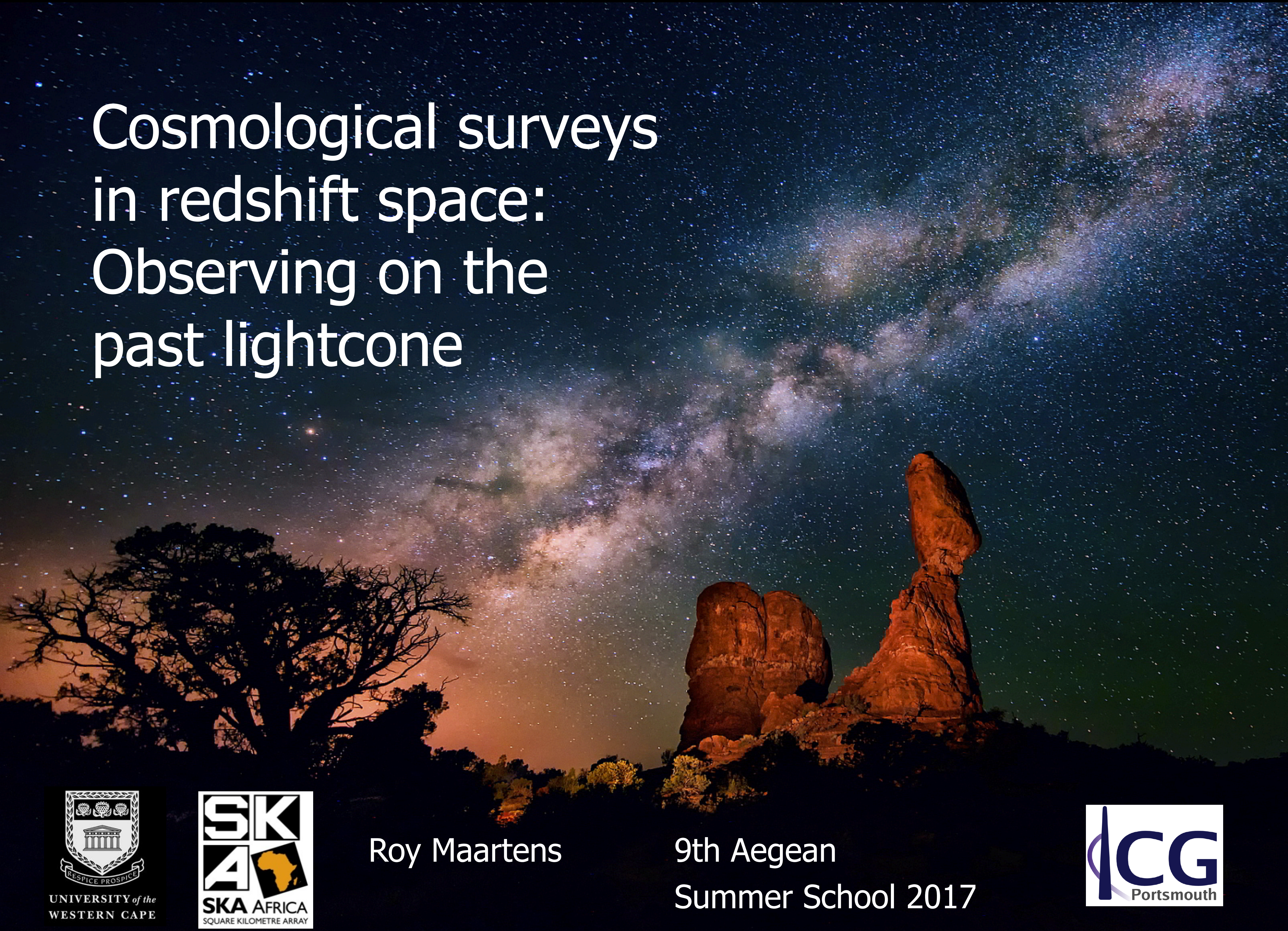
Cosmological surveys in redshift space: Observing on the past lightcone



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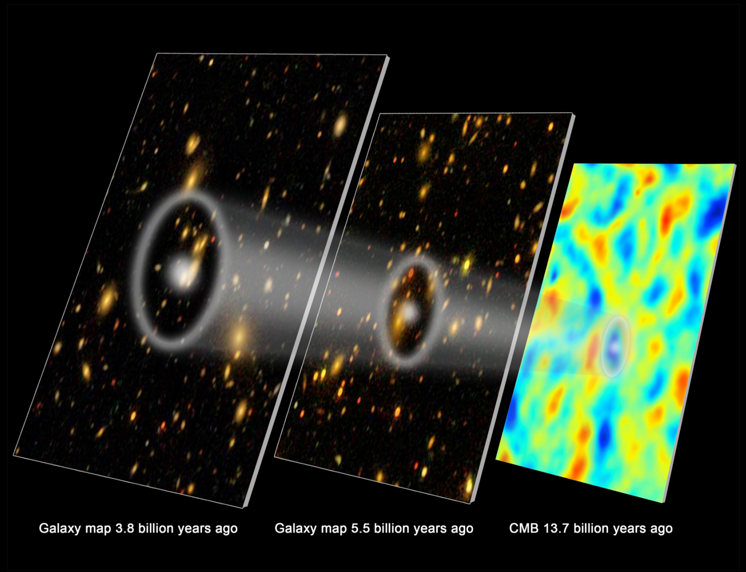
9th Aegean Summer School 2017

Ultra-large volume galaxy surveys – the next frontier

The next generation of surveys will map the matter distribution in ultra-large volumes:

These surveys will:

- advance ‘precision cosmology’
- sharpen tests of modified gravity
- lead to new and unexpected discoveries



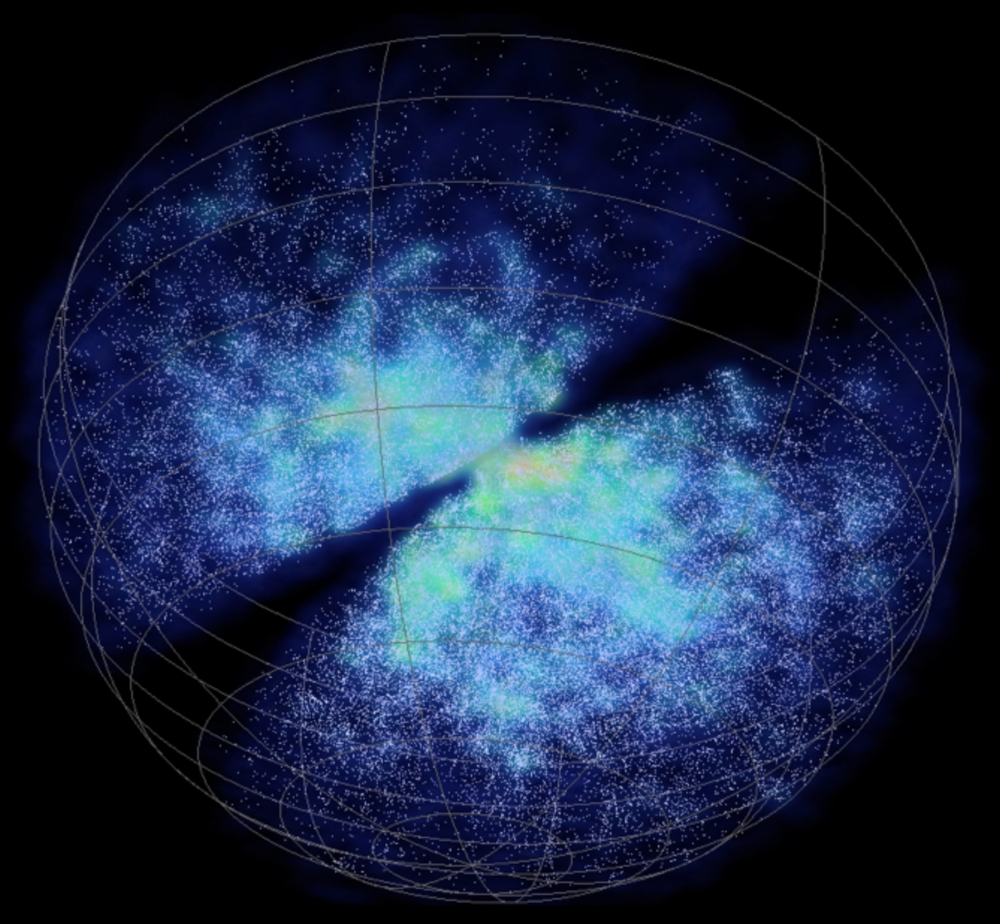
In order to exploit the enormous potential of future surveys, we need to ensure that theoretical precision matches observational precision.

Galaxy surveys: what do we measure?

Large-scale structure contains a wealth of information about the Universe – many more modes than the CMB.

Analysis of the counts, sizes and shapes of galaxies allow us to probe:

- Initial conditions of the Universe
- Content and geometry of the universe
- Theory of gravity



In order to do this, we need to understand what we are measuring.

Galaxy counts

We observe:

- angular position and redshift:

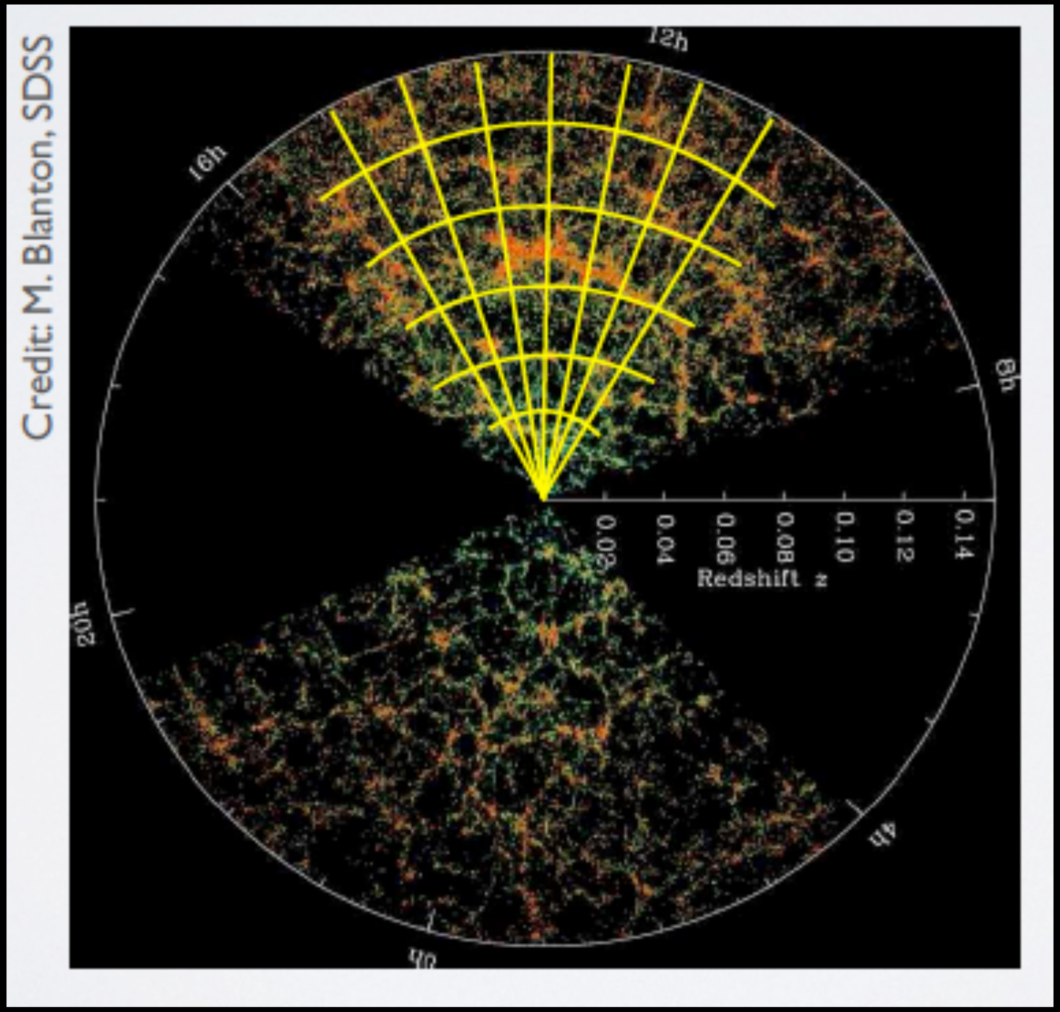
$$(\mathbf{n}, z) \qquad \mathbf{n}\cdot\mathbf{n} = 1$$

