Non-Unitary Mixing Matrices in Neutrino and Vector-like Quark Models

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In collaboration with:

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Cofinanciado por:

Overview

Motivation

Approximations are common in literature.

- Neutrinos: Seesaw approximation, Casas-Ibarra, Fernandez-Martinez et al.[1], ...
- VLQs: Common assumption that heavy VLQs only couple to third generation quarks.

What if you want to study a region... **where these approximations fail?**

What if you want to ... **perform a general scan of the parameter space,** without biases?

---

[1] arXiv:1605.08774 [hep-ph], mixing matrix written using an infinite power series, truncate at your taste approach.

Mass Matrices and their Diagonalisation

$$\mathcal{L}_{\mathrm{M}} = -\begin{pmatrix} \overline{d}^0_L & \overline{D}^0_L \end{pmatrix} \mathcal{M}_d \begin{pmatrix} d^0_R \\ D^0_R \end{pmatrix} - \begin{pmatrix} \overline{u}^0_L & \overline{U}^0_L \end{pmatrix} \mathcal{M}_u \begin{pmatrix} u^0_R \\ U^0_R \end{pmatrix} + \text{h.c.}\,,$$

$$\mathcal{L}_m = -\left[\frac{1}{2} n_L^T C^* \mathcal{M}^* n_L + \overline{l_L} d_l l_R\right] + \text{h.c.}\,,$$

$$\mathcal{M}_q = \left(\begin{array}{c:c} m_q & \overline{m}_q \\ \hdashline \underbrace{\overline{M}_q}_{3} & \underbrace{M_q}_{n_q} \end{array}\right) \begin{array}{l} \}\, 3 \\ \}\, n_q \end{array}\,, \qquad \mathcal{M} = \left(\begin{array}{c:c} \mathbf{0} & m \\ \hdashline \underbrace{m^T}_{3} & \underbrace{M}_{n_R} \end{array}\right) \begin{array}{l} \}\, 3 \\ \}\, n_R \end{array}\,,$$

Mass Matrices and their Diagonalisation

$$\mathcal{V}_L^{q\dagger}\ \mathcal{M}_q\ \mathcal{V}_R^q\ =\ \mathcal{D}_q$$

$$\mathcal{V}^T\mathcal{M}^*\mathcal{V}=\mathcal{D}\ ,$$

$$\mathcal{V}_\chi^q\ =\ \left(\begin{array}{c} A_\chi^q \\ \hdashline B_\chi^q \end{array}\right)\ , \qquad \mathcal{V}\ =\ \left(\begin{array}{c} A \\ \hdashline B \end{array}\right)\ ,$$

$$\chi=L,R\ \ ,\ q=u,d \qquad A=3\times(3+n)\ ,\ B=n\times(3+n)$$

Unitary $\mathcal{V}s$: equations relating A, B with $\mathbf{0}$ and $\mathbb{1}$ matrices.

Mass Matrices and their Diagonalisation

$$\begin{aligned} m_q &= A_L^q\,\mathcal{D}_q\,A_R^{q\,\dagger}\,, \\ \overline{m}_q &= A_L^q\,\mathcal{D}_q\,B_R^{q\,\dagger}\,, \\ \overline{M}_q &= B_L^q\,\mathcal{D}_q\,A_R^{q\,\dagger}\,, \\ M_q &= B_L^q\,\mathcal{D}_q\,B_R^{q\,\dagger}\,. \end{aligned}$$

$$\begin{aligned} \mathbf{0} &= A\,\mathcal{D}\,A^T\,, \\ m &= A\,\mathcal{D}\,B^T\,, \\ M &= B\,\mathcal{D}\,B^T\,. \end{aligned}$$

Interactions

Charged Currents:

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}} \begin{pmatrix} \overline{u}_L & \overline{U}_L \end{pmatrix} V \gamma^\mu \begin{pmatrix} d_L \\ D_L \end{pmatrix} W_\mu^+ + \text{h.c.}$$

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}} \bar{l}_L V \gamma^\mu \begin{pmatrix} n_L \\ N_L \end{pmatrix} W_\mu^+ + \text{h.c.} \ ,$$

