A far-from-equilibrium horizon

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Humboldt Kolleg on Quantum Gravity and Fundamental Interactions

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In memory of Theodore Tomaras

A *dear friend and outstanding scientist*

Based on 2101.12529; **2107.00965;** *& in progress*

with Vassilis Papadopoulos

& Zhongwu Chen

See also 2006.11333 *w.* Shira Chapman, Dongsheng Ge, Giuseppe Policastro

1. Introduction

(Near) static gravitational horizons, related to (near) equilibrium thermodynamical systems, are well studied and well understood.

Beckenstein, Hawking, . . . , Jacobson, . . .

Much less is known far from equilibrium

Local versus event horizon ?

Light rays one-way

Causally disconnected

Stationary states are a particular set of states where progress looks possible

In this talk I will describe a simple far-from-equilibrium system for which exact calculations are possible. The hope is that one will learn from it some more general lessons.

The system in question is that of a **gravitating domain wall** anchored at an AdS boundary

These are ubiquitous in quantum gravity for very different reasons:

-- bridges in the QG lanscape

(phase coexistence, bubble nucleation, cosmology)

-- Randall-Sundrum compactifications & localized gravity

-- Enter in recent toy models of the Page curve

I will not dwell on these issues in today's talk

Most gravitating domain walls are thick. But starting with the famous paper of Coleman & De Lucia, a frequently-used approximation is that of thin walls.

The minimal action

$$I_{\rm gr} = -\frac{1}{2}\int_{\mathbb{S}_1} d^3x\sqrt{g_1}\,(R_1 + \frac{2}{\ell_1^2}) - \frac{1}{2}\int_{\mathbb{S}_2} d^3x\sqrt{g_2}\,(R_2 + \frac{2}{\ell_2^2})$$

$$+\lambda\int_{\mathbb{W}} d^2s\sqrt{\hat{g}_w} + \text{GHY terms} + \text{ct}\,.$$

depends on 3 dimensionless parameters ℓ_1, ℓ_2, λ (with $8\pi G = 1$)

We will work in 2+1 dimensions. The thin wall is a simple form of 'matter.'

Take $\ell_1 \leq \ell_2$. A simple calculation shows that vacuum domain walls exist for

true vacuum (ℓ_1)

false vacuum (ℓ_2)

CB ’02

$$\underbrace{\frac{1}{\ell_1} - \frac{1}{\ell_2}}_{\lambda_{\text{min}}} < \lambda < \underbrace{\frac{1}{\ell_1} + \frac{1}{\ell_2}}_{\lambda_{\text{max}}}$$

← *False vacuum <u>unstable</u> to bubble nucleation*

→ *Domain wall <u>inflates</u>*

Vilenkin ’81
Ipser, Sikivie ’83
Karch, Randall ‘01

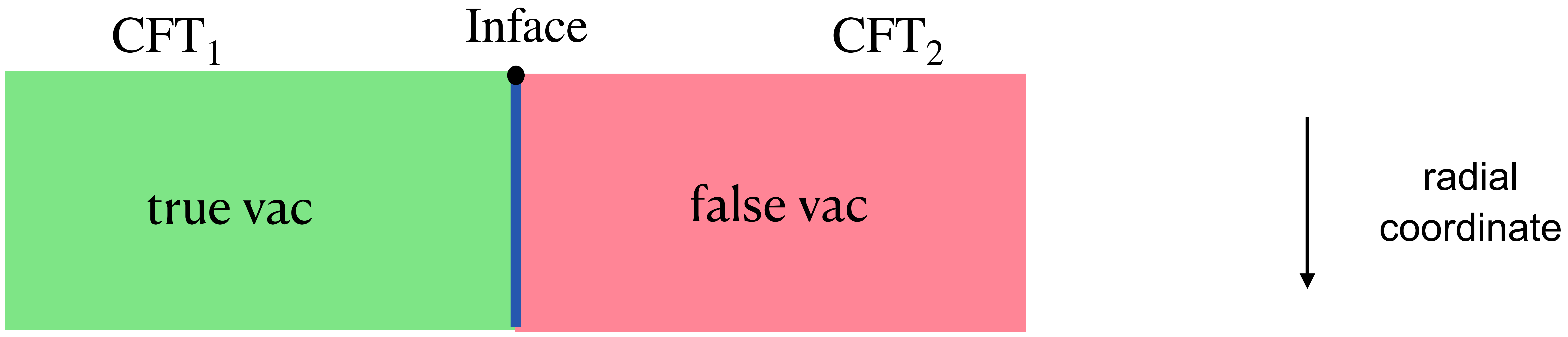
↓ *<u>BPS</u> values for <u>flat</u> walls*

Cvetic, Griffies, Rey ’92
Cardoso, Dall’Agata, Lust ’02
Ceresole *et al* ‘06

Holographic dual :

The wall hits the AdS boundary at the location of a conformal interface

Karch, Randall ’01
CB, de Boer, Dijkgraaf, Ooguri ‘02



Scales chararizing the state *(temperature, heat flow, volume)* deform the interior geometry of both the bulks and the wall away from AdS

Dictionary:

Central charges

$$c_j = 12\pi \ell_j$$

Brown, Henneaux '86

Entropy

$$\log g_{\rm I} = 2\pi\ell_1\ell_2 \left[\lambda_{\max} \tanh^{-1}\left(\frac{\lambda}{\lambda_{\max}}\right) - \lambda_{\min} \tanh^{-1}\left(\frac{\lambda_{\min}}{\lambda}\right)\right]$$

Simidzija, Van Raamsdonk '20

Azeyanagi, Karch, Takayanagi, Thompson '07

Energy transmission coeffs

$$\mathcal{T}_{1\to2} = \frac{\lambda_{\max} + \lambda_{\min}}{\lambda_{\max} + \lambda}, \qquad \mathcal{T}_{2\to1} = \frac{\lambda_{\max} - \lambda_{\min}}{\lambda_{\max} + \lambda}$$

