Understanding the Higgs mass in string theory

Steve Abel *(IPPP), Corfu* 08/21

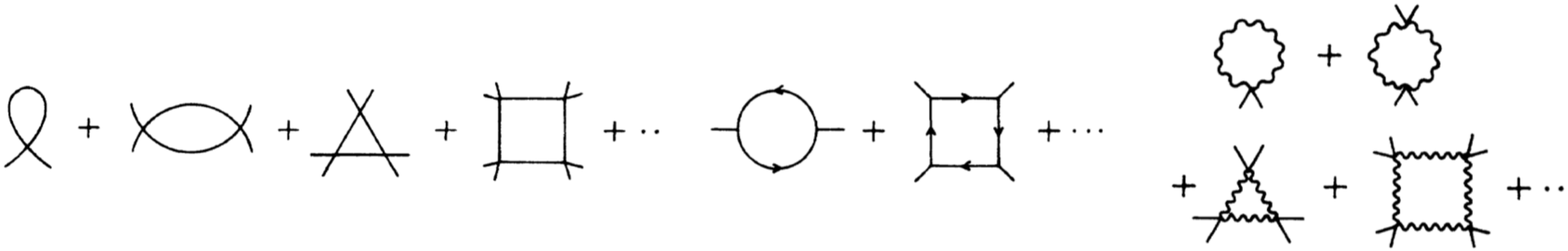
Mainly based on work with Keith Dienes arXiv:2106.04622 and related to …

- w/ Dienes+Mavroudi *Phys.Rev. D* 91, (2015) 126014, arXiv:1502.03087
- SAA JHEP 1611 (2016) 085, arXiv:1609.01311
- Aaronson, SAA, Mavroudi, *Phys. Rev. D* 95, (2016) 106001, arXiv:1612.05742
- w/ Stewart, *Phys.Rev.D* 96 (2017) 10, 106013 arXiv:1701.06629
- w/ Dienes+Mavroudi *Phys.Rev.D* 97 (2018) 12, 126017 arXiv: 1712.06894

Motivation: key questions for the UV completion

Effective field theories leave many unsolved problems for scalars like the Higgs:
e.g. hierarchy problem (essentially the statement that the EFT is very badly behaved)

Coleman-Weinberg effective potential:



Potential by doing one loop momentum integrals with a cut-off is

$$\Lambda(\phi) = \frac{M_{UV}^2}{32\pi^2} \underset{\text{eff}}{\text{Str}} M^2 - \frac{1}{64\pi^2} \underset{\text{eff}}{\text{Str}} M^4 \log\left(c \frac{M^2}{M_{UV}^2}\right)$$

where masses *M(*ϕ*)* are themselves functions of the field ϕ *and the cut off* M_{UV} *is …???*

The Higgs is maximally sensitive to both UV and IR: *think of it less as a problem and more as a "**canary in the coal mine**"*

Motivation: key questions for the UV completion

Q: "Suppose nature is a closed string theory. It is finite entirely because of its special symmetries (modular invariance) and that would be true even today. What does it tell us about the Higgs?"

What would we need to do to answer this question?

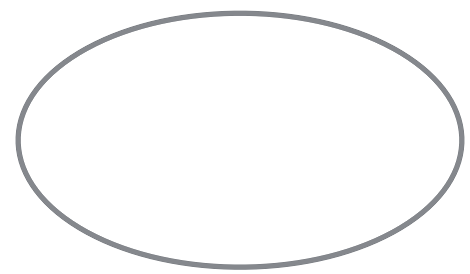
- *In most "string phenomenology" you start supersymmetric then jump to the EFT, and "abandon the beauties of number theory", which is what makes it all finite (NOT SUSY!!).*
- *But the world today cannot be blind to the beauties of number theory because it IS finite!*
- *String theory is UV/IR mixed so we will need to figure out how an EFT emerges from the string theory?*
- *Unless we missed it, no one ever wrote down the string equivalent of the CW effective potential!*
- ***Warning**: in this talk (much as in CW) I do not favour any particular model. I will just draw general conclusions about the properties the Higgs mass must have (even today) due to the theory's finiteness.*

Layout

- Background - the effective potential in a stringy way
- Modular invariance — the ultimate UV/IR mixer
- The Higgs mass and renormalisation

1. Background: the effective potential in a stringy way

Let's look at the one-loop cosmological constant (a.k.a. effective potential). Simplest way to derive it is as a trivial loop of massive propagators of mass *M(ϕ)* as follows:



For our discussion this can be written in a "stringy way" using a Schwinger worldline parameter, t :

$$\Lambda = \sum_i \int \frac{d^4k}{(2\pi)^4} (-1)^F \log(k^2 + M_i^2) = \sum_i \int \frac{d^4k}{(2\pi)^4} \int \frac{dt}{t} (-1)^F e^{-t(k^2+M_i^2)}$$

$$= \sum_i \int_{M_{UV}^{-2}}^{\mu_{UV}^{-2}} \frac{dt}{t^3} (-1)^F e^{-t\,M_i^2}$$

Can identify a "particle partition function" as a weighted sum over the spectral density:

$$g(t) = \mathrm{Str}\ \left(t^{-1} e^{-tM^2}\right)$$

Performing the integral of g indeed gives the effective potential:

$$\Lambda(\phi) = \frac{M_{UV}^2}{32\pi^2}\mathrm{Str}M^2 - \frac{1}{64\pi^2}\mathrm{Str}M^4 \log\left(c\frac{M^2}{M_{UV}^2}\right)$$

