Nonlocal gravity. Conceptual aspects and cosmological consequences

Michele Maggiore

UNIVERSITÉ DE GENÈVE
FACULTÉ DES SCIENCES
Département de physique théorique

Sifnos, Sept. 18-23, 2017

based on

Jaccard, MM, Mitsou, PRD 2013, 1305.3034

MM, PRD 2014, 1307.3898

Foffa, MM, Mitsou, PLB 2014, 1311.3421

Foffa, MM, Mitsou, IJMPA 2014, 1311.3435

Kehagias and MM, JHEP 2014, 1401.8289

MM and Mancarella, PRD 2014, 1402.0448

Dirian, Foffa, Khosravi, Kunz, MM, JCAP 2014, 1403.6068

Dirian, Foffa, Kunz, MM, Pettorino, JCAP 2015, 1411.7692

MM PRD 2016, 1603.01515

Cusin, Foffa, MM, PRD 2016, 1512.06373

Cusin, Foffa, MM, Mancarella, PRD 2016, 1602.01078

Dirian, Foffa, Kunz, MM, Pettorino, JCAP 2016, 1602.03558

MM (review) Springer, 1606.08784

Dirian, 1704.04075

Belgacem, Dirian, Foffa, MM, in prep.

Summary

- Conceptual aspects
  - nonlocality in the quantum effective action
  - scenarios for mass scale generation in gravity in the IR
  - FAQ
    - causality
    - degrees of freedoms, ghosts
    - localization and boundary conditions
    - freedom in the choice of nonlocal term

- Comparison with cosmological data
  - background evolution and dark energy
  - cosmological perturbations
  - Bayesian parameter estimation and model comparison

Nonlocality and the quantum effective action

At the fundamental level, the action in QFT is local

However, the quantum effective action is nonlocal

$$e^{iW[J]} \equiv \int D\varphi \; e^{iS[\varphi]+i\int J\varphi}$$

$$\frac{\delta W[J]}{\delta J(x)} = \langle 0|\varphi(x)|0\rangle_J \equiv \phi[J]$$

quantum effective action: $$\Gamma[\phi] \equiv W[J] - \int \phi J$$

$$\frac{\delta\Gamma[\phi]}{\delta\phi(x)} = -J(x)$$

the quantum EA gives the exact eqs of motion for the vev, which include the quantum corrections

$$e^{i\Gamma[\phi]} = \int D\varphi\; e^{iS[\phi+\varphi]-i\int \frac{\delta\Gamma[\phi]}{\delta\phi}\varphi}$$

- We are `integrating out' the quantum fluctuations, not the fields

  It is not a Wilsonian effective action

- The regime of validity of the quatum EA is the same as that of the fundamental theory

- light particles $\leftrightarrow$ nonlocalities in the quantum effective action

these nonolocalities are well understood in the UV.

E.g. in QED

$$\Gamma_{\rm QED}[A_\mu] = -\frac{1}{4}\int d^4x \left[F_{\mu\nu}\frac{1}{e^2(\Box)}F^{\mu\nu} + \mathcal{O}(F^4)\right] + \text{fermionic terms}$$

$$\frac{1}{e^2(\Box)} \simeq \frac{1}{e^2(\mu)} - \beta_0 \log\left(\frac{-\Box}{\mu^2}\right)$$

$$\log\left(\frac{-\Box}{\mu^2}\right) \equiv \int_0^\infty dm^2 \left[\frac{1}{m^2+\mu^2} - \frac{1}{m^2-\Box}\right]$$

it is just the running of the coupling constant in coordinate space

Note: we are not integrating out light particles from the theory!

The quantum effective action is especially useful in GR

$$e^{i\Gamma_m[g_{\mu\nu}]} = \int D\varphi \; e^{iS_m[g_{\mu\nu};\varphi]}$$

(vacumm quantum EA. We can also retain the vev's of the matter fields
ith the Legendre transform)

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_m}{\delta g^{\mu\nu}} \qquad \langle 0|T_{\mu\nu}|0\rangle = \frac{2}{\sqrt{-g}} \frac{\delta \Gamma_m}{\delta g^{\mu\nu}}$$

It gives the exact Einstein eqs including quantum matter loops

$$G_{\mu\nu} = 8\pi G \, \langle 0_{\rm in}|T_{\mu\nu}|0_{\rm in}\rangle$$

$\Gamma = S_{EH} + \Gamma_m$ is an action that, used at tree level, give the eqs of motion that include the quantum effects

The quantum effective action in GR can be computed perturbatively in an expansion in the curvature using heat-kernel techniques

Barvinsky-Vilkovisky 1985,1987

$$\Gamma = \frac{m_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\,R + \int d^4x\sqrt{-g}\,\left[R\,k_R(\Box)R + \frac{1}{2}C_{\mu\nu\rho\sigma}k_W(\Box)C^{\mu\nu\rho\sigma}\right]$$

The form factors due to a matter field of mass m_s are known in closed form

Gorbar-Shapiro 2003

For $m_s << E$

$$k(\Box) \simeq k(\mu) - \beta_k \log\left(\frac{-\Box}{\mu^2}\right) + c_1\frac{m_s^2}{\Box} + c_2\frac{m_s^4}{\Box^2} + \ldots$$

For $m_s >> E$

$$k(\Box) \simeq O(\Box/m_s^2)$$

However, these corrections are only relevant in the UV (ie for quantum gravity) and not in the IR (cosmology):

$$R\log(-\Box/m_s^2)R \ll m_{\rm Pl}^2 R \quad \text{unless} \quad R\sim m_{\rm Pl}^2$$

unavoidable, since these are one-loop corrections, and we pay a factor $1/m_{Pl}^2$

For application to cosmology, we rather need some strong IR effect

Dynamical mass generation in gravity in the IR?

The techniques for computing the quantum EA are well understood in the UV, but much less in the IR

- infrared divergences of massless fields in dS lead to dynamical mass generation, $m^2_{\rm dyn} \propto H^2\sqrt{\lambda}$ Rajaraman 2010,....

  the graviton propagator has exactly the same IR divergences

  Antoniadis and Mottola 1986,....

