

# Gravity above the Planck Scale

Corfu2021: Humboldt Kolleg on Quantum Gravity and  
Fundamental Interactions

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*European Institute for Sciences and Their Applications*



# Planck Scale Symmetry?

- What happens at the Planck scale? And beyond?
- Could there be a radically different theory beyond the Planck scale?
- New highly symmetric phase?
- Tensionless string? Gross-Mende?
- Fundamental theory conformal?
- Symmetries spontaneously broken

# Conformal Symmetry?

- Is fundamental theory conformal? Natural?
- Local scale invariance
- Planck scale (and other scales) emerging from symmetry breaking?
- Problem: Conformal gravity higher derivative

# Is Gravity a Theory of a Graviton?

- At low energies gravity is a theory of a graviton in 4D
- Could gravity be “emergent”?
- Is spacetime “emergent”?
- Could gravity have a UV origin that is a theory of different field in  $D > 4$  spacetime?
- Could this allow conformal invariance?

# Is YM a Theory of Gluons?

- 4D Maxwell can arise from a p-form gauge field in  $D > 4$
- 4D Yang-Mills can arise from 6D non-lagrangian theory, not a theory of a vector field

# 4D Super YM from 6D

- Free (2,0) theory in 6-D: 2-form  $B$ ,  $H=H^*$
- Reduces to 4D  $N=4$  Maxwell,  $F=dA$
- Interacting (2,0) SCFT, non-lagrangian, reduces to 4D SYM
- Conformal invariance in 6D
- S-duality arises from diffeos on  $T^2$
- Strong coupling limit of 5-D SYM: (2,0) SCFT

# 4D gravity from 6D?

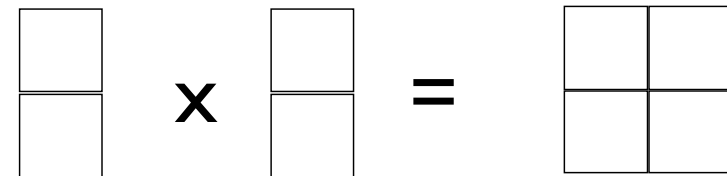
- Free (4,0) SCFT in 6-D: gauge field  $C_{mnpq}$
- Reduces to 5-D linearised N=8 SUGRA
- Is there an interacting (4,0) SCFT? Non-lagrangian, reducing to 5-D SUGRA?
- Strong coupling limit of 5-D SUGRA?
- Exotic conformal theory of gravity?
- Highly symmetric (4,0) phase of M-theory?

# Gravity = (YM)<sup>2</sup>

- Free SUGRA ~ Free (SYM)<sup>2</sup>



- Free (4,0) ~ Free ((2,0) theory)<sup>2</sup>



- Free (2,0) reduces to 5-D theory of photon + dual photon
- Free (4,0) reduces to 5-D theory of graviton + dual graviton + double dual graviton



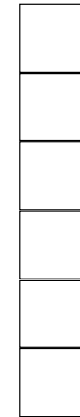
# 5-D Superalgebra

$$\{Q_\alpha^a, Q_\beta^b\} = \Omega^{ab} (\Gamma^\mu C)_{\alpha\beta} P_\mu + C_{\alpha\beta} (Z^{ab} + \Omega^{ab} K)$$

- Central charges Z,K
- Z Electric charges for Maxwell fields
- States with K ~ KK modes of 6-D (p,0) theory
- SYM: K carried by BPS solitons (from YM instantons)
- Does M-theory on  $T^6$  have BPS states with K?
- Do they become massless at strong coupling?

# Maxwell in D- dimensions

- Photon  $A_\mu$
- Dual photon:  $n=D-3$  form  $\tilde{A}_{\mu_1 \dots \mu_n}$   
 $F = *\tilde{F}$
- Magnetic charges: D-4 branes.  
A has Dirac strings, or connection on non-trivial bundle,  $\tilde{A}$  well-defined
- Electric charges: 0-branes.  
 $\tilde{A}$  has Dirac string singularities, A OK
- YM? No non-abelian theory for  $\tilde{A}$



