Bimetric Theory
(The notion of spacetime in Bimetric Gravity)

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SFH & Mikica Kocic (arXiv:1706.07806)

Also based on work with:

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Angnis Schmidt-May
Mikael von Strauss
Anders Lundkvist
Luis Apolo
Outline of the talk

Motivation & Problems

Gravity with an extra spin-2 field: Bimetric Theory

Potential Issues

Uniqueness and the local structure of spacetime

Discussion
Bimetric gravity:

Why study a theory of gravity with two metrics?

(Gravitational metric coupled to extra spin-2 fields)
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Motivation

- GR + Λ + CDM: Very successful
Motivation

▶ GR + Λ + CDM: Very successful
▶ Observations and theory indicate new physics

Dark matter, Dark energy (the cosmological constant problem), Origin of Inflation, The “trans-Planckian” problem, Quantum gravity, Matter-Antimatter asymmetry · · ·
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*Dark matter, Dark energy (the cosmological constant problem), Origin of Inflation, The “trans-Planckian” problem, Quantum gravity, Matter-Antimatter asymmetry...*

**Theoretical (top down) approaches:**

GUTs, Susy, String Theory (with or without Multiverse), Quantum Gravity, ···
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But why gravity with extra spin-2 fields?
Recall:

Spin \( (s) \Rightarrow \text{basic structure of field equations} \\
(Klein-Gordon, Dirac, Maxwell/Proca, Einstein)
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Spin in known physics:

- *Standard Model*: multiplets of $s = 0, \frac{1}{2}, 1$ fields
  
  Multiplet structure is crucial for the viability of SM
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- **General relativity**: single massless \(s = 2\) field \(g_{\mu\nu}\)
  
  Einstein-Hilbert for \(s = 2\) \(\sim\) Klein-Gordon for \(s = 0\)
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**Low-energy** String theory, etc: single massless spin-2 + plethora of spin 0, 1/2 and 1.
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**Low-energy** String theory, etc: single massless spin-2  
+ plethora of spin 0, 1/2 and 1.

*Unexplored corner*: gravity with more spin-2 fields
Challenges in adding spin-2 fields to GR

- No multiple massless spin-2 fields
- Linear theory of massive spin-2 fields (5 helicities)  
  \[Fierz, Pauli \ (1939)\]
- Ghosts in nonlinear theory (5 + 1 helicities)  
  \[Boulware, Deser \ (1972)\]

Consequences?
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- Ghosts in theories with multiple spin-2 fields
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Can such theories exist or is GR unique?

Consequences?
The Ghost Problem

Ghost: A field with negative kinetic energy

Example:

\[ \mathcal{L} = T - V = (\partial_t \phi)^2 \cdots \quad (healthy) \]

But

\[ \mathcal{L} = T - V = -(\partial_t \phi)^2 \cdots \quad (ghostly) \]

Consequences:
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Consequences:

- Instability: unlimited energy transfer from ghost to other fields
- Negative probabilities, violation of unitarity in quantum theory
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Discussion
GR with a generic spin-2 field

A dynamical theory for the metric $g_{\mu \nu}$ & spin-2 field $f_{\mu \nu}$

$$\mathcal{L} = m_p^2 \sqrt{-g} R -$$
GR with a generic spin-2 field

A dynamical theory for the metric $g_{\mu\nu}$ & spin-2 field $f_{\mu\nu}$

$$\mathcal{L} = m_p^2 \sqrt{-g} R - m^4 \sqrt{-g} V(g^{-1} f) +$$
GR with a generic spin-2 field

A dynamical theory for the metric $g_{\mu\nu}$ & spin-2 field $f_{\mu\nu}$

\[ \mathcal{L} = m_p^2 \sqrt{-g} R - m^4 \sqrt{-g} V(g^{-1}f) + \]

No dynamics for $f_{\mu\nu}$: Massive Gravity
GR with a generic spin-2 field

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No dynamics for $f_{\mu\nu}$: Massive Gravity

Overcoming the ghost in massive gravity:

[Creminelli, Nicolis, Papucci, Trincherini, (2005)]
[de Rham, Gabadadze, Tolley (2010)]
[SFH, Rosen (2011); SFH, Rosen, Schmidt-May (2011)]
GR with a generic spin-2 field

