Holographic self-tuning of the cosmological constant

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work with Elias Kiritsis and Christos Charmousis, 1704.05075

Introduction: the CC and QFT

The Cosmological Constant (CC) problem arises as a clash between classical GR and QFT (in the modern effective FT sense).

In classical GR:

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$$G_{\mu\nu} = \Lambda_{eff} \, g_{\mu\nu}, \quad \Lambda_{eff} = \Lambda_0 + 8\pi G_N \mathcal{E}_{vac}.$$

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Self-tuning: any mechanism which allow flat spcacetime solutions for generic values of E_{vac}.

Content of this talk

- Self-tuning possible in the a general framework of a dilatonic, asymmetric braneworld with general 2-derivative induced terms.
- Model based on holographic model building: dual of 4-dimensional, strongly coupled, non-gravitational fundamental theory. Previously explored around 2000: Arkani-Hamed *et al.* '00; Kachru,Schulz,Silverstein '00; Csaki *et al*, '00; All presented problems due to singularities or absence of localized 4d gravity on the brane

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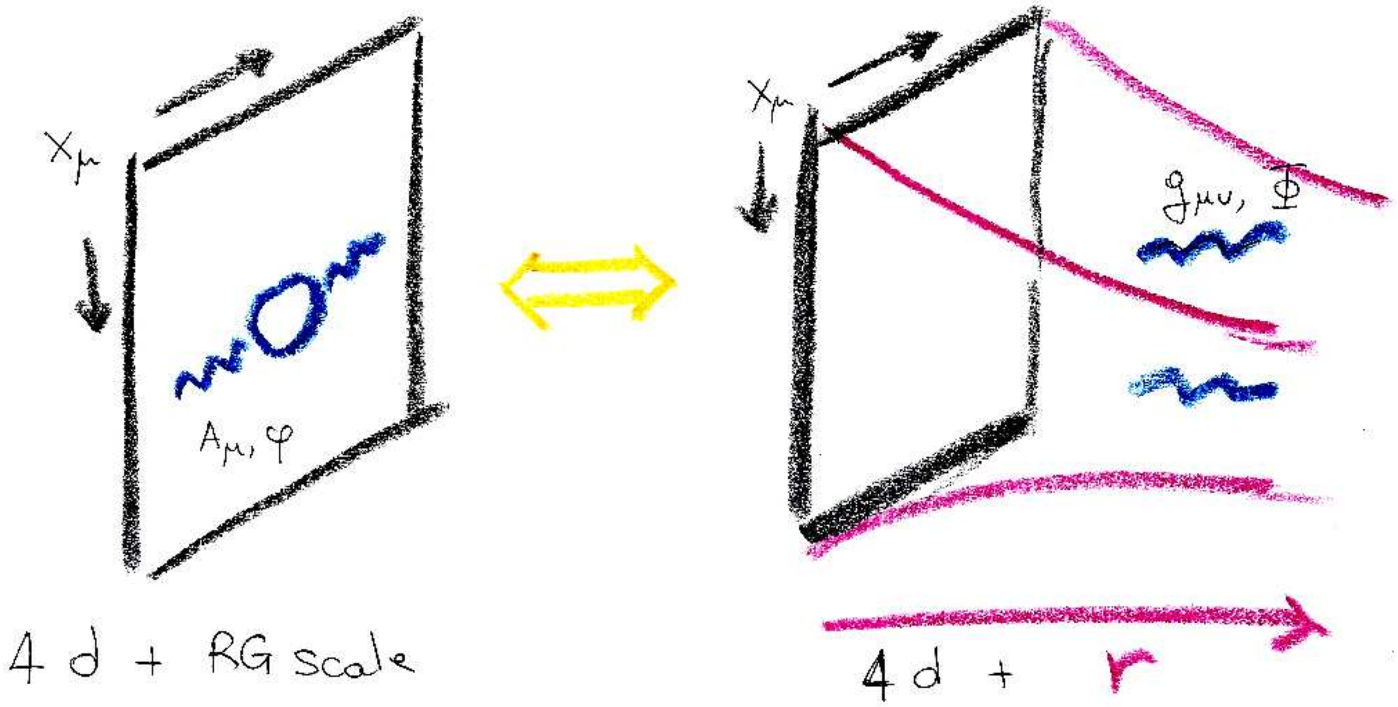
- Outline
  - AdS/CFT micro-review
  - Setup
  - Flat vacua: self-tuning
  - Tensor perturbations: emergent braneworld gravity
  - Scalar perturbations: stability
  - Perspectives

AdS/CFT detour

The AdS/CFT duality: conjecture that certain quantum field theories are equivalent to theories of gravity in higher dimensions Maldacena '98.



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- Conformal field theory in d dimension $\Leftrightarrow$
  Anti de Sitter spacetime AdS_{d+1}

$$ds^2 = du^2 + e^{-2u/\ell}\eta_{\mu\nu}dx^\mu dx^\nu$$

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- x^μ: QFT coordinates; u dual to energy scale $E \propto e^{-u/\ell}$.
- bulk scalar field $\varphi(u) \Leftrightarrow$ running coupling $g(E)$. The corresponding holographic RG-flow geometry breaks conformal invariance (except at fixed points where $\dot{\varphi} = 0$).

$$ds^2 = du^2 + e^{A(u)}\eta_{\mu\nu}dx^\mu dx^\nu, \quad \varphi = \varphi(u).$$

$$E \propto e^{A(u)}$$

Setup

Consider a 4d QFT with a UV conformal fixed point, made out of:

1. A strongly coupled large-N CFT, deformed by a relevant operator;
2. The weakly coupled Standard Model fields;
3. Some heavy messangers with mass scale Λ, coupling the first two.

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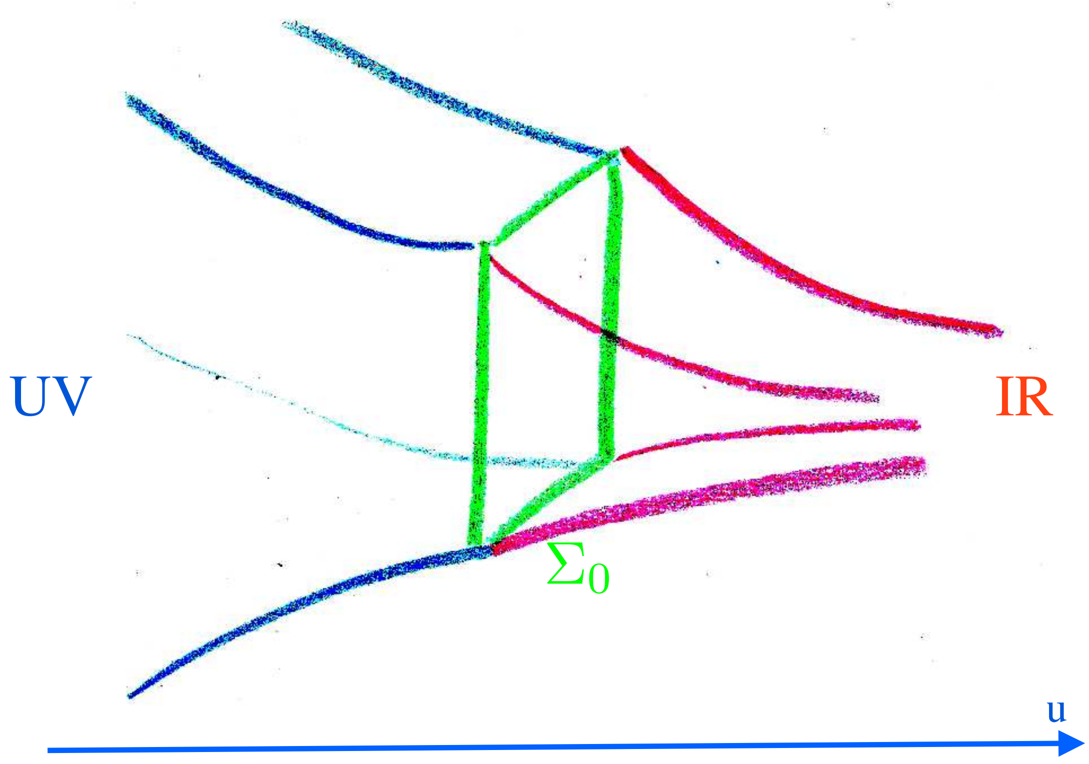
semi-holographic description:

- Describe the strongly coupled large-N theory by a 5d gravity dual with the metric g_{ab} and some bulk scalar fields φ_i, dual to the operators that drive the CFT to the IR.
- The weakly coupled SM fields have a standard field-theoretical description, and they sit on a 4d defect in th 5d dual geometry.