- quantum stability of dS. Decades of controversies....

- Euclidean lattice gravity suggests dynamical generation of a mass m, and a running of G_N

$$G(k^2) = G_N \left[1 + \left(\frac{m^2}{k^2} \right)^{\frac{1}{2\nu}} + \mathcal{O} \left(\frac{m^2}{k^2} \right)^{\frac{1}{\nu}} \right]$$

$\nu \approx 1/3$

Hamber 1999, .., 2017

- non-perturbative functional RG techniques find interesting fixed-point structure in the IR, and strong-gravity effects

Wetterich 2017

the dynamical emergences of a mass scale in the IR in gravity is a meaningful working hypothesis

Gauge-invariant (or diff-invariant) mass terms can be obtained with nonlocal operators

eg massive electrodynamics

Dvali 2006

$$\Gamma = -\frac{1}{4}\int d^4x \left(F_{\mu\nu}F^{\mu\nu} - m_\gamma^2 F_{\mu\nu}\frac{1}{\Box}F^{\mu\nu}\right)$$

in the gauge $\partial_\mu A^\mu = 0$ $\qquad \frac{1}{4}m_\gamma^2 F_{\mu\nu}\frac{1}{\Box}F^{\mu\nu} = \frac{1}{2}m_\gamma^2 A_\mu A^\mu$

a nonlocal but gauge-inv photon mass

- Numerical results on the gluon propagator from lattice QCD and OPE are reproduced by adding to the quantum effective action a term

$$\frac{m_g^2}{2}\mathrm{Tr}\int d^4x\, F_{\mu\nu}\frac{1}{\Box}F^{\mu\nu}$$

$$F_{\mu\nu}=F^a_{\mu\nu}T^a\,,\qquad \Box^{ab}=D^{ac}_{\mu}D^{\mu,cb}\,,\qquad D^{ab}_{\mu}=\delta^{ab}\partial_\mu-gf^{abc}A^c_\mu$$

(Boucaud et al 2001,Capri et al 2005,Dudal et al 2008)

it is a nonlocal but gauge invariant mass term for the gluons, generated dynamically by strong IR effects

Our approach: we will postulate some nonlocal effective action, which depends on a mass scale, and is supposed to catch IR effects in GR

- phenomenological approach. Identify a non-local modification of GR that works well

- attempt at a more fundamental understanding

Our prototype model will be

MM and M.Mancarella 2014

$$\Gamma_{\rm RR} = \int d^4x \sqrt{-g}\, \left[\frac{m_{\rm Pl}^2}{2} R - R \frac{\Lambda_{\rm RR}^4}{\Box^2} R \right]$$

``RR model"

- $\Lambda_{\rm RR}$ generated dynamically, analog to $\Lambda_{\rm QCD}$
- $g_{\mu\nu}(x) = e^{2\sigma(x)} \eta_{\mu\nu} \quad \rightarrow \quad R = -6\Box\sigma + \mathcal{O}(\sigma^2)$

  is a mass term for the conformal mode!
- $\Lambda_{\rm RR}^4 \sim m^2 m_{\rm Pl}^2 \qquad m \sim H_0 \rightarrow \Lambda_{\rm RR} \sim {\rm meV}$

  $\Lambda_{\rm RR}$ is the fundamental scale. No ultralight particle provides a solution to the naturalness problem

FAQ 1. Is the theory causal?

in a fundamental QFT nonlocality ➔ acausality. E.g.

$$\frac{\delta}{\delta\phi(x)}\int dx'\phi(x')(\Box^{-1}\phi)(x') = \frac{\delta}{\delta\phi(x)}\int dx'dx''\phi(x')G(x';x'')\phi(x'')$$

$$= \int dx'[G(x;x') + G(x';x)]\phi(x')$$

the quantum EA generates the eqs of motion of $\langle 0|g_{\mu\nu}|0\rangle$

$\langle 0_{\rm out}|g_{\mu\nu}|0_{\rm in}\rangle$ Feynman path integral, acausal eqs

$\langle 0_{\rm in}|g_{\mu\nu}|0_{\rm in}\rangle$ Schwinger-Keldish path integral, nonlocal but causal eqs

2. What is the domain of validity of the effective theory? Are you integrating out massless particles?

The eqs
$$e^{i\Gamma_m[g_{\mu\nu}]} = \int D\varphi \; e^{iS_m[g_{\mu\nu};\varphi]}$$

$$\langle 0|T_{\mu\nu}|0\rangle = \frac{2}{\sqrt{-g}}\frac{\delta\Gamma_m}{\delta g^{\mu\nu}}$$

are exact. It is not a Wilsonian approach

of course, the issue is to compute the EA, but if one can, it is valid in the whole regime of the fundamental action

3. What are the d.o.f. of your theory? Have you done a Hamiltonian analysis?

You cannot read the dof from the vacuum quantum effective action ! You need the fundamental theory

Example: Polyakov quantum EA in D=2

- is an example of the fact that we can get non-perturbative informations on the quantum EA

- since it can be computed exacly, it is useful to clarify conceptual aspects that create confusion in the literature

(dof, ghosts,propagating vs non-propagating fields, etc)

- consider gravity + N conformal matter fields in D=2

conformal anomaly:
$$\langle 0|T^a_a|0\rangle = \frac{N}{24\pi} R$$

this result is exact. Only the 1-loop term contributes.

