Corfu BSM Workshop **05.09.2021**

Minimal Flipped SU(5) from F-theory

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GREECE

Outline of the Talk

- Introductory remarks
- $\mathcal{F}$-Theory basics
- Building $\mathcal{F}$-Theory GUTs
- Minimal $\mathcal{F}$*lipped* $\mathcal{F}$-$SU(5)$
- Some Low Energy Implications
- Concluding Remarks

Ordinary GUTs vs F-GUTs

★ *Old* GUTs: **Interesting features**

▲ Gauge coupling unification

▲ Assembling of SM fermions in a few irreps.
($\in \bar{5}$, $10 \supset \underline{16}, \underline{10} \subset \underline{27} \cdots$.)

▲ Charge Quantisation

★ Deficiencies

▲ fermion mass hierarchy and mixing not predicted

▲ Yukawa Lagrangian poorly constrained

▲ Baryon number non-conservation

... Solution requires new insights ... such as:
Discrete and $U(1)$ symmetry extensions

F-GUTs: New Ingredients from F-theory

★ Discrete and $U(1)$ symmetries:

• necessary tools to suppress or eliminate undesired superpotential terms

★ Fluxes :

• induce chirality, ... truncate GUT irreps, ..., symmetry breaking

★ "Internal" Geometry :

• ... determines SM arbitrary parameters from a handful of topological properties

$\mathcal{B}$

★ **F-theory** and Elliptic Fibration★

$\Downarrow$

F-theory : Type II-B superstring with 7-branes

II-B: *closed string spectrum obtained by combining left and right moving open strings with NS and R-boundary conditions:*

$$(NS_+, NS_+),\ (R_-, R_-),\ (NS_+, R_-),\ (R_-, NS_+)$$

Bosonic spectrum, *notation***:**

(NS_+, NS_+): graviton, dilaton and 2-form KB-field:

$$g_{\mu\nu},\ \phi,\ B_{\mu\nu} \to B_2$$

(R_-, R_-): scalar, 2- and 4-index fields (*p-form potentials*)

$$C_0, C_{\mu\nu}, C_{\kappa\lambda\mu\nu} \to C_p,\ p = 0, 2, 4$$

Notations and Definitions (*bosonic part*)

1. The dilaton ϕ determines the *string coupling:*

$$g_{IIB} = e^{\phi}$$

2. *The RR axion C_0, and the dilaton ϕ are combined to one modulus, the axion-dilaton field:*

$$\boxed{\tau = C_0 + i\, e^{-\phi} \to C_0 + \frac{i}{g_{IIB}}}$$

3. The importance of τ is that it can be used to write the type **IIB** action in an $SL(2, Z)$ invariant way

$$S_{IIB} \quad \propto \quad \int d^{10}x\sqrt{-g}\left(R - \frac{1}{2}\frac{\partial_\mu \tau \partial^\mu \bar{\tau}}{(\mathrm{Im}\tau)^2} - \frac{1}{2}\frac{|G_3|^2}{\mathrm{Im}\tau} - \frac{1}{4}|F_5|^2\right)$$
$$-\frac{i}{4}\int \frac{1}{\mathrm{Im}\tau} C_4 + G_3 \wedge \tilde{G}_3$$

4. Indeed, it can be observed that this action in invariant under the trasformations:

$$\tau \to \frac{a\tau + b}{c\tau + d}$$

5. Due to $SL(2, Z)$ invariace, τ can vary accordingly, while leaving the action invariant.

6. Recall that the imaginary part is

$$\boxed{\mathrm{Im}\tau = \frac{1}{g_{IIB}}}$$

which implies that there exist values of τ leading to strongly coupled regions.