CB, Chapman, Ge, Policastro '20

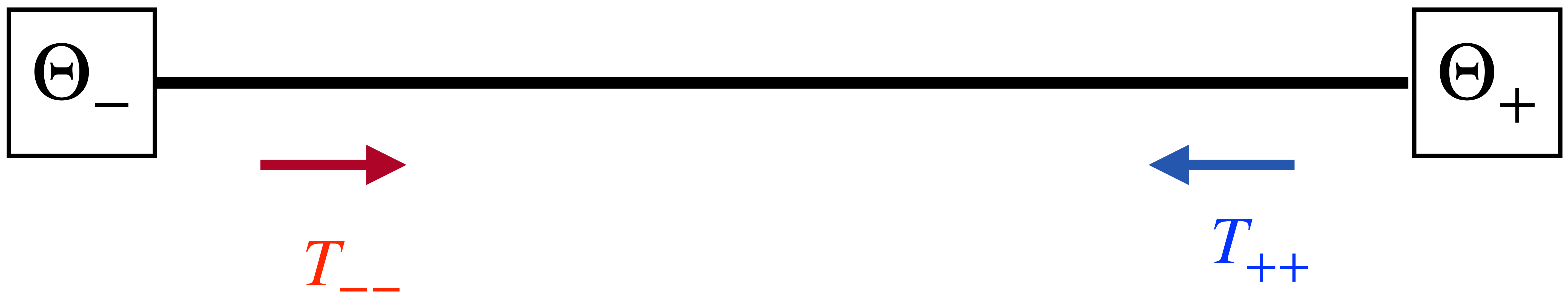
This minimal bottom-up model is at best a useful approximation to full-fledged top-down dual pairs. It captures however the 3 universal ICFT operators:

Bulk metrics	$g^{(1)}_{\mu\nu}$	$\longleftrightarrow$	$T^{(1)}_{ab}$	Stress tensors
	$g^{(2)}_{\mu\nu}$	$\longleftrightarrow$	$T^{(2)}_{ab}$	
Brane embedding	$x^\mu(\sigma_\alpha)$	$\longleftrightarrow$	$y^a(\tau)$	**Displacement operator**

Billo, Gonçalves, Lauria, Meineri ’16

In this talk I will describe some interesting **far-from-equilibrium** states of this system and compare with what is known/expected from the field theory side.

A simple set of states of a homogeneous q-wire



$$\langle T_{\pm\pm}\rangle = \frac{\pi c}{12}\Theta_\pm^2 \quad\Longrightarrow\quad \langle T^{tx}\rangle = \frac{\pi c}{12}(\Theta_-^2 - \Theta_+^2)$$

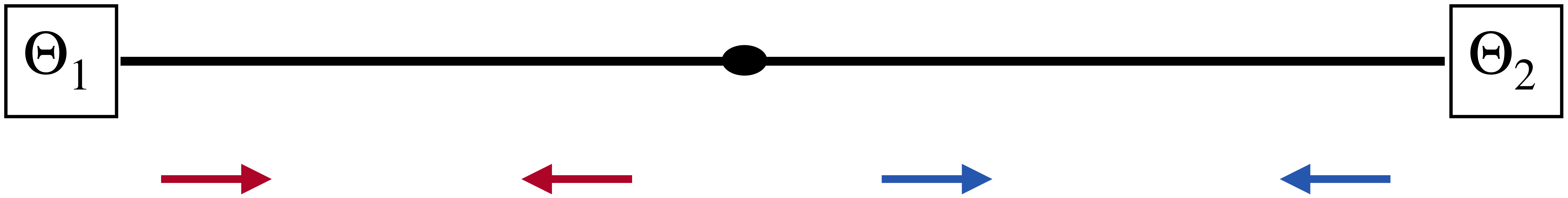
Heat flow

Stefan-Boltzman law

This is a fake out-of-equilibrium state, since left and right movers dont interact.

(*chemical potential for conserved momentum*)

To make things more interesting introduce a defect (or junction/interface):



$T^{(1)}_{--} = \frac{\pi c_1}{12}\Theta_1^2$ $\quad T^{(1)}_{++}$ $\quad T^{(2)}_{--}$ $\quad T^{(2)}_{++} = \frac{\pi c_2}{12}\Theta_2^2$

$$\langle T^{(1)}_{++}\rangle = \mathscr{R}_1 \frac{\pi c_1}{12}\Theta_1^2 + \mathscr{T}_2 \frac{\pi c_2}{12}\Theta_2^2$$

$$\langle T^{(2)}_{++}\rangle = \mathscr{T}_1 \frac{\pi c_1}{12}\Theta_1^2 + \mathscr{R}_2 \frac{\pi c_2}{12}\Theta_2^2$$

$\mathscr{R}_j\ ,\ \mathscr{T}_j$

reflection, transmission coefficients

These coefficients were introduced in Quella, Runkel, Watts '06

and shown to be universal in 2D in Meineri, Penedones, Rousset '19

They obey:

$$\mathcal{R}_j + \mathcal{T}_j = 1$$ **conservation of energy**

$$c_1 \mathcal{T}_1 = c_2 \mathcal{T}_2$$ **detailed balance**

So a simple calculation gives

$$\frac{dQ}{dt} = \frac{\pi}{12} c_1 \mathcal{T}_1 \left(\Theta_1^2 - \Theta_2^2 \right)$$

Bernard, Doyon, Viti '14

Agrees with special cases: $\mathcal{T}_1 = 0$ (boundary) $\mathcal{T}_1 = 1$ (Topological)

The energy currents do not suffice to describe the state of the outgoing fluids

We parametrize the entropy currents as follows:

thermal

$$\langle s_-^{(1)} \rangle = -\frac{\pi c_1}{6}\,\Theta_1\;, \qquad \langle s_+^{(1)} \rangle = \frac{\pi c_1}{6}\,\Theta_1^{\rm eff}$$

