Spinorial Perturbations in Galileon Black Holes

Bertha Cuadros-Melgar

University of São Paulo
Brazil

Work in collaboration with:

E. Abdalla (IFUSP), J. de Oliveira (UFMT), A. B. Pavan (UNIFEI),
and C. Pellicer (UFRN)
Goal and Motivation

♣ Stability of the geometry.

♣ Fermion fields have unique features that boson fields do not have.

♣ Gauge/gravity conjecture: spinorial perturbations describe matter fields at the boundary CFT.
The action is given by [M. Rinaldi, Phys. Rev. D86, 084048 (2012)]

\[
S = \frac{1}{2} \int dx^4 \sqrt{-g} \left[ \beta R - (g^{\mu\nu} - \bar{z}G^{\mu\nu}) \partial_\mu \phi \partial_\nu \phi \right].
\] (1)

where

\[
\beta = m_P^2, \quad \bar{z} = \frac{z}{m_P^2}.
\] (2)

For \( z > 0 \) a black hole solution can be found to be

\[
ds^2 = -F(r)dt^2 + H(r)dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2),
\] (3)
with

\[
F(r) = \frac{3}{4} + \frac{r^2}{L^2} - \frac{2M}{m_P^2 r} + \frac{\sqrt{\bar{z}}}{4r} \arctan \left( \frac{r}{\sqrt{\bar{z}}} \right),
\]

(4)

\[
H(r) = \frac{(r^2 + 2\bar{z})^2}{4(r^2 + \bar{z})^2 F(r)} ,
\]

\[
[\phi'(r)]^2 = -\frac{m_P^2 r^2 (r^2 + 2\bar{z})^2}{4\bar{z}(r^2 + \bar{z})^2 F(r)} ,
\]

(5)

being \( l^2 = 12\bar{z} \). Clearly, \( \bar{z} \) is a nonperturbative parameter that interpolates between \( S (z \to \infty) \) and SAdS solutions.
The spinorial field must obey Dirac equation

\[ i\gamma^{(a)} e_{(a)}^{\mu} \nabla_\mu \Psi - \mu_s \Psi = 0, \tag{6} \]

where

\[ \nabla_\mu = \partial_\mu + \frac{1}{4} \omega^{(a)(b)} e_{[a} \gamma_{b]} \gamma^{\mu}, \quad \omega^{(a)(b)} = e^{(a)}_\nu \partial_\mu e^{(b)}_\nu + e^{(a)}_\nu \Gamma^{\nu}_{\mu \sigma} e^{(b)}_\sigma. \tag{7} \]

Decoupling the radial part of Dirac equation and after some coordinate transformations the 2-spinors \( Z_{\pm} \) obey the following equation,

\[ \left( \frac{d^2}{dr^2_*} + \omega^2 \right) Z_{\pm} = V_{\pm} Z_{\pm}, \tag{8} \]
with the superpartner potentials

\[ V_\pm = W^2 \pm \frac{dW}{d\hat{r}_*}, \quad (9) \]

given in terms of the so-called superpotential

\[ W = \left[ F \left( \frac{K^2}{r^2} + \mu_s^2 \right) \right]^{1/2} \frac{1 + \frac{\mu_s K}{2\omega (K^2 + \mu_s^2 r^2)} \sqrt{\frac{F}{H}}}{1 + \frac{\mu_s K}{2\omega (K^2 + \mu_s^2 r^2)} \sqrt{\frac{F}{H}}}. \quad (10) \]

The coordinate \( \hat{r}_* \) can be calculated exactly in terms of a tortoise coordinate \( r_* \) both given by

\[ \hat{r}_* = r_* + \frac{1}{2\omega} \arctan \left( \frac{\mu_s r}{K} \right), \quad dr_* = \sqrt{\frac{H(r)}{F(r)}} \, dr. \quad (11) \]
Figure 1: Effective potential $V_+$ for fermion perturbations for different $K$ and $\bar{z}$.
Figure 2: Effective potential $V_-$ for fermion perturbations for different $K$ and $\bar{z}$. 
WKB results

<table>
<thead>
<tr>
<th>$K$</th>
<th>$\sqrt{\bar{z}} = 1$</th>
<th>$\sqrt{\bar{z}} = 0.8$</th>
<th>$\sqrt{\bar{z}} = 0.6$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Re(\omega)$</td>
<td>$-Im(\omega)$</td>
<td>$Re(\omega)$</td>
</tr>
<tr>
<td>1</td>
<td>0.420</td>
<td>0.357</td>
<td>0.394</td>
</tr>
<tr>
<td>2</td>
<td>0.832</td>
<td>0.340</td>
<td>0.858</td>
</tr>
<tr>
<td>3</td>
<td>1.273</td>
<td>0.326</td>
<td>1.361</td>
</tr>
<tr>
<td>4</td>
<td>1.712</td>
<td>0.321</td>
<td>1.845</td>
</tr>
<tr>
<td>5</td>
<td>2.148</td>
<td>0.319</td>
<td>2.321</td>
</tr>
</tbody>
</table>

Table 1: Dirac quasinormal frequencies with fixed horizon radius $r_+ = 1$ computed using the WKB technique.
Figure 3: Perturbations evolution for a superpartner potential $V_+$. 

Spinorial perturbations in galileon BH
Figure 4: Perturbations evolution for a superpartner potential $V_\mp$. 

Spinorial perturbations in galileon BH

B. Cuadros-Melgar
We considered fermion perturbations in the context of galileon black holes.

Due to potentials shape for massless fermions WKB method can be applied. For $\bar{z} \sim r_h$ convergence is poor. However, as $K$ increases, convergence is improved.

Numerical method shows very good agreement for $\omega_R$ at big $K$ and $\omega_I$ at small $K$. 

Spinorial perturbations in galileon BH

B.Cuadros-Melgar