A few words about

Elliptic Curves & Elliptic Fibration

An extremely important implication of the variation of the axion-dilaton τ *is that it gives rise to an elliptic fibration over the physical space-time*

In order to see this, let's start with II-B theory which is defined in 10-d space described by:

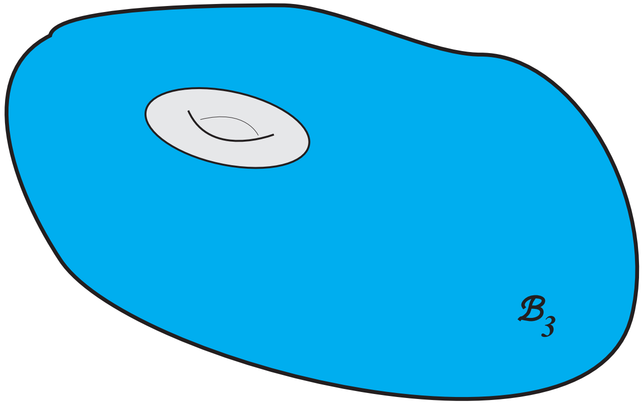
$$\mathcal{R}^{3,1} \times \mathcal{B}_3$$

where

▲ $\mathcal{R}^{3,1}$ is the usual 4-d space-time

▲ $\mathcal{B}_3$ Calabi-Yau (CY) manifold of 3 complex dimensions (3-fold)

▲ **IIB** *on* : $\mathcal{R}^{3,1} \times \mathcal{B}_3$



▲ ▲ F-theory is compactified on an **elliptically fibered manifold** where $\mathcal{B}_3$ is the base of the fibration.

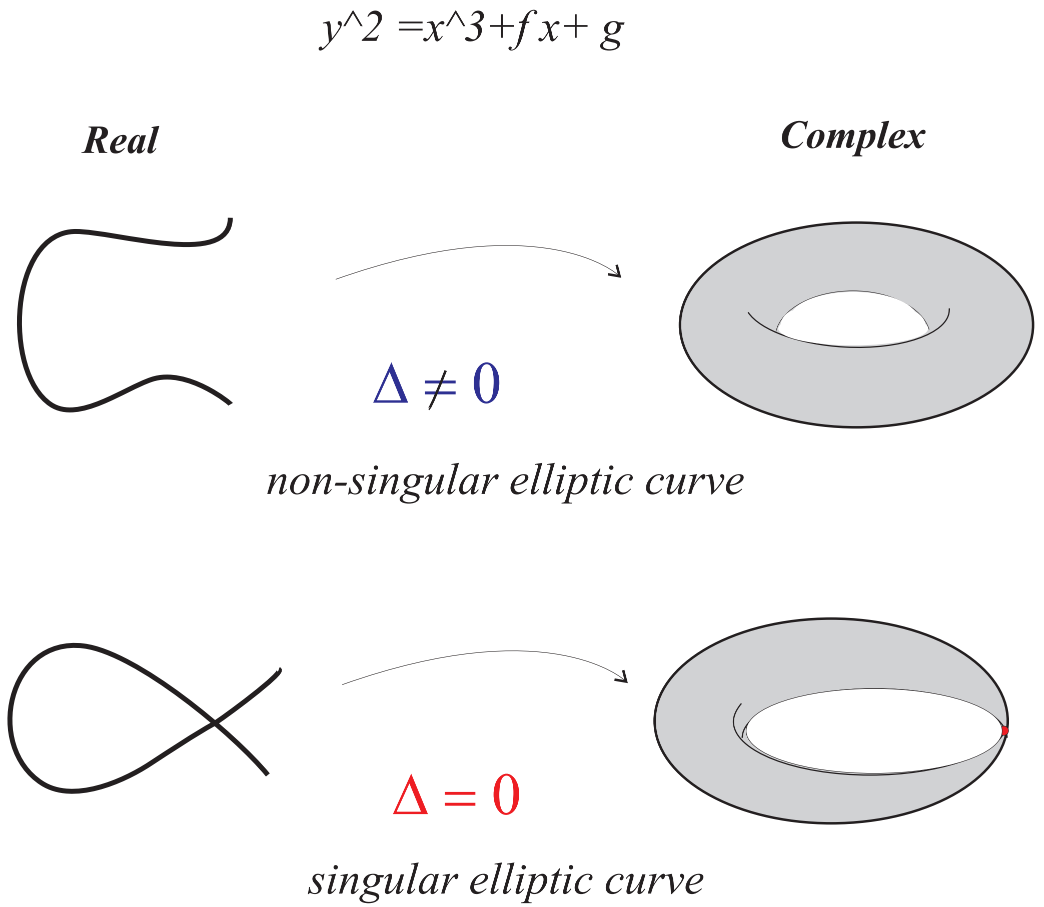
Mathematically, the **Elliptic Fibration** is described by the

