Asymptotic symmetries in asymptotically flat spacetimes

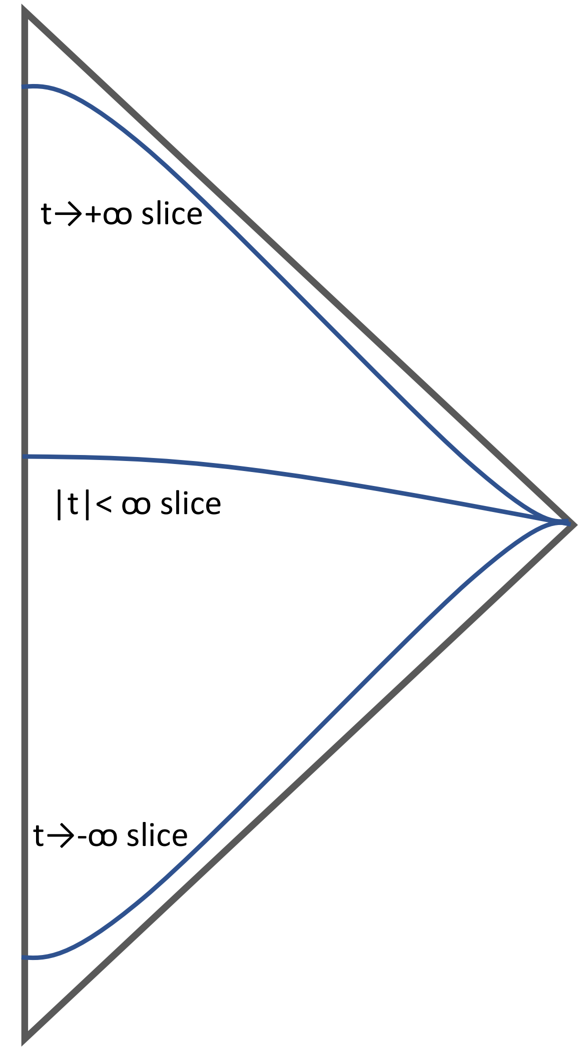
Miguel Campiglia

Instituto de Física, Facultad de Ciencias, UdelaR

Celestial Amplitudes and Flat Space Holography Workshop 2021

Field theories in (asymptotically) flat spacetimes are usually described by data at:

- t=const slice (e.g. Hamiltonian formulation)
- t →±∞ slice (e.g. scattering)
- Asymptotic symmetries may look different on each description but they should be compatible (not obvious!)
- In this talk I will mostly focus on t →±∞ description.



Asymptotically flat gravity in 4d

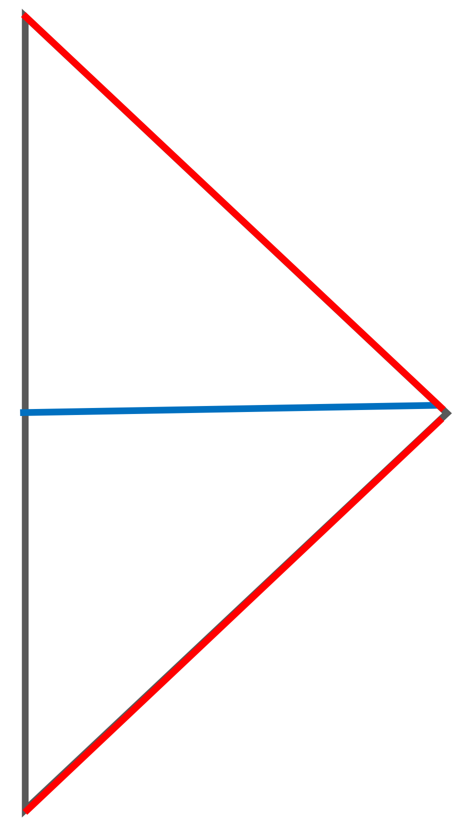
- The most elementary symmetries are (asymptotic) spacetime translations
- ➢ Notion of energy-momentum

- Understood in the 60's:
  - finite t formulation (Arnowitt Deser Misner)
  - $t \rightarrow \pm\infty$ formulation (Bondi Metzner Sachs)

- Compatibility of the 3 descriptions:

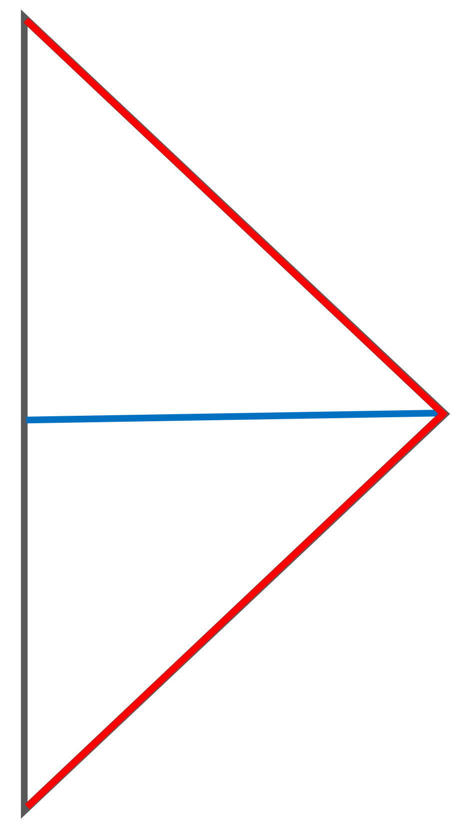
  [Ashtekar & Magnon-Ashtekar '79]

  (conservation of energy-momentum)



BMS found extra symmetries: supertranslations

- Compatibility between t →±∞ descriptions?
  [Strominger ‘13]
- Compatibility with ADM description?
  [Henneaux & Troessaert ‘18]
- BMS do not exhaust all asymptotic symmetries!
  →superrotations [Barnich & Troessaert ’10]
- What is the full asymptotic symmetry group of asymptotically flat gravity?



What is the full asymptotic symmetry group of asymptotically flat 4d gravity?

We can take guidance from

- Soft theorems ✓
- Simpler theories (e.g. d=4 QED and d≠4 gravity)
- Analysis of field equations near spatial infinity
- CCFT
- …..

- Soft theorems:
  - Classical and quantum
  - Tree vs loop level
  - Leading, subleading, sub-subleading,….

- Best (and first) stablished connection with asym. symmetries:

O(1/ω) Weinberg’s soft thm ↔ supermomentum conservation

[He, Lysov, Mitra, Strominger ’14]

  - Does not receive loop corrections
  - Valid at classical and quantum level

Higher order soft theorems

- Tree level: $O(\omega^n)$ n=0,1,…

[Cachazo Strominger '14, Hamada Shiu '18, Li Lin Zhang '18]

- Loop level: $O(\omega^n \log^{n+1}\omega)$ n=0,1,…

[Laddha Sen '18, Bhatkar Sahoo '18 Saha Sahoo Sen '19 Sahoo '20 Sahoo Sen '21]

- Clear asymptotic symmetry interpretation only for $O(\omega^{n=0})$ (superrotations) [Kapec Lysov Pasterski Strominger '14]

  Even then:

  - Classical phase-space description is quite subtle
  - Not known spatial infinity description

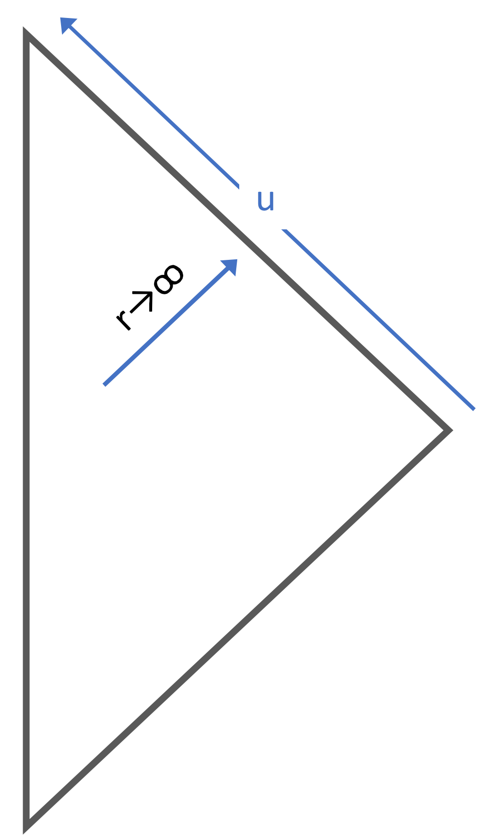
Plan for the remainder of the talk

1. Bondi expansion of spacetime metric and BMS
2. Superrotations and $O(\omega^0)$ tree-level soft thm
3. Symmetry interpretation for loop-level $O(\log \omega)$ soft thm?
4. Symmetry interpretation for tree-level $O(\omega^{n>0})$ soft thm?
5. Analogue problem in QED