In D=2:
$$g_{\mu\nu} = e^{2\sigma}\eta_{\mu\nu} \qquad R = -2\Box\sigma$$

$$\frac{\delta\Gamma}{\delta\sigma} = -\frac{N}{12\pi}\Box\sigma$$

$$\Gamma[\sigma] - \Gamma[\sigma=0] = -\frac{N}{24\pi}\int d^2x\, e^{2\sigma}\sigma\Box\sigma$$

in D=2, $\Gamma[0]=0$. This is the exact quantum EA we can covariantize $\Gamma[\sigma] = -\frac{N}{24\pi} \int d^2x \, e^{2\sigma} \sigma \Box \sigma$

using $\sqrt{-g} = e^{2\sigma}$

$$R = -2\Box\sigma$$

we get

$$\begin{aligned} \Gamma &= \frac{N}{48\pi} \int d^2x \sqrt{-g} \, \sigma R \\ &= -\frac{N}{96\pi} \int d^2x \sqrt{-g} \, R \Box^{-1} R \end{aligned}$$

Polyakov quantum effective action

More generally, if we also quantize gravity

$$S = \int d^2x \sqrt{-g}(\kappa R - \lambda) + S_m$$

$$\begin{aligned} \Gamma &= -\frac{N-25}{96\pi}\int d^2x\sqrt{-g}\, R\frac{1}{\Box}R - \lambda\int d^2x\sqrt{-g} \\ &= \int d^2x\sqrt{-\bar{g}}\left[\frac{N-25}{24\pi}\bar{g}^{ab}\partial_a\sigma\partial_b\sigma + \frac{N-25}{24\pi}\overline{R}\sigma - \lambda e^{2\sigma}\right] \end{aligned}$$

Apparently, for N>25, σ is a ghost!

But in fact the theory is healthy and there are no quanta of σ in the physical spectrum, once we impose the physical state conditions in the fundamental theory

(David 1988, Distler-Kawai 1989, Polchinski 1989)

- linearizing the RR model over flat space:

$$D^{\mu\nu\rho\sigma}(k) = \frac{i}{2k^2}\left(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\nu}\eta^{\rho\sigma}\right) + \frac{1}{6}\left(\frac{i}{k^2} - \frac{i}{k^2 - m^2}\right)\eta^{\mu\nu}\eta^{\rho\sigma},$$

a scalar ghost??

No! The degrees of freedom cannot be read from the quadratic part of a quantum EA!

The idea is that this should be the quantum EA of usual Einstein gravity plus matter, which includes massless fields (there are at least the graviton and the photon).

4. Is nonlocal gravity equivalent to a scalar-tensor theory?

The RR model can be put into local form introducing two auxiliary fields

$$U = -\Box^{-1} R \,, \qquad S = -\Box^{-1} U$$

then the eqs of motion read

$$\begin{aligned} G_{\mu\nu} &= \frac{m^2}{6} K_{\mu\nu}(U,S) + 8\pi G T_{\mu\nu} \\ \Box U &= -R \\ \Box S &= -U \end{aligned}$$

$$\Lambda^4_{\rm RR} = \frac{1}{12} m^2 m^2_{\rm Pl}$$

are U and S associated to real quanta? Then U would be a ghost consider again

$$\Gamma = \frac{N}{48\pi}\int d^2x\,\sqrt{-g}\,\sigma R$$
$$= -\frac{N}{96\pi}\int d^2x\,\sqrt{-g}\,R\Box^{-1}R$$

we could localize it introducing again $U = -\Box^{-1}R$

By definition $\Box U = -R$ ie $\Box U = 2\Box\sigma$

The most general solution is $U = 2\sigma + U_{\rm hom}$

however, only U=2σ gives back the original action

U is fixed in terms of the metric. There are no creation/annihilation operators associated to U

Similarly, consider

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m_\gamma^2 A_\mu A^\mu$$
$$= -\frac{1}{4}F_{\mu\nu}\left(1 - \frac{m_\gamma^2}{\Box}\right)F^{\mu\nu}$$

we could localize it, preserving gauge-invariance, introducing $U^{\mu\nu} = -\Box^{-1}F^{\mu\nu}$

however, there is no new dof associated to $U^{\mu\nu}$

The initial conditions on the original fields (metric,A_μ) fix in principle the initial conditions on the auxiliary fields

5. If we had a derivation of the RR model from a fundamental action, the inital conditions on the auxiliary fields would be fixed in terms of the initial conditions on the metric. However, we do not have a derivation. So, how do you choose in practice the initial conditions on U,S ?

in cosmology, at the background level initial conditions at early times are taken to be homogeneous: a priori the space of theories is parametrized by

$$\{U(t_0), \dot{U}(t_0), S(t_0), \dot{S}(t_0)\}$$

however, for the RR model the cosmological solutions are an attractor in this space : 3 stable directions, one marginal parameter $U=u_0$

For the Δ_4 model, 4 stable directions at the level of cosmological perturbations, what are the initial conditions on the auxiliary fields?

recall that for the Polyakov action $U = -\Box^{-1} R = 2\sigma$

in this case, writing $U(t,\mathbf{x}) = \bar{U}(t) + \delta U(t,\mathbf{x})$

$$\bar{U}(t_{\rm in}) = \bar{\sigma}(t_{\rm in})$$

$$\delta U(t_{\rm in},\mathbf{x}) = \delta\sigma(t_{\rm in},\mathbf{x})\,, \quad \delta\dot{U}(t_{\rm in},\mathbf{x}) = \delta\dot{\sigma}(t_{\rm in},\mathbf{x})$$

in general, δU, δV must vanish if the metric perturbation vanish (they are not independent dof!)

