BMS flux algebra in Celestial Holography

cf. 2108.11969 with Romain Ruzziconi (TU Vienna)

Laura Donnay (Technische Universität Wien)

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Starting point: gravitational solution space

[Bondi, van der Burg, Metzner][Sachs]
[Barnich, Troessaert]

Asymptotically flat spacetimes in Bondi gauge:

$$ds^2 = e^{2\beta}\frac{V}{r}du^2 - 2e^{2\beta}du\,dr + g_{AB}(dx^A - U^A du)(dx^B - U^B du)$$

$$\frac{V}{r} = -1 + \frac{2M(u,x^A)}{r} + O(r^{-2}) \qquad \beta = \frac{-1}{32r^2}C^{AB}C_{AB} + O(r^{-3})$$

$$g_{AB} = r^2\mathring{q}_{AB} + r\,C_{AB}(u,x^A) + O(r^{-1})$$

$$\mathring{q}_{AB}dx^A dx^B = 2(\Omega\bar{\Omega})^{-1}dz\,d\bar{z} \qquad \Omega = \frac{1+z\bar{z}}{\sqrt{2}} = \bar{\Omega}$$

$$U^A = -\frac{1}{2r^2}D_B C^{AB} - \frac{2}{3r^2}\left[N^A(u,x^B) - \frac{1}{2}C^{AB}D^C C_{BC}\right] + O(r^{-4})$$

Extended BMS symmetries

[Bondi, van der Burg, Metzner][Sachs]
[Barnich, Troessaert]

* $\xi^u = (\Omega\bar{\Omega})^{-1/2}\mathcal{T}(z,\bar{z}) + \frac{u}{2}(D_z Y + D_{\bar{z}}\bar{Y})$; $\xi^r = -\frac{r}{2}(D_z Y + D_{\bar{z}}\bar{Y}) + O(r^0)$

$\equiv f$

supertranslation parameter

$\xi^z = Y(z) + O(r^{-1})$ $\qquad$ $\xi^{\bar{z}} = \bar{Y}(\bar{z}) + O(r^{-1})$

superrotations

D_A : covariant derivative w.r.t. $\mathring{q}_{AB}$

* $\mathfrak{bms}_4$ algebra

$$[\xi(\mathcal{T}_1, Y_1, \bar{Y}_1), \xi(\mathcal{T}_2, Y_2, \bar{Y}_2)]_* = \xi(\mathcal{T}_{12}, Y_{12}, \bar{Y}_{12})$$

$$\mathcal{T}_{12} = Y_1\partial\mathcal{T}_2 - \frac{1}{2}\partial Y_1\,\mathcal{T}_2 - (1\leftrightarrow 2) + c.c.$$

$$Y_{12} = Y_1\partial Y_2 - (1\leftrightarrow 2) \;;\; \bar{Y}_{12} = \bar{Y}_1\bar{\partial}\bar{Y}_2 - (1\leftrightarrow 2)$$

$\Big\}$ (Witt $\oplus$ Witt) $\oplus\!\!\!\!\!\!\supset$ s^*

News and N^{vac}_{AB}

The news tensor $N_{AB} = \partial_u C_{AB}$ transforms inhomogeneously under superrotations

$(\delta_Y N_{AB} \ni D_A D_B D_C Y^C)$.

[Compère, Fiorucci, Ruzziconi]

The shifted news tensor, defined as

$$\hat{N}_{AB}(u,x) = N_{AB}(u,x) - N^{vac}_{AB}(x)$$

where $N^{vac}_{AB}(x) = \left[\frac{1}{2} D_A \Phi D_B \Phi - D_A D_B \Phi\right]^{TF}$; $\Phi(z,\bar{z}) = \varphi(z) + \bar{\varphi}(\bar{z}) - \ln\sqrt{\mathring{q}}$

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Stress-tensor for a 2d Euclidean Liouville theory

Liouville field $\Box\Phi = \mathring{R} = 2$

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now transforms homogeneously $\delta_{(f,Y)} \hat{N}_{AB} = (f\partial_u + \mathcal{L}_Y)\hat{N}_{AB}$

Any non-radiative configuration satisfies $\hat{N}_{AB} = 0$.

