Dark Matter – Dark Energy Interactions

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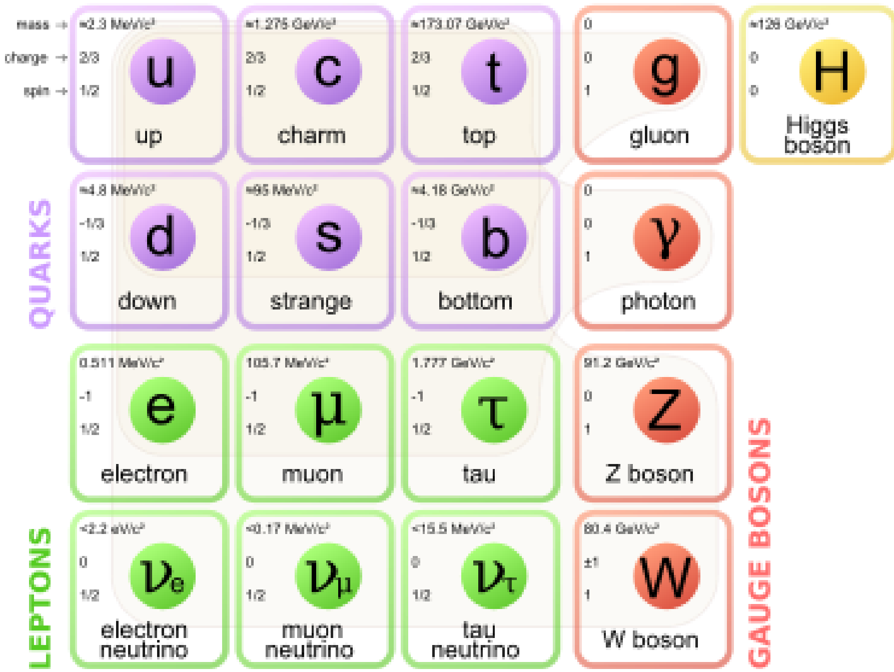
Goal

- We investigate cosmological scenarios in a universe where dark sectors are allowed to mutually interact
- Note:

  A consistent or interesting cosmology is not a proof for the consistency of the underlying gravitational theory

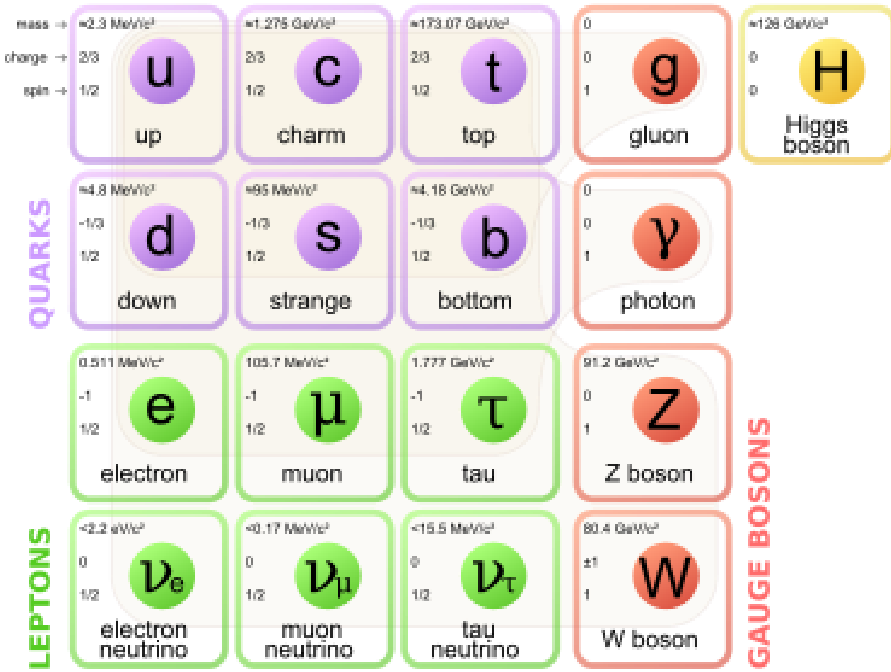
Why Modification?

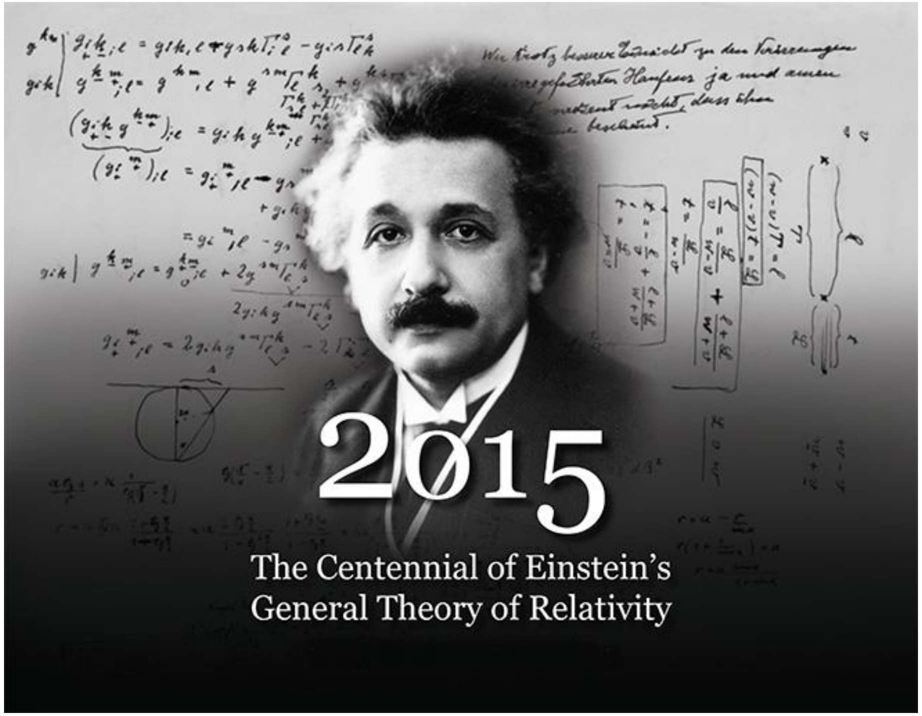
Knowledge of Physics: Standard Model



Why Modification?

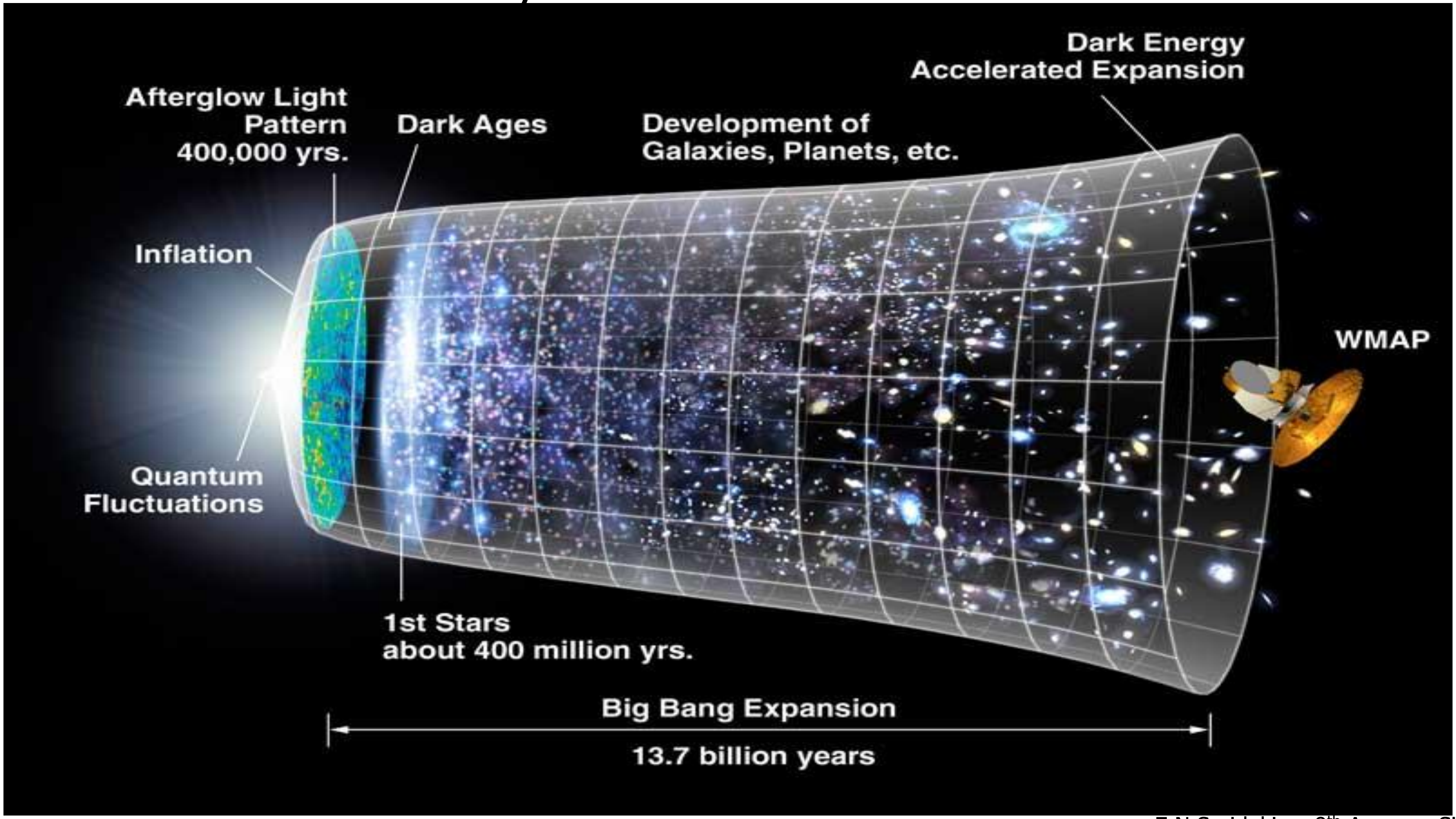
Knowledge of Physics: Standard Model + General Relativity



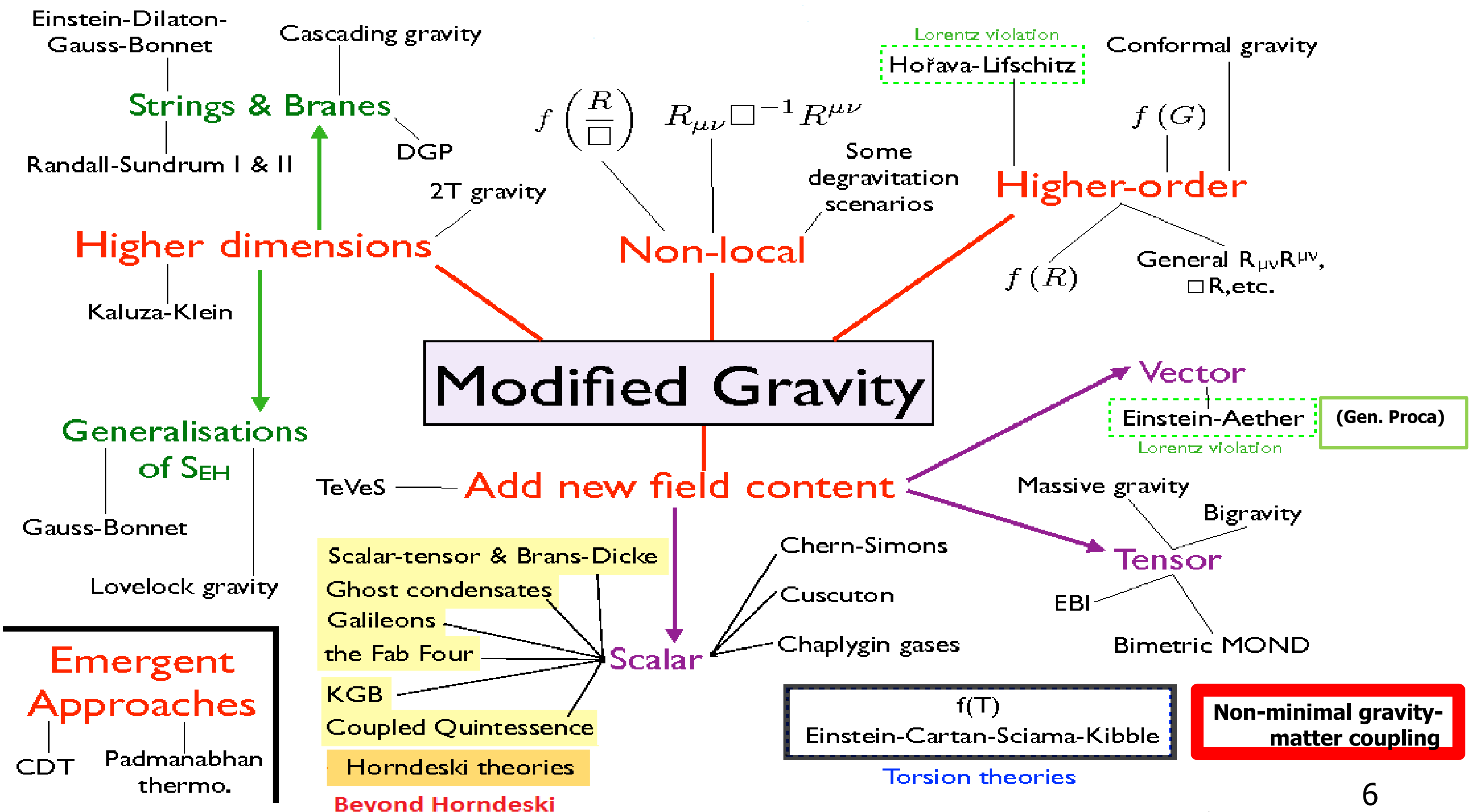


Why Modification?