Bondi expansion of the spacetime metric

$$ds^2 \overset{r\to\infty}{=} -du^2 - 2du dr + r^2\left(q_{ab} + \frac{1}{r}C_{ab}\right)dx^a dx^b + \cdots$$

- r radial coordinate
- $u \sim t - r$ retarded time
- $x = x^a$ coordinates on celestial sphere. E.g. $x = (z, \bar{z})$ or $x = (\theta, \varphi)$.
- q_{ab}: 2d metric at null infinity
- $C_{ab}(u, x)$: "free data" at null infinity = "shear" = transverse, trace-free part of metric perturbation: $h_{ij} \sim \frac{1}{r} D^a \hat{n}_i D^b \hat{n}_j C_{ab}$



Bondi expansion of the spacetime metric

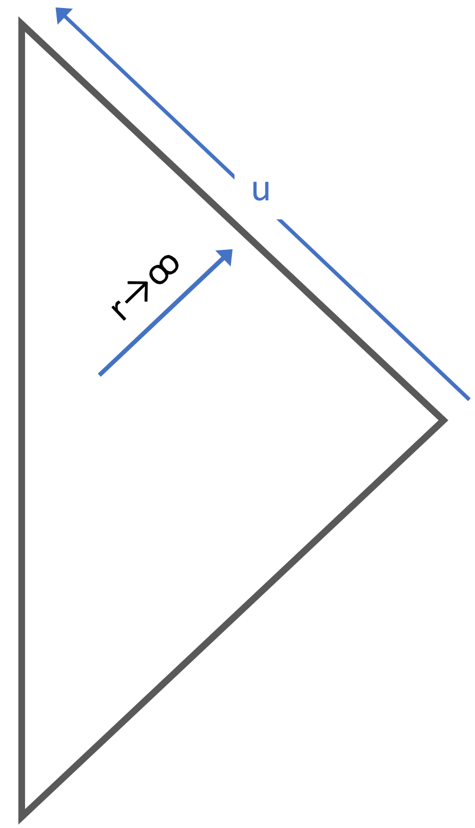
$$ds^2 \overset{r\to\infty}{=} -du^2 - 2dudr + r^2\left(q_{ab} + \frac{1}{r}C_{ab}\right)dx^a dx^b + \cdots$$

- To complete description we must specify u→±∞ fall-offs

- Most basic requirement:

$$\partial_u C_{ab}(u,x) \overset{|u|\to\infty}{\to} 0 \iff \widetilde{C}_{ab}(\omega,x) \overset{\omega\to 0}{=} O(1/\omega)$$

- Decay rate related to subleading terms in soft expansion



Large diffeomorphisms

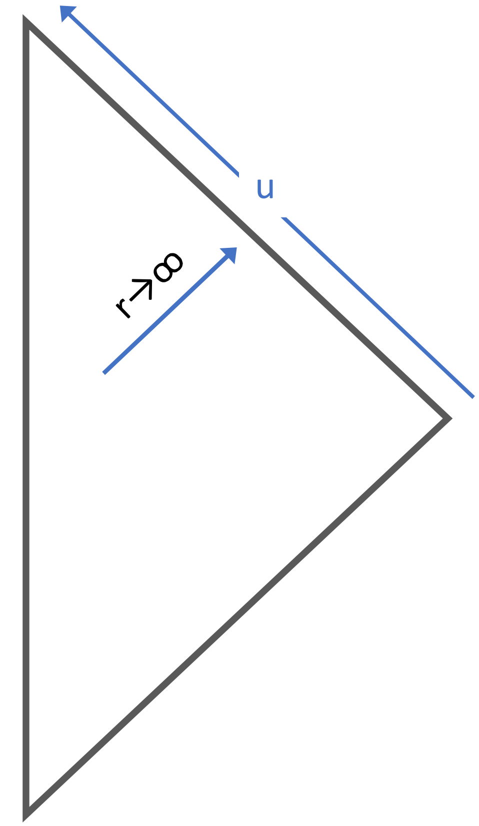
- Asymptotically Killing ↔ BMS
  - (super)translations: $\xi_f = f(x)\partial_u + \cdots$

$$\begin{aligned} \delta_f q_{ab} &= 0 \\ \delta_f C_{ab} &= f\partial_u C_{ab} - 2D_a D_b f \end{aligned}$$