- number of galaxies per pixel:

$$d\mathcal{N}(\mathbf{n}, z) = N(\mathbf{n}, z)\, dz\, d\Omega_{\mathbf{n}}$$

Then the galaxy number count contrast is

$$\Delta_g(\mathbf{n}, z) = \frac{N(\mathbf{n}, z) - \bar{N}(z)}{\bar{N}(z)}$$



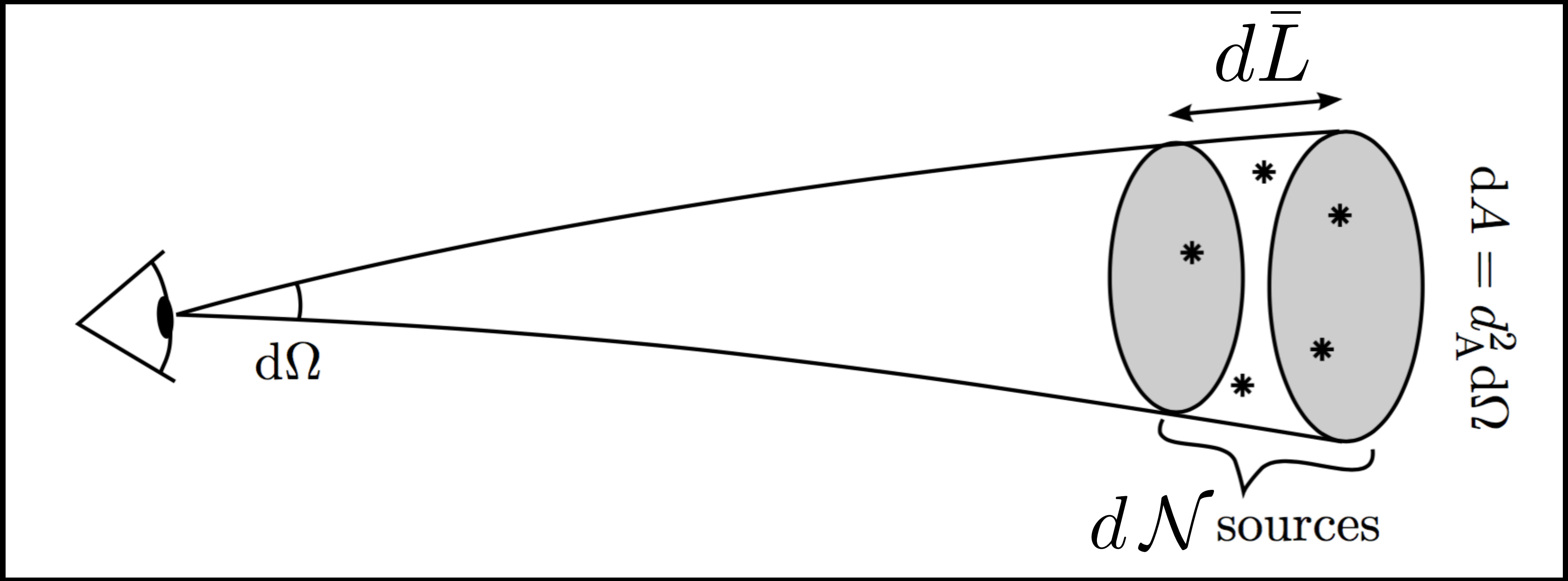
In the background:

$$\bar{N}(z) = \bar{n}_g \, \frac{\bar{d}_A^2}{(1+z)^2 \mathcal{H}}$$

proper number density

The other factors come from the volume element on the lightcone:

$$d\bar{V} = d\bar{L}\, d\bar{A} = (a d\chi)\big(\bar{d}_A^2 d\Omega_{\mathbf{n}}\big)$$
$$= a^2 \mathcal{H}^{-1} \bar{d}_A^2 \, dz \, d\Omega_{\mathbf{n}}$$



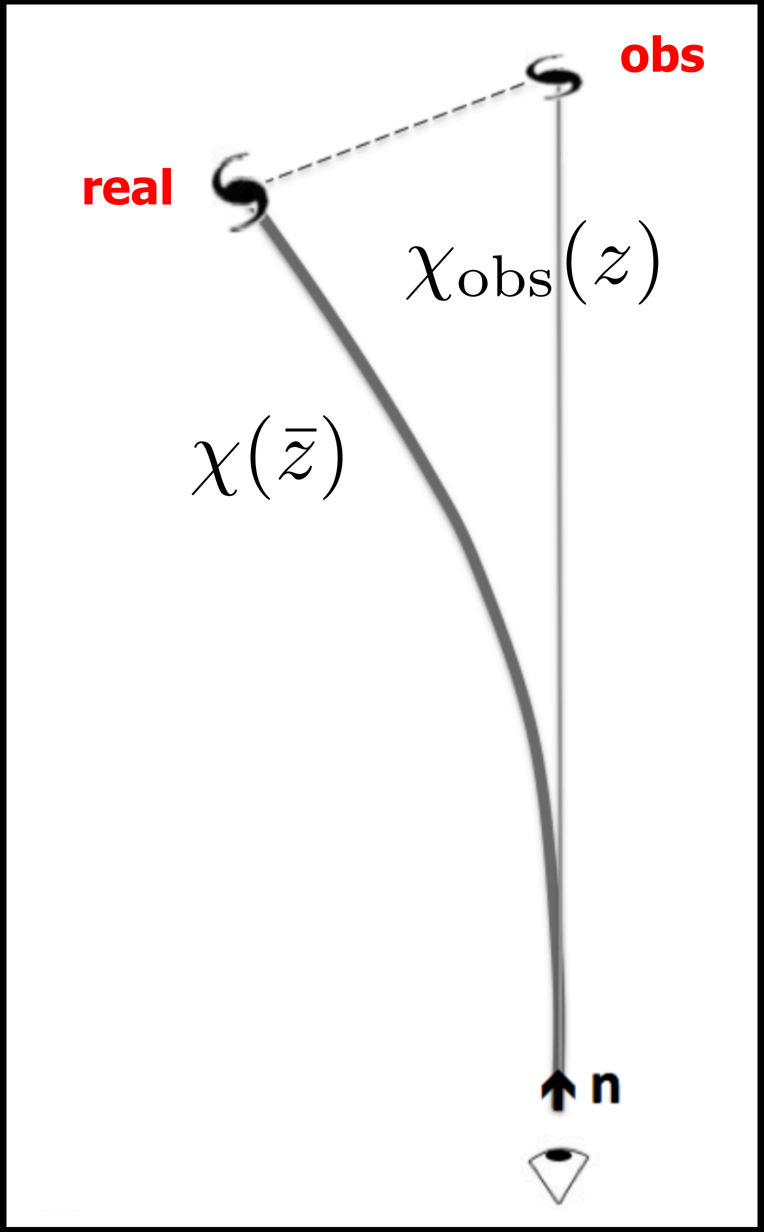
Distortions

In a homogeneous Friedmann universe:

- Light travels in straight lines
- Redshift is due purely to expansion: $z = \bar{z}$

This holds for galaxy counts if we assume that the observed and real positions of galaxies coincide. Then

$$\Delta_g \approx \frac{\delta n_g}{\bar{n}_g} = \delta_g = b\,\delta_m$$



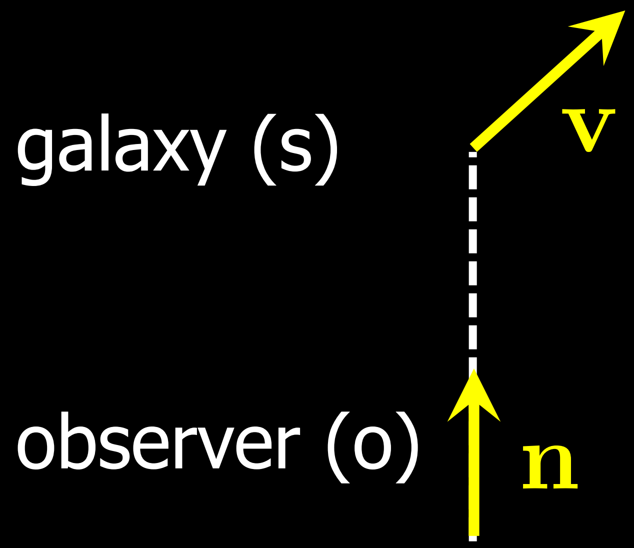
But inhomogeneities distort the redshift-distance relation and the lightray direction.

These relativistic effects lead to:

- Redshift-space distortions (RSD)
- Gravitational lensing distortions
- Other relativistic effects

Redshift-space distortions

The Hubble redshift is modified by a Doppler correction due to the peculiar velocity of galaxies:



In redshift space, overdensities in the linear regime are squeezed along the line of sight.

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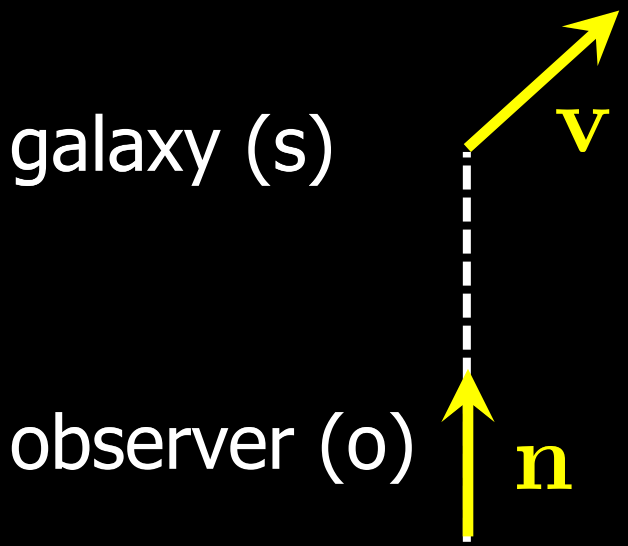
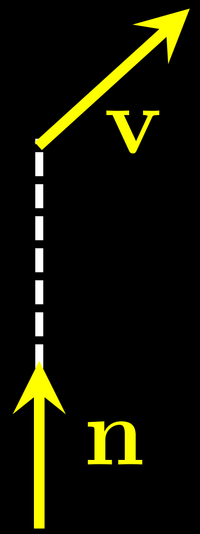


In redshift space, overdensities in the linear regime are squeezed along the line of sight. To leading order (ignoring metric perturbations):

$$1+z=\frac{(u_\mu k^\mu)_s}{(u_\mu k^\mu)_e}\approx\frac{\bar{E}_s(1+\mathbf{v}\cdot\mathbf{n})_s}{\bar{E}_o(1+\mathbf{v}\cdot\mathbf{n})_o}=(1+\bar{z})(1+\mathbf{v}_s\cdot\mathbf{n})$$

where we take $\mathbf{v}_o = 0$

Then the redshift perturbation is



$$\delta z \approx (1+\bar{z})\mathbf{v}\cdot\mathbf{n}$$

The real and observed comoving positions are

$$\mathbf{x} = \chi(\bar{z})\mathbf{n}, \qquad \mathbf{x}_{\rm obs} = \chi_{\rm obs}(z)\mathbf{n}$$

where

$$\chi_{\rm obs} = \chi(\bar{z}+\delta z) = \chi(\bar{z}) + \frac{1}{(1+\bar{z})\mathcal{H}}\,\delta z$$

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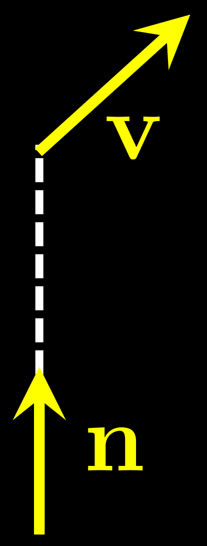

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From conservation of the number of galaxies:

$$\bar{n}_g(1+\delta_{g\,{\rm obs}})d^3\mathbf{x}_{\rm obs} = \bar{n}_g(1+\delta_g)d^3\mathbf{x}$$

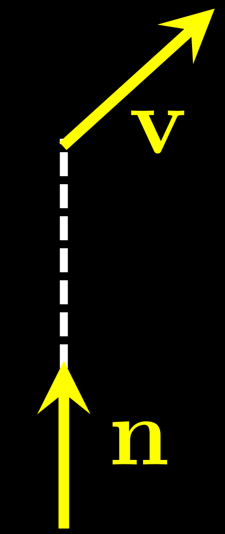
The Jacobian is

$$\frac{\partial\chi_{\rm obs}}{\partial\chi} = 1 + \frac{1}{(1+\bar{z})\mathcal{H}}\frac{\partial\delta z}{\partial\chi} + \frac{\delta z}{(1+\bar{z})}\frac{\partial\mathcal{H}}{\partial\chi}$$

The last term is much smaller than the second.

Then the redshift perturbation is

$$\delta z \approx (1+\bar{z})\mathbf{v}\cdot\mathbf{n}$$



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The last term is much smaller than the second.