Universe History:



Modified Gravity



Scalar-Tensor Theories

- Add a scalar field:

$$L=\frac{1}{16\pi}\sqrt{-g}\left[f(\phi)R-s(\phi)\nabla_{\mu}\phi\nabla^{\mu}\phi-2\Lambda(\phi)\right]+L_{m}\left(h(\phi)g_{\mu\nu},\psi\right)$$

Conformal Transf. to Jordan frame: $h(\phi)g_{\mu\nu}\rightarrow g_{\mu\nu}$

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Conformal Transf. to Jordan frame: $h(\phi)g_{\mu\nu}\rightarrow g_{\mu\nu}$

- Redefinition of ϕ :

$$L=\frac{1}{16\pi}\sqrt{-g}\left[\phi R-\frac{\omega(\phi)}{\phi}\nabla_{\mu}\phi\nabla^{\mu}\phi-2V(\phi)\right]+L_m\left(g_{\mu\nu},\psi\right)$$

- Brans-Dicke for $\omega\rightarrow const.,\ V\rightarrow 0$
- GR for $\omega\rightarrow\infty,\ \omega'/\omega^2\rightarrow 0,\ V\rightarrow const.$

[Brans,Dicke, PR 124] [Santos,Gregory, Annals Phys. 258]

Scalar-Tensor Theories

- Field equations:

$$\phi G_{\mu\nu} + \left[\Diamond\phi + \frac{\omega}{2\phi}(\nabla\phi)^2 + V \right] g_{\mu\nu} - \nabla_\mu \nabla_\nu \phi - \frac{\omega}{\phi}\nabla_\mu\phi\nabla_\nu\phi = 8\pi\, T_{\mu\nu}$$

$$(2\omega+3)\Box\phi + \omega'(\nabla\phi)^2 + 4V - 2\phi V' = 8\pi\, T$$

- For Brans-Dicke:

- PPN parameters: $\beta_{PPN} = 1,\ \gamma_{PPN} = \frac{1+\omega}{2+\omega} \Rightarrow \omega \geq 40000$

[D.F. Toress, PRD 66]

- Newton's constant: $G = \left(\frac{4+2\omega}{3+2\omega}\right)\frac{1}{\phi}$ with $\frac{\dot{G}}{G} \leq 1.7\ 10^{-12}\ yr^{-1}$

Brans-Dicke Cosmology

- Friedmann-Robertson-Walker metric: $ds^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j$
- Friedmann equations:

$$H^2 = \frac{8\pi}{3\phi}\rho_m - H\frac{\dot{\phi}}{\phi} + \frac{\omega}{6}\frac{\dot{\phi}^2}{\phi^2} + \frac{V}{3\phi}$$

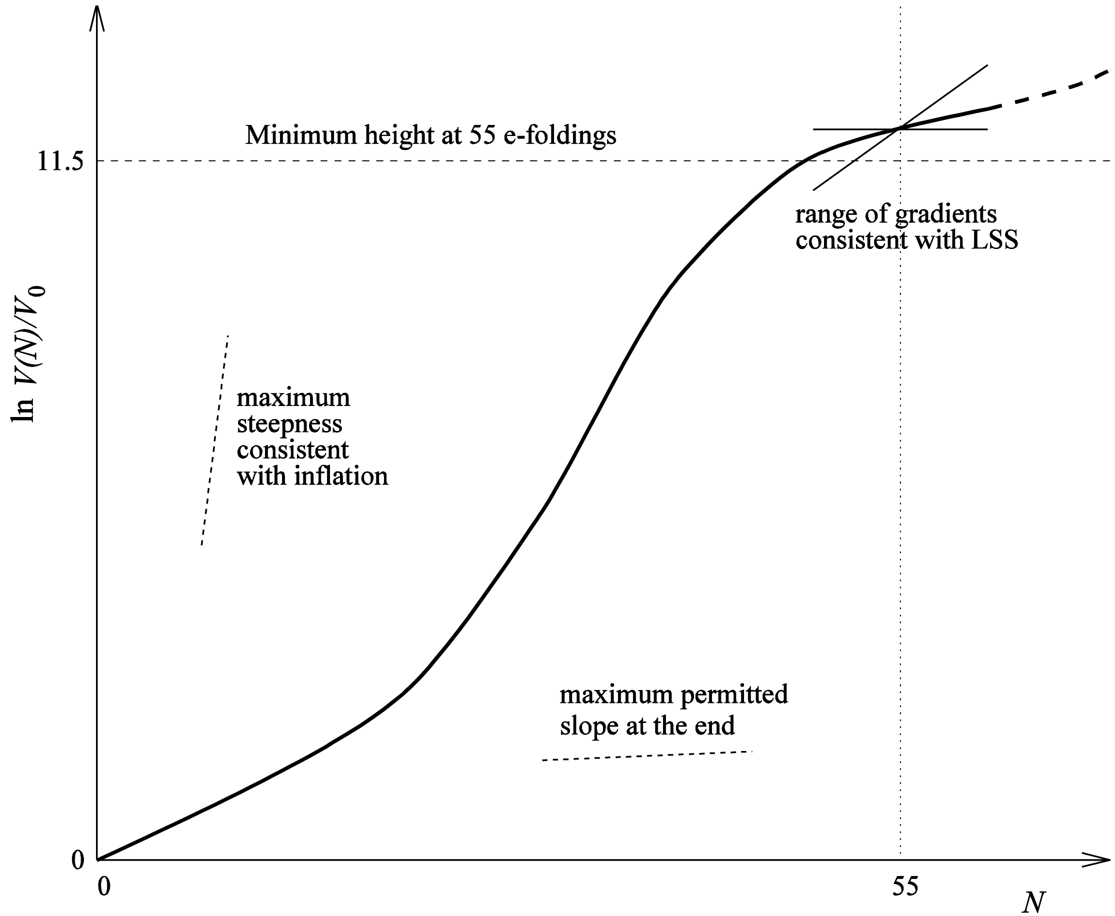
$$2\dot{H} + 3H^2 = -\frac{1}{\phi}\left(8\pi p_m + \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi} + 2H\dot{\phi} + \ddot{\phi}\right) + \frac{V}{\phi}$$

- Scalar-field equation:

$$\ddot{\phi} + 3H\dot{\phi} - \frac{8\pi}{2\omega+3}(\rho_m - 3p_m) = 0 + \frac{2}{2\omega+3}\left(2V - \phi\frac{dV}{d\phi}\right)$$

- Matter equation: $\dot{\rho}_m + 3H(\rho_m + p_m) = 0$

Inflation in Brans-Dicke Cosmology



[La,Steinhardt PRL 62], [Green, Liddle PRD 54]

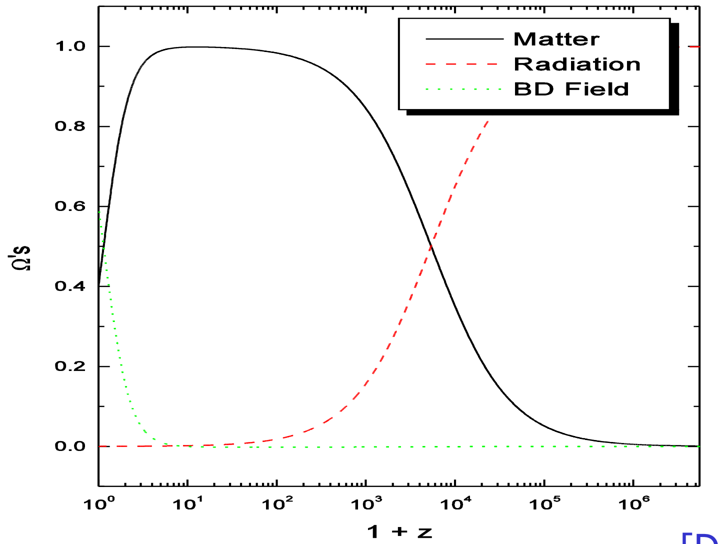
Dark Energy in Brans-Dicke Cosmology

- Effective Dark Energy sector:

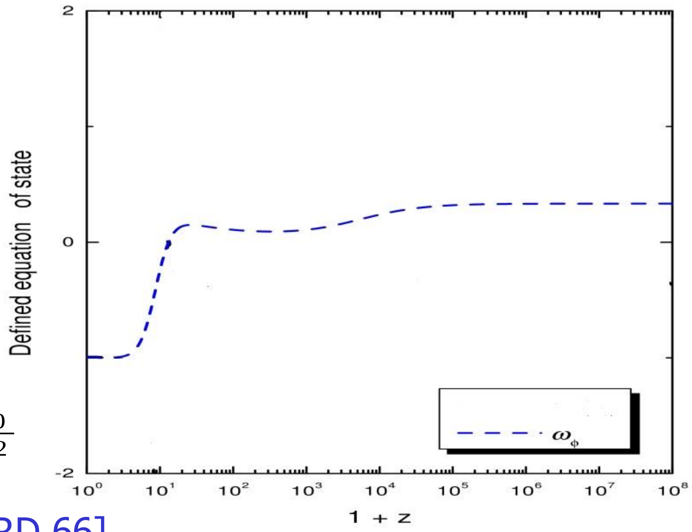
$$\rho_{DE} = \frac{3}{8\pi}\left(-H\dot{\phi} + \frac{\omega}{6}\frac{\dot{\phi}^2}{\phi}\right) + \frac{V}{8\pi}$$

$$p_{DE} = \frac{1}{8\pi}\left(\frac{\omega}{2}\frac{\dot{\phi}^2}{\phi} + 2H\dot{\phi} + \ddot{\phi}\right) - \frac{V}{8\pi}$$

$$\Rightarrow w_{DE} = \frac{p_{DE}}{\rho_{DE}}$$



$$V(\phi) = \frac{V_0}{\phi^2}$$



[D.F. Toress, PRD 66]

Horndeski Theories

- Most general 4D scalar-tensor theories having second-order field equations:

$$L_H = \sum_{i=2}^{5} L_i$$

$$L_2[K] = K(\phi, X)$$

$$L_3[G_3] = -G_3(\phi, X)\Diamond\phi$$

$$X = -\partial^\mu\phi\partial_\mu\phi/2$$

$$L_4[G_4] = G_4(\phi, X)R + G_{4,X}\left[(\Diamond\phi)^2 - (\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)\right]$$

$$L_5[G_5] = G_5(\phi, X)G_{\mu\nu}(\nabla^\mu\nabla^\nu\phi) - \frac{1}{6}G_{5,X}\left[(\Diamond\phi)^3 - 3(\Diamond\phi)(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi) + 2(\nabla^\mu\nabla_\alpha\phi)(\nabla^\alpha\nabla_\beta\phi)(\nabla^\beta\nabla_\mu\phi)\right]$$

[G. Horndeski, Int. J. Theor. Phys. 10]

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$$L_5[G_5] = G_5(\phi, X)G_{\mu\nu}(\nabla^\mu\nabla^\nu\phi) - \frac{1}{6}G_{5,X}\left[(\Diamond\phi)^3 - 3(\Diamond\phi)(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi) + 2(\nabla^\mu\nabla_\alpha\phi)(\nabla^\alpha\nabla_\beta\phi)(\nabla^\beta\nabla_\mu\phi)\right]$$

[G. Horndeski, Int. J. Theor. Phys. 10]

- Coincides with Generalized Galileon theories

$$\phi \to \phi + c, \ \ \partial_\mu\phi \to \partial_\mu\phi + b_\mu$$

[Nicolis, Rattazzi, Trincherini, PRD 79]

[Deffayet, Esposito-Farese, Vikman PRD 79]

Horndeski Cosmology (background)