$\mathcal{W}$eierstraß $\mathcal{E}$quation

the latter being a cubic equation with a rational point on it.

Recall that $\Rightarrow$

★ **Weierstraß** equation with complex coefficients defines a Torus



Non-singular curve "upgrades" to normal torus

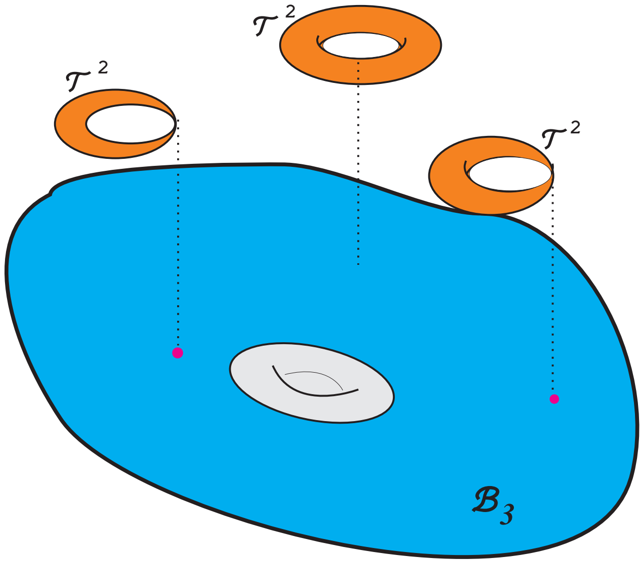
Singular curve corresponds to torus with a pinched radius.

Recall now that the axion-dilaton modulus $\tau = C_0 + \imath\, e^{-\phi}$ *can be thought as describing a torus*

Motivated by this, we make a continuous mapping of τ *to the points of the base* B_3. *We say that:*

▲ F-theory is defined on $\mathcal{R}^{3,1} \times \mathcal{X}$ ▲

where $\mathcal{X}$, elliptically fibered **CY** 4-fold over B_3



CY 4-fold: Red points: pinched torus

In $\mathcal{W}$eierstraß $\mathcal{F}$orm the **Elliptic Curve** is described by the vanishing locus of the polynomial

$$y^2 - (x^3 + f(z)\, x w^4 + g(z) w^6) = 0$$

... *with properties:*

1. equivalence relations of homogeneous (projective) coordinates $x, y, w \simeq (\lambda^2 x, \lambda^3 y, \lambda w)$
2. $f(z)$, $g(z) \rightarrow 8^{th}$ and 12^{th} degree polynomials.

Two Important Quantities characterise the fibration:

1. Discriminant: $\Delta(z) = 4\, f^3 + 27 g^2$
2. and j-invariant: $j(\tau) = \frac{4(24 f(z))^3}{\Delta(z)}$

- The zeros of the discriminant determine fiber singularities:

$$\Delta = \prod_{i=1}^{24}(z - z_i) = 0 \;\Rightarrow\; 24 \text{ roots } z_i$$

Coordinate z and modulus τ related through:

$$\begin{aligned} j(\tau) &= 4\,\frac{(24f(z))^3}{\Delta(z)} \propto e^{-2\pi i\tau} + 744 + \mathcal{O}(e^{2\pi i\tau}) \\ &\propto e^{2\pi/g_s} e^{-2\pi i C_0} + 744 + \cdots \end{aligned} \tag{1}$$

Solution gives τ around the zeros z_i of Δ:

$$\tau \approx \frac{1}{2\pi i}\log(z - z_i)$$

Due to multivalued log function, circling around z_i roots, τ shifts:

$$\tau \to \tau + 1 \;\Rightarrow C_0 \to C_0 + 1 \to$$

In other words, τ, C_0 undergo Monodromy.