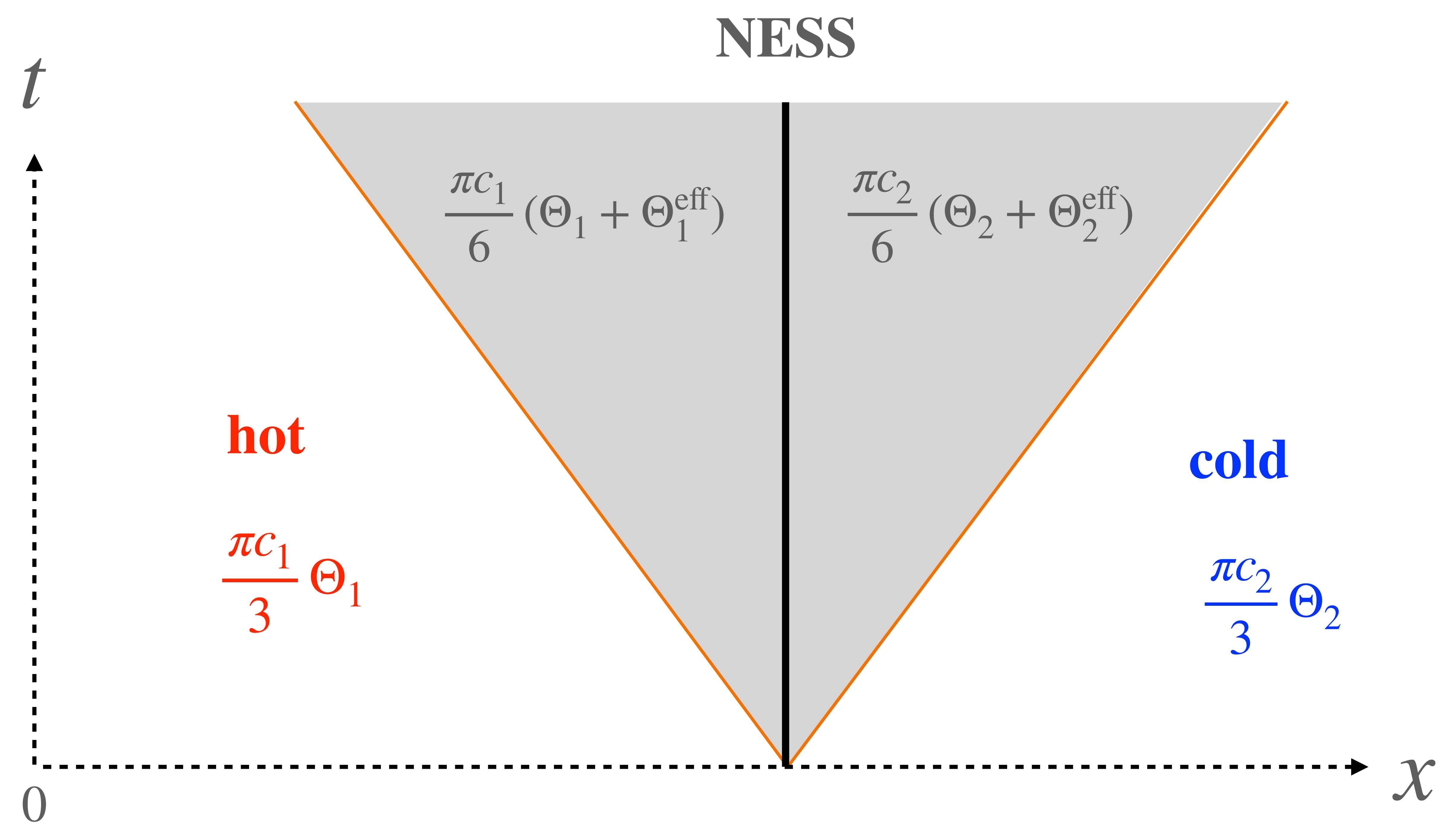
$$\langle s_+^{(2)} \rangle = \frac{\pi c_2}{6}\,\Theta_2\;, \qquad \langle s_-^{(2)} \rangle = -\frac{\pi c_2}{6}\,\Theta_2^{\rm eff}$$

? ?

Microcanonical bounds: $s \leq s_{\rm micro} = \left(\frac{\pi c}{3}\,\langle T \rangle\right)^{1/2} \Longrightarrow$

$$\Theta_1^{\rm eff} \leq \sqrt{\mathscr{R}_1 \Theta_1^2 + \mathscr{T}_1 \Theta_2^2} \qquad \text{and} \qquad \Theta_2^{\rm eff} \leq \sqrt{\mathscr{R}_2 \Theta_2^2 + \mathscr{T}_2 \Theta_1^2}$$

In general **entanglement** of scattering quanta leads to **production** of (coarse-grained) **entropy**:



Entropy densities

"partitioning protocol"

$$\frac{dS_{\text{tot}}}{dt} = \frac{\pi c_1}{6}(\Theta_1^{\text{eff}} - \Theta_1) + \frac{\pi c_2}{6}(\Theta_2^{\text{eff}} - \Theta_2) + \frac{dS_{\text{def}}}{dt}$$

A first-principles calculation of entropy production at an interface is lacking

In the minimal holographic model the interface is **maximally-mixing**,

i.e. outgoing quantum fluids are thermal & entropy production is maximal

In the homogeneous case, the dual state is given by the **BTZ metric**:

$$ds^2 = \frac{\ell^2 dr^2}{(r^2 - M\ell^2 + J^2\ell^2/4r^2)} - (r^2 - M\ell^2)dt^2 + r^2 dx^2 - J\ell\, dxdt$$

with

mass $$\frac{1}{2} M\ell = \langle T_{--}\rangle + \langle T_{++}\rangle$$

spin $$\frac{1}{2} J = \langle T_{--}\rangle - \langle T_{++}\rangle = \frac{dQ}{dt}$$