A dynamical theory for the metric $g_{\mu \nu}$ & spin-2 field $f_{\mu \nu}$

$$\mathcal{L} = m_p^2 \sqrt{-g} R - m^4 \sqrt{-g} V(g^{-1} f) + \mathcal{L}(f, \nabla f)$$
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- what is $V(g^{-1} f)$?
- what is $\mathcal{L}(f, \nabla f)$?
- proof of absence of the Boulware-Deser ghost

[SFH, Rosen (2011); de Rham, Gabadadze, Tolley (2010)]
GR with a generic spin-2 field

A dynamical theory for the metric $g_{\mu\nu}$ & spin-2 field $f_{\mu\nu}$

$$\mathcal{L} = m_p^2 \sqrt{-g} R - m^4 \sqrt{-g} V(g^{-1}f) + \mathcal{L}(f, \nabla f)$$

- what is $V(g^{-1}f)$ ?
- what is $\mathcal{L}(f, \nabla f)$ ?
- proof of absence of the Boulware-Deser ghost $V(g^{-1}f)$ for a ghost-free theory:

  [SFH, Rosen (2011); de Rham, Gabadadze, Tolley (2010)]
Digression: elementary symmetric polynomials $e_n(S)$

Consider matrix $S$ with eigenvalues $\lambda_1, \cdots, \lambda_4$.

\[
\begin{align*}
e_0(S) &= 1, & e_1(S) &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4, \\
e_2(S) &= \lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_1 \lambda_4 + \lambda_2 \lambda_3 + \lambda_2 \lambda_4 + \lambda_3 \lambda_4, \\
e_3(S) &= \lambda_1 \lambda_2 \lambda_3 + \lambda_1 \lambda_2 \lambda_4 + \lambda_1 \lambda_3 \lambda_4 + \lambda_2 \lambda_3 \lambda_4, \\
e_4(S) &= \lambda_1 \lambda_2 \lambda_3 \lambda_4.
\end{align*}
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e_4(S) = \lambda_1 \lambda_2 \lambda_3 \lambda_4.
\]

\[
e_0(S) = 1, \\
e_1(S) = \text{Tr}(S) \equiv [S], \\
e_2(S) = \frac{1}{2}([S]^2 - [S^2]), \\
e_3(S) = \frac{1}{6}([S]^3 - 3[S][S^2] + 2[S^3]), \\
e_4(S) = \det(S), \\
e_n(S) = 0 \quad (\text{for} \quad n > 4).
\]
\[ \det(\mathbb{1} + S) = \sum_{n=0}^{4} e_n(S) \]

Where:

\[ S = \sqrt{g - \mu\lambda\phi} \]

Square root of the matrix \( g_{\mu\lambda} \phi \)

Real? Unique?

How to make sense of it? (to be answered later)
$$\text{det}(1 + S) = \sum_{n=0}^{4} e_n(S)$$

$$V(S) = \sum_{n=0}^{4} \beta_n e_n(S)$$

Where:

$$S = \sqrt{g^{-1}f}$$

square root of the matrix $g^{\mu\lambda}f_{\lambda\nu}$

Real? Unique?

How to make sense of it? (to be answered later)
Ghost-free “bi-metric” theory

[SFH, Rosen (1109.3515,1111.2070)]

Ghost-free combination of kinetic and potential terms:

\[
\mathcal{L} = m_g^2 \sqrt{-g} R_g - m^4 \sqrt{-g} \sum_{n=0}^{4} \beta_n e_n(\sqrt{g^{-1}f}) + m_f^2 \sqrt{-f} R_f
\]

“bimetric” nature forced by the absence of ghost
Ghost-free “bi-metric” theory

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Ghost-free combination of *kinetic* and *potential* terms:

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\mathcal{L} = m_g^2 \sqrt{-g} R_g - m^4 \sqrt{-g} \sum_{n=0}^{4} \beta_n \ e_n(\sqrt{g^{-1} f}) + m_f^2 \sqrt{-f} R_f
\]

“bimetric” nature forced by the absence of ghost

Hamiltonian analysis:

\(7 = 2 + 5\) nonlinear propagating modes, no BD ghost!

\[C_1 = 0, \quad C_2 = \frac{d}{dt} C_1 = \{H, C\} = 0\]