From which we can infer the running Higgs mass-squared from the double derivative:

$$m_\phi^2 \;=\; \frac{M_{\mathrm{UV}}^2}{32\pi^2}\,\underset{\mathrm{eff}}{\mathrm{Str}}\;\partial_\phi^2 M^2 \;-\underset{\mathrm{eff}}{\mathrm{Str}}\;\partial_\phi^2\left[\frac{M^4}{64\pi^2}\log\left(c\frac{M^2}{M_{\mathrm{UV}}^2}\right)\right]$$

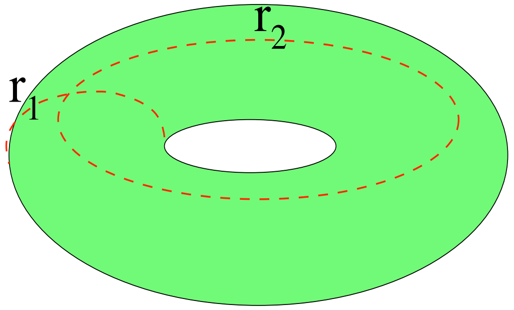
2. Modular invariance

Or: the ultimate UV/IR mixer

Let's understand how string theory does this but at the same time gets to be "finite":

Revisit the cosmological constant but now in string theory

Closed string theory instead maps out a torus:



Let’s understand how string theory does this but at the same time gets to be finite:

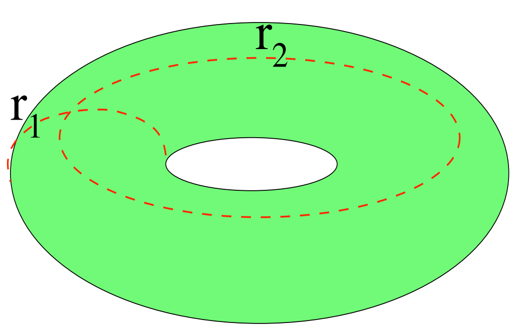
Revisit the cosmological constant but now in string theory

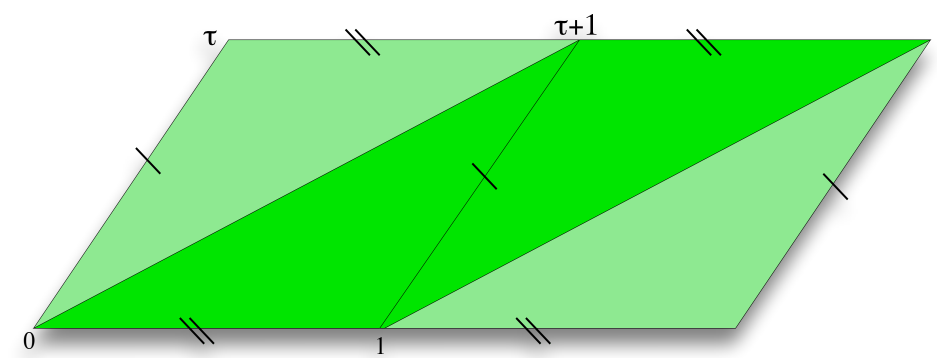
Closed string theory instead maps out a torus:

can be mapped to parallelogram in complex plane,

with single parameter τ , but theory invariant

under modular transformations:





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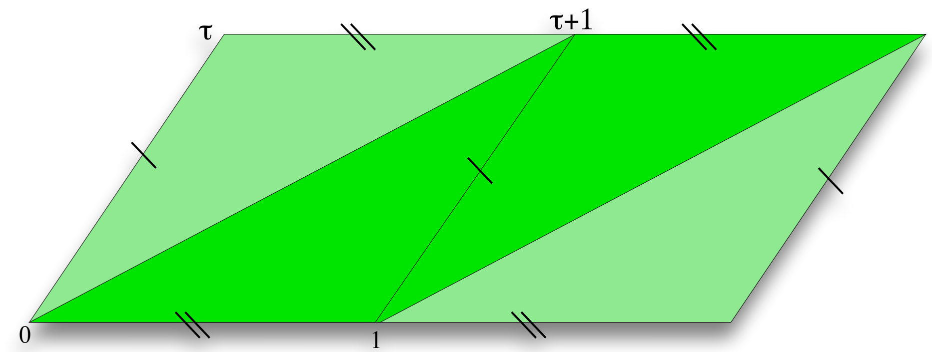