- Field Equations: $L.H.S = R.H.S$
- In flat FRW:
- $$2XK_{,X} - K + 6X\dot{\phi}HG_{3,X} - 2XG_{3,\phi} - 6H^2G_4 + 24H^2X(G_{4,X} + XG_{4,XX}) - 12HX\dot{\phi}G_{4,\phi X} - 6H\dot{\phi}G_{4,\phi} + 2H^3X\dot{\phi}(5G_{5,X} + 2XG_{5,XX}) - 6H^2X(3G_{5,\phi} + 2XG_{5,\phi X}) = -\rho_m$$

- $$K - 2X(G_{3,\phi} + \ddot{\phi}G_{3,X}) + 2(3H^2 + 2\dot{H})G_4 - 12H^2XG_{4,X} - 4H\dot{X}G_{4,X} - 8\dot{H}XG_{4,X} - 8HX\dot{X}G_{4,XX} + 2(\ddot{\phi} + 2H\dot{\phi})G_{4,\phi} + 4XG_{4,\phi\phi} + 4X(\ddot{\phi} - 2H\dot{\phi})G_{4,\phi X} - 2X(2H^3\dot{\phi} + 2H\dot{H}\dot{\phi} + 3H^2\ddot{\phi})G_{5,X} - 4H^2X^2\ddot{\phi}G_{5,XX} + 4HX(\dot{X} - HX)G_{5,\phi X} + 2\left[2(\dot{H}X + H\dot{X}) + 3H^2X\right]G_{5,\phi} + 4HX\dot{\phi}G_{5,\phi\phi} = -p_m$$

- $$\frac{1}{a^3}\frac{d}{dt}(a^3J) = P_\phi$$

with
$$J = \dot{\phi}K_{,X} + 6HXG_{3,X} - 2\dot{\phi}G_{3,\phi} + 6H^2\dot{\phi}(G_{4,X} + 2XG_{4,XX}) - 12HXG_{4,\phi X} + 2H^3X(3G_{5,X} + 2XG_{5,XX}) - 6H^2\dot{\phi}(G_{5,\phi} + XG_{5,\phi X})$$
$$P_\phi = K_{,\phi} - 2X(G_{3,\phi\phi} + \ddot{\phi}G_{3,\phi X}) + 6(2H^2 + \dot{H})G_{4,\phi} + 6H(\dot{X} + 2HX)G_{4,\phi X} - 6H^2XG_{5,\phi\phi} + 2H^3X\dot{\phi}G_{5,\phi X}$$

[De Felice,Tsujikawa JCAP 1202]

Horndeski Cosmology (perturbations)

- Scalar perturbations: $ds^2 = -(1+2\psi)dt^2 + a^2(1-2\phi)\delta_{ij}dx^i dx^j$ $\quad \Rightarrow L.H.S = R.H.S$
- No-ghost condition: $Q_S \equiv \frac{w_1\left(4w_1w_3+9w_2^2\right)}{3w_2^2} > 0$
- No Laplacian instabilities condition: $c_S^2 \equiv \frac{3\left(2w_1^2w_2H - 4w_2^2w_4 + 4w_1w_2\dot{w}_1 - 2w_1^2\dot{w}_2\right) - 6w_1^2(\rho_m + p_m)}{w_1\left(4w_1w_3+9w_2^2\right)} > 0$

with $\mathrm{w}_1 \equiv 2\left(G_4 - 2XG_{4,X}\right) - 2X\left(G_{5,X}\dot{\phi}H - G_{5,\phi}\right)$

$$\mathrm{w}_2 \equiv -2G_{3,X}X\dot{\phi} + 4G_4H - 16X^2G_{4,XX}H + 4\left(\dot{\phi}G_{4,\phi X} - 4HG_{4,X}\right)X + 2G_{4,\phi}\dot{\phi}$$
$$+8X^2G_{5,\phi X}H + 2HX\left(6G_{5,\phi} - 5HG_{5,X}\dot{\phi}\right) - 4G_{5,XX}\dot{\phi}X^2H^2$$

$$\mathrm{w}_3 \equiv 3X\left(K_{,X} + 2XK_{,XX}\right) + 6X\left(3X\dot{\phi}HG_{3,XX} - G_{3,\phi X}X - G_{3,\phi} + 6\dot{\phi}HG_{3,X}\right)$$
$$+18H\left(4HX^3G_{4,XXX} - HG_4 - 5X\dot{\phi}G_{4,\phi X} - G_{4,\phi}\dot{\phi} + 7HG_{4,X}X + 16HX^2G_{4,XX} - 2X^2\dot{\phi}G_{4,X\phi X}\right)$$
$$+6H^2X\left(2H\dot{\phi}G_{5,XXX}X^2 - 6X^2G_{5,\phi XX} + 13XH\dot{\phi}G_{5,XX} - 27G_{5,\phi X}X + 15H\dot{\phi}G_{5,X} - 18G_{5,\phi}\right)$$

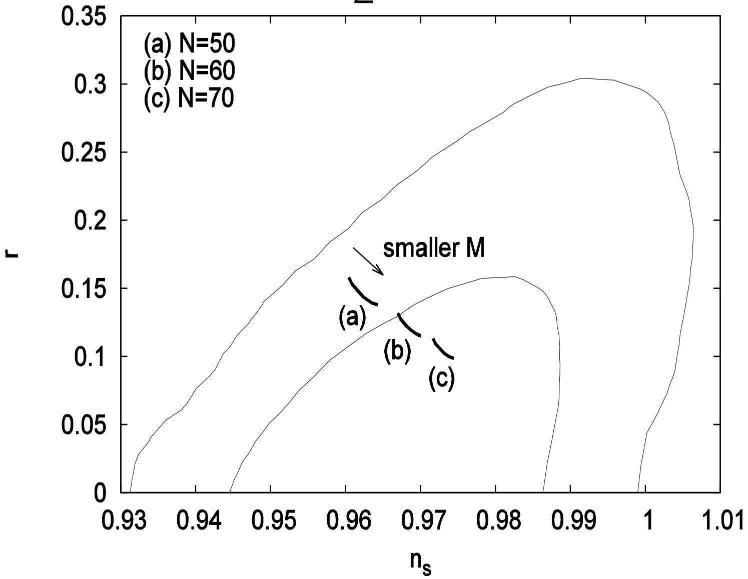
$$\mathrm{w}_4 \equiv 2G_4 - 2XG_{5,\phi} - 2XG_{5,X}\ddot{\phi}$$

[De Felice,Tsujikawa JCAP 1202]

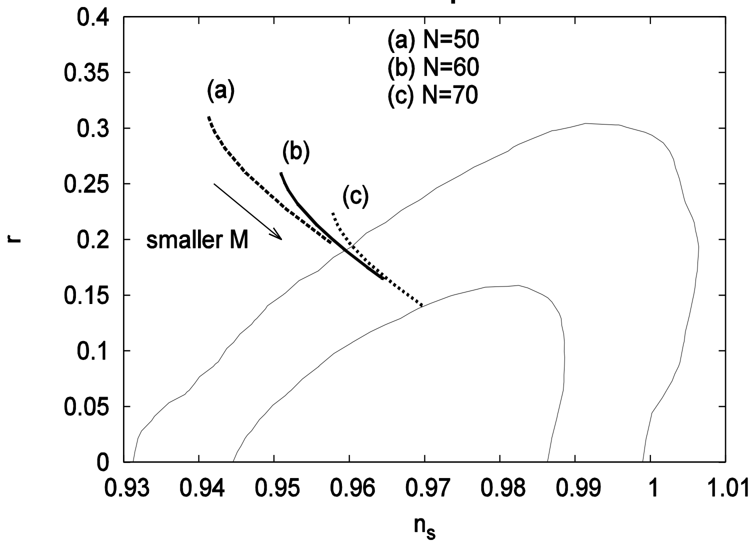
Inflation in Horndeski Theories

- $$K(\phi,X)=X-V(\phi),\ \ G_3(\phi,X)=\frac{c_3}{M^3}X,\ \ G_4=G_5=0$$ [Ohashi,Tsujikawa, JCAP 1210]

$$V(\phi)=\frac{1}{2}m^2\phi^2$$



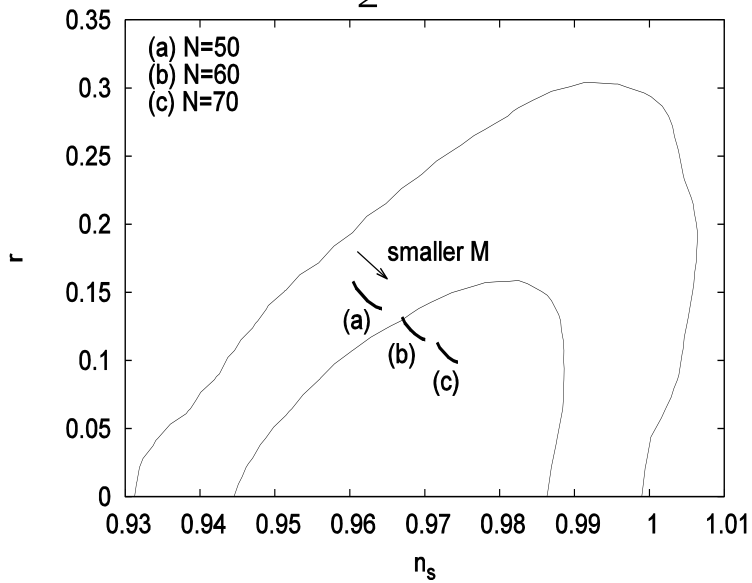
$$V(\phi)=\frac{1}{4}\lambda\phi^4$$



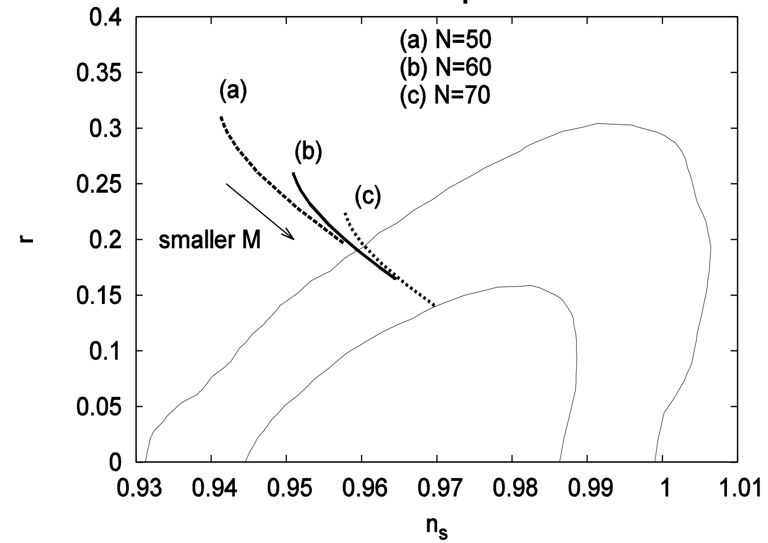
Inflation in Horndeski Theories

- $K(\phi, X) = X - V(\phi),\ G_3(\phi, X) = \frac{c_3}{M^3}X,\ G_4 = G_5 = 0$ [Ohashi,Tsujikawa, JCAP 1210]

$$V(\phi) = \frac{1}{2}m^2\phi^2$$



$$V(\phi) = \frac{1}{4}\lambda\phi^4$$



- G-Inflation (Shift-symmetric): $K(\phi, X) = X + \frac{X^2}{2M^3\mu},\ G_3(\phi, X) = \frac{1}{M^3}X,\ G_4 = G_5 = 0$

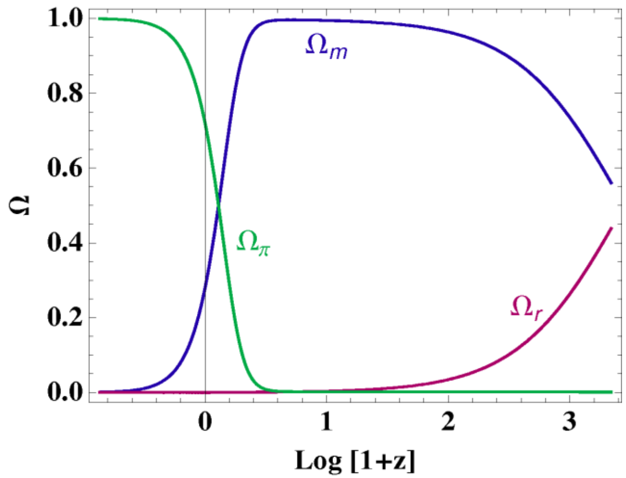
$r \approx 0.17$

[Kobayashi,Yamaguchi,Yokoyama PRL 105] [Banerjee, Saridakis PRD 95]

