Recent developments in Applied Newton-Cartan Geometry

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Based on work with Roel Andringa, Eric Bergshoeff, Athanasios Chatzistavrakidis, Jelle Hartong, Luca Romano, Ergin Sezgin, Thomas Zojer

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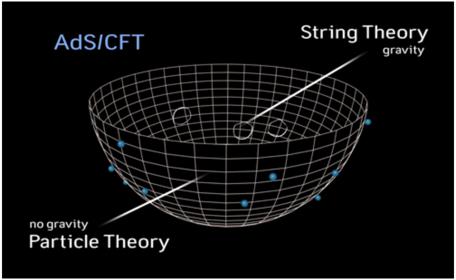
Introduction

- Newton-Cartan gravity is a diffeomorphism covariant description of Newtonian gravity, akin to General Relativity. Uses Newton-Cartan geometry.
- Old topic (Cartan, 1923) (Cartan, Dautcourt, Duval, Ehlers, Künzle, Trautman). Recently received renewed attention in the context of condensed matter physics. Why?
- Motivation 1 : holography, AdS/CFT. (Maldacena)

(quantum) gravity in asymptotically AdS_{d+1} = CFT in d dimensions



Introduction

- In practice, asymptotic values of fields of gravitational theory = sources for operators in CFT (holographic dictionary) (Gubser, Klebanov, Polyakov, Witten)

$$\langle e^{\int d^d x J(x)\mathcal{O}(x)} \rangle_{\text{CFT}} = \int \mathcal{D}\phi e^{iS_{\text{bulk}}[\phi(x,r)]}\Big|_{\phi(x,r\to\infty)=J(x)} .$$

  E.g. arbitrary Lorentzian metric $g^{(0)}_{\mu\nu}(x) = g_{\mu\nu}(x, r \to \infty)$ is source for $T_{\mu\nu}$.

- Generalized for non-relativistic CFTs (Kachru, Liu, Mulligan; Son). Instead of AdS, uses so-called Lifshitz space-times. Non-relativistic causal structure at the boundary. Absolute time.

- How to generalize the holographic dictionary? What is analog of $g^{(0)}_{\mu\nu}$? Leads to Newton-Cartan geometry with torsion (Christensen, Hartong, Obers, Rollier; Hartong, Kiritsis, Obers).

- Motivation 2: effective field theories for condensed matter systems. E.g. Unitary Fermi gas, Fractional Quantum Hall Effect. (Abanov, Geracie, Golkar, Gromov, Hoyos, Jensen, Prabhu, Roberts, Son, Wingate, Wu, Wu)

Introduction

- Geometric implementation of Galilean symmetry, via coupling a field theory to an arbitrary Newton-Cartan background, encodes transport properties (e.g. Hall viscosity).
- Inclusion of torsion is necessary for correct definition of energy current. How to do this?
- Motivation 3: extend useful techniques for relativistic field theories to non-relativistic realm.
- E.g. supersymmetric field theories on supersymmetric curved backgrounds can often be treated exactly via localization (Pestun). Supergravity techniques are handy for these constructions (Festuccia, Seiberg). Generalization to non-relativistic theories?
- For these applications, extensions of the usual Newton-Cartan framework are required. Here, discuss three extensions:
  - Inclusion of torsion
  - Inclusion of supersymmetry
  - Non-relativistic gravity with an action

Outline

- Torsion in non-relativistic gravity and its uses
- Torsionfull Newton-Cartan gravity
- $d = 3$ Newton-Cartan supergravity
- Extended Bargmann supergravity
- Conclusions and outline for future work

Torsionless Newton-Cartan geometry review

- Described in terms of time-like vielbein τ_μ, spatial vielbein $e_\mu{}^a$, gauge field m_μ associated to mass or particle number. Note: $\mu = 0, \cdots, d$ and $a = 1, \cdots, d$. Projective inverses τ^μ, $e^\mu{}_a$:

$$\tau^\mu \tau_\mu = 1\,, \quad \tau^\mu e_\mu{}^a = 0\,, \quad \tau_\mu e^\mu{}_a = 0\,,$$
$$e_\mu{}^a e^\nu{}_a = \delta^\nu_\mu - \tau^\nu \tau_\mu\,, \quad e^\mu{}_a e_\mu{}^b = \delta^b_a\,.$$

  Then: $V_0 = \tau^\mu V_\mu$, $V_a = e^\mu{}_a V_\mu$.

