Holographic aspects of Conformal (and Critical) Gravity

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(with G. Anastasiou and D. Rivera (UNAB))

Outline

1. Conformal Gravity: Action and Surface Terms

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2. Einstein Spaces in Conformal Gravity

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3. From Conformal to Einstein gravity (Maldacena)

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4. Higher-derivative Gravity Theories
5. Critical Gravity
6. Holographic Renormalization in Critical Gravity

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4. Higher-derivative Gravity Theories
5. Critical Gravity
6. Holographic Renormalization in Critical Gravity
7. Conclusions

Conformal Gravity: Field equations and surface terms

- **CG action:**

$$\begin{aligned} I_{CG} &= \alpha_{CG} \int_M d^4x \sqrt{-g}\, W^{\alpha\beta\mu\nu} W_{\alpha\beta\mu\nu} \\ &= \frac{\alpha_{CG}}{4} \int_M d^4x \sqrt{-g}\, \delta^{[\nu_1\nu_2\nu_3\nu_4]}_{[\mu_1\mu_2\mu_3\mu_4]} W^{\mu_1\mu_2}_{\nu_1\nu_2} W^{\mu_3\mu_4}_{\nu_3\nu_4} \end{aligned}$$

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where α_{CG} is an arbitrary dimensionless coupling constant

- **Variation of the Action**

$$\delta I_{CG} = \alpha_{CG} \int_M d^4x \sqrt{-g} B^{\nu}_{\mu} \left(g^{-1}\delta g\right)^{\mu}_{\nu} + \int_{\partial M} d^3x \Theta$$

where the Bach tensor is

$$B^{\nu}_{\mu} = -\delta^{[\nu\nu_1\nu_2\nu_3]}_{[\mu\mu_1\mu_2\mu_3]} \left[\nabla^{\mu_1} \nabla_{\nu_1} W^{\mu_2\mu_3}_{\nu_2\nu_3} + \frac{1}{2} R^{\mu_1}_{\nu_1} W^{\mu_2\mu_3}_{\nu_2\nu_3} \right]$$
$$= -4 \left[\nabla^{\alpha} \nabla_{\beta} W^{\beta\nu}_{\alpha\mu} + \frac{1}{2} R^{\alpha}_{\beta} W^{\beta\nu}_{\alpha\mu} \right]$$

Conformal Gravity: Field equations and surface terms

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I_{CG} &= \alpha_{CG}\int_M d^4x\sqrt{-g}\,W^{\alpha\beta\mu\nu}W_{\alpha\beta\mu\nu} \\
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\end{aligned}
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  where α_{CG} is an arbitrary dimensionless coupling constant

- **Variation of the Action**

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\delta I_{CG} = \alpha_{CG}\int_M d^4x\sqrt{-g}\,B^{\nu}_{\mu}\left(g^{-1}\delta g\right)^{\mu}_{\nu} + \int_{\partial M} d^3x\,\Theta
$$

  where the Bach tensor is

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\begin{aligned}
B^{\nu}_{\mu} &= -\delta^{[\nu\nu_1\nu_2\nu_3]}_{[\mu\mu_1\mu_2\mu_3]}\left[\nabla^{\mu_1}\nabla_{\nu_1}W^{\mu_2\mu_3}_{\nu_2\nu_3} + \frac{1}{2}R^{\mu_1}_{\nu_1}W^{\mu_2\mu_3}_{\nu_2\nu_3}\right] \\
&= -4\left[\nabla^{\alpha}\nabla_{\beta}W^{\beta\nu}_{\alpha\mu} + \frac{1}{2}R^{\alpha}_{\beta}W^{\beta\nu}_{\alpha\mu}\right]
\end{aligned}
$$

- **Bach tensor** is symmetric, traceless and covariantly conserved

- **Surface Term**

$$\Theta = \alpha_{CG}\sqrt{-h}\delta_{[\mu_1\mu_2\mu_3\mu_4]}^{[\nu_1\nu_2\nu_3\nu_4]}\left[n_{\nu_1}\delta\Gamma_{\kappa\nu_2}^{\mu_1}g^{\mu_2\kappa}W_{\nu_3\nu_4}^{\mu_3\mu_4}+\right.$$
$$\left.+n^{\mu_1}\nabla_{\nu_1}W_{\nu_2\nu_3}^{\mu_2\mu_3}\left(g^{-1}\delta g\right)_{\nu_4}^{\mu_4}\right]$$

- **Surface Term**

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$$\left.+n^{\mu_1}\nabla_{\nu_1}W^{\mu_2\mu_3}_{\nu_2\nu_3}\left(g^{-1}\delta g\right)^{\mu_4}_{\nu_4}\right]$$

- Einstein spacetimes are a (trivial) subset of Bach-flat solutions.