Fall-off conditions: $N_{AB} \underset{u\to\pm\infty}{=} N^{vac}_{AB} + O(u^{-2})$

[Campiglia, Laddha]

News and N^{vac}_{AB}

[Compère, Fiorucci, Ruzziconi]

$$\hat{N}_{AB}(u,x) = N_{AB}(u,x) - N^{vac}_{AB}(x)$$

One can also define

$$\hat{C}_{AB} = C_{AB} - u N^{vac}_{AB}$$

$$\lim_{u\to\pm\infty} \hat{C}_{AB} = C_{\pm} N^{vac}_{AB} - 2\left(D_A D_B C_{\pm}\right)^{TF}$$

$C_{\pm}$ is the value of the supertranslation field at $\mathcal{I}^{+}_{\pm}$ [Strominger, Zhiboedov]

These u-fall-offs imply that $\hat{N}_{AB} = 0$ at the corners $\mathcal{I}^{+}_{\pm}$.

& the shifted shear is electric

$$\left[\left(D_B D_C - \tfrac{1}{2} N^{vac}_{BC}\right)\hat{C}^{C}_{A} - \left(D_A D_C - \tfrac{1}{2} N^{vac}_{AC}\right)\hat{C}^{C}_{B}\right]\Big|_{\mathcal{I}^{+}_{\pm}} = 0$$

↳ These quantities were used in [Compère, Fiorucci, Ruzziconi] to prescribe a Hamiltonian that satisfies

$$\{H_{\xi_1}, H_{\xi_2}\}\Big|_{\mathcal{I}^+_\pm} = H_{[\xi_1,\xi_2]_*}\Big|_{\mathcal{I}^+_\pm}$$

namely closes under the standard Lie bracket at the corners $\mathcal{I}^+_+$ & $\mathcal{I}^+_-$ without any central extension.

→ the generalized bms_4 algebra is realized at spatial infinity.

(see also [Campiglia, Peraza])
& cf. [Henneaux, Troessaert]

BMS fluxes as conformal fields

* Conformal field $\phi_{h,\bar{h}}$: $\phi'_{h,\bar{h}}(x') = \left(\frac{\partial z}{\partial z'}\right)^{h} \left(\frac{\partial \bar{z}}{\partial \bar{z}'}\right)^{\bar{h}} \phi_{h,\bar{h}}(x)$

Infinitesimal action of superrotations

$$\delta_{\mathcal{Y}} \phi_{h,\bar{h}} = \mathcal{Y}\partial\phi_{h,\bar{h}} + h\,\partial\mathcal{Y}\,\phi_{h,\bar{h}} \qquad \delta_{\bar{\mathcal{Y}}} \phi_{h,\bar{h}} = \bar{\mathcal{Y}}\bar{\partial}\phi_{h,\bar{h}} + \bar{h}\,\bar{\partial}\bar{\mathcal{Y}}\,\phi_{h,\bar{h}}$$

$\mathfrak{bms}_4$ algebra

$$[\mathcal{Y}_1, \mathcal{Y}_2] = \mathcal{Y}_1\partial\mathcal{Y}_2 - \mathcal{Y}_2\partial\mathcal{Y}_1$$

$$[\mathcal{Y}_1, \mathcal{T}_2] = \mathcal{Y}_1\partial\mathcal{T}_2 - \frac{1}{2}\partial\mathcal{Y}_1\mathcal{T}_2$$

$$[\mathcal{T}_1, \mathcal{T}_2] = 0$$

(+c.c. relations)

$\rightarrow$

$(h, \bar{h})$

$\mathcal{Y} : (-1, 0)$

$\bar{\mathcal{Y}} : (0, -1)$

$\mathcal{T} : (-\frac{1}{2}, -\frac{1}{2})$

BMS fluxes

[Compère, Fiorucci, Ruzziconi]
[LD, Ruzziconi]

