Conformal Symmetry in Einstein-Cartan Gravity

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ArXiv: 1606.02677 & 1709.00330

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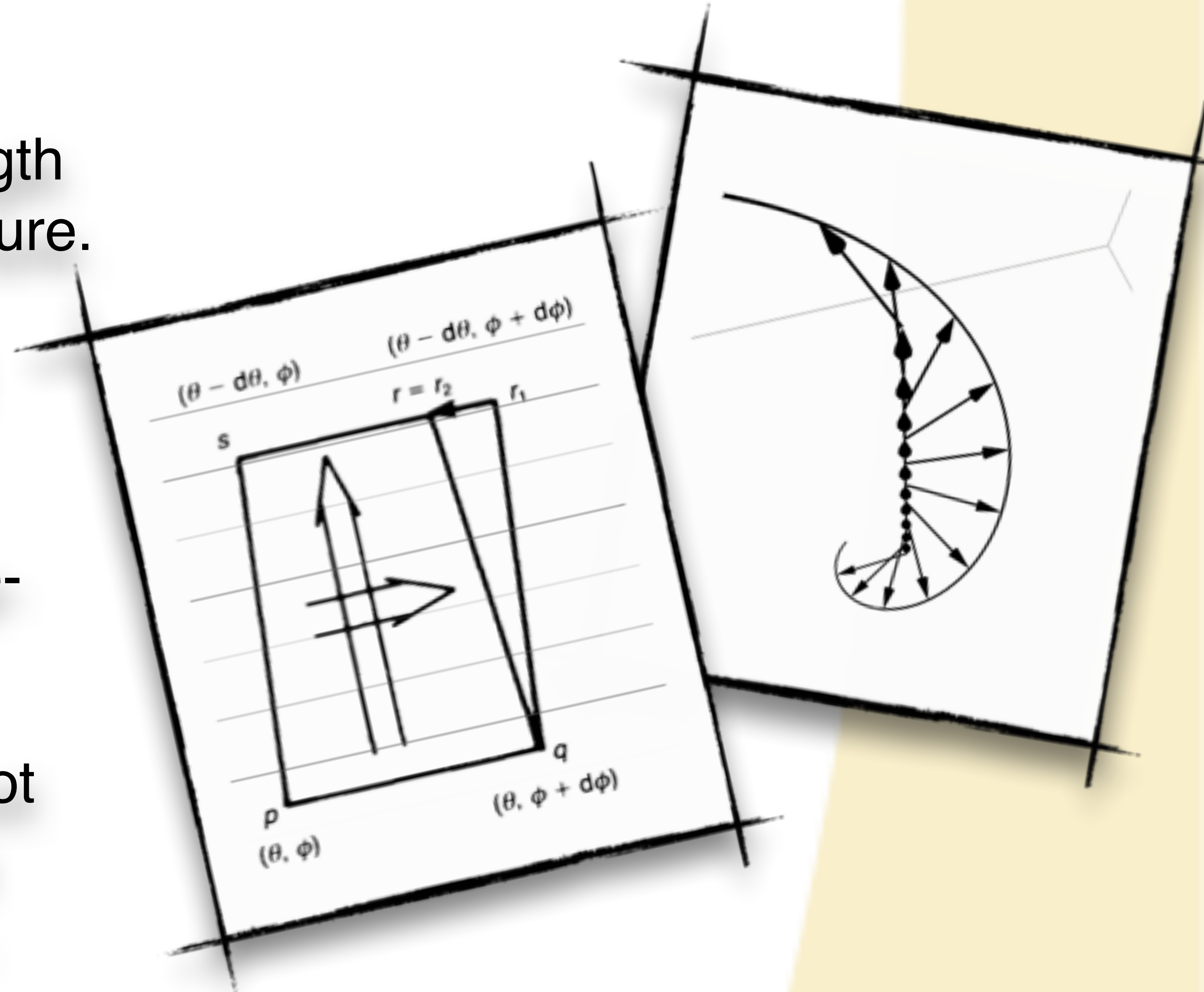
SIFNOS, September 2017

Some geometrical intuition

- Einstein: Gravity is a geometrical force, its strength given by space-time curvature.
- Cartan: adds an additional geometrical structure, separated from curvature, linked to “twisting” of space-time.
- Misconception: torsion is not just an external field. It is a geometrical universal field.