- 2 spin connections $\omega_\mu{}^{ab}$, $\omega_\mu{}^a$. Obtained by solving

$$R_{\mu\nu}(P^a) \equiv 2\partial_{[\mu} e_{\nu]}{}^a - 2\omega_{[\mu}{}^{ab} e_{\nu]b} - 2\omega_{[\mu}{}^a \tau_{\nu]} = 0\,,$$
$$R_{\mu\nu}(Z) \equiv 2\partial_{[\mu} m_{\nu]} - 2\omega_{[\mu}{}^a e_{\nu]a} = 0\,.$$

- Furthermore impose: $R_{\mu\nu}(H) = 2\partial_{[\mu}\tau_{\nu]} = 0$.

Torsion in non-relativistic gravity

- In relativistic gravity, torsion appears on the right-hand-side of the first Maurer-Cartan structure equation

$$R_{\mu\nu}(P^A) \equiv 2\partial_{[\mu}E_{\nu]}{}^A - 2\Omega_{[\mu}{}^{AB}E_{\nu]B} = T_{\mu\nu}{}^A \,.$$

Can be absorbed in the connection. Part of the specification of $\Omega_\mu{}^{AB}$.

- Non-relativistically, the analog of the first Maurer-Cartan structure equations with torsion reads

$$\begin{aligned} R_{\mu\nu}(H) &\equiv 2\partial_{[\mu}\tau_{\nu]} = T_{\mu\nu}{}^{(0)} \,, \\ R_{\mu\nu}(P^a) &\equiv 2\partial_{[\mu}e_{\nu]}{}^a - 2\omega_{[\mu}{}^{ab}e_{\nu]b} - 2\omega_{[\mu}{}^a\tau_{\nu]} = T_{\mu\nu}{}^a \,, \\ R_{\mu\nu}(Z) &\equiv 2\partial_{[\mu}m_{\nu]} - 2\omega_{[\mu}{}^a e_{\nu]a} = T_{\mu\nu}{}^{(c)} \,. \end{aligned}$$

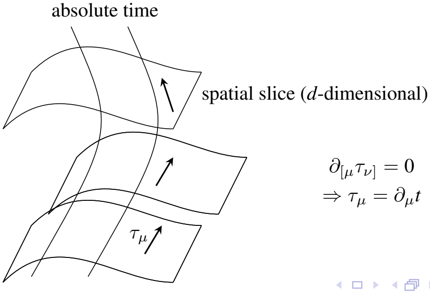
Only $T_{\mu\nu}{}^a$ and $T_{\mu\nu}{}^{(c)}$ can be absorbed in the connections. $T_{\mu\nu}{}^{(0)}$ represents an additional geometric datum! (Geracie, Prabhu, Roberts)

Newton-Cartan geometry without or with torsion

- There are three types of Newton-Cartan geometry that can be distinguished.
  - Torsionless : $\partial_{[\mu}\tau_{\nu]} = 0$.
  - Twistless torsional : $\tau_{[\mu}\partial_{\nu}\tau_{\rho]} = 0$.
  - Arbitrary torsion.
- Torsionless : $\partial_{[\mu}\tau_{\nu]} = 0$. This implies that the time interval between two events

$$\Delta t = \int_{\mathcal{C}} \mathrm{d}x^{\mu}\tau_{\mu}$$

is independent of the path $\mathcal{C}$ taken between the two events. Absolute time.

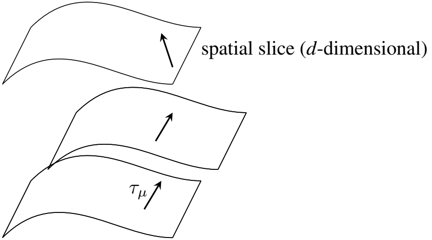


Newton-Cartan geometry without or with torsion

- Twistless torsional :

$$\tau_{[\mu}\partial_\nu\tau_{\rho]} = 0 \qquad \Leftrightarrow \qquad \tau_{ab} = e^\mu{}_a e^\nu{}_b \partial_{[\mu}\tau_{\nu]} = 0\,.$$

  Time interval between two events is now path dependent: no absolute time. The space-time still admits a foliation in spatial slices : absolute space. Newtonian causality still holds.



- Arbitrary torsion. A point p where $\tau_{[\mu}\partial_\nu\tau_{\rho]} \neq 0$ has a neighbourhood where each point lies in both the future and past of p. Acausal.

