

Bimetric Theory

(The notion of spacetime in Bimetric Gravity)

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9th Aegean Summer School

on Einstein's Theory of Gravity and its Modifications

Sept 18-23, 2017, Sifnos

SFH & Mikica Kocic (arXiv:1706.07806)

Also based on work with:

Rachel. A. Rosen

Angris Schmidt-May

Mikael von Strauss

Anders Lundkvist

Luis Apolo

Outline of the talk

Motivation & Problems

Gravity with an extra spin-2 field: Bimetric Theory

Potential Issues

Uniqueness and the local structure of spacetime

Discussion

Bimetric gravity:

Why study a theory of gravity with two metrics?

(Gravitational metric coupled to extra spin-2 fields)

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*Dark matter, Dark energy (the cosmological constant problem),
Origin of Inflation, The “trans-Planckian” problem,
Quantum gravity, Matter-Antimatter asymmetry . . .*

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Theoretical (top down) approaches:

GUTs, Susy, String Theory (with or without Multiverse),
Quantum Gravity, . . .

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But why gravity with extra spin-2 fields?

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Unexplored corner: gravity with more spin-2 fields

Challenges in adding spin-2 fields to GR

- ▶ No multiple massless spin-2 fields
- ▶ Linear theory of massive spin-2 fields (5 helicities)

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[Boulware, Deser (1972)]
- ▶ Ghosts in theories with multiple spin-2 fields

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Can such theories exist or is GR unique?

Consequences?

The Ghost Problem

Ghost: A field with **negative** kinetic energy

Example:

$$\mathcal{L} = T - V = (\partial_t \phi)^2 \dots \quad (\text{healthy})$$

But

$$\mathcal{L} = T - V = -(\partial_t \phi)^2 \dots \quad (\text{ghostly})$$

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Consequences:

- ▶ **Instability:** unlimited energy transfer from ghost to other fields
- ▶ **Negative probabilities, violation of unitarity in quantum theory**

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Overcoming the ghost in massive gravity:

[Creminelli, Nicolis, Papucci, Trincherini, (2005)]

[de Rham, Gabadadze, Tolley (2010)]

[SFH, Rosen (2011); SFH, Rosen, Schmidt-May (2011)]

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- ▶ what is $\mathcal{L}(f, \nabla f)$?
- ▶ proof of absence of the Boulware-Deser ghost

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$V(g^{-1} f)$ for a ghost-free theory:

[SFH, Rosen (2011); de Rham, Gabadadze, Tolley (2010)]

Digression: elementary symmetric polynomials $e_n(S)$

Consider matrix S with eigenvalues $\lambda_1, \dots, \lambda_4$.

$$e_0(S) = 1, \quad e_1(S) = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4,$$

$$e_2(S) = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4,$$

$$e_3(S) = \lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4,$$

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$$e_0(S) = 1,$$

$$e_1(S) = \text{Tr}(S) \equiv [S],$$

$$e_2(S) = \frac{1}{2}([S]^2 - [S^2]),$$

$$e_3(S) = \frac{1}{6}([S]^3 - 3[S][S^2] + 2[S^3]),$$

$$e_4(S) = \det(S),$$

$$e_n(S) = 0 \quad (\text{for } n > 4),$$

$$\det(\mathbb{1} + S) = \sum_{n=0}^4 e_n(S)$$

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$$V(S) = \sum_{n=0}^4 \beta_n e_n(S)$$

Where:

$$S = \sqrt{g^{-1}f}$$

square root of the matrix $g^{\mu\lambda} f_{\lambda\nu}$

Real? Unique?

