Problem 2

In[ ]:=
```
Clear[coord, metric, inversemetric, affine, riemann, lriemann, uriemann,
  ricci, scalar, einstein, weyl, geodesic, R, G, τ, i, j, k, l, s];
Clear[r, θ, ϕ, t, χ, a, m];

(*-----------------------------------------------------------------------------*)
(* This is what you need to set: *)
coord = {θ, ϕ};
n     = Length[coord];
metric = {
   {a^2,         0},
   {0 , a^2 Sin[θ]^2}};
(*-----------------------------------------------------------------------------*)

inversemetric = FullSimplify[Inverse[metric]];
Print["-----------------------------------------------------------------------------"];
Print["The Manifold has dimension   n= ",
  n, "\nCoordinate system:          ", coord];
Print["-----------------------------------------------------------------------------"];
Print["gμν=", metric // MatrixForm];
Print["g^μν=", inversemetric // MatrixForm];
Print["g  =", Det[metric] // FullSimplify];
affine := affine = FullSimplify[Table[
      (1 / 2) * Sum[
        (*                g^is (∂k gsj+∂j gsk-∂s gjk)            *)
        (inversemetric[[i, s]]) *
         (D[metric[[s, j]], coord[[k]]] +
            D[metric[[s, k]], coord[[j]]] - D[metric[[j, k]], coord[[s]]]),
        {s, 1, n}],
      {i, 1, n}, {j, 1, n}, {k, 1, n}]];
(*The non zero Christoffel symbols are computed and selected below: *)
listaffine := Table[
   If[
    UnsameQ[affine[[i, j, k]], 0],
    {Subscript[Superscript[Γ, i], j, k], affine[[i, j, k]]}
    ],
   {i, 1, n}, {j, 1, n}, {k, 1, j}];
Print["-----------------------------------------------------------------------------"];
Print["Christoffel Symbols:"];
```

```
Print[TableForm[
   Partition[DeleteCases[Flatten[listaffine], Null], 2], TableSpacing → {2, 2}]];
riemann := riemann = FullSimplify[Table[
     (* Rⁱⱼₖₗ=     ∂ₖΓⁱₗⱼ                         -          ∂ₗΓⁱₖⱼ*)
     D[   affine[[i, l, j]],  coord[[k]]     ] - D[affine[[i, k, j]], coord[[l]]    ] +
      (*           Γⁱₖₛ                  Γˢₗⱼ      -          Γⁱₗₛ                Γˢₖⱼ       *)
      Sum[affine[[i, k, s]] affine[[s, l, j]] -  affine[[i, l, s]] affine[[s, k, j]] ,
       {s, 1, n}],
     {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}]];
listriemann := Table[
   If[
    UnsameQ[riemann[[i, j, k, l]], 0],
    {Subscript[Superscript[R, i], j, k, l], riemann[[i, j, k, l]]}
   ],
   {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, k - 1}];
Print["--------------------------------------------------------------------"];
Print["Riemann Tensor:"];
Print[TableForm[
   Partition[DeleteCases[Flatten[listriemann], Null], 2], TableSpacing → {2, 2}]];
lriemann := lriemann = FullSimplify[Table[
     Sum[metric[[i, ii]] riemann[[ii, j, k, l]], {ii, 1, n}],
     {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}]];
listlriemann := Table[
   If[
    UnsameQ[lriemann[[i, j, k, l]], 0],
    {Subscript[R, i, j, k, l], lriemann[[i, j, k, l]]}
   ], {i, 1, n}, {j, 1, i - 1}, {k, 1, n}, {l, 1, k - 1}];
Print["--------------------------------------------------------------------"];
Print["Contravariant Riemann Tensor:"];
Print[TableForm[
   Partition[DeleteCases[Flatten[listlriemann], Null], 2], TableSpacing → {2, 2}]];
uriemann := uriemann = FullSimplify[Table[
     Sum[
      inversemetric[[j, jj]] inversemetric[[k, kk]] inversemetric[[l, ll]]
       riemann[[i, jj, kk, ll]], {jj, 1, n}, {kk, 1, n}, {ll, 1, n}
     ],
     {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}]];
listuriemann := Table[
   If[
    UnsameQ[uriemann[[i, j, k, l]], 0], {Superscript[
      Superscript[Superscript[Superscript[R, i], j], k], l], uriemann[[i, j, k, l]]}
   ], {i, 1, n}, {j, 1, i - 1}, {k, 1, n}, {l, 1, k - 1}];
```

```
Print["----------------------------------------------------------------------"];
Print["Covariant Riemann Tensor:"];
Print[TableForm[
   Partition[DeleteCases[Flatten[listuriemann], Null], 2], TableSpacing → {2, 2}]];
r2 = FullSimplify[Sum[lriemann[[i, j, k, l]] uriemann[[i, j, k, l]],
    {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}]];
Print["----------------------------------------------------------------------"];
Print["R^2= ", r2];
ricci := ricci = FullSimplify[Table[
      Sum[
       riemann[[i, j, i, l]],
       {i, 1, n}
      ], {j, 1, n}, {l, 1, n}]];
listricci := Table[
   If[
    UnsameQ[ricci[[j, l]], 0],
    {Subscript[R, j, l], ricci[[j, l]]}
   ], {j, 1, n}, {l, 1, j}];
Print["----------------------------------------------------------------------"];
Print["Ricci Tensor:"];
Print[TableForm[
   Partition[DeleteCases[Flatten[listricci], Null], 2], TableSpacing → {2, 2}]];
scalar = FullSimplify[Sum[inversemetric[[i, j]] ricci[[i, j]], {i, 1, n}, {j, 1, n}]];
Print["----------------------------------------------------------------------"];
Print["Curvature Scalar:"];
Print["R= ", scalar];
einstein := einstein = FullSimplify[ricci - (1/2) scalar * metric];
listeinstein := Table[
   If[
    UnsameQ[einstein[[j, l]], 0],
    {Subscript[G, j, l], einstein[[j, l]]}
   ], {j, 1, n}, {l, 1, j}];
Print["----------------------------------------------------------------------"];
Print["Einstein Tensor:"];
Print[TableForm[
   Partition[DeleteCases[Flatten[listeinstein], Null], 2], TableSpacing → {2, 2}]];
weyl := weyl = FullSimplify[Table[
      If[n > 3,
       lriemann[[i, j, k, l]]
        -   1/(n - 2)   (metric[[i, k]] ricci[[l, j]] - metric[[i, l]] ricci[[k, j]] -
            metric[[j, k]] ricci[[l, i]] + metric[[j, l]] ricci[[k, i]])
```

```
        + 1/((n - 1) (n - 2)) (metric[[i, k]] metric[[l, j]] - metric[[i, l]] metric[[k, j]]) scalar
       (*else, if n≤ 3 return 0:*), 0],
      {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}]];
listweyl := Table[
   If[
    UnsameQ[weyl[[i, j, k, l]], 0],
    {Subscript[C, i, j, k, l], weyl[[i, j, k, l]]}
   ] , {i, 1, n}, {j, 1, i - 1}, {k, 1, n}, {l, 1, k - 1}];
Print["-----------------------------------------------------------------------"];
Print["Weyl Tensor:"];
Print[TableForm[
   Partition[DeleteCases[Flatten[listweyl], Null], 2], TableSpacing → {2, 2}]];
geodesic := geodesic =
   Simplify[Table[-Sum[affine[[i, j, k]] u[j] u[k], {j, 1, n}, {k, 1, n}], {i, 1, n}]];
subst = Table[u[i] → Subscript[coord[[i]], τ], {i, 1, n}];
nlistgeodesic :=
  Table[{Subscript[coord[[i]], ττ], "+", -geodesic[[i]] /. subst, "= 0"}, {i, 1, n}];
Print["-----------------------------------------------------------------------"];
Print["Geodesic Equations:"];
Print[TableForm[nlistgeodesic, TableSpacing → {2}]];
```