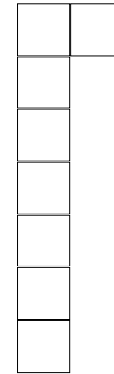
# Linearised Gravity

- **Graviton**  $h_{\mu\nu}$    $(1, 1)$

Field strength  $R_{\mu\nu\rho\sigma}$    $(2, 2)$

- **Dual Graviton**

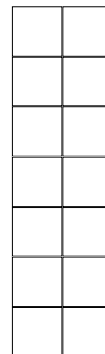
$\tilde{h}_{\mu_1 \dots \mu_n \nu}$   $(n, 1)$



$n=D-3$

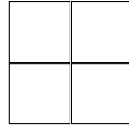
- **Double Dual Graviton**

$\tilde{\tilde{h}}_{\mu_1 \dots \mu_n \nu_1 \dots \nu_n}$   
 $(n, n)$



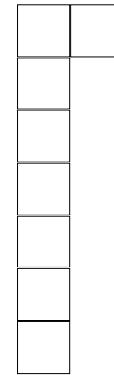
# Linearised Gravity

- **Graviton**  $h_{\mu\nu}$    $(1, 1)$

Field strength  $R_{\mu\nu\rho\sigma}$    $(2, 2)$

- **Dual Graviton**

$$\tilde{h}_{\mu_1 \dots \mu_n \nu} \quad (n, 1)$$



$$\tilde{R}_{\mu_1 \dots \mu_{n+1} \rho\sigma}$$

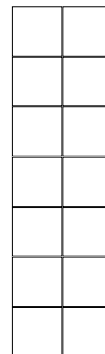
$n=D-3$

$$(n + 1, 2)$$

- **Double Dual Graviton**

$$\tilde{\tilde{h}}_{\mu_1 \dots \mu_n \nu_1 \dots \nu_n}$$

$$(n, n)$$



$$\tilde{\tilde{R}}_{\mu_1 \dots \mu_{n+1} \nu_1 \dots \nu_{n+1}}$$

$$(n + 1, n + 1)$$

Field strengths are Dual:

$$R \quad \tilde{R} = *R \quad \tilde{\tilde{R}} = *R*$$

Duality Exchanges field equals and Bianchis

$$R_{\mu\rho\nu}{}^\rho = 0 \quad \Leftrightarrow \quad \tilde{R}_{[\mu_1 \dots \mu_n \mu_{n+1} \nu] \rho} = 0$$

$$R_{[\mu\nu\rho]\sigma} = 0 \quad \Leftrightarrow \quad \tilde{R}_{\mu_1 \dots \mu_n \rho \nu}{}^\rho = 0$$

Electric and Magnetic Grav Sources  $T, \tilde{T}$  for  $h, \tilde{h}$

$$\begin{aligned} \tilde{T} &: \text{Dirac strings for } h \\ T &: \text{Dirac strings for } \tilde{h} \end{aligned}$$

## D=6 (2,0) free theory

R-symmetry  $Sp(2)=USp(4)$

Superconformal  $OSp(4/8^*) \supset USp(4) \times SO(6,2)$

$$B_{MN} \quad H = *H$$

5 scalars, 4 fermions

### Reduce to D=5

$$B_{\mu\nu}, B_{\mu 5} = A_{\mu} \quad H = *F$$

A, B dual, not independent

A, 5 scalars, 4 fermions: D=5 N=4 vector multiplet

### Reduce to D=4

2 vector fields  $B_{\mu i} = A_{\mu i} \quad i = 1, 2 \quad F_1 = *F_2$

SL(2,Z): diffeos on  $T^2$   $(A_1, A_2)$  doublet

Only one independent field, D=4 N=4 vector multiplet

SL(2,Z):  $(A_1, \tilde{A}_1)$  doublet, E-M duality

# The (4,0) Supermultiplet

D=6 little group  $SO(4) \sim SU(2) \times SU(2)$

States in representations of  $SU(2) \times SU(2) \times USp(8)$

$$(5, 1; 1) + (4, 1; 8) + (3, 1; 27) + (2, 1; 48) + (1, 1; 42)$$

Light-cone fields  $\alpha, \beta = 1, 2 \quad a, b = 1, \dots, 8$

$$C_{\alpha\beta\gamma\delta}, \psi_{\alpha\beta\gamma}^a, B_{\alpha\beta}^{ab}, \lambda_{\alpha}^{abc}, \phi^{abcd}$$

