A no-hair theorem for stars in Horndeski theories

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1. Horndeski theories
2. The no-hair theorem for shift-symmetric Horndeski theories
3. Breaking the hypotheses

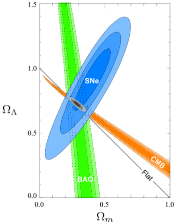
Modifying gravity: why?

1. Solving cosmological constant problems

2. Building alternatives; Establishing benchmarks

Modifying gravity: why?

1. Solving cosmological constant problems



2. Building alternatives; Establishing benchmarks

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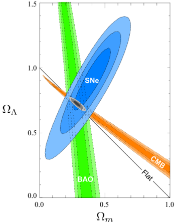
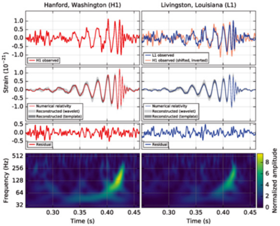


2. Building alternatives; Establishing benchmarks



Modifying gravity: how?

Horndeski theory in 4D Horndeski '74

$$S_{\mathrm{H}} = \int \mathrm{d}^4 x \mathcal{L}_{\mathrm{H}}(g_{\mu\nu}, g_{\mu\nu,i_1}, \ldots, g_{\mu\nu,i_1\ldots i_p};\ \phi, \phi_{,i_1}, \ldots, \phi_{,i_1\ldots i_q})$$

$$\Downarrow \quad \frac{1}{\sqrt{-g}} \frac{\delta\ \cdot}{\delta g_{\mu\nu}}$$

$$\mathcal{E}^{\mu\nu}(g_{\mu\nu}, g_{\mu\nu,i_1}, g_{\mu\nu,i_1,i_2};\ \phi, \phi_{,i_1}, \phi_{,i_1,i_2}) = 0$$

Modifying gravity: how?

Shift-symmetric Horndeski action

$$S_{\mathrm{H}} = \int \sqrt{-g}\,\mathrm{d}^4x\,(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5)$$

$$\begin{aligned}
\mathcal{L}_2 &= G_2(X), \\
\mathcal{L}_3 &= -G_3(X)\Box\phi, \\
\mathcal{L}_4 &= G_4(X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right], \\
\mathcal{L}_5 &= G_5(X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}\big[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 \\
&\quad + 2(\nabla_\mu\nabla_\nu\phi)^3\big],
\end{aligned}$$

where $X = -\partial^\mu\phi\partial_\mu\phi/2$

Modifying gravity: how?

Shift-symmetric (beyond) Horndeski action

$$S_{\rm bH} = \int \sqrt{-g}\, \mathrm{d}^4x \left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_4^{\rm bH} + \mathcal{L}_5^{\rm bH}\right)$$

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&\quad \left. + 2(\nabla_\mu\nabla_\nu\phi)^3\right], \\
\mathcal{L}_4^{\rm bH} &= F_4(X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma}{}_\sigma\nabla_\mu\phi\nabla_\alpha\phi\nabla_\nu\nabla_\beta\phi\nabla_\rho\nabla_\gamma\phi, \\
\mathcal{L}_5^{\rm bH} &= F_5(X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\delta}\nabla_\mu\phi\nabla_\alpha\phi\nabla_\nu\nabla_\beta\phi\nabla_\rho\nabla_\gamma\phi\nabla_\sigma\nabla_\delta\phi,
\end{aligned}$$

where $X = -\partial^\mu\phi\partial_\mu\phi/2$

Shift-symmetry

Action invariant under $\phi \to \phi + C$

Noether current

$$J^\mu = \frac{1}{\sqrt{-g}} \frac{\delta S[\phi]}{\delta(\partial_\mu \phi)}, \text{ such that } \nabla_\mu J^\mu = 0$$

The theorem

Assumptions

1. Spherical symmetry, staticity and regularity of the scalar field and the metric
2. Asymptotic flatness with a constant scalar field at spatial infinity
3. Finite norm of the current J^2
4. Canonical kinetic term $X \subseteq G_2$, and analyticity of the action around a trivial scalar field configuration

Theorem AL, Babichev, Charmousis '17

The solutions are identical to GR, with constant ϕ

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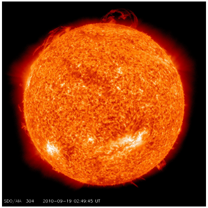
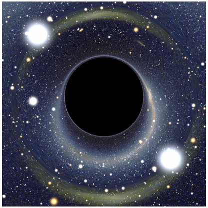
Step 1: No influx on stars

Spherically symmetric and static ansatz

$$\mathrm{d}s^2 = -h(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2), \qquad \phi = \phi(r)$$

Regularity everywhere (especially at the origin of spherical coordinates):





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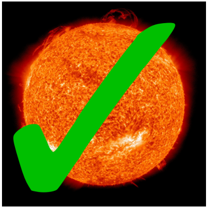

