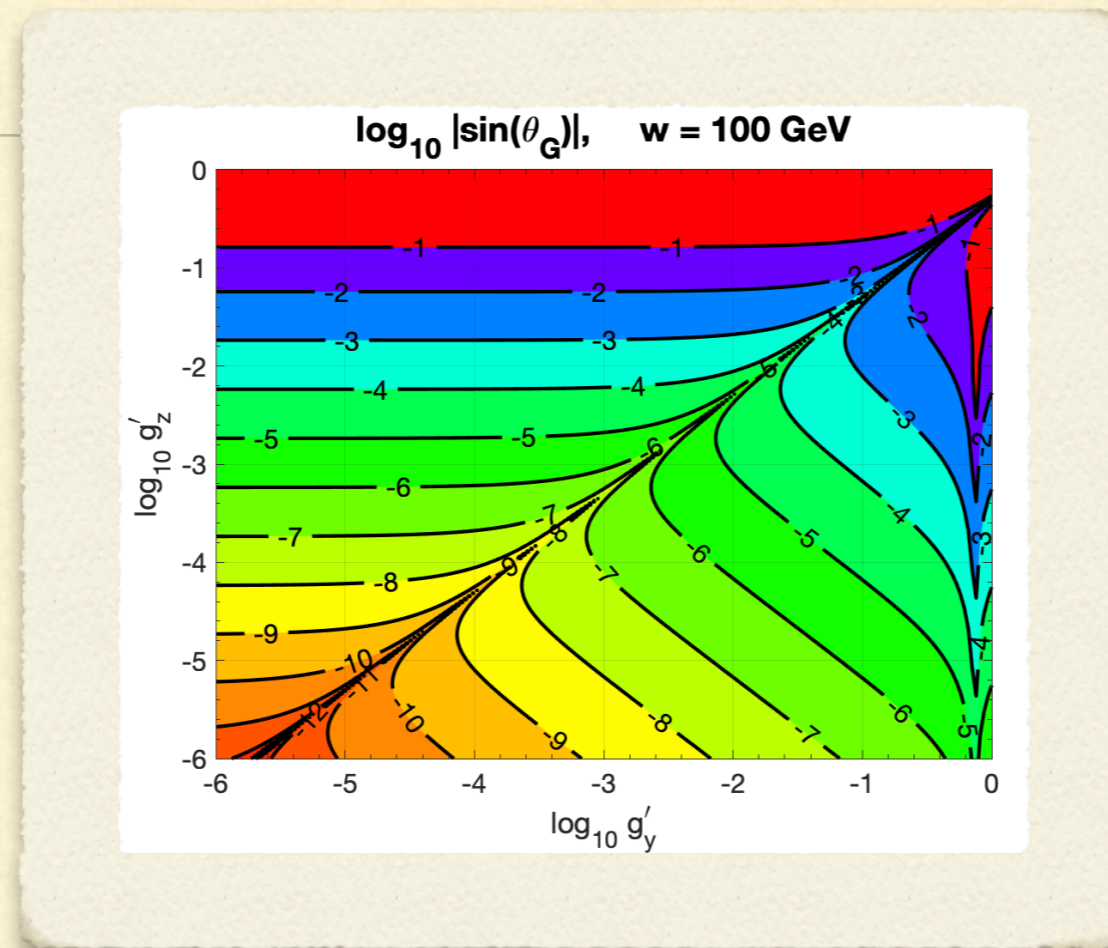
eigenvalues can be large, but coupling suppression tames the relative correction to the tree-level mass below percent level

Numerical estimates for the mass of Z' boson in logarithmic (g_y', g_z') plane

- assume large mixing in the scalar sector $\sin \theta_S = O(0.1)$
- Z' mass and Goldstone mixing are fixed by the gauge couplings $g_y' = \gamma_y' g_L$ and g_z' and ratio of VEVs, $\tan \beta \equiv w/v$
- $M_{Z'} \in [20, 200]$ MeV, relevant mass region for the super-weak model to reproduce the dark matter relic density [Seller et al: arXiv:2104.11248]