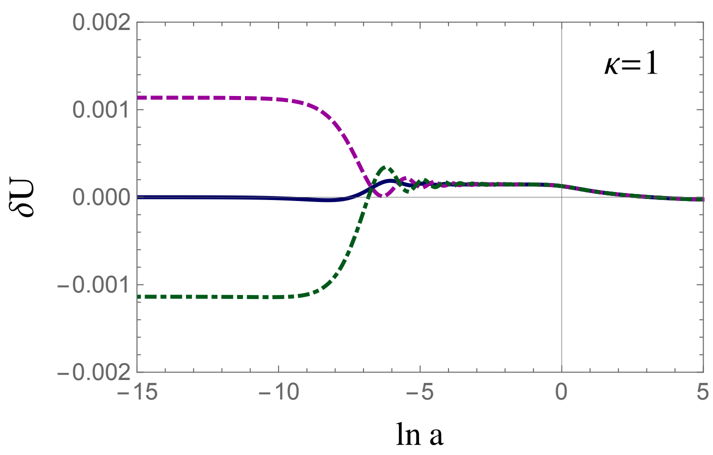
for the RR model we set

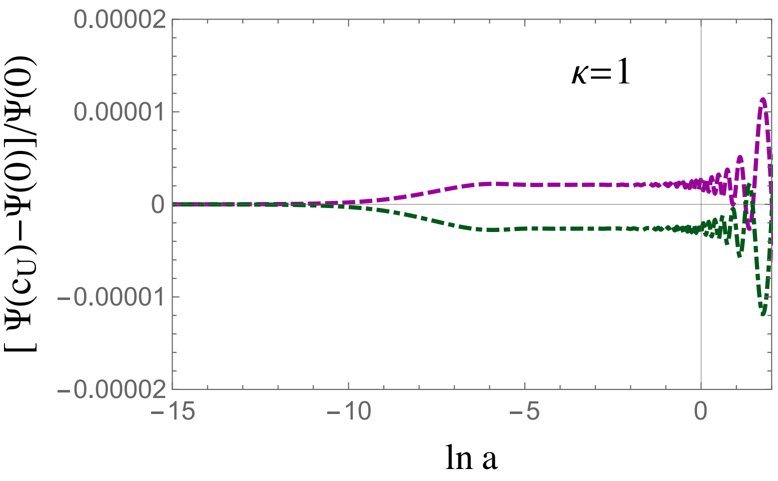
$$
\begin{aligned}
U(t_{\rm in},\mathbf{x}) &= c_U\,\Phi(t_{\rm in},\mathbf{x})\,, \qquad U'(t_{\rm in},\mathbf{x}) = c_U\,\Phi'(t_{\rm in},\mathbf{x})\,,\\
V(t_{\rm in},\mathbf{x}) &= c_V\,\Phi(t_{\rm in},\mathbf{x})\,, \qquad V'(t_{\rm in},\mathbf{x}) = c_V\,\Phi'(t_{\rm in},\mathbf{x})\,,
\end{aligned}
$$

$(V=H_0^2S)$ and vary c_U,c_V at a level $-10<c_U,c_V<10$

result: very little dependence of the final results

Belgacem, Dirian, Foffa, MM, in prep





6. How much freedom we have in the choice of the nonlocal term?

We have explored several possibilities

- massive photon: can be described replacing

$$\partial_\mu F^{\mu\nu} = j^\nu \quad \to \quad \left(1 - \frac{m^2}{\Box}\right) \partial_\mu F^{\mu\nu} = j^\nu$$

(Dvali 2006)

- a first guess for a massive deformation of GR could be

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \quad \to \quad \left(1 - \frac{m^2}{\Box_g}\right) G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

(Arkani-Hamed, Dimopoulos, Dvali and Gabadadze 2002)

however, we lose energy-momentum conservation $\nabla^\mu(\Box_g^{-1} G_{\mu\nu}) \neq 0$

- to preserve energy-momentum conservation:

$$S_{\mu\nu} = S^{\mathrm{T}}_{\mu\nu} + \frac{1}{2}(\boldsymbol{\nabla}_\mu S_\nu + \boldsymbol{\nabla}_\nu S_\mu)$$

$$G_{\mu\nu} - m^2(\Box^{-1}G_{\mu\nu})^T = 8\pi G T_{\mu\nu}$$

(Jaccard,MM, Mitsou, 2013)

however, instabilities in the cosmological evolution (Foffa,MM, Mitsou, 2013)

- $$G_{\mu\nu} - m^2(g_{\mu\nu}\Box^{-1}R)^T = 8\pi G T_{\mu\nu}$$

(MM 2013)

``RT model''

works very well if started from RD and fits well the data

however, cosmological perturbations are unstable if we start from deSitter inflation

(Belgacem, Cusin, MM and M.Mancarella, 2017, in prep.)

- a related model: (MM and M.Mancarella, 2014)

$$\Gamma_{\rm RR} = \frac{m_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\,\left[R - \frac{m^2}{6}R\frac{1}{\Box^2}R\right]$$

  - stable cosmological evolution and stable perturbations in all phases
  - fits very well the data

``RR model"

- a more general model (Cusin, Foffa, MM and M.Mancarella, 2016)

$$\Gamma = \frac{m_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\,\left[R - \mu_1 R\frac{1}{\Box^2}R - \mu_2 C^{\mu\nu\rho\sigma}\frac{1}{\Box^2}C_{\mu\nu\rho\sigma} - \mu_3 R^{\mu\nu}\frac{1}{\Box^2}R_{\mu\nu}\right]$$

stability requires $\mu_2=\mu_3=0$

- another useful variant (`Δ_4 model')

$$\Gamma_{\Delta_4} = \frac{m_{\rm Pl}^2}{2} \int d^4x \sqrt{-g} \left[R - \frac{1}{6} m^2 R \frac{1}{\Delta_4} R \right]$$

$$\Delta_4 = \Box^2 + 2R^{\mu\nu}\boldsymbol{\nabla}_\mu \boldsymbol{\nabla}_\nu - \frac{2}{3} R\Box + \frac{1}{3} g^{\mu\nu} \boldsymbol{\nabla}_\mu R \boldsymbol{\nabla}_\nu$$

(Cusin, Foffa, MM and M.Mancarella, 2016)
(Belgacem, Foffa, Dirian and MM, in prep.)

- not at all easy to construct a nonlocal model that works

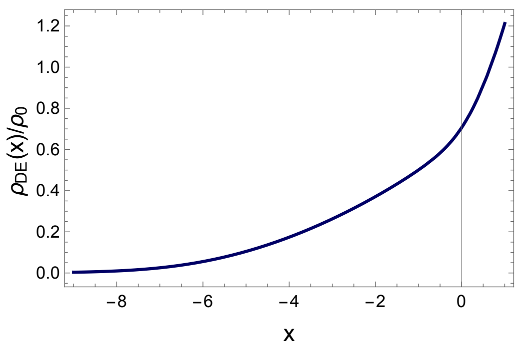
(viable background evolution, stable cosmological perturbations during RD, MD and also inflation)

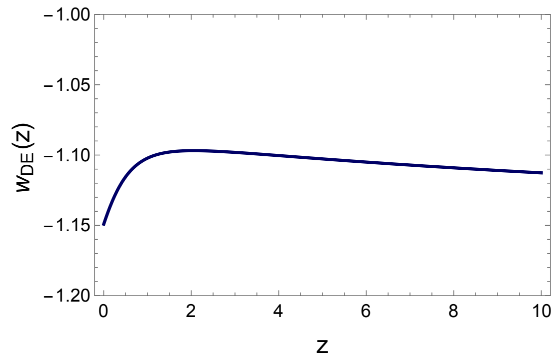
- the models that work correspond to dynamical mass generation for the conformal mode, rather than for the graviton

Cosmological background evolution

- the nonlocal term acts as an effective DE density!