Interpretation:

At $z = z_i$ $\exists$ source of RR-flux which is interpreted as a:

$D7$-brane at $z = z_i$, **normal** to the "tangent plane"

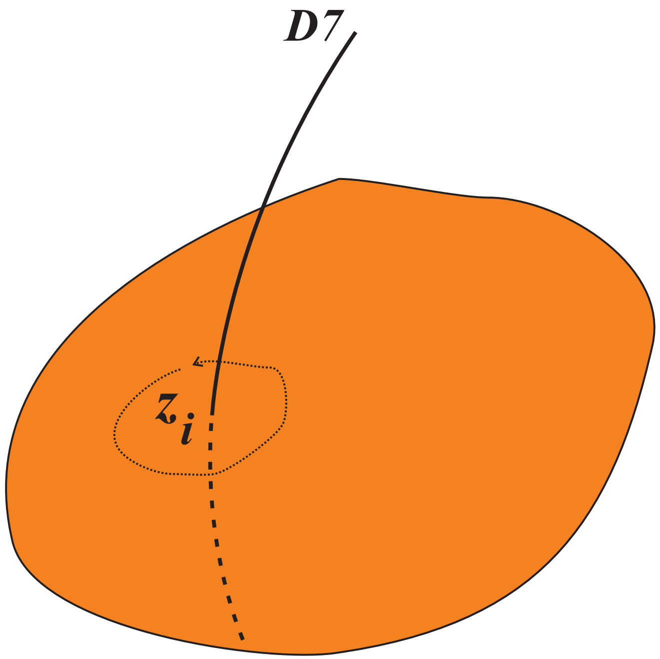


Figure 1: Moving around z_i, $\log(z) \to \log|z| + i(2\pi + \theta)$ and $\tau \to \tau + 1$

$D7$ branes are magnetic sources for the type IIB RR axion C_0

Geometric Singularities & Kodaira Classification:

- Type of Manifold **singularity** is specified by the vanishing order of Δ and the polynomials $f(z)$, $g(z)$ of Weierstrass eq:

$$y^2 = x^3 + f(z)\,x + g(z)$$

- **Singularities** are classified in terms of $\mathcal{A}\,\mathcal{D}\,\mathcal{E}$ Lie groups.

Interpretation of geometric singularities

$$\Downarrow$$

$$\boxed{CY_4\text{-}\textbf{Singularities} \rightleftarrows \textbf{gauge symmetries}}$$

$$\textbf{Groups} \quad \to \quad \begin{cases} SU(n) \\ SO(m) \\ \mathcal{E}_n \end{cases}$$

$\mathcal{C}$

The Non Abelian Sector

Rôle of Geometric Singularities on EFTs

The Kodaira classification: w.r.t. vanishing order of $f(z)$, $g(z)$ and $\Delta(z) = 4f(z)^3 + 27g(z)^2$. (see Morrison, Vafa hep-th/9603161)

ord($f(z)$)	ord($g(z)$)	ord($\Delta(z)$)	fiber type	Singularity
0	0	n	I_n	A_{n-1}
≥ 1	1	2	II	none
1	≥ 2	3	III	A_1
≥ 2	2	4	IV	A_2
2	≥ 3	$n+6$	I_n^*	D_{n+4}
≥ 2	3	$n+6$	I_n^*	D_{n+4}
≥ 3	4	8	IV^*	$\mathcal{E}_6$
3	≥ 5	9	III^*	$\mathcal{E}_7$
≥ 4	5	10	II^*	$\mathcal{E}_8$