Dark Energy in Horndeski Theories

- $K(\phi, X) = c_2 X,\ G_3(\phi, X) = c_3,\ G_4 = 1,\ G_5 = c_5$

- Background evolution: Universe thermal history



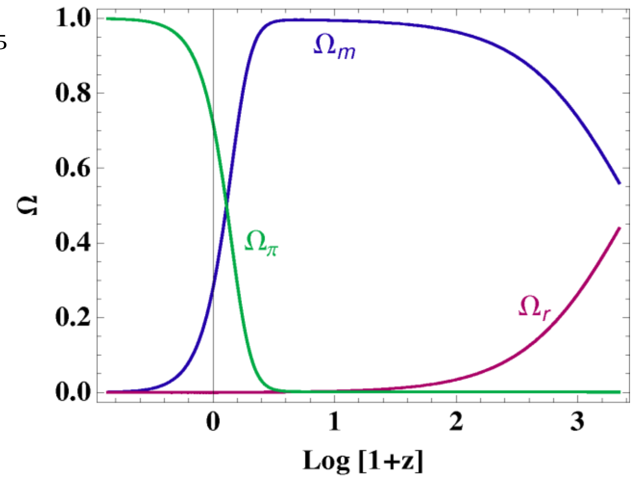
[Ali,Gannouji,Sami PRD 82]   [Leon, Saridakis JCAP 1303]

Dark Energy in Horndeski Theories

- $K(\phi, X) = c_2 X,\ G_3(\phi, X) = c_3,\ G_4 = 1,\ G_5 = c_5$

- Background evolution: Universe thermal history

[Leon, Saridakis JCAP 1303]



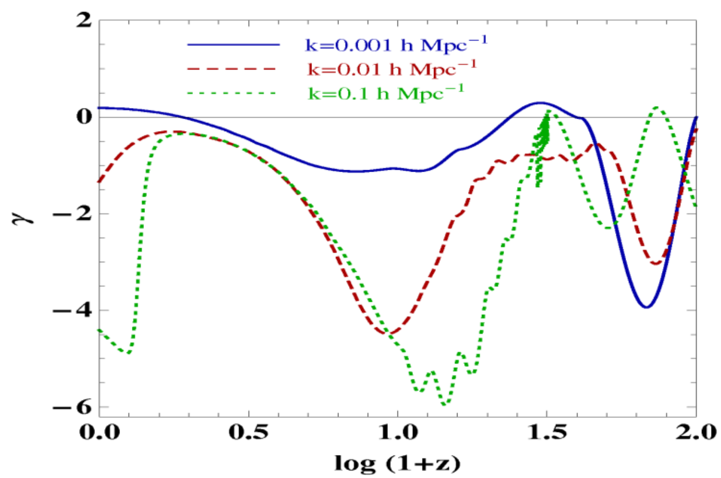
- Perturbations: $\ddot{\delta}_m + 2H\dot{\delta}_m = 4\pi G_{eff}\rho_m\delta_m$

  with $G_{eff} = G_{eff}(\phi, K, G_3, G_4, G_5)$

- Clustering growth rate: $\frac{d\ln\delta_m}{d\ln a} = \Omega_m^{\gamma}(a)$

  γ(z): Growth index.

[Ali,Gannouji,Sami PRD 82]



Fab Four

$$L_{FF} = L_{john} + L_{paul} + L_{george} + L_{ringo}$$

$$L_{john} = V_{john}(\phi) G^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi$$

$$L_{paul} = V_{paul}(\phi) P^{\mu\nu\alpha\beta} \nabla_\mu \phi \nabla_\alpha \phi \nabla_\nu \nabla_\beta \phi \qquad P^{\mu\nu}{}_{\alpha\beta} \equiv R^{\mu\nu}{}_{\alpha\beta} - 2R^{\mu}{}_{[\alpha}\delta^{\nu}{}_{\beta]} + 2R^{\nu}{}_{[\alpha}\delta^{\mu}{}_{\beta]} + R\delta^{\mu}{}_{[\alpha}\delta^{\nu}{}_{\beta]}$$

$$L_{george} = V_{george}(\phi) R$$

$$L_{ringo} = V_{ringo}(\phi) \hat{G} \qquad \hat{G} \equiv R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta} - 4R^{\mu\nu} R_{\mu\nu} + R^2$$

[Charmousis,Copeland,Padilla,Saffin PRL 108]

Case	cosmological behaviour	$V_j(\phi)$	$V_p(\phi)$	$V_g(\phi)$	$V_r(\phi)$
Stiff fluid	$H^2 \propto 1/a^6$	$c_1\phi^{\frac{4}{\alpha}-2}$	$c_2\phi^{\frac{6}{\alpha}-3}$	0	0
Radiation	$H^2 \propto 1/a^4$	$c_1\phi^{\frac{4}{\alpha}-2}$	0	$c_2\phi^{\frac{2}{\alpha}}$	$-\frac{\alpha^2}{8}c_1\phi^{\frac{4}{\alpha}}$
Curvature	$H^2 \propto 1/a^2$	0	0	0	$c_1\phi^{\frac{4}{\alpha}}$
Arbitrary	$H^2 \propto a^{2h}, \quad h \neq 0$	$c_1(1+h)\phi^{\frac{4}{\alpha}-2}$	0	0	$-\frac{\alpha^2}{16}h(3+h)c_1\phi^{\frac{4}{\alpha}}$

Case	cosmological behaviour	$V_j(\phi)$	$V_p(\phi)$	$V_g(\phi)$	$V_r(\phi)$
Matter I	$H^2 \propto 1/a^3$	$c_1\phi^{\hat{n}+4}$	$c_2\phi^{\hat{n}+6}$	0	$\frac{2\hat{n}-3}{16(2\hat{n}+7)(\hat{n}+6)}c_1\phi^{\hat{n}+6}$
Matter II	$H^2 \propto 1/a^3$	$c_1\phi^{\hat{n}+4}$	0	$c_2\phi^{\hat{n}+3}$	$-\frac{(\hat{n}+3)(2\hat{n}+5)}{8(2\hat{n}+7)(\hat{n}+6)}c_1\phi^{\hat{n}+6}$

[Copeland,Padilla,Saffin JCAP 1212]

Nonminimal Derivative Coupling

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R-\frac{1}{2}(g_{\mu\nu}-\zeta G_{\mu\nu})\partial^\mu\phi\partial^\nu\phi-V(\phi)\right]+S_m+S_r$$

- In flat FRW:

$$H^2=\frac{8\pi G}{3}\left[\frac{\dot{\phi}^2}{2}\left(1+9\zeta H^2\right)+V(\phi)+\rho_m+\rho_r\right]$$

$$2\dot{H}+3H^2=-8\pi G\left[\frac{\dot{\phi}^2}{2}\left[1-\zeta\left(2\dot{H}+3H^2+\frac{4H\ddot{\phi}}{\dot{\phi}}\right)\right]-V(\phi)+p_m+p_r\right]$$

[Saridakis,Suskov PRD 81]

Nonminimal Derivative Coupling – Dark Energy

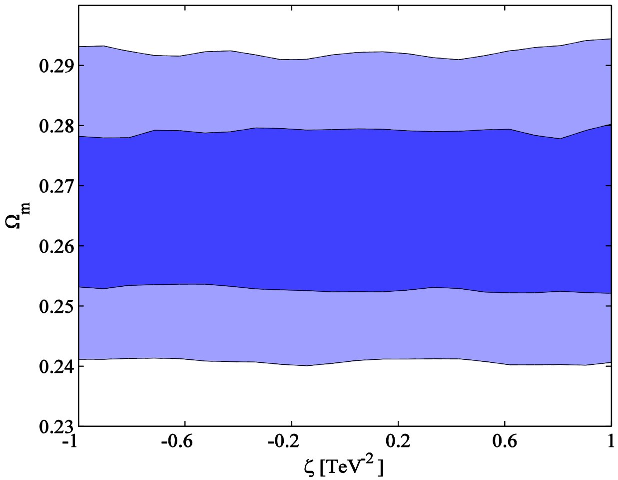
$$S=\int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R-\frac{1}{2}(g_{\mu\nu}-\zeta G_{\mu\nu})\partial^{\mu}\phi\partial^{\nu}\phi-V(\phi)\right]+S_m+S_r$$

- In flat FRW:

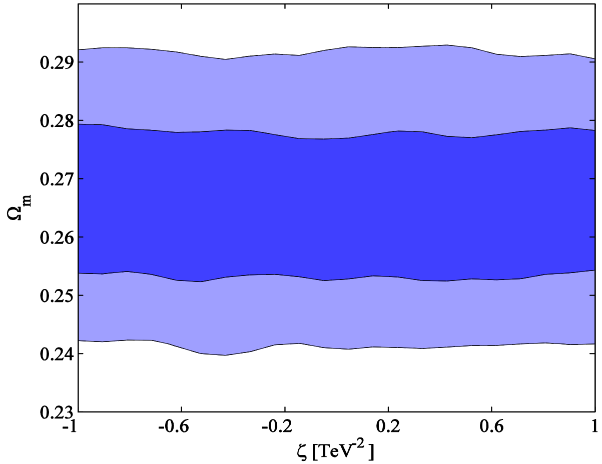
$$H^2=\frac{8\pi G}{3}\left[\frac{\dot\phi^2}{2}\left(1+9\zeta H^2\right)+V(\phi)+\rho_m+\rho_r\right]$$

$$2\dot H+3H^2=-8\pi G\left[\frac{\dot\phi^2}{2}\left[1-\zeta\left(2\dot H+3H^2+\frac{4H\ddot\phi}{\dot\phi}\right)\right]-V(\phi)+p_m+p_r\right]$$

[Saridakis,Suskov PRD 81]



$V(\phi)=V_0e^{\lambda\phi}$

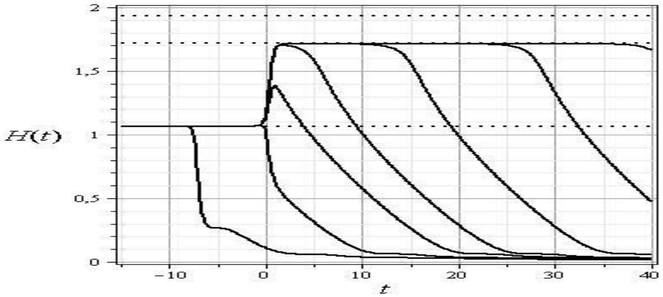


$V(\phi)=V_0\phi^n$

[Dent,Dutta,Saridakis,Xia JCAP 1311]

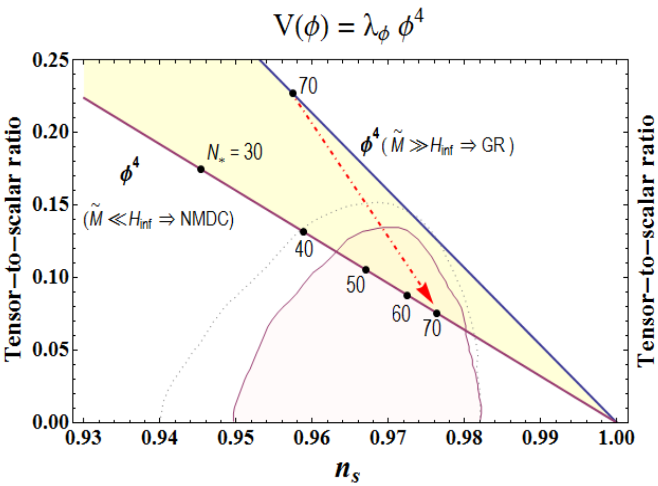
Nonminimal Derivative Coupling - Inflation

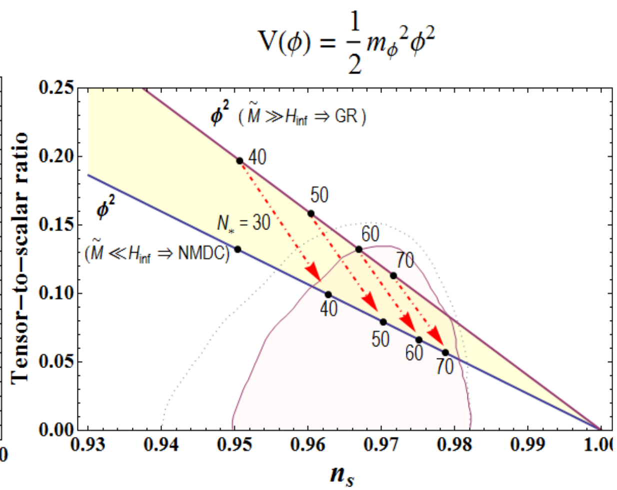
- New Higgs Inflation: $r \approx 0.05$ [Germani,Kehagias PRL 105]