Numerical estimates



$|\sin \theta_G|$ in logarithmic (g'_y, g'_z) plane

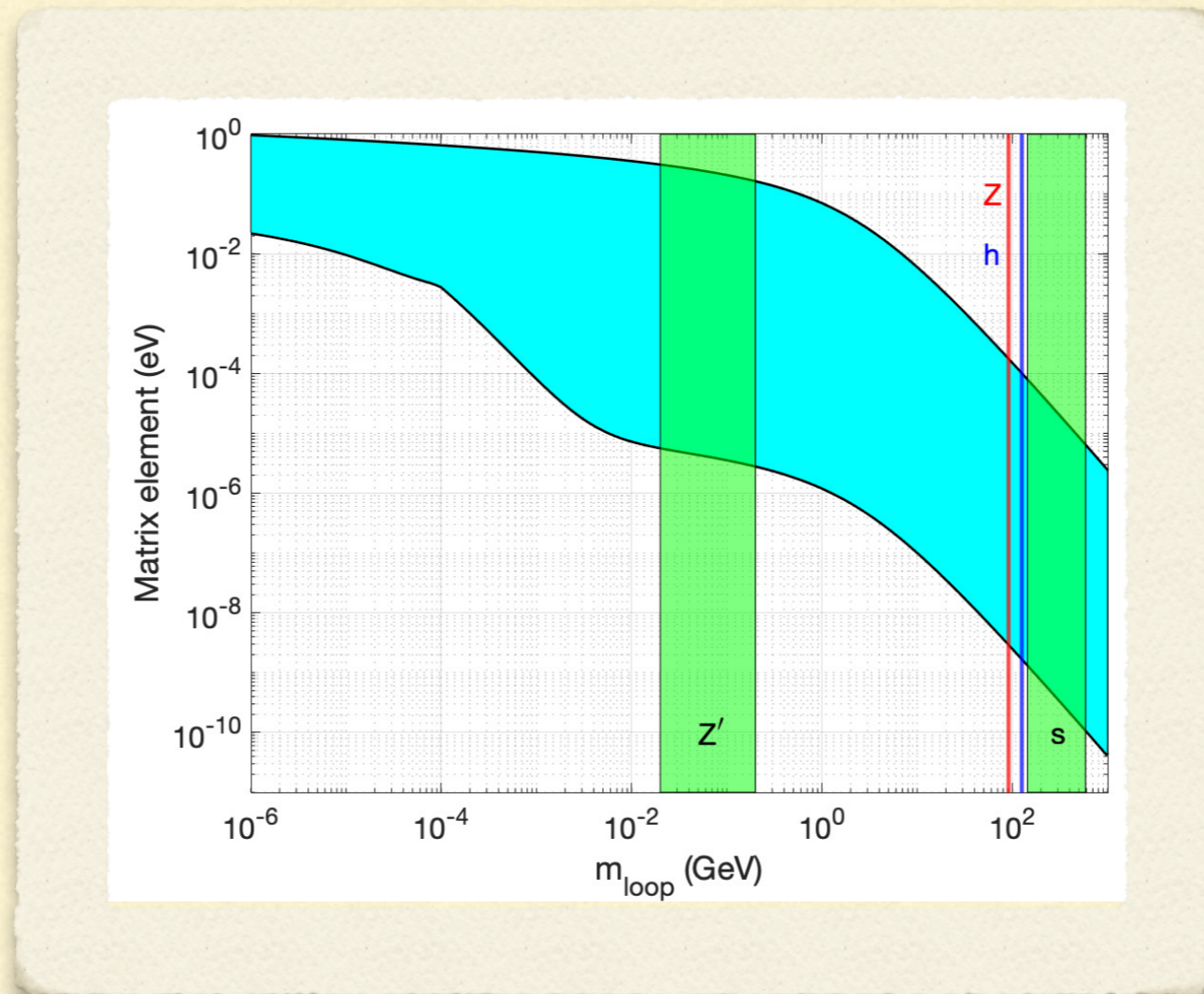
- hence a conservative upper limit: $|\sin \theta_G| < 10^{-6}$

- then
$$e^2(C_{Z\nu\nu}^L - C_{Z\nu\nu}^R)^2 = \cos^2 \theta_G \frac{M_Z^2}{v^2} \sim O(10^{-1})$$

$$e^2(C_{Z'\nu\nu}^L - C_{Z'\nu\nu}^R)^2 = \sin^2 \theta_G \frac{M_{Z'}^2}{v^2} \sim O(10^{-19}) \times \left(\frac{M_{Z'}}{100 \text{ MeV}} \right)^2$$

Numerical estimates

$$(\delta\mathbf{M}_L)_{ij} < O(10^{-7}) \text{ eV} + O(10^{-21}) \times \left(\frac{M_{Z'}}{100 \text{ MeV}} \right)^2 \mathbf{F}_{ij}(M_{Z'}^2)$$



Matrix elements F_{ij} as a function of the mass m_{loop} of the boson in the loop are confined to the blue band, assuming normal neutrino mass hierarchy, with vertical bands showing the relevant mass regions where the masses of the bosons in the loop lie. $144 < m_s/\text{GeV} < 558$, requiring stability of the vacuum. $m_1^{\text{tree}} = 0.01 \text{ eV}$, $m_4^{\text{tree}} = 30 \text{ keV}$, $m_5^{\text{tree}} \approx m_6^{\text{tree}} = 2.5 \text{ GeV}$