1) Supermomentum flux

$$\mathcal{P} = \frac{1}{4\pi G}\int du\, \partial_u \mathcal{M}\,, \qquad \mathcal{M} = (\Omega\bar{\Omega})^{-3/2}\left[M + \frac{1}{8}\left(C_{zz}N^{zz}_{vac} + C_{\bar{z}\bar{z}}N^{\bar{z}\bar{z}}_{vac}\right)\right]$$

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Using the phase space infinitesimal transformations

$$\delta_{(f,Y)}M = \left[f\partial_u + \mathcal{L}_Y + \frac{3}{2}D_C Y^C\right]M + \frac{1}{8}D_C D_B D_A Y^A C^{BC} + \frac{1}{4}N^{AB}D_A D_B f + \frac{1}{2}D_A f\, D_B N^{AB}$$

$$\delta_{(f,Y)}C_{AB} = \left[f\partial_u + \mathcal{L}_Y - \frac{1}{2}D_C Y^C\right]C_{AB} - 2D_A D_B f + \mathring{q}_{AB}D_C D^C f$$

$$\delta_{(f,Y)}N^{vac}_{AB} = \mathcal{L}_Y N^{vac}_{AB} - (D_A D_B D_C Y^C)^{TF}$$

One finds $\delta_{(\tau,y,\bar{y})}\mathcal{P} = \left[y\partial + \bar{y}\bar{\partial} + \frac{3}{2}\partial y + \frac{3}{2}\bar{\partial}\bar{y}\right]\mathcal{P} \leftarrow \left(\frac{3}{2}, \frac{3}{2}\right)$ $(h, \bar{h})$

cf. [Barnich, Ruzziconi]

BMS fluxes

1) Supermomentum flux

We can extract out of it a soft piece, $\mathcal{P}^{soft}$, which

can be rewritten in terms of the leading soft operator [He, Lysov, Mitra, Strominger]

$$\mathcal{N}^0 = \frac{1}{16\pi G}\int du\,(\Omega\bar{\Omega})^{1/2}\,\hat{N}_{zz} \qquad \left(\tfrac{3}{2}, -\tfrac{1}{2}\right)$$

& the "superrotation covariant derivative" [cf. Barnich, Ruzziconi & Campiglia, Laddha; Campiglia, Peraza]

$$\mathcal{D} = D_z - h\,\partial\Phi$$

$$\boxed{\mathcal{P}^{soft} = \mathcal{D}^2\bar{\mathcal{N}}^0 + \bar{\mathcal{D}}^2\mathcal{N}^0}$$

[LD, Ruzziconi]

BMS fluxes

2) Superangular momentum fluxes

$$\mathcal{J} = \frac{1}{8\pi G}\int_{-\infty}^{+\infty} du\, \partial_u \mathcal{N} \quad ; \quad \mathcal{N} = (\Omega\bar{\Omega})^{-1}\Big[N_{\bar{z}} - u\,\Omega^3 D_{\bar{z}} M + \frac{1}{4} C_{\bar{z}\bar{z}} D_{\bar{z}} C^{\bar{z}\bar{z}} + \frac{3}{16} D_{\bar{z}}(C_{zz}C^{zz}) + \frac{u}{4}\Big(D^z(D_z^2 - \frac{1}{2}N^{vac}_{zz})C^z{}_{\bar{z}} + c.c.\Big)\Big]$$

Using the transformation laws

$$\delta_{(f,Y)} C_{AB} = [f\partial_u + \mathcal{L}_Y - \frac{1}{2}D_C Y^C] C_{AB} - 2 D_A D_B f + \mathring{q}_{AB} D_C D^C f,$$

$$\delta_{(f,Y)} M = [f\partial_u + \mathcal{L}_Y + \frac{3}{2} D_C Y^C] M + \frac{1}{8} D_C D_B D_A Y^A C^{BC} + \frac{1}{4} N^{AB} D_A D_B f + \frac{1}{2} D_A f D_B N^{AB},$$