Tate's Algorithm

$$\boxed{y^2 + a_1 x\, y + a_3 y = x^3 + a_2\, x^2 + a_4 x + a_6}$$

Table: *Classification of Elliptic Singularities w.r.t. vanishing order of Tate's form coefficients* a_i*:*

Group	a_1	a_2	a_3	a_4	a_6	Δ
$SU(2n)$	0	1	n	n	$2n$	$2n$
$SU(2n+1)$	0	1	n	$n+1$	$2n+1$	$2n+1$
$SU(5)$	0	1	2	3	5	5
$SO(10)$	1	1	2	3	5	7
$\mathcal{E}_6$	1	2	3	3	5	8
$\mathcal{E}_7$	1	2	3	3	5	9
$\mathcal{E}_8$	1	2	3	4	5	10

$\mathcal{EXAMPLE}$

Choose "Tate" coefficients as follows:

$$a_1 = -b_5,\; a_2 = b_4 z,\; a_3 = -b_3 z^2,\; a_4 = b_2 z^3,\; a_6 = b_0 z^5$$

Vanishing orders of $a_i's:\; z^0,\; z^1,\; z^2,\; z^3,\; z^5$ & $\Delta \sim z^7$

$\Rightarrow$ Weierstraß' equation for the $SU(5)$ singularity

$$y^2 = x^3 + b_0 z^5 + b_2 x z^3 + b_3 y z^2 + b_4 x^2 z + b_5 x y$$

Associated spectral cover obtained by defining homogeneous coordinates $z \to U,\; x \to V^2,\; y \to V^3$ and affine parameter $s = \frac{U}{V},\; \Rightarrow$

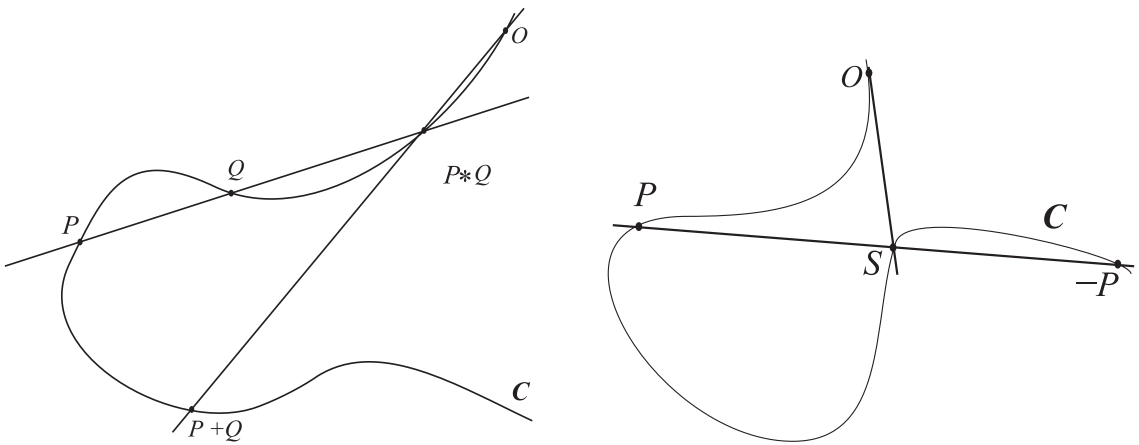
$$\mathcal{C}_5 :\; \boxed{0 \;=\; b_0 s^5 + b_2 s^3 + b_3 s^2 + b_4 s + b_5}$$

$\mathcal{D}$

The Abelian Sector

Rôle of Rational Points on Elliptic Curves

The Group Law on Elliptic Curves



The addition law: $P + Q$ (left).