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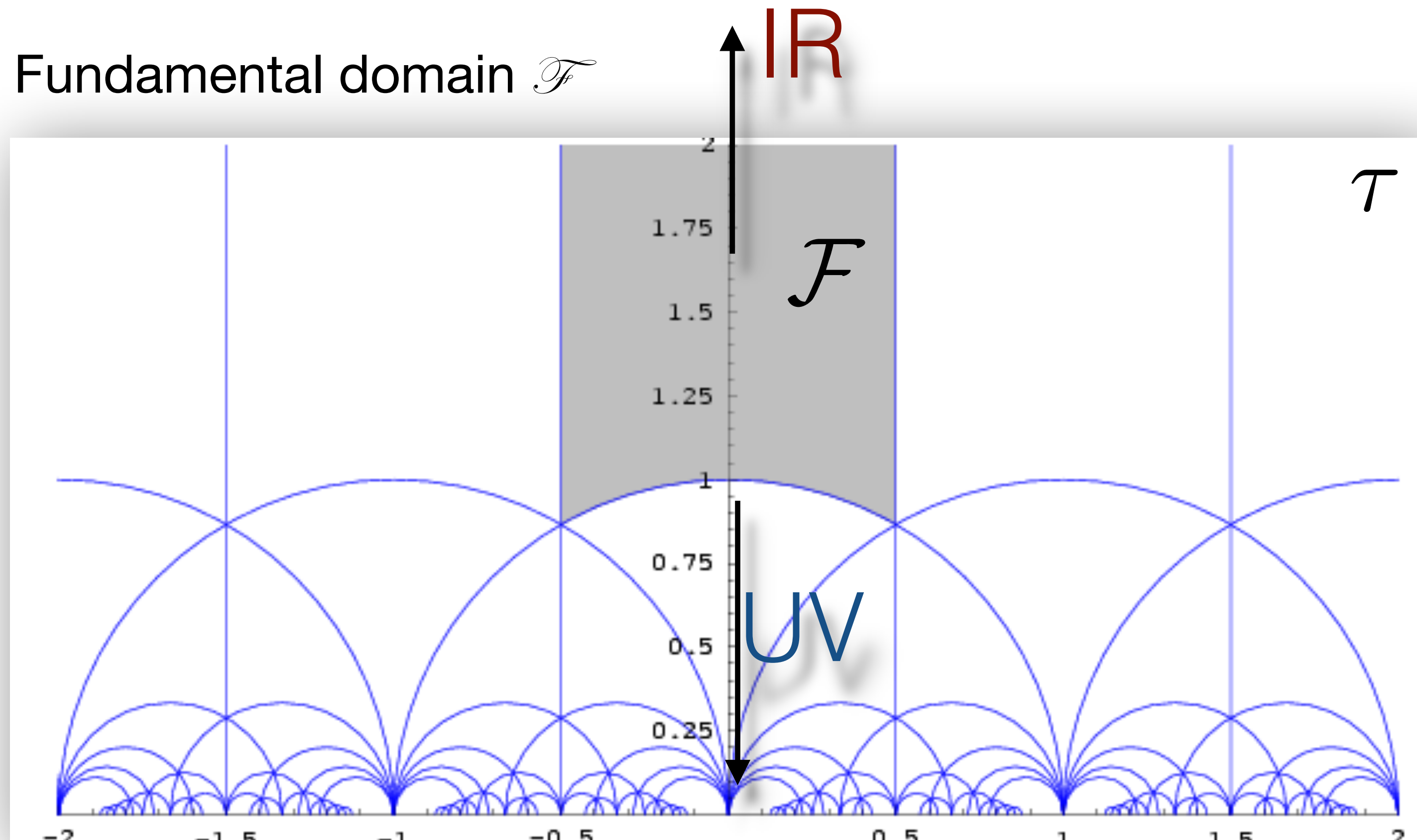


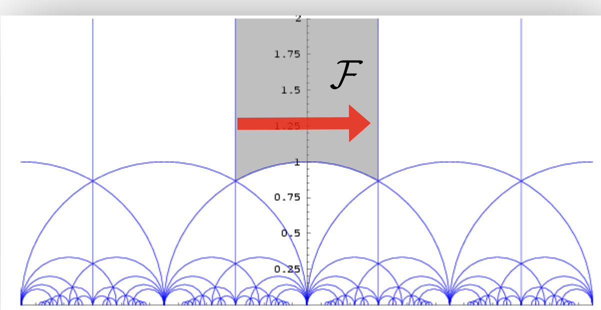
So then we have to integrate over all inequivalent tori, i.e. over τ

$$\tau = \tau_1 + i\tau_2 \qquad \left(\mathcal{M}^2 = \frac{1}{4\pi^2\alpha'} = \frac{M_s^2}{4\pi^2}\right)$$

$$\Lambda \equiv -\tfrac{1}{2}\,\mathcal{M}^D \int_{\mathcal{F}} \frac{d^2\tau}{{\tau_2}^2} Z(\tau) \qquad q = e^{2\pi i\tau}$$

$$= -\frac{1}{2}\mathcal{M}^D \int_{\mathcal{F}} \frac{d^2\tau}{{\tau_2}^{\frac{D}{2}+1}} \sum_{m,n} a_{mn}\overline{q}^m q^n$$

Counts physical (level matched) states weighted by statistics at each level



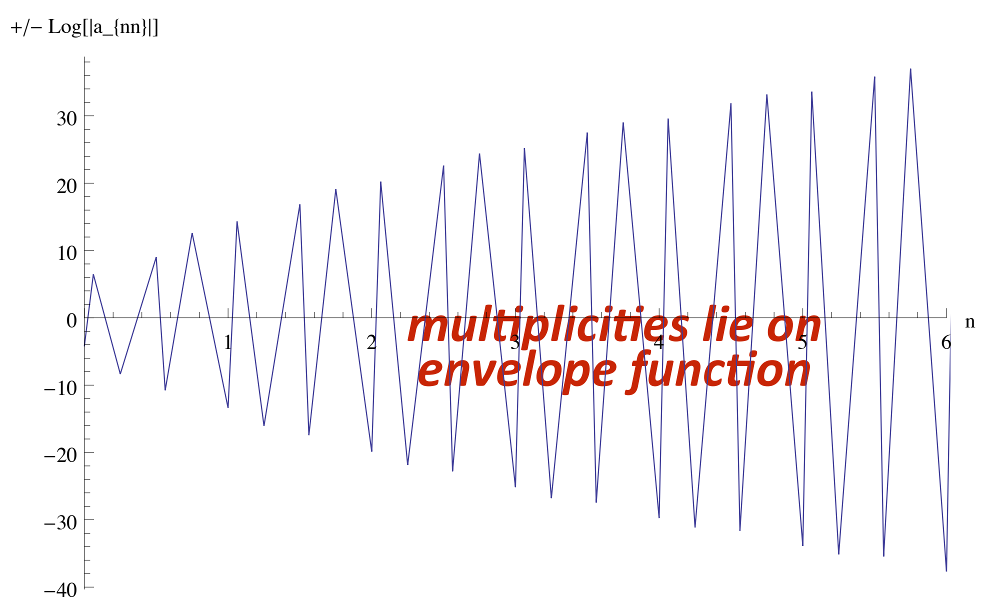
$$\approx -\frac{1}{2}\mathcal{M}^D \int_{M_{UV}^{-2}}^{\mu_{IR}^{-2}} \frac{d\tau_2}{{\tau_2}^{\frac{D}{2}+1}} \sum_n a_{nn}\, e^{-\pi\tau_2\alpha' M_n^2}$$

Due to modular invariance: there's an important way to rewrite this as a supertrace over the infinite tower of physical states. Much more natural and general for what we want to do. Superficially even looks similar to the field theory:

$$\Lambda = \frac{1}{24}\mathcal{M}^2 \mathrm{STr} M^2$$

- Dienes, Misaligned SUSY, 1994
- Kutasov, Seiberg, 1994
- Dienes, Moshe, Myers 1995

But note this definitely is *not* a field theory object — this supertrace is over the *infinite* string tower of states!!