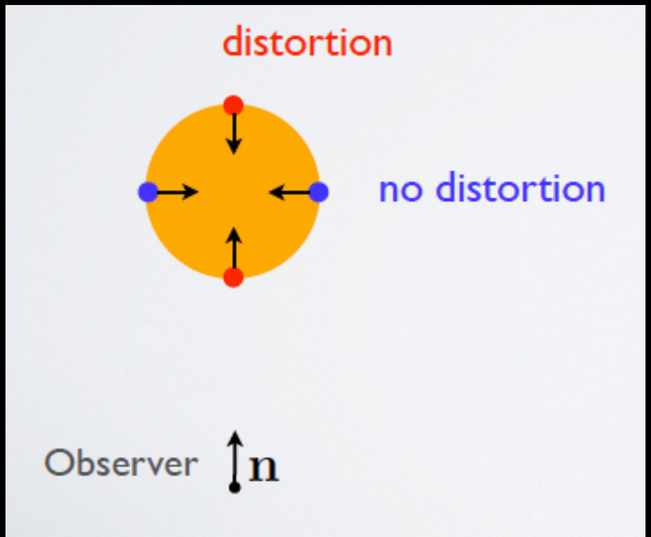
Finally, we get

$$(1+\delta_{g\,\mathrm{obs}})\left(1+\frac{1}{\mathcal{H}}\frac{\partial}{\partial\chi}\mathbf{v}\cdot\mathbf{n}\right)d^3\mathbf{x}=(1+\delta_g)d^3\mathbf{x}$$

which leads to the Kaiser formula

$$\delta_{g\,\mathrm{obs}}=\delta_g-\frac{1}{\mathcal{H}}\frac{\partial}{\partial\chi}\mathbf{v}\cdot\mathbf{n}$$

a better approximation to Δ_g .



The observed fluctuations in number counts are ‘contaminated’ by velocities. Is this a problem?

No – we can separate out the distortion and effectively measure the peculiar velocities and overdensity.

This gives a key test of modified gravity.

$$\delta_{g\,\mathrm{obs}} = \delta_g - \frac{1}{\mathcal{H}}\frac{\partial}{\partial\chi}\mathbf{v}\cdot\mathbf{n} \qquad\qquad \delta_g = b\delta_m$$

In order to access the extra information, we need to relate the Doppler term to the overdensity, as follows:

- galaxy velocity = DM velocity: $\mathbf{v} = \mathbf{v_m}$

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- time derivative of overdensity from growth rate:

$$\delta_m' = f\mathcal{H}\delta_m \quad \text{where} \quad f = \frac{d\ln\delta_m}{d\ln a}$$

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- radial velocity term:

$$\partial_\chi \mathbf{n}\cdot\mathbf{v} = n^i\partial_i(n^j\partial_j V_m) \to -(\mathbf{n}\cdot\mathbf{k})^2 V_m$$

The final result

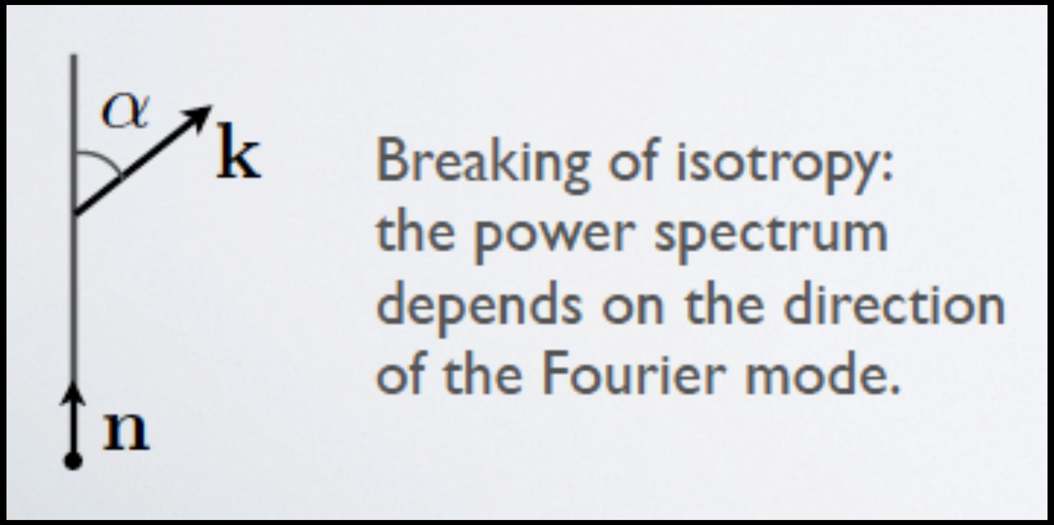
$$\delta_{g\,\mathrm{obs}} = (b + f\mu^2)\delta_m$$

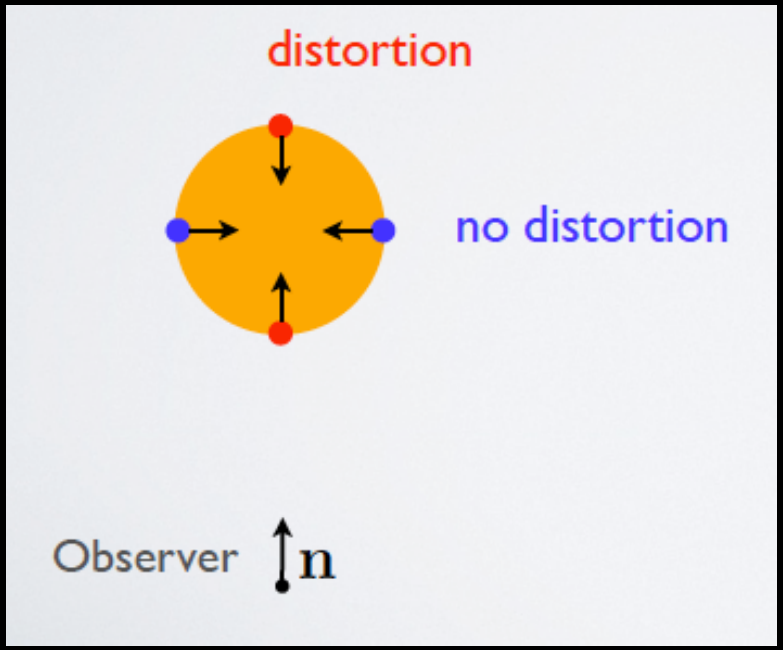
where

$$\mu = \mathbf{n}\cdot\hat{\mathbf{k}} = \frac{k_\parallel}{k} = \cos\alpha$$

leading to the power spectrum:

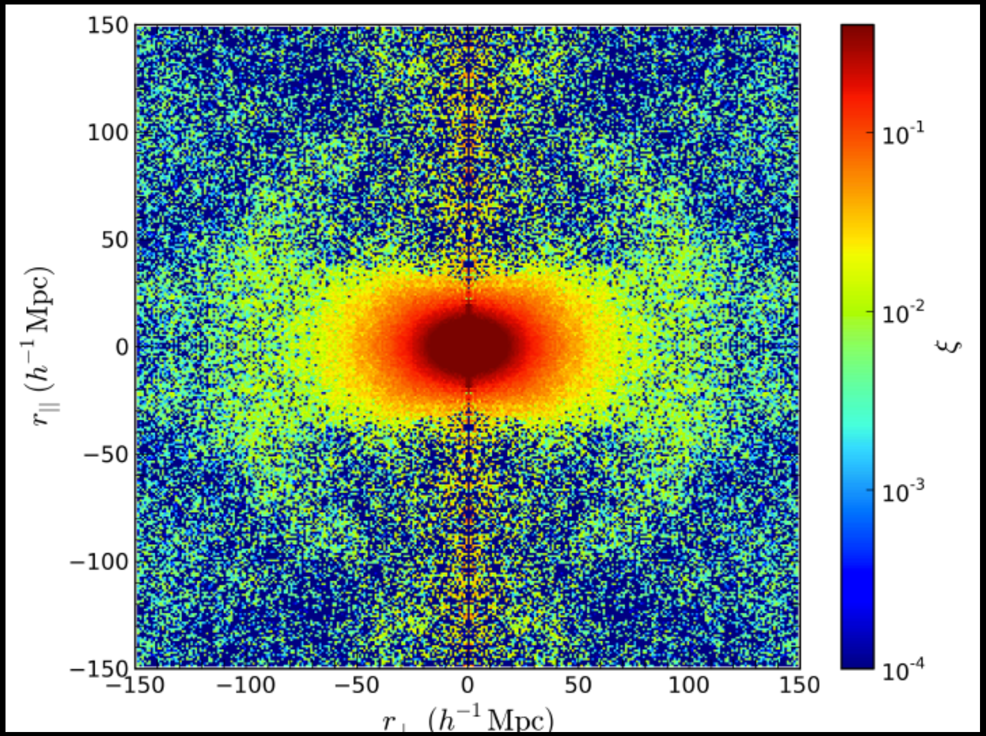
$$P_{g\,\mathrm{obs}}(\eta, k, \mu) = (b + f\mu^2)^2 P_m(\eta, k)$$





$$P_{g\,\mathrm{obs}}(\eta, k, \mu) = (b + f\mu^2)^2 P_m(\eta, k)$$

How can we separate out the information due to anisotropy?



$$P_{g\,\mathrm{obs}}(\eta,k,\mu) = (b + f\mu^2)^2 P_m(\eta,k)$$

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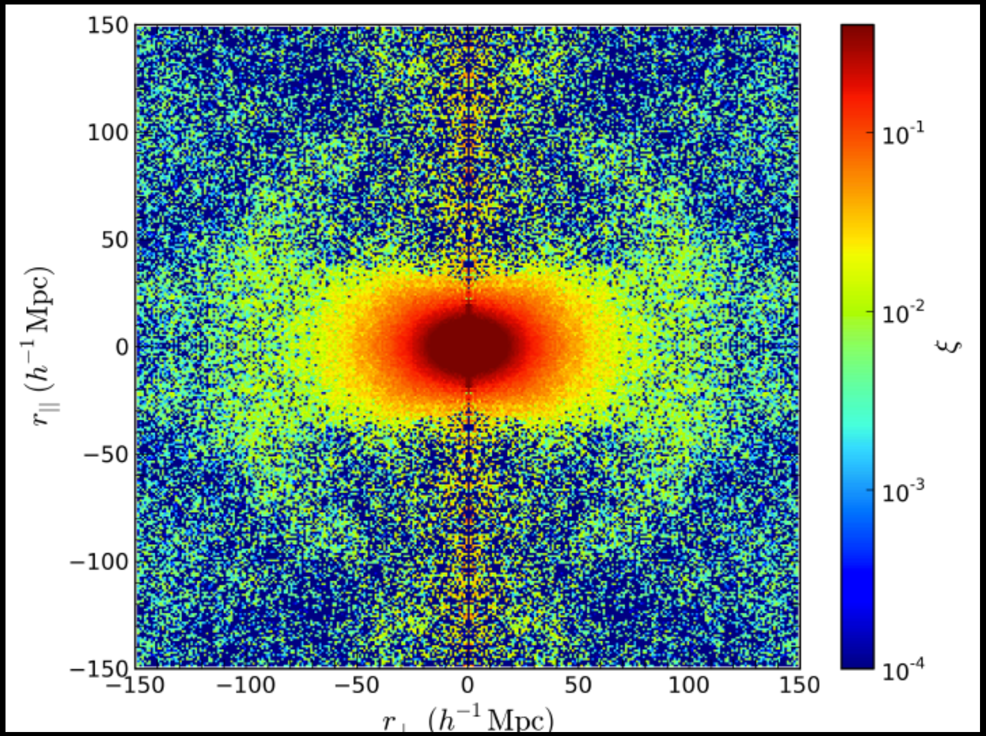


Using an expansion in Legendre polynomials, we get the monopole and quadrupole of the power spectrum:

$$P^0_{\mathrm{g,obs}}(\eta,k) = \frac{1}{2}\int_{-1}^{1} d\mu\, P_{\mathrm{g,obs}}(\eta,k,\mu) = \underline{\left(b^2 + \frac{2}{3}bf + \frac{1}{5}f^2\right) P_m(\eta,k)}$$

$$P^2_{\mathrm{g,obs}}(\eta,k) = \frac{5}{2}\int_{-1}^{1} d\mu\, \mathcal{P}_2(\mu) P_{\mathrm{g,obs}}(\eta,k,\mu) = \underline{\left(\frac{4}{3}bf + \frac{4}{7}f^2\right) P_m(\eta,k)}$$