Conformal Gravity: Field equations and surface terms

- **Surface Term**

$$\Theta = \alpha_{CG}\sqrt{-h}\delta^{[\nu_1\nu_2\nu_3\nu_4]}_{[\mu_1\mu_2\mu_3\mu_4]}\left[n_{\nu_1}\delta\Gamma^{\mu_1}_{\kappa\nu_2}g^{\mu_2\kappa}W^{\mu_3\mu_4}_{\nu_3\nu_4}+ +n^{\mu_1}\nabla_{\nu_1}W^{\mu_2\mu_3}_{\nu_2\nu_3}\left(g^{-1}\delta g\right)^{\mu_4}_{\nu_4}\right]$$

- Einstein spacetimes are a (trivial) subset of Bach-flat solutions.
- In general, solutions will be non-Einstein, when CG action is added on top of Einstein-Hilbert action

$$R_{\mu\nu} = \beta g_{\mu\nu} + \gamma B_{\mu\nu}$$

From Conformal to Einstein Gravity

- CG with Neumann boundary conditions is equivalent to Einstein gravity with cosmological constant.
  `J.Maldacena, [arXiv:1105.5632]`

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- Holographic argument

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- Decomposition of the Weyl tensor
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- Ricci tensor

$$R^{\mu}_{\nu} = -\frac{3}{\ell^2}\delta^{\mu}_{\nu} + \gamma B^{\mu}_{\nu}$$

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$$S^{\mu}_{\nu} = -\frac{1}{2}\left(\frac{1}{\ell^2}\delta^{\mu}_{\nu} - \gamma B^{\mu}_{\nu}\right)$$

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- Schouten tensor

$$S_{\nu}^{\mu}=-\frac{1}{2}\left(\frac{1}{\ell^{2}}\delta_{\nu}^{\mu}-\gamma B_{\nu}^{\mu}\right)$$

- Weyl tensor is separable

$$W_{\mu\nu}^{\alpha\beta}=W_{(E)\mu\nu}^{\alpha\beta}+W_{(NE)\mu\nu}^{\alpha\beta}$$

$$W_{(NE)\mu\nu}^{\alpha\beta}=-\frac{\gamma}{2}\left(B_{\mu}^{\alpha}\delta_{\nu}^{\beta}-B_{\mu}^{\beta}\delta_{\nu}^{\alpha}-B_{\nu}^{\alpha}\delta_{\mu}^{\beta}+B_{\nu}^{\beta}\delta_{\mu}^{\alpha}\right)$$

From Conformal To Einstein Gravity

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From Conformal To Einstein Gravity

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- For a given CG coupling $\alpha_{CG} = \ell^2 / 64\pi G$

$$I_{CG} = I^{(E)}_{ren} - \frac{\ell^2}{16\pi G} \gamma \int_M d^4x \sqrt{-g} \delta^{[\nu_1\nu_2]}_{[\mu_1\mu_2]} \big(G^{\mu_1}_{\nu_1} - \frac{\gamma}{2} B^{\mu_1}_{\nu_1} \big) B^{\mu_2}_{\nu_2} \ ,$$

–

- **In any** $D = 2n$

Holographic Renormalization $\Leftrightarrow$ Addition of Topological Invariants

Topological Regularization in 4D Einstein AdS Gravity

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- **In 4D AdS** $(\Lambda = -3/\ell^2)$

$$I_{\text{ren}}^{(E)} = \frac{1}{16\pi G}\int\limits_{M} d^4x\sqrt{-g}\left[R - 2\Lambda + \frac{\ell^2}{4}\left(R^{\alpha\beta\mu\nu}R_{\alpha\beta\mu\nu} - 4R^{\mu\nu}R_{\mu\nu} + R^2\right)\right]$$