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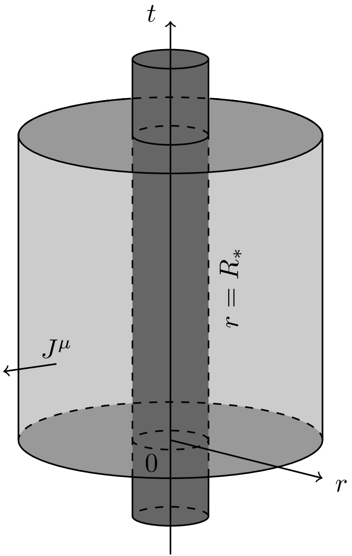

$$0 = \nabla_\mu J^\mu = \partial_\mu(\sqrt{-g}J^\mu)$$

Hence,

$$J^r = \frac{Q}{r^2}\sqrt{\frac{f}{h}}$$

$$0 = J^2 = J_\mu J^\mu = \frac{Q^2}{hr^4}$$

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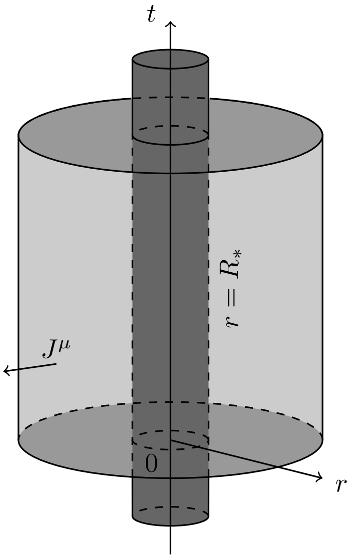

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Therefore $Q = 0$ and $J^r = 0$: no influx

Comparison with black holes [Hui and Nicolis '12]

$$J^2 = \frac{Q^2}{hr^4}$$

Stars

Origin of spherical coordinates is a spacetime point

Black holes

Presence of a horizon. Black holes no-hair theorem

$J^r \neq 0$? [Sotiriou and Zhou '14]

$$G_5 = \alpha \ln|X| \Leftrightarrow -\alpha\phi\hat{G}/4 + \text{boundary terms}$$

with $\hat{G}$ the Gauss-Bonnet topological invariant. Regular black holes solutions

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Step 2: $J^r = 0$ + assumptions 2-3 $\Rightarrow \phi' = 0$

Dependence of J^r on the scalar field

$$\begin{aligned} J^r = & - f\phi' G_{2X} - f\frac{rh'+4h}{rh}XG_{3X} + 2f\phi'\frac{fh-h+rfh'}{r^2h}G_{4X} \\ & + 4f^2\phi'\frac{h+rh'}{r^2h}XG_{4XX} - fh'\frac{1-3f}{r^2h}XG_{5X} + 2\frac{h'f^2}{r^2h}X^2G_{5XX} \\ & + 8f^2\phi'\frac{h+rh'}{r^2h}X(2F_4+XF_{4X}) - 12\frac{f^2h'}{rh}X^2(5F_5+2XF_{5X}) \end{aligned}$$

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Dependence of J^r on the scalar field

$$J^r = \phi' \mathcal{J}(\phi', f, h'/h, r)$$

with $\mathcal{J}$ regular around $\phi = \mathrm{constant}$.

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- Trivial branch $\phi' = 0$ as $r \to \infty$
- Then, progressively dialing the radius to 0, $\mathcal{J} = -f \neq 0$
- One cannot leave the trivial branch $\phi' = 0$

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$\phi' = 0$ everywhere. Solutions identical to GR

Complementary result: emission of gravitational waves

Gravitational waves and star binaries [Barausse and Yagi '15]

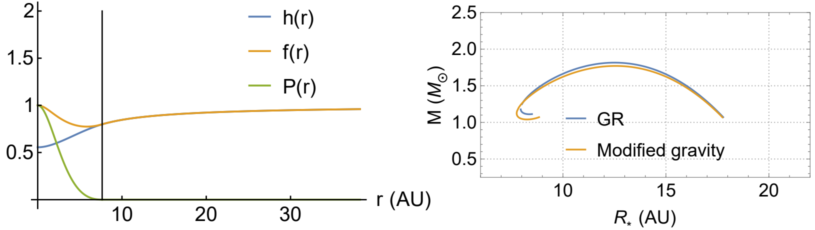
- Very similar assumptions, without staticity
- No modification with respect to GR at leading PN order

Breaking assumption 1: $\phi(t,r) = qt + \psi(r)$

Example: John Lagrangian — Cisterna *et al.* '15

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\zeta R + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right) + S_{\rm PF}$$

Matching with the 'stealth' Schwarzschild solution outside



Metric and pressure profile

Mass-radius relation

Breaking assumption 2: de Sitter solutions

- (a)dS asymptotics: kills step 2 of the proof
- Potentially, stars embedded in self-tuned expanding universe
- Already existing solutions for black holes Rinaldi '12

Breaking assumption 3: non-analytic G_i functions

Theories potentially containing hairy compact objects (either stars or black holes) in asymptotically flat spacetime:

[AL, Babichev, Charmousis '17]

- $G_2 \supset \sqrt{-X}$
- $G_3 \supset \ln|X|$
- $G_4 \supset \sqrt{-X}$
- $G_5 \supset \ln|X|$
- $F_4 \supset (-X)^{-3/2}$
- $F_5 \supset X^{-2}$

Conclusions

- No non-trivial scalar solutions under reasonable assumptions
- Classification of more exotic models: where to look for solutions or not
- Extension to stationary solutions?