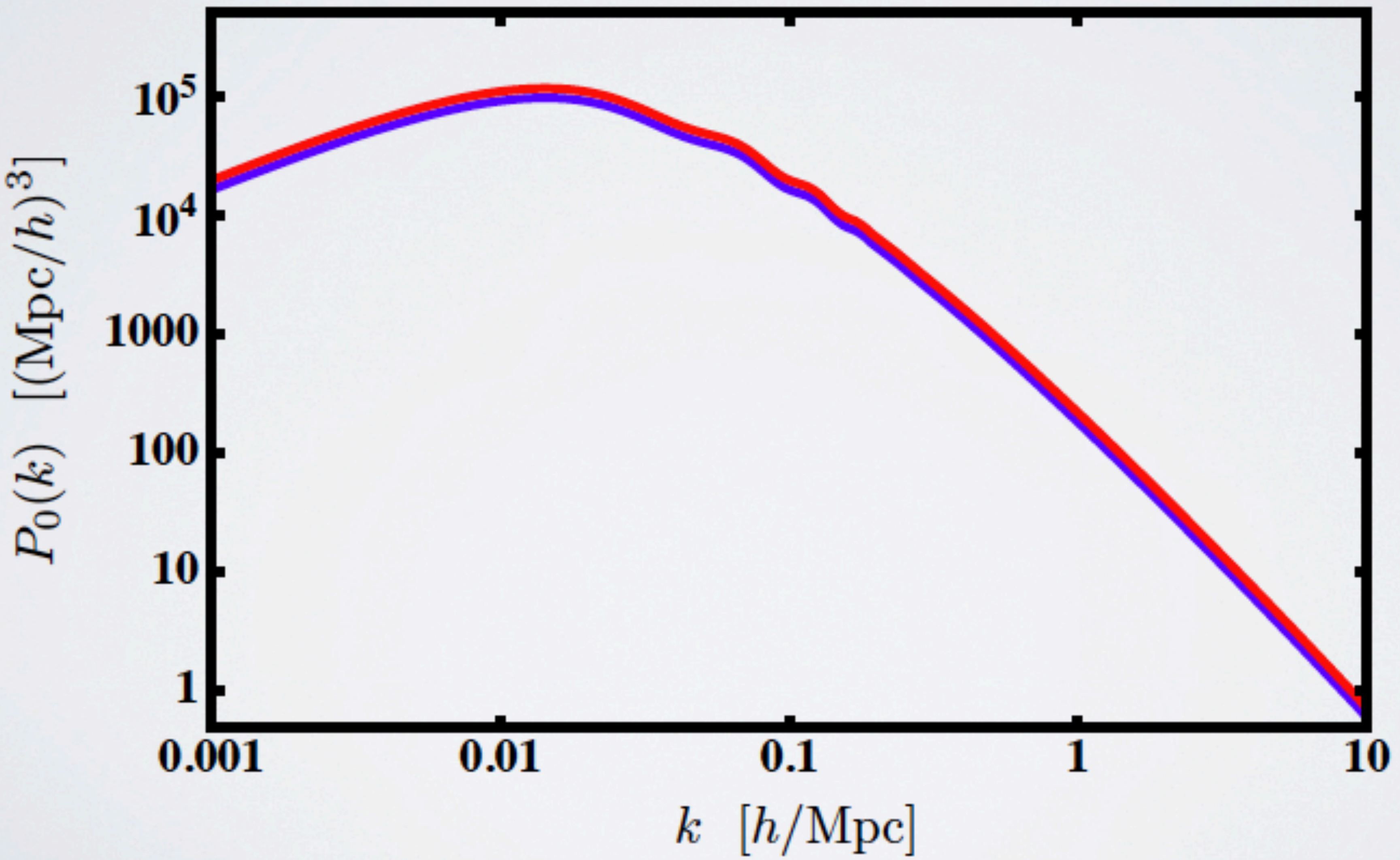
Monopole of the galaxy power spectrum in redshift space:

without redshift distortions

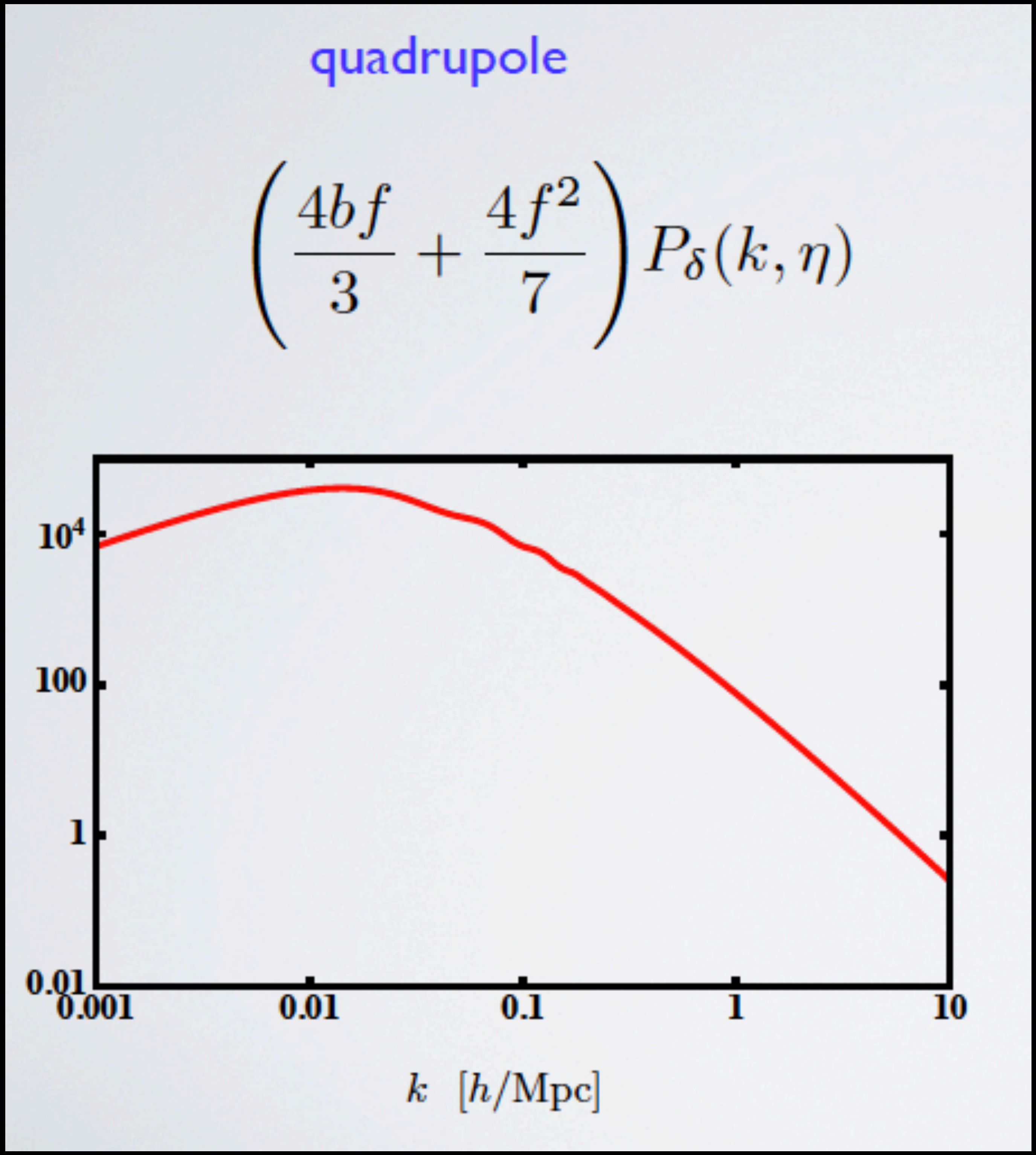
$$b^2 P_\delta(k,\eta)$$

with redshift distortions

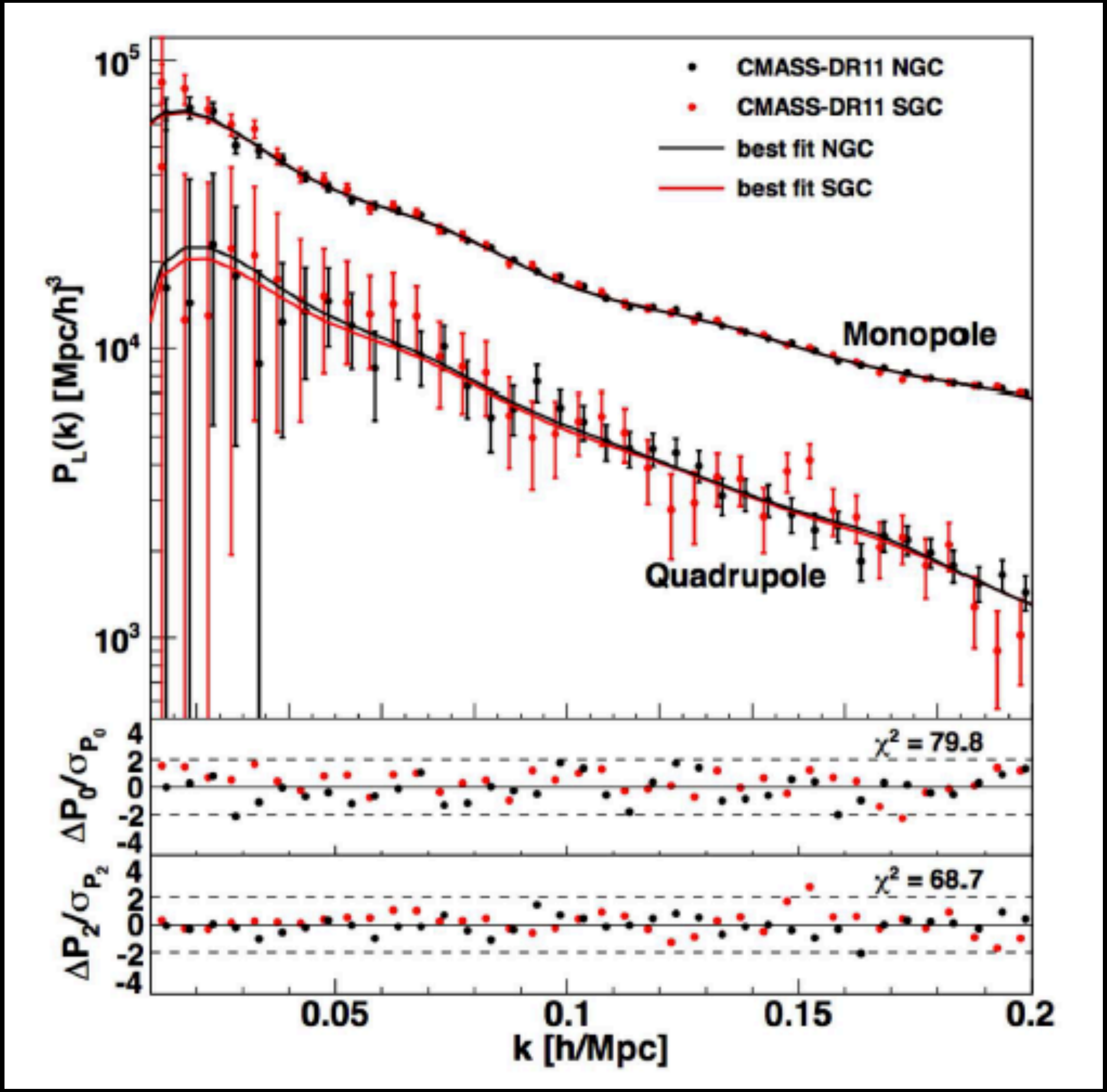
$$\left(b^2 + \frac{2bf}{3} + \frac{f^2}{5}\right) P_\delta(k,\eta)$$



Quadrupole of the galaxy power spectrum in redshift space:



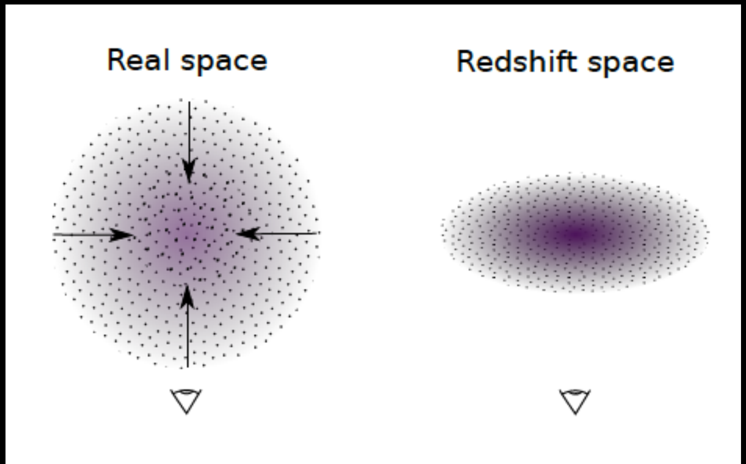
From the BOSS survey



Beutler et al 2014

Measuring the monopole and quadrupole allow us to separately extract *b* and *f* (up to a normalization of the power spectrum).