How to make sense of it? (to be answered later)

Ghost-free “bi-metric” theory

[SFH, Rosen (1109.3515,1111.2070)]

Ghost-free combination of *kinetic* and *potential* terms:

$$\mathcal{L} = m_g^2 \sqrt{-g} R_g - m^4 \sqrt{-g} \sum_{n=0}^4 \beta_n e_n(\sqrt{g^{-1}f}) + m_f^2 \sqrt{-f} R_f$$

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“bimetric” nature forced by the absence of ghost

Hamiltonian analysis:

$7 = 2 + 5$ nonlinear propagating modes, **no BD ghost!**

▶ $C_1 = 0$, $C_2 = \frac{d}{dt} C_1 = \{H, C\} = 0$

Detailed analysis of constraints [SFH, Lundkvist (to appear)]

Mass spectrum & Limits

$$\bar{f} = c^2 \bar{g}, \quad g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}, \quad f_{\mu\nu} = \bar{f}_{\mu\nu} + \delta f_{\mu\nu}$$

Linear modes:

Mass spectrum & Limits

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Linear modes:

$$\text{Massless spin-2:} \quad \delta G_{\mu\nu} = \left(\delta g_{\mu\nu} + \frac{m_f^2}{m_g^2} \delta f_{\mu\nu} \right) \quad (2)$$

$$\text{Massive spin-2:} \quad \delta M_{\mu\nu} = \left(\delta f_{\mu\nu} - c^2 \delta g_{\mu\nu} \right) \quad (5)$$

$g_{\mu\nu}, f_{\mu\nu}$ are mixtures of *massless* and *massive* modes

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The General Relativity limit: $m_g = M_P$, $m_f/m_g \rightarrow 0$
(more later)

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The General Relativity limit: $m_g = M_P$, $m_f/m_g \rightarrow 0$
(more later)

Massive gravity limit: $m_g = M_P$, $m_f/m_g \rightarrow \infty$

Can Bimetric be a fundamental theory?

- ▶ Similar to Proca theory in curved background,

$$\sqrt{|\det g|}(F_{\mu\nu}F^{\mu\nu} - m^2 g^{\mu\nu}A_\mu A_\nu + R_g)$$

- ▶ May need the equivalent of Higgs mechanism with the extra fields for better quantum or even classical behaviour

Some features

1) Ghost-free Matter couplings, same as in GR:

$$\mathcal{L}_g(\mathbf{g}, \phi) + \mathcal{L}_f(\mathbf{f}, \tilde{\phi})$$

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(protected by symmetry, like fermion masses)

3) Massive spin-2 particles: dark matter (stable enough)

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3) Massive spin-2 particles: dark matter (stable enough)

4) Some blackhole and cosmological solutions explored

5) Gravitational waves ?

6) Generalization to more than 2 fields

GR limit

The General Relativity limit:

$$m_g = M_P, \quad \alpha = m_f/m_g \rightarrow 0$$

Cosmological solutions in the GR limit:

$$3H^2 = \frac{\rho}{M_{Pl}^2} - \frac{2}{3} \frac{\beta_1^2}{\beta_2} m^2 - \alpha^2 \frac{\beta_1^2}{3\beta_2^2} H^2 + \mathcal{O}(\alpha^4)$$

The GR approximation breaks down at sufficiently strong fields

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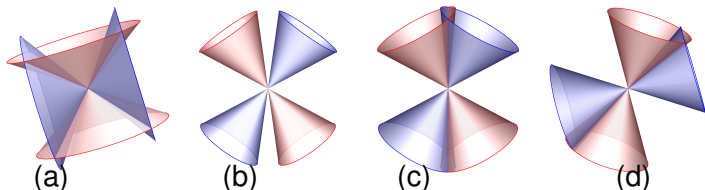
Potential Issues

- ▶ (1) g, f : incompatible notions of space and time?
- ▶ (2) nonunique $\sqrt{g^{-1}f}$: ambiguity in defining the action?
- ▶ Causality: local closed timelike curves (CTC's) ?
- ▶ The initial value problem?
- ▶ Faster than light possible in the gravitational sector (good or bad?)
- ▶ Analogue of “energy conditions” and global “bi”hyperbolicity?

Problem of incompatible spacetimes

Problem 1: $g_{\mu\nu}$ & $f_{\mu\nu}$ have Lorentzian signature (1, 3).

But may not admit compatible 3+1 decompositions



Then, no consistent time evolution equations,
no Hamiltonian formulation.