The need for torsion

- In holographic applications, the need for torsion is easily understood. Asymptotic data are only defined up to local dilatations

$$\delta\tau_\mu = \lambda_D^2(x)\tau_\mu\,, \qquad \delta e_\mu{}^a = \lambda_D(x) e_\mu{}^a\,.$$

The condition $\partial_{[\mu}\tau_{\nu]} = 0$ is however not invariant such dilatations, hence the need for (twistless) torsion.

- In effective field theories, one can implement Galilean symmetry via a non-relativistic version of minimal coupling to a gravitational background. E.g.

$$\partial_t \qquad \to \qquad \tau^\mu D_\mu \qquad (D_\mu = \partial_\mu - \mathrm{i}\, M\, m_\mu)\,.$$

One can then consider a matter Lagrangian density $\mathcal{L}(\Phi, \tau_\mu, e_\mu{}^a, m_\mu)$ and define an energy-current via

$$t^\mu = \frac{\delta\mathcal{L}}{\delta\tau_\mu}\,.$$

This variation is unconstrained and hence presupposes torsion. In order to calculate Ward identities, transport properties,... one should couple to torsional backgrounds.

Newton-Cartan gravity with torsion

- Gauging the Bargmann algebra leads to torsionless Newton-Cartan geometry.

$$R_{0a}(G^a) = 0\,, \qquad R_{0b}(J^{ba}) = 0\,, \qquad R_{ac}(J^{cb}) = 0\,,$$

  only consistent in torsionless case.

- Newton-Cartan gravity can be obtained from a null reduction of GR (Julia, Nicolai; Bergshoeff, Chatzistavrakidis, Romano, Rosseel). Ansatz in coordinates adapted to a null Killing vector $\xi = \partial_v$:

$$E_M{}^A = \begin{matrix} \\ \mu \\ v \end{matrix} \begin{matrix} a & - & + \\ \left(\begin{matrix} e_\mu{}^a \\ 0 \end{matrix} \right. & \begin{matrix} \tau_\mu \\ 0 \end{matrix} & \left. \begin{matrix} -m_\mu \\ 1 \end{matrix} \right) \end{matrix} .$$

  Breaks Lorentz transformations to spatial rotations and Galilean boosts.

- Off-shell, there is no constraint on τ_μ.
- Reducing the equations of motion however leads to conditions that put the torsion to zero. Needed for consistency (boost invariance of the e.o.m.s).
- Other strategy needed!

The conformal method

- Relativistic gravity is equivalent to a real, compensating scalar ϕ coupled to conformal gravity (gauging of conformal algebra)

$$\delta\phi = w\lambda_D\phi\,, \qquad D_M\phi = (\partial_M - w b_M)\,\phi\,.$$

Action

$$S = -\frac{1}{2}\int \mathrm{d}^D x\, E\, \phi\Box^C\phi\,,$$

$$\Box^C\phi = E^{AM}\left(\partial_M D_A\phi - (w+1) b_M D_A\phi + \Omega_{MAB} D^B\phi + 2w f_{MA}\phi\right)\,,$$

$$E_A{}^M f_M{}^A = -\frac{1}{4(D-1)} R\,.$$

- The Einstein-Hilbert action is then obtained by gauge fixing dilatations via the choice ϕ = constant. GR = gauge equivalent to a conformally coupled scalar.
- Non-relativistic case : consider a complex scalar, coupled to non-relativistic 'conformal' gravity, obtained by gauging the Schroedinger algebra.

Torsion and the Schrödinger algebra

- The $z = 2$ Schrödinger algebra is an extension of the Bargmann algebra, with a dilatation generator D and special conformal transformation K.

$$[D, H] = -2H\,, \quad [H, K] = D\,, \quad [D, K] = 2K\,,$$
$$[D, P_a] = -P_a\,, \quad [D, G_a] = G_a\,, \quad [K, P_a] = -G_a\,.$$

- Gauging (Bergshoeff, Hartong, Rosseel)

symmetry	generators	gauge field	parameters	curvatures
time translations	H	τ_μ	$\zeta(x^\nu)$	$R_{\mu\nu}(H)$
space translations	P^a	$e_\mu{}^a$	$\zeta^a(x^\nu)$	$R_{\mu\nu}{}^a(P)$
boosts	G^a	$\omega_\mu{}^a$	$\lambda^a(x^\nu)$	$R_{\mu\nu}{}^a(G)$
spatial rotations	J^{ab}	$\omega_\mu{}^{ab}$	$\lambda^{ab}(x^\nu)$	$R_{\mu\nu}{}^{ab}(J)$
central charge transf.	Z	m_μ	$\sigma(x^\nu)$	$R_{\mu\nu}(Z)$
dilatations	D	b_μ	$\Lambda_D(x^\nu)$	$R_{\mu\nu}(D)$
spec. conf. transf.	K	f_μ	$\Lambda_K(x^\nu)$	$R_{\mu\nu}(K)$