  - Rotations/Boosts: $\xi_V = V^a(x)\partial_a + \frac{u}{2}D_c V^c \partial_u + \cdots, \quad V^a : \text{CKV of } q_{ab}$

$$\begin{aligned} \delta_V q_{ab} &= \mathcal{L}_V q_{ab} - D_c V^c q_{ab} = 0 \\ \delta_V C_{ab} &= \left(\mathcal{L}_V - \tfrac{1}{2}D_c V^c + \tfrac{u}{2}D_c V^c \partial_u\right) C_{ab} - u\overbrace{D_a D_b D \cdot V}^{=0} \end{aligned}$$

- Superrotations:
  - Relax CKV at isolated points [Barnich Troessaert '10]
  - Drop CKV condition altogether [MC Laddha '15]
  - Can get one from the other by appropriate limit/smearing [Donnay Pasterski Puhm '20]



Superrotations

- Need to relax Bondi expansion by
  - $O(r^2)$ terms in angular metric and
  - $O(u)$ terms in shear

$$\begin{aligned} \delta_V q_{ab} &\neq 0 \\ \delta_V C_{ab} &= (\mathcal{L}_V + \cdots) C_{ab} - u D_a D_b D \cdot V \end{aligned}$$

- Relaxed expansion implicit in [Geroch '76].
- $O(u)$ term encoded in a 2d tensor defined by

$$D^b T_{ab} = -\frac{1}{2} D_a \mathcal{R}[q]$$

- Usefully written in terms of a ``Liouville field'' [Compere Long '16, Compere Fiorucci Ruzziconi '18]

$$\begin{aligned} \mathcal{R} &= 2(e^{-2\psi} - D^2\psi) \\ T_{ab} &= 2(D_a\psi D_b\psi + D_a D_b \psi) \end{aligned}$$

- Used to define a ``renormalized'' shear

$$\hat{C}_{ab} := C_{ab} - uT_{ab} \implies \delta_V \hat{C}_{ab} = (\mathcal{L}_V + \cdots) \hat{C}_{ab}, \quad \partial_u \hat{C}_{ab} \overset{|u|\to\infty}{\to} 0$$

Large diffeo charges

- ``Canonical'' BMS charges can be obtained from symplectic structure on C_{ab} [Ashtekar Streubel '81]
- Superrotations require renormalized symplectic structure [Compere Fiorucci Ruzziconi '18] with non-local/non-covariant counterterms [Flanagan Prabhu Shehzad '19]
- So far this has only been treated with ``tree-level'' fall-offs

$$\partial_u C_{ab}(u,x) \overset{|u|\to\infty}{\to} O(1/u^{2+\delta}) \iff \widetilde{C}_{ab}(\omega,x) \overset{\omega\to 0}{=} O(1/\omega) + O(\omega^0)$$

- Resulting charge consistent with tree-level subleading soft theorem [Kapec Lysov Pasterski Strominger '14]

- Superrotation charge (and symplectic structure) can be improved to obtain charge algebra closure of superrotation/supertranslations without affecting soft theorem consistency [MC Peraza, MC Laddha] (see Alok's talk)

Beyond (tree-level) superrotations: Higher order in soft expansion

- $O(\omega^n)$ $n>0$ tree-level soft thms could arise by further relaxation of Bondi fall-offs to allow for certain $O(r^n)$ vector fields [MC Laddha '16 Compere '19]

- Phase space/renormalized symplectic structure has not been worked out
  - ➢ No handle on vector field/charge algebra

  (Recent progress from the CCFT perspective in the single-helicity sector [Guevara Himwich Pate Strominger '21])

- Similar structure in tree-level QED [MC Laddha '18, Peraza to appear] and tree-level YM [MC Peraza to appear]

Beyond (tree-level) superrotations: Loop-corrected charges?