The growth rate *f* is a good diagnostic of deviations from GR with standard (non-clustering) Dark Energy.



Parametrization:

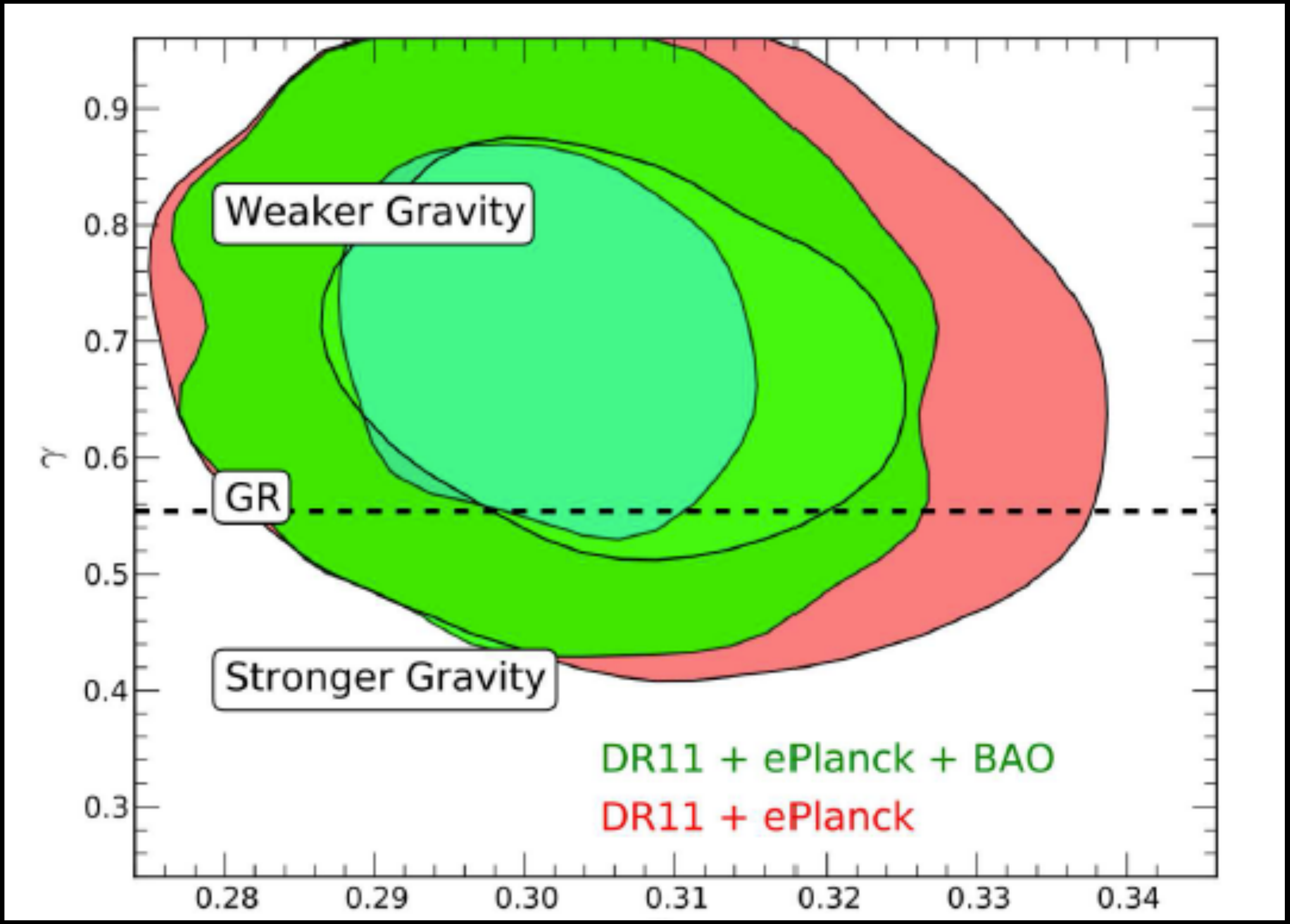
$$f(\eta, \mathbf{k}) = \left[\Omega_m(\eta)\right]^{\gamma(\eta, \mathbf{k})}$$

In LCDM, and dynamical DE where the clustering of DE is negligible,

$$\gamma \approx 0.55$$

A significant deviation from this value could indicate a breakdown of GR.

Probing deviations from GR with BOSS:



Ω_{m0}

Samushia et al 2014

Data is consistent with GR.

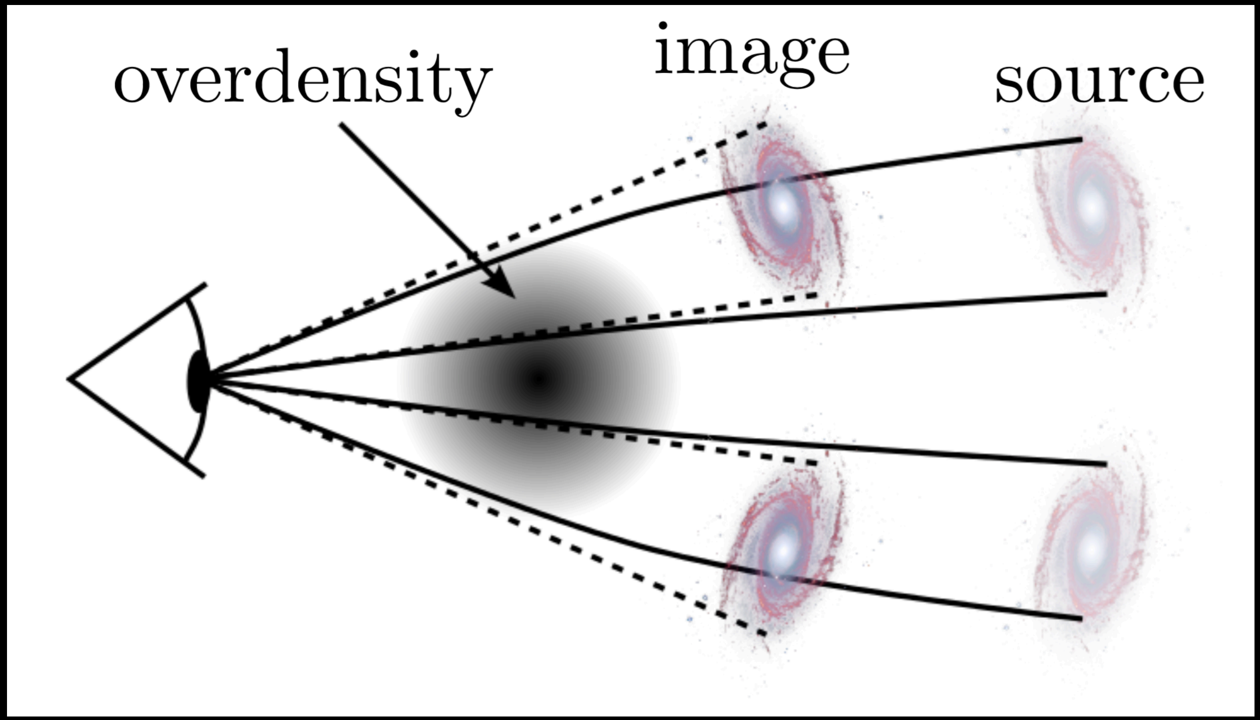
Lensing distortion of number counts

Lensing displaces the images of galaxies away from their true position.

Intervening matter leads to an increase in solid angle:

$$d\tilde{\Omega} = \mathcal{M}\, d\Omega$$

magnification



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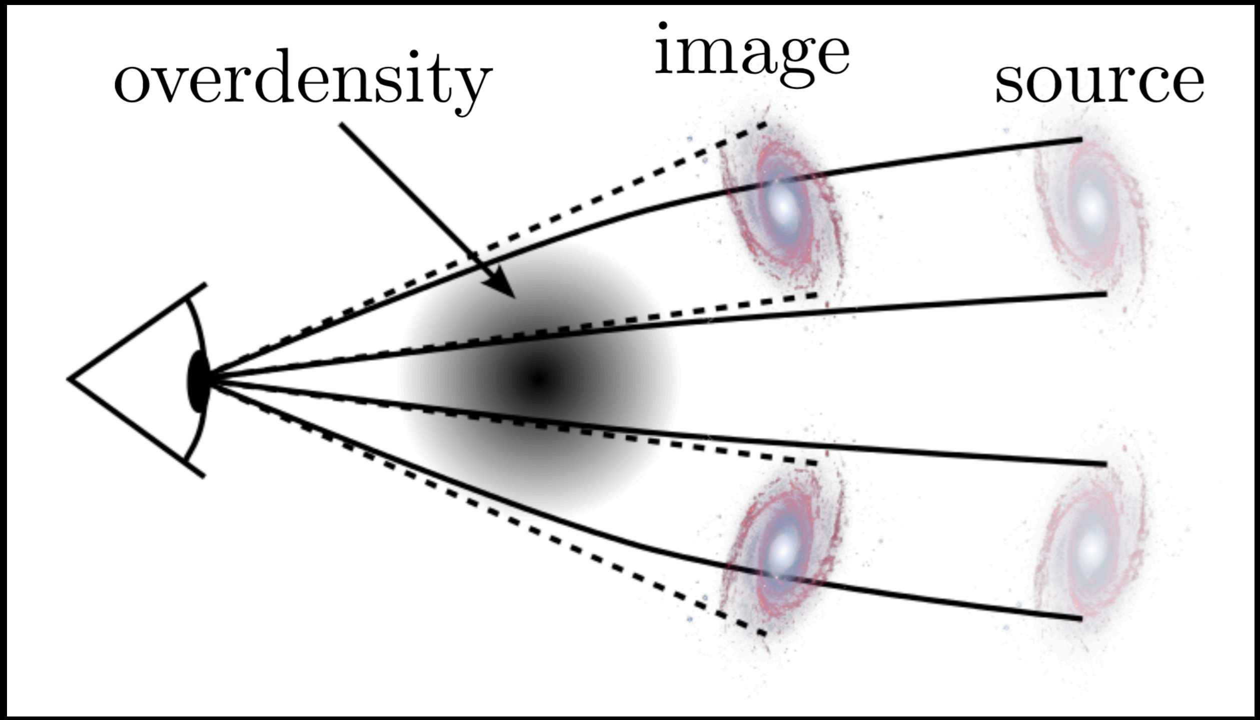

This reduces the number of galaxies per solid angle per redshift:

$$\tilde{N} = \mathcal{M}^{-1} N$$

so that the number count contrast changes:

$$\Delta_g \;\rightarrow\; \Delta_g + \mathcal{M}^{-1} - 1$$

The magnification is determined by the lensing convergence:

$$\mathcal{M} = 1 + 2\kappa$$

$$\kappa = \frac{1}{2} \int_0^{\chi_s} d\chi \, (\chi_s - \chi) \frac{\chi}{\chi_s} \nabla_\perp^2 (\Phi + \Psi)$$

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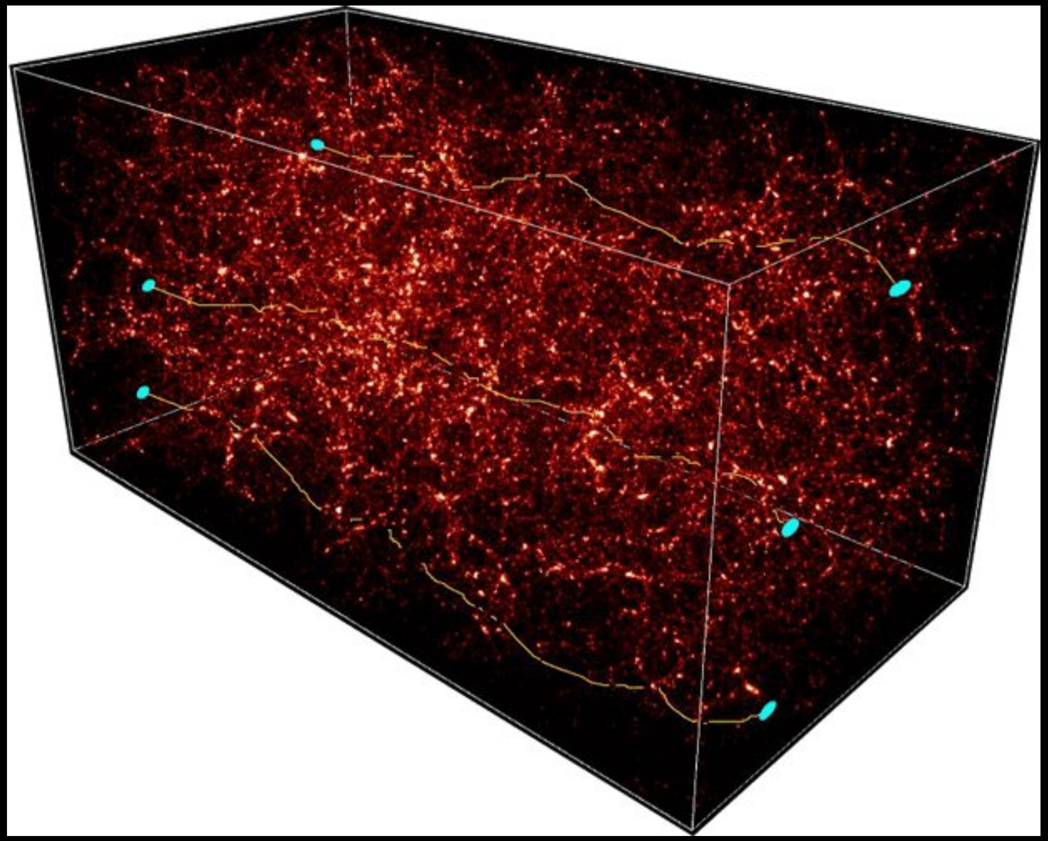
So we have the second correction to the number count contrast:

$$\Delta_g \approx b\,\delta_m - \frac{1}{\mathcal{H}}\frac{\partial}{\partial\chi}\left(\mathbf{v}\cdot\mathbf{n}\right) - 2\,\kappa$$

Similar to RSD – the distortion from lensing contains new information:

- Lensing convergence allows us to effectively measure the lensing potential from number counts
- This is a new way to measure the lensing potential – without the need to measure shapes of galaxies as in lensing shear surveys.
- It can also measure the lensing potential on very large scales.

