Analysis of asymptotic symmetries at spatial infinity

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Analysis of asymptotic symmetries at spatial infinity - The case of supergravity

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the asymptotic symmetry superalgebra of which is an extension of the BMS_4 algebra with an infinite number of fermionic generators, which are the "square roots" of all the supertranslations.

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Can one understand why such non-linear terms arise in the algebra ?

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- Symmetries are phase space transformations that leave the action invariant and for which there is therefore a well-defined moment map.
- One can accordingly apply standard theorems of Hamiltonian mechanics.

Finite-dimensional supersymmetric extension

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An equivalent analysis can be performed at spatial infinity. (MH + J. Matulich + T. Neogi, arXiv :2004.07299 [hep-th])

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$$T = i\epsilon_1^T \epsilon_2, \quad W = i\epsilon_1^T \gamma_i \epsilon_2 \, n^i \, .$$

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(Hamiltonian parametrization of BMS_4, see C. Troessaert, arXiv :1704.06223 [hep-th] and MH+CT, arXiv :1801.03718 [gr-qc] : $\xi^\perp \sim T(\theta,\varphi)$, $\xi^i \sim \partial^i (rW(\theta,\varphi)$, T even, W odd, $(T,W) \sim \alpha$ where $\alpha(\theta,\varphi)$ is the parameter of BMS_4 supertranslations in the null infinity description)

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The parameters $T(\theta,\varphi)$ and $W(\theta,\varphi)$ of BMS_4 supertranslations and the fermionic parameters $\epsilon(\theta,\varphi)$ of their "square roots" transform in such representations.

Lorentz generators

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The Lorentz algebra $so(3,1)$ reads

$$[A_1, A_2] = A_3, \quad [A_2, A_3] = A_1, \quad [A_3, A_1] = A_2 \quad \Leftrightarrow \quad [A_i, A_j] = \epsilon_{ijk} A_k,$$

$$[A_i, B_j] = \epsilon_{ijk} B_k,$$

$$[B_i, B_j] = -\epsilon_{ijk} A_k.$$

The A_k's are the generators of spatial rotations, while the B_k's are the generators of boosts. They transform as vectors under spatial rotations. It is useful to define

$$H_3 = iA_3, \qquad H_+ = iA_1 - A_2, \qquad H_- = iA_1 + A_2$$

and similarly

$$F_3 = iB_3, \qquad F_+ = iB_1 - B_2, \qquad F_- = iB_1 + B_2\,.$$

Irreducible representations

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General results on irreducible (infinite-dimensional, not necessarily unitary) representations of $so(3,1)$ by Gel'fand, Naimark, Harish-Chandra.

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$$R = \oplus R_l$$

where l (the weight/$so(3)$-spin of R_l) is an non-negative integer or half-integer. The sum can be infinite.

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If the representation R of $so(3,1)$ is irreducible, each representation R_l of $so(3)$ that appears in R occurs at most once, i.e., is non degenerate.

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Let $l_0 \geq 0$ be the lowest $so(3)$-spin that appears in the decomposition of R. Then the $so(3)$-spins are $l = l_0, l_0 + 1, \cdots$, with a maximum value $l_0 + n$ (n integer) if the representation R is finite-dimensional, and no upper bound if R is infinite-dimensional.

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In a standard basis ξ_{lm} of R_l, one has

$$H_3 \xi_{lm} = m \xi_{lm},$$
$$H_- \xi_{lm} = \sqrt{(l+m)(l-m+1)}\, \xi_{l,m-1},$$
$$H_+ \xi_{lm} = \sqrt{(l+m+1)(l-m)}\, \xi_{l,m+1},$$

($m = -l, -l+1, \cdots, l-1, l$). These relations are unaffected if we rescale all ξ_{lm}'s by the same m-independent factor $h(l)$, $\xi_{lm} \to h(l) \xi_{lm}$.

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The B_i's are vector operators, so that by the Wigner-Eckart theorem, they can only change the $so(3)$-spin l by ± 1, i.e. $B_i \xi_{lm}$ is a linear combination of vectors $\xi_{l'm'}$ with $l' = l-1, l, l+1$.

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One finds explicitly, upon suitable normalization of the ξ_{lm}'s,

$$F_3 \xi_{lm} = c_l \sqrt{l^2 - m^2}\, \xi_{l-1,m} - a_l m \xi_{l,m} - c_{l+1} \sqrt{(l+1)^2 - m^2}\, \xi_{l+1,m}$$

$$F_+ \xi_{lm} = c_l \sqrt{(l-m)(l-m-1)}\, \xi_{l-1,m+1} - a_l \sqrt{(l-m)(l+m+1)}\, \xi_{l,m+1} + c_{l+1} \sqrt{(l+m+1)(l+m+2)}\, \xi_{l+1,m+1}$$

$$F_- \xi_{lm} = -c_l \sqrt{(l+m)(l+m-1)}\, \xi_{l-1,m-1} - a_l \sqrt{(l+m)(l-m+1)}\, \xi_{l,m-1} - c_{l+1} \sqrt{(l-m+1)(l-m+2)}\, \xi_{l+1,m-1}$$

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$$a_l = \frac{i l_0 l_1}{l(l+1)}, \qquad c_l = \frac{i}{l}\sqrt{\frac{(l^2 - l_0^2)(l^2 - l_1^2)}{4l^2 - 1}}$$

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It is important to realize that l_0 and l_1 enter symmetrically the formulas giving the coefficients a_l and c_l. However, only l_0 is required to be a non-negative integer or half-integer.

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A representation with minimum $so(3)$spin l_0 contains the weights $l_0, l_0+1, \cdots$.

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The representation is finite-dimensional if this series stops. Assume that it stops at $l_{max} = l_0 + n$ for some integer n. This will occur if and only if $c_{l_{max}+1} = c_{l_0+n+1} = 0$, which will be the case if and only if $l_0 + n + 1 = |l_1|$. Thus, a representation is finite-dimensional if and only if $|l_1| - l_0 - 1$ is a non-negative integer. $|l_1| - 1$ is then the highest $so(3)$-spin occuring in the representation.

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Only the finite-dimensional trivial (scalar) representation is unitary ($l_0 = 0$, $l_1 = \pm 1$)

BMS$_4$ supertranslations

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It is a multiplier representation of the type

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with $k = 1$.

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One can make the change of parametrization (basis) $\tau \leftrightarrow T, W$.

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where $\xi^\perp = b_i x^i$ (boosts) and ξ^k is a rotation Killing vector.

The parity of χ is preserved by the Lorentz transformations. The representation that contains the constant spinors is described by even χ's and we take therefore χ to be even.

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but as a representation of the Lorentz algebra, it is indecomposable.

Paradox - Closure of fermionic symmetries and Jacobi identity

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But as we saw, this is not so !

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Only the heads have.

Conclusions and comments

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Both contain the BMS_4 algebra and the super-Poincaré algebra, with $[\text{Head}^F, \text{Head}^F] = \text{Head}^B$.

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