$(P, Q = \textit{rational} \to P + Q \ \textit{rational}.)$

The opposite element $P + (-P) = \mathcal{O}$ (right)

Mordell Theorem

$$\Downarrow$$

The Rational Points on Elliptic Curve constitute a finitely generated Abelian Group

$$\Downarrow$$

Mordell - Weil Group

In elliptic fibration:

The Rational Points $\Rightarrow$ *Rational* Sections

★ *A new class of Abelian Symmetries associated with Rational Sections of elliptic curves*

Mordell-Weil group ... finitely generated:

$$\underbrace{\mathbb{Z} \oplus \mathbb{Z} \oplus \cdots \oplus \mathbb{Z}}_{r} \oplus \mathcal{G}$$

Abelian group: Rank - r (*unknown*). *Torsion* part: $\mathcal{G} \rightarrow$:

$$\mathcal{G} = \begin{cases} \mathbb{Z}_n & n = 1, 2, \ldots, 10, 12 \\ \mathbb{Z}_k \times \mathbb{Z}_2 & k = 2, 4, 6, 8 \end{cases}$$

(*Cvetic et al 1210.6094,1307.6425; Mayhofer et al, 1211.6742; Borchmann et al 1307.2902; Krippendorf et al, 1401.7844*. For some aspects see *I. Antoniadis and G.K.L., PLB735 (2014)226*)

to wrap things up:
In $\mathcal{F}$-Theory, Abelian gauge symmetries (other than those embedded in E_8) are encoded in rational sections of the Elliptic Fibration and constitute the so called Mordell-Weil group.

Simplest (*and perhaps most viable*) Case:
Rank-1 Mordell-Weil

$\mathcal{E}$

F-theory Model Building

(*Original papers :Vafa et al, arXiv:0802.3391, 0806.0102*
Donagi et al 0808.2223, 0904.1218)

A Class of 'semi-local' constructions

Manifold , Fluxes & Monodromies

The role of the manifold:

▲ Candidate GUT embedded in maximal exceptional group:

$$\mathcal{E}_8 \to \mathbf{G_{GUT}} \times \mathcal{C}$$

Example: *Assuming a Manifold with $SU(5)$ divisor:*

$$\begin{aligned} \mathcal{E}_8 &\to SU(5) \times SU(5)_\perp \\ &\to SU(5) \times U(1)_\perp^4 \end{aligned}$$

Matter descends from the Adjoint:

$$248 \to (24,1) + (1,24) + (10,5) + (\overline{5},10) + (\overline{10},\overline{5}) + (5,\overline{10})$$

The role of fluxes:

Three important implications

- $SU(5)$ Chirality
- $SU(5)$ Symmetry Breaking

(fluxes act as the surrogate of the Higgs vev)

- **Splitting of $SU(5)$-reps**

Two types of fluxes:

i) M_{10}, M_5: (*associated with* $U(1)_\perp$*'s*)

determine the chirality of complete $10, 5 \in SU(5)$

ii) N_Y: *(turned on along* $U(1)_Y \in SU(5)$*)*

... **split** $SU(5)$*-representations*

$SU(5)$ chirality from $U(1)_\perp$ Flux

$U(1)_\perp$–Flux on $\in$ **10**'s:

$$\#10 - \#\overline{10} \quad = \quad M_{10}$$

$U(1)_\perp$– Flux on $\in$ **5**'s:

$$\#5 - \#\overline{5} \quad = \quad M_5$$

SM chirality form Hypercharge Flux

$U(1)_Y-$**Flux**-splitting of **10**'s:

$$\begin{aligned} n_{(3,2)_{\frac{1}{6}}} - n_{(\bar{3},2)_{-\frac{1}{6}}} &= M_{10} \\ n_{(\bar{3},1)_{-\frac{2}{3}}} - n_{(3,1)_{\frac{2}{3}}} &= M_{10} - N_{Y_{10}} \\ n_{(1,1)_1} - n_{(1,1)_{-1}} &= M_{10} + N_{Y_{10}} \end{aligned}$$

$U(1)_Y-$ **Flux**-splitting of **5**'s:

$$\begin{aligned} n_{(3,1)_{-\frac{1}{3}}} - n_{(\bar{3},1)_{\frac{1}{3}}} &= M_5 \\ n_{(1,2)_{\frac{1}{2}}} - n_{(1,2)_{-\frac{1}{2}}} &= M_5 + N_{Y_5} \end{aligned}$$