$$\Gamma^{\lambda}_{[\mu\nu]}$$

$$T^a = \mathrm{d}e^a + \omega^a_b \wedge e^b$$

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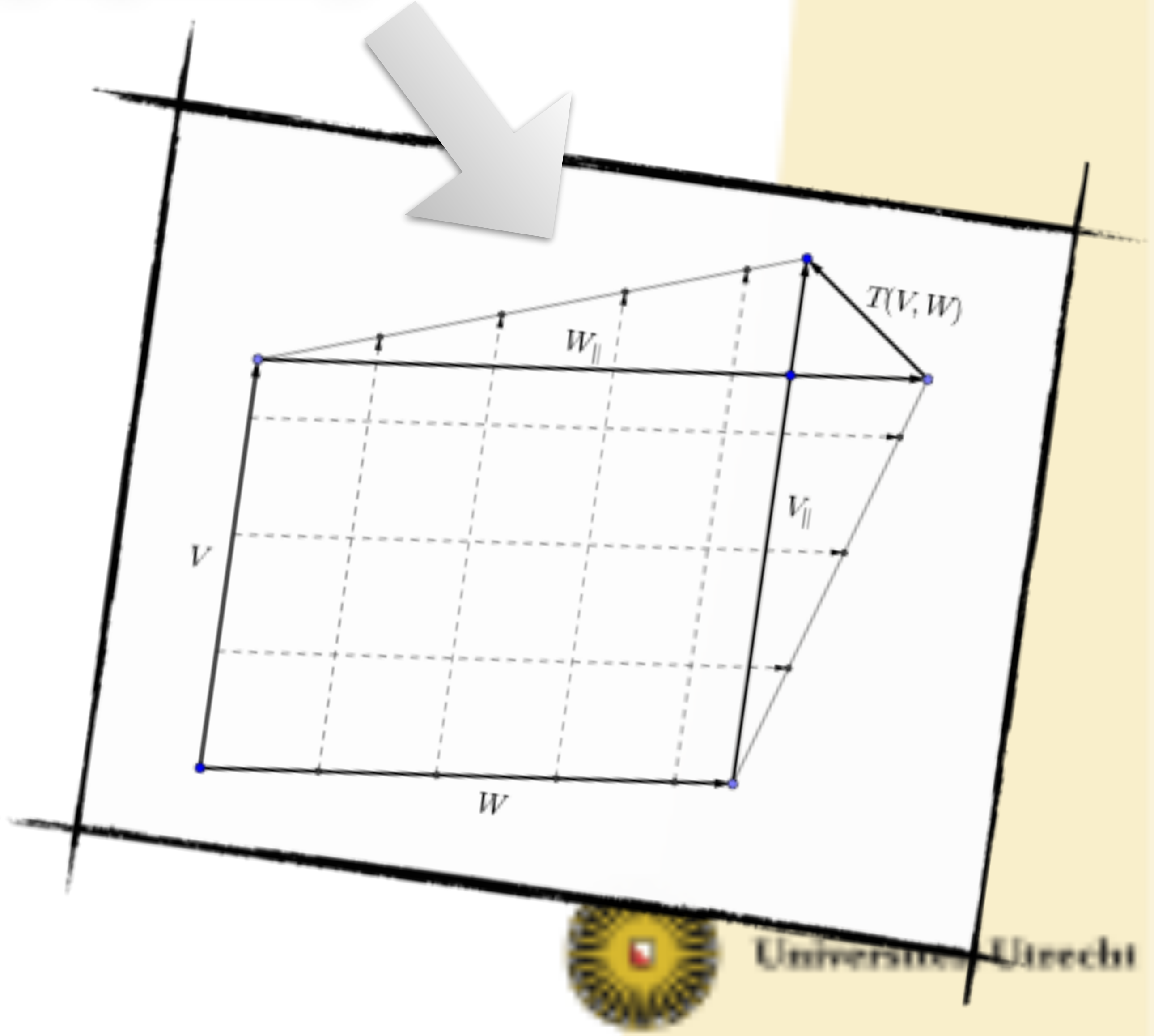
The link between torsion and Weyl symmetry

- Why should torsion be linked to Weyl symmetry?

$$\omega^a_b \to \omega^a_b$$

$$e^a_\mu \to e^{\theta(x)} e^a_\mu$$

$$T^a \to T^a + e^a \wedge \mathrm{d}\theta$$



- The torsion trace is naturally linked to scale transformations.
- Transforming torsion and vierbein leaves the Cartan connection invariant.