Semi-holographic setup

$$S = M^3 \int d^4x \int du \sqrt{-g} \left[R - \frac{1}{2} g^{ab} \partial_a \varphi \partial_b \varphi - V(\varphi) \right]$$ ← 5d Gravity dual of 4d CFT

$$+ \int_{\Sigma_0} d^4\sigma \sqrt{-\gamma} \mathcal{L}(\psi_i, H, W^a, \ldots, \varphi, \gamma_{\mu\nu}).$$ ← Weakly coupled 4d QFT



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- Action is the most general up to two derivates preserving 4d diffeos.

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↑ Localized Effective action induced by quantum effects of weakly coupled QFT

(up to two derivative in the bulk fields)

We take this class of actions as the starting point and the definition of our model

The unknown functions appearing in the localized action can be taken as a phenomenologicalinput or motivated by weakly coupled calculation.

work in progress with E. Kiritsis and L. Witkowski

Field equations and matching conditions

$$S = M^3 \int d^4x \int du \sqrt{-g} \left[R - \frac{1}{2} g^{ab} \partial_a \varphi \partial_b \varphi - V(\varphi) \right]$$

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Einstein equations + Israel junction conditions ($[\ ] \equiv$ jump across Σ_0):

$$G_{ab} = \frac{1}{2} \partial_a \varphi \partial_b \varphi - \frac{1}{2} g_{ab} \left(\frac{1}{2} g^{cd} \partial_c \varphi \partial_d \varphi + V(\varphi) \right),$$

$$\left[\gamma_{\mu\nu} \right] = \left[\varphi \right] = 0; \quad \left[K_{\mu\nu} - \gamma_{\mu\nu} K \right] = \frac{1}{\sqrt{-\gamma}} \frac{\delta S_{\Sigma_0}}{\delta \gamma^{\mu\nu}}; \quad \left[n^a \partial_a \varphi \right] = -\frac{1}{\sqrt{-\gamma}} \frac{\delta S_{\Sigma_0}}{\delta \varphi}$$

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Self tuning if $\exists$ solutions with flat defect for generic $W_B \sim \Lambda^4$.

Bulk equations

$$S_5 = M^3 \int d^4x \int du \sqrt{-g} \left[R - \frac{1}{2} g^{ab} \partial_a \varphi \partial_b \varphi - V(\varphi) \right]$$

Vacuum (Poincaré invariant) solutions:

$$ds^2 = du^2 + e^{2A(u)} \eta^{\mu\nu} dx_\mu dx_\nu, \qquad \varphi = \varphi(u)$$

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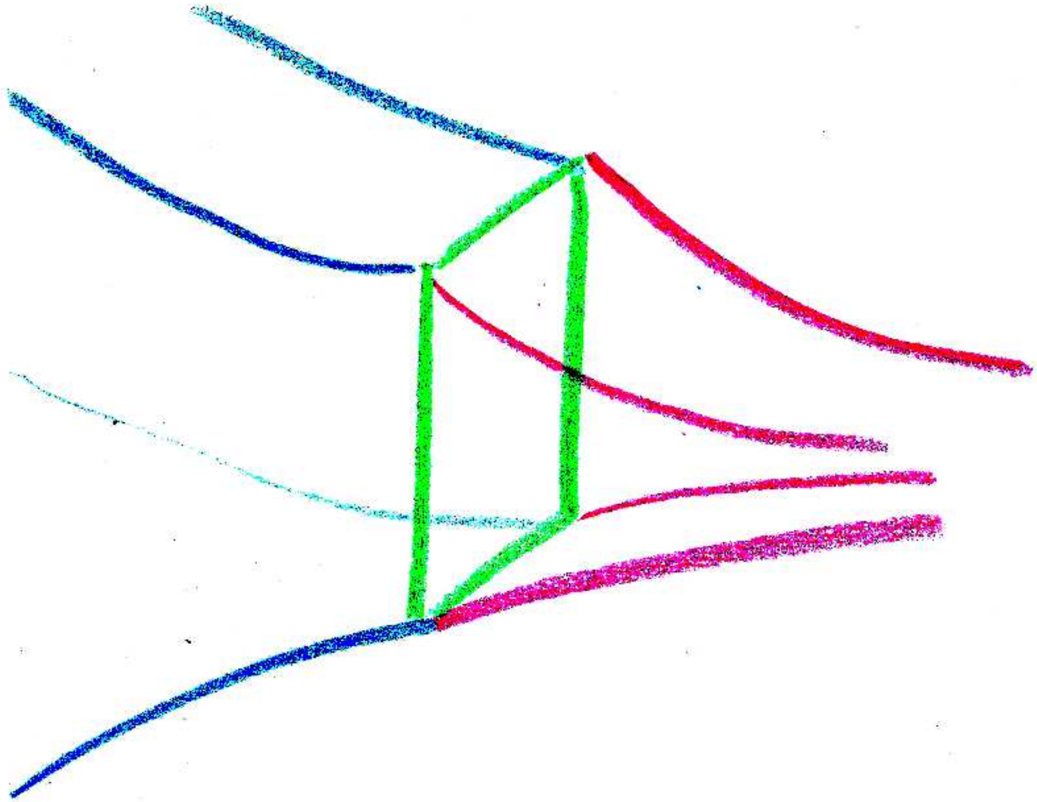
One has to *solve independently on each side* of the defect (at $u = u_0$), and glue the solutions using Israel junction conditions:

$$\Big[A\Big] = \Big[\varphi\Big] = 0; \quad \Big[\dot{A}\Big] = -\frac{1}{6} W_B(\varphi(u_0)); \quad \Big[\dot{\varphi}\Big] = \frac{dW_B}{d\varphi}(\varphi(u_0))$$

Vacuum Geometry



$A_{UV}(u), \varphi_{UV}(u)$ $A_{IR}(u), \varphi_{IR}(u)$

$e^{A_{UV}} \to +\infty, \ \varphi_{UV} \to 0$

UV-AdS boundary

$e^{A_{IR}} \to 0, \ \varphi_{IR} \to \varphi_*$

Interior of IR-AdS space

Superpotential

Write Einstein's equations as first order flow equations, with an auxiliary scalar function $W(\varphi)$ $\quad$ $({}' = d/d\varphi)$:

$$\dot{A} = -\frac{1}{6}W(\varphi) \qquad \dot{\Phi} = W'(\varphi),$$

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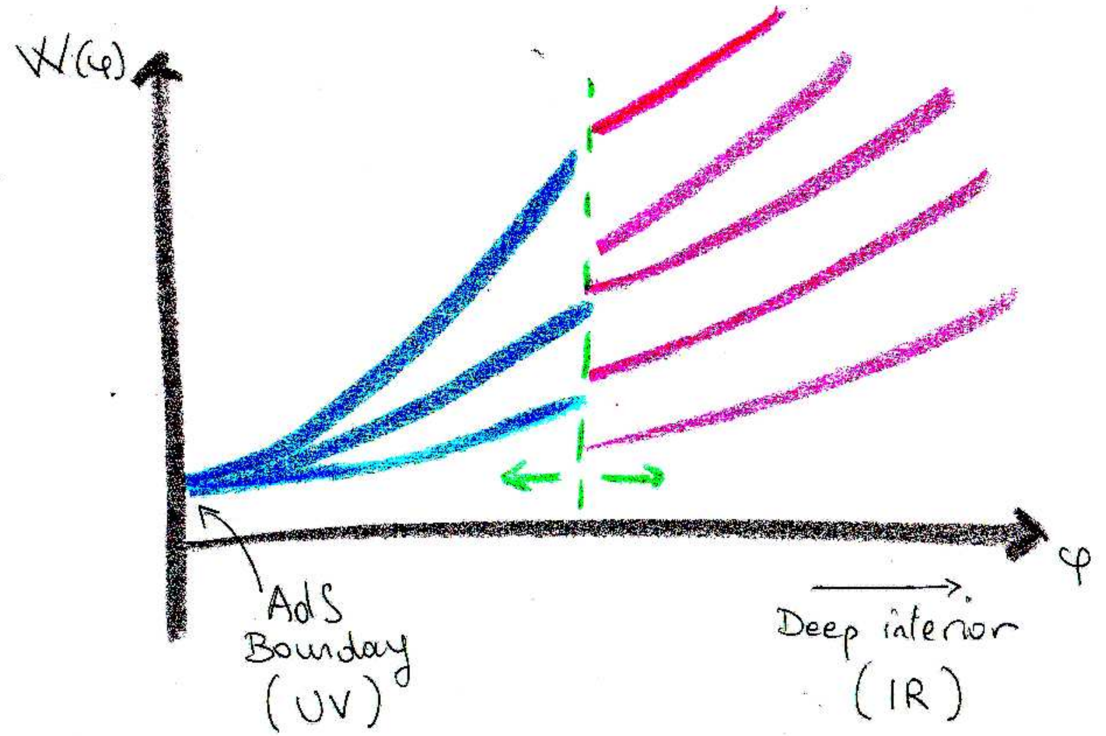
$$W(\varphi) = \begin{cases} W^{UV}(\varphi) & \varphi < \varphi_0 \\ W^{IR}(\varphi) & \varphi > \varphi_0 \end{cases}$$

- On each side of the interface ($\varphi = \varphi_0$), W is determined by one integration consntant C.