This has **outer and inner horizons**, and an **ergosphere** (cf Kerr BH):

$$r_\pm^2 = \frac{1}{2} M\ell^2 \pm \frac{1}{2}\sqrt{M^2\ell^4 - J^2\ell^2}$$

$$r_{\rm ergo} = \sqrt{M}\,\ell \geq r_+$$

4. Dual gravitational state

In the homogeneous case, the dual state is given by the **BTZ metric**:

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In infalling **Eddington-Finkelstein** coordinates

$$dv = dt + \frac{\ell\, dr}{h(r)} \qquad \text{and} \qquad dy = dx + \frac{J\ell^2 dr}{2r^2 h(r)} \ ,$$

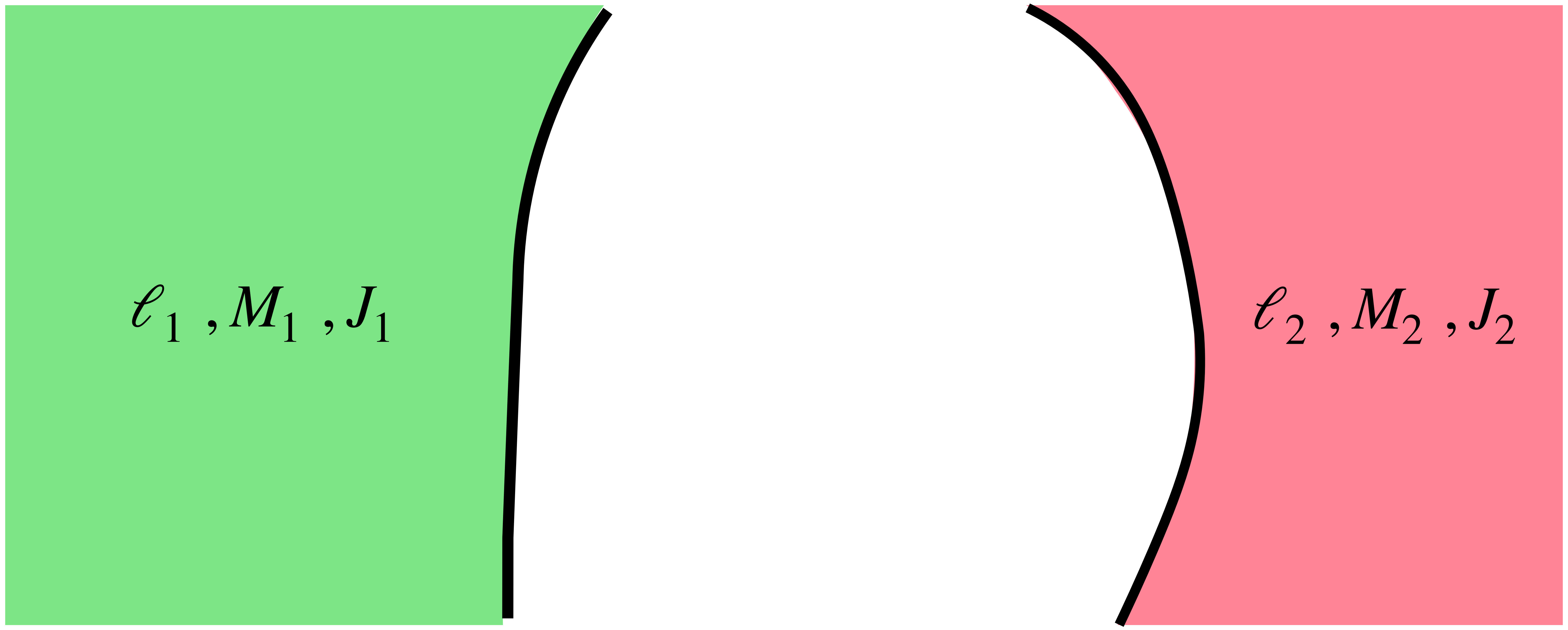
$$ds^2 = -\, h(r)\, dv^2 + 2\ell\, dv\, dr + r^2 \Big(dy - \frac{J\ell}{2r^2}\, dv\Big)^2$$

$$\frac{1}{r^2}(r^2 - r_+^2)(r^2 - r_-^2)$$

the metric is smooth at the **future horizon.**

Infalling light rays have $v, y = \text{constant}$

We glue two BTZ backgrounds along a **brane** with tension λ



The embedding of the stationary brane is described by six functions of one variable σ :

$$x_j(\sigma), \quad r_j(\sigma), \quad t_j = \tau + f_j(\sigma)$$

The brane equations are

-- Continuity of the induced metric $\hat{g}_{ab}(\sigma)$ *3 eqs*

-- Israel-Lanczos conditions: $[K_{\alpha\beta}] = -\lambda\, \hat{g}_{\alpha\beta}$ *1 eq & 2 momentum constraints*

Consistency requires $\boxed{J_1 = -J_2}$ *Energy conservation in ICFT*

Gauge fixing:

$$\sigma = r_1^2 - M_1\ell_1^2 = r_2^2 - M_2\ell_2^2$$

$$\Delta t' \equiv f_2' - f_1' = \frac{J_1}{2\sigma}(\ell_1 x_1' + \ell_2 x_2')$$

time advance/delay

Solution of remaining eqs:

$$\frac{x_1}{\ell_1} = -\int \frac{\mathrm{sgn}(\sigma)\left[(\lambda^2 + \lambda_0^2)\,\sigma^2 + (M_1 - M_2)\sigma\right]}{2(\sigma - \sigma_+^{\mathrm{H1}})(\sigma - \sigma_-^{\mathrm{H1}})\sqrt{A\sigma(\sigma-\sigma_+)(\sigma-\sigma_-)}}$$

$$\frac{x_2}{\ell_2} = -\int \frac{\mathrm{sgn}(\sigma)\left[(\lambda^2 - \lambda_0^2)\,\sigma^2 - (M_1 - M_2)\sigma\right]}{2(\sigma - \sigma_+^{\mathrm{H2}})(\sigma - \sigma_-^{\mathrm{H2}})\sqrt{A\sigma(\sigma-\sigma_+)(\sigma-\sigma_-)}}$$