Topological Regularization in 4D Einstein AdS Gravity

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- **Renormalized Action in AdS/CFT**

$$I_{\text{ren}}^{(E)} = I_{EH} - \frac{1}{8\pi G} \int_{\partial M} d^3x \sqrt{-h} K + \frac{1}{8\pi G} \int\limits_{\partial M} d^3x \sqrt{-h} \left(\frac{2}{\ell} + \frac{\ell}{2} \mathcal{R}(h) \right)$$

Proof: Euler Theorem

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- Boundary term

$$B_3 = 4\sqrt{-h}\delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]} K^{j_1}_{i_1} \left(\frac{1}{2} \mathcal{R}^{j_2 j_3}_{i_2 i_3} - \frac{1}{3} K^{j_2}_{i_2} K^{j_3}_{i_3} \right)$$

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- **Extrinsic counterterms (Kounterterms)**
  `R.O., [hep-th/0504233,hep-th/0610230]`

$$I = I_{EH} + \frac{\ell^2}{16\pi G}\int_{\partial M} d^3x\sqrt{-h}\,\delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]} K^{j_1}_{i_1}\left(\frac{1}{2}\mathcal{R}^{j_2 j_3}_{i_2 i_3}(h) - \frac{1}{3}K^{j_2}_{i_2}K^{j_3}_{i_3}\right).$$

- **Add zero**

$$I = I_{EH} - \frac{1}{8\pi G} \int\limits_{\partial M} d^3x \sqrt{-h}\, K + \int\limits_{\partial M} d^3x\, \mathcal{L}_{ct}.$$

$$\mathcal{L}_{ct} = \frac{\ell^2}{16\pi G} \sqrt{-h} \delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]} K^{j_1}_{i_1} \left(\frac{1}{2} \mathcal{R}^{j_2 j_3}_{i_2 i_3}(h) - \frac{1}{3} K^{j_2}_{i_2} K^{j_3}_{i_3} + \frac{1}{\ell^2} \delta^{j_2}_{i_2} \delta^{j_3}_{i_3} \right)$$

Proof: Extrinsic counterterms

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- Expansion of K^j_i for any AAdS spacetime

$$K^i_j = \frac{1}{\ell}\delta^i_j - \ell S^i_j(h) + \mathcal{O}(\mathcal{R}^2)$$

$$S^i_j(h) = \frac{1}{D-3}\left(\mathcal{R}^i_j(h) - \frac{1}{2(D-2)}\delta^i_j \mathcal{R}(h)\right)$$

Proof: From Extrinsic to Intrinsic Formulation

- **From Extrinsic to Intrinsic Counterterms**
  O. Miskovic and R.O., [arXiv:0902.2082]

$$\mathcal{L}_{ct} = \frac{\ell^2}{16\pi G}\sqrt{-h}\delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]}\left(\frac{\delta^{j_1}_{i_1}}{\ell} - \ell S^{i_1}_{j_1}\right)\times$$
$$\times\left(\frac{1}{2}\mathcal{R}^{j_2 j_3}_{i_2 i_3}(h) - \frac{1}{3}\left(\frac{\delta^{j_2}_{i_2}}{\ell} - \ell S^{i_2}_{j_2}\right)\left(\frac{\delta^{j_3}_{i_3}}{\ell} - \ell S^{i_3}_{j_3}\right) + \frac{1}{\ell^2}\delta^{j_2}_{i_2}\delta^{j_3}_{i_3}\right) + \ldots$$

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$$\times\left(\frac{1}{2}\mathcal{R}^{j_2 j_3}_{i_2 i_3}(h) - \frac{1}{3}\left(\frac{\delta^{j_2}_{i_2}}{\ell} - \ell S^{i_2}_{j_2}\right)\left(\frac{\delta^{j_3}_{i_3}}{\ell} - \ell S^{i_3}_{j_3}\right) + \frac{1}{\ell^2}\delta^{j_2}_{i_2}\delta^{j_3}_{i_3}\right) + \ldots$$