- This crazy spectrum has finite Λ

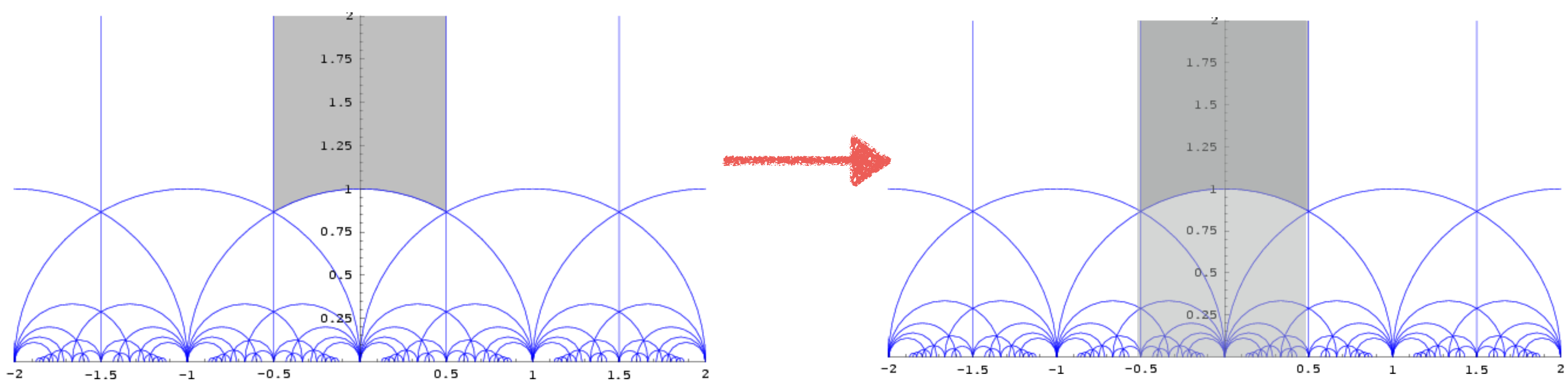
How does this identity emerge?: We claim this supertrace is equal to:

$$\Lambda \quad = -\frac{\mathcal{M}^4}{2}\int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2}\,\mathcal{Z}(\tau)$$

But the Str expression is in terms of physical (level-matched) states, i.e. the *particle* partition function which is just what we are left with after doing the tau 1 integral:

$$\begin{aligned} g(\tau_2) &= -\frac{\mathcal{M}^4}{2}\int_{-\frac{1}{2}}^{\frac{1}{2}} d\tau_1 \mathcal{Z}(\tau) \\ &= -\frac{\mathcal{M}^4}{2}\tau_2^{-1}\mathrm{Str} e^{-\pi\tau_2\alpha' M^2} \end{aligned}$$

In other words the whole integral must have been recast as the integral of a related function over the "critical strip" (by various number theory tricks: unfolding, Rankin-Selberg method (1940) etc)

