Gauge Field Theory Vacuum and Cosmological Inflation without Scalar Field

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Zero-Point Energy Contribution to the Cosmological Constant

$$E_0 = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2}\omega_p \sim \frac{1}{16\pi^2}\Lambda^4$$

The contribution of zero-point energy exceeds by many orders of magnitude the observational cosmological upper bound on the energy density of the universe. However the recent covariant calculation of all components of the energy-momentum tensor performed by Donoghue demonstrated that the contribution of the vacuum zero-point fluctuations has the form [27]

$$T_{\mu\nu} \propto diag(1, \frac{1}{3}, \frac{1}{3}, \frac{1}{3})$$

and is therefore traceless: $T_{\mu\mu} = 0$. It follows that the contribution of the vacuum zero-point energy of quantised fields to the cosmological constant is equal to zero: $\Lambda_{cc} \propto T_{\mu\mu} = 0$.

J. F. Donoghue, *The cosmological constant and the use of cutoffs*, [arXiv:2009.00728 [hep-th]].

Contribution of Vacuum Fluctuations to the Cosmological Constant

The calculation of the effective Lagrangian in QED by Heisenberg and Euler was the first example of a well-defined physically motivated prescription allowing to obtain a finite, gauge and renormalisation group-invariant result when investigating the vacuum fluctuations of quantised fields [29]. It appears that only the difference between vacuum energy in the presence and in the absence of external sources has a well-defined physical meaning [29, 30, 31, 32, 33, 34, 35, 36, 1, 2, 3, 4, 5]. Here we will follow this prescription and will derive the quantum equation of state for non-Abelian gauge fields by using the effective Lagrangian approach [37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54,

Heisenberg-Euler, 1936; Schwinger 1951;Coleman-Weinberg 1973; Vanyashin-Terentev 1965; Skalozub:1975; Brown-Duff,1975; Duff — Ramon-Medrano,1975; Nielsen and Olesen 1978; Skalozub 1978; Nielsen 1978; Ambjorn-Nielsen-Olesen1979; Nielsen and Ninomiya, 1979; Nielsen and Olesen 1979; Nielsen-Ninomiya 1980; Nielsen-Olesen 1979; Ambjorn-Olesen 1980; Ambjorn-Olesen 1980; Skalozub1980; Leutwyler 1980; Leutwyler 1981; Duff 1977 ; G.S 1976, 1977, 2018

Heisenberg-Euler Effective Lagrangian

$$\mathcal{L}_{eff} = \frac{\mathcal{E}^2 - \mathcal{H}^2}{2} - \pi mc^2 (\frac{mc}{h})^3 \int_0^\infty \frac{ds}{s^3} e^{-s} \{ \frac{as\cos(as)}{\sin(as)} \; \frac{bs\cosh(bs)}{\sinh(bs)} - 1 + \frac{a^2 - b^2}{3} s^2 \}$$

where dimensionless fields are

$$a = \frac{e\hbar\mathcal{E}}{m^2c^3}, \qquad b = \frac{e\hbar\mathcal{H}}{m^2c^3}$$

$$mc^2 = 8.2 \cdot 10^{-7} \; \frac{g \; cm^2}{s^2} \qquad \lambda_c = \frac{\hbar}{mc} = 3.86 \cdot 10^{-11} cm \qquad \frac{mc^2}{(\frac{\hbar}{mc})^3} = 1.43 \cdot 10^{25} \frac{g}{cm \; s^2}$$

$$\mathcal{E}_c = \frac{m^2c^3}{e\hbar} \sim 10^{16} \; Volt/cm \qquad \mathcal{H}_c = \frac{m^2c^3}{e\hbar} \sim 4.4 \cdot 10^{13} \; Gauss$$

Heisenberg-Euler Effective Lagrangian

Limit of massless Fermions GS Ann.Phys.2018

$$\mathcal{L}_e = -\mathcal{F} + \frac{e^2 \mathcal{F}}{24\pi^2}\Big[\ln(\frac{2e^2\mathcal{F}}{\mu^4}) - 1\Big], \qquad \mathcal{F} = \frac{\vec{\mathcal{H}}^2 - \vec{\mathcal{E}}^2}{2}, \quad \mathcal{G} = \vec{\mathcal{E}}\vec{\mathcal{H}} = 0,$$

the energy momentum tensor by using the formula derived by Schwinger in [5]:

$$T_{\mu\nu} = (F_{\mu\lambda}F_{\nu\lambda} - g_{\mu\nu}\frac{1}{4}F^2_{\lambda\rho})\frac{\partial\mathcal{L}}{\partial\mathcal{F}} - g_{\mu\nu}(\mathcal{L} - \mathcal{F}\frac{\partial\mathcal{L}}{\partial\mathcal{F}} - \mathcal{G}\frac{\partial\mathcal{L}}{\partial\mathcal{G}}).$$

In massless QED using the one-loop expression (1.2) for $T_{\mu\nu}$ one can get

$$T_{\mu\nu} = T^M_{\mu\nu}\Big[1 - \frac{e^2}{24\pi^2}\ln\frac{2e^2\mathcal{F}}{\mu^4}\Big] + g_{\mu\nu}\frac{e^2}{24\pi^2}\mathcal{F}, \qquad \mathcal{G} = 0.$$

Effective Lagrangian in Yang-Mills theory

The YM effective Lagrangian take the following form

$$\mathcal{L}^{(1)} = -\frac{1}{8\pi^2}\int \frac{ds}{s^3} e^{-i\mu^2 s} \frac{(gF_1 s)\ (gF_2 s)}{\sinh(gF_1 s)\ \sinh(gF_2 s)} - \\ -\frac{1}{4\pi^2}\int \frac{ds}{s^3} e^{-i\mu^2 s} (gF_1 s)\ (gF_2 s)[\frac{\sinh(gF_1 s)}{\sinh(gF_2 s)} + \frac{\sinh(gF_2 s)}{\sinh(gF_1 s)}]$$

$$F_1^2 = -\mathcal{F} - (\mathcal{F}^2 + \mathcal{G}^2)^{1/2}, \qquad F_2^2 = -\mathcal{F} + (\mathcal{F}^2 + \mathcal{G}^2)^{1/2}$$

Vanyashin and Terentev 1965
Duff and Ramon-Medrano 1975
Skalozub 1976

Bartalin, Matinyan and Savvidy 1976
Savvidy 1977
Matinyan and Savvidy 1978

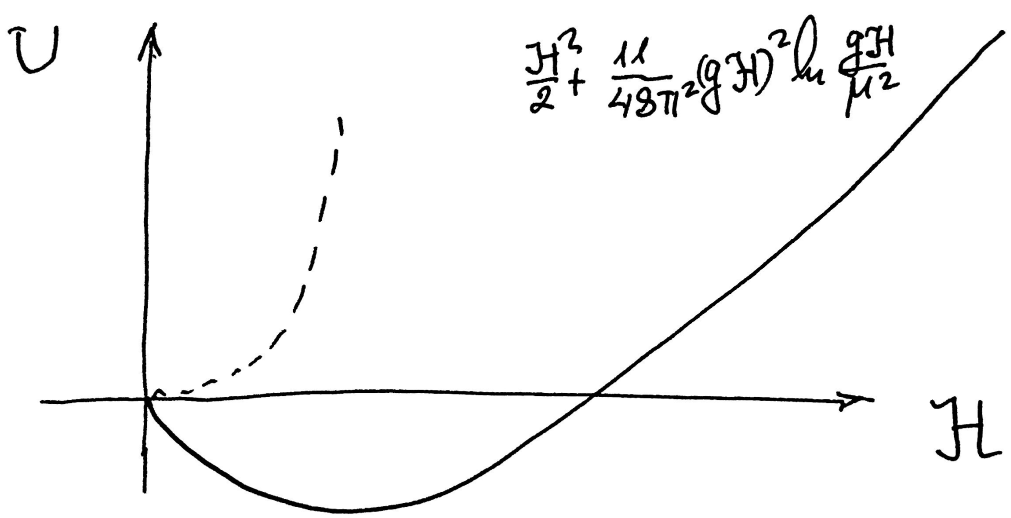
N.Nielsen and Olesen 1978
Ambjorn, N.Nielsen and Olesen 1979
H.Nielsen and Ninomia 1979
H.Nielsen and Olesen 1979
Ambjorn and Olesen1980