- ``Loop-level'' fall-offs (Laddha, Sahoo, Sen, …)

$$\partial_u C_{ab}(u,x) \overset{|u|\to\infty}{=} O(1/u^2) \iff \widetilde{C}_{ab}(\omega,x) \overset{\omega\to 0}{=} O(1/\omega) + O(\ln\omega)$$

- Tree-level superrotation charge ill-defined under these fall-offs

$$Q_V^{\text{soft}} \propto \int du u \partial_u C_{ab}$$

- Expectation: loop-level renormalization of symplectic structure/charge should be consistent with loop-level O(ln ω) soft theorem [Sahoo Sen '18]
- Expectation based on analogue problem for subleading soft photons [MC Laddha '19] and existence of loop-corrected celestial stress tensor [He Kapec Raclariu Strominger '17]

Subleading photons

- Tree-level $O(\omega^0)$ subleading soft photon theorem can be written as a conservation of certain charges Q_Y parametrized by sphere vector fields [Lysov, Pasterski, Strominger '14]
- Charges can be interpreted as O(r) large gauge charges [MC Laddha '16]

$$\Lambda(r,u,x) \stackrel{r\to\infty}{=} r\lambda(x) + \cdots , \quad Y^a = D^a\lambda$$

- Similar to superrotations, these require an extension of standard $r\to\infty$ and $u\to\pm\infty$ fall-offs of gauge fields and consequent renormalization of symplectic structure [Peraza to appear]
- Loop-level $O(\ln \omega)$ soft photon theorem can be written in terms of a loop-corrected version of Q_Y charges [MC Laddha '19 Bhatkar '20]
- Symmetry/phase space interpretation still not understood

Soft factorization and loop corrections

- Consider the amplitudes

$$\mathcal{A}_{n+1}(\underbrace{\vec{p}_1,\ldots,\vec{p}_n}_{\text{charged particles}},\underbrace{(\omega\hat{q},\epsilon)}_{\text{photon}}) \quad \text{and} \quad \mathcal{A}_n(\vec{p}_1,\ldots,\vec{p}_n)$$

  in the limit $\omega \to 0$

- Beyond tree-level $\mathcal{A}_{n+1}$ and $\mathcal{A}_n$ are IR divergent
- However ratio $\mathcal{A}_{n+1}/\mathcal{A}_n$ is IR finite at leading [Weinberg '65] and subleading [Sahoo, Sen '18] order

$$\frac{\mathcal{A}_{n+1}(\vec{p}_1,\ldots,\vec{p}_n,(\omega\hat{q},\epsilon))}{\mathcal{A}_n(\vec{p}_1,\ldots,\vec{p}_n)} \overset{\omega\to 0}{=} \frac{1}{\omega}S^{(0)} + \ln\omega\, S^{(\ln)} + O(\omega^0)$$

- $S^{(0)} = O(e)$ does not receive loop corrections
- $S^{(\ln)} = O(e^3)$ appears at 1-loop and does not receive higher order corrections

Soft factors

- $S^{(0)} = \sum_{i \in \text{in,out}} e_i \frac{\epsilon \cdot p_i}{q \cdot p_i}$ $\qquad q^\mu = (1, \hat{q})$
- $S^{(\ln)} = S_I^{(\ln)} + S_{II}^{(\ln)}$ with

$$S_I^{(\ln)} = \sum_{i,j \in \text{in}} f(p_i, p_j) + \sum_{i,j \in \text{out}} f(p_i, p_j)$$

$$S_{II}^{(\ln)} = \sum_{i,j \in \text{in,out}} g(p_i, p_j)$$

- $S_I^{(\ln)}$ determines $\ln \omega$ in classical radiation

Soft factorization as a Ward identity

- From

$$\mathcal{A}_{n+1}(\omega\hat{q}) \overset{\omega\to 0}{=} \frac{1}{\omega} S^{(0)} \mathcal{A}_n + \ln\omega\, S^{(\ln)} \mathcal{A}_n + O(\omega^0)$$

we can write two identities:

$$\lim_{\omega\to 0} \omega \mathcal{A}_{n+1}(\omega\hat{q}) = S^{(0)} \mathcal{A}_n \quad \text{(leading factorization)}$$

$$\lim_{\omega\to 0} \partial_\omega[\omega^2 \partial_\omega \mathcal{A}_{n+1}(\omega\hat{q})] = S^{(\ln)} \mathcal{A}_n \quad \text{(subleading factorization)}$$