The Manifold has dimension   n= 2

Coordinate system: $\{\theta, \phi\}$

$$g_{\mu\nu}=\begin{pmatrix} a^2 & 0 \\ 0 & a^2\,\text{Sin}[\theta]^2 \end{pmatrix}$$

$$g^{\mu\nu}=\begin{pmatrix} \frac{1}{a^2} & 0 \\ 0 & \frac{\text{Csc}[\theta]^2}{a^2} \end{pmatrix}$$

$$g \;=a^4\,\text{Sin}[\theta]^2$$

Christoffel Symbols:

$\Gamma^1{}_{2,2}$ $-\text{Cos}[\theta]\,\text{Sin}[\theta]$

$\Gamma^2{}_{2,1}$ $\text{Cot}[\theta]$

Riemann Tensor:

$R^1{}_{2,2,1}$ $-\text{Sin}[\theta]^2$

$R^2{}_{1,2,1}$ 1

Contravariant Riemann Tensor:

$R_{2,1,2,1}$ $a^2 \text{Sin}[\theta]^2$

Covariant Riemann Tensor:

R^{2121} $\frac{\text{Csc}[\theta]^2}{a^6}$

$R^2 = \frac{4}{a^4}$

Ricci Tensor:

$R_{1,1}$ 1

$R_{2,2}$ $\text{Sin}[\theta]^2$

Curvature Scalar:

$R = \frac{2}{a^2}$

Einstein Tensor:

{}

Weyl Tensor:

{}

Geodesic Equations:

$\theta_{\tau\tau}$ + $-\text{Cos}[\theta]\,\text{Sin}[\theta]\,\phi_\tau^2$ = 0

$\phi_{\tau\tau}$ + $2\,\text{Cot}[\theta]\,\theta_\tau\,\phi_\tau$ = 0

Problem 4

In[ ]:=

```
Clear[coord, metric, inversemetric, affine, riemann, lriemann, uriemann,
  ricci, scalar, einstein, weyl, geodesic, R, G, τ, i, j, k, l, s];
Clear[r, θ, ϕ, t, χ, a, m, Ω];

(*---------------------------------------------------------------------------*)
(* This is what you need to set: *)
coord = {t, x, y, z};
n     = Length[coord];
```

```
metric = {
   {-(1 - Ω^2 (x^2 + y^2)), Ω y, -Ω x, 0},
   { Ω y            ,  1 ,  0  , 0},
   {-Ω x            ,  0,  1  , 0},
   { 0              ,  0,  0 ,  1}
  };
(*------------------------------------------------------------------------*)

inversemetric = FullSimplify[Inverse[metric]];
Print["------------------------------------------------------------------------"];
Print["The Manifold has dimension   n= ",
  n, "\nCoordinate system:          ", coord];
Print["------------------------------------------------------------------------"];
Print["gμν=", metric // MatrixForm];
Print["g^μν=", inversemetric // MatrixForm];
Print["g  =", Det[metric] // FullSimplify];
affine := affine = FullSimplify[Table[
     (1 / 2) * Sum[
       (*              g^is (∂k gsj+∂j gsk-∂s gjk)            *)
       (inversemetric[[i, s]]) *
        (D[metric[[s, j]], coord[[k]] ] +
          D[metric[[s, k]], coord[[j]] ] - D[metric[[j, k]], coord[[s]] ]),
       {s, 1, n}],
     {i, 1, n}, {j, 1, n}, {k, 1, n}] ];
(*The non zero Christoffel symbols are computed and selected below: *)
listaffine := Table[
   If[
    UnsameQ[affine[[i, j, k]], 0],
    {Subscript[Superscript[Γ, i], j, k], affine[[i, j, k]]}
     ],
   {i, 1, n}, {j, 1, n}, {k, 1, j}];
Print["------------------------------------------------------------------------"];
Print["Christoffel Symbols:"];
Print[TableForm[
   Partition[DeleteCases[Flatten[listaffine], Null], 2], TableSpacing → {2, 2}]];
riemann := riemann = FullSimplify[Table[
     (* R^i_jkl=    ∂k Γ^i_lj                          -          ∂l Γ^i_kj*)
     D[   affine[[i, l, j]],  coord[[k]]     ] - D[affine[[i, k, j]],  coord[[l]]    ] +
      (*         Γ^i_ks                Γ^s_lj       -          Γ^i_ls              Γ^s_kj       *)
      Sum[affine[[i, k, s]] affine[[s, l, j]] -   affine[[i, l, s]] affine[[s, k, j]] ,
       {s, 1, n}],
     {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}] ];
```

```
listriemann := Table[
   If[
    UnsameQ[riemann[[i, j, k, l]], 0],
    {Subscript[Superscript[R, i], j, k, l], riemann[[i, j, k, l]]}
   ],
   {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, k - 1}];
Print["------------------------------------------------------------------------"];
Print["Riemann Tensor:"];
Print[TableForm[
   Partition[DeleteCases[Flatten[listriemann], Null], 2], TableSpacing → {2, 2}]];
lriemann := lriemann = FullSimplify[Table[
     Sum[metric[[i, ii]] riemann[[ii, j, k, l]], {ii, 1, n}],
     {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}]];
listlriemann := Table[
   If[
    UnsameQ[lriemann[[i, j, k, l]], 0],
    {Subscript[R, i, j, k, l], lriemann[[i, j, k, l]]}
   ], {i, 1, n}, {j, 1, i - 1}, {k, 1, n}, {l, 1, k - 1}];
Print["------------------------------------------------------------------------"];
Print["Contravariant Riemann Tensor:"];
Print[TableForm[
   Partition[DeleteCases[Flatten[listlriemann], Null], 2], TableSpacing → {2, 2}]];
uriemann := uriemann = FullSimplify[Table[
     Sum[
      inversemetric[[j, jj]] inversemetric[[k, kk]] inversemetric[[l, ll]]
       riemann[[i, jj, kk, ll]], {jj, 1, n}, {kk, 1, n}, {ll, 1, n}
     ],
     {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}]];
listuriemann := Table[
   If[
    UnsameQ[uriemann[[i, j, k, l]], 0], {Superscript[
      Superscript[Superscript[Superscript[R, i], j], k], l], uriemann[[i, j, k, l]]}
   ], {i, 1, n}, {j, 1, i - 1}, {k, 1, n}, {l, 1, k - 1}];
Print["------------------------------------------------------------------------"];
Print["Covariant Riemann Tensor:"];
Print[TableForm[
   Partition[DeleteCases[Flatten[listuriemann], Null], 2], TableSpacing → {2, 2}]];
r2 = FullSimplify[Sum[lriemann[[i, j, k, l]] uriemann[[i, j, k, l]],
    {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}]];
Print["------------------------------------------------------------------------"];
Print["R^2= ", r2];
ricci := ricci = FullSimplify[Table[
     Sum[
```

```
      riemann[[i, j, i, l]],
      {i, 1, n}
     ], {j, 1, n}, {l, 1, n}]];
listricci := Table[
   If[
    UnsameQ[ricci[[j, l]], 0],
    {Subscript[R, j, l], ricci[[j, l]]}
   ], {j, 1, n}, {l, 1, j}];
Print["-------------------------------------------------------------------------"];
Print["Ricci Tensor:"];
Print[TableForm[
   Partition[DeleteCases[Flatten[listricci], Null], 2], TableSpacing → {2, 2}]];
scalar = FullSimplify[Sum[inversemetric[[i, j]] ricci[[i, j]], {i, 1, n}, {j, 1, n}]];
Print["-------------------------------------------------------------------------"];
Print["Curvature Scalar:"];
Print["R= ", scalar];
einstein := einstein = FullSimplify[ricci - (1/2) scalar * metric];
listeinstein := Table[
   If[
    UnsameQ[einstein[[j, l]], 0],
    {Subscript[G, j, l], einstein[[j, l]]}
   ], {j, 1, n}, {l, 1, j}];
Print["-------------------------------------------------------------------------"];
Print["Einstein Tensor:"];
Print[TableForm[
   Partition[DeleteCases[Flatten[listeinstein], Null], 2], TableSpacing → {2, 2}]];
weyl := weyl = FullSimplify[Table[
     If[n > 3,
      lriemann[[i, j, k, l]]
       - 1/(n - 2) (metric[[i, k]] ricci[[l, j]] - metric[[i, l]] ricci[[k, j]] -
           metric[[j, k]] ricci[[l, i]] + metric[[j, l]] ricci[[k, i]])
       + 1/((n - 1) (n - 2)) (metric[[i, k]] metric[[l, j]] - metric[[i, l]] metric[[k, j]]) scalar
      (*else, if n≤ 3 return 0:*), 0],
     {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}]];
listweyl := Table[
   If[
    UnsameQ[weyl[[i, j, k, l]], 0],
    {Subscript[C, i, j, k, l], weyl[[i, j, k, l]]}
   ], {i, 1, n}, {j, 1, i - 1}, {k, 1, n}, {l, 1, k - 1}];
Print["-------------------------------------------------------------------------"];
```

```
Print["Weyl Tensor:"];
Print[TableForm[
   Partition[DeleteCases[Flatten[listweyl], Null], 2], TableSpacing → {2, 2}]];
geodesic := geodesic =
   Simplify[Table[-Sum[affine[[i, j, k]] u[j] u[k], {j, 1, n}, {k, 1, n}], {i, 1, n}]];
subst = Table[u[i] → Subscript[coord[[i]], τ], {i, 1, n}];
nlistgeodesic :=
  Table[{Subscript[coord[[i]], ττ], "+", -geodesic[[i]] /. subst, "= 0"}, {i, 1, n}];
Print["------------------------------------------------------------------------"];
Print["Geodesic Equations:"];
Print[TableForm[nlistgeodesic, TableSpacing → {2}]];
```