Dimensional Transmutation and Condensation

G.S. 1977

$$\mathcal{L}_g = -\mathcal{F} - \frac{11N}{96\pi^2} g^2 \mathcal{F} \Big(\ln \frac{2g^2\mathcal{F}}{\mu^4} - 1 \Big) \,, \qquad \mathcal{F} = \frac{\vec{\mathcal{H}}_a^2 - \vec{\mathcal{E}}_a^2}{2} > 0, \quad \mathcal{G} = \vec{\mathcal{E}}_a \vec{\mathcal{H}}_a = 0 \,.$$

$$\mathcal{L}_q = -\mathcal{F} + \frac{N_f}{48\pi^2} g^2 \mathcal{F} \Big[\ln(\frac{2g^2\mathcal{F}}{\mu^4}) - 1 \Big]$$



$$2g^2\mathcal{F}_{vac} = \mu^4 \exp\left(-\frac{96\pi^2}{b\ g^2(\mu)}\right) = \Lambda_{YM}^4,$$

where $b = 11N - 2N_f$.

$$T_{\mu\nu} = T_{\mu\nu}^{YM} \Big[1 + \frac{b\ g^2}{96\pi^2} \ln \frac{2g^2\mathcal{F}}{\mu^4} \Big] - g_{\mu\nu} \frac{b\ g^2}{96\pi^2} \mathcal{F}, \qquad \mathcal{G} = 0.$$

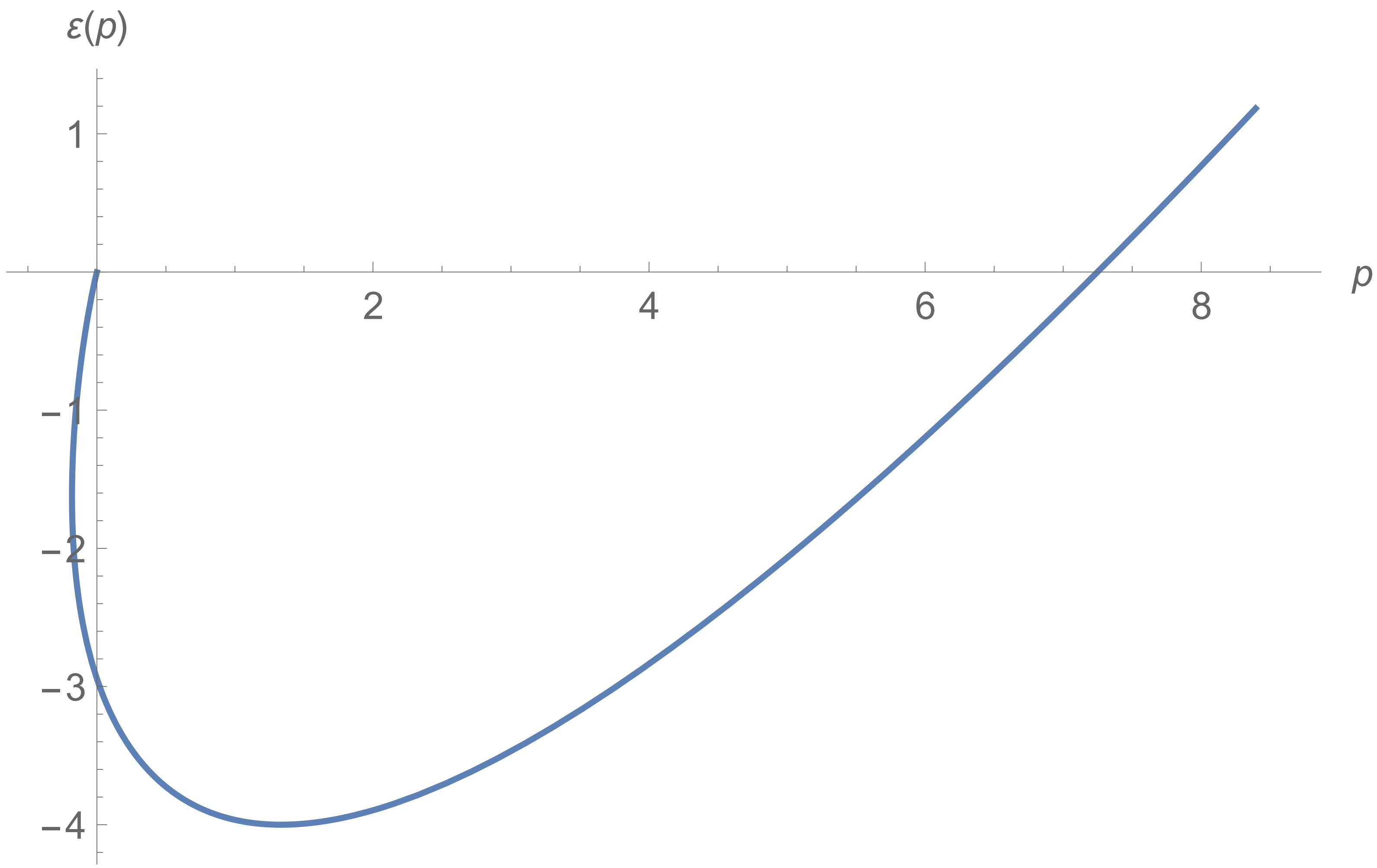
Quantum Energy Momentum Tensor

$$T_{\mu\nu} = T_{\mu\nu}^{YM}\Big[1 + \frac{b\ g^2}{96\pi^2}\ln\frac{2g^2\mathcal{F}}{\mu^4}\Big] - g_{\mu\nu}\frac{b\ g^2}{96\pi^2}\mathcal{F}, \qquad \mathcal{G} = 0,$$

$$T_{00} \equiv \epsilon(\mathcal{F}) = \ \mathcal{F} + \frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} - 1\Big) \qquad T_{ij} = \delta_{ij}\ \Big[\frac{1}{3}\mathcal{F} + \frac{1}{3}\frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} + 3\Big)\Big] = \delta_{ij}\ p(\mathcal{F}).$$

$$\epsilon(\mathcal{F}) = \ \mathcal{F} + \frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} - 1\Big), \qquad p(\mathcal{F}) = \frac{1}{3}\mathcal{F} + \frac{1}{3}\frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} + 3\Big).$$

Yang-Mills Quantum Equation of State



$$\epsilon(\mathcal{F}) = \mathcal{F} + \frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} - 1\Big), \qquad p(\mathcal{F}) = \frac{1}{3}\mathcal{F} + \frac{1}{3}\frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} + 3\Big).$$

Yang-Mills Quantum Equation of State

$$p = \frac{1}{3}\epsilon + \frac{4}{3}\frac{b\ g^2\mathcal{F}}{96\pi^2}\Lambda^4_{YM} \qquad \text{and} \qquad w = \frac{p}{\epsilon} = \frac{\ln\frac{2g^2\mathcal{F}}{\Lambda^4_{YM}} + 3}{3\Big(\ln\frac{2g^2\mathcal{F}}{\Lambda^4_{YM}} - 1\Big)}$$

general parametrisation of the equation of state $p = w\epsilon$

Hilbert -Einstein Action

$$S = -\frac{c^3}{16\pi G}\int R\sqrt{-g}d^4x + \int (\mathcal{L}_q + \mathcal{L}_g)\,\sqrt{-g}d^4x.$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}\Big[T^{YM}_{\mu\nu}\Big(1 + \frac{b\ g^2}{96\pi^2}\ln\frac{2g^2\mathcal{F}}{\mu^4}\Big) - g_{\mu\nu}\frac{b\ g^2}{96\pi^2}\mathcal{F}\Big].$$

$$\Lambda_{eff} = \frac{8\pi G}{3c^4}\ \epsilon_{vac} = -\frac{8\pi G}{3c^4}\frac{b}{192\pi^2}2g^2\mathcal{F}_{vac} = -\frac{8\pi G}{3c^4}\frac{b}{192\pi^2}\Lambda^4_{YM}\ .$$

Friedmann Evolution Equations

$$\dot{\epsilon} + 3\frac{\dot{a}}{a}(\epsilon + p) = 0, \longrightarrow \epsilon + p = \frac{4\mathcal{A}}{3}\ (2g^2\mathcal{F})\ \log\frac{2g^2\mathcal{F}}{\Lambda_{YM}^4},$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^4}(\epsilon + 3p). \longrightarrow \epsilon + 3p = 2\mathcal{A}\ (2g^2\mathcal{F})\ \Big(\log\frac{2g^2\mathcal{F}}{\Lambda_{YM}^4} + 1\Big).$$

the first equation can be solved for the field strength

$$2g^2\dot{\mathcal{F}} + 4(2g^2\mathcal{F})\frac{\dot{a}}{a} = 0 \qquad\qquad 2g^2\mathcal{F}\ a^4 = const \equiv \Lambda_{YM}^4\ a_0^4,$$