Junction conditions for the superpotential



Junction conditions take a simple form:

$$W^{IR}(\varphi_0) - W^{UV}(\varphi_0) = W_B(\varphi_0), \quad \frac{dW^{UV}}{d\varphi}(\varphi_0) - \frac{dW^{IR}}{d\varphi}(\varphi_0) = \frac{dW_B}{d\varphi}(\varphi_0)$$

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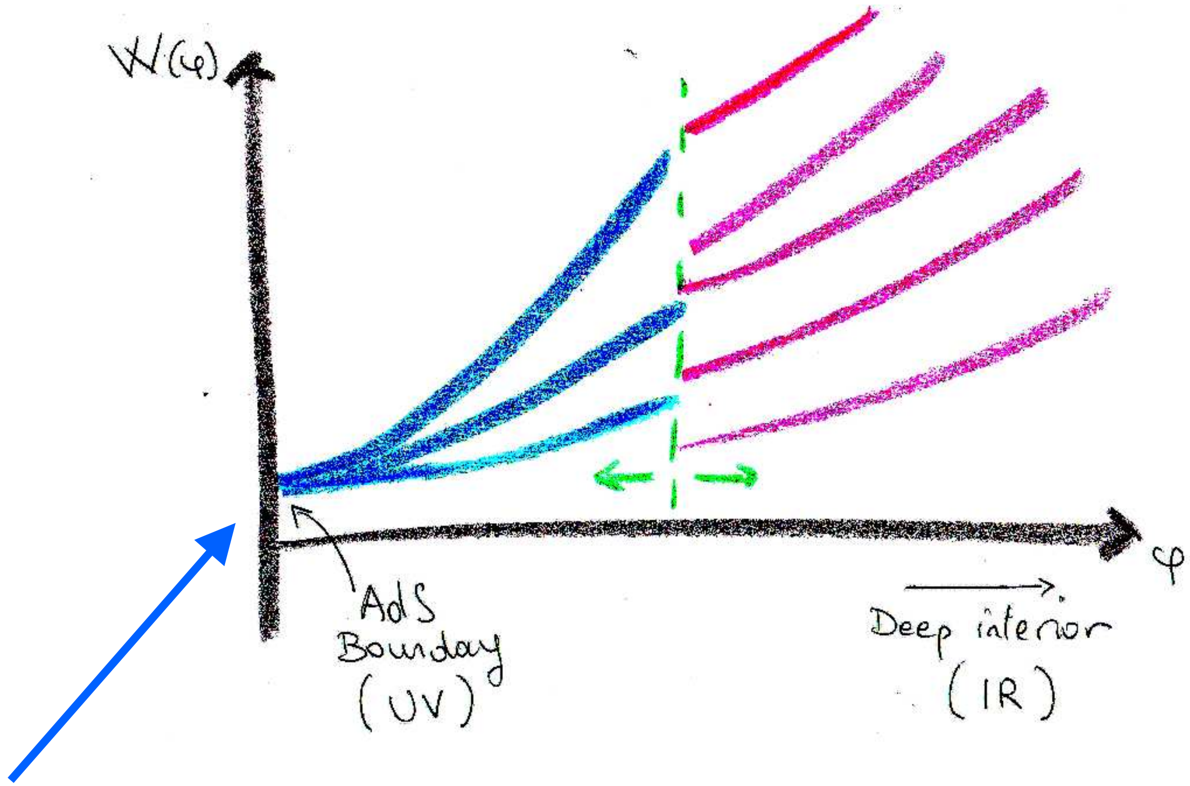

UV side: Solutions arrive at the AdS fixed point for all values of the integration constant C_{UV}: UV fixed point is an attractor.

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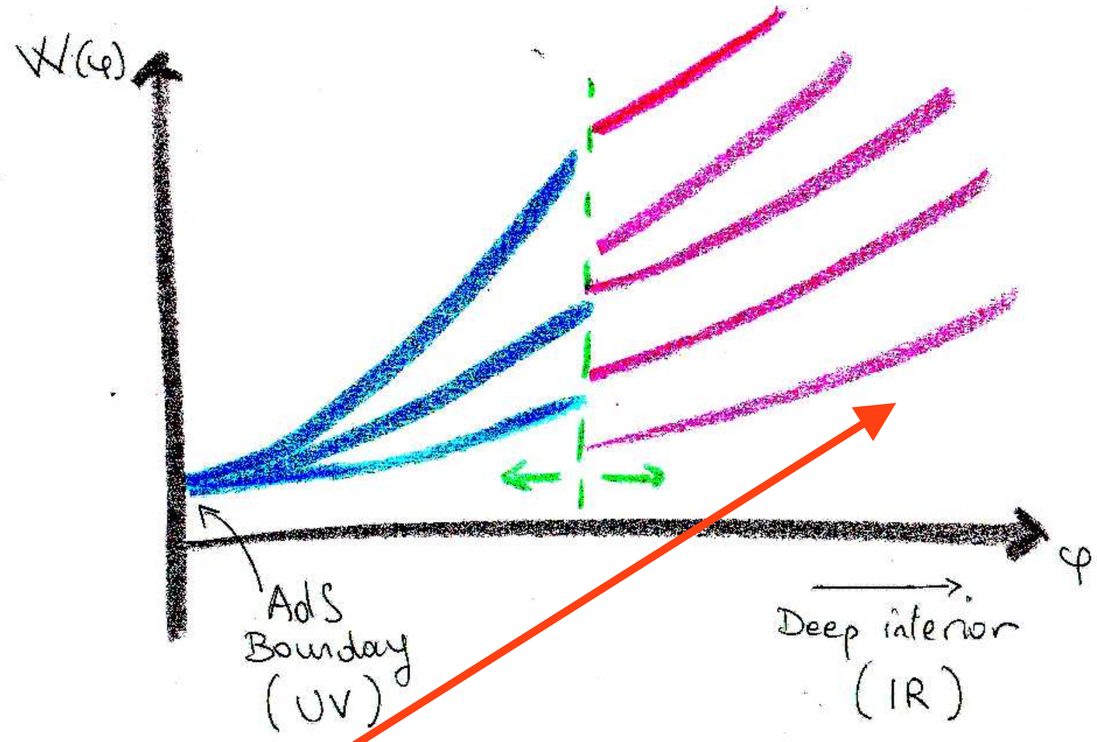

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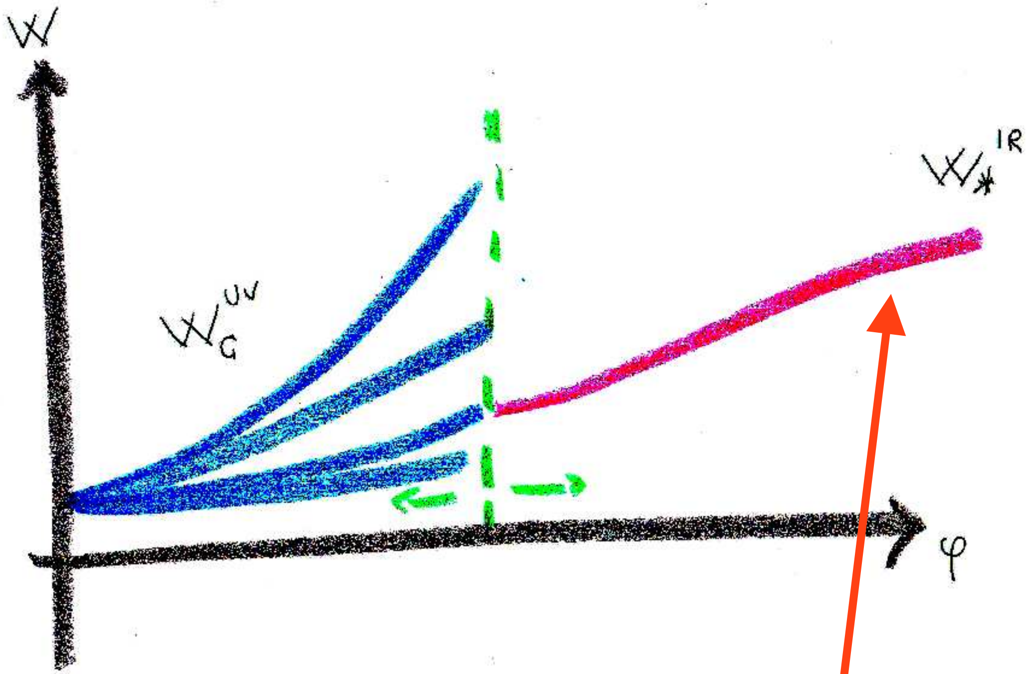
IR side: Only certain IRs are acceptable (e.g. IR AdS fixed point) This picks out a single solution W_*^{IR} and fixes $C_{IR} = C_*$