$$\begin{aligned}\delta_{(f,Y)} N_A = {} & [f\partial_u + \mathcal{L}_Y + D_C Y^C] N_A + 3 M D_A f - \frac{3}{16} D_A f N_{BC} C^{BC} \\ & - \frac{1}{32} D_A D_B Y^B C_{CD} C^{CD} + \frac{1}{4}(2 D^B f + D^B D_C D^C f) C_{AB} \\ & - \frac{3}{4} D_B f (D^B D^C C_{AC} - D_A D_C C^{BC}) + \frac{3}{8} D_A (D_C D_B f C^{BC}) \\ & + \frac{1}{2}(D_A D_B f - \frac{1}{2} D_C D^C f \mathring{q}_{AB}) D_C C^{BC} + \frac{1}{2} D_B f N^{BC} C_{AC}.\end{aligned}$$

one gets

$$\delta_{(\mathcal{T},\mathcal{Y},\bar{\mathcal{Y}})}\mathcal{J} = \mathcal{Y}\partial\mathcal{J} + \bar{\mathcal{Y}}\bar{\partial}\bar{\mathcal{J}} + \partial\mathcal{Y}\,\bar{\mathcal{J}} + 2\bar{\partial}\bar{\mathcal{Y}}\,\mathcal{J} + \frac{1}{2}\mathcal{T}\bar{\partial}\mathcal{P} + \frac{3}{2}\bar{\partial}\mathcal{T}\mathcal{P}.$$

$(1,2)$

BMS fluxes

2) Superangular momentum fluxes

Nicer form : $\mathcal{J}^{soft} = -\bar{\mathcal{D}}^3 \mathcal{N}^{(1)} - \bar{\mathcal{D}}^3 \mathcal{C} \mathcal{N}^{(0)} - 3\bar{\mathcal{D}}^2 \mathcal{C} \bar{\mathcal{D}} \mathcal{N}^{(0)}$

$\mathcal{N}^{(1)}(z,\bar{z}) = \frac{1}{16\pi G}\int_{-\infty}^{+\infty} du\, (\Omega\bar{\Omega})\, u\, \hat{N}_{\bar{z}\bar{z}}$: subleading soft mode $(1,-1)$ [Kapec, Lysov, Pasterski, Strominger]

$\mathcal{C}(z,\bar{z}) = (\Omega\bar{\Omega})^{1/2} C_-$: $\left(-\frac{1}{2}, -\frac{1}{2}\right)$

$$F_{(\mathcal{T},\mathcal{Y},\bar{\mathcal{Y}})} = \int dz d\bar{z} \, [\mathcal{T}\mathcal{P} + \mathcal{Y}\bar{\mathcal{J}} + \bar{\mathcal{Y}}\mathcal{J}]$$

$$= \frac{1}{8\pi G}\int du dz d\bar{z} \, \partial_u [2\mathcal{T}\mathcal{M} + \mathcal{Y}\bar{\mathcal{N}} + \bar{\mathcal{Y}}\mathcal{N}]$$

This is the pairing between BMS generators & momenta :

$$\mathfrak{bms}_4^* \times \mathfrak{bms}_4 \mapsto \mathbb{R} : ((\mathcal{P},[\mathcal{J}],[\bar{\mathcal{J}}]),(\mathcal{T},\mathcal{Y},\bar{\mathcal{Y}})) \mapsto F_{(\mathcal{T},\mathcal{Y},\bar{\mathcal{Y}})} = \langle (\mathcal{P},[\mathcal{J}],[\bar{\mathcal{J}}]),(\mathcal{T},\mathcal{Y},\bar{\mathcal{Y}}) \rangle$$

$$F_{(\mathcal{T},\mathcal{Y},\bar{\mathcal{Y}})} = \int dz d\bar{z} \, [\mathcal{T}\mathcal{P} + \mathcal{Y}\bar{\mathcal{J}} + \bar{\mathcal{Y}}\mathcal{J}]$$
$$= \frac{1}{8\pi G}\int du dz d\bar{z} \, \partial_u [2\mathcal{T}M + \mathcal{Y}\bar{\mathcal{N}} + \bar{\mathcal{Y}}\mathcal{N}]$$