where $\lambda_0^2 = \lambda_{\min}\lambda_{\max}$ and putative singularities

$$\sigma_\pm^{\mathrm{H}j} = -\frac{M_j \ell_j^2}{2} \pm \frac{1}{2}\sqrt{M_j^2 \ell_j^4 - J_j^2 \ell_j^2}$$

horizons

$$\sigma_\pm = \frac{-B \pm \sqrt{B^2 - AC}}{A}$$

$$A = (\lambda_{\max}^2 - \lambda^2)(\lambda^2 - \lambda_{\min}^2), \quad B = \lambda^2(M_1 + M_2) - \lambda_0^2(M_1 - M_2),$$

$$C = -(M_1 - M_2)^2 + \lambda^2 J_1^2 \ .$$

Exchange space and time, J=0

Simidzija, Van Raamsdonk ’20

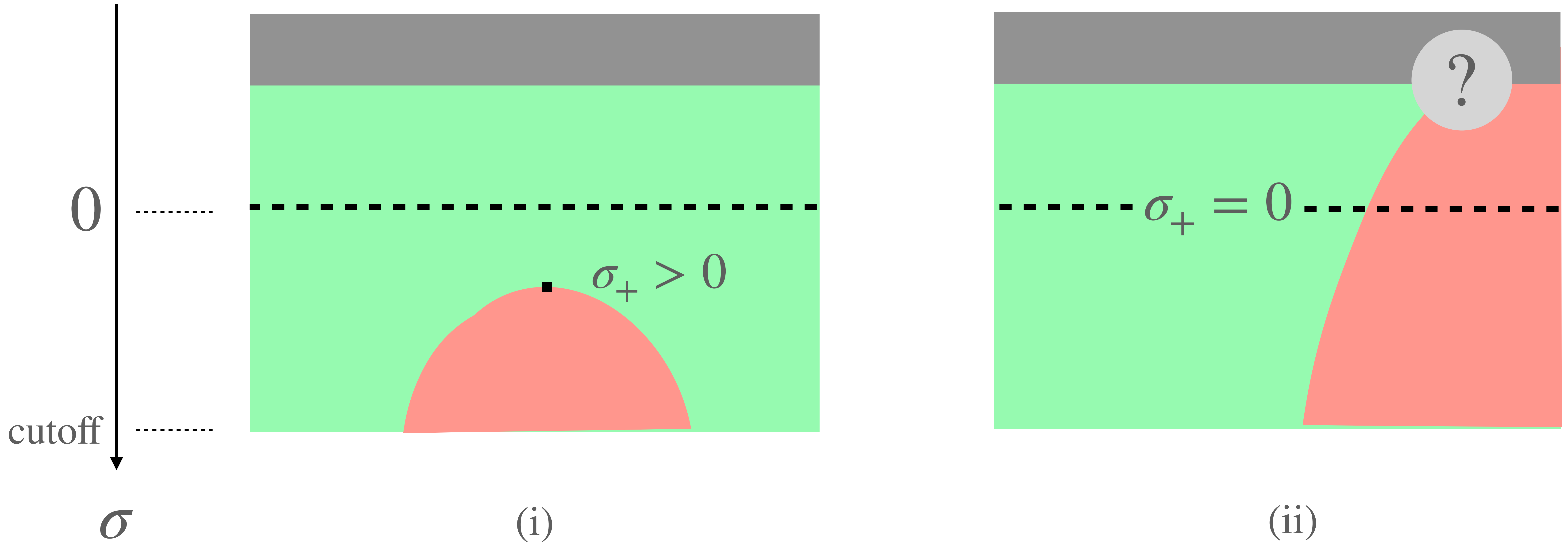
CB, Chen, Papadopoulos '21

One finds two types of brane solution:

$\sigma_+ > 0$ turning point

$\sigma_+ = 0$ smooth entry in ergoregion

$\sigma_+ < 0$ wrong signature



avoids ergoregion

enters ergoregion

$\sigma_+ = 0$ implies

$$\boxed{M_1 - M_2 = \pm\lambda J_1 = \mp\lambda J_2} \quad \text{and} \quad \lambda^2(M_1 + M_2) \geq \lambda_0^2(M_1 - M_2)$$

Using the holographic dictionary and the fact that the incoming fluxes are thermal gives:

$$M_j = 4\pi^2\Theta_j^2 - \frac{J_j}{\ell_j} \implies M_1 - M_2 = 4\pi^2(\Theta_1^2 - \Theta_2^2) - J_1\left(\frac{1}{\ell_1} + \frac{1}{\ell_2}\right)$$

$$\boxed{\implies \frac{dQ}{dt} = \frac{\pi}{12}c_1\mathcal{T}_{1\to 2}\left(\Theta_1^2 - \Theta_2^2\right)} \quad \text{with} \quad \boxed{\mathcal{T}_{1\to 2} = \frac{\lambda_{\max} + \lambda_{\min}}{\lambda_{\max} \pm \lambda}}$$

cf black/white. hole

For + sign, recover the expected Stefan-Boltzman constant

With $\sigma_+ = 0$ the embedding functions read

$$\frac{x_1'}{\ell_1} = -\frac{(\lambda^2 + \lambda_0^2)\,\sigma + (M_1 - M_2)}{2(\sigma - \sigma_+^{\rm H1})(\sigma - \sigma_-^{\rm H1})\sqrt{A(\sigma - \sigma_-)}}$$

$$\frac{x_2'}{\ell_2} = -\frac{(\lambda^2 - \lambda_0^2)\,\sigma - (M_1 - M_2)}{2(\sigma - \sigma_+^{\rm H2})(\sigma - \sigma_-^{\rm H2})\sqrt{A(\sigma - \sigma_-)}}$$