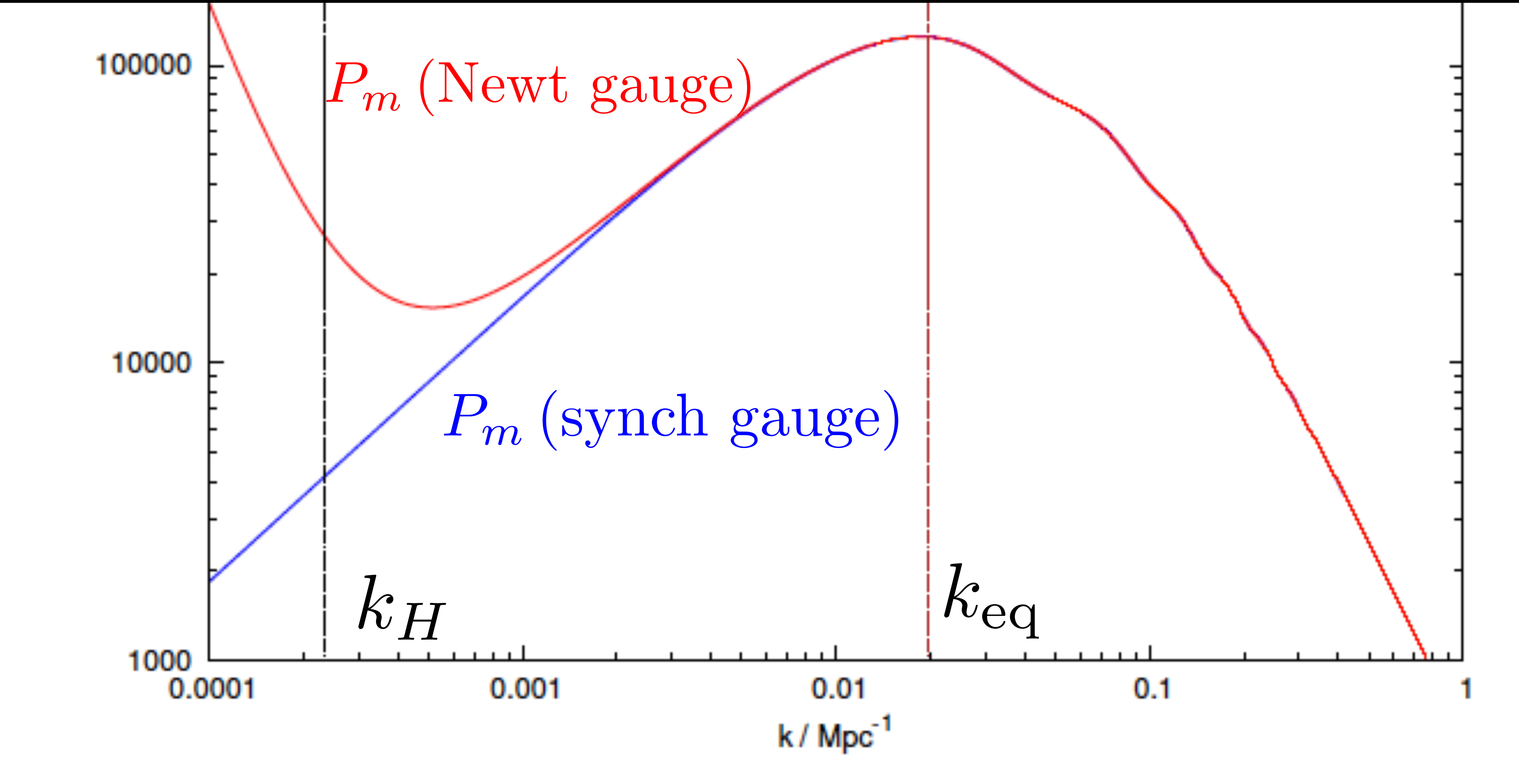


Galaxy bias

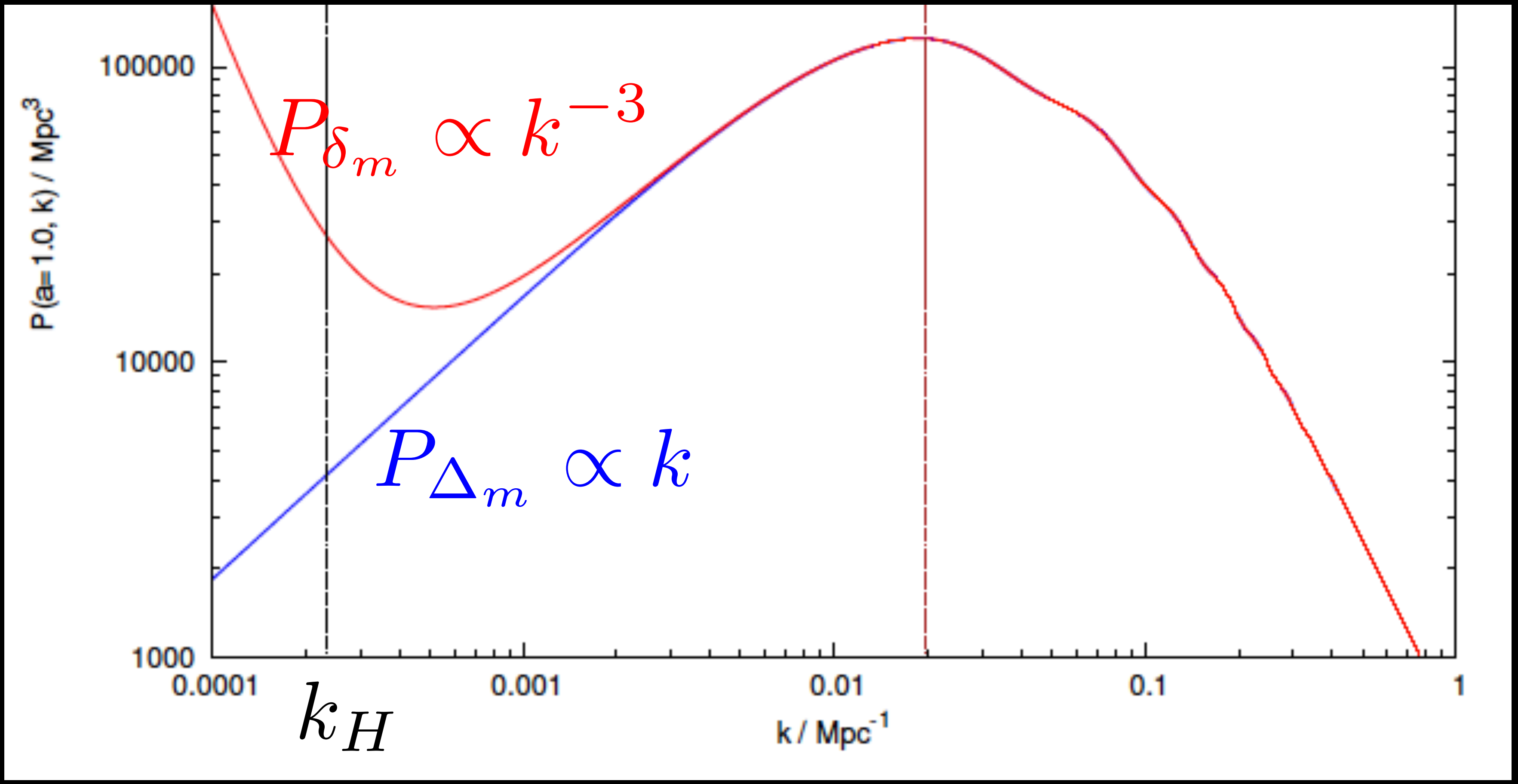
Simple scale-independent bias model for large scales:

$$\delta_g(\eta, \mathbf{k}) = b(\eta)\delta_m(\eta, \mathbf{k})$$

This is gauge-dependent – since δ_m is:



Gauge dependence of $P_m(k)$ on super-Hubble scales



$$\nabla^2\Phi = 4\pi G a^2 \rho_m \delta_m + 3\mathcal{H}(\Phi' + \mathcal{H}\Phi)$$

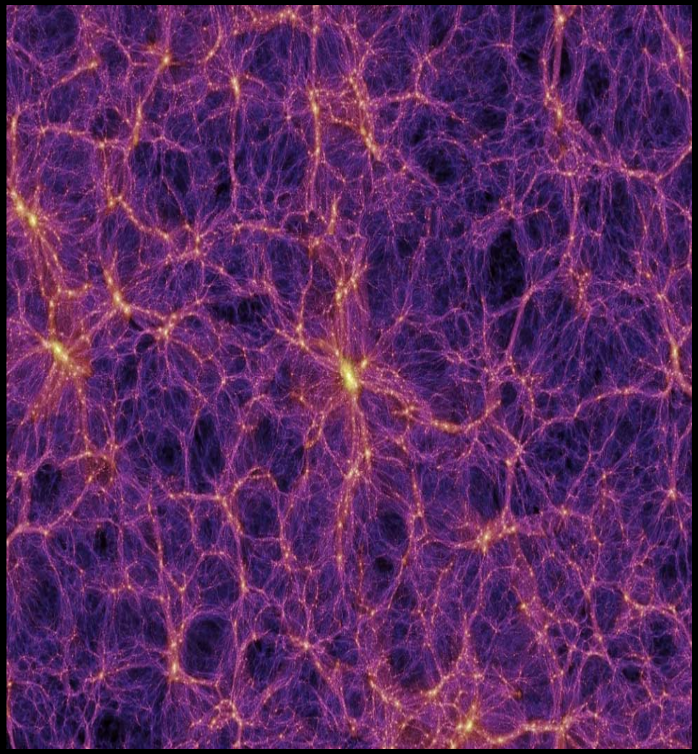
$$\nabla^2\Phi \to \frac{3}{2}\mathcal{H}^2\delta_m + 3\mathcal{H}^2\Phi \quad (\Omega_m = 1,\ \Phi' = 0)$$

$$\delta_m \propto -\frac{2k^2}{3\mathcal{H}^2}\Phi_p - 2\Phi_p \to -2\Phi_p \propto k^{-3/2} \quad (T \to 1)$$

Covariant definition of linear bias:

Identify the physical frame where the relation holds.