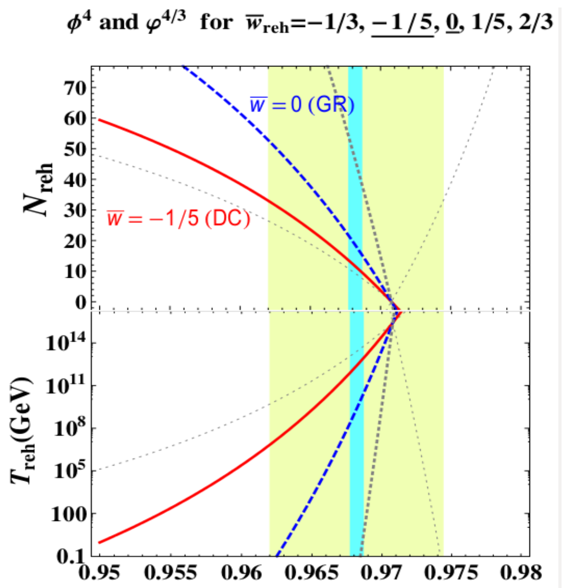


$$V(\phi) = V_0 \phi^2$$

[Skugoreva,Sushkov,Toporensky PRD 88]







[Dalianis,Koutsoumbas,Ntrekis,Papantonopoulos JCAP 1702]

Beyond Horndeski Theories

- Beyond Horndeski, free from Ostrogradski instabilities but still propagating 2+1 dof's:

$$L_{BH} = \sum_{i=2}^{5} L_i$$

$$X = -\partial^\mu \phi \partial_\mu \phi / 2$$

$$A_i = A_i(\phi, X)$$

$$B_i = B_i(\phi, X)$$

$$L_2 = L_2^H[A_2]$$

$$L_3 = L_3^H[C_3 + 2XC_{3,X}] + L_2^H[XC_{3,\phi}]$$

$$L_4 = L_4^H[B_4] + L_3^H[C_4 + 2XC_{4,X}] + L_2^H[XC_{4,\phi}] - \frac{B_4 + A_4 - 2XB_{4,X}}{X^2} L^{gal1}$$

$$L_5 = L_5^H[G_4] + L_4^H[C_5] + L_3^H[D_5 + 2XD_{5,X}] + L_2^H[XD_{5,\phi}] + \frac{XB_{5,X} + 3A_5}{3(-X)^{5/2}} L^{gal2}$$

with

$$L^{gal1} = X\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)\right] - 2\left[(\nabla^\mu\phi\nabla^\nu\phi)(\nabla_\mu\nabla_\nu\phi)(\Box\phi) - (\nabla^\mu\phi)(\nabla_\mu\nabla_\nu\phi)(\nabla_\lambda\phi)(\nabla^\lambda\nabla^\nu\phi)\right]$$

$$L^{gal2} = X\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi) + 2(\nabla_\mu\nabla_\nu\phi)(\nabla^\nu\nabla^\rho\phi)(\nabla^\mu\nabla_\rho\phi)\right]$$
$$-3\begin{bmatrix}(\Box\phi)^2(\nabla_\mu\phi)(\nabla^\mu\nabla^\nu\phi)(\nabla_\nu\phi) - 2(\Box\phi)(\nabla_\mu\phi)(\nabla^\mu\nabla^\nu\phi)(\nabla_\nu\nabla_\rho\phi)(\nabla^\rho\phi) \\ -(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)(\nabla_\rho\phi)(\nabla^\rho\nabla^\lambda\phi)(\nabla_\lambda\phi) + 2(\nabla_\mu\phi)(\nabla^\mu\nabla^\nu\phi)(\nabla_\nu\nabla_\rho\phi)(\nabla^\rho\nabla^\lambda\phi)(\nabla_\lambda\phi)\end{bmatrix}$$

$$C_3 = \frac{1}{2}\int A_3(-X)^{-3/2}dX \qquad C_5 = -\frac{1}{4}X\int B_{5,\phi}(-X)^{-3/2}dX$$

$$C_4 = -\int B_{4,\phi}(-X)^{-1/2}dX \qquad D_5 = -\int C_{5,\phi}(-X)^{-1/2}dX \qquad G_5 = -\int B_{5,X}(-X)^{-1/2}dX$$

- Primary constraint prevents the propagation of extra degrees of freedom

[Gleyzes,Langlois,Piazza,Vernizzi, PRL 114], [Crisostomi,Hull,Koyama,Tasinato, JCAP 1603]

Bi-scalar Theories

- Modified gravity propagating 2+2 dof's $S=\int d^4x\sqrt{-g}\,f\left(R,(\nabla R)^2,\Diamond R\right)$

- For $f\left(R,(\nabla R)^2,\Diamond R\right)=K\left(R,(\nabla R)^2\right)+Q\left(R,(\nabla R)^2\right)\Diamond R$

[Naruko,Yoshida,Mukohyama CQG 33]

$$\Rightarrow S=\int d^4x\sqrt{-g}\left[\frac{1}{2}\hat{R}-\frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi-\frac{1}{\sqrt{6}}e^{-\sqrt{2/3}\chi}\hat{g}^{\mu\nu}Q\partial_\mu\chi\partial_\nu\phi+\frac{1}{4}e^{-2\sqrt{2/3}\chi}K+\frac{1}{2}e^{-\sqrt{2/3}\chi}Q\hat{\Diamond}\phi-\frac{1}{4}e^{-\sqrt{2/3}\chi}\phi\right]$$

$$K=K(\phi,B),\ G=G(\phi,B),\ B=2e^{\sqrt{2/3}\chi}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$$

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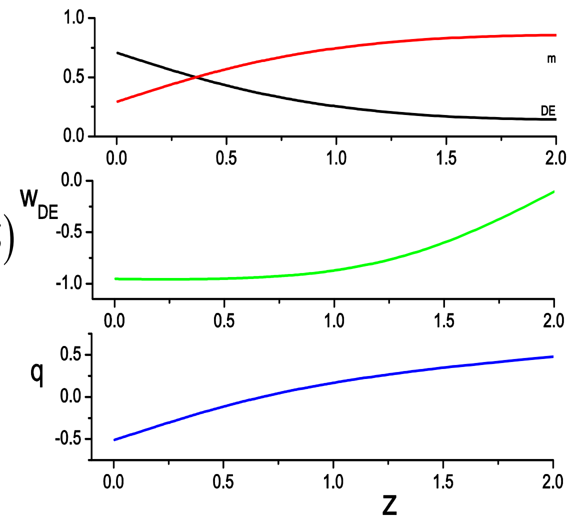
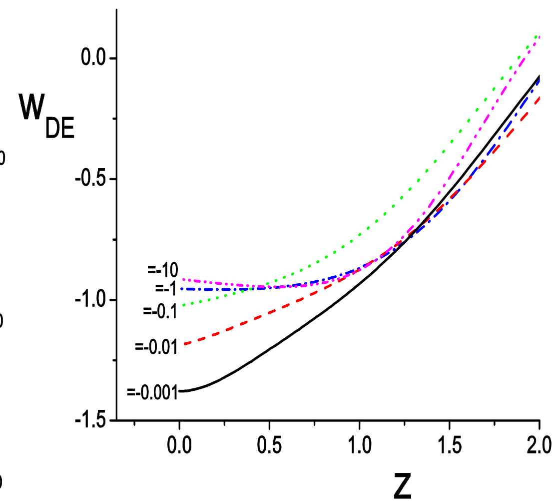
$$K=K(\phi,B),\ \ G=G(\phi,B),\ \ B=2e^{\sqrt{2/3}\chi}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$$

- eg.: $K(\phi,B)=\frac{\phi}{2},\ \ G(\phi,B)=\xi B$

$$\rho_{DE}=\frac{1}{2}\dot{\chi}^2-\frac{1}{8}e^{-2\sqrt{2/3}\chi}\left(1-2e^{\sqrt{2/3}\chi}\right)\phi-\xi\dot{\phi}^3\left(\sqrt{6}\dot{\chi}-6H\right)$$

$$p_{DE}=\frac{1}{2}\dot{\chi}^2+\frac{1}{8}e^{-2\sqrt{2/3}\chi}\left(1-2e^{\sqrt{2/3}\chi}\right)\phi-\frac{1}{3}\xi\dot{\phi}^2\left(\sqrt{6}\dot{\phi}\dot{\chi}+6\ddot{\phi}\right)$$





[Saridakis,Tsoukalas PRD 93]

Dark Matter – Dark Energy Interaction

- Theoretical argument: In principle, since the underlying theory and the microphysics of both dark energy and dark matter is unknown, possible mutual interactions cannot be excluded.

Dark Matter – Dark Energy Interaction

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- Phenomenological argument: Alleviate the coincidence problem: Why are the DE and DM densities nearly equal today, although they scale independently through the expansion history

[Billyard, Coley, PRD 61]
[Wang, Gong, Abdalla, PLB 624]
[Caldera-Cabral, Maartens, Urena-Lopez, PRD 79]
[Mimoso, Nunes, Pavon, PRD 73]
[Chen, Gong, Saridakis JCAP 0904]
[Clifton, Barrow, PRD 73]

DM – DE Interaction

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} R \right] + S_\phi + S_{DM} + S_b$$

- Assume that DE and DM are effectively described by perfect fluids.

$$H^2 = \frac{8\pi G}{3} (\rho_{DE} + \rho_{DM})$$

$$\dot{H} = -4\pi G (\rho_{DE} + p_{DE} + \rho_{DM} + p_{DM})$$

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- Equations give only the total conservation, namely

$$\nabla^b T_{ab}^{(tot)} = \nabla^b \left[T_{ab}^{(DE)} + T_{ab}^{(DM)}\right] = 0$$

- If we assume DM conservation, i.e $\nabla^b T_{ab}^{(DM)} = 0$ then DE is also conserved: $\nabla^b T_{ab}^{(DE)} = 0$

$$\Rightarrow \dot{\rho}_{DM} + 3H(\rho_{DM} + p_{DM}) = 0$$

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DM – DE Interaction

- However, it is not forbidden to assume DM – DE interaction by arbitrarily splitting as:

$$\nabla^b T_{ab}^{(DM)} = Q_a$$

$$\nabla^b T_{ab}^{(DE)} = -Q_a$$

with Q_a a phenomenological descriptor of the interaction (positive Q_a corresponds to energy transfer from DE to DM and vice versa).

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with Q_a a phenomenological descriptor of the interaction (positive corresponds to energy transfer from DE to DM and vice versa).

- Despite possible pathologies (curvature perturbation blowing up in super-Hubble scales [Valiviita,Majerotto,Maartens, JCAP 0807]) it leads to interesting cosmological behavior.

Phenomenological Models

- I) $Q = Q_0 = 3H\left(\alpha_{DE}\rho_{DE} + \alpha_{DM}\rho_{DM}\right)$
- II) $Q = Q_0 = \Gamma\rho_{DM}$
- III) $Q = Q_0 = \alpha\kappa^{2n}H^{3-2n}\rho^{n}_{DM}$
- etc…

Phenomenological Models

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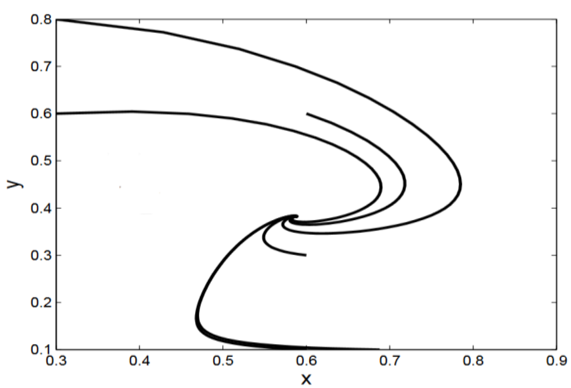
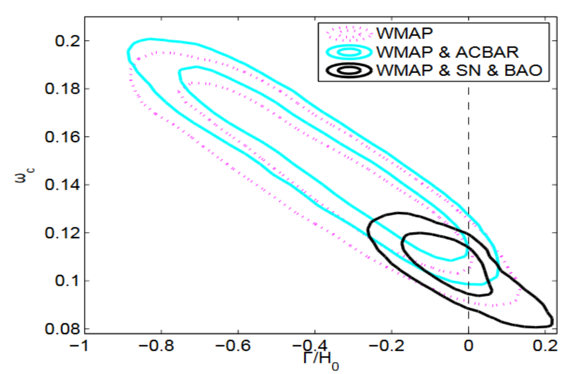
- II) $Q = Q_0 = \Gamma\rho_{DM}$
- III) $Q = Q_0 = \alpha\kappa^{2n}H^{3-2n}\rho_{DM}^{n}$
- etc…
- Obtain late time attractors with $R \equiv \rho_{DE}/\rho_{DM} \sim 1$
- 