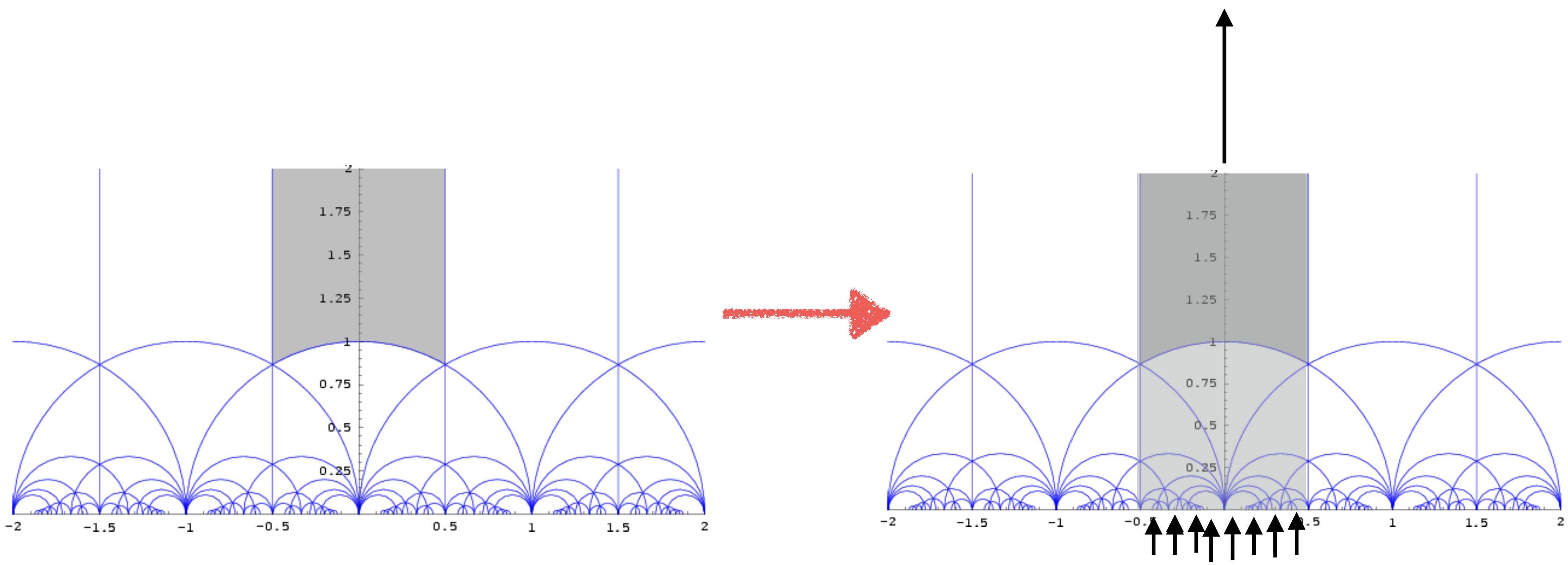


The incredible fact that this infinite supertrace is finite can then be put down to the fact that the particle partition function … behaves as follows in the UV (i.e. as $\tau_2 \to 0$):

$$g(\tau_2) \sim \tau_2^{-1} \mathrm{Str}\,(e^{-\tau_2 M^2}) \longrightarrow c_0$$

In other words *Str(1)=0*. In other words the nett spectrum "behaves" like a ***2 dimensional theory in the UV.*** *Unlike* supersymmetry however there is no level by level cancellation and the nett (Boson-Fermion) numbers of states in each level are completely crazy!

Note the important difference from the string-theory-textbook picture. There is not really a single "IR cusp". All cusps contribute equally to the integral:



All cusps are equivalent under modular transformations. In a modular invariant integral there is only IR: there is no "ultra UV" anywhere.

3. The Higgs mass

First assume that the partition function is a function of the higgs. Then begin with the naive expression:

$$m_\phi^2 \;\equiv\; \left.\frac{d^2\Lambda(\phi)}{d\phi^2}\right|_{\phi=0}$$

So double-differentiating the $\mathcal{Z}$ that is in Λ by ϕ, the relevant integral is (**almost**) given by just inserting the M

$$X \;=\; -\,\pi\alpha'\tau_2\,\partial_\phi^2 M^2 + \left(\pi\alpha'\tau_2\right)^2\left(\partial_\phi M^2\right)^2$$

Almost but not quite: the shifts in $\mathcal{Z}$ induced by the Higgs correspond to coordinate shifts of the modular forms (actually the Higgs *is* a linear combination of these coordinates). For the Higgs double-derivative to be modular *covariant* we require a modular completion which is found to be ***universal***:

$$X \longrightarrow X + \frac{\xi}{4\pi^2\mathcal{M}^2} \qquad\qquad \xi = -\mathrm{T}r(\mathcal{T}_{21}\mathcal{T}_{12})$$

Note that this is cosmological constant contribution due to the modular anomaly of the original naive $\boldsymbol{X}$. This universal term would in most practical cases be identified as a Higgs dependent shift in the volume modulus of the compactification space (e.g. 10D —> 4D compactification) with ξ being the quadratic Casimir (e.g. Cardoso, Lust, Mohaupt; Antoniadis, Taylor).

So. Putting this into the integral we get … ta da !

$$m_\phi^2 = \frac{\xi}{4\pi^2}\frac{\Lambda^{(1)}}{\mathcal{M}^2} + \frac{1}{24}\mathcal{M}^2 \mathrm{Str}\partial_\phi^2 M^2 + \mathrm{STr}_{M=0}(\partial_\phi M^2)^2 \times \infty + \mathrm{STr}_{M>0}(\partial_\phi M^2)^2 \times 0$$

What? Wait! Of course the integral must still be *logarithmically* divergent for massless states!

Regularisation and renormalisation

The quartic terms are precisely those terms that should be *logarithmically* dependent on RG scale. But we didn't yet put in any physical RG scale! So at the moment the integral returns infinity if the state is massless (or zero if it is massive).

