Axial Anomaly in Very Special Relativity

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Outline

- Motivation
- Very Special Relativity SM (VSR SM)
- Mandelstam-Leibbrandt regularization (ML)
- Axial Anomaly in VSR Electrodynamics
- Based in J.A. Phys.Rev.D 103 (2021) 7, 075011

Neutrino Oscillations

- Nobel Prize in Physics 2015

The proportion of each flavor in the same neutrino beam changes in time. It is proportional to the distance to the source.

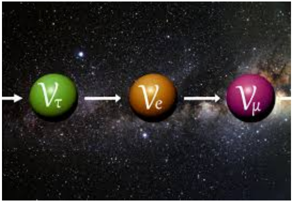


Figure: Neutrino oscillations

In the Standard Model, neutrinos are masless and have left handed quirality (L).

Neutrinos can oscillate only if their mass is not zero ->Physics beyond the SM.

Very Special Relativity

A. Cohen and S. Glashow, Phys.Rev.Lett.97:021601,2006:
All kinematical effects associated to invariance under the Lorentz group(6 parameters) can be obtained from four parameters subgroups of the Lorentz group, opening the road to new predictions which violate Lorentz symmetry, but preserve the symmetry under such subgroups.

- VSR implies special relativity (SR) in the context of local quantum field theory or of CP conservation.
- Most interesting Subgroup of the Lorentz Group:Hom(2), 3 parameters; Sim(2) , 4 parameters.
  There are no invariant tensors for these cases. So SR kinematics is preserved.
  No local Lorentz symmetry-breaking operator preserving either of these groups exists.

$$T_1 = K_x + J_y, T_2 = K_y - J_x \tag{1}$$
$$Hom(2) generators : T_1, T_2, K_z \tag{2}$$
$$Sim(2) generators : T_1, T_2, K_z, J_z \tag{3}$$

$n = (1, 0, 0, 1)$, $n.n = 0$, n is invariant under T_1, T_2, J_z, but under boosts in the z-direction (generated by K_z), $n \rightarrow e^{\phi} n$
p_1, p_2 particle momenta: $\frac{p_1.n}{p_2.n}$ is VSR but not SR invariant.

Neutrino mass in VSR:

$$\left(\not{p} - \frac{m_v^2}{2} \frac{\not{n}}{n.p}\right) \nu_L = 0, \left(\not{p} - \frac{m_v^2}{2} \frac{\not{n}}{n.p}\right)^2 \nu_L =$$

$$(p^2 - m_\nu^2) \nu_L = 0$$

Very Special Relativity Standard Model

J.A., R. Avila and P. González, Electroweak standard model with very special relativity,PHYSICAL REVIEW D 91, 105007 (2015)

- As yet LHC do not see new particles or symmetries, beyond the SM ones.
- Neutrino are massless in the SM, but in nature they are massive(neutrino oscillations).
- We want to keep the particles and symmetries of the SM, but provide masses for neutrinos
- The VSR SM is a simple model with $SU(2)_L x U(1)_R$ symmetry, with the same number of leptons and gauge fields as in the SM.
- It is renormalizable and unitarity is preserved.
- New non-local terms that violate Lorentz invariance are able to describe in a straightforward manner the observed neutrino oscillations.
- We predict new processes such as the decay μ->$e+\gamma$,which are forbidden in the SM. $B = 4.43(4.56) \times 10^{-25}[eV^2](NH(IH))$.

Weinberg-Salam lagrangian

- **Gauge Lagrangian:** Two kind of gauge fields, B_μ and A^i_μ.

$$\mathcal{L}_{gauge} = -\frac{1}{4}F^i_{\mu\nu}F_i^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}, \tag{4}$$

- **Leptonic Lagrangian:** Three $SU(2)$ doublets $L_a = \begin{pmatrix} \nu^0_{aL} \\ e^0_{aL} \end{pmatrix}$, and three $SU(2)$ singlet R_a. There is no right-handed neutrino. The index a represent the different families;the index 0 denotes the fermionic fields before SSB.

$$\mathcal{L}_{lepton} = i\bar{L}^a \not{D} L_a + \frac{i}{2}\bar{L}_b \not{n}[m_L^2]^{ba}(n^\alpha D_\alpha)^{-1}L_a + i\bar{R}^a \not{D} R_a + \frac{i}{2}\bar{R}_b \not{n}[m_R^2]^{ba}(n^\alpha D_\alpha)^{-1}R_a, \tag{5}$$

m_L^2 and m_R^2 are hermitian matrices in family indices, (ba), and they could depend on γ^5. The doublets have a hypercharge $Y = -1$ and the singlets have $Y = -2$.