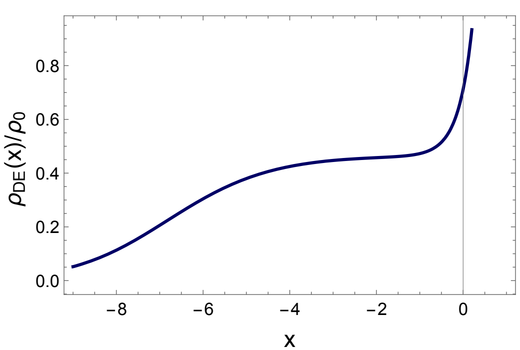
define w_{DE} from $\dot{\rho}_{\rm DE} + 3(1 + w_{\rm DE}) H \rho_{\rm DE} = 0$

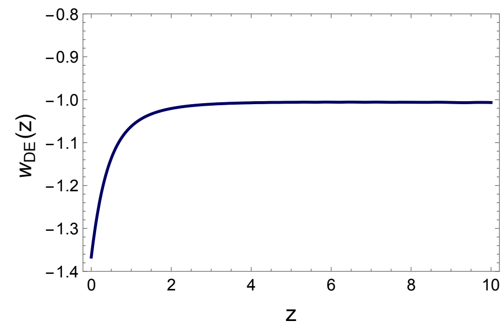




RR model
with u_0=0

x=ln a





Δ_4 model

a phantom DE equation of state !

Cosmological perturbations

- well-behaved? YES

Dirian, Foffa, Khosravi, Kunz, MM
JCAP 2014

this step is already non-trivial, see e.g. DGP or bigravity

- consistent with data? YES

- comparison with ΛCDM

Dirian, Foffa, Kunz, MM, Pettorino,
JCAP 2015, 2016

Dirian, 2017

implement the perturbations in a Boltzmann code
compute likelihood, χ^2, perform parameter estimation

Boltzmann code analysis and comparison with data

- We test the non-local models against
  - Planck 2015 TT, TE, EE and lensing data,
  - isotropic and anisotropic BAO data,
  - JLA supernovae,
  - local H_0 measurements,
  - growth rate data

  and we perform Bayesian parameter estimation and model comparison
- we modified the CLASS code and use Montepython MCMC
- we vary

$$\omega_b = \Omega_b h_0^2,\ \omega_c = \Omega_c h_0^2,\ H_0,\ A_s,\ n_s,\ z_{\rm re}$$

In ΛCDM, Ω_Λ is a derived parameter, fixed by the flatness condition. Similarly, in our model the mass parameter m^2 is a derived parameter, fixed again from $\Omega_{tot}=1$

we have the same free parameters as in ΛCDM

- A crucial point: neutrino masses (Dirian 2017)

Oscillation and β-decay experiments give $0.06\ \mathrm{eV} \leq \Sigma_\nu m_\nu \leq 6.6\ \mathrm{eV}$

The Planck baseline analysis from ΛCDM fixes $\Sigma_\nu m_\nu = 0.06\ \mathrm{eV}$

If we $\Sigma_\nu m_\nu$ vary, in ΛCDM we hit the lower bound

CMB Planck data, interpreted with ΛCDM, only give an upper bound $\Sigma_\nu m_\nu < 0.23\ \mathrm{eV}$

Planck coll., Ade et al 2015

The RR and Δ_4 nonlocal models predict a non-zero value of $\Sigma_\nu m_\nu$ the interval $0.06\ \mathrm{eV} \leq \Sigma_\nu m_\nu \leq 6.6\ \mathrm{eV}$.

It is wrong to keep fixed $\Sigma_\nu m_\nu = 0.06\ \mathrm{eV}$ in the nonlocal models

- # Parameter estimation. Results

the most interesting predictions are on H_0 and $\Sigma_\nu m_\nu$

from CMB+BAO+SNae:

	ΛCDM	$\nu\Lambda$CDM	RR	Δ_4
H_0	$67.67^{+0.47}_{-0.50}$	$67.60^{+0.66}_{-0.55}$	$69.49^{+0.79}_{-0.80}$	$70.27^{+0.95}_{-0.94}$
$\sum_\nu m_\nu$ (eV)	0.06 (fixed)	< 0.10	$0.219^{+0.083}_{-0.084}$	$0.185^{+0.087}_{-0.096}$

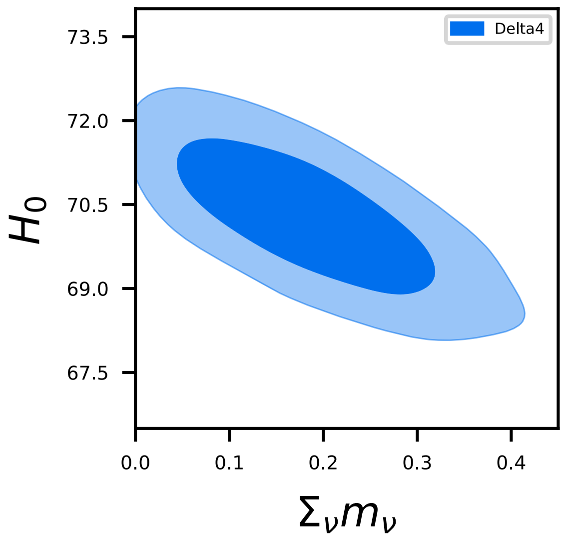
- prediction for neutrino masses

(consistent with the limits $0.06\ \text{eV} \leq \Sigma_\nu m_\nu \leq 6.6\ \text{eV}$)