IR Selection



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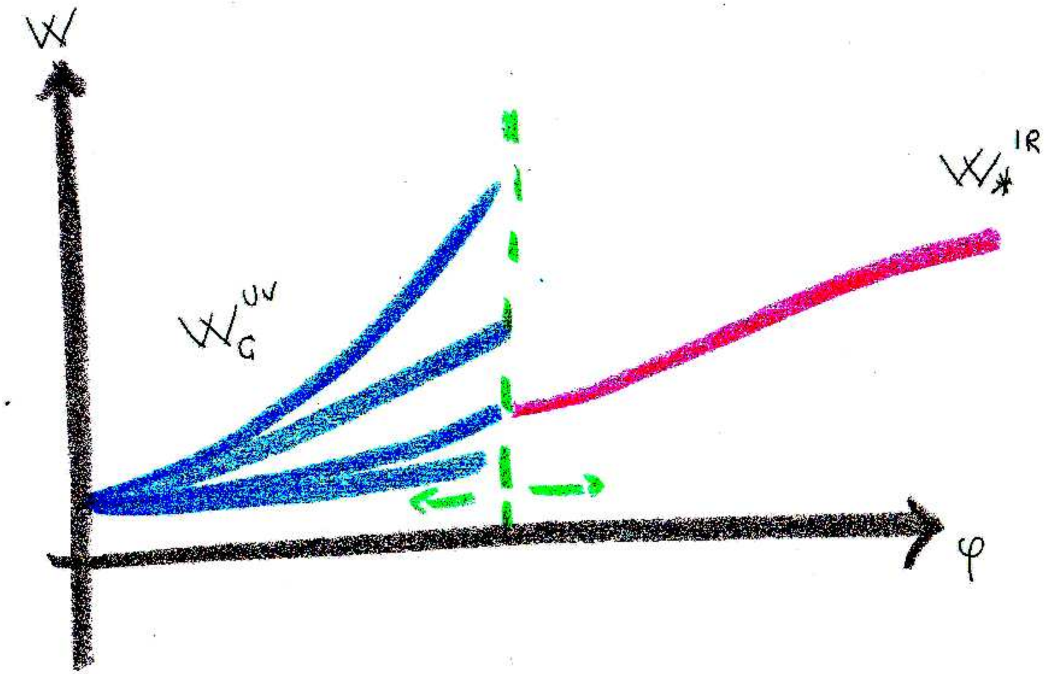
Equilibrium solution



$$W^{UV}(\varphi_0) = W_*^{IR}(\varphi_0) - W_B(\varphi_0), \qquad \frac{dW^{UV}}{d\varphi}(\varphi_0) = \frac{dW_*^{IR}}{d\varphi}(\varphi_0) - \frac{dW_B}{d\varphi}(\varphi_0)$$

Two equations for two unknowns C_{UV}, φ_0. Generically there exist a unique (or a discrete set of) solutions with C_{UV}, φ_0 determined.

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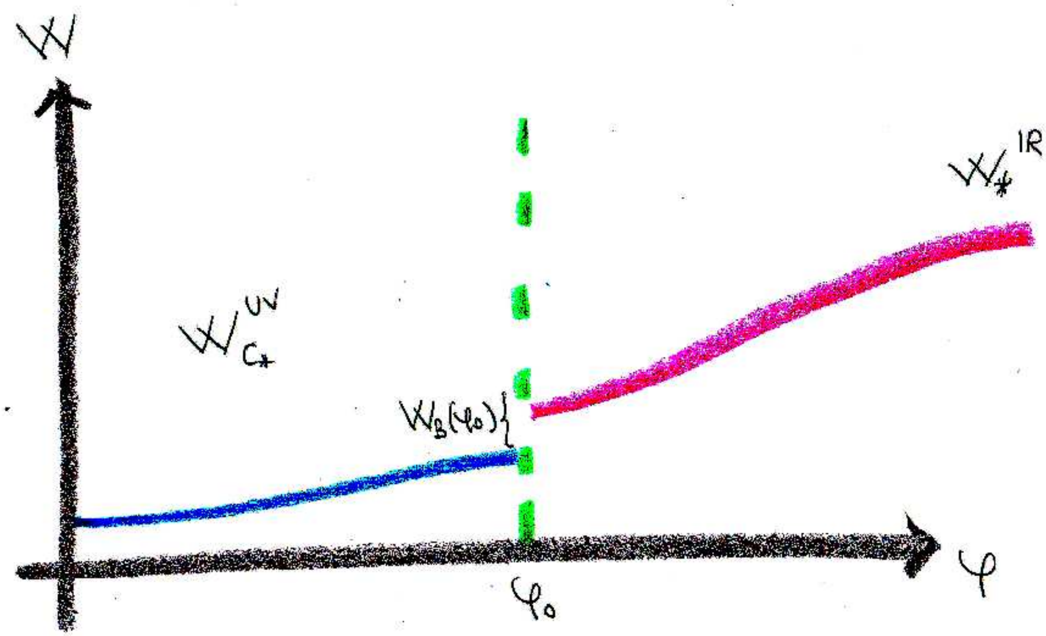

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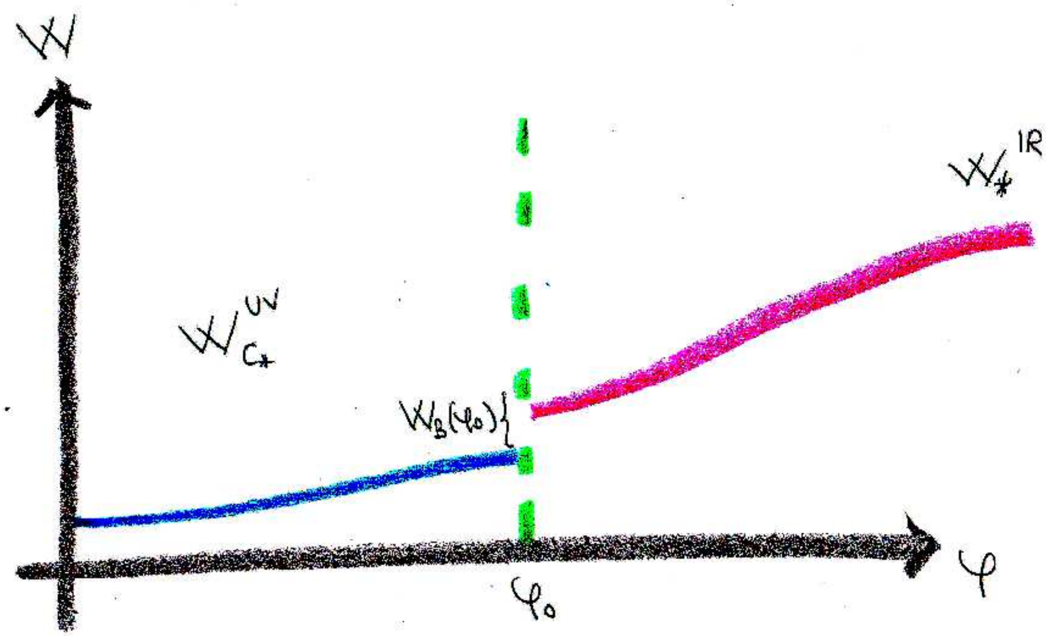

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For generic brane vacuum energy $\sim \Lambda^4$, geometry (VEVs and brane position) adjusts so that the brane is flat and the UV glues to the regular IR through the junction (*self-tuning*).