- Leading factorization can be understood as a "Ward identity" [Strominger et al '14]:

$$\langle \text{out}| Q_+^{(0)} \mathcal{S} - \mathcal{S} Q_-^{(0)} |\text{in}\rangle = 0$$

- Does subleading factorization admit similar interpretation?

$$\langle \text{out}| Q_+^{(\ln)} \mathcal{S} - \mathcal{S} Q_-^{(\ln)} |\text{in}\rangle = 0 \quad ?$$

$Q^{(\ln)}_{\mathscr{I}^+}$

$x^A = (z, \bar{z})$

- $\mathcal{E}_z(u, \hat{x}) = \int_0^\infty \omega(a_+(\omega\hat{x})e^{-i\omega u} + a_-^\dagger(\omega\hat{x})e^{i\omega u})d\omega$

  $(z, \bar{z})$ stereographic coordinates for $\hat{x}$

$$Q^{(\ln)}_{\mathscr{I}^+} = -\int Y^A(\hat{x})\partial_u(u^2\mathcal{E}_A(u, \hat{x}))d^2\hat{x}du$$

$\mathcal{E}_A = \partial_u A_A$

$$= \lim_{\omega\to 0} \partial_\omega \omega^2 \partial_\omega \int d^2\hat{q}\left[Y^z(\hat{x})a_+(\omega\hat{x}) + Y^{\bar{z}}(\hat{x})a_-^\dagger(\omega\hat{x})\right] + h.c.$$

- This will produce LHS of

$$\lim_{\omega\to 0} \partial_\omega[\omega^2\partial_\omega \mathcal{A}_{n+1}(\omega\hat{q})] = S^{(\ln)}\mathcal{A}_n$$

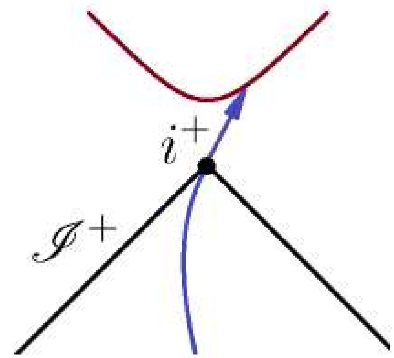
- RHS should come from $Q^{(\ln)}_{i^\pm}$

Fields at i^+

- To construct $Q^{(\ln)}_{i^+}$ we study fields at time-like infinity

$$x^\mu = \tau X^\mu, \quad \tau \to \infty \quad \text{with} \quad X^\mu X_\mu = -1$$

In the Penrose diagram this can be thought of as a 'blow-up' of i^+ into a unit hyperbolid as in [Ashtekar Hansen '78] for i^0



- Charged scalar field asymptotics:

$$\varphi(\tau, X) \overset{\tau\to\infty}{=} \tau^{-3/2} e^{ie\ln\tau V(X)} \left(b(X)e^{-i\tau m} + c^*(X)e^{i\tau m}\right) + \cdots$$

- $V(X) = \lim_{\tau\to\infty} \tau X^\mu A_\mu(\tau X)$
- $b(X), c(X)$: Fourier modes of free field evaluated at $p^\mu = mX^\mu$

- Asymptotics of current $j_\mu = ie\varphi^*\partial_\mu\varphi + c.c.$

$$j_\tau(\tau,X) \overset{\tau\to\infty}{=} \frac{1}{\tau^3}\rho(X) + \cdots\,, \quad j_\alpha(\tau,X) \overset{\tau\to\infty}{=} \frac{\ln\tau}{\tau^3}J_\alpha(X) + \cdots$$

$$\rho = e[b^*b - c^*c], \qquad J_\alpha = e^2[b^*b + c^*c]\partial_\alpha V$$

- 'Gauss Law' at i^+:

$$\mathcal{D}^2 V(X) = \rho(X) \implies V = \frac{1}{\mathcal{D}^2}\rho + V_{\mathsf{hom}}(X) \quad \text{with} \quad \mathcal{D}^2 V_{\mathsf{hom}} = 0$$