Galaxies and CDM have the same velocity on linear scales.
The common rest-frame is the correct frame to define scale-independent bias:



$$\delta_g^{\rm synch}(\eta, \mathbf{k}) = b(\eta)\, \delta_m^{\rm synch}(\eta, \mathbf{k})$$

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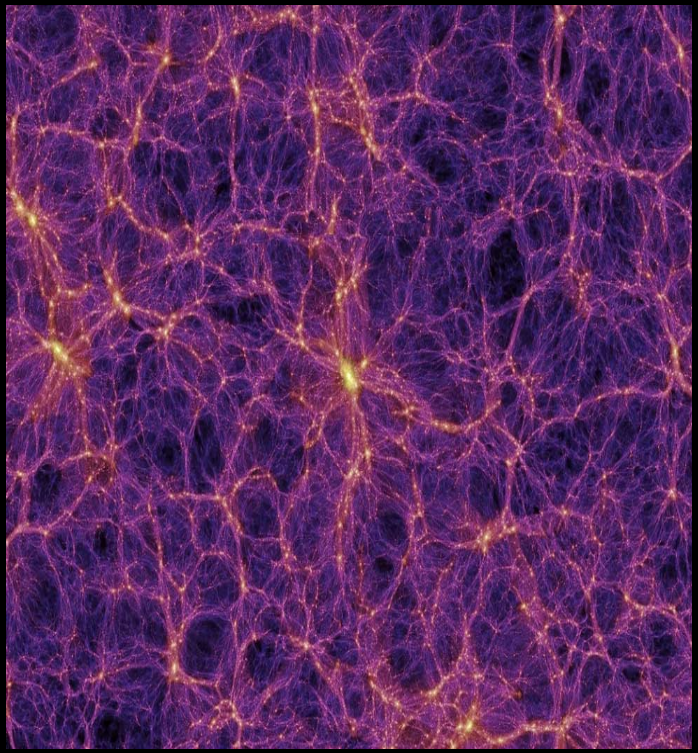


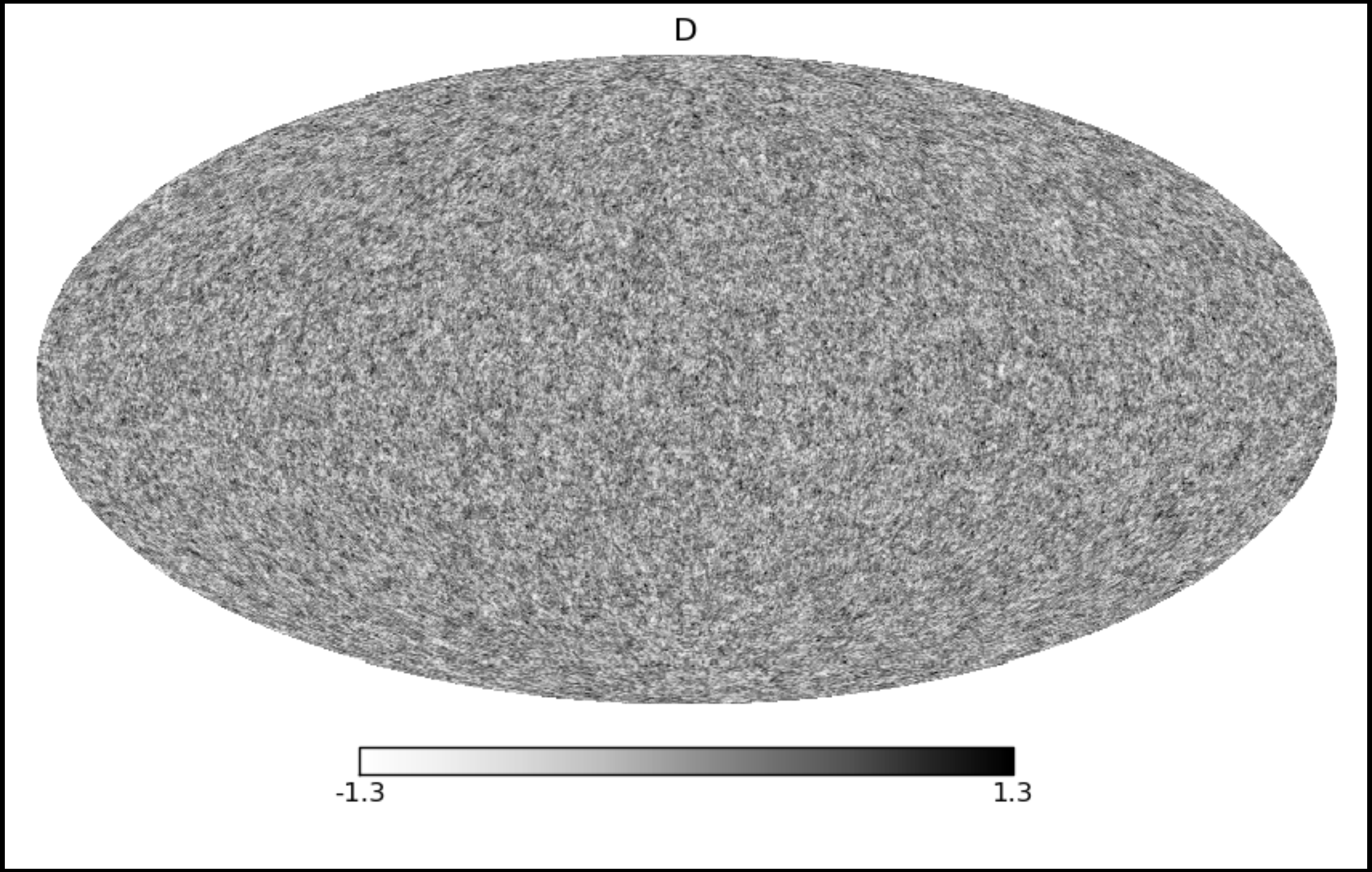
$$\delta_g^{\rm synch}(\eta,\mathbf{k}) = b(\eta)\,\delta_m^{\rm synch}(\eta,\mathbf{k})$$

Then the number count contrast becomes

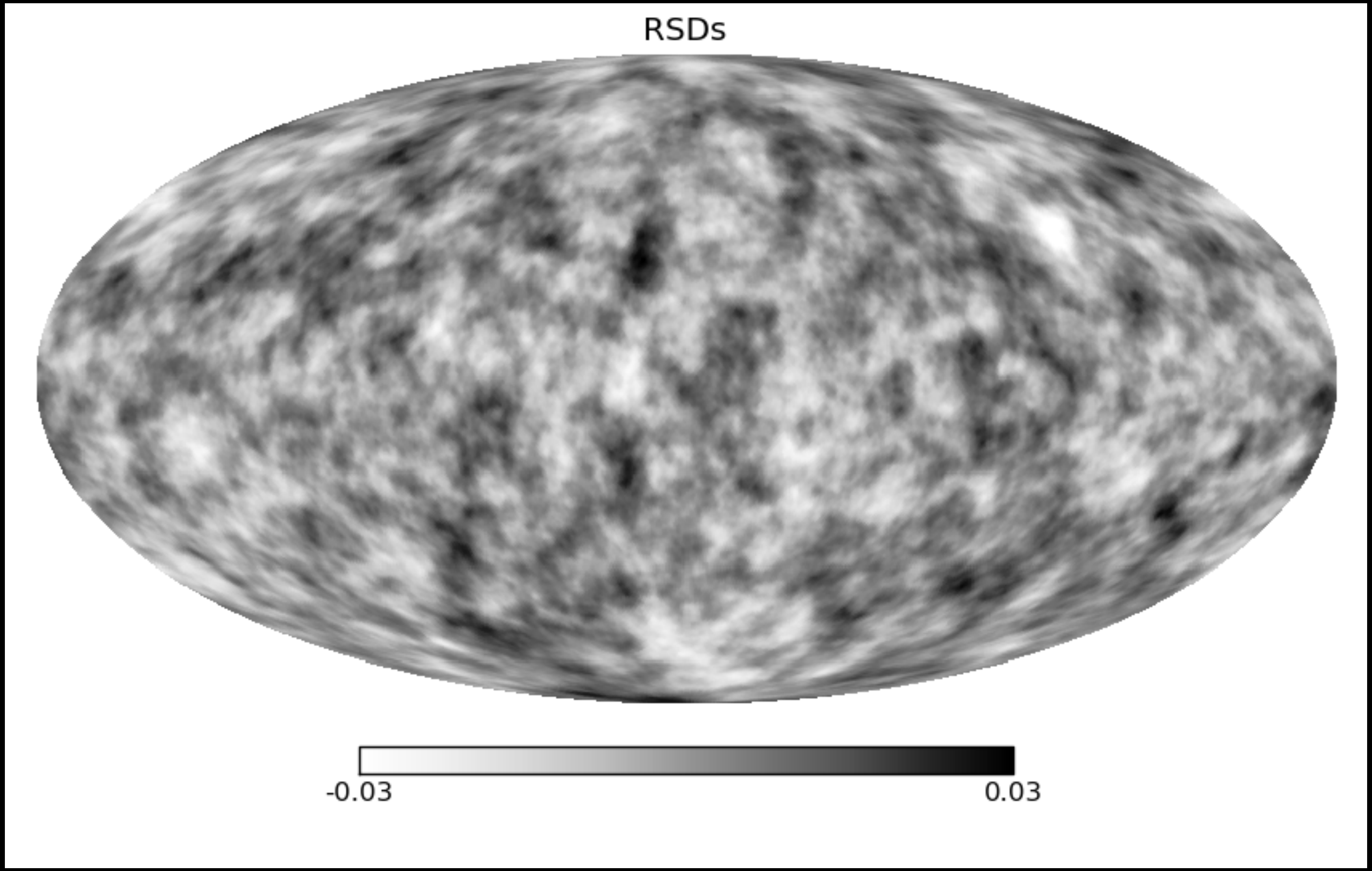
$$\Delta_g \approx b\,\delta_m^{\rm synch} - \frac{1}{\mathcal{H}}\frac{\partial}{\partial\chi}\left(\mathbf{v}\cdot\mathbf{n}\right) - 2\,\kappa + 3\mathcal{H}V$$

from Newt to synch
gauge transformation

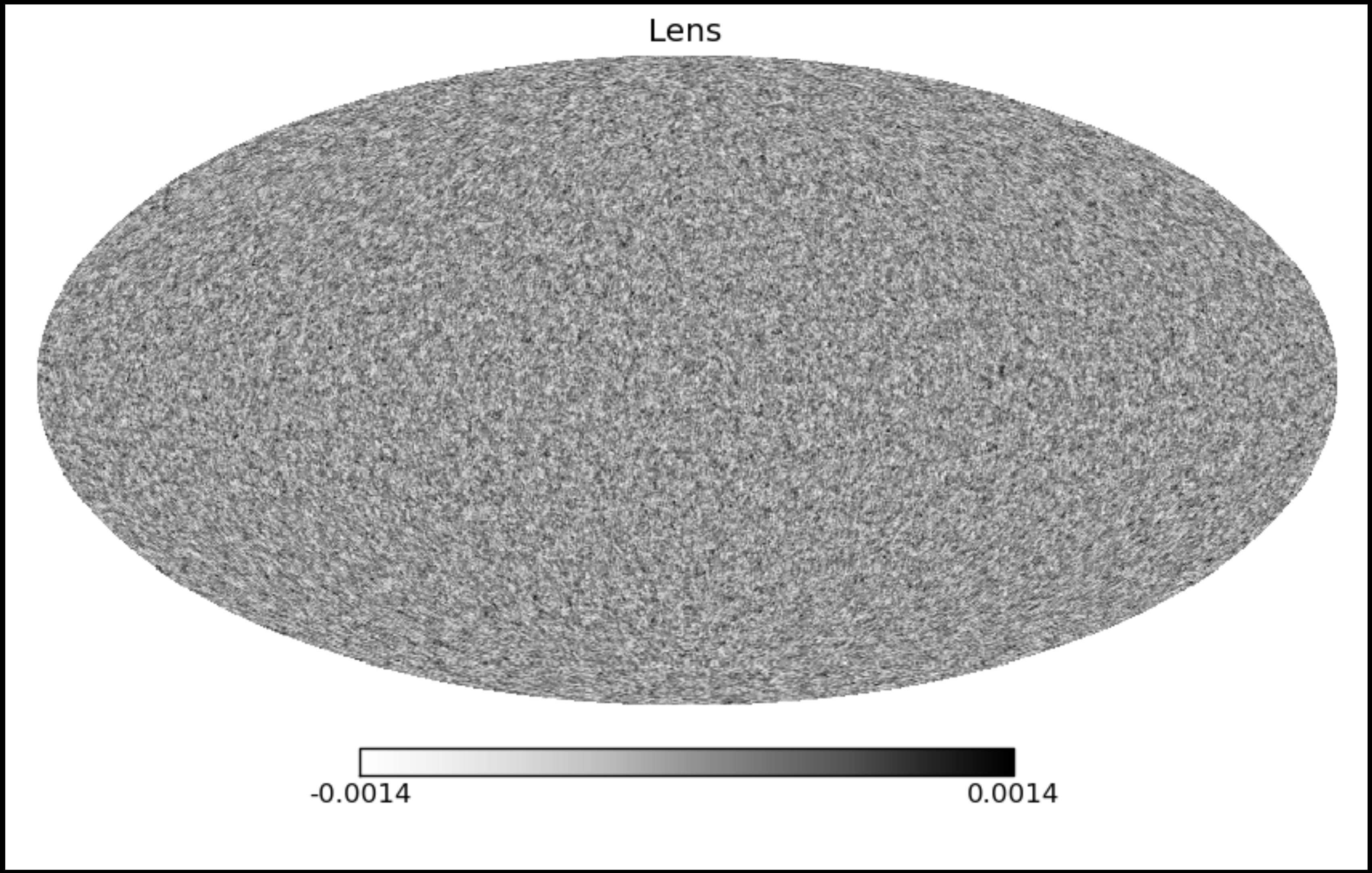
All sky map of density perturbations in Δ_g



All sky map of RSD perturbations in Δ_g



All sky map of lensing perturbations in Δ_g



What other contributions are there to Δ_g ?