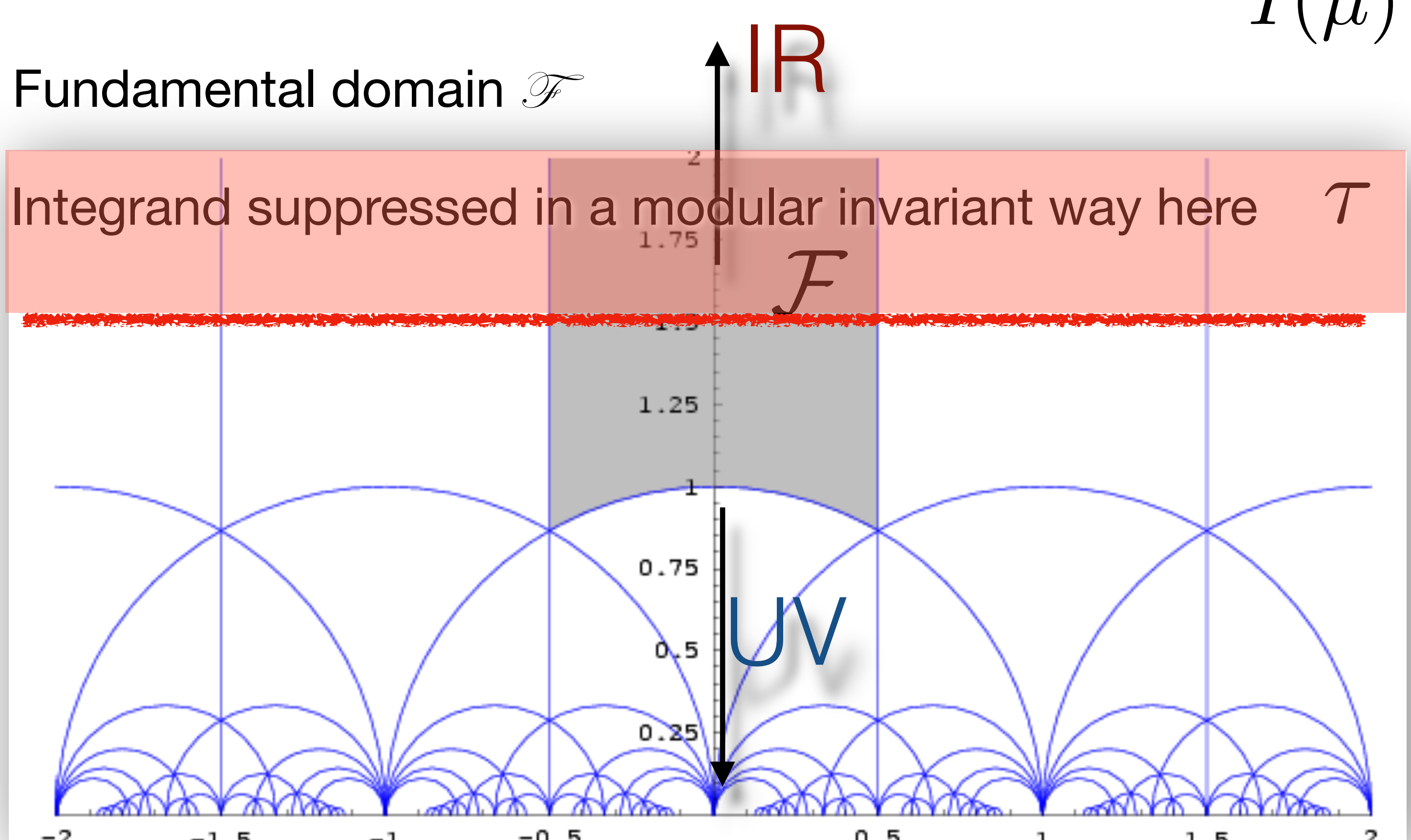
Generally need to find a way to regulate the theory at some IR scale μ to extract a physical "running"

To do this, as I said at the start, traditionally, we would think of stringy “threshold corrections” and match them to an effective field theory (EFT) whose contributions would then be subtracted from the string integral as if they were exactly massless (Zagier 1981). But that traditional approach …

- could never yield a fully modular invariant answer as the EFT is by definition not modular invariant
- could not give “Wilsonian renormalisation” *(i.e. address the question of how small and large energy scales get separated)*: my choice of if the neutrino is light enough to be called “massless” and be in the effective theory is completely arbitrary and will always break modular invariance

Instead we must abandon the idea of selecting an EFT by hand, and introduce a modular invariant RG cut-off procedure instead:

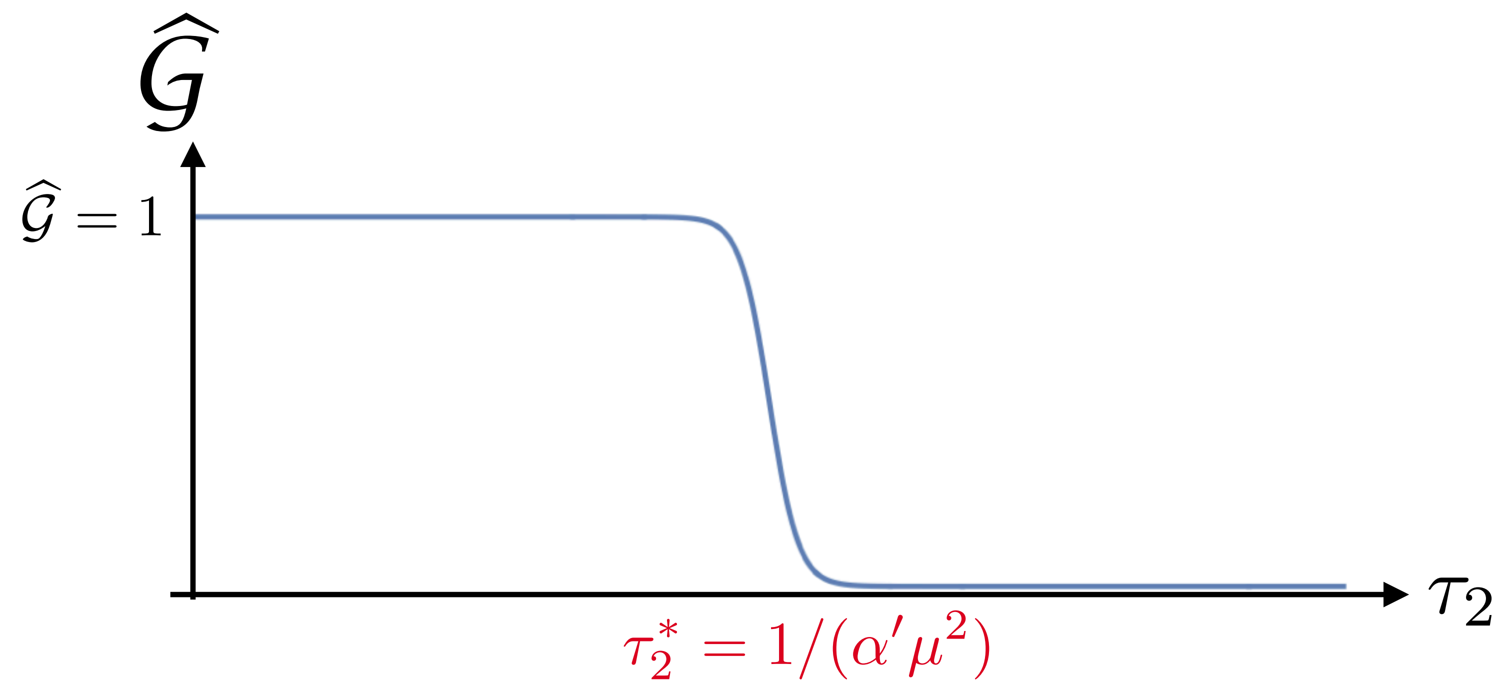
$$\widehat{I}(\mu) = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2}\, \widehat{\mathcal{G}}(\mu,\tau,\overline{\tau})\, F(\tau,\overline{\tau})$$