Friedmann Evolution Equations

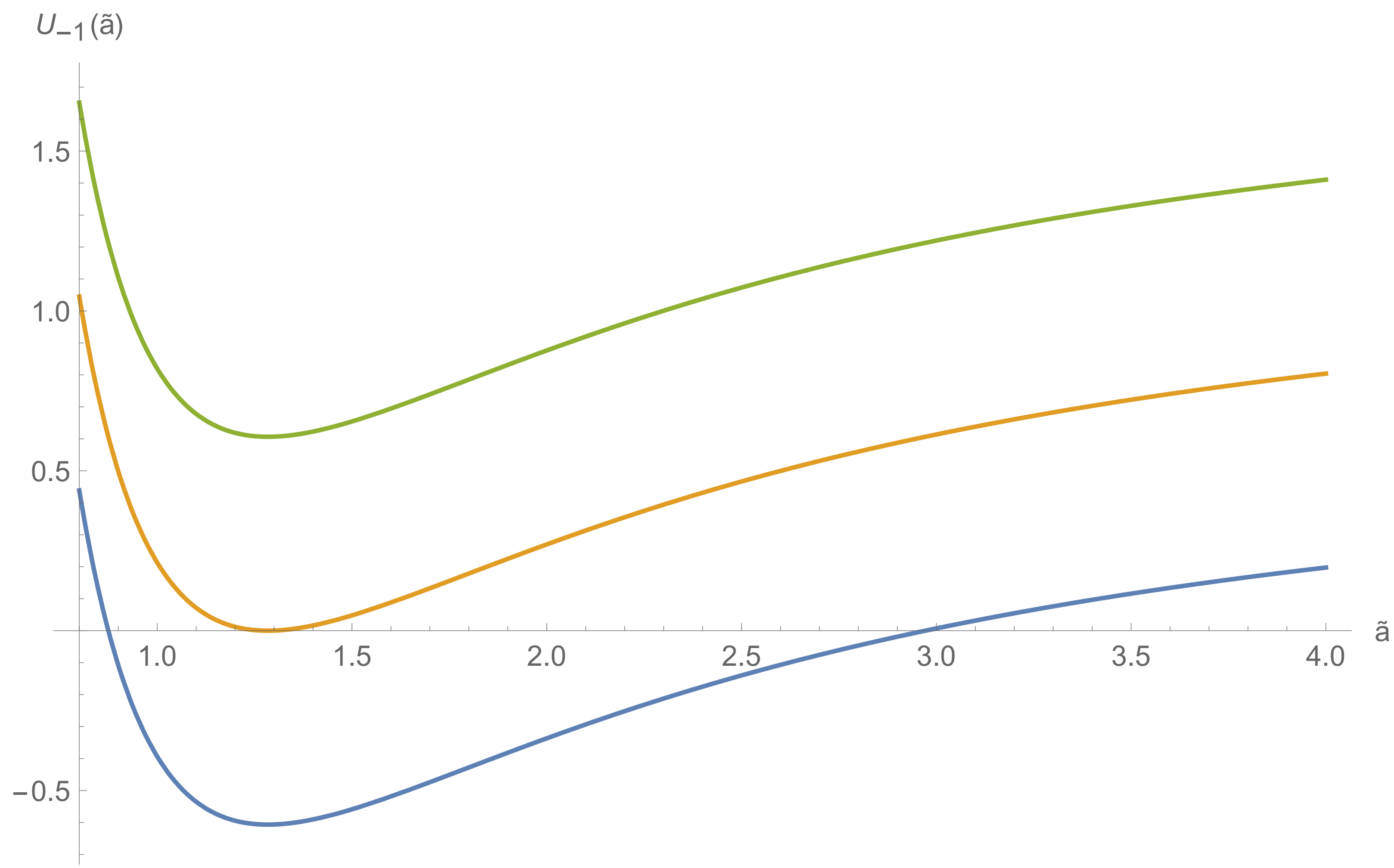
$$a(\tau) = a_0 \ \tilde{a}(\tau), \qquad ct = L \ \tau,$$

$$\frac{d\tilde{a}}{d\tau} = \pm\sqrt{\frac{1}{\tilde{a}^2}\Big(\log\frac{1}{\tilde{a}^4} - 1\Big) - k\gamma^2}, \qquad k = 0, \pm 1, \qquad \gamma^2 = \Big(\frac{L}{a_0}\Big)^2.$$

$$\frac{1}{L^2} = \frac{8\pi G}{3c^4} \ \mathcal{A} \ \Lambda_{YM}^4 \equiv \Lambda_{eff} \ ,$$

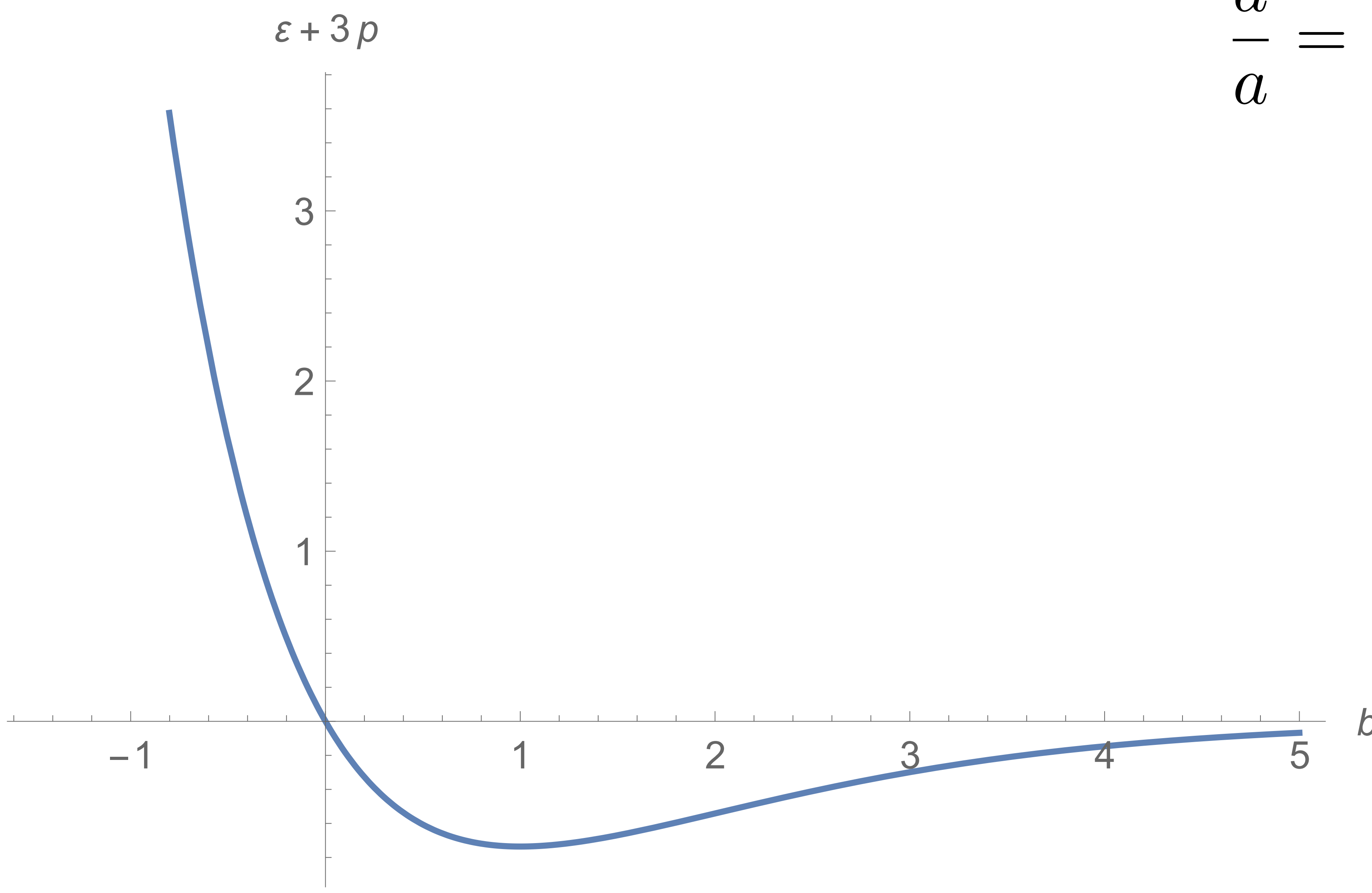
$$\mathcal{A} = \frac{b}{192\pi^2} = \frac{11N - 2N_f}{192\pi^2}.$$

$$U_{-1}(\tilde{a}) \equiv \frac{1}{\tilde{a}^2}\left(\log\frac{1}{\tilde{a}^4} - 1\right) + \gamma^2.$$



Strong Energy Dominance Condition is Violated

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^4}(\epsilon + 3p).$$



$\epsilon + 3p$ of the Friedmann acceleration equation is positive when $b < 0$ and is negative when $b > 0$.