The Manifold has dimension   n= 4

Coordinate system: {t, x, y, z}

$$g_{\mu\nu}=\begin{pmatrix} -1+(x^2+y^2)\Omega^2 & y\,\Omega & -x\,\Omega & 0 \\ y\,\Omega & 1 & 0 & 0 \\ -x\,\Omega & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$g^{\mu\nu}=\begin{pmatrix} -1 & y\,\Omega & -x\,\Omega & 0 \\ y\,\Omega & 1-y^2\Omega^2 & x\,y\,\Omega^2 & 0 \\ -x\,\Omega & x\,y\,\Omega^2 & 1-x^2\Omega^2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

g =-1

Christoffel Symbols:

$\Gamma^2{}_{1,1}$ $-x\,\Omega^2$

$\Gamma^2{}_{3,1}$ Ω

$\Gamma^3{}_{1,1}$ $-y\,\Omega^2$

$\Gamma^3{}_{2,1}$ $-\Omega$

Riemann Tensor:

{}

Contravariant Riemann Tensor:

{}

Covariant Riemann Tensor:

{}

```
----------------------------------------------------------------------
R²= 0
----------------------------------------------------------------------
Ricci Tensor:
{}
----------------------------------------------------------------------
Curvature Scalar:
R= 0
----------------------------------------------------------------------
Einstein Tensor:
{}
----------------------------------------------------------------------
Weyl Tensor:
{}
----------------------------------------------------------------------
Geodesic Equations:
```

$t_{\tau\tau} \quad + \quad 0 \quad = 0$

$x_{\tau\tau} \quad + \quad -\Omega\, t_\tau\,(x\,\Omega\, t_\tau - 2\, y_\tau) \quad = 0$

$y_{\tau\tau} \quad + \quad -\Omega\, t_\tau\,(y\,\Omega\, t_\tau + 2\, x_\tau) \quad = 0$