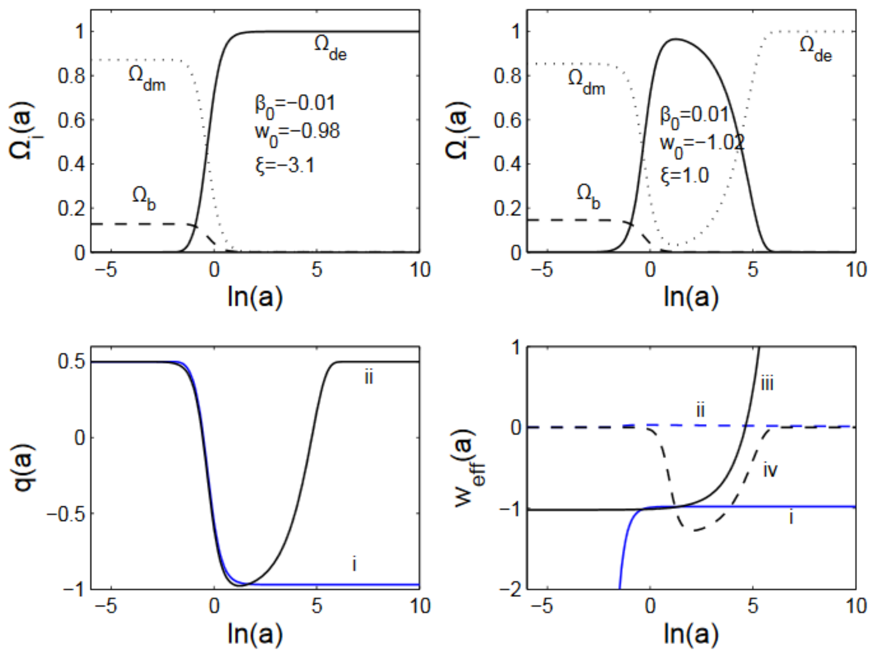
[Chen, Gong, Saridakis JCAP 0904]

[Valiviita,Majerotto,Maartens, MNRAS 402]

[Caldera-Cabral, Maartens, Urena-Lopez, PRD 79]

More general phenomenological models

- $Q = 3H\beta(a)\rho_{DE}$ with $\beta(a) = \beta_0 a^{\xi}$. $\Rightarrow \rho_{DE}(a)$ known



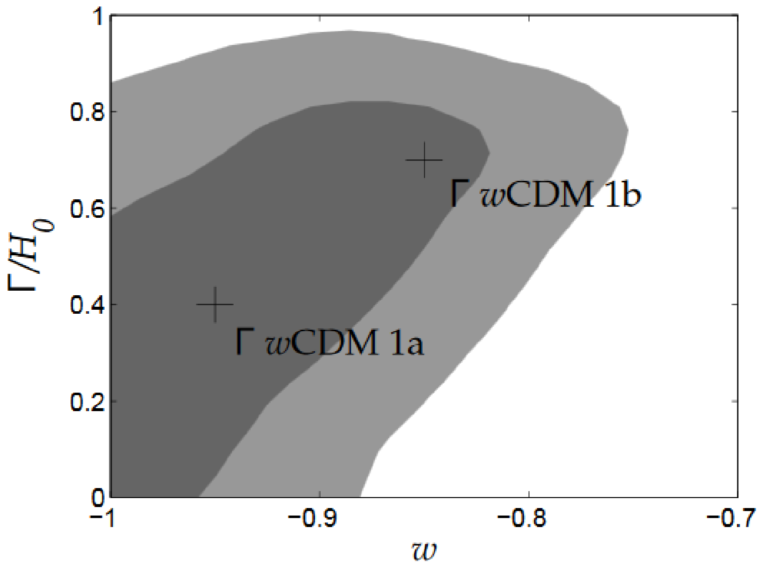
- Solve coincidence problem, can lead to intermediate acceleration

  [Chen, Gong, Saridakis JCAP 0904]

Observational constraints

- Impose SNIa, BAO and CMB observational constraints

Model	Q^μ_A	$\Delta\chi^2$	Γ/H_0	w	H_0	$\Omega_b h^2$	$\Omega_c h^2$	n_s	A_s	τ_{rei}
ΛCDM best-fit	-	0	-	-1	69.8	0.0223	0.113	0.960	2.16×10^{-9}	0.0844
ΛCDM69	-	0.774	-	-1	69.0	0.0221	0.114	0.958	2.18×10^{-9}	0.0855
ΛCDM70	-	-0.0200	-	-1	70.0	0.0224	0.112	0.962	2.16×10^{-9}	0.0844
wCDM best-fit	-	-0.220	-	-1.03	70.7	0.0222	0.113	0.960	2.18×10^{-9}	0.0883
ΓwCDM A	$Q\|u_c$	-	0	-0.98	70.0	0.0226	0.112	0.960	2.10×10^{-9}	0.0900
ΓwCDM B	$Q\|u_c$	-	0.2	-0.98	70.0	0.0226	0.112	0.960	2.10×10^{-9}	0.0900
ΓwCDM C	$Q\|u_c$	-	0.4	-0.98	70.0	0.0226	0.112	0.960	2.10×10^{-9}	0.0900
ΓwCDM 1a	$Q\|u_c$	-0.00830	0.4	-0.95	70.9	0.0222	0.0702	0.961	2.16×10^{-9}	0.0816
ΓwCDM 1b	$Q\|u_c$	0.702	0.7	-0.85	70.0	0.0223	0.0311	0.963	2.15×10^{-9}	0.0832
ΓwCDM 2a	$Q\|u_x$	-0.236	0.4	-0.95	71.0	0.0224	0.0701	0.966	2.19×10^{-9}	0.0870
ΓwCDM 2b	$Q\|u_x$	-0.0420	0.7	-0.85	70.2	0.0224	0.0305	0.966	2.15×10^{-9}	0.0819
$\Gamma \geq 0, w \geq -1$ best-fit	$Q\|u_c$	-0.0522	0.366	-0.964	71.0	0.0224	0.0748	0.963	2.18×10^{-9}	0.0849
$\Gamma \geq 0, w \geq -1$ best-fit	$Q\|u_x$	-0.322	0.798	-0.851	70.4	0.0224	0.0194	0.965	2.18×10^{-9}	0.0870



[Clemson, Koyama, Zhao, Maartens, Valiviita PRD 85]

- Incorporate relativistic effects in the large-scale power spectrum.

[Duniya, Bertacca, Maartens, PRD 91]

Another approach to phenomenological models

- If Q=0 then $\rho_{DM}=\rho_{DM0}/a^{3}$. Instead of imposing Q one can parametrize its effect assuming:

$\rho_{DM}=\rho_{DM0}/a^{3-\delta}$ (perturbations can also be studied; obtain matter overdensity) [Wang, Meng CQG 22]

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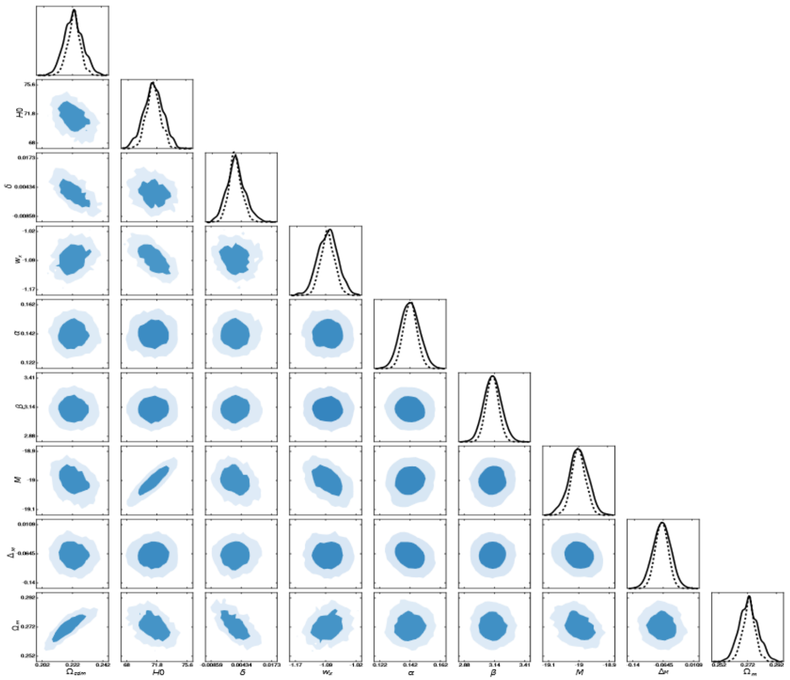
Param.	best-fit	mean$\pm\sigma$	95% lower	95% upper
Ω_{cdm0}	0.2246	$0.2229^{+0.0063}_{-0.0069}$	0.2099	0.2365
H_0	71.17	$71.37^{+1.3}_{-1.3}$	68.67	74.01
δ	0.00099	$0.00196^{+0.0038}_{-0.0046}$	−0.00631	0.01085
w	−1.085	$-1.087^{+0.027}_{-0.028}$	−1.139	−1.032
α	0.143	$0.1422^{+0.0065}_{-0.007}$	0.1291	0.1556
β	3.117	$3.126^{+0.079}_{-0.083}$	2.966	3.29
M	−19.04	$-19.04^{+0.041}_{-0.037}$	−19.12	−18.96
Δ_M	−0.0721	$-0.0680^{+0.024}_{-0.023}$	−0.116	−0.0211
Ω_{m0}	0.2746	$0.2729^{+0.0063}_{-0.0069}$	0.2599	0.2865

H0+SNIa+BAO+CMB



- Slight tendency towards interacting DE

δ<0 implies energy flow DM -> DE

[Nunes, Pan, Saridakis PRD 94]

Lagrangian? Covariant formulation?

- Microscopic Lagrangian of DM-DE interaction is unknown. Effective Lagrangians are also absent.

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- Two interacting fluids:

$$\dot{\rho}_1 + 3H(\rho_1 + p_1) = Q$$
$$\dot{\rho}_2 + 3H(\rho_2 + p_2) = -Q$$

- Covariant approach (two “not-tilted” fluids, i.e with common 4-velocity):

$$T_{ab}^{(1)} = (p_1 + \rho_1)u_a u_b + p_1 g_{ab} + q_a u_b + q_b u_a$$
$$T_{ab}^{(2)} = (p_2 + \rho_2)u_a u_b + p_2 g_{ab} - q_a u_b - q_b u_a$$

[Faraoni, Dent Saridakis PRD 90]

- $q^c = \alpha(t) u^c$ is a current energy density that describes the energy transfer between the fluids (time dependent due to spacial isotropy)

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- $q^c = \alpha(t) u^c$ is a current energy density that describes the energy transfer between the fluids (time dependent due to spacial isotropy)
- Imperfect fluids with $T^{(i)} = -\rho_i + 3p_i \mp 2\alpha$

$$\nabla^b T_{ab}^{(i)} = u_a u^b \nabla_b p_i + u_a u_b \nabla^b(\rho_i \pm 2\alpha) + \nabla_a p_i + (p_i + \rho_i \pm 2\alpha) u^b \nabla_b u_a + (p_i + \rho_i \pm 2\alpha) u_a \nabla^b u_b$$

- Hence, not a robust Lagrangian description for imperfect fluids

[Faraoni, Dent Saridakis PRD 90]

Lagrangian? Covariant formulation?

- Inspired by the Lagrangian formulation of classical dissipative oscillator we can remove the "imperfectness" by transforming the metric as:

$$\bar{g}_{ab} = g_{ab} + 2\lambda\alpha u_a u_b$$

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- Hence: $T_{ab} = (p + \rho - 2\lambda\alpha p + 2\alpha) u_a u_b + p\bar{g}_{ab}$
- Describes a perfect fluid with $\bar{\rho} = \rho - 2\lambda\alpha p + 2\alpha$ and $\bar{p} = p$ in spacetime metric $\bar{g}_{ab}$

$$\overline{\nabla}^b T_{ab} = 0$$

- $\Rightarrow L = \sqrt{-\bar{g}}\ p$: Lagrangian description in a fictitious metric that depends on the fluid
- Still not ideal for multiple fluids.