$V = A_L^{u^\dagger} A_L^d$

$V = A$

Interactions

Neutral Interactions:

$$\mathcal{L}_Z = -\frac{g}{2\cos\theta_W} Z_\mu [\begin{pmatrix} \overline{q_L} & \overline{Q_L} \end{pmatrix} F^q \gamma^\mu \begin{pmatrix} q_L \\ Q_L \end{pmatrix} + \text{h.c.}$$

$$\mathcal{L}_H = -\frac{h}{v}\left[\begin{pmatrix} \overline{q}_L & \overline{Q}_L \end{pmatrix} F^q \mathcal{D}_q \begin{pmatrix} q_R \\ Q_R \end{pmatrix} \right] + \text{h.c.}\,.$$

$$\mathcal{L}_Z = -\frac{g}{2\cos\theta_W} Z_\mu [\begin{pmatrix} \overline{n_L} & \overline{N_L} \end{pmatrix} F \gamma^\mu \begin{pmatrix} n_L \\ N_L \end{pmatrix}] + \text{h.c.}$$

$$\mathcal{L}_H = -\frac{h}{v}\left[\begin{pmatrix} \overline{n}_L & \overline{N}_L \end{pmatrix} F \mathcal{D} \begin{pmatrix} n_L^c \\ N_L^c \end{pmatrix} \right] + \text{h.c.}\,.$$

$F^q = A_L^{q\dagger} A_L^q$

$F = A^\dagger A$

Parameterisation

$$\mathcal{V}^q_\chi = \left(\begin{array}{c|c} K^q_\chi & K^q_\chi \, X^{q\dagger}_\chi \\ \hline \underbrace{-\overline{K}^q_\chi \, X^q_\chi}_{3} & \underbrace{\overline{K}^q_\chi}_{n_q} \end{array}\right) \begin{array}{l} \left.\vphantom{K^q_\chi}\right\}3 \\ \left.\vphantom{K^q_\chi}\right\}n_q \end{array} . \; \mathcal{V} = \left(\begin{array}{c|c} K & K \, X^\dagger \\ \hline \underbrace{-\overline{K} \; X}_{3} & \underbrace{\overline{K}}_{n_R} \end{array}\right) \begin{array}{l} \left.\vphantom{K^q_\chi}\right\}3 \\ \left.\vphantom{K^q_\chi}\right\}n_R \end{array} .$$

Non-singular general complex matrices K and $\overline{K}$.

$$A = (K \quad KX^\dagger) \, , \; B = (-\overline{K} \, X \quad \overline{K} \,)$$

Parameterisation

$$
\begin{aligned}
m_q &= K_L^q \left(d_q + X_L^{q\dagger} D_q X_R^q \right) K_R^{q\dagger}, \\
\overline{m}_q &= K_L^q \left(X_L^{q\dagger} D_q - d_q X_R^{q\dagger} \right) \overline{K}_R^{q\,\dagger}, \\
\overline{M}_q &= \overline{K}_L^{q} \left(D_q X_R^q - X_L^q d_q \right) K_R^{q\dagger}, \\
M_q &= \overline{K}_L^{q} \left(D_q + X_L^q d_q X_R^{q\dagger} \right) \overline{K}_R^{q\,\dagger}.
\end{aligned}
$$

WB where $\overline{m}_q$ is $\mathbf{0}$ (always possible, same for $\overline{M}_q$)

$$
\begin{aligned}
X_L^q &= \sqrt{D^{-1}} P^q \sqrt{d} \\
X_R^q &= \sqrt{D} P^q \sqrt{d^{-1}}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{0} &= d + X^\dagger D X^* \ , \\
m &= K \, X^\dagger D \left(Z^{-1} \right)^* \ , \\
M &= Z \left(D + X \, d \, X^T \right) Z^T \, .
\end{aligned}
$$

$$X = \pm \, i \sqrt{D^{-1}} \, O_c \sqrt{d} \, ,$$

Parameterisation

$$K_\chi^q = U_K(\mathbb{1}_3 + X_L^{q\dagger} X_L^q)^{-1/2}, \qquad K = U_K(\mathbb{1}_3 + X^\dagger X)^{-1/2}$$

$$\overline{K}_\chi^{\,q} = U_{\overline{K}}(\mathbb{1}_{n_q} + X_L^q X_L^{q\dagger})^{-1/2}. \qquad \overline{K} = U_{\overline{K}}(\mathbb{1}_{n_R} + XX^\dagger)^{-1/2},$$