Detailed analysis of constraints [SFH, Lundkvist (to appear)]
Mass spectrum & Limits

\[ \bar{f} = c^2 \bar{g}, \quad g_{\mu \nu} = \bar{g}_{\mu \nu} + \delta g_{\mu \nu}, \quad f_{\mu \nu} = \bar{f}_{\mu \nu} + \delta f_{\mu \nu} \]

Linear modes:

Massless spin-2: 
\[ \delta G_{\mu \nu} = (\delta g_{\mu \nu} + m^2 f_{\mu \nu}) (\bar{g}_{\mu \nu} + \bar{f}_{\mu \nu}) \]

Massive spin-2: 
\[ \delta M_{\mu \nu} = (\delta f_{\mu \nu} - c^2 \delta g_{\mu \nu}) (\bar{g}_{\mu \nu} + \bar{f}_{\mu \nu}) \]

The General Relativity limit: 
\[ m_g = M_P, \quad m_f / m_g \to 0 \]

Massive gravity limit: 
\[ m_g = M_P, \quad m_f / m_g \to \infty \]
Mass spectrum & Limits

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Linear modes:

Massless spin-2: \[
\delta G_{\mu\nu} = \left( \delta g_{\mu\nu} + \frac{m_f^2}{m_g^2} \delta f_{\mu\nu} \right) \quad (2)
\]

Massive spin-2: \[
\delta M_{\mu\nu} = \left( \delta f_{\mu\nu} - c^2 \delta g_{\mu\nu} \right) \quad (5)
\]

\(g_{\mu\nu}, f_{\mu\nu}\) are mixtures of massless and massive modes
Mass spectrum & Limits

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(more later)

Massive gravity limit: \( m_g = M_P, \quad m_f / m_g \to \infty \)
Can Bimetric be a fundamental theory?

- Similar to Proca theory in curved background,

\[
\sqrt{|\text{det } g|} \left( F_{\mu\nu} F^{\mu\nu} - m^2 g^{\mu\nu} A_\mu A_\nu + R_g \right)
\]

- May need the equivalent of Higgs mechanism with the extra fields for better quantum or even classical behaviour
Some features

1) Ghost-free Matter couplings, same as in GR:

\[ \mathcal{L}_g(g, \phi) + \mathcal{L}_f(f, \tilde{\phi}) \]
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$$\mathcal{L}_g(g, \phi) + \mathcal{L}_f(f, \tilde{\phi})$$

2) $\beta_1, \beta_2, \beta_3$: effective cosmological constant at late times

(protected by symmetry, like fermion masses)

3) Massive spin-2 particles: dark matter (stable enough)
Some features

1) Ghost-free Matter couplings, same as in GR:

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2) \( \beta_1, \beta_2, \beta_3 \): effective cosmological constant at late times

(protected by symmetry, like fermion masses)

3) Massive spin-2 particles: dark matter (stable enough)

4) Some blackhole and cosmological solutions explored

5) Gravitational waves ?

6) Generalization to more than 2 fields
The General Relativity limit:

\[ m_g = M_P, \quad \alpha = m_f/m_g \to 0 \]

Cosmological solutions in the GR limit:

\[ 3H^2 = \frac{\rho}{M_{Pl}^2} - \frac{2}{3} \frac{\beta_1}{\beta_2} m^2 - \alpha^2 \frac{\beta_1^2}{3\beta_2^2} H^2 + \mathcal{O}(\alpha^4) \]

The GR approximation breaks down at sufficiently strong fields.
Outline of the talk

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Gravity with an extra spin-2 field: Bimetric Theory

Potential Issues

Uniqueness and the local structure of spacetime

Discussion
Potential Issues

- (1) $g, f$: incompatible notions of space and time?
- (2) nonunique $\sqrt{g^{-1}} f$: ambiguity in defining the action?
- Causality: local closed timelike curves (CTC’s) ?
- The initial value problem?
- Faster than light possible in the gravitational sector (good or bad?)
- Analogue of “energy conditions” and global “bi”hyperbolicity?
Problem 1: $g_{\mu\nu}$ & $f_{\mu\nu}$ have Lorentzian signature $(1, 3)$. But may not admit compatible 3+1 decompositions.