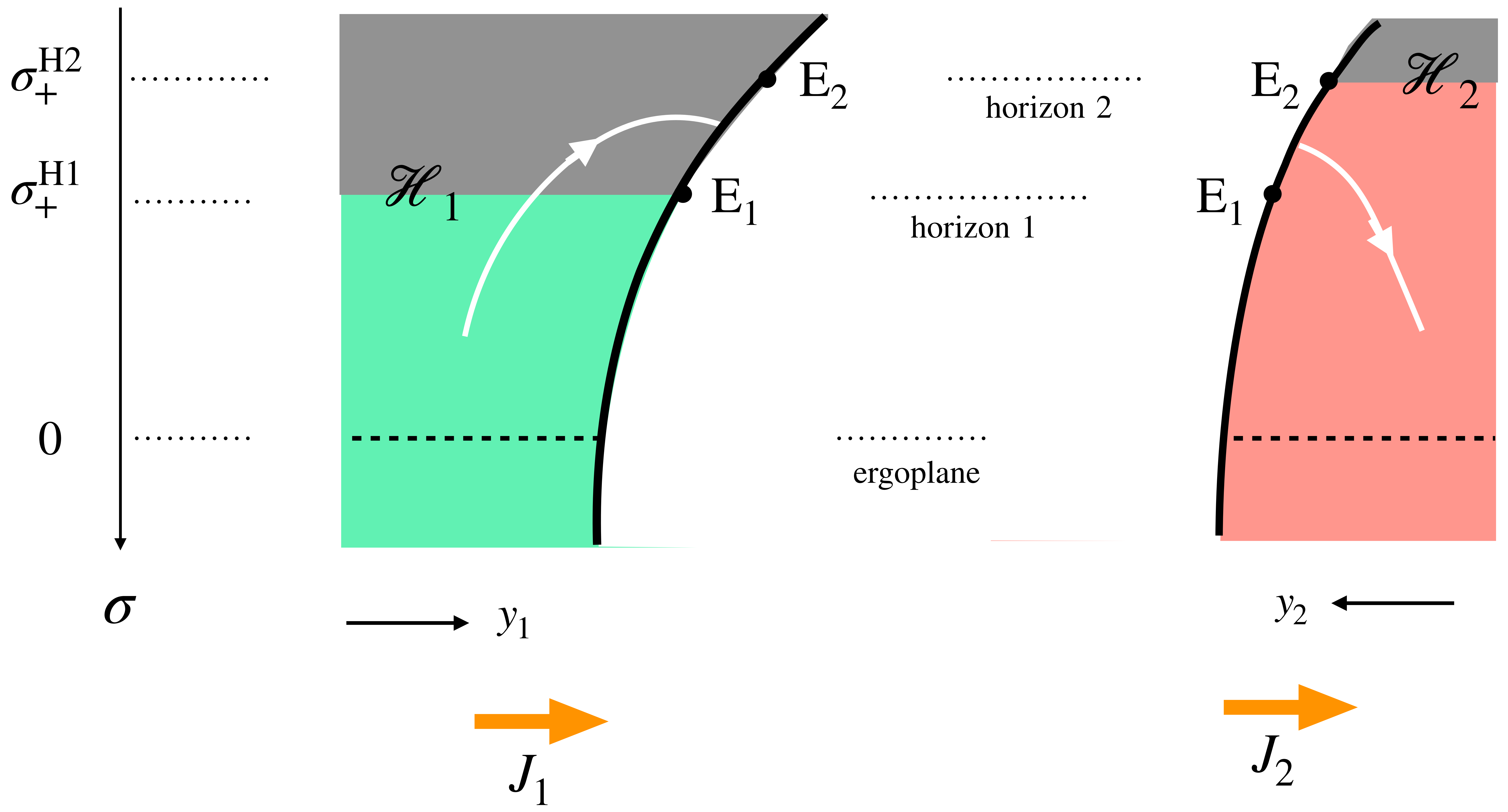
Can be shown that σ_- lies behind the inner (Cauchy) horizons

where the classical solution cannot be trusted

cf Dias, Reall, Santos ’19
Papadodimas et al '19
Balasubramanian et al '19
Emparan, Tomasevic '20

$\therefore$ Beyond the ergoplane the brane cannot turn around and exit the horizon

The solution looks like this (Eddington-Finkelstein coordinates):



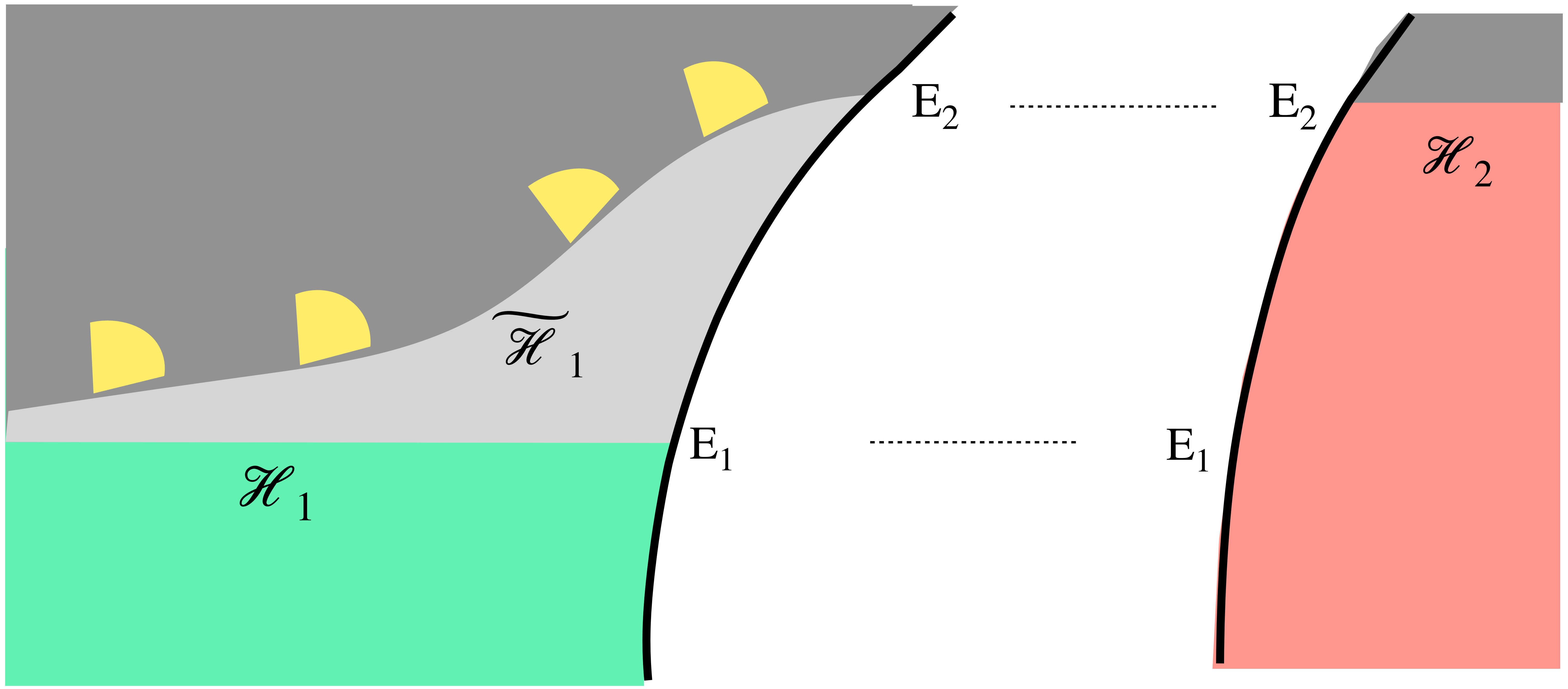
The local (apparent) horizon $\mathscr{H}_1 \cup \mathscr{H}_2$ lies outside the event (causal) horizon