$$K_{CKM} = K_L^{u\dagger} K_L^d \qquad K_{PMNS} = K$$

$$F^q = \begin{pmatrix} (\mathbb{1}_3 + X_L^{q\dagger} X_L^q)^{-1} & (\mathbb{1}_3 + X_L^{q\dagger} X_L^q)^{-1} X_L^{q\dagger} \\ X_L^q (\mathbb{1}_3 + X_L^{q\dagger} X_L^q)^{-1} & X_L^q (\mathbb{1}_3 + X_L^{q\dagger} X_L^q)^{-1} X_L^{q\dagger} \end{pmatrix}$$

$$F = \begin{pmatrix} (\mathbb{1}_3 + X^\dagger X)^{-1} & (\mathbb{1}_3 + X^\dagger X)^{-1} X^\dagger \\ X(\mathbb{1}_3 + X^\dagger X)^{-1} & X(\mathbb{1}_3 + X^\dagger X)^{-1} X^\dagger \end{pmatrix}$$

Parameterisation

$$\mathcal{V} = \begin{pmatrix} U_K(\mathbb{1}_3 + X^\dagger X)^{-1/2} & U_K(\mathbb{1}_3 + X^\dagger X)^{-1/2} X^\dagger \\ -U_{\overline{K}}(\mathbb{1}_{n_R} + X X^\dagger)^{-1/2} X & U_{\overline{K}}(\mathbb{1}_{n_R} + X X^\dagger)^{-1/2} \end{pmatrix}$$

Parameterisations in the leptonic sector with a similar structure existed in the literature prior to this work [2] [3], but are either approximations or a special case of this one.

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[2] J.G. Korner, A. Pilaftsis and K. Schilcher, Leptonic CP asymmetries in flavor changing H0 decays, Phys. Rev. D 47 (1993) 1080 [hep-ph/9301289] [INSPIRE]

[3] W. Grimus and L. Lavoura, JHEP 11, 042 (2000), arXiv:hep-ph/0008179 [hep-ph].

Procedure

- Start with d, D, U_K and O_c/P^q
- Calculate X
- Calculate mass matrices, V and F

Done. Everything at tree level is exact.

Things to worry about:

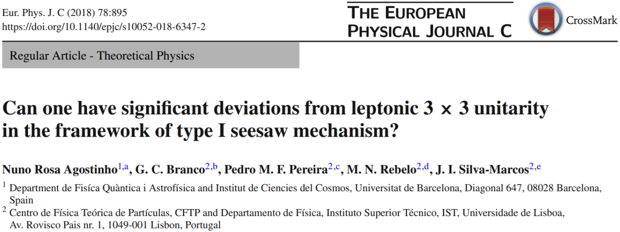
- Radiative Corrections on the light neutrino masses
- Perturbativity (Heavy masses and deviations from unitarity are in a "seesaw")

Usefulness

- Exact Formulas at tree level.
- Easy to implement numerically.
- (In principle) Extendable to any model with non-unitary mixing matrices: Inverse Seesaw, Linear Seesaw, type-II and type-III seesaw, models with vector like fermions and scalars, ...

Used in: Neutrinos

Neutrino spectra were considered where the seesaw approximation fails:



Eur. Phys. J. C (2018) 78:895
https://doi.org/10.1140/epjc/s10052-018-6347-2

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Regular Article - Theoretical Physics

Can one have significant deviations from leptonic 3 × 3 unitarity in the framework of type I seesaw mechanism?

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All $M_i \sim TeV$.

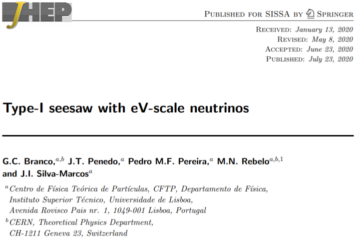
Deviations from Unitarity matching the experimentally allowed upper bounds.

---

[4] Agostinho, N.R., Branco, G.C., Pereira, P.M.F. et al. Eur. Phys. J. C 78, 895 (2018). https://doi.org/10.1140/epjc/s10052-018-6347-2

Used in: Neutrinos

and where 1st order approximations deviate from the exact result:



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Type-I seesaw with eV-scale neutrinos

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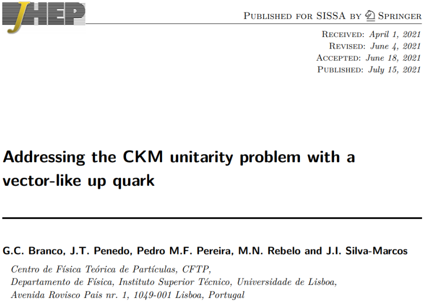
$M_i \sim$ eV, eV, GUT; $M_i \sim$ eV,KeV,GUT; $M_i \sim$ eV, TeV, TeV;
Spectra that could explain the ShortBaseline Anomaly;
Effects on CP asymmetries measurable in LongBaseline Experiments.