Type II Solution — Initial Acceleration of Finite Duration

$$\frac{d\tilde{a}}{d\tau} = \pm\sqrt{\frac{1}{\tilde{a}^2}\Big(\log\frac{1}{\tilde{a}^4} - 1\Big) - k\gamma^2}, \qquad k = 0, \pm 1, \qquad \gamma^2 = \Big(\frac{L}{a_0}\Big)^2.$$

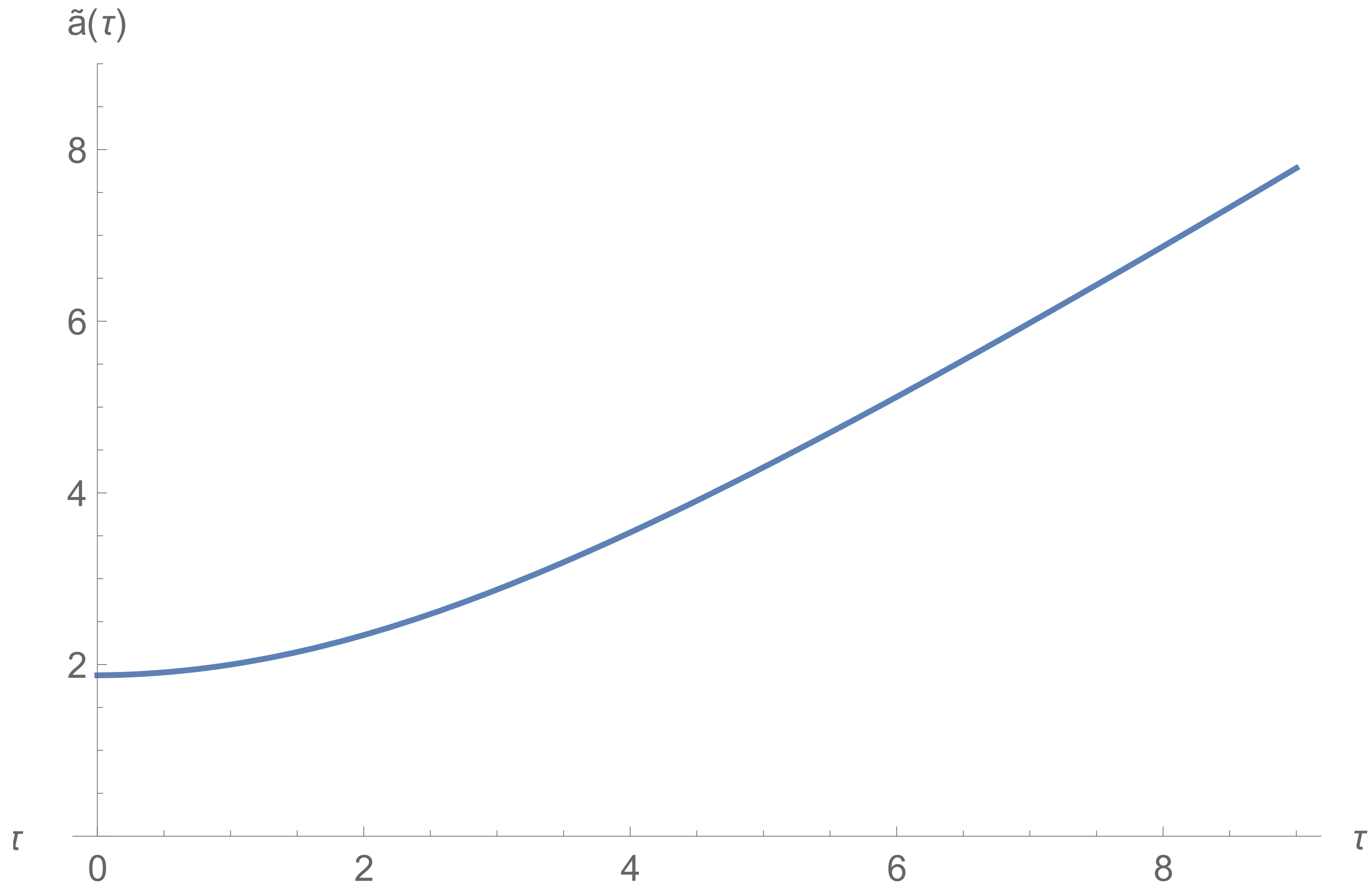
$$\tilde{a}^4 = \mu_2^4 e^{b^2}, \qquad b \in [0, \infty],$$

$$\frac{db}{d\tau} = \frac{2}{\mu_2^2}\; e^{-\frac{b^2}{2}}\Big(\frac{\gamma^2\mu_2^2}{b^2}(e^{\frac{b^2}{2}} - 1) - 1\Big)^{1/2}.$$

$$\mu_2^2 = -\frac{2}{\gamma^2}\; W_-\Big(-\frac{\gamma^2}{2\sqrt{e}}\Big), \qquad 0 \leq \gamma^2 < \frac{2}{\sqrt{e}} \text{ and } \tilde{a} \geq \mu_2.$$

Type II Solution Initial Acceleration of Finite Duration

$$\frac{db}{d\tau} = \frac{2}{\mu_2^2}\, e^{-\frac{b^2}{2}} \Big(\frac{\gamma^2 \mu_2^2}{b^2} (e^{\frac{b^2}{2}} - 1) - 1 \Big)^{1/2}. \qquad \tilde{a}^4 = \mu_2^4 e^{b^2}, \qquad b \in [0, \infty],$$