Geometrical properties

- Riemann curvature and geodesics trajectories are frame invariant.

$$R^{\lambda}{}_{\sigma\mu\nu} \to R^{\lambda}{}_{\sigma\mu\nu}$$

$$\dot{\gamma}^{\mu} \to e^{-\theta(x)}\dot{\gamma}^{\mu}$$

$$\nabla_{\dot{\gamma}}\dot{\gamma}^{\mu} \to e^{-\theta(x)}\nabla_{\dot{\gamma}}\dot{\gamma}^{\mu}$$

Proper time reparametriz-ation

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu}$$

$$T_{\mu\nu} \to e^{-(D-2)\theta(x)}T_{\mu\nu}$$

$$\kappa \equiv \frac{\alpha}{\Phi^2(x)} \to e^{(D-2)\theta(x)}\frac{\alpha}{\Phi^2(x)}$$

- Trajectories of free falling bodies invariant up to a reparametrization of time.
- Absence of dimension-full parameters requires dynamical Planck Mass.

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Scale symmetry and dilatation current

- Scale invariant theory possess a Noether charge, the dilatation current

$$\Pi^\mu = -\frac{D-2}{2}\phi\partial^\mu\phi$$

- If scale invariance is exact on the state of the field, the scale current is conserved and energy tensor is traceless

$$\partial_\mu \Pi^\mu = 0$$

$$T^\mu_\mu = 0$$

- If the theory is scale invariant, the equation of motion imply

$$T^\mu_\mu = -\partial_\mu \Pi^\mu$$

$$\Longrightarrow \quad \Pi^\mu = T^\mu_\nu x^\nu \text{ if } g_{\mu\nu} = \eta_{\mu\nu}$$

Interactions in scalar theory

- For scalars the dilatation current is:

$$\Pi^\mu = \frac{D-2}{2}\phi\partial^\mu\phi \implies \partial_\mu\Pi^\mu = T^\mu_\mu$$

- Idea: couple dilatation current to torsion trace (and complete theory by requiring symmetry).

$$\mathcal{L}_{int} = T_\mu\Pi^\mu + \left(\frac{D-2}{2}\right)^2 T_\mu T^\mu \phi^2$$

- Extension of gravitational field sources. Equation of motion imply the fundamental equation:

$$\Pi^\mu = \frac{1}{\sqrt{-g}}\frac{\delta S}{\delta T_\mu}$$

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}}$$

$$\nabla_\mu\Pi^\mu + T^\mu_\mu = 0$$

Modified gravity theory

- By symmetry all operators are marginal (dimensionless couplings)

$$\mathcal{L}_\phi = -\frac{1}{2}\left(\bar{\nabla}\phi\right)^2 + \frac{\xi}{2}\phi^2 R + \lambda\phi^4$$

- Any curvature coupling is allowed, as long as the coupling constant is dimensionless

$$\mathcal{L}_{grav} = \alpha\bar{R}^2 + \beta\bar{R}_{\mu\nu}\bar{R}^{\mu\nu} + \gamma F_{\mu\nu}F^{\mu\nu}$$

- Second order in curvature possible and automatically satisfy the fundamental equation

$$T^{grav}_{\mu\nu}g^{\mu\nu} + \nabla_\mu \Pi^\mu_{grav} = 0$$

- The rest of the SM is ok in D=4!

$$\frac{i}{2}\bar{\psi}\gamma^\mu \overset{\leftrightarrow}{\nabla}_\mu \psi + \phi\bar{\psi}\psi + \mathrm{Tr}\left(\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}\right)$$

Weyl symmetry in the quantum theory(formally)

- Phase space quantisation is manifestly Weyl invariant:

$$\pi = \frac{\delta S}{\delta \dot{\phi}}$$

$$[\phi, \pi] = i\hbar\delta^{(D-1)}(\vec{x} - \vec{x}')$$

- This means that the Weyl symmetry Ward identities are preserved:

$$\int \mathcal{D}\phi\mathcal{D}\pi \exp\left(iS[\phi,\pi]\right) =$$
$$= \int \mathcal{D}\phi \det{}^{\frac{1}{2}} \left(\sqrt{-g}g^{00}\delta^{(D-1)}\left(\vec{x} - \vec{x}'\right)\right) \exp\left(iS[\phi]\right)$$

$$\langle \nabla_\mu \hat{\Pi}^\mu \rangle + \langle \hat{T}^\mu_\mu \rangle = 0$$