---

[5] Branco, G.C., Penedo, J.T., Pereira, P.M.F. et al. J. High Energ. Phys. 2020, 164 (2020). https://doi.org/10.1007/JHEP07(2020)164

Used in: VLQs



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Addressing the CKM unitarity problem with a vector-like up quark

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- In [6], one up VLQ was introduced to explain the CKM unitarity problem. Still tractable in the standard PDG parameterisation. Our parameterisation is useful when $n_q > 1$.
- A Review on VLQs, in collaboration with C.C. Nishi and A.L. Cherchiglia, expected 2022 on arXiv.

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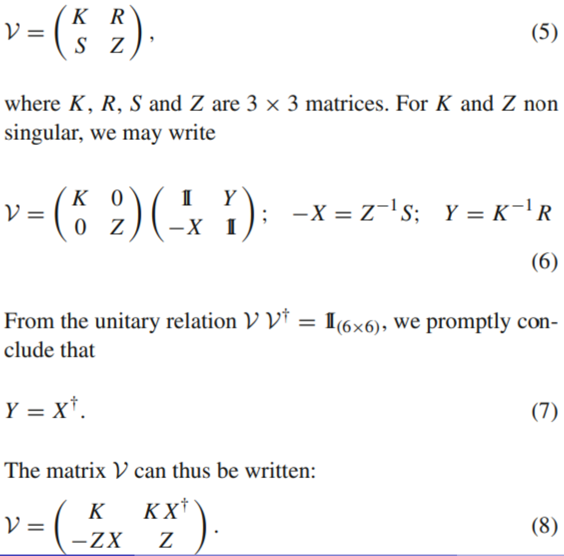
[6] Branco, G.C., Penedo, J.T., Pereira, P.M.F. et al. J. High Energ. Phys. 2021, 99 (2021). https://doi.org/10.1007/JHEP07(2021)099

The End

Thank You!

Backup Slides



$$\mathcal{V} = \begin{pmatrix} K & R \\ S & Z \end{pmatrix}, \tag{5}$$

where K, R, S and Z are 3×3 matrices. For K and Z non singular, we may write

$$\mathcal{V} = \begin{pmatrix} K & 0 \\ 0 & Z \end{pmatrix} \begin{pmatrix} \mathbb{1} & Y \\ -X & \mathbb{1} \end{pmatrix}; \quad -X = Z^{-1} S; \quad Y = K^{-1} R \tag{6}$$

From the unitary relation $\mathcal{V}\,\mathcal{V}^\dagger = \mathbb{1}_{(6\times6)}$, we promptly conclude that

$$Y = X^\dagger. \tag{7}$$

The matrix $\mathcal{V}$ can thus be written:

$$\mathcal{V} = \begin{pmatrix} K & K X^\dagger \\ -ZX & Z \end{pmatrix}. \tag{8}$$

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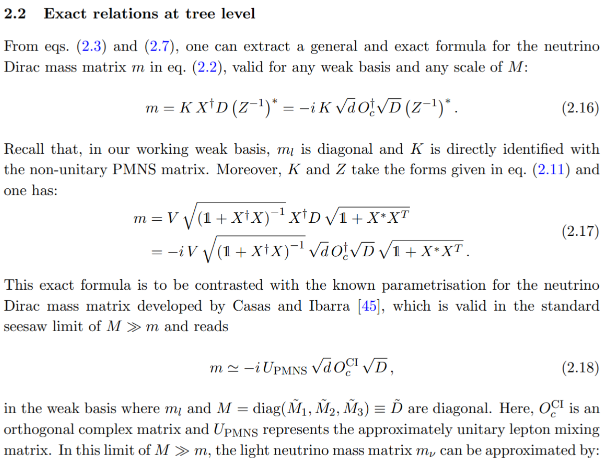

2.2 Exact relations at tree level

From eqs. (2.3) and (2.7), one can extract a general and exact formula for the neutrino Dirac mass matrix m in eq. (2.2), valid for any weak basis and any scale of M:

$$m = K\, X^\dagger D \left(Z^{-1}\right)^* = -i\, K\, \sqrt{d}\, O_c^\dagger \sqrt{D} \left(Z^{-1}\right)^* . \tag{2.16}$$