- nonlocal models consistent with the value of H_0 obtained by local measurements $H_0 = 73.02 \pm 1.79$ (Riess et al 1604.01424)

discrepancy 3.1σ for ΛCDM, 2.0σ for RR, 1.5σ for Δ_4

- the predictions for $\Sigma_\nu m_\nu$ and H_0 are correlated



- with CMB+BAO+SNa, RR and ΛCDM have comparable χ^2, while Δ_4 is disfavored
- currently running chains including also H_0

Conclusion: at the phenomenological level, these non-local models are quite satisfying

- solar system tests OK
- generates dynamically a dark energy
- cosmological perturbations work well
- comparison with CMB,SNe,BAO with modified Boltzmann code ok
- pass tests of structure formation
- higher value of H0

Same number of parameters as ΛCDM, and competitive with ΛCDM from the point of view of fitting the data

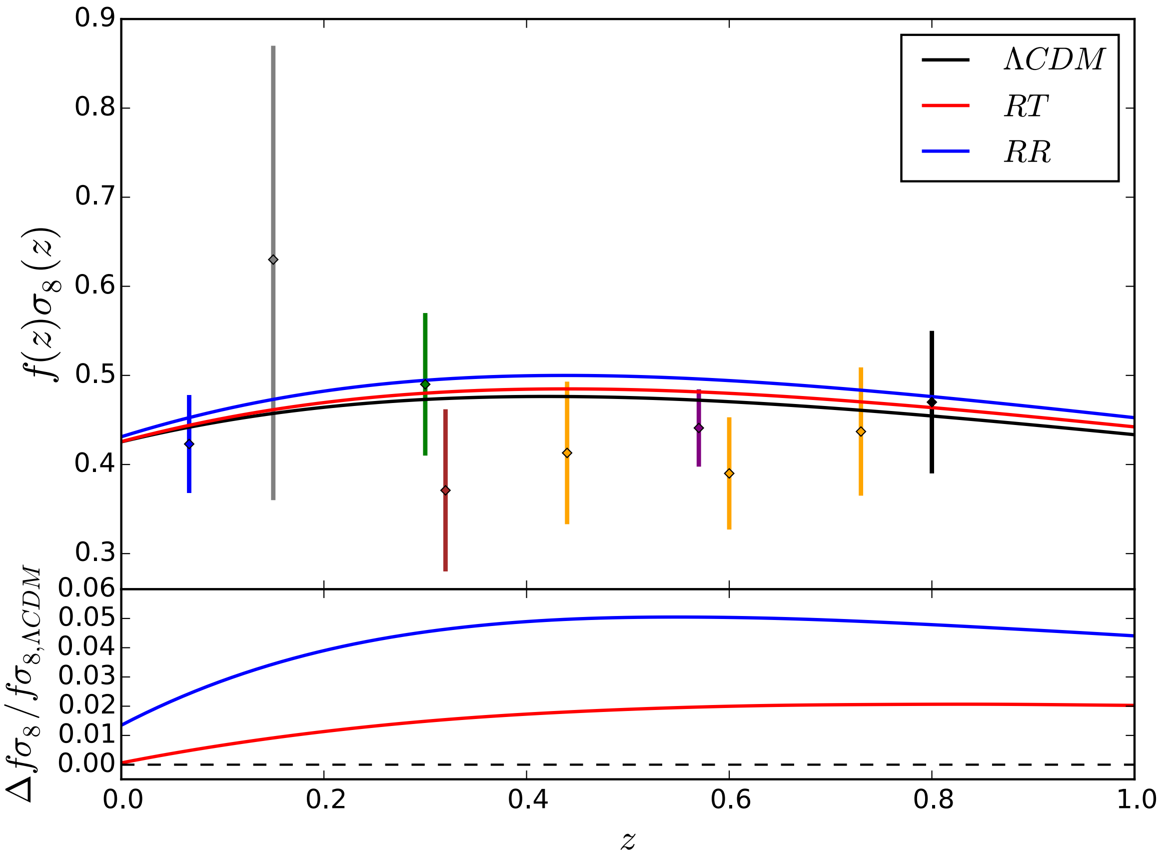
Next step:

understanding from first principles where such non-local term comes from

Thank you!

bkup slides

growth rate and structure formation



Absence of vDVZ discontinuity

- the propagator is continuous for m=0

$$D^{\mu\nu\rho\sigma}(k) = \frac{i}{2k^2}\left(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\nu}\eta^{\rho\sigma}\right) + \frac{1}{6}\left(\frac{i}{k^2} - \frac{i}{k^2 - m^2}\right)\eta^{\mu\nu}\eta^{\rho\sigma},$$

- write the eqs of motion of the non-local theory in spherical symmetry:

A. Kehagias and MM 2014

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

for mr <<1: low-mass expansion

for $r >> r_S$: Newtonian limit (perturbation over Minowski)

match the solutions for $r_S << r << m^{-1}$ (this fixes all coefficients)

- result: for $r \gg r_s$
$$A(r) = 1 - \frac{r_S}{r}\left[1 + \frac{1}{3}(1 - \cos mr)\right]$$
$$B(r) = 1 + \frac{r_S}{r}\left[1 - \frac{1}{3}(1 - \cos mr - mr\sin mr)\right]$$

for $r_s \ll r \ll m^{-1}$: $A(r) \simeq 1 - \frac{r_S}{r}\left(1 + \frac{m^2r^2}{6}\right)$

the limit $m \to 0$ is smooth !