- i^+ contribution to $Q^{(\ln)}$

$$Y^\alpha(X) = \int d^2\hat{q}\, G^\alpha_A(X,\hat{q}) Y^A(\hat{q})$$

$$Q^{(\ln)}_{i^+} = \int d^3X Y^\alpha(X) J_\alpha(X) \quad (\star)$$

- Quantum charge given by $(\star)$ with $J_\alpha \to \widehat{J}_\alpha$,

$$\widehat{J}_\alpha = e^3 : [b^\dagger b + c^\dagger c]\partial_\alpha \frac{1}{\mathcal{D}^2}[b^\dagger b - c^\dagger c] : + e^2[b^\dagger b + c^\dagger c]\partial_\alpha \widehat{V}_{\mathsf{hom}}$$

- $Q_+^{(\ln)} = Q_{i^+}^{(\ln)} + Q_{\mathscr{I}^+}^{(\ln)}$

- $Q_-^{(\ln)}$ constructed similarly

- If $\widehat{V}_{\text{hom}} = 0$, evaluating

$$\langle \text{out}| Q_+^{(\ln)} \mathcal{S} - \mathcal{S} Q_-^{(\ln)} |\text{in}\rangle = 0$$

leads to

$$\lim_{\omega \to 0} \partial_\omega [\omega^2 \partial_\omega \mathcal{A}_{n+1}(\omega \hat{q})] = S_I^{(\ln)} \mathcal{A}_n$$

- We are missing $S_{II}^{(\ln)}$ piece

- V_{hom} comes from the free part of the photon field

$$V_{\text{hom}}(X) = \lim_{\tau\to\infty} \tau X^\mu A_\mu^{\text{free}}(\tau, X) \quad (\star)$$

where

$$A_\mu^{\text{free}}(x) = \sum_{h=\pm} \int d^3\vec{p}\, \epsilon_\mu^h a_h(\vec{p}) e^{ip\cdot x} + h.c.$$

- Evaluating $(\star)$ one finds $\qquad q^\mu = (1, \hat{q})$

$$\hat{V}_{\text{hom}}(X) = \sum_{h=\pm} \int d^2\hat{q} \frac{X\cdot\epsilon^h}{X\cdot q}\left[i \lim_{\omega\to 0} \omega \left(\hat{a}_h(\omega\hat{q}) - \hat{a}_h^\dagger(\omega\hat{q})\right)\right]$$

- Classically $V_{\text{hom}} = 0$ due to $u\to\infty$ fall-offs.

- In quantum theory $\langle \text{out}|\widehat{V}_{\text{hom}}\mathcal{S}|\text{in}\rangle \neq 0$

- $e^2[b^\dagger b + c^\dagger c]\partial_\alpha \widehat{V}_{\text{hom}} \subset \widehat{J}_\alpha$ produces missing $S_{II}^{(\ln)} \quad \Longrightarrow$

$$\langle \text{out}|Q_+^{(\ln)}\mathcal{S} - \mathcal{S}Q_-^{(\ln)}|\text{in}\rangle = 0 \iff \lim_{\omega\to 0}\partial_\omega[\omega^2\partial_\omega \mathcal{A}_{n+1}(\omega\hat{q})] = S^{(\ln)}\mathcal{A}_n$$

Other relevant topics I did not get time/knowledge to review:

- Memory effects in GW from binary coalescence [Nichols, …]
- Asym symm in asym flat gravity in d=3 [Barnich, Compere, Geiller, Gomberoff, Gonzalez, Oblak, Troessaert, …] and d>4 [Hollands Ishibashi '03 Tanabe, S. Kinoshita, and T. Shiromizu '11 Kapec Lysov Pastersk Strominger '17 Aggarwal '18 Colferai Lionetti '20 Campoleoni Francia Heissenberg '21 Capone '21]
- Group theoretical aspects of (extended) BMS [Barnich Ruzziconi '21 Prinz Schmeding '21]
- Asym symm from hyperboloid description of spatial infinity [Ashtekar Hansen '78 Virmani '11 Compere Dehouck '11 Troessaert '17 Prabhu '19, …]
- Asym symm in ADM formulation (Henneaux's talk)
- ….

Thank you for listening!