→

- Source dilatation current by generating Energy momentum trace

- Identity "broken" by terms which vanish upon regularisation, e.g.

$$(D-4)\lambda\langle\hat{\phi}^4\rangle$$

Local anomaly, topological terms

- This violates something we learned in the past years:

$$S_{eff} = \int \mathrm{d}^D x \frac{\sqrt{-g}}{D-4} \left(\bar{R}^2 - 4\bar{R}_{\mu\nu}\bar{R}^{\nu\mu} + \bar{R}_{\alpha\beta\gamma\delta}\bar{R}^{\gamma\delta\alpha\beta}\right)$$

$$\langle \hat{T}^{\mu}_{\mu} \rangle = C\left(\bar{R}^2 - 4\bar{R}_{\mu\nu}\bar{R}^{\nu\mu} + \bar{R}_{\alpha\beta\gamma\delta}\bar{R}^{\gamma\delta\alpha\beta}\right) \neq 0$$

→ Local anomaly

- Including torsion trace this is compensated, and does not violate the fundamental Ward identity.

$$T^{\mu}_{\mu} + \nabla_{\mu}\Pi^{\mu} = 0$$

- This is because the Gauss Bonnet integral is a boundary term, and gets absorbed in the divergence of the dilatation current.

$$\bar{R}^2 - 4\bar{R}_{\mu\nu}\bar{R}^{\nu\mu} + \bar{R}_{\alpha\beta\gamma\delta}\bar{R}^{\gamma\delta\alpha\beta} = \nabla_{\mu}\mathcal{V}^{\mu}$$

$$\Longrightarrow \Pi^{\mu} \to \Pi^{\mu} + \mathcal{V}^{\mu}$$

Running of coupling constants and all that

- One can show that in the renormalised theory the trace of the energy momentum tensor gets a contribution from the beta functions of the theory.

$$\langle \hat{T}^{\mu}_{\mu} \rangle = \sum_i \beta_i(\lambda_j;\mu)\mathcal{L}^i_{int}$$

- The same terms must source the divergence of dilatation current, by Noether theorem.

$$\langle \nabla_\mu \hat{\Pi}^{\mu} \rangle = -\sum_i \beta_i(\lambda_j;\mu)\mathcal{L}^i_{int}$$

- The fundamental Ward identity must stay true in the renormalised theory. How can we achieve this?

- Natural solution:

$$\mu = e^{-\theta}\ ,\ \theta\,\text{goldstone}$$

$$T_\mu = T^{\mathcal{T}}_\mu + \partial_\mu \theta$$

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Perspective: Conformal symmetry breaking

- Generically, quantum effects dynamically generate a scale.

- Can only break the global symmetry but not the gauge symmetry.

$$\langle \hat{T}^\mu_\mu \rangle \neq 0$$

$$\langle \nabla_\mu \hat{\Pi}^\mu \rangle \neq 0$$

Goldstone mode

$$g_{\mu\nu} = e^{2\theta}\eta_{\mu\nu}\,,\; T_\mu = \partial_\mu \theta$$

$$\langle \hat{T}^\mu_\mu \rangle = \frac{\delta S_{\text{eff}}}{\delta\theta}$$

$$\langle \nabla_\mu \hat{\Pi}^\mu \rangle = -\partial_\mu \frac{\delta S_{\text{eff}}}{\delta \partial_\mu \theta}$$

- Effective action depends on the goldstone mode and matter fields.

- On-shell we can set the goldstone to a constant, and recover usual effective theory.

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Conclusions

- We constructed a theory of gravity and torsion which is locally Weyl invariant.
- Formal arguments led us to propose that the trace anomaly is actually just a manifestation of sourcing the dilatations current, but does not actually break the local symmetry, just the global part.
- The goldstone mode for the broken symmetry can be used as “probing scale”, which restores the Ward identities in the effective theory.
- Setting this scale to a constant is possible, after variation has been performed, which just defines a unitary gauge in which the goldstone mode is eaten by the metric.
- Next question: conformal symmetry breaking, mass generation, inflation.

Thanks for attention

Questions?