Then, no consistent time evolution equations, no Hamiltonian formulation.
Nonuniqueness, Reality and Covariance

Problem 2:

- $S = \sqrt{g^{-1}}f$ is not unique,
- May not be real, covariant

To properly define the theory ($V(S)$):

(a) $S^\mu_\nu$ needs to be specified uniquely,
(b) Restrict $g_{\mu\nu}, f_{\mu\nu}$ so that $S$ is real, covariant

What are the restrictions on $g$ & $f$?
Can they be imposed meaningfully & consistent with dynamics?
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Uniqueness and the local structure of spacetime

Natural requirements:

- General Covariance: $S^{\mu}_{\nu}$ must be a (1,1) tensor
- $S$ must be real

Implication:

Problem 2 (reality, uniqueness) has a natural solution.

This also solves Problem 1 (compatible spacetimes).

[SFH, M. Kocic (arXiv:1706.07806)]
Solution to the uniqueness problem of $V(S)$

Matrix square roots:
- **Primary roots**: Max 16 distinct roots, generic
- **Nonprimary roots**: Infinitely many, non-generic
  (when eigenvalues in different Jordan blocks coincide)
Solution to the uniqueness problem of $V(S)$

Matrix square roots:
- **Primary roots**: Max 16 distinct roots, generic
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  (when eigenvalues in different Jordan blocks coincide)

General Covariance: $A_{\mu \nu} = g^{\mu \rho} f_{\rho \nu}$ is a (1,1) tensor,

$$x^\mu \rightarrow \tilde{x}^\mu \Rightarrow A \rightarrow Q^{-1}AQ,$$

for $Q_{\mu \nu} = \frac{\partial x^\mu}{\partial \tilde{x}^\nu}$
Uniqueness of $S$

$$S_{\mu \nu}^{\mu \nu} = (\sqrt{A})_{\mu \nu}^{\mu \nu} :$$

- Primary roots: \[ \sqrt{A} \rightarrow \sqrt{Q^{-1}AQ} = Q^{-1}\sqrt{A}Q \]
- Nonprimary roots: \[ \sqrt{Q^{-1}AQ} \neq Q^{-1}\sqrt{A}Q \]

**Step 1:**
General covariance $\Rightarrow$ only primary roots are allowed.

*A Consequence: Examples of backgrounds with local CTC's correspond to nonprimary roots and are excluded*
Uniqueness of $S$

$S_{\mu\nu}^{\mu} = (\sqrt{A})_{\mu\nu}^{\mu}$:

- Primary roots: $\sqrt{A} \rightarrow \sqrt{Q^{-1}AQ} = Q^{-1}\sqrt{A}Q$
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Step 1:
General covariance $\Rightarrow$ only primary roots are allowed.

_A Consequence: Examples of backgrounds with local CTC's correspond to nonprimary roots and are excluded_

Step 2:
Only the _principal root_ is always primary. Hence, $S$ must be a principal root.

(Nonprincipal roots degenerate to nonprimary roots when some eigenvalues coincide).
Reality of $S$

Real $S = \sqrt{g^{-1}f}$ ⇒ simple classification of allowed $g$, $f$ configurations

[SFH, M. Kocic (arXiv:1706.07806)]
Reality of $S$

Real $S = \sqrt{g^{-1}f} \Rightarrow$ simple classification of allowed $g, f$ configurations

[SFH, M. Kocic (arXiv:1706.07806)]

**Theorem:** Real $S = \sqrt{g^{-1}f}$ exist if and only if the null cones of $g$ and $f$ (i) intersect in open sets, or, (ii) have no common space nor common time directions (Type IV)
Reality of $S$

Real $S = \sqrt{g^{-1}f}$ ⇒ simple classification of allowed $g$, $f$ configurations

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*Types I-III: proper 3+1 decompositions, primary roots, allowed.
*Type IV: only nonprimary real roots! (excluded).
Choice of the square root

**Reality + General Covariance $\Rightarrow$**

* Real principal square root (unique),
* Compatible 3+1 decomposition

\[ h_{\mu \nu} = g_{\mu \rho} (\sqrt{g^{-1} f})^\rho_{\nu} \]

$h$ null-cones for the principal root (except for the last one)