The former is discontinuous and **non-compact**

$\Longrightarrow$ no contradiction with general theorems

cf Hawking & Ellis

The (non-Killing) event horizon in region 1 is the boundary of the causal past of $E_2 \times \text{time}$



Define global timelike unit vector field:

$$t^\mu \partial_\mu = \frac{\partial}{\partial v_j} + \frac{h_j(r_j) - 1}{2\ell_j} \frac{\partial}{\partial r_j} + \frac{J_j \ell_j}{2r_j^2} \frac{\partial}{\partial y_j} \qquad \text{in the } j\text{th region} .$$

Future-directed null curves obey

$$\dot{x}^\mu = (\dot{v}, \dot{r}, \dot{y}) \qquad \text{where} \quad \dot{x}^\mu \dot{x}_\mu = 0 \quad \text{and} \quad \dot{x}^\mu t_\mu < 0$$

$$\Longrightarrow \quad \dot{r} = \frac{h(r)}{2\ell}\dot{v} - \frac{r^2}{2\ell \dot{v}}\left(\dot{y} - \frac{J\ell}{2r^2}\dot{v}\right)^2 \quad \text{and} \quad \dot{v} > 0 \; .$$

$\therefore$ Arrow of time defined by inceasing v, & behind the horizon ($h < 0$) r is monotone decreasing

So $\mathscr{H}_2$ Is part of the event horizon

The projection of $\tilde{\mathscr{H}}_1$ on a Cauchy slice

Is a curve through E_2 everywhere tangent to the local lightcone

$\Longrightarrow$ Minimize the angle between projection of null curves and positive y_1 axis

$$\Longrightarrow \boxed{\left.\frac{dy}{dr}\right|_{\widetilde{\mathscr{H}}_1} = \frac{2\ell}{J\ell - 2r\sqrt{M\ell^2 - r^2}}}$$

near BTZ horizon, $r = r_+ + \epsilon$, behaves as ϵ^{-1}

$\Longrightarrow$ event and BTZ horizons approach asymptotically each other

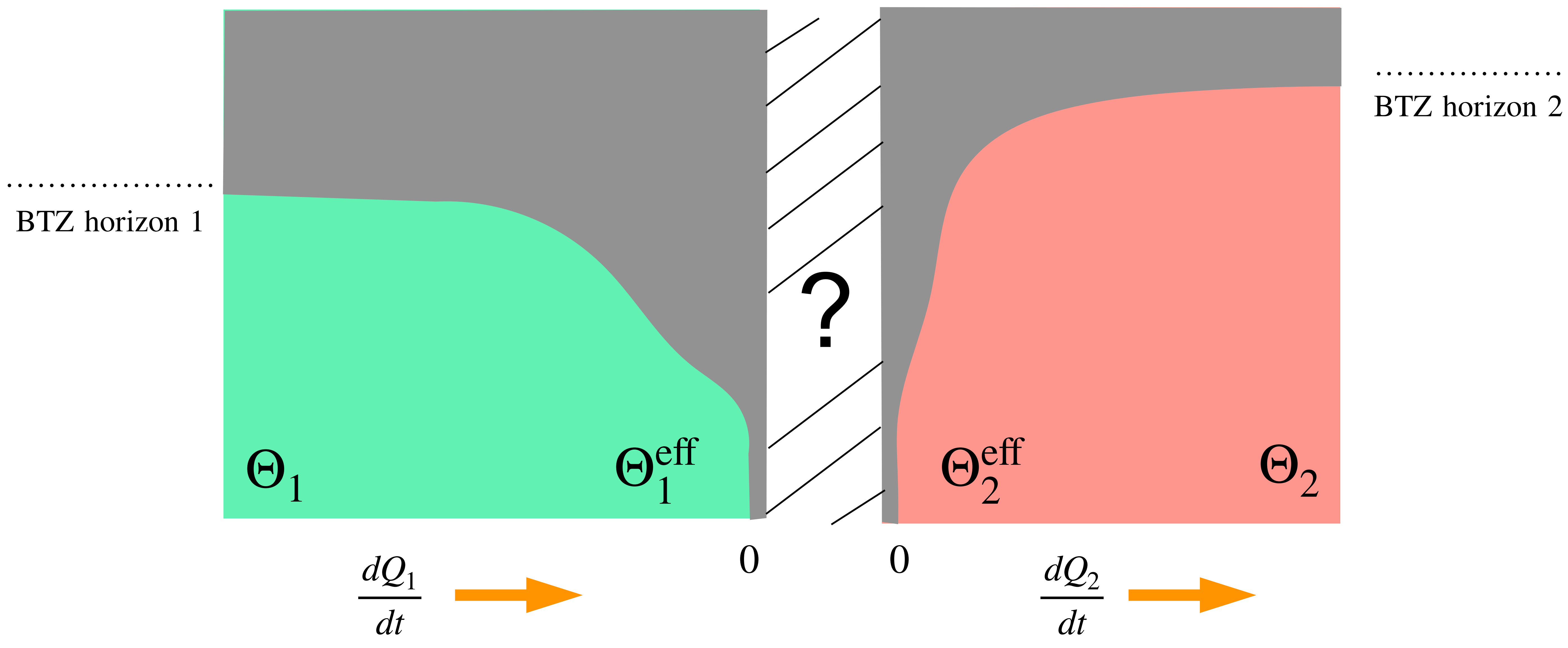
Implies that outgoing fluxes are thermal in both directions