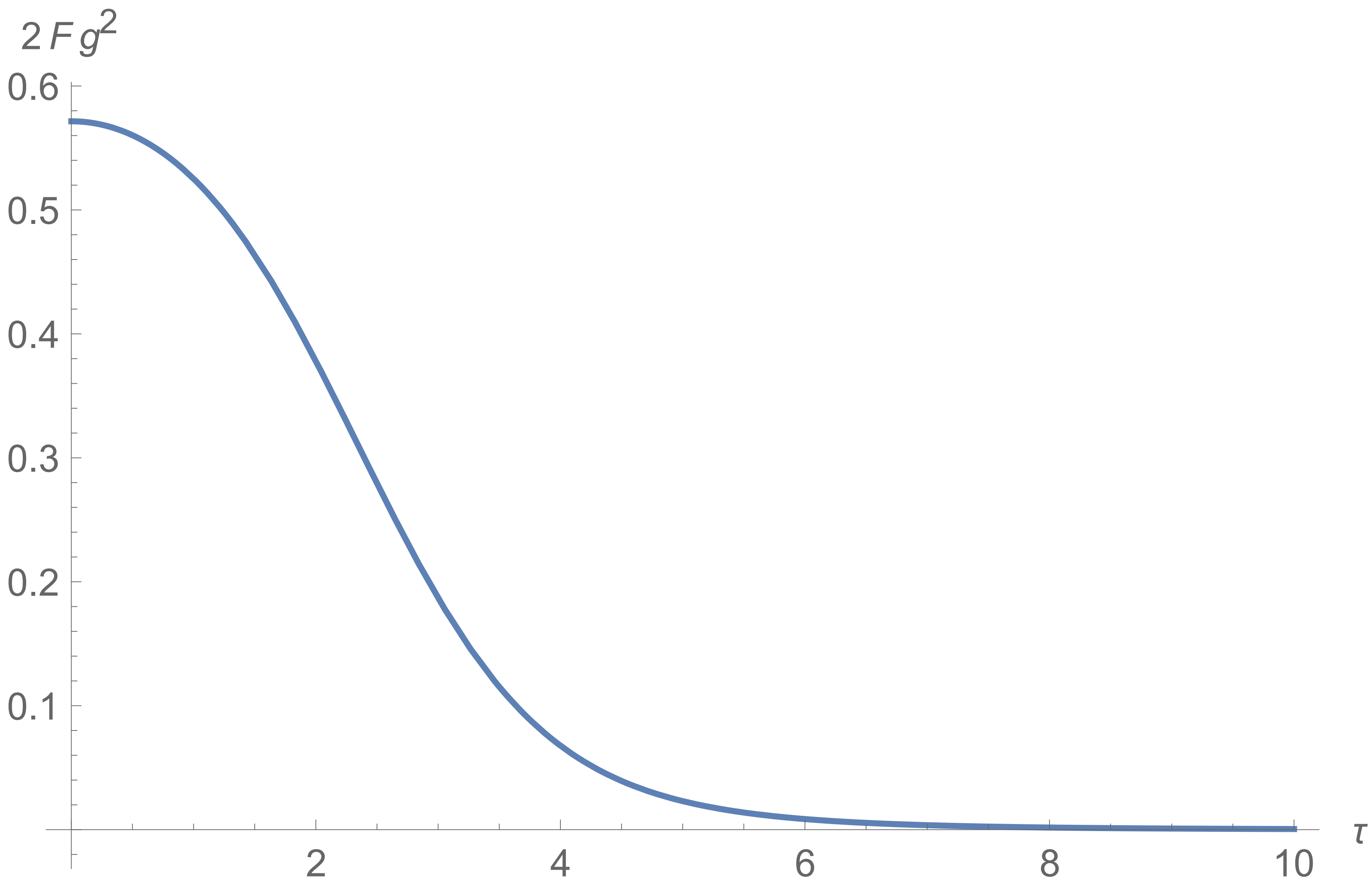


The regime of the exponential growth will continuously transformed into the linear in time growth of the scale factor‡

$$a(t) \simeq \; ct, \qquad a(\eta) \simeq a_0 e^{\eta}. \tag{5.87}$$

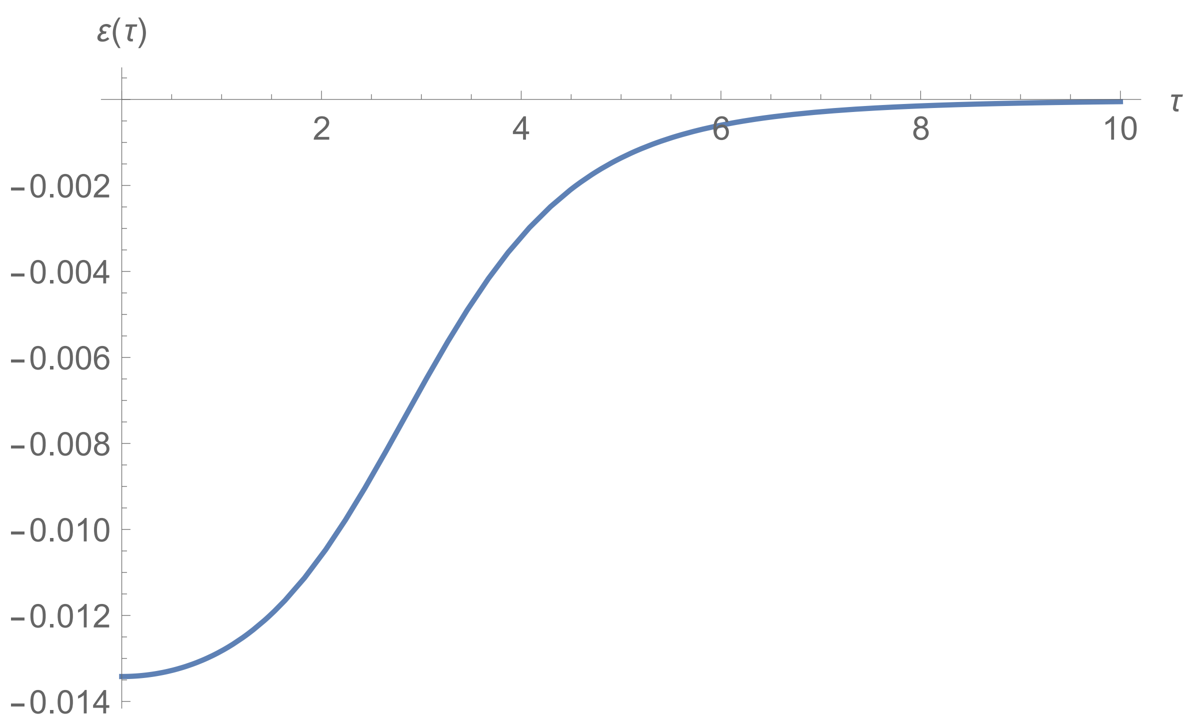
Evolution of the Field Strength

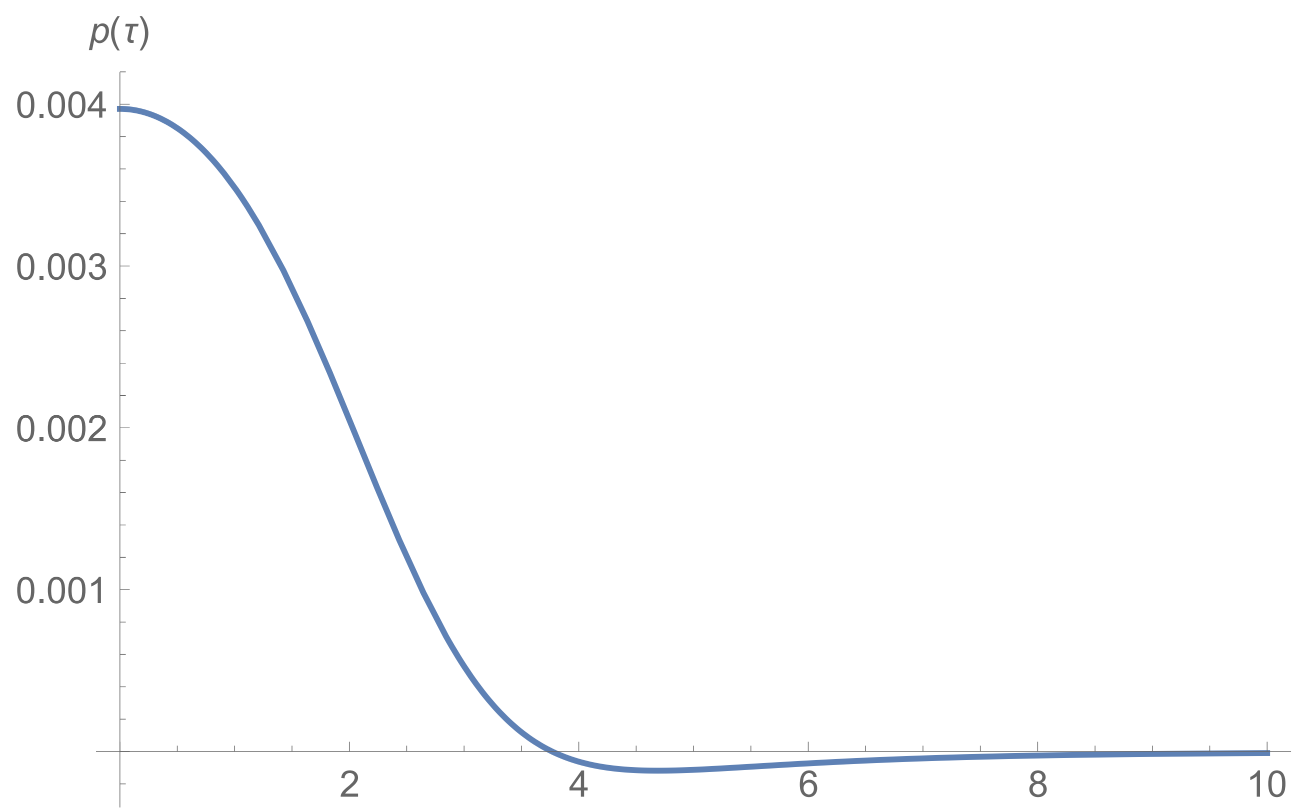
$$2g^2\mathcal{F} \;= \frac{\Lambda_{YM}^4}{\tilde{a}^4(\tau)}$$



Evolution of Energy Density and Pressure

$$\epsilon = \frac{\mathcal{A}}{\tilde{a}^4(\tau)}\Big(\log\frac{1}{\tilde{a}^4(\tau)} - 1\Big)\Lambda_{YM}^4, \qquad p = \frac{\mathcal{A}}{3\tilde{a}^4(\tau)}\Big(\log\frac{1}{\tilde{a}^4(\tau)} + 3\Big)\Lambda_{YM}^4.$$