$z_{\tau\tau} \quad + \quad 0 \quad = 0$

Problem 12

In[ ]:=

```
Clear[coord, metric, inversemetric, affine, riemann, lriemann, uriemann,
  ricci, scalar, einstein, weyl, geodesic, R, G, τ, i, j, k, l, s];
Clear[r, θ, ϕ, t, χ, a, m, x, y];

(*--------------------------------------------------------------------------------*)
(* This is what you need to set: *)
coord = {x, y};
n     = Length[coord];
metric = {
    {y^-2,  0},
    {0  , y^-2}};
(*--------------------------------------------------------------------------------*)

inversemetric = FullSimplify[Inverse[metric]];
Print["----------------------------------------------------------------------"];
```

```
Print["The Manifold has dimension   n= ",
  n, "\nCoordinate system:          ", coord];
Print["-----------------------------------------------------------------------"];
Print["gμν=", metric // MatrixForm];
Print["gμν=", inversemetric // MatrixForm];
Print["g  =", Det[metric] // FullSimplify];
affine := affine = FullSimplify[Table[
     (1 / 2) * Sum[
       (*                gis (∂k gsj+∂j gsk-∂s gjk)              *)
       (inversemetric[[i, s]]) *
        (D[metric[[s, j]], coord[[k]] ] +
          D[metric[[s, k]], coord[[j]] ] - D[metric[[j, k]], coord[[s]] ]),
       {s, 1, n}],
     {i, 1, n}, {j, 1, n}, {k, 1, n}] ];
(*The non zero Christoffel symbols are computed and selected below: *)
listaffine := Table[
   If[
    UnsameQ[affine[[i, j, k]], 0],
    {Subscript[Superscript[Γ, i], j, k], affine[[i, j, k]]}
    ] ,
   {i, 1, n}, {j, 1, n}, {k, 1, j}];
Print["-----------------------------------------------------------------------"];
Print["Christoffel Symbols:"];
Print[TableForm[
   Partition[DeleteCases[Flatten[listaffine], Null], 2], TableSpacing → {2, 2}]];
riemann := riemann = FullSimplify[Table[
     (* Ri jkl=     ∂k Γi lj                          -          ∂l Γi kj*)
     D[   affine[[i, l, j]],  coord[[k]]     ] - D[affine[[i, k, j]], coord[[l]]    ] +
      (*          Γi ks                 Γs lj      -          Γi ls                 Γs kj       *)
      Sum[affine[[i, k, s]] affine[[s, l, j]] -  affine[[i, l, s]] affine[[s, k, j]] ,
       {s, 1, n}],
     {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}] ];
listriemann := Table[
   If[
    UnsameQ[riemann[[i, j, k, l]], 0],
    {Subscript[Superscript[R, i], j, k, l], riemann[[i, j, k, l]]}
   ] ,
   {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, k - 1}];
Print["-----------------------------------------------------------------------"];
Print["Riemann Tensor:"];
Print[TableForm[
   Partition[DeleteCases[Flatten[listriemann], Null], 2], TableSpacing → {2, 2}]];
```

```
lriemann := lriemann = FullSimplify[Table[
     Sum[metric⟦i, ii⟧ riemann⟦ii, j, k, l⟧, {ii, 1, n}],
     {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}]];
listlriemann := Table[
   If[
    UnsameQ[lriemann⟦i, j, k, l⟧, 0],
    {Subscript[R, i, j, k, l], lriemann⟦i, j, k, l⟧}
   ], {i, 1, n}, {j, 1, i - 1}, {k, 1, n}, {l, 1, k - 1}];
Print["-----------------------------------------------------------------"];
Print["Contravariant Riemann Tensor:"];
Print[TableForm[
   Partition[DeleteCases[Flatten[listlriemann], Null], 2], TableSpacing → {2, 2}]];
uriemann := uriemann = FullSimplify[Table[
     Sum[
      inversemetric⟦j, jj⟧ inversemetric⟦k, kk⟧ inversemetric⟦l, ll⟧
       riemann⟦i, jj, kk, ll⟧, {jj, 1, n}, {kk, 1, n}, {ll, 1, n}
     ],
     {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}]];
listuriemann := Table[
   If[
    UnsameQ[uriemann⟦i, j, k, l⟧, 0], {Superscript[
      Superscript[Superscript[Superscript[R, i], j], k], l], uriemann⟦i, j, k, l⟧}
   ], {i, 1, n}, {j, 1, i - 1}, {k, 1, n}, {l, 1, k - 1}];
Print["-----------------------------------------------------------------"];
Print["Covariant Riemann Tensor:"];
Print[TableForm[
   Partition[DeleteCases[Flatten[listuriemann], Null], 2], TableSpacing → {2, 2}]];
r2 = FullSimplify[Sum[lriemann⟦i, j, k, l⟧ uriemann⟦i, j, k, l⟧,
    {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}]];
Print["-----------------------------------------------------------------"];
Print["R^2= ", r2];
ricci := ricci = FullSimplify[Table[
     Sum[
      riemann⟦i, j, i, l⟧,
      {i, 1, n}
     ], {j, 1, n}, {l, 1, n}]];
listricci := Table[
   If[
    UnsameQ[ricci⟦j, l⟧, 0],
    {Subscript[R, j, l], ricci⟦j, l⟧}
   ], {j, 1, n}, {l, 1, j}];
Print["-----------------------------------------------------------------"];
Print["Ricci Tensor:"];
```

```
Print[TableForm[
   Partition[DeleteCases[Flatten[listricci], Null], 2], TableSpacing → {2, 2}]];
scalar = FullSimplify[Sum[inversemetric[[i, j]] ricci[[i, j]], {i, 1, n}, {j, 1, n}]];
Print["------------------------------------------------------------------"];
Print["Curvature Scalar:"];
Print["R= ", scalar];
einstein := einstein = FullSimplify[ricci - (1/2) scalar * metric];
listeinstein := Table[
   If[
    UnsameQ[einstein[[j, l]], 0],
    {Subscript[G, j, l], einstein[[j, l]]}
   ], {j, 1, n}, {l, 1, j}];
Print["------------------------------------------------------------------"];
Print["Einstein Tensor:"];
Print[TableForm[
   Partition[DeleteCases[Flatten[listeinstein], Null], 2], TableSpacing → {2, 2}]];
weyl := weyl = FullSimplify[Table[
     If[n > 3,
      lriemann[[i, j, k, l]]
       - 1/(n - 2) (metric[[i, k]] ricci[[l, j]] - metric[[i, l]] ricci[[k, j]] -
          metric[[j, k]] ricci[[l, i]] + metric[[j, l]] ricci[[k, i]])
       + 1/((n - 1) (n - 2)) (metric[[i, k]] metric[[l, j]] - metric[[i, l]] metric[[k, j]]) scalar
      (*else, if n≤ 3 return 0:*), 0],
     {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}]];
listweyl := Table[
   If[
    UnsameQ[weyl[[i, j, k, l]], 0],
    {Subscript[C, i, j, k, l], weyl[[i, j, k, l]]}
   ], {i, 1, n}, {j, 1, i - 1}, {k, 1, n}, {l, 1, k - 1}];
Print["------------------------------------------------------------------"];
Print["Weyl Tensor:"];
Print[TableForm[
   Partition[DeleteCases[Flatten[listweyl], Null], 2], TableSpacing → {2, 2}]];
geodesic := geodesic =
   Simplify[Table[-Sum[affine[[i, j, k]] u[j] u[k], {j, 1, n}, {k, 1, n}], {i, 1, n}]];
subst = Table[u[i] → Subscript[coord[[i]], τ], {i, 1, n}];
nlistgeodesic :=
  Table[{Subscript[coord[[i]], ττ], "+", -geodesic[[i]] /. subst, "= 0"}, {i, 1, n}];
Print["------------------------------------------------------------------"];
Print["Geodesic Equations:"];
```

The Manifold has dimension n= 2
Coordinate system: {x, y}

$$g_{\mu\nu}=\begin{pmatrix} \frac{1}{y^2} & 0 \\ 0 & \frac{1}{y^2} \end{pmatrix}$$

$$g^{\mu\nu}=\begin{pmatrix} y^2 & 0 \\ 0 & y^2 \end{pmatrix}$$

$$g \ =\frac{1}{y^4}$$

Christoffel Symbols:

$\Gamma^1{}_{2,1} \quad -\frac{1}{y}$

$\Gamma^2{}_{1,1} \quad \frac{1}{y}$

$\Gamma^2{}_{2,2} \quad -\frac{1}{y}$

Riemann Tensor:

$R^1{}_{2,2,1} \quad \frac{1}{y^2}$

$R^2{}_{1,2,1} \quad -\frac{1}{y^2}$

Contravariant Riemann Tensor:

$R_{2,1,2,1} \quad -\frac{1}{y^4}$

Covariant Riemann Tensor:

$R^{2121} \quad -y^4$

$R^2= 4$

Ricci Tensor:

$R_{1,1} \quad -\frac{1}{y^2}$

$R_{2,2} \quad -\frac{1}{y^2}$

Curvature Scalar:

R= −2

```
----------------------------------------------------------------------
Einstein Tensor:
{}
----------------------------------------------------------------------
Weyl Tensor:
{}
----------------------------------------------------------------------
Geodesic Equations:
```

$$x_{\tau\tau} + -\frac{2\, x_\tau\, y_\tau}{y} = 0$$

$$y_{\tau\tau} + -\frac{-x_\tau^2 + y_\tau^2}{y} = 0$$

In[ ]:=

```
Integrate[y/R (1 - y^2/R^2)^(-1/2), y, Assumptions → R > 0 && y > 0 && y < R]
```