Recall that, in our working weak basis, m_l is diagonal and K is directly identified with the non-unitary PMNS matrix. Moreover, K and Z take the forms given in eq. (2.11) and one has:

$$\begin{aligned} m &= V \sqrt{\left(\mathbb{1} + X^\dagger X\right)^{-1}}\, X^\dagger D \sqrt{\mathbb{1} + X^* X^T} \\ &= -i\, V \sqrt{\left(\mathbb{1} + X^\dagger X\right)^{-1}}\, \sqrt{d}\, O_c^\dagger \sqrt{D} \sqrt{\mathbb{1} + X^* X^T} . \end{aligned} \tag{2.17}$$

This exact formula is to be contrasted with the known parametrisation for the neutrino Dirac mass matrix developed by Casas and Ibarra [45], which is valid in the standard seesaw limit of $M \gg m$ and reads

$$m \simeq -i\, U_{\text{PMNS}} \sqrt{d}\, O_c^{\text{CI}} \sqrt{D} , \tag{2.18}$$

in the weak basis where m_l and $M = \text{diag}(\tilde{M}_1, \tilde{M}_2, \tilde{M}_3) \equiv \tilde{D}$ are diagonal. Here, O_c^{CI} is an orthogonal complex matrix and U_{PMNS} represents the approximately unitary lepton mixing matrix. In this limit of $M \gg m$, the light neutrino mass matrix m_ν can be approximated by:

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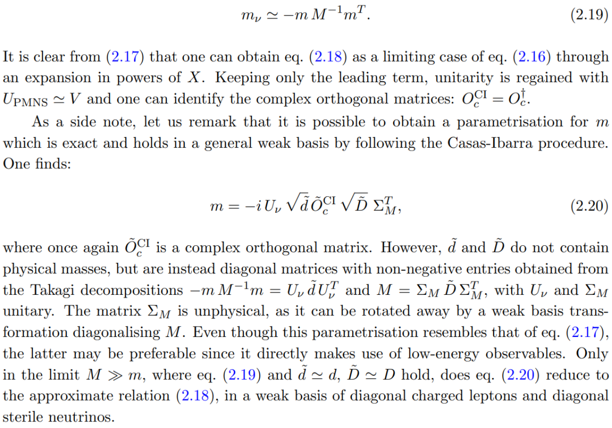

$$m_\nu \simeq -m\, M^{-1} m^T. \tag{2.19}$$

It is clear from (2.17) that one can obtain eq. (2.18) as a limiting case of eq. (2.16) through an expansion in powers of X. Keeping only the leading term, unitarity is regained with $U_{\text{PMNS}} \simeq V$ and one can identify the complex orthogonal matrices: $O_c^{\text{CI}} = O_c^\dagger$.

As a side note, let us remark that it is possible to obtain a parametrisation for m which is exact and holds in a general weak basis by following the Casas-Ibarra procedure. One finds:

$$m = -i\, U_\nu \sqrt{\tilde{d}}\, \tilde{O}_c^{\text{CI}} \sqrt{\tilde{D}}\, \Sigma_M^T, \tag{2.20}$$

where once again $\tilde{O}_c^{\text{CI}}$ is a complex orthogonal matrix. However, $\tilde{d}$ and $\tilde{D}$ do not contain physical masses, but are instead diagonal matrices with non-negative entries obtained from the Takagi decompositions $-m\, M^{-1} m = U_\nu\, \tilde{d}\, U_\nu^T$ and $M = \Sigma_M\, \tilde{D}\, \Sigma_M^T$, with U_ν and Σ_M unitary. The matrix Σ_M is unphysical, as it can be rotated away by a weak basis transformation diagonalising M. Even though this parametrisation resembles that of eq. (2.17), the latter may be preferable since it directly makes use of low-energy observables. Only in the limit $M \gg m$, where eq. (2.19) and $\tilde{d} \simeq d$, $\tilde{D} \simeq D$ hold, does eq. (2.20) reduce to the approximate relation (2.18), in a weak basis of diagonal charged leptons and diagonal sterile neutrinos.