i.e. incoming fluxes are **thermalized by single scattering at interface !**

Is this possible in boundary CFT ?

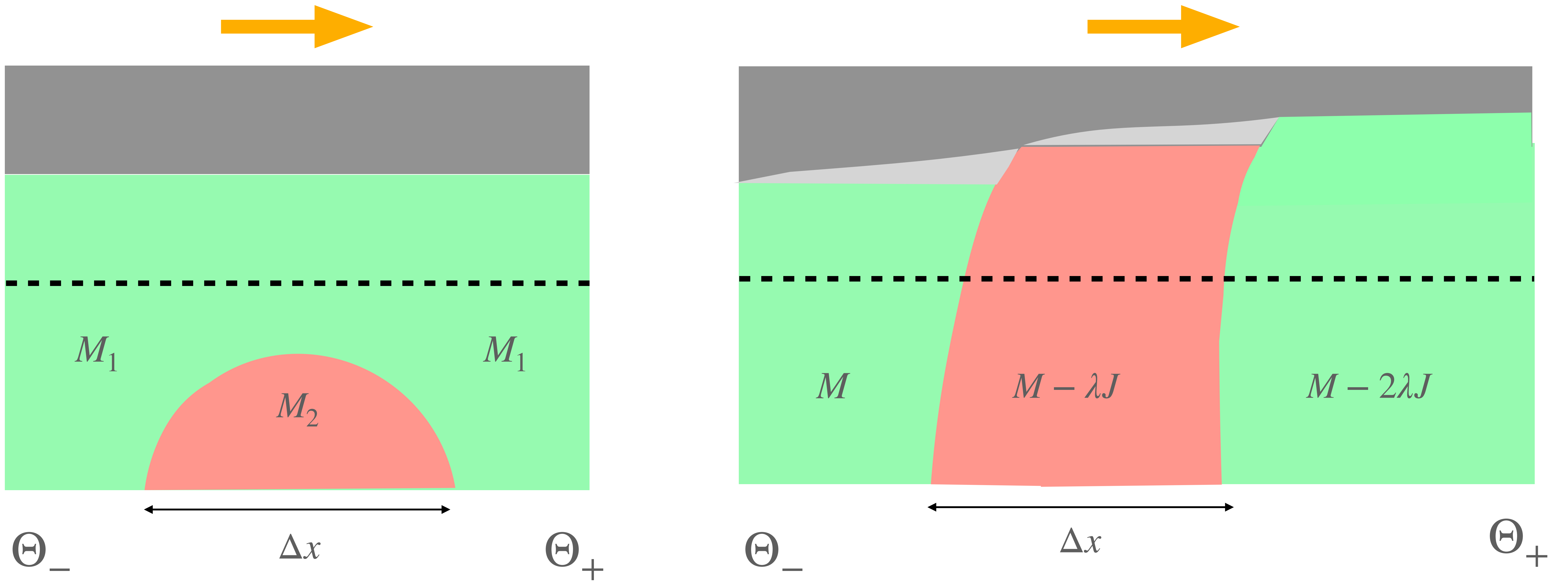
cf Hubeny, Marolf, Rangamani, Fiscetti, Emparan, Martinez, Wiseman, Santos, . . .

Resembles double-sided funnel solution, but who ordered fine-tuning of temperatures at two horizon points?



When $\sigma_+ > 0$ the brane has a turning point outside the ergoregion.

The solution is shown on the left:



Near thermal equilibrium $(\Theta_+ \simeq \Theta_-)$ the system undergoes a phase transition at a critical value of $\Delta x\,\Theta$

CB, Papadopoulos '21

This is a Hawking-Page type of transition (possibly signaling the deconfinement of the middle CFT

Witten '98; . . .

At **high temperature**, the **thermal conductivity** is the same as for a brane with tension 2λ

Using the expression for the transport coefficients one finds:

$$\mathcal{T}_{\text{pair}} = \mathcal{T}_1 (1 + \mathcal{R}_2^2 + \mathcal{R}_2^4 + \cdots)\, \mathcal{T}_2$$

classical scatterers

At **low temperature**, the system behaves as in the homogeneous system

$$\mathscr{T}_{\text{pair}} = 1$$

perfect constructive interference

(as if the two branes have merged into a tensionless one)

This phase does not exist when $c_1 > 3c_2$

i.e. when the island CFT has too few degrees of freedom

Reassuringly, this includes the limit of an "empty CFT"

7. Outlook

Many questions raised by this simple model. Most urgently (in progress) :

-- Compute Ryu-Takayanagi-Hubeny-Rangamani surfaces;

understand how entropy production is related to spike in event horizon

At the same time, many of the features may be related to the bottom-up thin-brane approximation:

-- Extend to top-down, thick-brane models

-- Compute entropy production in ICFT; maximal mixing ?

Thank you