VSR QED

- In the VSR SM the electron neutrino and the electron belong to a doublet under $SU(2)_L$
- m is the VSR mass of both electron and neutrino. After spontaneous symmetry breaking(SSB), the electron acquires an additional mass $M = \frac{G_e v}{\sqrt{2}}$, where G_e is the electron Yukawa coupling and v is the VEV of the Higgs.. The electron mass is $M_e = \sqrt{M^2 + m^2}$.
- The neutrino mass is not affected by SSB:$M_{\nu_e} = m$.
- Restricting the VSRSM after SSB to the interactions between photon and electron alone, we get the VSR QED action.ψ is the electron field. A_μ is the photon field. We use the Feynman gauge.

$$\mathcal{L} = \bar{\psi}(i(\not{D} + \frac{1}{2}\not{n} m^2 (n \cdot D)^{-1}) - M)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{(\partial_\mu A_\mu)^2}{4}$$

$$D_\mu = \partial_\mu - ieA_\mu, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, n.n = 0$$

Currents

- The vector current(electric charge conservation) is:

$$j^{\mu} = \bar{\psi}\gamma^{\mu}\psi + \frac{1}{2}m^2\left(\frac{1}{n\cdot D^{\dagger}}\bar{\psi}\right)\not{n}n^{\mu}\left(\frac{1}{n\cdot D}\psi\right)$$

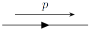
- The axial vector current is:

$$j^{\mu 5} = \bar{\psi}\gamma^{\mu}\gamma^{5}\psi + \frac{1}{2}m^2\left(\frac{1}{n\cdot D^{\dagger}}\bar{\psi}\right)\not{n}n^{\mu}\gamma^{5}\left(\frac{1}{n\cdot D}\psi\right)$$

Both currents are conserved at the classical level. We are interested in computing expectation values of these currents.
To get the Feynman rules we use the expansion of $(n.D)^{-1}$ both in the currents and the lagrangian.

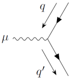
$$\begin{aligned}(n.D)^{-1} = (1 + ie(n.\partial)^{-1}(n.A) + (ie)^2(n.\partial)^{-1}(n.A)(n.\partial)^{-1}(n.A) + \\ (ie)^3(n.\partial)^{-1}(n.A)(n.\partial)^{-1}(n.A)(n.\partial)^{-1}(n.A))(n.\partial)^{-1} + \dots\end{aligned}$$

Feynman rules



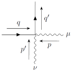
$$= i\frac{\not{p} + M - \frac{m^2}{2}\frac{\not{n}}{n\cdot p}}{p^2 - M^2 - m^2 + i\varepsilon}$$

(a) Electron propagator



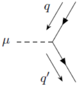
$$= -ie(\gamma_\mu + \frac{1}{2}m^2\frac{\not{n}n_\mu}{n\cdot q\ n\cdot q'})$$

(b) $e - e - A_\mu$ vertex



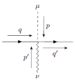
$$= -ie^2\frac{1}{2}\ \not{n}m^2\frac{n_\mu n_\nu}{n\cdot q\ n\cdot q'}[\frac{1}{n\cdot(p+q)} + \frac{1}{n\cdot(p'+q)}]$$

(c) $e - e - A_\mu - A_\nu$ vertex



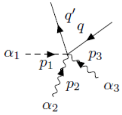
$$= -ie(\gamma_\mu + \frac{1}{2}m^2\frac{\not{n}n_\mu}{n\cdot q\ n\cdot q'})\gamma_5$$

(d) axial-e-e vertex



$$= -(ie)^2\frac{1}{2}\ \not{n}m^2\frac{n_\mu n_\nu}{n\cdot q\ n\cdot q'}[\frac{1}{n\cdot(p+q)} + \frac{1}{n\cdot(p'+q)}]\gamma_5$$

(e) *axial* $- A_\nu - e - e$ vertex



$$= V(p_1, p_2, p_3, q)$$

(f) *axial* $- A_{\alpha_2} - A_{\alpha_3} - e - e$ vertex

Mandelstam-Leibbrandt(ML)prescription

- $\int dp \frac{1}{[p^2+2p.q-m^2]^a}\frac{1}{n.p}$ has an infrared divergence when $n.p = 0$
- Light-Cone gauge quantization of gauge and string theories.
- Mandelstam-Leibbrandt prescription(ML): $\frac{1}{n.p} = \lim_{\varepsilon\to 0}\frac{p.\bar{n}}{n.pp.\bar{n}+i\varepsilon}$, $\bar{n}.\bar{n} = 0$, $n.\bar{n} = 1$
- ML has very nice properties: The poles in the p_0 complex plane are situated such that the Wick's rotation from Euclidean to Minkowsky space is justified; it preserves naive power counting of loop integrals; and in gauge theories, it maintains the Ward identities of the gauge symmetry.