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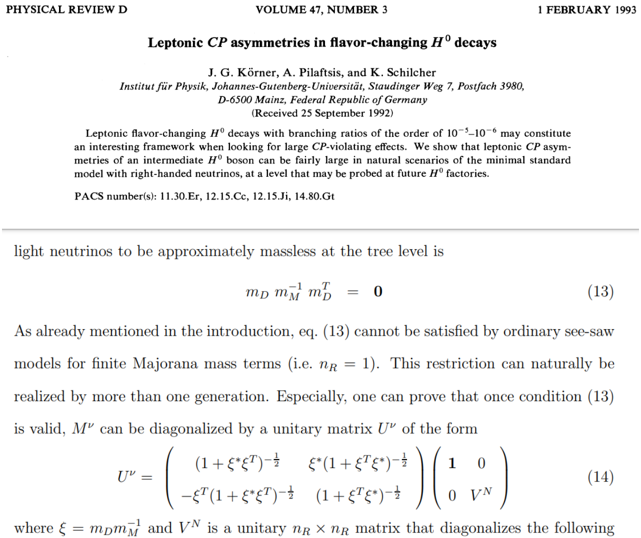

PHYSICAL REVIEW D VOLUME 47, NUMBER 3 1 FEBRUARY 1993

Leptonic *CP* asymmetries in flavor-changing H^0 decays

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(Received 25 September 1992)

Leptonic flavor-changing H^0 decays with branching ratios of the order of 10^{-5}–10^{-6} may constitute an interesting framework when looking for large *CP*-violating effects. We show that leptonic *CP* asymmetries of an intermediate H^0 boson can be fairly large in natural scenarios of the minimal standard model with right-handed neutrinos, at a level that may be probed at future H^0 factories.

PACS number(s): 11.30.Er, 12.15.Cc, 12.15.Ji, 14.80.Gt

light neutrinos to be approximately massless at the tree level is

$$m_D\, m_M^{-1}\, m_D^T \;=\; \mathbf{0} \qquad (13)$$

As already mentioned in the introduction, eq. (13) cannot be satisfied by ordinary see-saw models for finite Majorana mass terms (i.e. $n_R = 1$). This restriction can naturally be realized by more than one generation. Especially, one can prove that once condition (13) is valid, M^ν can be diagonalized by a unitary matrix U^ν of the form

$$U^\nu = \begin{pmatrix} (1+\xi^*\xi^T)^{-\frac{1}{2}} & \xi^*(1+\xi^T\xi^*)^{-\frac{1}{2}} \\ -\xi^T(1+\xi^*\xi^T)^{-\frac{1}{2}} & (1+\xi^T\xi^*)^{-\frac{1}{2}} \end{pmatrix} \begin{pmatrix} \mathbf{1} & 0 \\ 0 & V^N \end{pmatrix} \qquad (14)$$

where $\xi = m_D m_M^{-1}$ and V^N is a unitary $n_R \times n_R$ matrix that diagonalizes the following

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$$\mathcal{V} = \begin{pmatrix} U_K(\mathbb{1}_3 + X^\dagger X)^{-1/2} & U_K(\mathbb{1}_3 + X^\dagger X)^{-1/2} X^\dagger \\ -U_{\overline{K}}(\mathbb{1}_{n_R} + XX^\dagger)^{-1/2} X & U_{\overline{K}}(\mathbb{1}_{n_R} + XX^\dagger)^{-1/2} \end{pmatrix}$$

$$\mathcal{V}^T = \begin{pmatrix} (\mathbb{1}_3 + X^T X^*)^{-1/2} & -X^T(\mathbb{1}_{n_R} + X^* X^T)^{-1/2} \\ X^*(\mathbb{1}_3 + X^T X^*)^{-1/2} & (\mathbb{1}_{n_R} + X^* X^T)^{-1/2} \end{pmatrix} \begin{pmatrix} U_K^T & 0 \\ 0 & U_{\overline{K}}^T \end{pmatrix}$$

$$\xi^* = -X^T \quad ?$$

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$$-X^T = \mp i\sqrt{d}O_c^T\sqrt{D^{-1}}$$

But $\xi^* \equiv (mM^{-1})^*$...

Exact result:

$$(\mathrm{m}\ \mathrm{M}^{-1})^* = \left(\pm iK^*\sqrt{d}O_c^T\sqrt{D}(Z^{-1})\right)\left((Z^\dagger)^{-1}\left(D + X^* dX^\dagger\right)^{-1}(Z^*)^{-1}\right)$$

ξ^* and $-X^T$ roughly the same when

$$K \sim \mathbb{1}, Z \sim \mathbb{1}, X^*\,d\,X^\dagger \sim 0$$