Covariant fields

$$C_{MNPQ}, \psi_{MN}^a, B_{MN}^{ab}, \lambda^{abc}, \phi^{abcd}$$

# D=6 Free (4,0) Theory

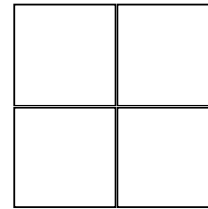
Hull

42 scalars

27 self-dual  $B_2$ :  $H = *H$

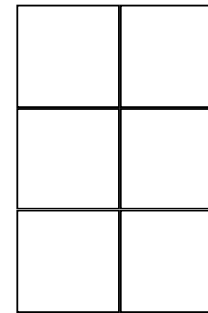
Gauge field

$C_{MNPQ}$



Curvature

$G_{MNPQRS}$



Self-dual:  $G = *G = G*$

“Supergravity without a graviton”

Superconformal  $OSp(8/8^*) \supset USp(8) \times SO(6,2)$



## Reduce to D=5

27  $B_2 \rightarrow 27$  vectors  $A_1$ , 42 scalars  $\rightarrow 42$  scalars

$$C_{\mu 5 \nu 5} = h_{\mu \nu}$$

$$C_{\mu \nu \rho 5} = \tilde{h}_{\mu \nu \rho}$$

$$C_{\mu \nu \rho \sigma} = \tilde{\tilde{h}}_{\mu \nu \rho \sigma}$$

Self-duality: Only one of these independent, dual gravitons

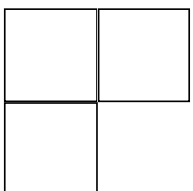
## Spectrum of D=5 N=8 SUGRA!

Graviton, 27 vectors, 42 scalars Diffeos

Vectors from  $B_{MN}$

Graviton from  $C_{MNPQ}$

Diffeos from  $C$  gauge transformations. Parameter



## Reduce to D=4

42 scalars  $\rightarrow$  42 scalars, Dual vector doublets  $B_{\mu i} = A_{\mu i}$

Metrics  $C_{\mu(ij)\nu} = -(h_{\mu\nu})_{ij}$

Curvatures:  $R_{21} = *R_{11}$ ,  $R_{12} = R_{11}^*$ ,  $R_{22} = *R_{11}^*$

$$h_{21} = \tilde{h}_{11}, \quad h_{22} = \tilde{\tilde{h}}_{11}$$

Just  $h_{11}$  independent

SL(2,Z) on torus:

$(A_1, A_2)$  doublets, E-M duality

Triplet  $h_{ij}$ : gravitational triality symmetry of free theory

# 5-D SYM at Strong Coupling

$$\{Q_\alpha^a, Q_\beta^b\} = \Omega^{ab} (\Gamma^\mu C)_{\alpha\beta} P_\mu + C_{\alpha\beta} (Z^{ab} + \Omega^{ab} K)$$

Z electric charges: carried by W-bosons etc

YM instanton in  $R^4$  lifts to BPS soliton in 5-D

K proportional to instanton number  $n$ , (2,0) short mult.

$$M \propto \frac{n}{g_{YM}^2}$$

Light at strong coupling: KK tower for 6'th dimension

Decompactifies to (2,0) theory in 6D as  $g_{YM}^2 \rightarrow \infty$

# (2,0) Interacting CFT

D=5 non-renormalizable, defined within string theory  
e.g. D4 brane theory

Strong coupling limit defined within string theory  
e.g. multiple D4 branes  $\rightarrow$  multiple M5 branes

No direct construction of interacting (2,0) theory.

Reduce on  $T^2$  gives interacting N=4 SYM  
and  $SL(2, \mathbb{Z})$  S-duality from torus diffeos

$g_{YM}$  dimensionful. Limit is one to high energies

$$E \gg (g_{YM})^{-2}$$

$$E(g_{YM})^2 \rightarrow \infty$$

# SUGRA at Strong Coupling

$$\{Q_\alpha^a, Q_\beta^b\} = \Omega^{ab} (\Gamma^\mu C)_{\alpha\beta} P_\mu + C_{\alpha\beta} (Z^{ab} + \Omega^{ab} K)$$

If there are BPS states carrying  $K$ , with spectrum

$$M \propto \frac{n}{l_{Plank}}$$

Become light in strong coupling (high energy) limit

$$E \times l_{Plank} \rightarrow \infty$$

Decompactification limit with  $K$ -states as a KK tower?  
If so, must decompactify to a (4,0) theory in 6D as (4,0)  
short multiplet