A significant simplification of ML

- J.A., PRD93 (2016)065033,Erratum: PRD94 (2016)049901
- Let us compute the following simple integral:

$$A_\mu = \int dp \frac{f(p^2)p_\mu}{n.p}$$

where f is an arbitrary function.dp is the integration measure in d dimensional space and n_μ is a fixed null vector($n.n = 0$). This integral is infrared divergent when $n.p = 0$.

- The ML is:

$$\frac{1}{n.p} = \lim_{\varepsilon \to 0} \frac{p.\bar{n}}{n.pp.\bar{n} + i\varepsilon} \tag{6}$$

where $\bar{n}_\mu$ is a new null vector with the property $n.\bar{n} = 1$.

- To compute A_μ we have to know the specific form of f, provide an specific form of n_μ and $\bar{n}_\mu$, and evaluate the residues of all poles of $\frac{f(p^2)}{n.p}$ in the p_0 complex plane, a rather formidable task for an arbitrary f.

- Instead we want to point out the following symmetry:

$$n_\mu \to \lambda n_\mu, \bar{n}_\mu \to \lambda^{-1}\bar{n}_\mu, \lambda \neq 0, \lambda \varepsilon R \tag{7}$$

- It preserves the definitions of n_μ and $\bar{n}_\mu$:

$$\begin{aligned} 0 = n.n \to \lambda^2 n.n = 0 \\ 0 = \bar{n}.\bar{n} \to \lambda^{-2}\bar{n}.\bar{n} = 0 \\ 1 = n.\bar{n} \to n.\bar{n} = 1 \end{aligned}$$

- We see from (6) that:

$$\frac{1}{n.p} \to \frac{1}{n.p}\lambda^{-1}$$

- Now we compute A_μ, based on its symmetries. It is a Lorentz vector which scales under (7) as λ^{-1}. The only Lorentz vectors we have available in this case are n_μ and $\bar{n}_\mu$. But (7) forbids n_μ. That is:

$$A_\mu = a\bar{n}_\mu$$

- Multiply by n_μ to find $A.n = a$. Thus $a = \int dpf(p^2)$. Finally:

$$\int dp \frac{f(p^2)p_\mu}{n.p} = \bar{n}_\mu \int dpf(p^2)$$

- By the same token we find

$$A_{\mu\nu\lambda} = \int dp \frac{f(p^2)p_\mu p_\nu p_\lambda}{n.p} = a(\bar{n}_\mu g_{\nu\lambda})_S + b(\bar{n}_\mu \bar{n}_\nu n_\lambda)_S$$

where $()_S$ means symmetric in all Lorentz indices.

- We compute $A_{\mu\nu\lambda}n^\lambda = \frac{1}{d} g_{\mu\nu} \int dpf(p^2)p^2$ to get:

$$\int dp \frac{f(p^2)p_\mu p_\nu p_\lambda}{n.p} = \frac{1}{d} \int dpf(p^2)p^2 \{(\bar{n}_\mu g_{\nu\lambda})_S - (\bar{n}_\mu \bar{n}_\nu n_\lambda)_S\}$$

The integrals on p_μ are dimensionally regularized.

Application to loop integrals

1. Using dimensional regularization, we obtain($\omega = d/2$):

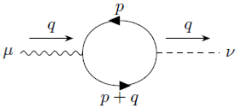
$$\int dp \frac{1}{[p^2 + 2p.q - m^2]^a} \frac{1}{(n.p)^b} = (-1)^b i(\pi)^\omega (2)^b \frac{\Gamma(a+b-\omega)}{\Gamma(a)\Gamma(b)}$$
$$(\bar{n}.q)^b \int_0^1 dt t^{b-1} \frac{1}{(m^2 + q^2 - 2n.q\bar{n}.qt)^{a+b-\omega}}, \quad (8)$$

2. Other integrals can be obtained deriving respects to q_μ:

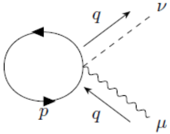
- We compute the expectation value of the axial vector current in a background field A_ν:

$$< j^{5\nu}(q) >= \int d^2x < j^{5\nu}(x) > e^{iqx} = (-ie)^{-1} i\Pi^{5\mu\nu}(q) A_\mu \quad (9)$$

- It is given by the two graphs (Figure 3a and Figure 3b):