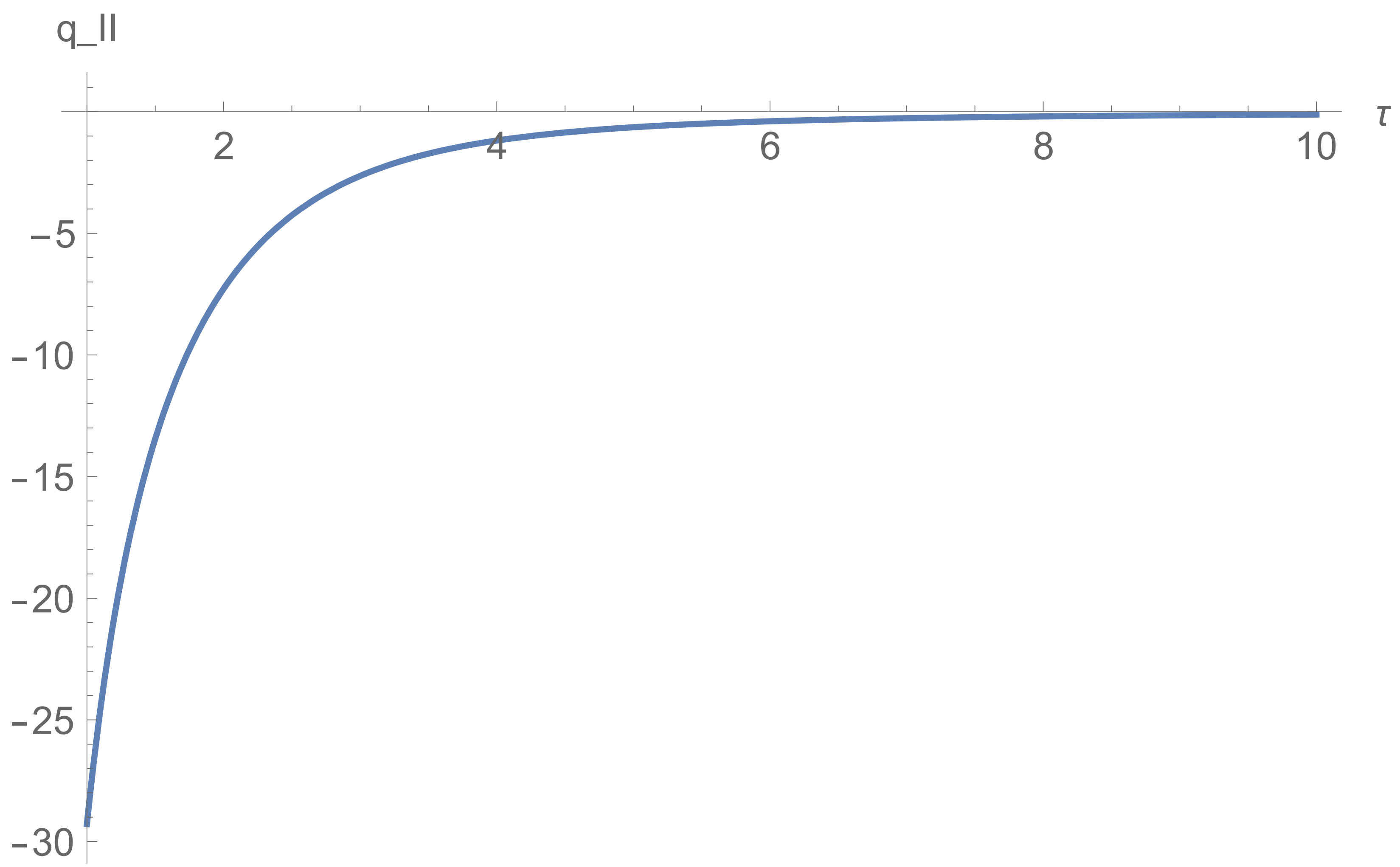
Evolution of the Hubble parameter

deceleration parameter

$$q = -\frac{\ddot{a}}{a}\frac{1}{H^2}.$$

$$q = \frac{\frac{1}{\tilde{a}^4}\left(\log\frac{1}{\tilde{a}^4} + 1\right)}{\frac{1}{\tilde{a}^4}\left(\log\frac{1}{\tilde{a}^4} - 1\right) - \frac{k\gamma^2}{\tilde{a}^2}}$$

Type II Solution *Deceleration of finite duration*



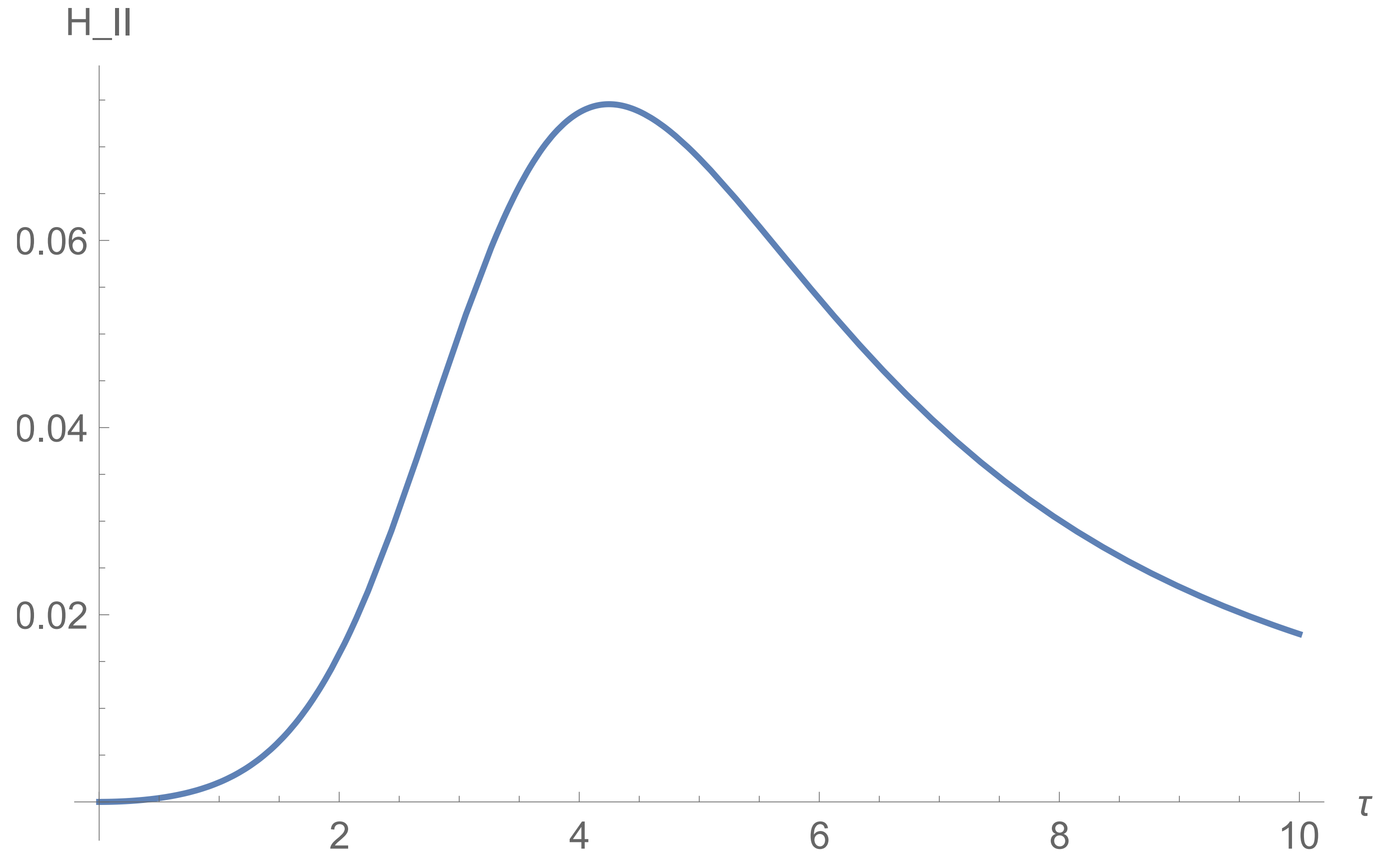
The deceleration parameter of the Type II solution is always negative:

$$q_{II} = \frac{b^2 + \gamma^2\mu_2^2 - 2}{b^2 + \gamma^2\mu_2^2(1 - e^{b^2/2})} < 0 \qquad q_{II} \propto -\frac{2}{b^2} \qquad q_{II} \propto -\frac{b^2}{\gamma^2\mu_2^2} e^{-b^2/2} \to 0.$$