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- The induced Einstein term on the defect allows for the existence of a 4d-like graviton resonance (Dvali,Gabadadze,Porrati, '00)

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- Bulk curvature $\Rightarrow$ 4d massive graviton at *very* large distances.

Scales of braneworld gravity

Two competing scales:

1. "DGP" transition length: $r_c \approx U(\varphi_0)$
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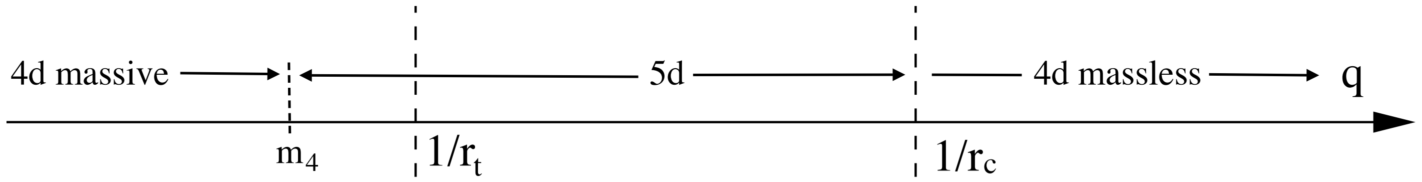

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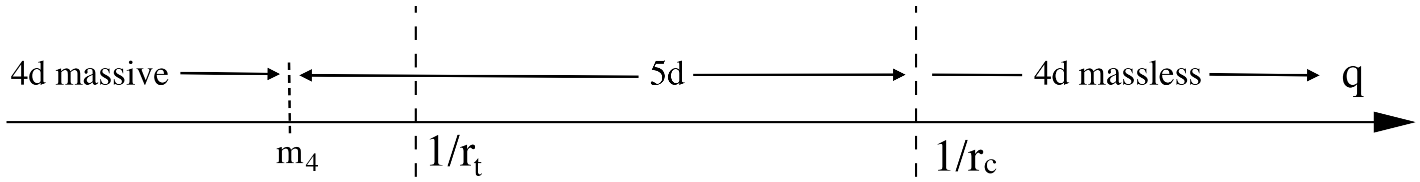

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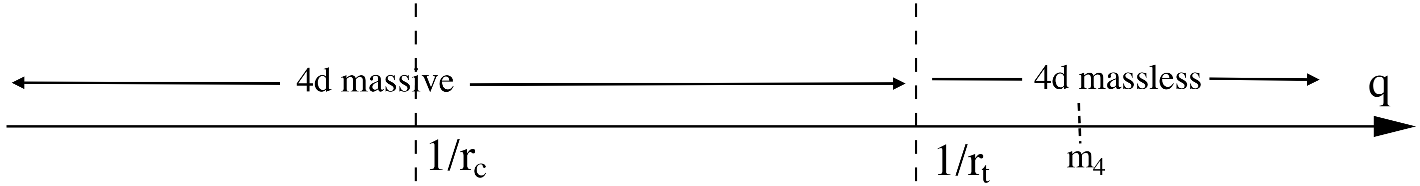

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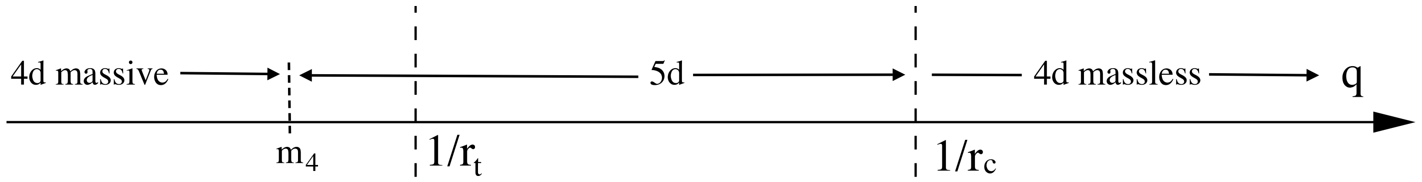

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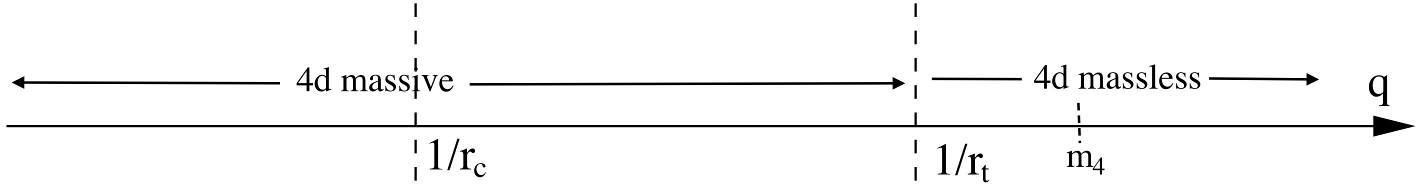

$$M_p^2 \approx M^3 U_0, \qquad m_g^2 \approx \frac{\mathcal{R}_0}{U_0}$$

Scalar perturbations

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1.

$$\tau_0 > 0, \quad Z_0 > 0, \quad Z_0\tau_0 > 36\left(\frac{dU_B}{d\varphi}\Big|_{\varphi_0}\right)^2$$

$$\tau_0 \equiv 6\left(6\frac{W_B}{W_{IR}W_{UV}} - U\right)_{\varphi_0}, \quad Z_0 \equiv Z(\varphi_0)$$

$\Rightarrow$ No ghost instabilities

Scalar perturbations

- Determine whether vacuum solution (flat brane at $r = r_0$) is stable.
- Possible light scalar mediated interactions (fifth force, violations of equivalence principle) $\Rightarrow$ pheno constraints.
- Analysis of linear flucutations show that there exist conditions on the background solution which guarantee stability.

2.

$$\tilde{\mathcal{M}}^2 \equiv \left(\frac{d^2 W_B}{d\varphi^2}(\varphi_0) - \left[\frac{d^2 W}{d\varphi^2} \right]_{UV}^{IR} \right) \geq 0$$

$\Rightarrow$ No tachyonic instabilities.

Conclusion and outlook

We constructed a framework where Self-tuning of the CC is generically realized. Challenge now is model-building: construct phenomenologically viable model.

- Acceptable values of M_p, r_c, m_g given large UV cutoff;
- Compliance with stability requirements;
- Deal with vDVZ discontinuity (Role of non-linearities, Veinshtein mechanism);
- Avoidance of fifth force constraints;

Conclusion and outlook

We constructed a framework where Self-tuning of the CC is generically realized. Challenge now is model-building: construct phenomenologically viable model.

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If this all goes through, one can do more phenomenology:

- Add SM and Higgs field (see Lukas Witkowski's talk)
- Study the space of solutions: non-flat brane, time-dependent solutions (cosmology) (ongoing work with Lukas Witkowski and Jewek Ghosh)
- The framework can potentially addess EW hierarchy problem (via stabilized warped extra dimensions) and late-time acceleration (cosmology close to the equilibrium position).

Example

$$V(\varphi) = -12 - \left(\frac{\Delta(4-\Delta)}{2} - \frac{b^2}{4}\right)\varphi^2 - V_1 \sinh^2 \frac{b\varphi}{2},$$