(a) $i\Pi^{15\mu\nu}$



(b) $i\Pi^{25\mu\nu}$

$$i\Pi^{15\mu\nu} = -(-ie)^2 \int dp \mathrm{Tr}\{\left[\gamma^\mu + \frac{1}{2}n^\mu \left(\not{n}\right) m^2 (n.(p+q))^{-1}(n.p)^{-1}\right]$$

$$\frac{i\left(\not{p} + M - \frac{m^2}{2}\frac{\not{n}}{n\cdot p}\right)}{p^2 - M^2 - m^2 + i\varepsilon}\left[\gamma^\nu + \frac{1}{2}n^\nu \left(\not{n}\right) m^2 (n.(p+q))^{-1}(n.p)^{-1}\right]$$

$$\gamma^5 \frac{i\left(\left(\not{p} + \not{q}\right) + M - \frac{m^2}{2}\frac{\not{n}}{n\cdot(p+q)}\right)}{(p+q)^2 - M^2 - m^2 + i\varepsilon}\} \quad (10)$$

$$i\Pi^{25\mu\nu} = -(ie)^2 n^\mu n^\nu i \int dp (n.p)^{-2}[(n.(q+p))^{-1} + (n.(-q+p))^{-1}]$$

$$\mathrm{Tr}(\frac{1}{2}\not{n} m^2 \frac{i\left(\not{p} + M - \frac{m^2}{2}\frac{\not{n}}{n\cdot p}\right)}{p^2 - M^2 - m^2 + i\varepsilon}\gamma^5) \quad (11)$$

Pauli- Villars regularization

To compute the axial anomaly we will use Pauli-Villars regularization and Mandelstam-Leibbrandt prescription to treat infrared divergences.
Notice that equation (10) is logarithmically divergent and equation (11) is finite.
It is easy to check that formally:

$$q_\mu(\Pi^{15\mu\nu} + \Pi^{25\mu\nu}) = 0$$

if shift of the integration variable $p \to p + k$ is allowed. Here k is a constant vector. This would be true if the integral (10) would be finite.

Pauli-Villars

- Introduce a Pauli-Villars particle of mass $\bar{M}$ and define the regularized amplitude:

$$\Pi^{5R\mu\nu}(M,\bar{M},q) = \Pi^{15\mu\nu}(M,q) + \Pi^{25\mu\nu}(M,q) - \Pi^{15\mu\nu}(\bar{M},q) - \Pi^{25\mu\nu}(\bar{M},q)$$

- Since $\Pi^{5R\mu\nu}(M,\bar{M},q)$ is finite, it satisfies the naive Ward identity(electric charge conservation):

$$q_\mu \Pi^{5R\mu\nu}(M,\bar{M},q) = 0$$

- On the other hand, the axial Ward identity is:

$$i(\Pi^{15\mu\nu} + \Pi^{25\mu\nu})q_\nu = 2M\mathcal{A}(M,q)^\mu$$

$$= 2M(-ie)^2 \int dp \mathrm{Tr}[\gamma^\mu + \frac{1}{2} n^\mu \not{n} m^2 (n.(p+q))^{-1}(n.p)^{-1}] \frac{i(\not{p}+M-\frac{m^2}{2}\frac{\not{n}}{n\cdot p})}{p^2-M^2-m^2+i\varepsilon}\gamma^5 \frac{i\left((\not{p}+\not{q})+M-\frac{m^2}{2}\frac{\not{n}}{n\cdot(p+q)}\right)}{(p+q)^2-M^2-m^2+i\varepsilon} \quad (12)$$

- The regularized axial amplitude satisfies:

$$i\Pi^{5R\mu\nu}(M,\bar{M},q)q_\nu = 2M\mathcal{A}(M,q)^\mu - 2\bar{M}\mathcal{A}(\bar{M},q)^\mu$$

Since the original amplitude is obtained formally as $lim_{\bar{M}\to\infty}$, the axial anomaly is given by:

$$B^\mu = lim_{\bar{M}\to\infty}(-2\bar{M}\mathcal{A}(\bar{M},q)^\mu)$$

Computation of the anomaly

Now, we compute (12). First notice that after computing the trace, the integral is finite. A typical term containing the vector n^μ is of the form:

$$C^\mu = 2M^2 m^2(-ie)^2 \varepsilon^{\mu\alpha} n_\alpha \int dp \frac{1}{p^2 - M^2 - m^2 + i\varepsilon} \frac{1}{(p+q)^2 - M^2 - m^2 + i\varepsilon$$