By comparison, in massive gravity the same computation gives

$$A(r) = 1 - \frac{4}{3}\frac{r_S}{r}\left(1 - \frac{r_S}{12m^4r^5}\right)$$

vDVZ discontinuity

breakdown of linearity below $r_V = (r_s/m^4)^{1/5}$

Background evolution

- consider $$\Gamma_{\rm RR} = \frac{m_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\,\left[R - \frac{m^2}{6}R\frac{1}{\Box^2}R\right]$$

localize using $$U = -\Box^{-1}R\,, \quad S = -\Box^{-1}U$$

in FRW we have 3 variables: H(t), U(t), W(t)=H^2(t)S(t).

define $x = \ln a(t)$, $h(x) = H(x)/H_0$,

$\gamma = (m/3H_0)^2$ $\zeta(x) = h'(x)/h(x)$

$$h^2(x) = \Omega_M e^{-3x} + \Omega_R e^{-4x} + \gamma Y(U, U', W, W')$$
$$U'' + (3+\zeta)U' = 6(2+\zeta)$$
$$W'' + 3(1-\zeta)W' - 2(\zeta' + 3\zeta - \zeta^2)W = U$$

- there is an effective DE term, with

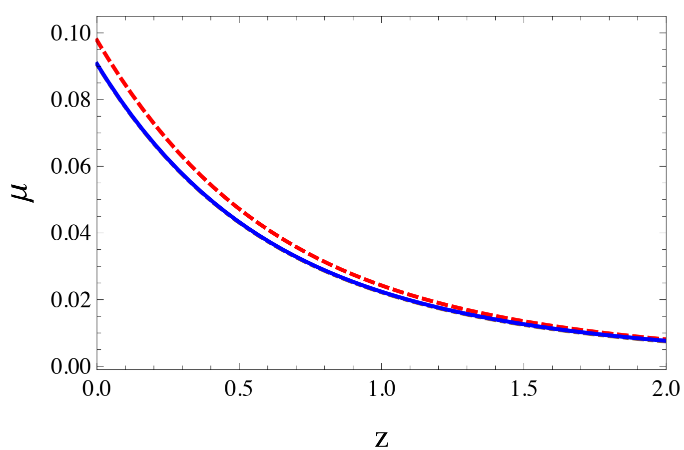
$$\rho_{\rm DE}(x) = \rho_0 \gamma Y(x) \qquad\qquad \rho_0 = 3H_0^2/(8\pi G)$$

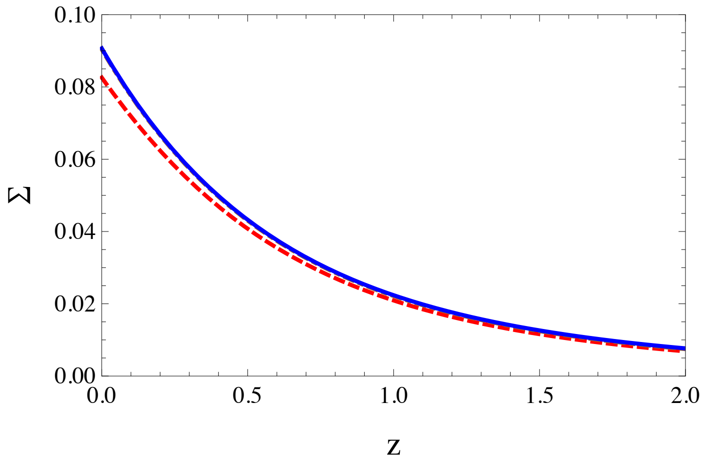
- define w_{DE} from $\dot{\rho}_{\rm DE} + 3(1+w_{\rm DE})H\rho_{\rm DE} = 0$

- the model has the same number of parameters as ΛCDM, with $\Omega_\Lambda \leftrightarrow \gamma$ (plus the inital conditions, parametrized by u_0, c_U, c_W)

- the perturbations are well-behaved and differ from ΛCDM at a few percent level

$$\Psi = [1+\mu(a;k)]\Psi_{\rm GR}$$
$$\Psi-\Phi = [1+\Sigma(a;k)](\Psi-\Phi)_{\rm GR}$$

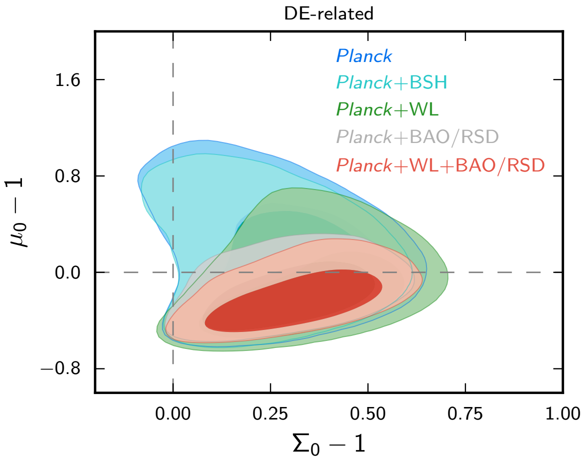




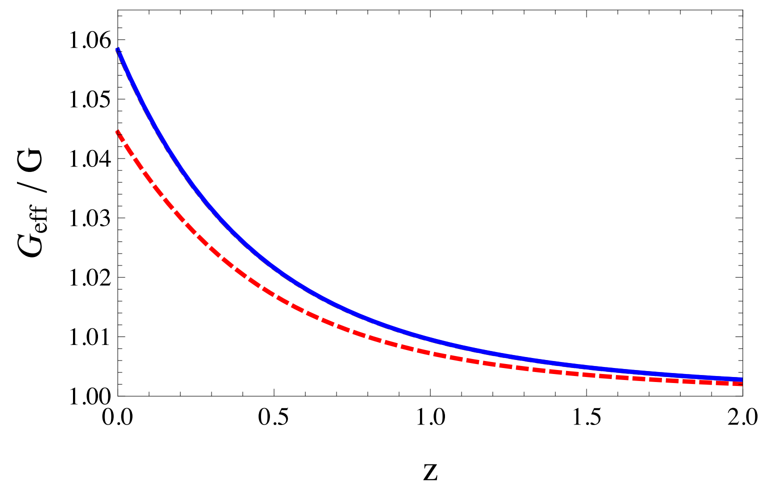
- deviations at z=0.5 of order 4%

- consistent with data:

(Ade et al., Planck XV, 2015)