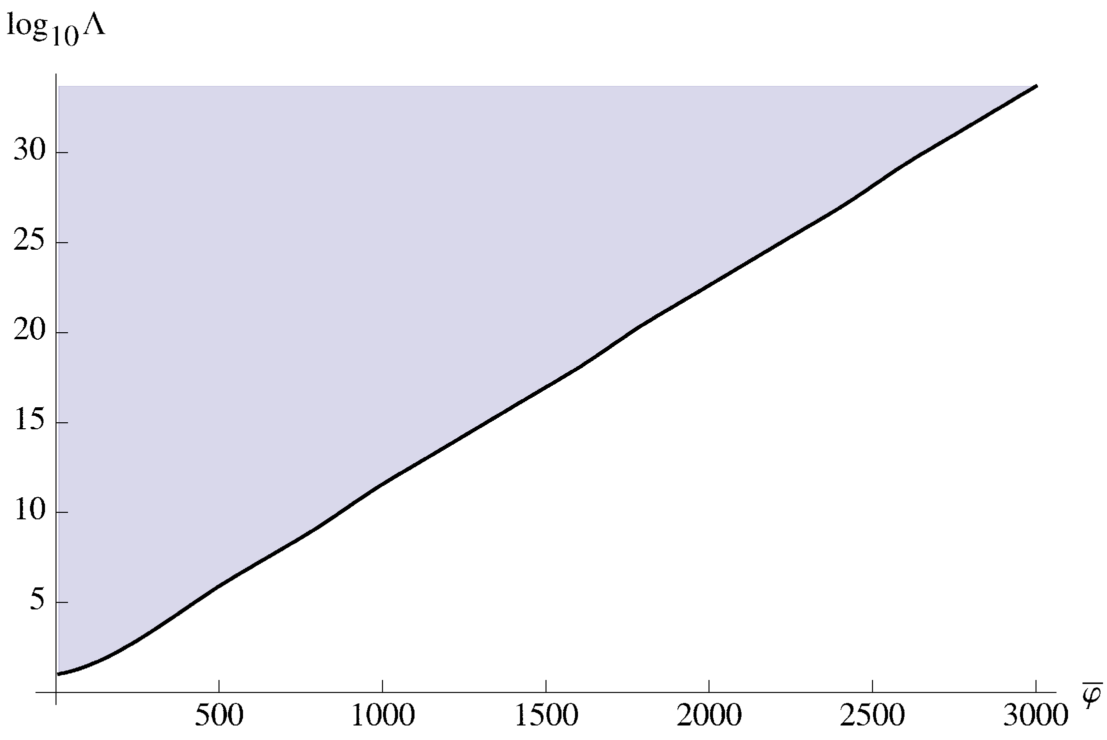
- supports an AdS fixed point at $\varphi = 0$ ($\ell_{UV} = 1$)
- good IR solution:

$$W_{IR}(\varphi) \sim \sqrt{\frac{2}{(32/3) - b^2}} \exp \frac{b\varphi}{2}, \qquad \varphi \to +\infty.$$

How large can Λ be?

$$W_B(\varphi) = \Lambda^4 \left[-1 - \frac{\varphi}{s} + \left(\frac{\varphi}{s}\right)^2 \right]$$

$$b = \frac{1}{\sqrt{6}},\ \Delta = 3,\ V_1 = 1$$



$$\varphi_0 \simeq \bar{\varphi} \approx 1.6\, s$$

Effective 4d Green's function

Introuce tensor perturbations:

$$\delta g_{\mu\nu} = e^{2A(r)} h_{\mu\nu}(r, x^{\alpha}), \quad h^{\mu}_{\mu} = \partial^{\mu} h_{\mu\nu} = 0$$

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Solve classical linearized equation for tensor fluctuations with localized source:

$$h_{\mu\nu}(x, r) = \int d^4x G_{\mu\nu}{}^{\rho\sigma}(x - x'; r, r_0) T_{\rho\sigma}(x', r_0),$$

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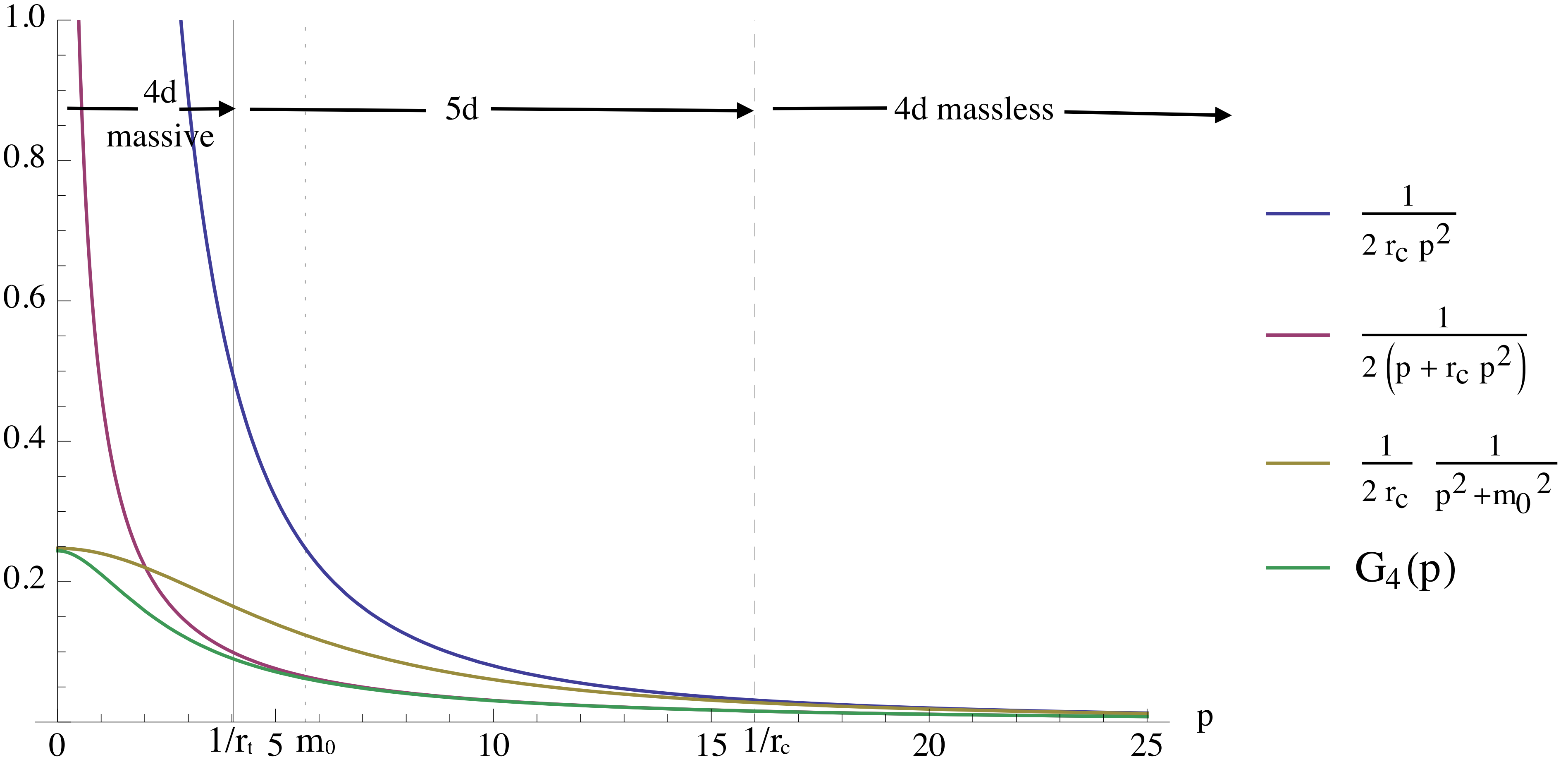
Tree-level interaction described in purely 4d terms by an effective Green's function:

$$S_{int}(T) = \int \frac{d^4p}{(2\pi)^4} \tilde{G}_4(p) \left[T_{\mu\nu}(p) T^{\mu\nu}(-p) - \frac{1}{3} T(p) T(-p) \right]$$