Now we recall an important property of ML prescription. It preserves naive power counting. According to this, $C^\mu \sim M^{-1}$ for large M.
Following the same argument, we can easily check that all terms containing n^μ vanish when $M \to \infty$.
It remains the Lorentz invariant term:

$$i\Pi^{5\mu\nu}(q)q_\nu = lim_{\bar{M}\to\infty} \quad -4e^2 \bar{M}^2 \varepsilon^{\alpha\mu} q_\alpha \int dp \frac{1}{p^2 - \bar{M}^2 - m^2 + i\varepsilon} \frac{}{(p+q)^2}$$

$$q_\nu < j^{5\nu} >= \frac{1}{-ie} i\Pi^{5\mu\nu}(q) A_\mu q_\nu = \frac{e}{\pi} \varepsilon^{\alpha\mu} q_\alpha A_\mu \qquad (14)$$

Equation (14) is the standard Lorentz invariant result.

Anomaly in more dimensions and other regularizations

- $4d$ anomaly, using PV+ML regularization gives the standard Lorentz invariant term.
- $2d$ and $4d$ anomaly, using Dimensional Regularization+ML regularization gives the standard Lorentz invariant term.
- Fujikawa's method plus ML, produce the standard Lorentz invariant anomaly in even dimensions.
- This implies that cancellation of anomalies within families of quarks and leptons still holds in the VSR SM.The VSR SM is anomaly free.

Schwinger Dyson equation for the product of three currents in VSR

The vector current is:

$$j^\mu = \bar{\psi}\gamma^\mu\psi + \frac{1}{2}m^2 n^\mu((n\cdot\partial)^{-1}\bar{\psi})\not{n}(n\cdot\partial)^{-1}\psi$$

The axial vector current is:

$$j^{\mu 5} = \bar{\psi}\gamma^\mu\gamma^5\psi + \frac{1}{2}m^2\left(\frac{1}{n\cdot\partial}\bar{\psi}\right)\not{n}n^\mu\gamma^5\left(\frac{1}{n\cdot\partial}\psi\right)$$

Consider the path integral,

$$Z = \int \mathcal{D}\psi\mathcal{D}\bar{\psi}e^{iS}j^{\alpha 5}(y)j^\beta(z)$$

where the action is $S = \int d^4x\bar{\psi}\left(i\left(\not{\partial} + \frac{1}{2}\not{n}m^2(n\cdot\partial)^{-1}\right) - M\right)\psi$

Three vector currents identity

- Make the following local transformations

$$\delta\psi(x) = i\alpha(x)\psi(x); \qquad \delta\bar{\psi}(x) = -i\alpha(x)\bar{\psi}(x) \tag{15}$$

The integration measure is invariant under this transformation.

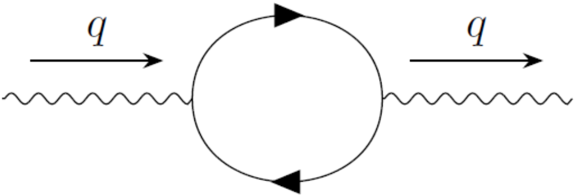
- We get the following Ward identity:

$$\partial^x_\mu < 0|T(j^\mu(x)j^{\alpha 5}(y)j^\beta(z))|0 > = < 0|T\delta_x j^{\alpha 5}(y)j^\beta(z))|0 > + \\ < 0|T(j^{\alpha 5}(y)\delta_x j^\beta(z))|0 > \tag{16}$$

where $\delta_x j^{\alpha 5}(y)$ and $\delta_x j^\beta(z)$ are the variations of the current under (15).

- The non-locality of the action and currents modify the Ward identity for the triangle graph.
- In a similar way we can derive the Ward identity for the divergence of the axial vector current. It contains extra terms.

Recovering $Sim(2)$



We trade $\bar{n}_\mu$ by q_μ. i.e. $\bar{n}_\mu = an_\mu + bq_\mu$. From the conditions: $\bar{n}.\bar{n} = 0$, $\bar{n}.n = 1$ we get $\bar{n}_\mu = -\frac{q^2}{2(n.q)^2} n_\mu + \frac{q_\mu}{n.q}$

Conclusions

- We have examined the appearance of axial anomalies in VSR electrodynamics, using Pauli-Villars and dimensional regularization of ultraviolet divergences and Mandelstam-Leibbrandt regularization of infrared divergences.
- Since ML preserves naive power counting in loop integrals, we have shown that the usual form for the anomaly of the axial current appears, without corrections from VSR terms.
- No anomaly is present in the vector current conservation.
- We derived the Ward identity for the product of two vectors and one axial vector current in VSR. The nonlocality of the model introduces new contact terms.
- According to our results, the VSRSM must be free from local chiral anomalies.
- For more details please see J.A. Phys.Rev.D 103 (2021) 7, 075011

THANK YOU