- Balasubramanian-Kraus counterterms in 4D

$$\mathcal{L}_{ct} = \frac{1}{8\pi G}\sqrt{-h}\left(\frac{2}{\ell} + \frac{\ell}{2}\mathcal{R}(h)\right) + \ldots$$

Key features of Renormalized AdS Action

- **EH AdS + GB in 4D:**

$$I_{ren}^{(E)} = \frac{1}{16\pi G}\int_M d^4x\sqrt{-g}\left[(R-2\Lambda)+\frac{\ell^2}{16}\delta^{[\nu_1\nu_2\nu_3\nu_4]}_{[\mu_1\mu_2\mu_3\mu_4]}R^{\mu_1\mu_2}_{\nu_1\nu_2}R^{\mu_3\mu_4}_{\nu_3\nu_4}\right]$$

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- Stelle-West (MacDowell-Mansouri) form of the Renormalized Action

$$I_{ren}^{(E)} = \frac{\ell^2}{256\pi G}\int\limits_M d^4x\sqrt{-g}\delta_{[\mu_1\mu_2\mu_3\mu_4]}^{[\nu_1\nu_2\nu_3\nu_4]}\left[R_{\nu_1\nu_2}^{\mu_1\mu_2}+\frac{\delta_{[\nu_1\nu_2]}^{[\mu_1\mu_2]}}{\ell^2}\right]\left[R_{\nu_3\nu_4}^{\mu_3\mu_4}+\frac{\delta_{[\nu_3\nu_4]}^{[\mu_3\mu_4]}}{\ell^2}\right]$$

Weyl Tensor for Einstein Spaces

- Weyl tensor

$$
\begin{aligned}
W_{\mu\nu}^{\alpha\beta} &= R_{\mu\nu}^{\alpha\beta} - \left(S_{\mu}^{\alpha}\delta_{\nu}^{\beta} - S_{\mu}^{\beta}\delta_{\nu}^{\alpha} - S_{\nu}^{\alpha}\delta_{\mu}^{\beta} + S_{\nu}^{\beta}\delta_{\mu}^{\alpha} \right) \\
S_{\mu}^{\alpha} &= \frac{1}{D-2}\left(R_{\mu}^{\alpha} - \frac{1}{2(D-1)}\delta_{\mu}^{\alpha}R\right)
\end{aligned}
$$

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S^{\alpha}_{\mu} &= \frac{1}{D-2}\big(R^{\alpha}_{\mu} - \frac{1}{2(D-1)}\delta^{\alpha}_{\mu}R\big)
\end{aligned}
$$

- Weyl tensor for Einstein spaces $R_{\mu\nu} = -\frac{(D-1)}{\ell^2}g_{\mu\nu}$

$$
W^{\alpha\beta}_{(E)\mu\nu} = R^{\alpha\beta}_{\mu\nu} + \frac{1}{\ell^2}\delta^{[\alpha\beta]}_{[\mu\nu]}
$$

Key features of Renormalized AdS Action

- Renormalized Action
  O. Miskovic and R.O., [arXiv:0902.2082]

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Key features of Renormalized AdS Action

- Renormalized Action

  O. Miskovic and R.O., [arXiv:0902.2082]

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- Variation of the action

$$\delta I_{ren}^{(E)}=\frac{\ell^2}{64\pi G}\int_{\partial M} d^3x\sqrt{-h}\,\delta_{[\mu_1\mu_2\mu_3\mu_4]}^{[\nu_1\nu_2\nu_3\nu_4]}n_{\nu_1}\delta\Gamma_{\kappa\nu_2}^{\mu_1}g^{\mu_2\kappa}W_{(E)\nu_3\nu_4}^{\mu_3\mu_4}$$