$$G_4(x) \equiv G(x, r_0, r_0).$$

4d-5d transition

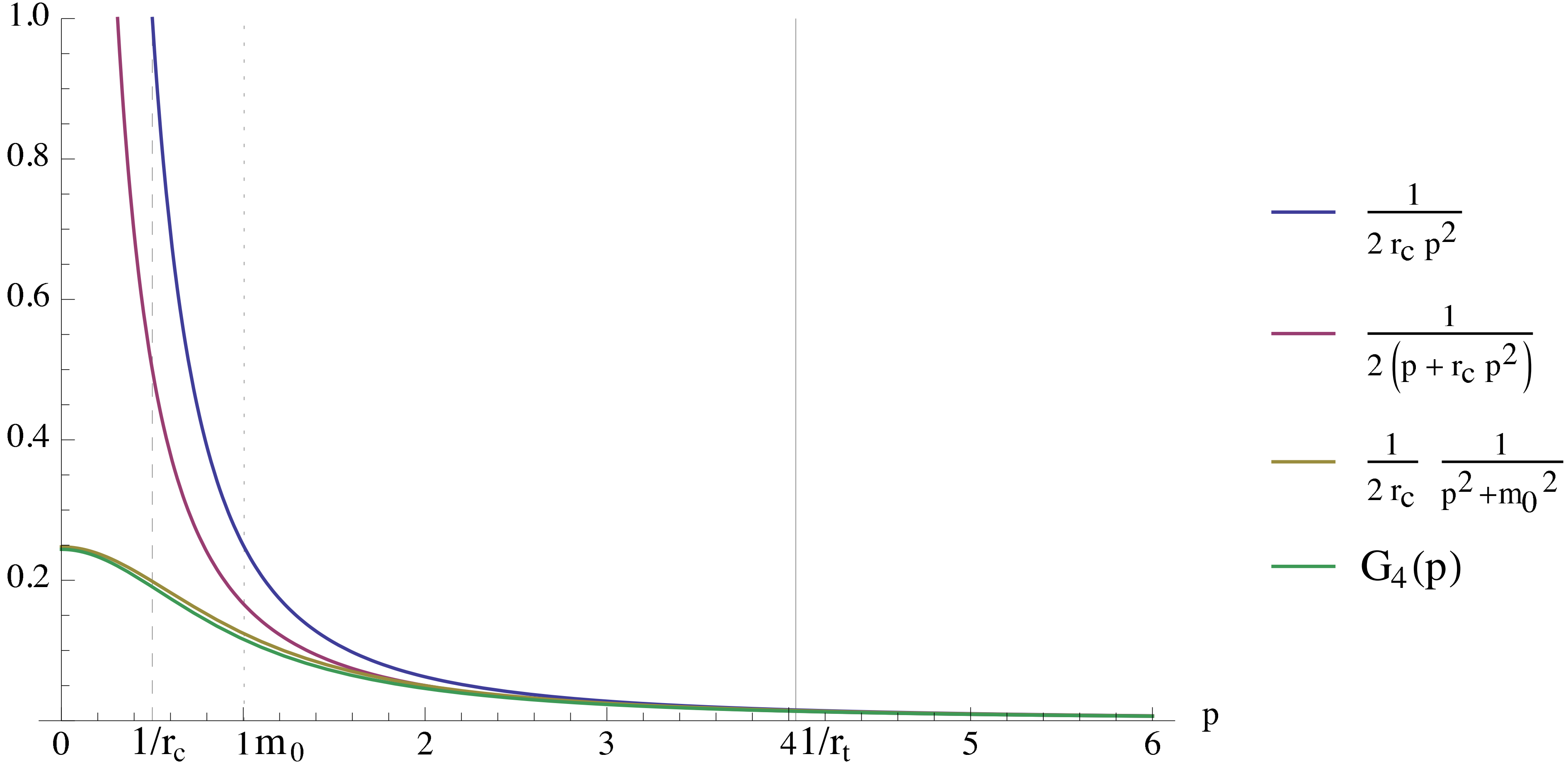
$r_c < r_t$: DGP-like transition, at intermediate distances.



$$r_c = U_0, \; r_t = \frac{e^{-A_0}}{\mathcal{R}_0}, \quad M_p^2 \approx M^3 U_0, \quad m_0^2 \approx \frac{\mathcal{R}_0}{U_0},$$

Massless/Massive gravity transition

$r_c > r_t$ massive graviton propagator all the way.



$$r_c = U_0,\ r_t = \frac{e^{-A_0}}{\mathcal{R}_0}, \quad M_p^2 \approx M^3 U_0, \quad m_0^2 \approx \frac{\mathcal{R}_0}{U_0},$$

Looking for solutions

Junction conditions can be rewritten as a non-linear equation for φ_0:

$$-\frac{Q^2}{2}\Big(W^{IR}(\varphi_0) - W^B(\varphi_0)\Big)^2 + \frac{1}{2}\left(\frac{dW^{IR}}{d\varphi} - \frac{dW^B}{d\varphi}\right)^2_{\varphi_0} = V(\varphi_0),$$

$$Q \equiv \sqrt{\frac{d}{2(d-1)}}$$

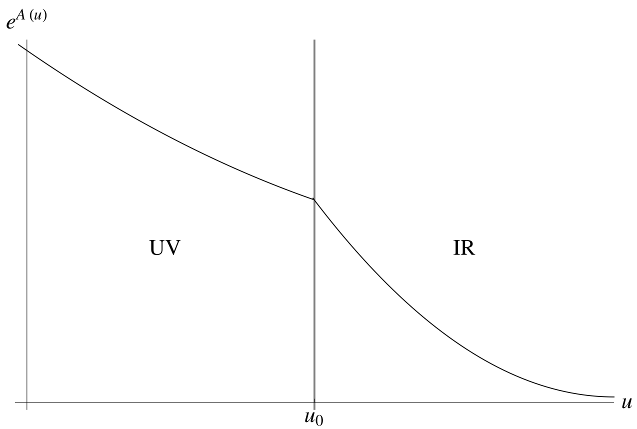
V, W^B and W^{IR} are *fixed functions* of φ.

1. Solve for φ_0
2. Solve superpotential equation for $W_{UV}(\varphi)$ with initial condition:

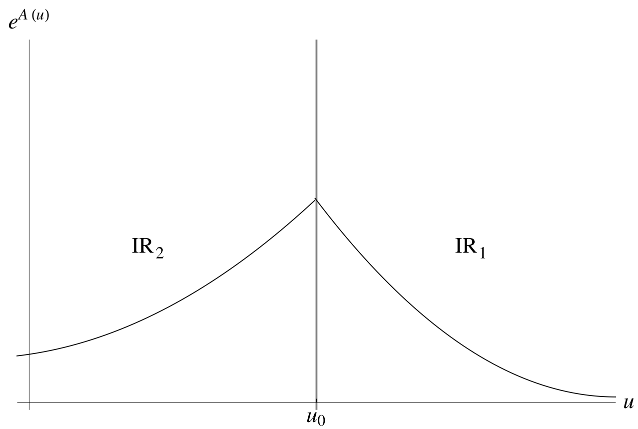
$$W^{UV}(\varphi_0) = W^{IR}(\varphi_0) - W^B(\varphi_0)$$

Consistent self-tuining

Two possibilities:



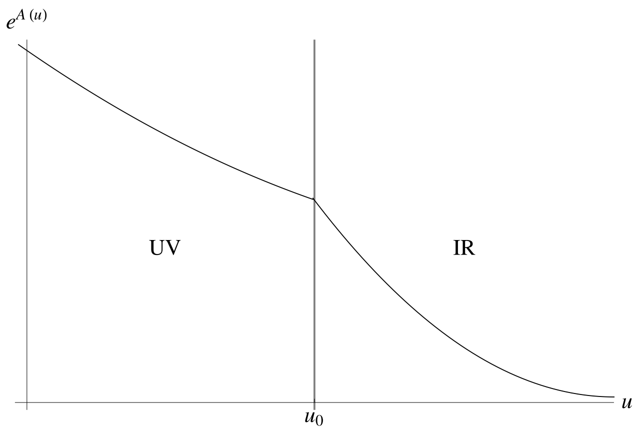
$W_{UV} > 0$



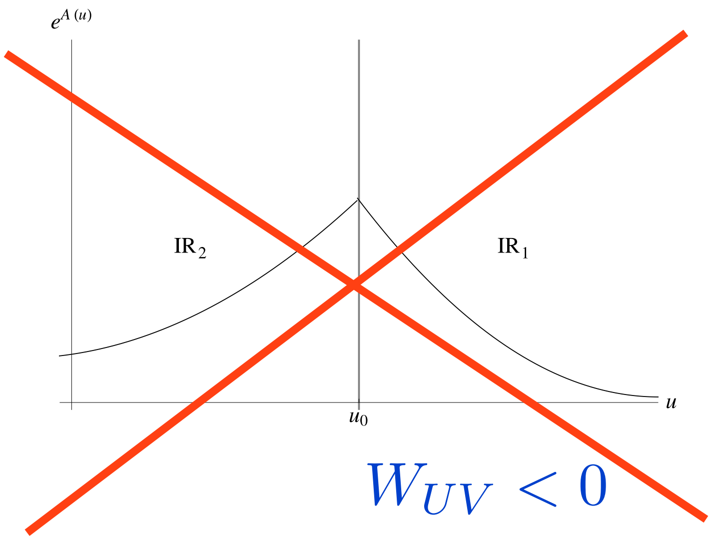
$W_{UV} < 0$

Consistent self-tuning

Two possibilities:



$W_{UV} > 0$



$W_{UV} < 0$

Needs fine tuning of the brane potential to join two "special" solutions

Cfr. Randall-Sundrum setup

Genericity

As we will see, it is desirable (but not strictly necessary) that $W_B(\varphi_0) > 0$, i.e. $0 < W_{UV}(\varphi_0) < W_{IR}(\varphi_0)$ (in this case, the solution is manifestly ghost-free).
It turns out that that for such solutions to exist, it is enough that

$$W(\bar{\varphi}) = 0, \qquad W'(\bar{\varphi}) > 0$$

for some $\bar{\varphi}$. Then the equations are solved, with $W_B(\varphi_0) > 0$, for:

$$\varphi_0 \approx \bar{\varphi} + \left.\frac{\partial_\varphi (W_{IR}^2)}{4|V|}\right|_{\varphi=\bar{\varphi}}$$

provided:

$$\frac{W_B(\varphi_0)}{W_{IR}(\varphi_0)} \ll 1$$

Relating scales

- We can relate bulk parameters M, e^{A_0}, ℓ_{UV} to those of the dual field theory N, g_0, Δ:

$$e^{A_0} \propto (\ell_{UV} g_0)^{1/(d-\Delta)}, \quad (M\ell_{UV})^3 \propto N^2$$

- Bulk superpotentials set the scale of the bulk curvature scale: $W(\varphi(u)) \propto \mathcal{R}(u)$

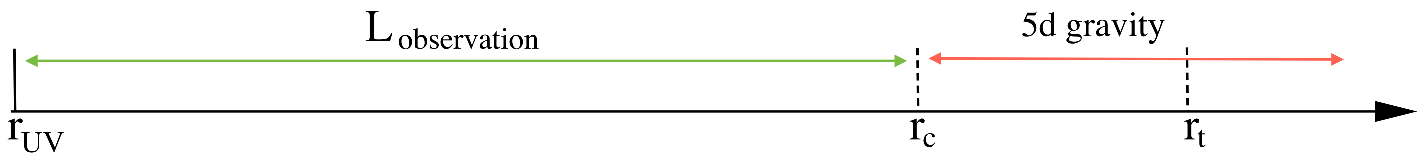
$$\Rightarrow \qquad \frac{M}{\mathcal{R}_0} \sim \frac{N^{2/3}}{\ell_{UV} W_{UV}(\varphi_0)}$$

- The scale of brane potentials is set by the UV cut-off Λ:

$$W_B \sim \frac{\Lambda^4}{M^3}, \qquad U_B \sim \frac{\Lambda^2}{M^3}$$

DGP scenario

Requires $r_t > r_c$



- Gravity must be modified at cosmological distances:

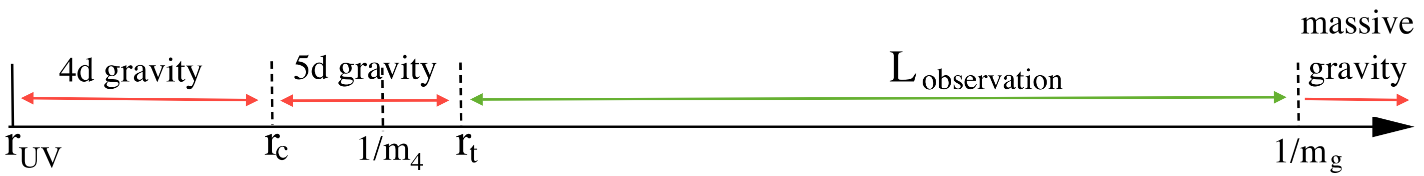
$$M_p r_c = \left(\frac{MU_0}{4}\right)^{3/2} \approx \left(\frac{\Lambda}{M}\right)^3 u^3(\varphi_0) \approx 10^{60}$$

- The assumption $r_t > r_c$ translates into:

$$e^{-A_0} U_0 \mathcal{R}_0 \lesssim 1 \quad \Rightarrow \quad \left(\frac{\Lambda}{M}\right)^2 u(\varphi_0)\, \frac{\ell_{UV} W_{UV}(\varphi_0)}{N^{2/3} \left(\ell_{UV} g_0\right)^{\frac{1}{(d-\Delta)}}} \lesssim 1$$

Massive gravity scenario 1

Requires $r_t > r_c$



- Large distance modification (graviton mass) must be at cosmological scales

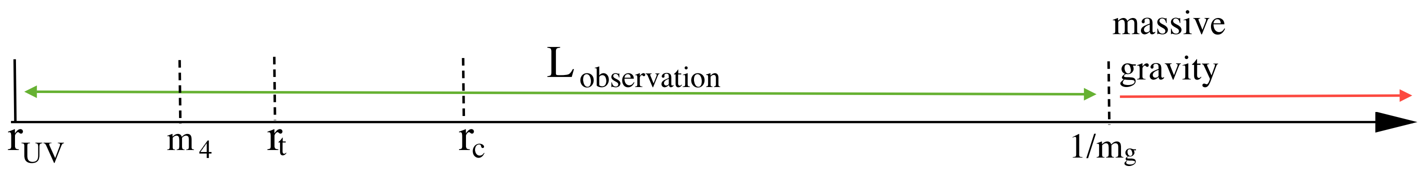
$$\frac{m_g}{M_p} \lesssim 10^{-60} \quad \Rightarrow \left(\frac{M}{\Lambda}\right)^2 \frac{(\ell_{UV} W_{UV}(\varphi_0))^{1/2}}{u(\varphi_0)} \frac{1}{N^{1/3}} \quad < \quad 10^{-60}$$

- Short distance modification must be below (tenths of) mm:

$$r_t M_p \lesssim 10^{-30} \quad \Rightarrow \quad \left(\frac{M}{\Lambda}\right) \frac{\ell_{UV} W_{UV}(\varphi_0)}{u^{1/2}(\varphi_0)\,(\ell_{UV} g_0)^{\frac{1}{(d-\Delta)}}\, N^{2/3}} \quad > \quad 10^{-30}$$

Massive gravity scenario 2

Alternatively, $r_t < r_c$ (no DGP regime)



- Same large scale condition:

$$\frac{m_g}{M_p} \lesssim 10^{-60} \quad \Rightarrow \left(\frac{M}{\Lambda}\right)^2 \frac{(\ell_{UV} W_{UV}(\varphi_0))^{1/2}}{u(\varphi_0)} \frac{1}{N^{1/3}} \quad < \quad 10^{-60}$$

- No short distance modification until the UV cut-off.

Scalar-mediated interaction

Define metric and dilaton sources:

$$T_{\mu\nu} = -\frac{2}{\sqrt{\gamma}}\frac{\delta S_m}{\gamma^{\mu\nu}}, \quad O = \frac{\delta S_m}{\delta\varphi}.$$

Interaction between brane-localized sources:

$$S_{int} = -\frac{1}{2}\int \frac{d^4q}{(2\pi)^4}\mathcal{T}^\dagger(q)G_s(q)\mathcal{T}(-q), \qquad \mathcal{T} \equiv \left(T^\mu_\mu,\ O\right)$$

$$G_s(q) \equiv \frac{1}{2M^3}P\left[\Sigma\left(\Gamma_1 + q^2\Gamma_2\right) + \mathcal{D}^{-1}(r_0;q)\right]^{-1}P^\dagger$$

$$P \equiv -\frac{z_{IR}z_{UV}}{[z]}\begin{pmatrix} \frac{1}{z_{IR}} & -\frac{1}{z_{UV}} \\ 1 & 1 \end{pmatrix}.$$

- Modes coupling to O can be parametrically heavy, $m \simeq \mathcal{M}$.
- Modes coupling to T remain light.

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Localized term Bulk contribution

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localized mass (Γ_1) — localized kinetic term ($q^2\Gamma_2$)

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DGP regime (short distance)

Overall interaction for light modes in the 4d regime:

$$\mathcal{V}(q) \simeq \frac{1}{q^2}\left[\frac{1}{2M^3U_0}\left(T_{\mu\nu}(q)T^{\mu\nu}(-q) - \frac{1}{3}T^{\mu}_{\mu}(q)T^{\nu}_{\nu}(-q)\right) + \frac{1}{2M^3\tau_0}T^{\mu}_{\mu}(q)T^{\nu}_{\nu}(-q)\right]$$

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Something interesting happens if

$$\left.\frac{W_B}{W_{IR}W_{UV}}\right|_{\varphi_0} \ll U_0, \quad \Rightarrow \quad \tau_0 \simeq -6U_0.$$

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$$\Rightarrow \quad \mathcal{V}(q) \simeq \frac{1}{q^2}\left[\frac{1}{2M_p^2}\left(T_{\mu\nu}(q)T^{\mu\nu}(-q) - \frac{1}{2}T^{\mu}_{\mu}(q)T^{\nu}_{\nu}(-q)\right)\right], M_p^2 = M^3U_0$$

- Tensor Structure becomes that of a 4d massless graviton !
- Leftover interaction is light scalar with ultra-weak coupling
- Warning: need to check explcitly about ghosts