For the **Higgs 'curve'** *in particular*:

Hyper-Flux Doublet-Triplet splitting :

$U(1)_Y-$ **Flux**-splitting of $\mathbf{5_{H_u}}$:

$$n_{(3,1)_{-\frac{1}{3}}} - n_{(\bar{3},1)_{\frac{1}{3}}} = M_5 = 0$$

$$n_{(1,2)_{\frac{1}{2}}} - n_{(1,2)_{-\frac{1}{2}}} = M_5 + N_{Y_5} = 0 + 1 = 1 \ (H_u)$$

$U(1)_Y-$ **Flux**-splitting of $\mathbf{\bar{5}_{H_d}} \rightarrow$:

$$n_{(3,1)_{-\frac{1}{3}}} - n_{(\bar{3},1)_{\frac{1}{3}}} = M_5 = 0$$

$$n_{(1,2)_{\frac{1}{2}}} - n_{(1,2)_{-\frac{1}{2}}} = M_5 + N_{Y_5} = 0 - 1 = -1 \ (H_d)$$

$\swarrow\searrow$

General Property

by virtue of Hyperflux, members of the same family, may no longer be components of the same 5-plet

$\Downarrow$

simple way to realise:

Doublet-Triplet splitting

Flipped SU(5) from F-theory

(with V. Basiouris)

It follows according to the following breaking pattern:

$$E_8 \supset SO(10) \times SU(4)_\perp \supset [SU(5) \times U(1)_\chi] \times SU(4)_\perp \ , \qquad (2)$$

MONODROMIES

focusing on $SU(4)_\perp \rightarrow$ *locally described by Cartan roots:*

$$t_i \ = \ SU(4)_\perp - \text{roots} \rightarrow \ \sum_{i=1}^{4} t_i = 0$$

$SU(5)_{GUT}$ *representations in Effective Theory transform according to:*

$$(10, 4) \to 10_{t_i} \quad (\overline{5}, 6) \to \overline{5}_{t_i+t_j}$$

roots t_i *obey a* 4^{th}*-degree polynomial* ($SU(4)$ spectral cover)

$$\sum_{k=0}^{4} b_k t^{4-k} = 0$$

with b_k *'conveying' topological properties to the effective model*

Solving for $t_i = t_i(b_k)$ $\Rightarrow$ possible branchcuts: $\to$ Monodromies
Minimum case :

$$Z_2 : \ t_1 \leftrightarrow t_2 \Rrightarrow U(1)_{\perp}^{3} \to U(1)_{\perp}^{2}$$

A few remarks

▲ **In del Pezzo surfaces**, *Higgs adjoints cannot be accommodated.*
Georgi-Glashow $SU(5)$ can break only with $U(1)_Y$ flux.

▲ **Flipped $SU(5)$ needs only $10+\overline{10}$ for symmetry breaking**.

▲ *No need to turn on $U(1)_{Y_0} \in SU(5)$ flux which requires special conditions to keep $U(1)_{Y_0}$-boson massless. Under these assumptions:*

▲ **"Flipped" $SU(5)$ one of the few possible viable choices!**

$$10_{t_1} \to F_i,\ \bar{5}_{t_1} \to \bar{f}_i,\ 1_{t_1} \to e^c_j, \tag{3}$$

$$\mathbf{5_{-t_1-t_4}} \to \mathbf{h},\ \mathbf{\bar{5}_{t_3+t_4}} \to \mathbf{\bar{h}}\ . \tag{4}$$

$$10_{t_3} \to H,\ \overline{10}_{-t_4} \to \overline{H}, \tag{5}$$

$$1_{t_3} \to E^c_m,\ 1_{-t_4} \to \bar{E}^c_n, \tag{6}$$

The model predictes the existence of singlets

$$1_{t_i - t_j} \to \theta_{ij},\ i,j = 1,2,3,4$$

(modulo the Z_2 monodromy $t_1 \leftrightarrow t_2$), dubbed here:

$$\theta_{12} \equiv \theta_{21} = S,\ \theta_{13} = \chi,\ \theta_{31} = \bar{\chi},$$

$$\theta_{14} \to \psi,\ \theta_{41} = \bar{\psi},\ \theta_{34} \to \zeta,\ \theta_{43} \to \bar{\zeta}$$