Nonuniqueness, Reality and Covariance

Problem 2:

- ▶ $S = \sqrt{g^{-1}f}$ is not unique,
- ▶ May not be real, covariant

To properly define the theory ($V(S)$):

- (a) S^μ_ν needs to be specified uniquely,
- (b) Restrict $g_{\mu\nu}, f_{\mu\nu}$ so that S is real, covariant

What are the restrictions on g & f ?

Can they be imposed meaningfully & consistent with dynamics?

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Natural requirements:

- ▶ General Covariance: S^μ_ν must be a (1,1) tensor
- ▶ S must be real

Implication :

Problem 2 (*reality, uniqueness*) has a natural solution.

This also solves **Problem 1** (*compatible spacetimes*).

[SFH, M. Kocic (arXiv:1706.07806)]

Solution to the uniqueness problem of $V(S)$

Matrix square roots:

- ▶ **Primary roots:** Max 16 distinct roots, generic
- ▶ **Nonprimary roots:** Infinitely many, non-generic
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General Covariance: $A^\mu{}_\nu = g^{\mu\rho} f_{\rho\nu}$ is a (1,1) tensor,

$$x^\mu \rightarrow \tilde{x}^\mu \Rightarrow A \rightarrow Q^{-1} A Q, \quad \text{for} \quad Q^\mu{}_\nu = \frac{\partial x^\mu}{\partial \tilde{x}^\nu}$$

Uniqueness of S

$$S^\mu{}_\nu = (\sqrt{A})^\mu{}_\nu :$$

- ▶ Primary roots: $\sqrt{A} \rightarrow \sqrt{Q^{-1}AQ} = Q^{-1}\sqrt{A}Q$
- ▶ Nonprimary roots: $\sqrt{Q^{-1}AQ} \neq Q^{-1}\sqrt{A}Q$

Step 1:

General covariance \Rightarrow only primary roots are allowed.

A Consequence: Examples of backgrounds with local CTC's correspond to nonprimary roots and are excluded

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General covariance \Rightarrow only primary roots are allowed.

A Consequence: Examples of backgrounds with local CTC's correspond to nonprimary roots and are excluded

Step 2:

Only the *principal root* is always primary. Hence, S must be a *principal root*.

(Nonprincipal roots degenerate to nonprimary roots when some eigenvalues coincide).

Reality of S

Real $S = \sqrt{g^{-1}f} \Rightarrow$ simple classification of allowed g, f
configurations

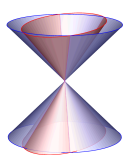
[SFH, M. Kocic (arXiv:1706.07806)]

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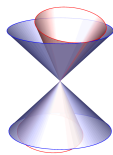
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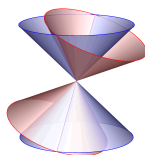
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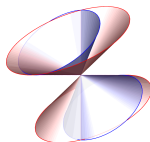
Type I



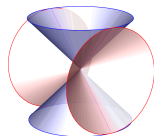
Type IIa



Type IIb



Type III



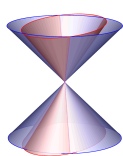
Type IV

Reality of S

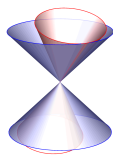
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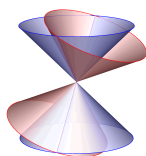
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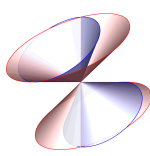
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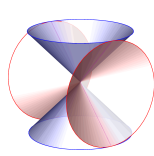
Type IIa



Type IIb



Type III



Type IV

*Types I-III: proper 3+1 decompositions, primary roots, allowed.

*Type IV: only nonprimary real roots! (excluded).

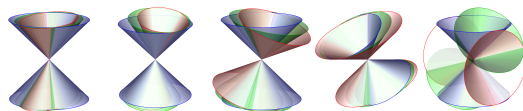
Choice of the square root

Reality + General Covariance \Rightarrow

- * Real principal square root (unique),
- * Compatible 3+1 decomposition

$$h_{\mu\nu} = g_{\mu\rho}(\sqrt{g^{-1}f})^{\rho}_{\nu}$$

h null-cones for the principal root (except for the last one)



Useful for choosing good coordinate systems,
The specific, existing local CTC's in massive gravity ruled out.