Useful for choosing good coordinate systems,
The specific, existing local CTC's in massive gravity ruled out.
Consistency with dynamics

Type IV metrics arise as a limit of Type IIb metrics. But in the limit, $S$ has a branch cut. Then, for the principal root, the variation $\delta S$ is not defined at the cut $\Rightarrow$ Eqns. of motion not valid for Type IV. Hence cannot arise dynamically.
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A simple mechanical example:

$$A = \int dt \left( \dot{x}^2/2 - \lambda \sqrt{x^2} \right), \quad \sqrt{x^2} = |x| \quad (1)$$

$$\ddot{x} = -\lambda (x > 0), \quad \ddot{x} = \lambda (x < 0) \quad (2)$$

(no equation at $x = 0$)

Implication for some acausality arguments (CTC’s) in massive gravity
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The beginning of understanding theories of spin-2 fields beyond General Relativity.
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The beginning of understanding theories of spin-2 fields beyond General Relativity.

- Superluminality? (yes, but not necessarily harmful, \textit{replacement for inflation}?)
- Causality? (needs to be investigated further)
- Energy conditions, global “bi-hyperbolicity”
- A more fundamental formulation
- Application to cosmology, blackholes, GW, etc.
- Extra symmetries $\Rightarrow$ \textbf{Modified kinetic terms}? much less understood. [SFH, Apolo (2016)]
Thank you!
The Hojman-Kuchar-Teitelboim Metric

General Relativity in 3+1 decomposition \((g_{\mu\nu} : \gamma_{ij}, N, N_i)\):

\[
\sqrt{g} R \sim \pi^{ij} \partial_t \gamma_{ij} - NR^0 - N_i R^i
\]

Constraints: \(R^0 = 0\), \(R^i = 0\).

Algebra of General Coordinate Transformations (GCT):

\[
\begin{align*}
\{ R^0(x), R^0(y) \} &= - \left[ R^i(x) \frac{\partial}{\partial x^i} \delta^3(x - y) - R^i(y) \frac{\partial}{\partial y^i} \delta^3(x - y) \right] \\
\{ R^0(x), R_i(y) \} &= - R^0(y) \frac{\partial}{\partial x^i} \delta^3(x - y) \\
\{ R_i(x), R_j(y) \} &= - \left[ R_j(x) \frac{\partial}{\partial x^i} \delta^3(x - y) - R_i(y) \frac{\partial}{\partial y^j} \delta^3(x - y) \right]
\end{align*}
\]

\(R_i = \gamma_{ij} R^j\), \(\gamma_{ij}\): metric of spatial 3-surfaces.

- Any generally covariant theory contains such an algebra.
- HKT: The tensor that lowers the index on \(R^i\) is the physical metric of 3-surfaces.
The HKT metric in bimetric theory

Consider \( g_{\mu\nu} = (\gamma_{ij}, N, N_i) \) and \( f_{\mu\nu} = (\phi_{ij}, L, L_i) \),

\[
\mathcal{L}_{g,f} \sim \pi^{ij} \gamma_{ij} + p^{ij} \phi_{ij} - M \tilde{R}^0 - M_i \tilde{R}^i
\]

On the surface of second class Constraints.

GCT Algebra:

\[
\{ \tilde{R}^0(x), \tilde{R}^0(y) \} = - \left[ \tilde{R}^i(x) \frac{\partial}{\partial x^i} \delta^3(x - y) - \tilde{R}^i(y) \frac{\partial}{\partial y^i} \delta^3(x - y) \right]
\]

\[
\{ \tilde{R}^0(x), \tilde{R}_i(y) \} = - \tilde{R}^0(y) \frac{\partial}{\partial x^i} \delta^3(x - y)
\]

\( \tilde{R}_i = \phi_{ij} \tilde{R}^j \), \( \phi_{ij} \): the 3-metric of \( f_{\mu\nu} \), or

\( \tilde{R}_i = \gamma_{ij} \tilde{R}^j \), \( \gamma_{ij} \): the 3-metric of \( g_{\mu\nu} \).

The HKT metric of bimetric theory is \( g_{\mu\nu} \) or \( f_{\mu\nu} \), consistent with ghost-free matter couplings

[SFH, A. Lundkvist (to appear)]