Torsion and the Schrödinger algebra

- Impose constraints such that $\omega_\mu{}^{ab}$, $\omega_\mu{}^a$, $b_a = e^\mu{}_a b_\mu$ and f_μ become dependent. Transformation rules of independent fields τ_μ, $e_\mu{}^a$, m_μ, $b_0 = \tau^\mu b_\mu$:

$$\delta\tau_\mu = 2\lambda_D\tau_\mu\,, \qquad \delta e_\mu{}^a = \lambda^a{}_b e_\mu{}^b + \lambda^a\tau_\mu + \lambda_D e_\mu{}^a\,,$$
$$\delta m_\mu = \partial_\mu\sigma + \lambda^a e_{\mu a}\,, \qquad \delta b_0 = \tau^\mu\partial_\mu\lambda_D + \lambda_K - \lambda^a b_a\,.$$

- In particular, one imposes

$$R_{0a}(H) \equiv \tau^\mu e^\nu{}_a\left(2\partial_{[\mu}\tau_{\nu]} - 4b_{[\mu}\tau_{\nu]}\right) = 0 \quad \Rightarrow \quad b_a = -\tau^\mu e^\nu{}_a\partial_{[\mu}\tau_{\nu]}\,.$$

  One still keeps $R_{ab}(H) = 2e^\mu{}_a e^\nu{}_b\partial_{[\mu}\tau_{\nu]} \neq 0$!

- Set of (in)dependent gauge fields that can be used to covariantize derivatives.

- Only implement J^{ab} and G^a and fix K, D and Z. Fix K by imposing $b_0 = 0$. To fix D and Z we introduce two scalars ϕ and χ

$$\delta\phi = \lambda_D\phi \qquad \Rightarrow D_\mu\phi = \partial_\mu\phi - b_\mu\phi\,,$$
$$\delta\chi = M\sigma \qquad \Rightarrow D_\mu\chi = \partial_\mu\chi - Mm_\mu\,.$$

Newton-Cartan gravity with torsion

- There exists a boost invariant connection for spatial rotations $\tilde{\omega}_\mu{}^{ab}$. Build curvature $R_{\mu\nu}(\tilde{\omega}^{ab})$ and impose

$$R_{0b}(\tilde{\omega}^{ba}) = 0\,, \qquad R_{ac}(\tilde{\omega}^{cb}) = 0\,,$$

as torsionful equivalent of corresponding Newton-Cartan e.o.m.s.

- In three dimensions

$$D_0 D_0 \phi - \frac{2}{M}(D_0 D_a \phi)(D^a \chi) + \frac{1}{M^2}(D_a D_b \phi)(D^a \chi)(D^b \chi)$$
$$- \frac{1}{M^3}(D^a \chi)(D_a \chi)(D^b \chi)\tau_b{}^c D_c \phi + \frac{1}{4M^4}(D^a \chi)(D_a \chi)(D^b \chi)(D^c \chi)\tau_b{}^d \tau_{cd} \phi = 0$$

is Schroedinger invariant. Note that $D_0 D_0 \phi$ contains a term

$$\tau^\mu f_\mu \phi \qquad \text{with} \qquad \tau^\mu f_\mu = \frac{1}{d-1} R'_{0a}(G^a) + \text{torsion terms}\,.$$

- Other e.o.m. found by imposing gauge fixing conditions

$$\phi = 1\,, \qquad \chi = 0\,.$$

Newton-Cartan supergravity

- Inclusion of supersymmetry. Done by algebra gauging in 3d.
- Look at $\mathcal{N} = 2$ version (Lukierski,Prochnicka,Stichel,Zakrzewski):

$$
\begin{aligned}
[J_{ab}, P_c] &= -2\delta_{c[a}P_{b]}\,, & [J_{ab}, G_c] &= -2\delta_{c[a}G_{b]}\,,\\
[G_a, H] &= -P_a\,, & [G_a, P_b] &= -\delta_{ab}Z\,,\\
[J_{ab}, Q^{\pm}] &= -\frac{1}{2}\gamma_{ab}Q^{\pm}\,, & [G_a, Q^{+}] &= -\frac{1}{2}\gamma_{a0}Q^{-}\,,\\
\{Q^{+}_{\alpha}\,,\, Q^{+}_{\beta}\} &= 2\delta_{\alpha\beta}H\,, & \{Q^{+}_{\alpha}\,,\, Q^{-}_{\beta}\} &= -\,[\gamma^{a0}]_{\alpha\beta}P_a\,,\\
\{Q^{-}_{\alpha}\,,\, Q^{-}_{\beta}\} &= 2\delta_{\alpha\beta}\,Z\,.
\end{aligned}
$$