# D=5 N=8 Superalgebra

$$\{Q_\alpha^a, Q_\beta^b\} = \Omega^{ab} (\Gamma^\mu C)_{\alpha\beta} P_\mu + C_{\alpha\beta} (Z^{ab} + \Omega^{ab} K)$$

K carried by KK monopoles

Gibbons & Perry

$Z^{ab}$  carried by charged 0-branes (from wrapped M-branes)

BPS bound  $M \geq |K|$

Full D=5 M-theory on  $S^1$ :

Has  $E_7(\mathbb{Z})$  U-duality symmetry

Includes duality  $P^5 \leftrightarrow K$

D>5: D-5 form charge K carried by KK monopoles CMH

# K-Charge in D=5

Spacetime M asymptotic to  $\bar{M}$

k asymptotic to Killing vector on  $\bar{M}$

$$\Delta\omega = \omega - \bar{\omega}$$

$$e^A = e^A_{\mu} dx^{\mu}$$

Difference in spin connections: Asymptotic tensor  
ADM Momentum:

$$P[k] = \frac{1}{16\pi^2} \int_{\Sigma^3} *(e^A_{\wedge} e^B_{\wedge} k)_{\wedge} \Delta\omega_{AB}$$

Nester

K-charge

Sphere at spatial infinity  $\Sigma_3$

$$K = \frac{1}{16\pi^2} \int_{\Sigma^3} e^A_{\wedge} e^B_{\wedge} \Delta\omega_{AB}$$

CH

# M-Theory

- M-theory on  $T^6$  has  $D=5$   $N=8$  SUGRA as low energy limit
- $D=5$  branes lift to self-dual strings in  $D=6$ . Tension to zero in strong coupling limit
- Is strong coupling limit a 6D theory with  $(4,0)$  SUSY, with exotic conformal gravity?
- Highly symmetric phase of M-theory?



# K in 5D M-Theory?

- (4,0) theory on  $S^1$ : KK modes are 5D BPS, charge K is momentum on  $S^1$
- Does M-theory on  $\mathbb{R}^{4,1} \times T^6$  have BPS states carrying K?
- A question about BPS spectrum in M-theory
- Necessary for there to be strong coupling limit to 6D (4,0)

# Symmetry of (4,0)

## Free theory:

Conventional field theory in flat background

Background diffeomorphisms + gauge trans

$$\delta C_{MNPQ} = \partial_{[M}\chi_{N]PQ} + \partial_{[P}\chi_{Q]MN} - 2\partial_{[M}\chi_{NPQ]}$$

## Reduce to D=5 or D=4:

Combine  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

2 Symmetries are the same for  $g_{\mu\nu}$

On  $T^2$ , background diffeos give  $SL(2, \mathbb{Z})$  S-duality of both spin-1 and spin-2 fields in D=4

## Interacting D=6 theory:

Can't combine background  $\eta_{MN}$  & field  $C_{MNPQ}$

Don't expect D=6 diffeos, but exotic symmetries that give D=5 diffeomorphisms

Without D=6 diffeomorphisms, no reason to expect  $SL(2, \mathbb{Z})$  and hence no "derivation" of gravitational S-duality (unlike free case)

Without D=6 diffeomorphisms, should spacetime be replaced by something more exotic?

This should be consistent with free limit being a conventional field theory

# (2,0) & (4,0) 6-D CFTs

- No local covariant interacting field theory
- D=5 BPS electric 0-branes and magnetic strings lift to self-dual strings in D=6. Tension to zero in conformal limit
- Large superconformal symmetry: (4,0) has 32+32 susys
- YM and graviton in D=5 lift to self-dual tensor gauge fields
- D=5  $g_{\text{YM}}$  &  $l_{\text{planck}}$  from  $R_6$  as no scale in 6-D

# Conclusions

- Dual gravitons and gravitational S-duality work well for free theory
- For  $D \geq 5$ , charge  $K$  carried by KK monopoles, and branes from  $D=4$  instantons. Related to NUT charge and magnetic charge of KK monopoles
- For  $D=4$  SYM or linearised SUGRA, S-duality from  $(2,0)$  or  $(4,0)$  theory on  $T^2$

# (4,0): All For Nothing?

- Key issue: spectrum of BPS states with K
- Extra dimension from strong coupling?
- (4,0) theory as a limit of M-theory?  
Vast symmetry and unusual features
- Not usual spacetime, no metric or diffeos