Out[ ]= $-\sqrt{R^2 - y^2}$

In[ ]:=

```
Integrate[y^-1 (1 - y^2/R^2)^(-1/2), y, Assumptions → R > 0 && y > 0 && y < R]
```

Out[ ]= $R\left(-\frac{\mathrm{Log}\left[R + \sqrt{R^2 - y^2}\right]}{2\,R} + \frac{\mathrm{Log}\left[-R^2 + R\,\sqrt{R^2 - y^2}\right]}{2\,R}\right)$

Problem 8, Carroll

In[ ]:=

```
Clear[coord, metric, inversemetric, affine, riemann, lriemann, uriemann,
  ricci, scalar, einstein, weyl, geodesic, R, G, τ, i, j, k, l, s];
Clear[r, θ, ϕ, t, χ, a, m, x, y, ψ];

(*------------------------------------------------------------------------*)
(* This is what you need to set: *)
coord = {ψ, θ, ϕ};
n     = Length[coord];
metric = {
   {1 , 0       , 0              },
   {0 , Sin[ψ]^2, 0              },
   {0 , 0       , Sin[ψ]^2 Sin[θ]^2}};
(*------------------------------------------------------------------------*)
```

```
inversemetric = FullSimplify[Inverse[metric]];
Print["----------------------------------------------------------------------"];
Print["The Manifold has dimension   n= ",
  n, "\nCoordinate system:          ", coord];
Print["----------------------------------------------------------------------"];
Print["gμν=", metric // MatrixForm];
Print["gμν=", inversemetric // MatrixForm];
Print["g  =", Det[metric] // FullSimplify];
affine := affine = FullSimplify[Table[
     (1 / 2) * Sum[
       (*              g^is (∂k gsj+∂j gsk-∂s gjk)            *)
       (inversemetric[[i, s]]) *
        (D[metric[[s, j]], coord[[k]] ] +
          D[metric[[s, k]], coord[[j]] ] - D[metric[[j, k]], coord[[s]] ]),
       {s, 1, n}],
     {i, 1, n}, {j, 1, n}, {k, 1, n}] ];
(*The non zero Christoffel symbols are computed and selected below: *)
listaffine := Table[
   If[
    UnsameQ[affine[[i, j, k]], 0],
    {Subscript[Superscript[Γ, i], j, k], affine[[i, j, k]]}
     ] ,
   {i, 1, n}, {j, 1, n}, {k, 1, j}];
Print["----------------------------------------------------------------------"];
Print["Christoffel Symbols:"];
Print[TableForm[
   Partition[DeleteCases[Flatten[listaffine], Null], 2], TableSpacing → {2, 2}]];
riemann := riemann = FullSimplify[Table[
     (* R^i_jkl=    ∂k Γ^i_lj                        -          ∂l Γ^i_kj*)
     D[   affine[[i, l, j]],  coord[[k]]     ] - D[affine[[i, k, j]],  coord[[l]]    ] +
      (*          Γ^i_ks                Γ^s_lj      -          Γ^i_ls                Γ^s_kj       *)
      Sum[affine[[i, k, s]] affine[[s, l, j]] -  affine[[i, l, s]] affine[[s, k, j]] ,
       {s, 1, n}],
     {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}] ];
listriemann := Table[
   If[
    UnsameQ[riemann[[i, j, k, l]], 0],
    {Subscript[Superscript[R, i], j, k, l], riemann[[i, j, k, l]]}
   ] ,
   {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, k - 1}];
Print["----------------------------------------------------------------------"];
```

```
Print["Riemann Tensor:"];
Print[TableForm[
   Partition[DeleteCases[Flatten[listriemann], Null], 2], TableSpacing → {2, 2}]];
lriemann := lriemann = FullSimplify[Table[
     Sum[metric⟦i, ii⟧ riemann⟦ii, j, k, l⟧, {ii, 1, n}],
     {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}]];
listlriemann := Table[
   If[
    UnsameQ[lriemann⟦i, j, k, l⟧, 0],
    {Subscript[R, i, j, k, l], lriemann⟦i, j, k, l⟧}
   ], {i, 1, n}, {j, 1, i - 1}, {k, 1, n}, {l, 1, k - 1}];
Print["----------------------------------------------------------------"];
Print["Contravariant Riemann Tensor:"];
Print[TableForm[
   Partition[DeleteCases[Flatten[listlriemann], Null], 2], TableSpacing → {2, 2}]];
uriemann := uriemann = FullSimplify[Table[
     Sum[
      inversemetric⟦j, jj⟧ inversemetric⟦k, kk⟧ inversemetric⟦l, ll⟧
       riemann⟦i, jj, kk, ll⟧, {jj, 1, n}, {kk, 1, n}, {ll, 1, n}
     ],
     {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}]];
listuriemann := Table[
   If[
    UnsameQ[uriemann⟦i, j, k, l⟧, 0], {Superscript[
      Superscript[Superscript[Superscript[R, i], j], k], l], uriemann⟦i, j, k, l⟧}
   ], {i, 1, n}, {j, 1, i - 1}, {k, 1, n}, {l, 1, k - 1}];
Print["----------------------------------------------------------------"];
Print["Covariant Riemann Tensor:"];
Print[TableForm[
   Partition[DeleteCases[Flatten[listuriemann], Null], 2], TableSpacing → {2, 2}]];
r2 = FullSimplify[Sum[lriemann⟦i, j, k, l⟧ uriemann⟦i, j, k, l⟧,
    {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}]];
Print["----------------------------------------------------------------"];
Print["R²= ", r2];
ricci := ricci = FullSimplify[Table[
     Sum[
      riemann⟦i, j, i, l⟧,
      {i, 1, n}
     ], {j, 1, n}, {l, 1, n}]];
listricci := Table[
   If[
    UnsameQ[ricci⟦j, l⟧, 0],
    {Subscript[R, j, l], ricci⟦j, l⟧}
```

```
   ], {j, 1, n}, {l, 1, j}];
Print["------------------------------------------------------------------"];
Print["Ricci Tensor:"];
Print[TableForm[
   Partition[DeleteCases[Flatten[listricci], Null], 2], TableSpacing → {2, 2}]];
scalar = FullSimplify[Sum[inversemetric[[i, j]] ricci[[i, j]], {i, 1, n}, {j, 1, n}]];
Print["------------------------------------------------------------------"];
Print["Curvature Scalar:"];
Print["R= ", scalar];
einstein := einstein = FullSimplify[ricci - (1/2) scalar * metric];
listeinstein := Table[
   If[
    UnsameQ[einstein[[j, l]], 0],
    {Subscript[G, j, l], einstein[[j, l]]}
   ], {j, 1, n}, {l, 1, j}];
Print["------------------------------------------------------------------"];
Print["Einstein Tensor:"];
Print[TableForm[
   Partition[DeleteCases[Flatten[listeinstein], Null], 2], TableSpacing → {2, 2}]];
weyl := weyl = FullSimplify[Table[
     If[n > 3,
      lriemann[[i, j, k, l]]
       -    1/(n - 2)    (metric[[i, k]] ricci[[l, j]] - metric[[i, l]] ricci[[k, j]] -
            metric[[j, k]] ricci[[l, i]] + metric[[j, l]] ricci[[k, i]])
       + 1/((n - 1) (n - 2)) (metric[[i, k]] metric[[l, j]] - metric[[i, l]] metric[[k, j]]) scalar
      (*else, if n≤ 3 return 0:*), 0],
     {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}]];
listweyl := Table[
   If[
    UnsameQ[weyl[[i, j, k, l]], 0],
    {Subscript[C, i, j, k, l], weyl[[i, j, k, l]]}
   ], {i, 1, n}, {j, 1, i - 1}, {k, 1, n}, {l, 1, k - 1}];
Print["------------------------------------------------------------------"];
Print["Weyl Tensor:"];
Print[TableForm[
   Partition[DeleteCases[Flatten[listweyl], Null], 2], TableSpacing → {2, 2}]];
geodesic := geodesic =
   Simplify[Table[-Sum[affine[[i, j, k]] u[j] u[k], {j, 1, n}, {k, 1, n}], {i, 1, n}]];
subst = Table[u[i] → Subscript[coord[[i]], τ], {i, 1, n}];
nlistgeodesic :=
```

```
  Table[{Subscript[coord[[i]], ττ], "+", -geodesic[[i]] /. subst, "= 0"}, {i, 1, n}];
Print["-----------------------------------------------------------------------"];
Print["Geodesic Equations:"];
Print[TableForm[nlistgeodesic, TableSpacing → {2}]];
```