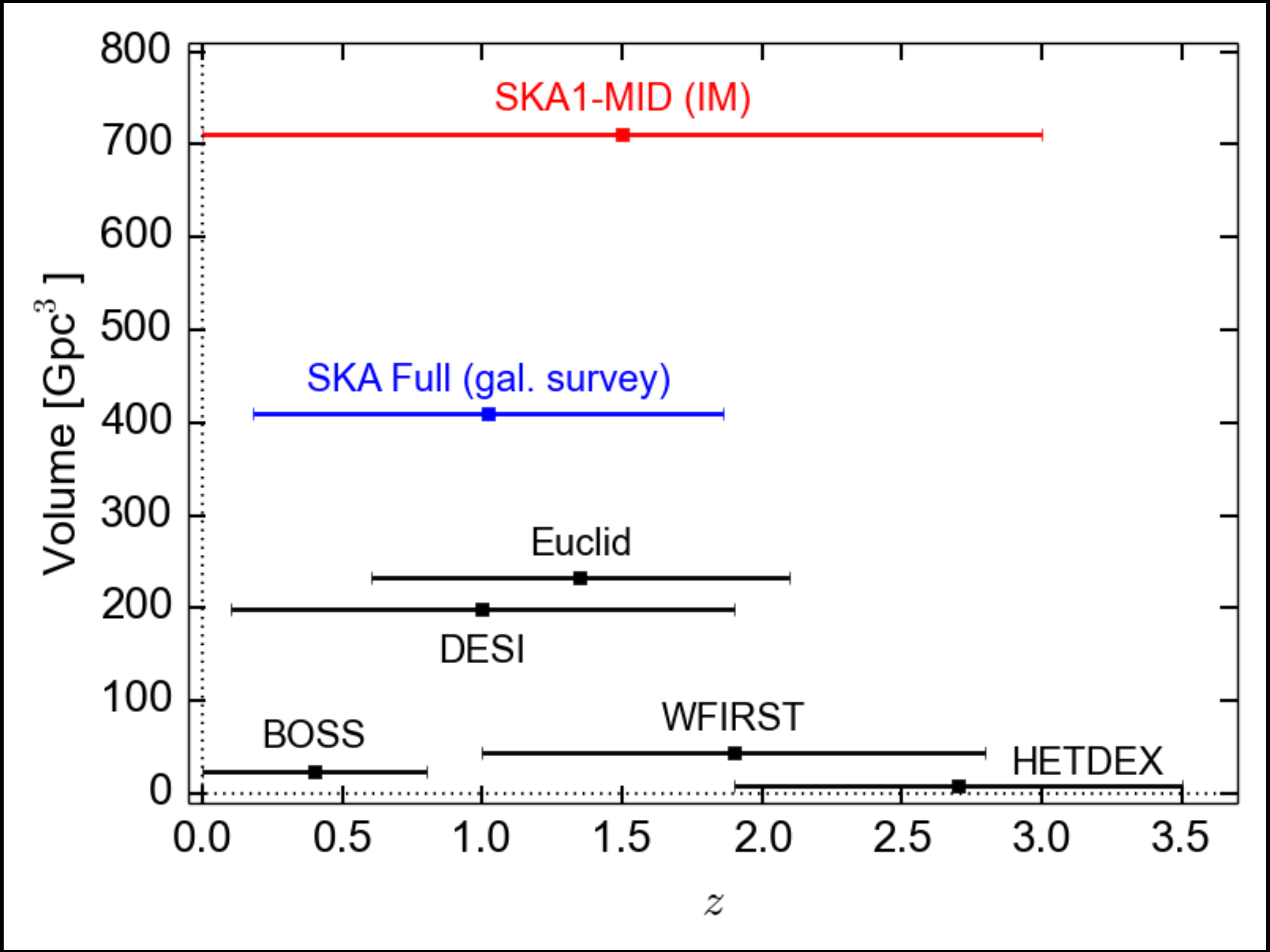
Gravitational redshift?

Thinking of the CMB – what about Sachs-Wolfe and ISW effects?
And time-delay?

These (and some other terms) are *all* present – but they are only non-negligible on ultra-large scales:

$$k_H \leq k \lesssim k_{\rm eq}$$

Future surveys will cover huge volumes, and include ultra-large scale modes:



The ultra-large scale effects on number counts have been derived.

(Yoo, Fitzpatrick, Zaldarriaga 2009; Yoo 2010; Bonvin, Durrer 2011; Challinor, Lewis 2011)

$$\text{Notation:} \quad r \equiv \chi, \; D \equiv \delta_m^{\text{synch}}, \; \Delta_\Omega \equiv r^2 \nabla_\perp^2$$

density

redshift-space distortion

$$\Delta(z, \mathbf{n}) = b \cdot D - \frac{1}{\mathcal{H}} \partial_r (\mathbf{V} \cdot \mathbf{n})$$

lensing

$$- \int_0^r dr' \frac{r - r'}{r r'} \Delta_\Omega (\Phi + \Psi)$$

Doppler

gravitational redshift

$$+ \left(1 - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - \frac{2}{r\mathcal{H}}\right) \mathbf{V} \cdot \mathbf{n} + \frac{1}{\mathcal{H}} \dot{\mathbf{V}} \cdot \mathbf{n} + \frac{1}{\mathcal{H}} \partial_r \Psi$$

$$+ \Psi - 2\Phi + \frac{1}{\mathcal{H}} \dot{\Phi} - 3 \frac{\mathcal{H}}{k} V + \frac{2}{r} \int_0^r dr' (\Phi + \Psi)$$

$$+ \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{r\mathcal{H}}\right) \left[\Psi + \int_0^r dr' (\dot{\Phi} + \dot{\Psi})\right]$$

→ potential

standard expression

$$\Delta(z,\mathbf{n}) = b\cdot D - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n})$$

lensing: important at high z

$$- \int_0^r dr' \frac{r-r'}{rr'}\Delta_\Omega(\Phi+\Psi)$$

relativistic contributions: important at large scale

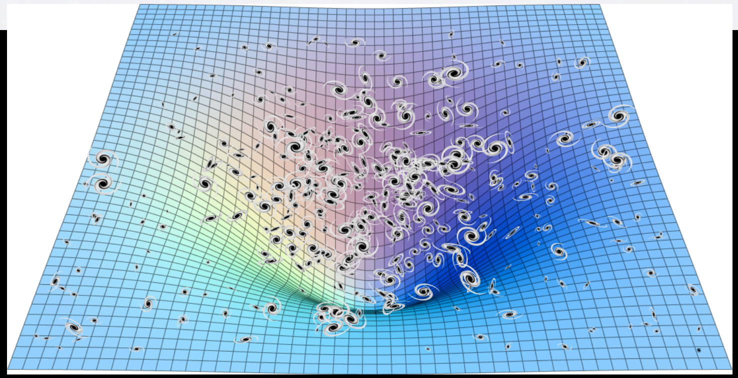
$$+ \left(1-\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}-\frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\partial_r\Psi$$

$$\frac{\mathcal{H}}{k}D$$

$$+ \Psi - 2\Phi + \frac{1}{\mathcal{H}}\dot{\Phi} - 3\frac{\mathcal{H}}{k}V + \frac{2}{r}\int_0^r dr'(\Phi+\Psi)$$

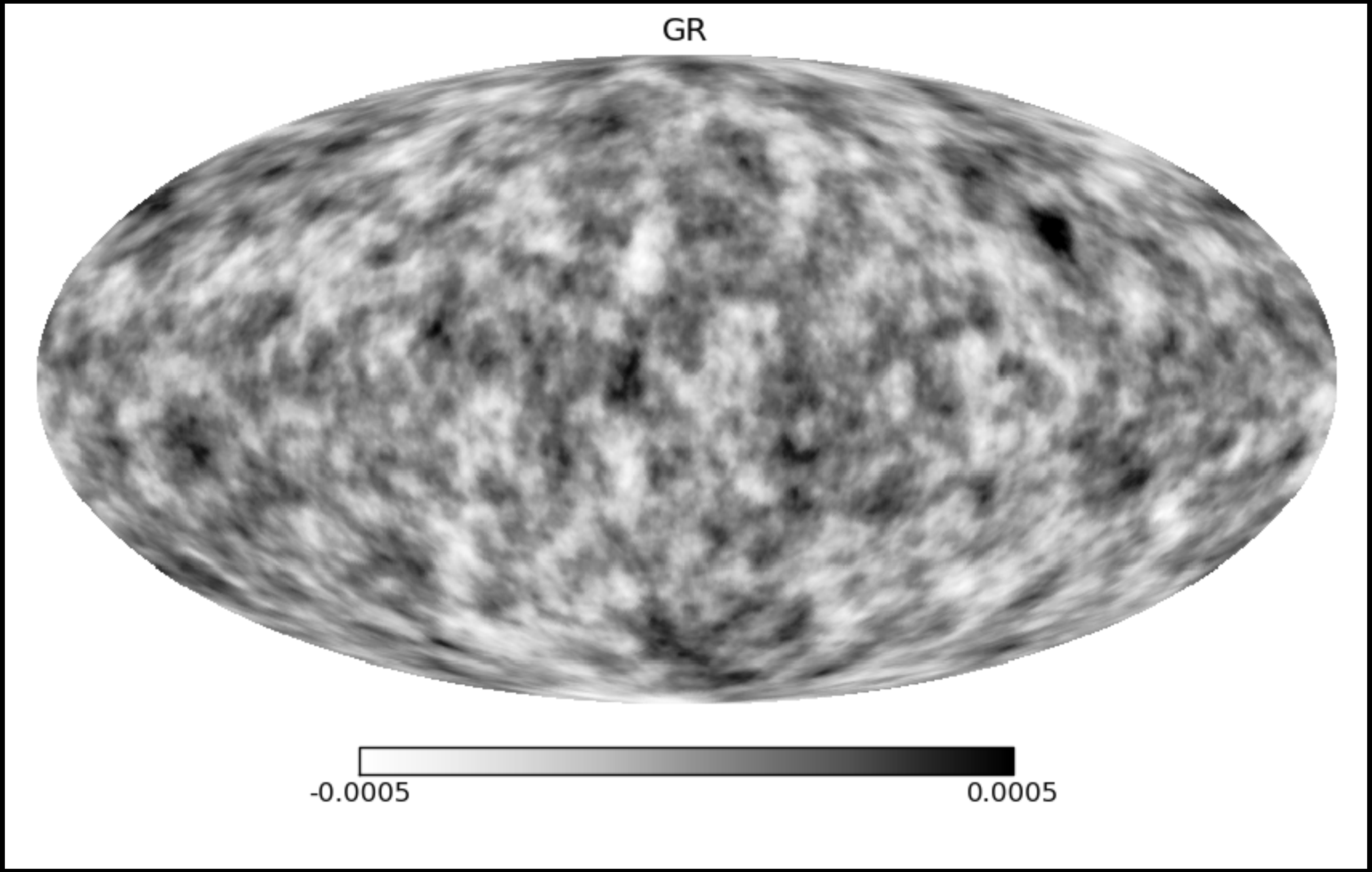
$$+ \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{2}{r\mathcal{H}}\right)\left[\Psi + \int_0^r dr'(\dot{\Phi}+\dot{\Psi})\right]$$

$$\left(\frac{\mathcal{H}}{k}\right)^2 D$$



There is new information in the ultra-large scale GR terms.
These terms grow on very large scales – but so does cosmic variance.

All sky map of horizon-scale GR terms in Δ_g



The ultra-large scale effects contain additional information.

But even the biggest and best future galaxy surveys – Euclid, LSST, SKA – will be unable to measure these effects *on their own,* because of cosmic variance.

(Yoo et al 2013; Alonso, RM et al 2015; Raccanelli et al 2015)

However, with the multi-tracer method – i.e. using 2 different tracers of the stochastic DM distribution – we can detect the horizon-scale GR terms.

(Alonso & Ferreira 2015; Fonseca, RM et al 2015)

See talk 2.