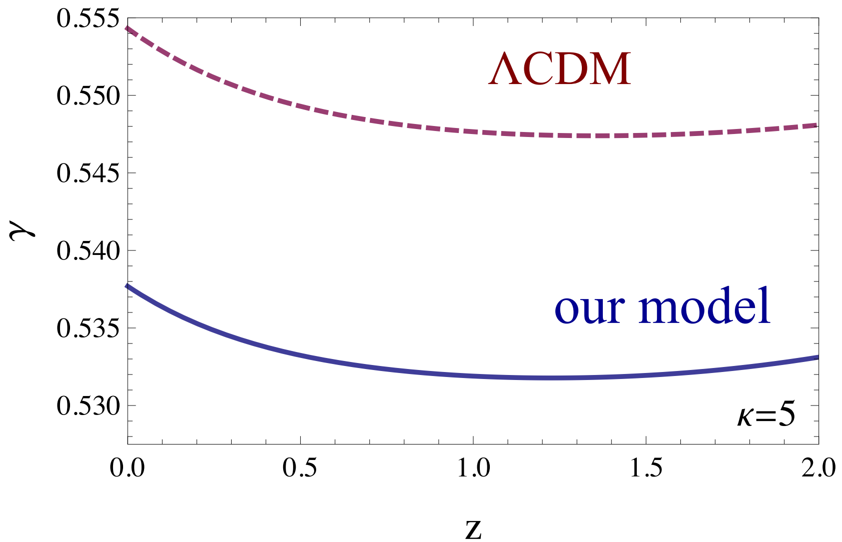


effective Newton constant (RR model)



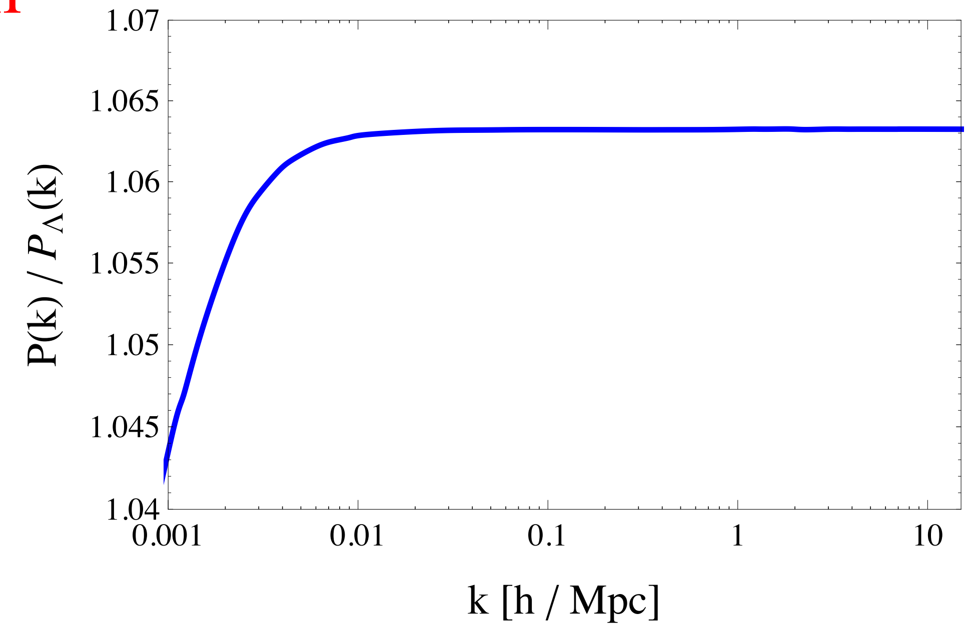
growth index:

$$\frac{d \log \delta_M(a;k)}{d \ln a} = [\Omega_M]^{\gamma(z;k)}$$

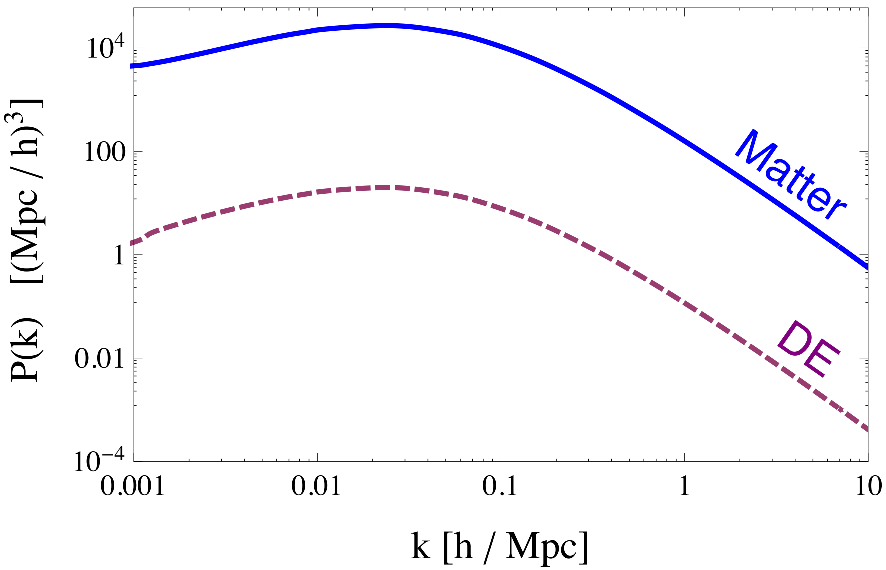


- linear power spectrum

matter power spectrum compared to ΛCDM



DE clusters but its linear power spectrum is small compared to that of matter



- sufficiently close to ΛCDM to be consistent with existing data, but distinctive prediction that can be clearly tested in the near future

  - phantom DE eq of state: w(0)= - 1.14 (RR) + a full prediction for w(z)
    - DES Δw=0.03
    - EUCLID Δw=0.01
  - linear structure formation

    $$\mu(a) = \mu_s a^s \to \mu_{\mathbf{s}} = \mathbf{0.09}, \mathbf{s} = \mathbf{2}$$

    - Forecast for EUCLID, $\Delta\mu$=0.01
  - non-linear structure formation: 10% more massive halos

    Barreira, Li, Hellwing, Baugh, Pascoli 2014
  - lensing: deviations at a few %