Hubble Parameter

$$L^2H^2 = L^2\Big(\frac{\dot{a}}{a}\Big)^2 = \frac{1}{\tilde{a}^2}\Big(\frac{d\tilde{a}}{d\tau}\Big)^2 = \frac{1}{\tilde{a}^4(\tau)}\Big(\log\frac{1}{\tilde{a}^4(\tau)} - 1\Big) - \frac{k\gamma^2}{\tilde{a}^2(\tau)}$$

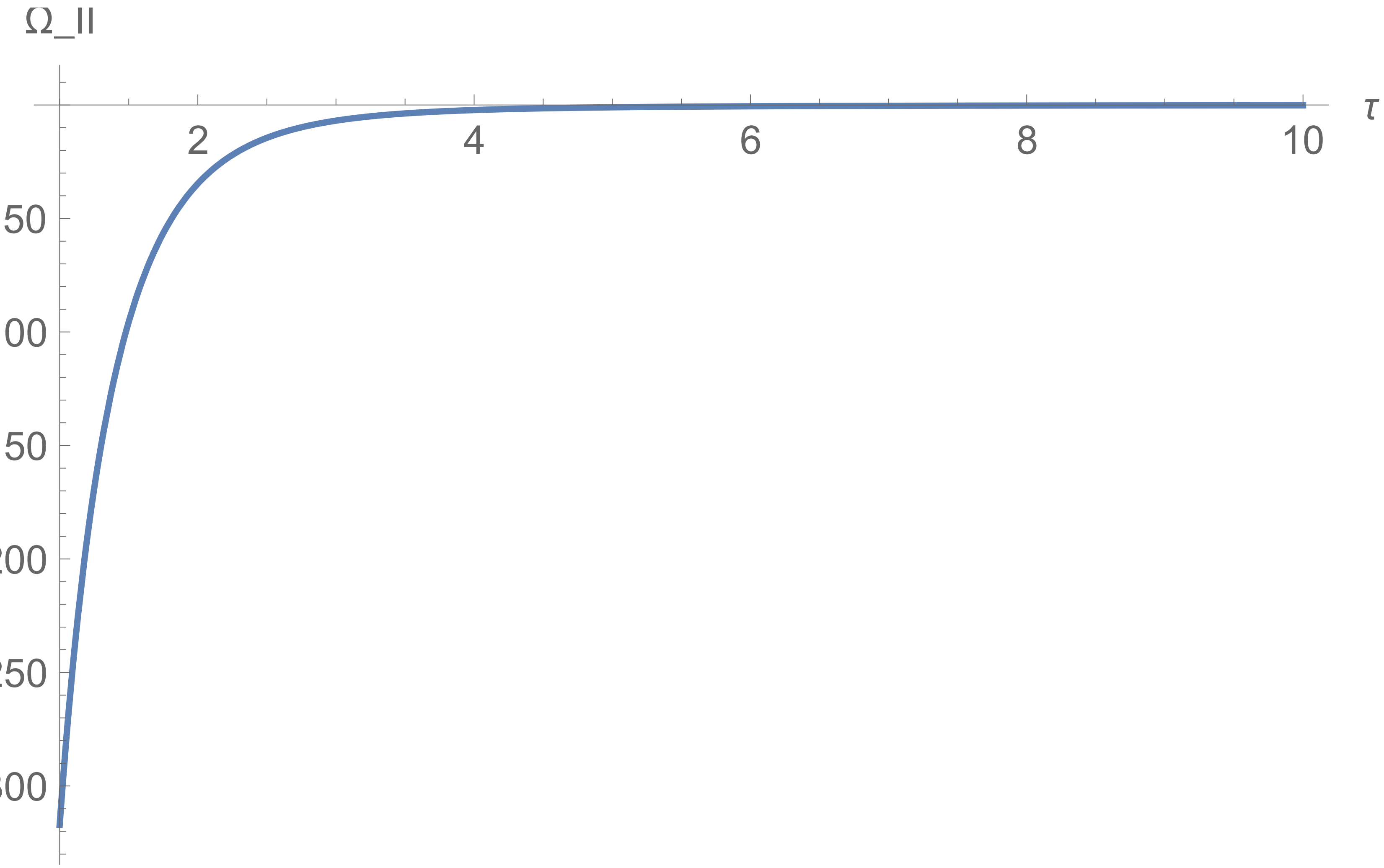
$$L^2H^2 = \frac{e^{-b^2}}{\mu_2^4}\Big(\gamma^2\mu_2^2(e^{b^2/2} - 1) - b^2\Big).$$



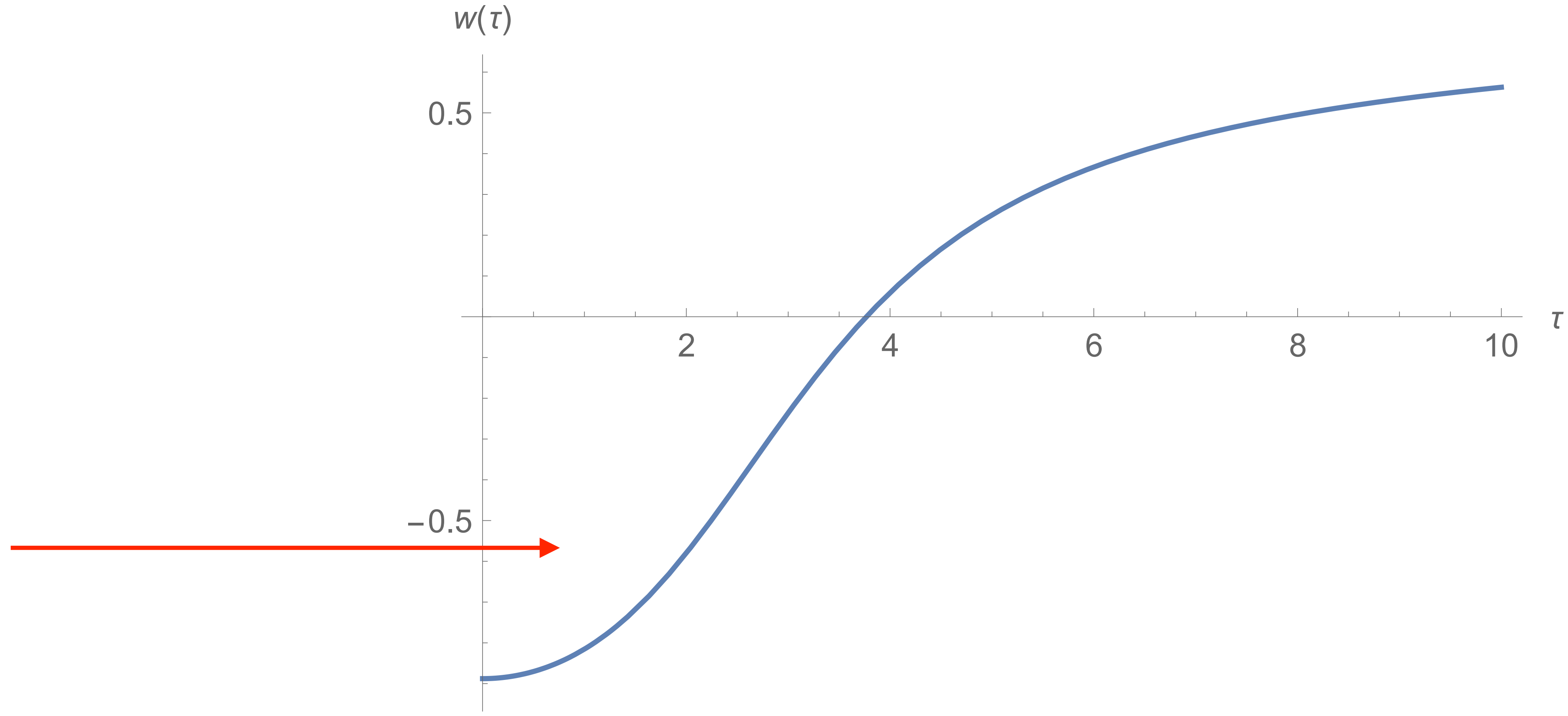
Type II Solution *Density Parameter*

$$\Omega_{vac} \equiv \frac{8\pi G}{3c^4}\frac{\epsilon}{H^2}$$

$$\Omega_{vac} - 1 = -\frac{\gamma^2}{(\frac{d\tilde{a}}{d\tau})^2} = -\frac{\gamma^2 \mu_2^2 e^{b^2/2}}{\gamma^2 \mu_2^2 (e^{b^2/2} - 1) - b^2}$$