Required properties of “Wilsonian” regulator, $\widehat{\mathcal{G}}$:

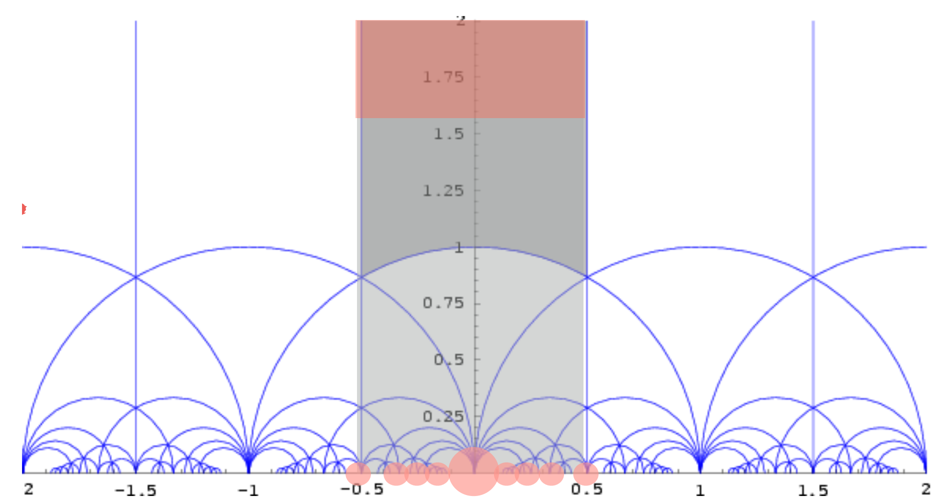
$$\widehat{I}(\mu) = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2}\, \widehat{\mathcal{G}}(\mu,\tau,\overline{\tau})\, F(\tau,\overline{\tau})$$

- a) Is itself a modular function
- b) Should look roughly like this ….



- c) As our goal is to write everything as a supertrace which ultimately means an integral over the critical strip …This only makes sense if actually all the cusps are crushed equally. In other words: all the cusps are equivalent IR cusps, implying…

$$\tau_2^* \equiv 1/\tau_2^* \implies \widehat{\mathcal{G}}(\mu,\tau,\overline{\tau}) = \widehat{\mathcal{G}}(M_s^2/\mu,\tau,\overline{\tau})$$



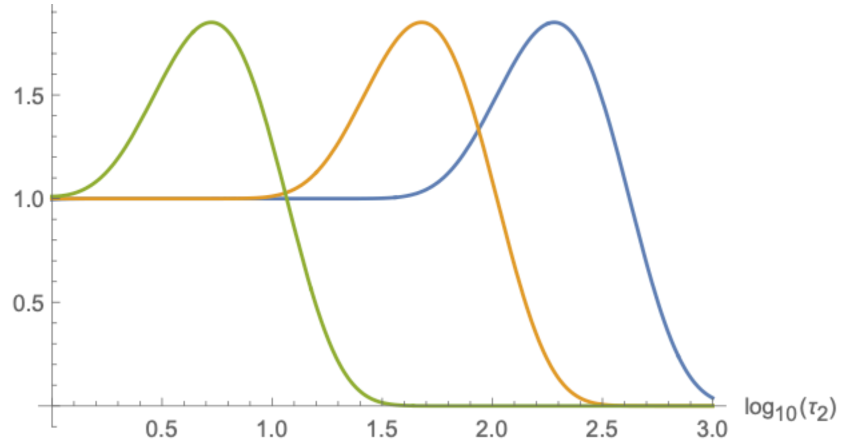
We can adapt a (geometrically derived) modular invariant regulator that already exists: (Kiritsis, Kounnas, Petropoulos, Rizos)

- Take the circle partition function with radius defined by parameter $a \equiv \sqrt{\alpha'}/R$:

$$Z_{\text{circ}}(a,\tau) = \sqrt{\tau_2} \sum_{m,n\in\mathbb{Z}} \overline{q}^{(ma-n/a)^2/4}\, q^{(ma+n/a)^2/4}$$

- Then a suitable cut-off function that obeys *all* these properties is …

$$\widehat{\mathcal{G}}(a,\tau) = \frac{2a^2}{1+2a^2}\frac{\partial}{\partial a}\left(Z_{\text{circ}}(2a,\tau) - Z_{\text{circ}}(a,\tau)\right)$$



$$\mu^2(a) = \frac{2a^2}{\alpha'} \implies \tau_2^* = 1/2a^2$$

The result is a smooth modular invariant running answer:

Complicated infinite sum of Bessel functions, but it has the following magical behaviour …

$$\widehat{m}_\phi^2 = \frac{\xi}{4\pi^2}\frac{\widehat{\Lambda}(\mu)}{\mathcal{M}^2} + \partial_\phi^2 \widehat{\Lambda}(\mu)$$

$$\widehat{\Lambda}(\mu,\phi) = \frac{1}{24}\mathcal{M}^2\,\mathrm{Str}\,M^2 - c'\underset{M\gtrsim\mu}{\mathrm{Str}}\,M^2\mu^2 \;-\underset{0\leq M\lesssim\mu}{\mathrm{Str}}\left[\frac{M^4}{64\pi^2}\log\left(c\frac{M^2}{\mu^2}\right) + c''\mu^4\right]$$

$c = 2e^{2\gamma+1/2}$, $c' = 1/(96\pi^2)$, and $c'' = 7c'/10$

This is a fully UV complete effective potential which holds for any modular invariant theory.