Consistency with dynamics

Type IV metrics arise as a limit of Type IIb metrics. But in the limit, S has a branch cut. Then, for the principal root, the variation δS is not defined at the cut \Rightarrow Eqns. of motion not valid for Type IV. Hence cannot arise dynamically.

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A simple mechanical example:

$$A = \int dt \left(\dot{x}^2/2 - \lambda\sqrt{x^2} \right), \quad \sqrt{x^2} = |x| \quad (1)$$

$$\ddot{x} = -\lambda(x > 0), \quad \ddot{x} = \lambda(x < 0) \quad (2)$$

(no equation at $x = 0$)

Implication for some acausality arguments (CTC's) in massive gravity

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The beginning of understanding theories of spin-2 fields beyond General Relativity.

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The beginning of understanding theories of spin-2 fields beyond General Relativity.

- ▶ Superluminality? (yes, but not necessarily harmful, *replacement for inflation?*)
- ▶ Causality? (needs to be investigated further)
- ▶ Energy conditions, global “bi-hyperbolicity”
- ▶ A more fundamental formulation
- ▶ Application to cosmology, blackholes, GW, etc.
- ▶ Extra symmetries \Rightarrow **Modified kinetic terms?**
much less understood. [SFH, Apolo (2016)]

Thank you!

The Hojman-Kuchar-Teitelboim Metric

General Relativity in 3+1 decomposition ($g_{\mu\nu} : \gamma_{ij}, N, N_i$):

$$\sqrt{g}R \sim \pi^{ij} \partial_t \gamma_{ij} - NR^0 - N_i R^i$$

Constraints: $R^0 = 0, R^i = 0$.

Algebra of General Coordinate Transformations (GCT):

$$\{R^0(x), R^0(y)\} = - \left[R^i(x) \frac{\partial}{\partial x^i} \delta^3(x-y) - R^i(y) \frac{\partial}{\partial y^i} \delta^3(x-y) \right]$$

$$\{R^0(x), R_i(y)\} = -R^0(y) \frac{\partial}{\partial x^i} \delta^3(x-y)$$

$$\{R_i(x), R_j(y)\} = - \left[R_j(x) \frac{\partial}{\partial x^i} \delta^3(x-y) - R_i(y) \frac{\partial}{\partial y^j} \delta^3(x-y) \right]$$

$R_i = \gamma_{ij} R^j$, γ_{ij} : metric of spatial 3-surfaces.

- ▶ Any generally covariant theory contains such an algebra.
- ▶ HKT: The tensor that lowers the index on R^i is the physical metric of 3-surfaces.

The HKT metric in bimetric theory

Consider $g_{\mu\nu} = (\gamma_{ij}, N, N_i)$ and $f_{\mu\nu} = (\phi_{ij}, L, L_i)$,

$$\mathcal{L}_{g,f} \sim \pi^{ij} \gamma_{ij} + p^{ij} \phi_{ij} - M \tilde{R}^0 - M_i \tilde{R}^i$$

On the surface of second class Constraints.

GCT Algebra:

$$\{\tilde{R}^0(x), \tilde{R}^0(y)\} = - \left[\tilde{R}^i(x) \frac{\partial}{\partial x^i} \delta^3(x-y) - \tilde{R}^i(y) \frac{\partial}{\partial y^i} \delta^3(x-y) \right]$$

$$\{\tilde{R}^0(x), \tilde{R}_i(y)\} = -\tilde{R}^0(y) \frac{\partial}{\partial x^i} \delta^3(x-y)$$

$\tilde{R}_i = \phi_{ij} \tilde{R}^j$, ϕ_{ij} : the 3-metric of $f_{\mu\nu}$, or

$\tilde{R}_i = \gamma_{ij} \tilde{R}^j$, γ_{ij} : the 3-metric of $g_{\mu\nu}$.

The HKT metric of bimetric theory is $g_{\mu\nu}$ or $f_{\mu\nu}$, consistent with ghost-free matter couplings

[SFH, A. Lundkvist (to appear)]