Type II Solution — Effective Parameter w

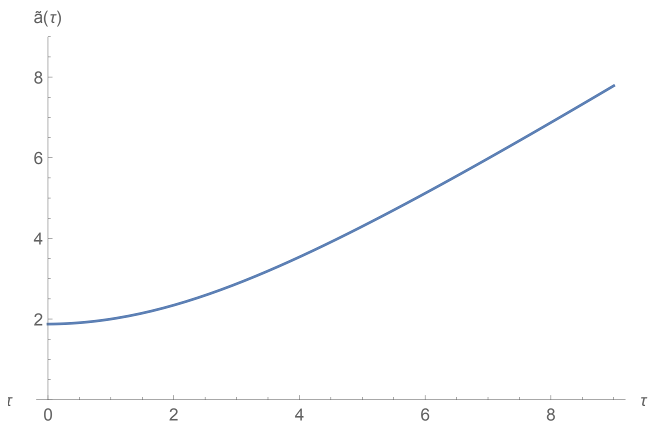


For the equation of state $p = w\epsilon$ one can find the behaviour of the effective parameter w

$$w_{II} = \frac{b^2(\tau) + \gamma^2\mu_2^2 - 4}{3\Big(b^2(\tau) + \gamma^2\mu_2^2\Big)}, \qquad -1 \leq w_{II},$$

$$w = \frac{p}{\epsilon} = \frac{\log\frac{1}{\tilde{a}^4(\tau)} + 3}{3\Big(\log\frac{1}{\tilde{a}^4(\tau)} - 1\Big)}.$$

Type II Solution Initial Acceleration of Finite Duration



The number of e-foldings

typical parameters around $\gamma^2 = 1.211$, $\mu_2^2 \simeq 1.75$ we get $\tau_s = 10^{23}$ and $\mathcal{N} \simeq 53$. $\mathcal{N} = \ln \frac{a(\tau_s)}{a(0)}$.

$$t_s^{GUM} = \frac{L_{GUM}}{c}\tau_s \simeq 4.2 \times 10^{-13}\ sec, \qquad \text{where } L_{GUM} \simeq 1.25 \times 10^{-25} cm$$

$$a(0) = L_{GUM}\frac{\mu_2}{\gamma} \simeq 1.5 \times 10^{-25} cm, \qquad a(t_s) = L_{GUM}\frac{\mu_2}{\gamma}e^{\mathcal{N}} \simeq 1.25 \times 10^{-2} cm,$$

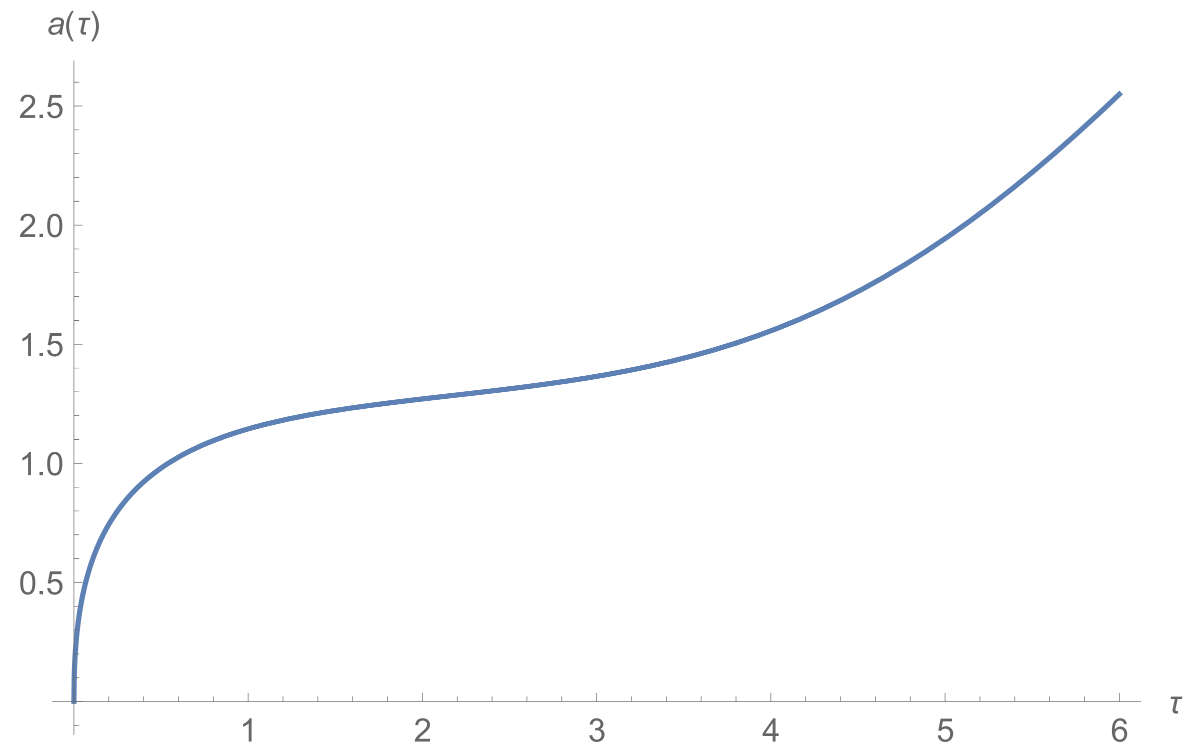
The regime of the exponential growth will continuously transformed into the linear in time growth of the scale factor‡

$$a(t) \simeq\ ct, \qquad a(\eta) \simeq a_0 e^{\eta}. \tag{5.87}$$

Type IV Solution - Late time Acceleration

The Type IV solution is defined in the region $\gamma^2 > \gamma_c^2$

$$\gamma_c^2 = \frac{2}{\sqrt{e}},$$



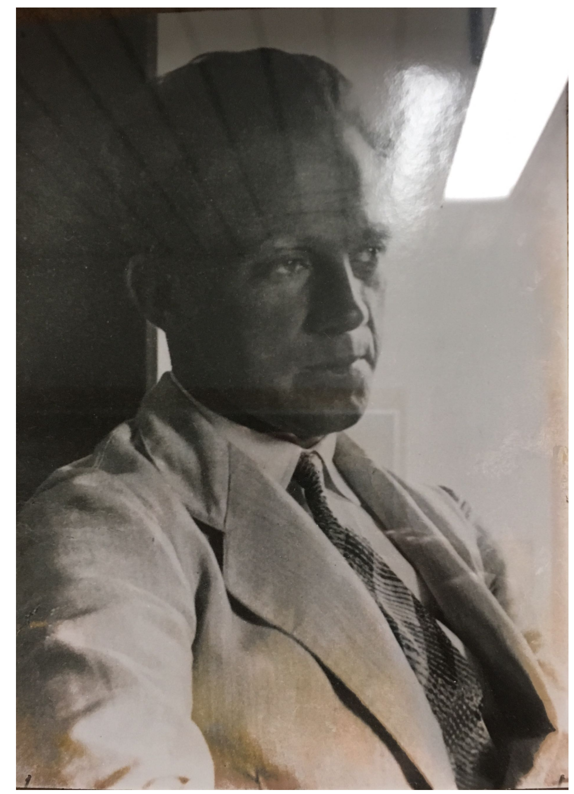
$$q_{IV} = \frac{b}{b + \frac{1}{2}\left(1 - \frac{\gamma^2}{\gamma_c^2} e^{2b}\right)},$$

$$H = \sqrt{\frac{2}{e}} \frac{e^{-2b}}{L} \left(\frac{\gamma^2}{\gamma_c^2} e^{2b} - 1 - 2b\right)^{1/2} \simeq \frac{1}{ct}.$$

$$\Omega_{vac} = 1 - \frac{\gamma^2}{(\frac{d\tilde{a}}{d\tau})^2} = 1 - \frac{\gamma^2 e^{2b}}{\gamma_c^2 \left(\frac{\gamma^2}{\gamma_c^2} e^{2b} - 1 - 2b\right)} \to 0.$$

Euler and Kockel 1935.
Heisenberg and Euler 1936

Hans Euler



Werner Heisenberg

Pair Creation in Electric Field

Arnold Sommerfeld Werner Heisenberg

Werner Heisenberg in Demokritos National Research Center
Athens, 1956 -1957

Thank You !