[Faraoni, Dent Saridakis PRD 90]

Another approach to phenomenological models

- Matter fluid: $L_M = -\sqrt{-g}\,\rho(n,s) + J^{\mu}\left(\varphi_{,\mu} + s\theta_{,\mu} + \beta_A \alpha^{A}_{,\mu}\right)$

  φ, θ, β_A are Lagrange multipliers, and α_A are the Lagrange coordinates of the fluid

  J^{μ} vector-density particle-number flux

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- DM-DE interaction:

  Algebraic coupling: $L_M + L_{\text{int}} = -\sqrt{-g}\rho(n,s,\phi) + J^{\mu}\left(\varphi_{,\mu} + s\theta_{,\mu} + \beta_A \alpha^A_{,\mu}\right)$

  Derivative Coupling: $L_M + L_{\text{int}} = -\sqrt{-g}\rho(n,s) + f(n,s,\phi)J^{\mu}\partial_{\mu}\phi + J^{\mu}\left(\varphi_{,\mu} + s\theta_{,\mu} + \beta_A \alpha^A_{,\mu}\right)$

  $\Rightarrow \nabla^{\mu}T^{(\phi)}_{\mu\nu} = Q_{\nu} = -\nabla^{\mu}T^{(dm)}_{\mu\nu}$

- Al. coupl.: $Q_{\nu} = \frac{\partial\rho(n,\phi)}{\partial\phi}\partial_{\nu}\phi$

- Der. Coupl.: $Q_{\nu} = -n^2\frac{\partial f(n,\phi)}{\partial n}\nabla_{\lambda}u^{\lambda}\partial_{\nu}\phi$

[Koivisto, Saridakis, Tamanini JCAP 1509]

- Perturbations, structure formation, quasi-static limit etc

Dark energy - dark matter interaction/unification from generalized Galileons

- Most general 4D scalar-tensor theories having second-order field equations:

$$L_H = \sum_{i=2}^{5} L_i$$

$$L_2[K] = K(\phi, X)$$

$$L_3[G_3] = -G_3(\phi, X)\Diamond\phi$$

$$X = -\partial^\mu\phi\partial_\mu\phi/2$$

$$L_4[G_4] = G_4(\phi, X)R + G_{4,X}\left[(\Diamond\phi)^2 - (\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)\right]$$

$$L_5[G_5] = G_5(\phi, X)G_{\mu\nu}(\nabla^\mu\nabla^\nu\phi) - \frac{1}{6}G_{5,X}\left[(\Diamond\phi)^3 - 3(\Diamond\phi)(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi) + 2(\nabla^\mu\nabla_\alpha\phi)(\nabla^\alpha\nabla_\beta\phi)(\nabla^\beta\nabla_\mu\phi)\right]$$

[G. Horndeski, Int. J. Theor. Phys. 10]

- Coincides with Generalized Galileon theories

$$\phi \rightarrow \phi + c, \;\; \partial_\mu\phi \rightarrow \partial_\mu\phi + b_\mu$$

[Nicolis,Rattazzi,Trincherini, PRD 79]

[Deffayet, Esposito-Farese, Vikman PRD 79]

Dark energy - dark matter interaction/unification from generalized Galileons

- Field Equations In flat FRW:

- $$2XK_{,X}-K+6X\dot{\phi}HG_{3,X}-2XG_{3,\phi}-6H^2G_4+24H^2X(G_{4,X}+XG_{4,XX})-12HX\dot{\phi}G_{4,\phi X}-6H\dot{\phi}G_{4,\phi}+2H^3X\dot{\phi}(5G_{5,X}+2XG_{5,XX})-6H^2X(3G_{5,\phi}+2XG_{5,\phi X})=-\rho_m$$

- $$K-2X(G_{3,\phi}+\ddot{\phi}G_{3,X})+2(3H^2+2\dot{H})G_4-12H^2XG_{4,X}-4H\dot{X}G_{4,X}-8\dot{H}XG_{4,X}-8HX\dot{X}G_{4,XX}+2(\ddot{\phi}+2H\dot{\phi})G_{4,\phi}+4XG_{4,\phi\phi}+4X(\ddot{\phi}-2H\dot{\phi})G_{4,\phi X}-2X(2H^3\dot{\phi}+2H\dot{H}\dot{\phi}+3H^2\ddot{\phi})G_{5,X}-4H^2X^2\ddot{\phi}G_{5,XX}+4HX(\dot{X}-HX)G_{5,\phi X}+2\left[2(\dot{H}X+H\dot{X})+3H^2X\right]G_{5,\phi}+4HX\dot{\phi}G_{5,\phi\phi}=-p_m$$

- $$\frac{1}{a^3}\frac{d}{dt}(a^3J)=P_\phi$$

with
$$J=\dot{\phi}K_{,X}+6HXG_{3,X}-2\dot{\phi}G_{3,\phi}+6H^2\dot{\phi}(G_{4,X}+2XG_{4,XX})-12HXG_{4,\phi X}+2H^3X(3G_{5,X}+2XG_{5,XX})-6H^2\dot{\phi}(G_{5,\phi}+XG_{5,\phi X})$$
$$P_\phi=K_{,\phi}-2X(G_{3,\phi\phi}+\ddot{\phi}G_{3,\phi X})+6(2H^2+\dot{H})G_{4,\phi}+6H(\dot{X}+2HX)G_{4,\phi X}-6H^2XG_{5,\phi\phi}+2H^3X\dot{\phi}G_{5,\phi X}$$

[De Felice,Tsujikawa JCAP 1202]

Dark energy - dark matter interaction/unification from generalized Galileons

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}R+\eta X+\eta_2 X^2+\lambda_3\phi\Diamond\phi+\frac{\lambda_5}{2}G_{\mu\nu}\partial^\mu\phi\partial^\nu\phi\right]$$

- In flat FRW:

$$3H^2-\eta X-3\eta_2 X^2+\lambda_3 X-9\lambda_5 H^2 X=0$$

$$2\dot{H}+3H^2+\eta X+\eta_2 X^2-\lambda_3 X-\lambda_5\left[\left(2\dot{H}+3H^2\right)X+2H\dot{X}\right]=0$$

$$(\eta-\lambda_3)\left(3HX+\frac{\dot{X}}{2}\right)+\eta_2\left(6HX\dot{H}+3X\dot{X}\right)2+\lambda_5\left(9H^3X+6HX^2+\frac{3}{2}X\dot{X}\right)=0$$

[Koutsoumbas,Ntrekis,Papantonopoulos,Saridakis, 1704.08640]

Dark energy - dark matter interaction/unification from generalized Galileons

- We can rewrite as:

$$H^2 = \frac{8\pi G}{3}\rho$$

$$\dot{H} = -4\pi G(\rho + p)$$

with

$$\rho = \eta X + 3\eta_2 X^2 - \lambda_3 X + 9\lambda_5 H^2 X$$

$$p = \eta X + \eta_2 X^2 - \lambda_3 X - \lambda_5\left[\left(2\dot{H} + 3H^2\right)X + 2H\dot{X}\right]$$

- Klein-Gordon becomes:

$$\dot{\rho} + 3H(\rho + p) = 0$$

- Define Equation-of-State parameter: $w = p/\rho$

[Koutsoumbas,Ntrekis,Papantonopoulos,Saridakis, 1704.08640]

Dark energy - dark matter interaction/unification from generalized Galileons

- Shift symmetry allows to write:

$$p(\rho) = -\frac{12\eta_2\lambda^2 f(\rho) - \left[2\lambda(7\eta_2 + 2\lambda\lambda_5) + 6\eta_2\lambda_5\rho\right] f^2(\rho) + (2\eta_2 - 3\lambda^2\rho) f^3(\rho)}{72\eta_2^2\lambda + 36\eta_2(-2\eta_2 + \lambda\lambda_5 f(\rho)) + 6\lambda_5(-7\eta_2 + 2\lambda\lambda_5) f^2(\rho) - 9\lambda_5^2 f^3(\rho)}$$

with $f(\rho) = \lambda + 3\lambda_5\rho + \sqrt{12\eta_2\rho + (\lambda + 3\lambda_5\rho)^2}$ and $\lambda = \eta - \lambda_3$

$$w = p/\rho$$

[Koutsoumbas,Ntrekis,Papantonopoulos,Saridakis, 1704.08640]

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$$w=p/\rho$$

- Allows for a unified description of universe evolution.
- (Generalized) Chaplygin gas:

$$p=-A/\rho^{\beta}$$

$$\dot{\rho}+3H(\rho+p)=0\Rightarrow \quad \rho=\rho_0\left[A_0+\frac{1-A_0}{a^{3(1+\beta)}}\right]^{1/1+\beta}$$

$$p=-\rho_0 A_0\left[A_0+\frac{1-A_0}{a^{3(1+\beta)}}\right]^{-\beta/1+\beta}$$

$$z=-1+\frac{a_0}{a}$$

[Koutsoumbas,Ntrekis,Papantonopoulos,Saridakis, 1704.08640]

Dark energy - dark matter interaction/unification from generalized Galileons

- Simplest case: $\lambda \neq 0,\ \eta_2 = \lambda_5 = 0 \qquad \Rightarrow w = 1$
- Model I : $\boxed{\lambda \neq 0,\ \eta_2 \neq 0,\ \lambda_5 = 0}$

$$\Rightarrow w(z) = \frac{\lambda + \frac{1}{6}\left(-\lambda + \sqrt{\lambda^2 + 36\eta_2 H^2(z)}\right)}{\lambda + \frac{1}{2}\left(-\lambda + \sqrt{\lambda^2 + 36\eta_2 H^2(z)}\right)}$$

[Koutsoumbas,Ntrekis,Papantonopoulos,Saridakis, 1704.08640]

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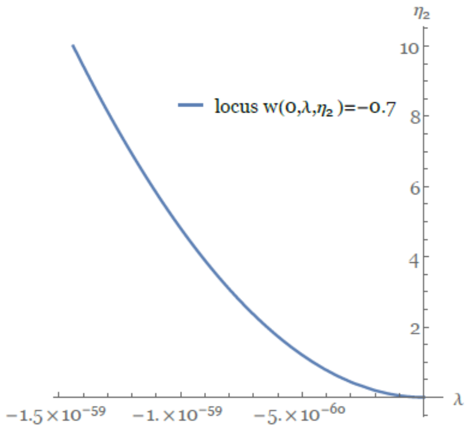
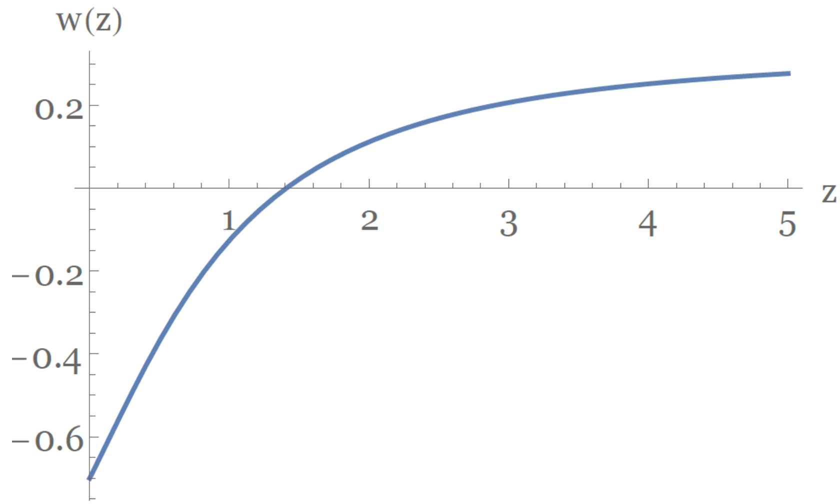
we demand $\mathrm{w(z=0) = \text{-}0.7}$ and $\mathrm{H(z=0) = H_0}$





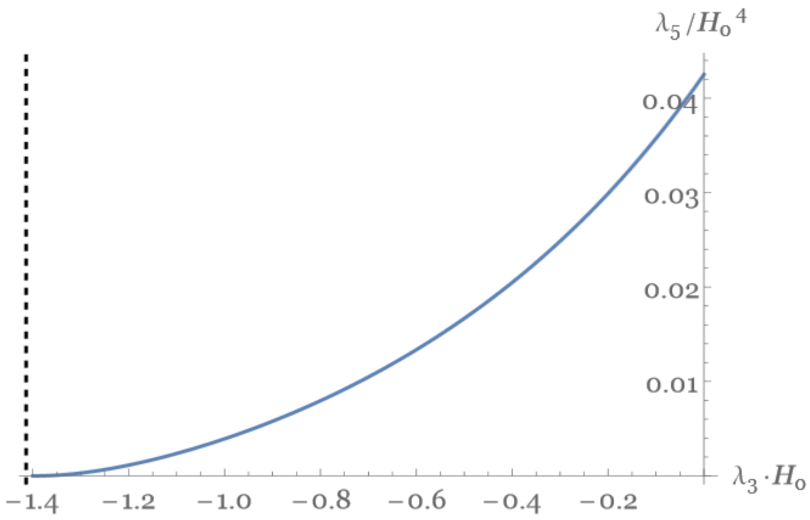
[Koutsoumbas,Ntrekis,Papantonopoulos,Saridakis, 1704.08640]

Dark energy - dark matter interaction/unification from generalized Galileons

- Model II : $\lambda \neq 0, \; \eta_2 = 0, \; \lambda_5 \neq 0$

$$\Rightarrow w(z) = \frac{\lambda\left(\lambda + 15\lambda_5 H^2(z)\right)}{\lambda^2 + 9\lambda_5 H^2(z)\left(\lambda + 6\lambda_5 H^2(z)\right)}$$

we demand $\mathrm{w(z=0) = -0.7}$ and $\mathrm{H(z=0) = H_0}$



[Koutsoumbas,Ntrekis,Papantonopoulos,Saridakis, 1704.08640]