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↳ one can then interpret the transformation laws

$$\delta\mathcal{P} = [\mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial} + \tfrac{3}{2}\partial\mathcal{Y} + \tfrac{3}{2}\bar{\partial}\bar{\mathcal{Y}}]\,\mathcal{P}$$
$$\delta\mathcal{J} = [\mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial} + \partial\mathcal{Y} + 2\bar{\partial}\bar{\mathcal{Y}}]\,\mathcal{J} + \tfrac{1}{2}\mathcal{T}\bar{\partial}\mathcal{P} + \tfrac{3}{2}\bar{\partial}\mathcal{T}\mathcal{P}$$

as the coadjoint representation of $\mathfrak{bms}_4$ [Barnich, Ruzziconi]

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$$\delta\mathcal{P} = [\mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial} + \tfrac{3}{2}\partial\mathcal{Y} + \tfrac{3}{2}\bar{\partial}\bar{\mathcal{Y}}]\,\mathcal{P}$$
$$\delta\mathcal{J} = [\mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial} + \partial\mathcal{Y} + 2\bar{\partial}\bar{\mathcal{Y}}]\,\mathcal{J} + \tfrac{1}{2}\mathcal{T}\bar{\partial}\mathcal{P} + \tfrac{3}{2}\bar{\partial}\mathcal{T}\mathcal{P}$$

as the coadjoint representation of $\mathfrak{bms}_4$ [Barnich, Ruzziconi]

- One can define a soft/hard sector $F^{\text{soft/hard}}_{(\mathcal{T},\mathcal{Y},\bar{\mathcal{Y}})}$ with $\mathcal{P}^{\text{soft}}, \mathcal{J}^{\text{soft}}, \bar{\mathcal{J}}^{\text{soft}}$ → also transform in the coadjoint.

Conclusion

$\phi_{h,\bar{h}}$	$\mathcal{T}$	$\mathcal{Y}$	$\mathcal{P}$	$\mathcal{J}$	$\mathcal{N}^{(0)}$	$\mathcal{N}^{(1)}$
h	$-\frac{1}{2}$	-1	$\frac{3}{2}$	1	$\frac{3}{2}$	1
$\bar{h}$	$-\frac{1}{2}$	0	$\frac{3}{2}$	2	$-\frac{1}{2}$	-1
J	0	-1	0	-1	2	2
Δ	-1	-1	3	3	1	0

* We constructed the "BMS fluxes" with a prescription such that
  - they vanish for non-radiative solutions
  - the u-falloffs make them finite
* They are given by some non-local combinations of the gravity solution space that transform in the coadjoint representation of the extended BMS group (superrotations involve a careful treatment of the Liouville field)

Conclusion

$\phi_{h,\bar{h}}$	$\mathcal{T}$	$\mathcal{Y}$	$\mathcal{P}$	$\mathcal{J}$	$\mathcal{N}^{(0)}$	$\mathcal{N}^{(1)}$	$\mathcal{C}$	P	T
h	$-\frac{1}{2}$	-1	$\frac{3}{2}$	1	$\frac{3}{2}$	1	$-\frac{1}{2}$	$\frac{3}{2}$	2
$\bar{h}$	$-\frac{1}{2}$	0	$\frac{3}{2}$	2	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	$\frac{1}{2}$	0
J	0	-1	0	-1	2	2	0	1	2
Δ	-1	-1	3	3	1	0	-1	2	2

CCFT currents

* We constructed the "BMS fluxes" with a prescription such that
    - they vanish for non-radiative solutions
    - the u-falloffs make them finite
* They are given by some non-local combinations of the gravity solution space that transform in the coadjoint representation of the extended BMS group (superrotations involve a careful treatment of the Liouville field)

also

* We gave the relations with the CCFT supertranslation current & stress tensor
* We deduced the CCFT OPEs from the BMS flux algebra. [2108.11969]

Thank you very much!