The Z_2 monodromy allows a tree-level top-Yukawa coupling.
The superpotential terms are

$$\begin{aligned} \mathcal{W} \ = \ & \lambda^u_{ij} F_i \bar{f}_j \bar{h} + \lambda^d_{ij} F_i F_j h\, \bar{\psi} + \lambda^e_{ij} e^c_i \bar{f}_j h\, \bar{\psi} + \kappa_i \overline{H} F_i S\, \bar{\psi} \\ & + \alpha_{mj} \bar{E}^c_m e^c_j\, \bar{\psi} + \beta_{mn} \bar{E}^c_m E^c_n\, \bar{\zeta} + \gamma_{nj} E^c_n \bar{f}_j h\, \bar{\zeta} \qquad (7) \\ & + \lambda_{\bar{H}} \overline{H H} \bar{h} \bar{\zeta} + \lambda_H H H h \bar{\zeta} (\chi + \bar{\zeta}\psi) + \lambda_\mu (\chi + \lambda' \bar{\zeta}\psi) \bar{h}\, h \end{aligned}$$

Issue: fine-tuning or extra symmetries required to deal with μ term.

Mechanisms for Fermion mass hierarchy

▼ If families are distributed on different matter curves:
Implementation of Froggatt-Nielsen mechanism (*Nucl.Phys. B147 (1979) 277*) in F-models:
Dudas and Palti, 0912.0853
GKL and G.G. Ross, 1009.6000

▼ If all three families are on the same matter curve, masses to lighter families can be generated by:
i) non-commutative fluxes *Cecotti et al, 0910.0477*
ii) non-perturbative effects, *Aparicio et al, 1104.2609*

▼ Using Modular Invariance to derive the mass textures
(*with Charalambous, SF King, Ye-Ling Zhou, to appear*)

▲▲ We adopt the second mechanism since:

All families reside on the same matter curve

Mass terms

$$\lambda^u_{ij} F_i \bar{f}_j \bar{h} \to Q\, u^c\, h_u + \ell \nu^c\, h_u \to m_D^T = m_u \propto \lambda^u \langle h_u \rangle$$

$$\lambda^d_{ij} F_i\, F_j h \bar{\psi} \to m_d = \lambda^d \langle h_d \rangle$$

$$H\, H\, h + \bar{H}\, \bar{H}\, \bar{h} \to \langle H \rangle d^c_H D + \langle \bar{H} \rangle \bar{d}^c_H \bar{D}$$

see-saw with extra (sterile) 'neutrino' S:

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_{\nu_D} & 0 \\ m_{\nu_D}^T & 0 & M_{\nu^c S} \\ 0 & M_{\nu^c S}^T & M_S \end{pmatrix} \qquad (8)$$

★ Conclusions ★

• in F-theory $\exists$ *interesting connections between*:

GUT Symmetry and *Elliptically fibred* Internal Manifold

Abelian Symmetries and Rational sections

• Interesting Predictions of *Effective String F-Theory Models*

Flipped ($SU(5)\times U(1)$) model *encompasses interesting features*

BSM Physics predictions:

• Vector-like $E^c+\bar{E}^c$, $D+\bar{D}\cdots$

Z' bosons non-universaly coupled to families, possibly related to potential SM-deviations (B*-meson anomalies, ...*)

★ Thank you for your attention ★

University of Ioannina

$\mathcal{SUSY}$ 2022 Conference

27 June- 1 July 2022 (pre-SUSY school 24-26 June)