The Manifold has dimension   n= 3

Coordinate system: $\{\psi, \theta, \phi\}$

$$g_{\mu\nu}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & \text{Sin}[\psi]^2 & 0 \\ 0 & 0 & \text{Sin}[\theta]^2\,\text{Sin}[\psi]^2 \end{pmatrix}$$

$$g^{\mu\nu}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & \text{Csc}[\psi]^2 & 0 \\ 0 & 0 & \text{Csc}[\theta]^2\,\text{Csc}[\psi]^2 \end{pmatrix}$$

$g\ =\text{Sin}[\theta]^2\,\text{Sin}[\psi]^4$

Christoffel Symbols:

$\Gamma^1{}_{2,2}$ $-\text{Cos}[\psi]\,\text{Sin}[\psi]$

$\Gamma^1{}_{3,3}$ $-\text{Cos}[\psi]\,\text{Sin}[\theta]^2\,\text{Sin}[\psi]$

$\Gamma^2{}_{2,1}$ $\text{Cot}[\psi]$

$\Gamma^2{}_{3,3}$ $-\text{Cos}[\theta]\,\text{Sin}[\theta]$

$\Gamma^3{}_{3,1}$ $\text{Cot}[\psi]$

$\Gamma^3{}_{3,2}$ $\text{Cot}[\theta]$

Riemann Tensor:

$R^1{}_{2,2,1}$ $-\text{Sin}[\psi]^2$

$R^1{}_{3,3,1}$ $-\text{Sin}[\theta]^2\,\text{Sin}[\psi]^2$

$R^2{}_{1,2,1}$ 1

$R^2{}_{3,3,2}$ $-\text{Sin}[\theta]^2\,\text{Sin}[\psi]^2$

$R^3{}_{1,3,1}$ 1

$R^3{}_{2,3,2}$ $\text{Sin}[\psi]^2$

Contravariant Riemann Tensor:

$R_{2,1,2,1}$ $\text{Sin}[\psi]^2$

$R_{3,1,3,1}$ $\text{Sin}[\theta]^2\,\text{Sin}[\psi]^2$

$R_{3,2,3,2}$ $\text{Sin}[\theta]^2\,\text{Sin}[\psi]^4$

```
Covariant Riemann Tensor:
```

R^{2121} $\quad \mathrm{Csc}[\psi]^2$

R^{3131} $\quad \mathrm{Csc}[\theta]^2\,\mathrm{Csc}[\psi]^2$

R^{3232} $\quad \mathrm{Csc}[\theta]^2\,\mathrm{Csc}[\psi]^4$

```
----------------------------------------------------------------------
```

R^2= 12

```
----------------------------------------------------------------------

Ricci Tensor:
```

$R_{1,1}$ $\quad 2$

$R_{2,2}$ $\quad 2\,\mathrm{Sin}[\psi]^2$

$R_{3,3}$ $\quad 2\,\mathrm{Sin}[\theta]^2\,\mathrm{Sin}[\psi]^2$

```
----------------------------------------------------------------------

Curvature Scalar:

R= 6

----------------------------------------------------------------------

Einstein Tensor:
```

$G_{1,1}$ $\quad -1$

$G_{2,2}$ $\quad -\mathrm{Sin}[\psi]^2$

$G_{3,3}$ $\quad -\mathrm{Sin}[\theta]^2\,\mathrm{Sin}[\psi]^2$

```
----------------------------------------------------------------------

Weyl Tensor:

{}

----------------------------------------------------------------------

Geodesic Equations:
```

$\psi_{\tau\tau} \quad + \quad -\mathrm{Cos}[\psi]\,\mathrm{Sin}[\psi]\left(\theta_\tau^2 + \mathrm{Sin}[\theta]^2\,\phi_\tau^2\right) \quad = \ 0$

$\theta_{\tau\tau} \quad + \quad -\mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta]\,\phi_\tau^2 + 2\,\mathrm{Cot}[\psi]\,\theta_\tau\,\psi_\tau \quad = \ 0$

$\phi_{\tau\tau} \quad + \quad 2\,\phi_\tau\left(\mathrm{Cot}[\theta]\,\theta_\tau + \mathrm{Cot}[\psi]\,\psi_\tau\right) \quad = \ 0$