Below the mass of all states (that couple to the Higgs) they do not contribute to the running.

At some intermediate energy scale the result is a sum over all states *as **if they had all logarithmically run up from their mass***.

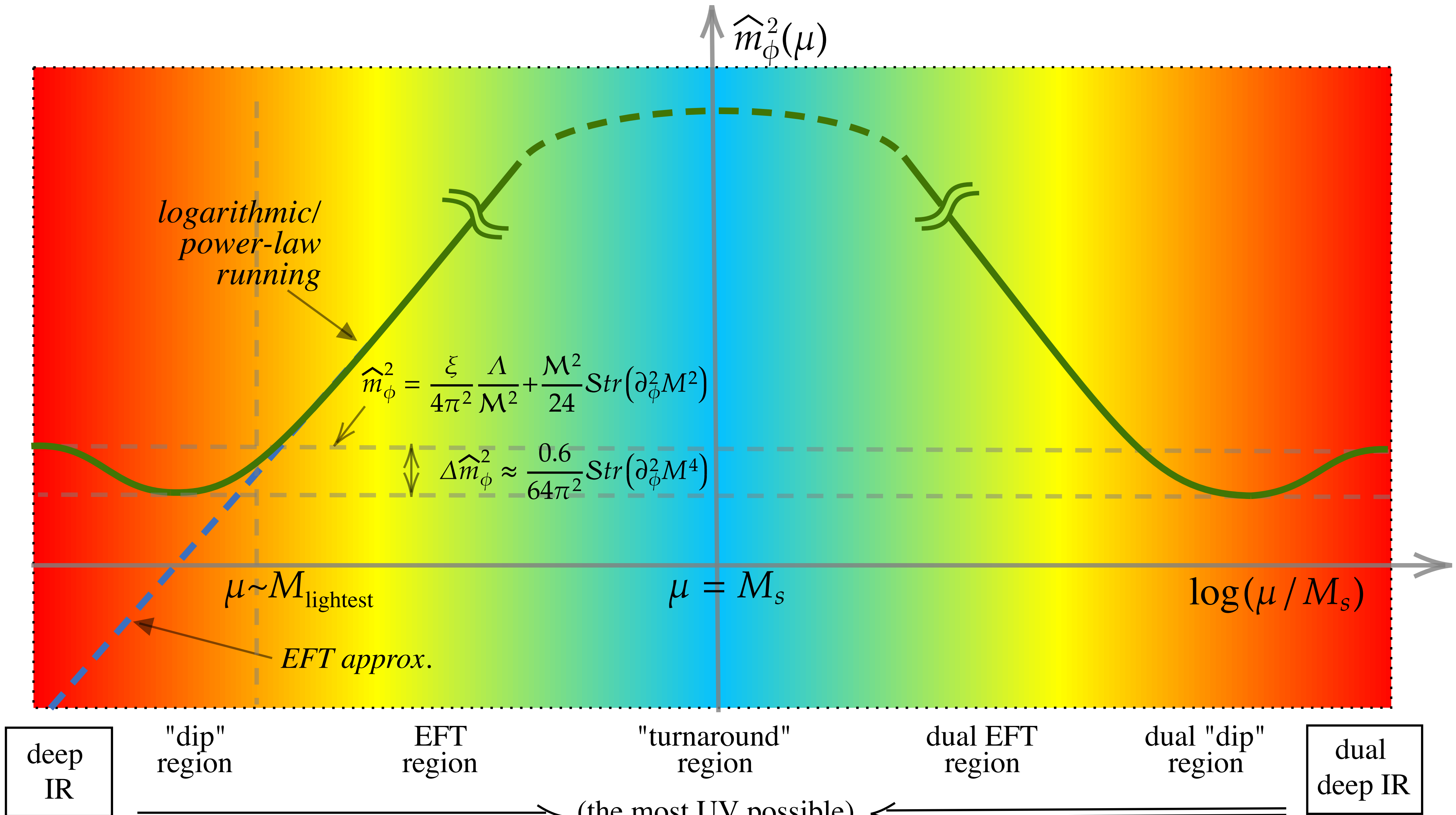
It is by construction symmetric around the string scale.

This is a fully UV complete effective potential which holds for any modular invariant theory.

Below the mass of all states (that couple to the Higgs) they do not contribute to the running.

At intermediate energy scales the result is a sum over all lighter states *as* ***if they had all logarithmically run up from their mass*****.**

It is by construction symmetric around the string scale.



$$\lim_{\mu\to 0} \hat{m}_\phi^2(\mu) = \frac{\xi}{4\pi^2}\frac{\Lambda}{\mathcal{M}^2} + \frac{1}{24}\mathcal{M}^2 \mathrm{Str}\partial_\phi^2 M^2$$

5. Conclusions

- We have developed a general supertrace formula for the Higgs, that plays the role for all generic modular invariant theories that the effective potential plays in field theory.
- A modular invariant regulator provides a natural “Wilsonian energy cut-off” and a definition of RG scale. Gives meaning where the EFT fails, and retains the predictivity of the UV complete theory.
- The stringy CW potential is the sum of an infinite tower of particle potentials.
- Operators such as the Higgs mass can be thought of as “running” to its predetermined IR value: this is actually both a UV and IR asymptote as it should be.
- The Weak/Planck and cosmological constant hierarchy problems are connected in this one operator.
- Relevant for many old and new pheno ideas: e.g. a stringy naturalness (Veltman) condition:

$$\mathrm{Str}\,\partial_\phi^2 M^2 \lesssim \frac{24}{\mathcal{M}^2} M_W^2$$