- $\mathcal{N} = 1$ subalgebra, consisting of bosonic generators and Q_-.
- Gauging proceeds along very similar lines as in ordinary Bargmann case, upon introducing two extra gauge fields $\psi_{\mu\pm}$ and corresponding curvatures $\hat{\psi}_{\mu\nu\pm}$.

Newton-Cartan supergravity

- Spin connections solved by imposing conventional constraints

$$\hat{R}_{\mu\nu}(P^a) = 0\,, \qquad \hat{R}_{\mu\nu}(Z) = 0\,.$$

- Also impose

$$\hat{R}_{\mu\nu}(H) \equiv 2\partial_{[\mu}\tau_{\nu]} - \frac{1}{2}\bar{\psi}_{\mu+}\gamma^0\psi_{\nu+} = 0\,.$$

Consistency with supersymmetry implies

$$\hat{R}_{\mu\nu}(H) = 0 \qquad \Rightarrow \qquad \hat{\psi}_{\mu\nu+} = 0 \qquad \Rightarrow \qquad \hat{R}_{\mu\nu}(J^{ab}) = 0\,.$$

Last equation is stronger analog of some bosonic e.o.m.s. Implies flat space.

- Supersymmetry algebra closes upon using

$$\gamma^\mu\tau^\nu\hat{\psi}_{\mu\nu-} = 0\,, \qquad e^\mu{}_a e^\nu{}_b\hat{\psi}_{\mu\nu-} = 0\,.$$

First can be interpreted as fermionic e.o.m. and varies to bosonic e.o.m.

$$\hat{R}_{0a}(G^a) = 0\,.$$

A consistent set of constraints can be found ⇒ on-shell Newton-Cartan supergravity.

Newton-Cartan supergravity

- Gauge fixing to Newtonian supergravity. Most fields are set to constant values except for

$$m_0 \equiv \Phi \qquad \psi_{0-} \equiv \Psi \,.$$

  Ψ is superpartner of the Newton force $\Phi_i = \partial_i \Phi$:

$$\delta \Phi_i = \bar{\epsilon}_-(t)\gamma^0 \partial_i \Psi + \frac{1}{2}\bar{\epsilon}_+ \gamma_i \dot{\Psi} \,, \qquad \delta \Psi = \dot{\epsilon}_-(t) - \frac{1}{2}\Phi^i \gamma_{i0}\epsilon_+ \,.$$

- This does not include bosonic torsion : $\partial_{[\mu}\tau_{\nu]} = 0$ when fermions are set to zero.
- One can however construct Schroedinger supergravity. (Bergshoeff, Rosseel, Zojer)
- Supersymmetry $\Rightarrow \hat{R}_{\mu\nu}(J^{ab}) = 0$. Matter coupling is not likely to help, since only the 'Laplace equation' $\hat{R}_{0a}(G^a) = 0$ gets sourced. Moreover, no satisfactory action principle. For e.g. localization applications this is unwanted.
- Can one do better? At least in 3d one can!

Extended Bargmann gravity

- In $3d$, the Bargmann algebra admits an extra central charge: (Lévy-Leblond)

$$[G_a, P_b] = \epsilon_{ab} Z \,, \qquad [G_a, G_b] = \epsilon_{ab} S \,.$$

- This 'extended Bargmann algebra' admits a non-degenerate, invariant bilinear form (trace)

$$< G_a, P_b >= \delta_{ab} \,, \quad < H, S >=< Z, J >= -1 \,.$$

- A proposal for an action exists : a Chern-Simons action (Papageorgiou, Schroers)

$$S_{\mathrm{EBG}} = \frac{k}{4\pi} \int \mathrm{d}^3 x \, \varepsilon^{\mu\nu\rho} \left(e_\mu{}^a R_{\nu\rho}(G_a) - m_\mu R_{\nu\rho}(J) - \tau_\mu R_{\nu\rho}(S) \right) \,.$$