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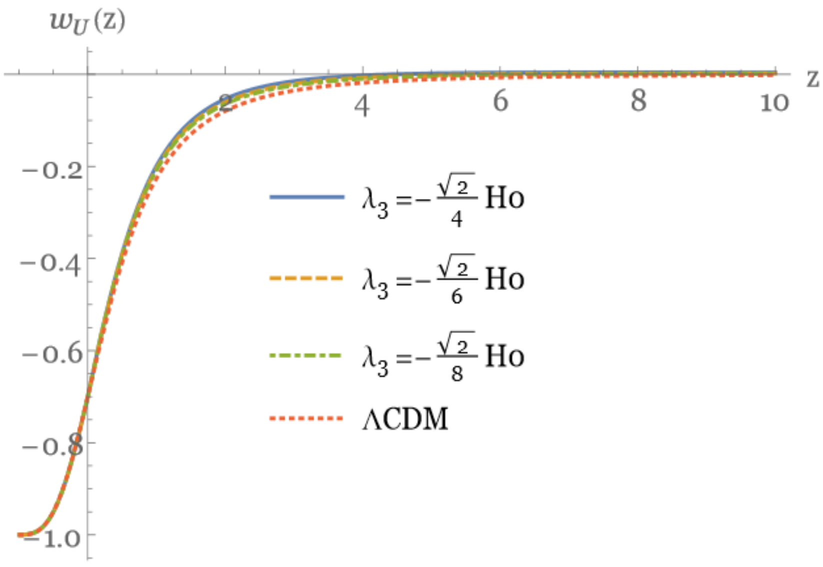

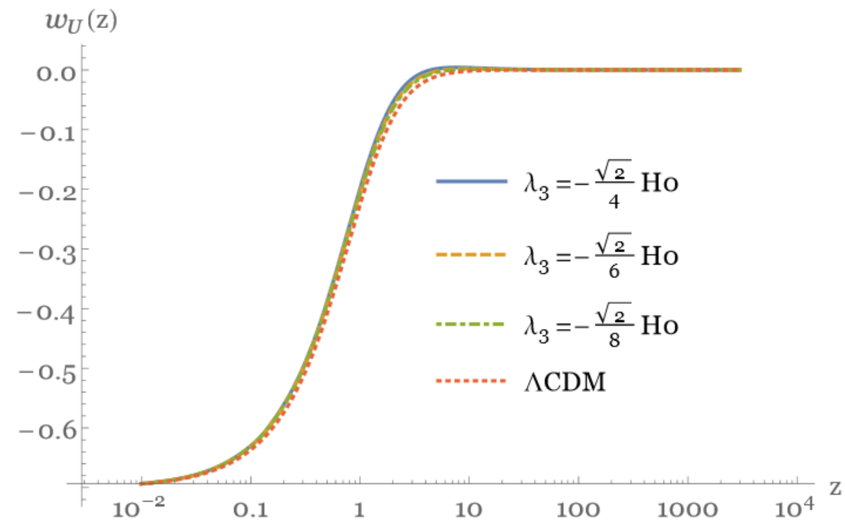

[Koutsoumbas,Ntrekis,Papantonopoulos,Saridakis, 1704.08640]

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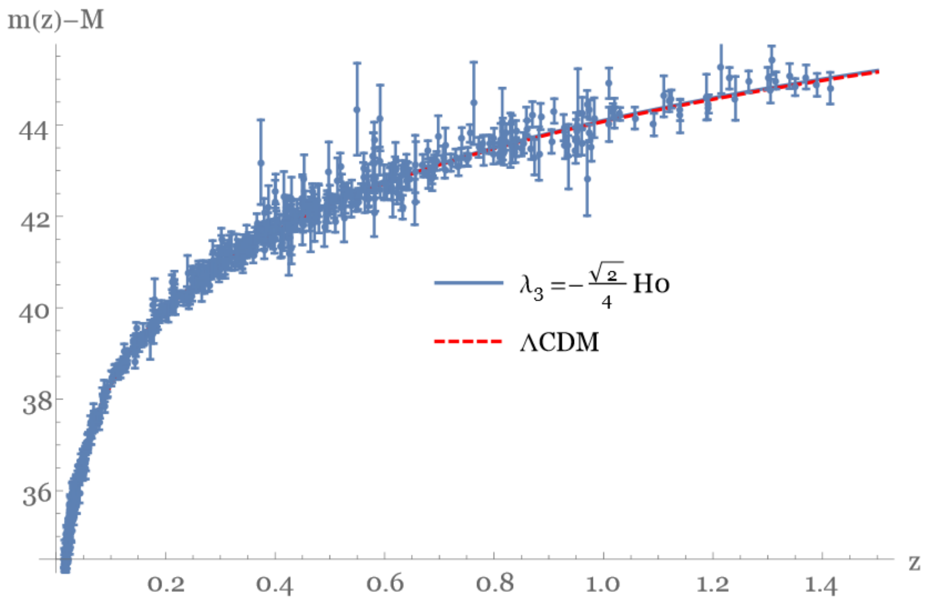

580 SN Ia data points

[Koutsoumbas,Ntrekis,Papantonopoulos,Saridakis, 1704.08640]

Dark energy - dark matter interaction/unification from generalized Galileons

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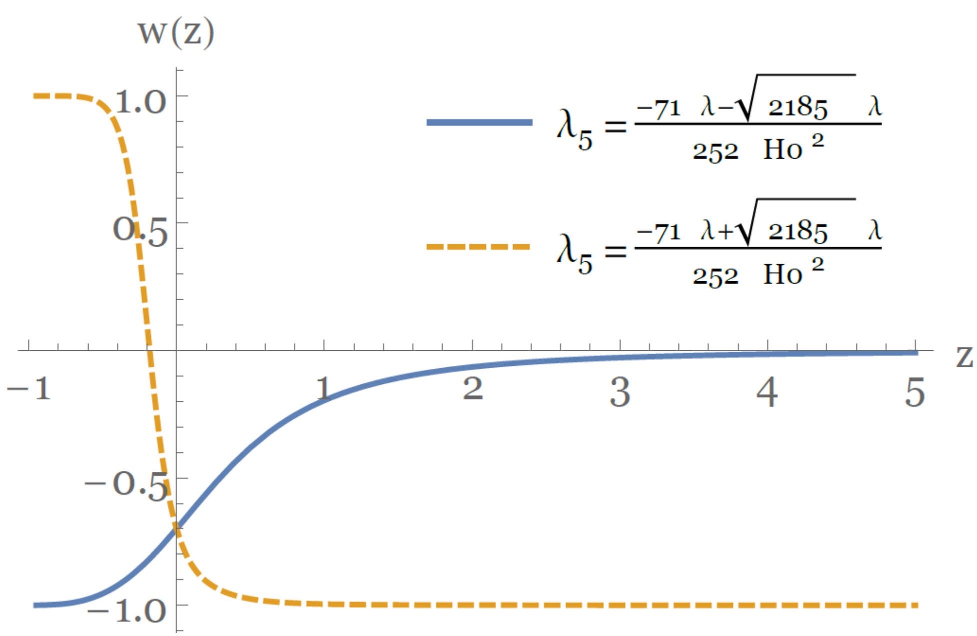
we demand $w(z=0) = -0.7$ and $H(z=0) = H_0$



[Koutsoumbas,Ntrekis,Papantonopoulos,Saridakis, 1704.08640]

Dark energy - dark matter interaction/unification from generalized Galileons

- Scalar perturbations: $ds^2 = -(1+2\psi)dt^2 + a^2(1-2\phi)\delta_{ij}dx^i dx^j$ $\Rightarrow L.H.S = R.H.S$
- No-ghost condition: $Q_S \equiv \frac{w_1\left(4w_1w_3+9w_2^2\right)}{3w_2^2} > 0$
- No Laplacian instabilities condition: $c_S^2 \equiv \frac{3\left(2w_1^2w_2H - 4w_2^2w_4 + 4w_1w_2\dot{w}_1 - 2w_1^2\dot{w}_2\right) - 6w_1^2(\rho_m + p_m)}{w_1\left(4w_1w_3+9w_2^2\right)} > 0$

with $w_1 \equiv 2\left(G_4 - 2XG_{4,X}\right) - 2X\left(G_{5,X}\dot{\phi}H - G_{5,\phi}\right)$

$$w_2 \equiv -2G_{3,X}X\dot{\phi} + 4G_4H - 16X^2G_{4,XX}H + 4\left(\dot{\phi}G_{4,\phi X} - 4HG_{4,X}\right)X + 2G_{4,\phi}\dot{\phi} + 8X^2G_{5,\phi X}H + 2HX\left(6G_{5,\phi} - 5HG_{5,X}\dot{\phi}\right) - 4G_{5,XX}\dot{\phi}X^2H^2$$

$$w_3 \equiv 3X\left(K_{,X} + 2XK_{,XX}\right) + 6X\left(3X\dot{\phi}HG_{3,XX} - G_{3,\phi X}X - G_{3,\phi} + 6\dot{\phi}HG_{3,X}\right) + 18H\left(4HX^3G_{4,XXX} - HG_4 - 5X\dot{\phi}G_{4,\phi X} - G_{4,\phi}\dot{\phi} + 7HG_{4,X}X + 16HX^2G_{4,XX} - 2X^2\dot{\phi}G_{4,X\phi X}\right) + 6H^2X\left(2H\dot{\phi}G_{5,XXX}X^2 - 6X^2G_{5,\phi XX} + 13XH\dot{\phi}G_{5,XX} - 27G_{5,\phi X}X + 15H\dot{\phi}G_{5,X} - 18G_{5,\phi}\right)$$

$$w_4 \equiv 2G_4 - 2XG_{5,\phi} - 2XG_{5,X}\ddot{\phi}$$

[De Felice,Tsujikawa JCAP 1202]

Dark energy - dark matter interaction/unification from generalized Galileons

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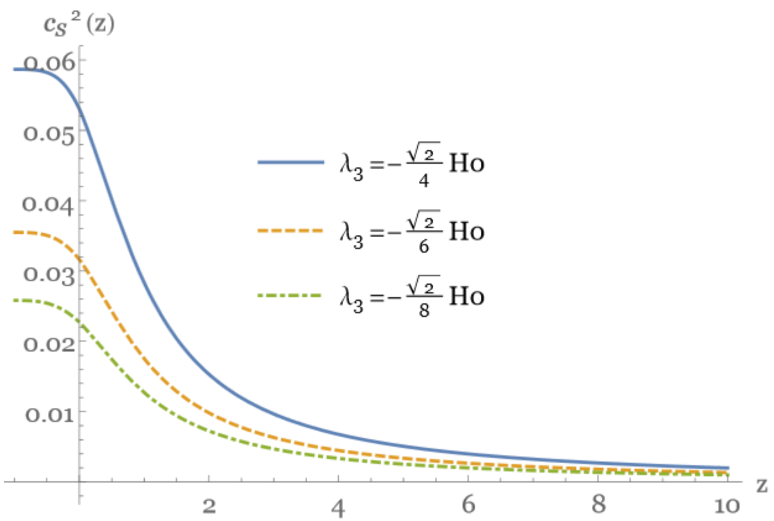

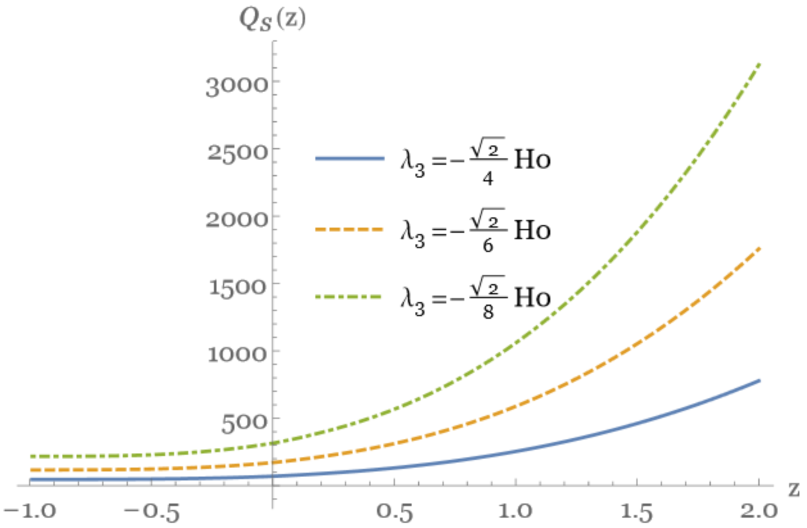

Healthy scalar perturbations. Necessary to see tensor perturbations, and the speed of gravitational waves.

[Koutsoumbas,Ntrekis,Papantonopoulos,Saridakis, 1704.08640]

Conclusions

- i) Modification of our knowledge is probably required for the explanation of cosmological evolution.
- ii) There is a huge variety of modifications.
- iii) Dark Energy (or Modified Gravity) - Dark Matter interaction cannot be excluded, and it can alleviate the coincidence problem.
- iv) Many phenomenological approaches. Can become Covariant. A full Lagrangian description is still missing.
- v) DE - DM interaction/unification from generalized Galileons with shift-symmetry. Unified universe evolution.
- vi) SN Ia data OK. Necessary: Confront with CMB, BAO, and LSS data. Need to add baryonic matter separately. Perform full perturbation analysis, confront with data.

THANK YOU!