- **Surface Term**

$$\Theta = \frac{\ell^2}{64\pi G}\sqrt{-h}\delta^{[\nu_1\nu_2\nu_3\nu_4]}_{[\mu_1\mu_2\mu_3\mu_4]}[n_{\nu_1}\delta\Gamma^{\mu_1}_{\lambda\nu_2}g^{\mu_2\lambda}(W^{\mu_3\mu_4}_{(E)\nu_3\nu_4}+W^{\mu_3\mu_4}_{(NE)\nu_3\nu_4})+ +n^{\mu_1}\nabla_{\nu_1}W^{\mu_2\mu_3}_{(NE)\nu_2\nu_3}\left(g^{-1}\delta g\right)^{\mu_4}_{\nu_4}]$$

where we have used the Bianchi identity

- **Surface Term**

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- More explicit form

$$\delta I_{CG} = \frac{\ell^2}{64\pi G}\int_{\partial M} d^3x\sqrt{-h}\delta^{[\nu_1\nu_2\nu_3\nu_4]}_{[\mu_1\mu_2\mu_3\mu_4]}(n_{\nu_1}\delta\Gamma^{\mu_1}_{\lambda\nu_2}g^{\mu_2\lambda}W^{\mu_3\mu_4}_{(E)\nu_3\nu_4}- \\ -2\gamma[n_{\nu_1}\delta\Gamma^{\mu_1}_{\lambda\nu_2}g^{\mu_2\lambda}B^{\mu_3}_{\nu_3}+n^{\mu_1}\nabla_{\nu_1}B^{\mu_2}_{\nu_2}\left(g^{-1}\delta g\right)^{\mu_3}_{\nu_3}])$$

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where we have used the Bianchi identity

- More explicit form

$$\delta I_{CG} = \frac{\ell^2}{64\pi G}\int_{\partial M} d^3x\sqrt{-h}\delta^{[\nu_1\nu_2\nu_3\nu_4]}_{[\mu_1\mu_2\mu_3\mu_4]}(n_{\nu_1}\delta\Gamma^{\mu_1}_{\lambda\nu_2}g^{\mu_2\lambda}W^{\mu_3\mu_4}_{(E)\nu_3\nu_4}- \\ -2\gamma[n_{\nu_1}\delta\Gamma^{\mu_1}_{\lambda\nu_2}g^{\mu_2\lambda}B^{\mu_3}_{\nu_3}+n^{\mu_1}\nabla_{\nu_1}B^{\mu_2}_{\nu_2}\left(g^{-1}\delta g\right)^{\mu_3}_{\nu_3}])$$

- For Einstein spaces

$$\delta I_{CG} = \delta I^{(E)}_{ren}$$

Higher-derivative gravity theories

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- Not a suitable candidate for a quantum theory of gravity
- In the low-energy limit of String Theory non-linear terms in the curvature appear
- Interest in higher-derivative theories: gravity with a better UV behavior
- Examples in 3D: Topologically Massive Gravity (TMG), New Massive Gravity (NMG)

Higher-derivative gravity theories in 4D

- Examples in 4D: Conformal Gravity and Critical Gravity

Higher-derivative gravity theories in 4D

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- Four-derivative gravity theories, more chances to be renormalizable

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- Examples in 4D: Conformal Gravity and Critical Gravity
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Higher-derivative gravity theories in 4D

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- More insight on the logarithmic CFTs living at the boundary - AdS / LCFT correspondence

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- Fine-tuned parameters $(\alpha = -3\beta, \beta = -1/2\Lambda)$: massive scalar mode disappears and the massive spin-2 field turns massless$\Longrightarrow$ **Criticality**

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- At the critical point [H. Lu and C.N. Pope, arXiv:1101.1971]

$$I_{critical} = \frac{1}{16\pi G}\int_M d^4x\sqrt{-g}\left[R - 2\Lambda + \frac{3}{2\Lambda}\left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right)\right]$$

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- In particular, the holographic stress tensor vanishes for Einstein spaces
  `N. Johansson, A. Naseh and T. Zojer, [arXiv:1205.5804 ]`

Asymptotically AdS spacetimes with Log Terms

- The conformal boundary of *ALAdS* spacetimes, located at $\rho = 0$ (Fefferman-Graham expansion)

$$ds^2 = \frac{\ell^2}{4\rho^2}\ d\rho^2 + \frac{1}{\rho} g_{ij}\left(\rho, x\right)\ dx^i dx^j$$

$$g_{ij}(x, \rho) = g_{(0)ij}(x) + \rho g_{(2)ij}(x) + \cdots + \rho^{d/2} g_{(d)} + \rho^{d/2} h_{(d)} \log \rho$$

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- Critical Gravity in 4D:

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- Holographic information for Log CFT
  [G. Anastasiou and R.O., work in progress]

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