- Variation with respect to ω_μ, $\omega_\mu{}^a$ and s_μ leads to

$$R_{\mu\nu}(Z) = 0 \,, \qquad R_{\mu\nu}(P^a) = 0 \,, \qquad R_{\mu\nu}(H) = 0 \,.$$

- Arises as a non-relativistic limit of Einstein-Hilbert + a CS action for two abelian vectors. (Bergshoeff, Rosseel)

Extended Bargmann gravity

- Since we have an action, matter coupling is straightforward. E.g.

$$S = S_{\text{EBG}} + \int \mathrm{d}^3x\, e\, \left[\frac{\mathrm{i}}{2}\left(\Phi^* \tau^\mu D_\mu \Phi - \Phi \tau^\mu D_\mu \Phi^*\right) - \frac{1}{2m} e^\mu{}_a e^{\nu a} D_\mu \Phi^* D_\mu \Phi\right] .$$

  with $D_\mu \Phi = \partial_\mu \Phi + \mathrm{i}\, m\, m_\mu \Phi$.

- Defining currents

$$t^\mu = \frac{4\pi}{k}\frac{\delta}{\delta \tau_\mu}\left(e\mathcal{L}_{\text{m}}\right) , \qquad t^\mu{}_a = \frac{4\pi}{k}\frac{\delta}{\delta e_\mu{}^a}\left(e\mathcal{L}_{\text{m}}\right) , \qquad j^\mu = \frac{4\pi}{k}\frac{\delta}{\delta m_\mu}\left(e\mathcal{L}_{\text{m}}\right) ,$$

  one finds

$$\epsilon^{\mu\nu\rho} R_{\nu\rho}(S) = t^\mu , \qquad \epsilon^{\mu\nu\rho} R_{\nu\rho}(J) = j^\mu , \qquad \epsilon^{\mu\nu\rho} R_{\nu\rho}(G_a) = -t^\mu{}_a .$$

- Bianchi identities imply consistency conditions

$$e_\mu{}^a j^\mu = -\tau_\mu t^\mu{}_a , \qquad e_\mu{}^{[a} t^{|\mu|b]} = 0 ,$$
$$D_\mu t^\mu = 0 , \qquad D_\mu t^\mu{}_a = 0 , \qquad D_\mu j^\mu = 0 .$$

Extended Bargmann Supergravity

- Note that in general all components of the curvature tensors are sourced by matter. In contrast to Newton-Cartan gravity where only R_{00} is sourced.
- Supersymmetric version exists!
- Based on superalgebra with non-degenerate supertrace

$$
\begin{aligned}
&[J, Q^{\pm}] = -\frac{1}{2}\gamma_0 Q^{\pm}\,, && [J, R] = -\frac{1}{2}\gamma_0 R\,,\\
&[G_a, Q^{+}] = -\frac{1}{2}\gamma_a Q^{-}\,, && [G_a, Q^{-}] = -\frac{1}{2}\gamma_a R\,,\\
&[S, Q^{+}] = -\frac{1}{2}\gamma_0 R\,, && \{Q^{+}_{\alpha}, Q^{+}_{\beta}\} = (\gamma_0 C^{-1})_{\alpha\beta} H\,,\\
&\{Q^{+}_{\alpha}, Q^{-}_{\beta}\} = -(\gamma^a C^{-1})_{\alpha\beta} P_a\,, && \{Q^{-}_{\alpha}, Q^{-}_{\beta}\} = (\gamma_0 C^{-1})_{\alpha\beta} Z\,,\\
&\{Q^{+}_{\alpha}, R_{\beta}\} = (\gamma_0 C^{-1})_{\alpha\beta} Z\,.
\end{aligned}
$$

- Good starting point to study matter couplings and see whether non-relativistic supersymmetric field theories on non-trivial supersymmetric backgrounds can be obtained.

Conclusions

- Newton-Cartan gravity gives a geometric picture of non-relativistic gravity, that can be useful in modern applications as well.
- E.g. useful in AdS/CFT, effective condensed matter field theories.
- Often require extensions of Newton-Cartan geometry and gravity. Requires a new look.
- Inclusion of torsion, supersymmetry and consideration of other non-relativistic gravities (e.g. with action) is important.
- Can be constructed in a modern fashion, by considering algebra gaugings, non-relativistic limits, conformal methods.
- Future: extensions to off-shell supergravity, higher-dimensional supergravity cases, inclusion of matter, massive higher spins.