Problem 6, Carroll

In[ ]:=
```
Clear[coord, metric, inversemetric, affine, riemann, lriemann, uriemann,
  ricci, scalar, einstein, weyl, geodesic, R, G, τ, i, j, k, l, s];
Clear[r, θ, ϕ, t, χ, a, m, x, y, ψ];

(*---------------------------------------------------------------------------*)
(* This is what you need to set: *)
```

```
coord = {t, r, θ, ϕ};
n     = Length[coord];
metric = {
   {-(1 - 2 M/r), 0      , 0  , 0          },
   {0         , 1 + 2 M/r , 0  , 0          },
   {0         , 0      , r^2 , 0          },
   {0         , 0      , 0  , r^2 Sin[θ]^2}};
(*-----------------------------------------------------------------------*)

inversemetric = FullSimplify[Inverse[metric]];
Print["-----------------------------------------------------------------------"];
Print["The Manifold has dimension   n= ",
  n, "\nCoordinate system:           ", coord];
Print["-----------------------------------------------------------------------"];
Print["gμν=", metric // MatrixForm];
Print["g^μν=", inversemetric // MatrixForm];
Print["g  =", Det[metric] // FullSimplify];
affine := affine = FullSimplify[Table[
      (1 / 2) * Sum[
        (*              g^is (∂k gsj+∂j gsk−∂s gjk)            *)
        (inversemetric[[i, s]]) *
         (D[metric[[s, j]], coord[[k]] ] +
           D[metric[[s, k]], coord[[j]] ] - D[metric[[j, k]], coord[[s]] ]),
        {s, 1, n}],
      {i, 1, n}, {j, 1, n}, {k, 1, n}] ];
(*The non zero Christoffel symbols are computed and selected below: *)
listaffine := Table[
    If[
     UnsameQ[affine[[i, j, k]], 0],
     {Subscript[Superscript[Γ, i], j, k], affine[[i, j, k]]}
      ],
    {i, 1, n}, {j, 1, n}, {k, 1, j}];
Print["-----------------------------------------------------------------------"];
Print["Christoffel Symbols:"];
Print[TableForm[
    Partition[DeleteCases[Flatten[listaffine], Null], 2], TableSpacing → {2, 2}]];
riemann := riemann = FullSimplify[Table[
      (* R^i_jkl=     ∂k Γ^i_lj                      -           ∂l Γ^i_kj*)
      D[   affine[[i, l, j]],  coord[[k]]     ] - D[affine[[i, k, j]], coord[[l]]    ] +
```

```
(*          Γ^i_ks               Γ^s_lj      -         Γ^i_ls               Γ^s_kj        *)
Sum[affine⟦i, k, s⟧ affine⟦s, l, j⟧ -  affine⟦i, l, s⟧ affine⟦s, k, j⟧ ,
 {s, 1, n}],
{i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}]];
listriemann := Table[
   If[
    UnsameQ[riemann⟦i, j, k, l⟧, 0],
    {Subscript[Superscript[R, i], j, k, l], riemann⟦i, j, k, l⟧}
   ],
   {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, k - 1}];
Print["-----------------------------------------------------------------"];
Print["Riemann Tensor:"];
Print[TableForm[
   Partition[DeleteCases[Flatten[listriemann], Null], 2], TableSpacing → {2, 2}]];
lriemann := lriemann = FullSimplify[Table[
     Sum[metric⟦i, ii⟧ riemann⟦ii, j, k, l⟧, {ii, 1, n}],
     {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}]];
listlriemann := Table[
   If[
    UnsameQ[lriemann⟦i, j, k, l⟧, 0],
    {Subscript[R, i, j, k, l], lriemann⟦i, j, k, l⟧}
   ], {i, 1, n}, {j, 1, i - 1}, {k, 1, n}, {l, 1, k - 1}];
Print["-----------------------------------------------------------------"];
Print["Contravariant Riemann Tensor:"];
Print[TableForm[
   Partition[DeleteCases[Flatten[listlriemann], Null], 2], TableSpacing → {2, 2}]];
uriemann := uriemann = FullSimplify[Table[
     Sum[
      inversemetric⟦j, jj⟧ inversemetric⟦k, kk⟧ inversemetric⟦l, ll⟧
       riemann⟦i, jj, kk, ll⟧, {jj, 1, n}, {kk, 1, n}, {ll, 1, n}
     ],
     {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}]];
listuriemann := Table[
   If[
    UnsameQ[uriemann⟦i, j, k, l⟧, 0], {Superscript[
      Superscript[Superscript[Superscript[R, i], j], k], l], uriemann⟦i, j, k, l⟧}
   ], {i, 1, n}, {j, 1, i - 1}, {k, 1, n}, {l, 1, k - 1}];
Print["-----------------------------------------------------------------"];
Print["Covariant Riemann Tensor:"];
Print[TableForm[
   Partition[DeleteCases[Flatten[listuriemann], Null], 2], TableSpacing → {2, 2}]];
r2 = FullSimplify[Sum[lriemann⟦i, j, k, l⟧ uriemann⟦i, j, k, l⟧,
    {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}]];
```

```
Print["--------------------------------------------------------------------------"];
Print["R²= ", r2];
ricci := ricci = FullSimplify[Table[
      Sum[
       riemann[[i, j, i, l]],
       {i, 1, n}
      ], {j, 1, n}, {l, 1, n}]];
listricci := Table[
   If[
    UnsameQ[ricci[[j, l]], 0],
    {Subscript[R, j, l], ricci[[j, l]]}
   ], {j, 1, n}, {l, 1, j}];
Print["--------------------------------------------------------------------------"];
Print["Ricci Tensor:"];
Print[TableForm[
   Partition[DeleteCases[Flatten[listricci], Null], 2], TableSpacing → {2, 2}]];
scalar = FullSimplify[Sum[inversemetric[[i, j]] ricci[[i, j]], {i, 1, n}, {j, 1, n}]];
Print["--------------------------------------------------------------------------"];
Print["Curvature Scalar:"];
Print["R= ", scalar];
einstein := einstein = FullSimplify[ricci - (1/2) scalar * metric];
listeinstein := Table[
   If[
    UnsameQ[einstein[[j, l]], 0],
    {Subscript[G, j, l], einstein[[j, l]]}
   ], {j, 1, n}, {l, 1, j}];
Print["--------------------------------------------------------------------------"];
Print["Einstein Tensor:"];
Print[TableForm[
   Partition[DeleteCases[Flatten[listeinstein], Null], 2], TableSpacing → {2, 2}]];
weyl := weyl = FullSimplify[Table[
      If[n > 3,
       lriemann[[i, j, k, l]]
        - 1/(n - 2) (metric[[i, k]] ricci[[l, j]] - metric[[i, l]] ricci[[k, j]] -
            metric[[j, k]] ricci[[l, i]] + metric[[j, l]] ricci[[k, i]])
        + 1/((n - 1) (n - 2)) (metric[[i, k]] metric[[l, j]] - metric[[i, l]] metric[[k, j]]) scalar
       (*else, if n≤ 3 return 0:*), 0],
      {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}]];
listweyl := Table[
   If[
```

```
     UnsameQ[weyl[[i, j, k, l]], 0],
     {Subscript[C, i, j, k, l], weyl[[i, j, k, l]]}
    ], {i, 1, n}, {j, 1, i - 1}, {k, 1, n}, {l, 1, k - 1}];
Print["---------------------------------------------------------------------"];
Print["Weyl Tensor:"];
Print[TableForm[
   Partition[DeleteCases[Flatten[listweyl], Null], 2], TableSpacing → {2, 2}]];
geodesic := geodesic =
   Simplify[Table[-Sum[affine[[i, j, k]] u[j] u[k], {j, 1, n}, {k, 1, n}], {i, 1, n}]];
subst = Table[u[i] → Subscript[coord[[i]], τ], {i, 1, n}];
nlistgeodesic :=
  Table[{Subscript[coord[[i]], ττ], "+", -geodesic[[i]] /. subst, "= 0"}, {i, 1, n}];
Print["---------------------------------------------------------------------"];
Print["Geodesic Equations:"];
Print[TableForm[nlistgeodesic, TableSpacing → {2}]];
```

The Manifold has dimension   n= 4

Coordinate system: {t, r, θ, ϕ}

$$g_{\mu\nu}=\begin{pmatrix} -1+\frac{2M}{r} & 0 & 0 & 0 \\ 0 & 1+\frac{2M}{r} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\,\mathrm{Sin}[\theta]^2 \end{pmatrix}$$

$$g^{\mu\nu}=\begin{pmatrix} \frac{r}{2M-r} & 0 & 0 & 0 \\ 0 & \frac{r}{2M+r} & 0 & 0 \\ 0 & 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 0 & \frac{\mathrm{Csc}[\theta]^2}{r^2} \end{pmatrix}$$

$$g \;=-r^2\left(-4M^2+r^2\right)\mathrm{Sin}[\theta]^2$$

Christoffel Symbols:

$\Gamma^{1}{}_{2,1}$ $\frac{M}{r(-2M+r)}$

$\Gamma^{2}{}_{1,1}$ $\frac{M}{2Mr+r^2}$

$\Gamma^{2}{}_{2,2}$ $-\frac{M}{2Mr+r^2}$

$\Gamma^{2}{}_{3,3}$ $-\frac{r^2}{2M+r}$

$\Gamma^{2}{}_{4,4}$ $-\frac{r^2 \mathrm{Sin}[\theta]^2}{2M+r}$

$\Gamma^{3}{}_{3,2}$ $\frac{1}{r}$

$\Gamma^{3}{}_{4,4}$ $-\mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta]$

$\Gamma^{4}{}_{4,2}$ $\frac{1}{r}$

$\Gamma^{4}{}_{4,3}$ $\mathrm{Cot}[\theta]$

Riemann Tensor:

$R^{1}{}_{2,2,1}$ $\frac{4M^3-2Mr^2}{r^2(-2M+r)^2(2M+r)}$

$R^{1}{}_{3,3,1}$ $\frac{Mr}{-4M^2+r^2}$

$R^{1}{}_{4,4,1}$ $\frac{Mr\,\mathrm{Sin}[\theta]^2}{-4M^2+r^2}$

$R^{2}{}_{1,2,1}$ $-\frac{2M(-2M^2+r^2)}{r^2(-2M+r)(2M+r)^2}$

$R^{2}{}_{3,3,2}$ $\frac{Mr}{(2M+r)^2}$

$R^{2}{}_{4,4,2}$ $\frac{Mr\,\mathrm{Sin}[\theta]^2}{(2M+r)^2}$

$R^{3}{}_{1,3,1}$ $\frac{M}{r^2(2M+r)}$

$R^{3}{}_{2,3,2}$ $-\frac{M}{r^2(2M+r)}$

$R^{3}{}_{4,4,3}$ $-\frac{2M\,\mathrm{Sin}[\theta]^2}{2M+r}$

$R^{4}{}_{1,4,1}$ $\frac{M}{r^2(2M+r)}$

$R^{4}{}_{2,4,2}$ $-\frac{M}{r^2(2M+r)}$

$R^{4}{}_{3,4,3}$ $\frac{2M}{2M+r}$

Contravariant Riemann Tensor:

$R_{2,1,2,1}$ $\frac{4 M^3-2 M r^2}{-4 M^2 r^3+r^5}$

$R_{3,1,3,1}$ $\frac{M}{2 M+r}$

$R_{3,2,3,2}$ $-\frac{M}{2 M+r}$

$R_{4,1,4,1}$ $\frac{M \operatorname{Sin}[\theta]^2}{2 M+r}$

$R_{4,2,4,2}$ $-\frac{M \operatorname{Sin}[\theta]^2}{2 M+r}$

$R_{4,3,4,3}$ $\frac{2 M r^2 \operatorname{Sin}[\theta]^2}{2 M+r}$

Covariant Riemann Tensor:

R^{2121} $\frac{-4 M^3 r+2 M r^3}{(4 M^2-r^2)^3}$

R^{3131} $\frac{M}{r^2 (-2 M+r)^2 (2 M+r)}$

R^{3232} $-\frac{M}{r^2 (2 M+r)^3}$

R^{4141} $\frac{M \operatorname{Csc}[\theta]^2}{r^2 (-2 M+r)^2 (2 M+r)}$

R^{4242} $-\frac{M \operatorname{Csc}[\theta]^2}{r^2 (2 M+r)^3}$

R^{4343} $\frac{2 M \operatorname{Csc}[\theta]^2}{r^6 (2 M+r)}$

$$R^2 = \frac{16 M^2 \left(64 M^6 - 64 M^5 r + 4 M^4 r^2 + 16 M^3 r^3 - 8 M r^5 + 3 r^6\right)}{\left(-4 M^2 r + r^3\right)^4}$$

Ricci Tensor:

$R_{1,1}$ $-\frac{4 M^3}{r^2 (-2 M+r) (2 M+r)^2}$

$R_{2,2}$ $\frac{4 M^2 (-3 M+2 r)}{r^2 (-2 M+r)^2 (2 M+r)}$

$R_{3,3}$ $\frac{8 M^3}{(2 M-r) (2 M+r)^2}$

$R_{4,4}$ $\frac{8 M^3 \operatorname{Sin}[\theta]^2}{(2 M-r) (2 M+r)^2}$

Curvature Scalar:

$$R = \frac{8 M^2 \left(4 M^2 - 3 M r + r^2\right)}{\left(-4 M^2 r + r^3\right)^2}$$

Einstein Tensor:

$G_{1,1}$ $\frac{4 M^2 (-2 M+r)}{r^3 (2 M+r)^2}$

$G_{2,2}$ $-\frac{4 M^2}{(2 M-r) r^3}$

$G_{3,3}$ $\frac{4 M^2 (M-r) r}{(-4 M^2+r^2)^2}$

$G_{4,4}$ $\frac{4 M^2 (M-r) r \mathrm{Sin}[\theta]^2}{(-4 M^2+r^2)^2}$

Weyl Tensor:

$C_{2,1,2,1}$ $-\frac{2 M (4 M-3 r) (2 M^2-r^2)}{3 r^4 (-4 M^2+r^2)}$

$C_{3,1,3,1}$ $-\frac{M (4 M-3 r) (2 M^2-r^2)}{3 (2 M-r) r (2 M+r)^2}$

$C_{3,2,3,2}$ $-\frac{M (4 M-3 r) (2 M^2-r^2)}{3 r (-2 M+r)^2 (2 M+r)}$

$C_{4,1,4,1}$ $-\frac{M (4 M-3 r) (2 M^2-r^2) \mathrm{Sin}[\theta]^2}{3 (2 M-r) r (2 M+r)^2}$

$C_{4,2,4,2}$ $-\frac{M (4 M-3 r) (2 M^2-r^2) \mathrm{Sin}[\theta]^2}{3 r (-2 M+r)^2 (2 M+r)}$

$C_{4,3,4,3}$ $\frac{2 M (4 M-3 r) r^2 (2 M^2-r^2) \mathrm{Sin}[\theta]^2}{3 (-4 M^2+r^2)^2}$

Geodesic Equations:

$t_{\tau\tau} + -\frac{2 M r_\tau t_\tau}{2 M r-r^2} = 0$

$r_{\tau\tau} + -\frac{M r_\tau^2 - M t_\tau^2 + r^3 \theta_\tau^2 + r^3 \mathrm{Sin}[\theta]^2 \phi_\tau^2}{2 M r+r^2} = 0$

$\theta_{\tau\tau} + \frac{2 r_\tau \theta_\tau}{r} - \mathrm{Cos}[\theta] \mathrm{Sin}[\theta] \phi_\tau^2 = 0$

$\phi_{\tau\tau} + \frac{2 (r_\tau + r \mathrm{Cot}[\theta] \theta_\tau) \phi_\tau}{r} = 0$