The effect of Universal Extra Dimensions on Cosmological Evolution

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Overview

- Brief intro to K-K extra dimensions and motivation
- Setup of a UED scenario in Cosmology
- Solutions of the Field Equations
- A specific model
- An interesting pair of EoS parameters
- Conclusions

Motivation

- Cosmological observations offer a testing ground for extra dimensional scenarios.
- Possible Dark Matter candidates in K-K modes (LKPs)
- The dynamics of the extra space could have offered an alternative to the cosmological constant.
- We will look for the circumstances under which a UED scenario could be an alternative to Λ-CDM.

Introduction to K-K extra dimensions

- Initially K-K wanted to unify E/M and Gravity by introducing an extra compactified dimension. In UED scenarios every SM particle is allowed to propagate everywhere.
- For example

$$S = \int d^5x \frac{1}{2} \partial^{\mathrm{M}} \Phi(x^\mu, \mathrm{y}) \partial_{\mathrm{M}} \Phi(x^\mu, \mathrm{y}) \quad \text{with} \quad \Phi(x^\mu, \mathrm{y} + 2\pi L) = \Phi(x^\mu, \mathrm{y})$$

giving:

$$S = \int d^4x \left\{ \frac{1}{2} \partial^{\mu} \varphi^{\dagger(0)} \partial_{\mu} \varphi^{(0)} + \sum_{n=1}^{\infty} \left[\partial^{\mu} \varphi^{\dagger(n)} \partial_{\mu} \varphi^{(n)} - \frac{n^2}{L^2} \varphi^{\dagger(n)} \varphi^{(n)} \right] \right\}$$

$$m_{(n)}^2 = \frac{n^2}{L^2}$$

Setup

- Our metric

$$g_{MN}dx^M dx^N = -dt^2 + a^2(t)\gamma_{ij}dx^i dx^j + b^2(t)\tilde{\gamma}_{pq}dx^p dx^q$$

- The expanded E-H action

$$S_{4+n} = \frac{1}{8\pi G_{4+n}} \int d^{4+n}x \sqrt{-g}[R + \mathfrak{L}_m]$$

- We can bring this in an equivalent 4-d Einstein frame form by performing a Weyl transformation:

$$\hat{g}_{\mu\nu} = b^n \bar{g}_{\mu\nu}$$

giving us Gravity+Radion field:

$$S_4 = \int d^4x \sqrt{-\hat{g}} \left[\frac{1}{2\hat{k}^2}\hat{R} - \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi + V_{eff}(\Phi)\right]$$

by introducing:

$$\Phi = \sqrt{\frac{n(n+2)}{2\hat{k}^2}} lnb \ \ , V_{eff}(\Phi) = C_1 \exp\{-\Phi\} - C_2\widetilde{\mathrm{V}}\, \mathfrak{L}_m \exp\{-\Phi\},$$

$$\hat{k}^2 = \frac{k^2}{\widetilde{V}}, \widetilde{\mathrm{V}} = \int d^n y \sqrt{\tilde{g}} \propto L^n$$

$\Longrightarrow$Stabilization!

Friedmann Equations

- We get

$$3\left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{k_a^2}{a^2}\right] + 3n\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{n(n-1)}{2}\left[\left(\frac{\dot{b}}{b}\right)^2 + \frac{k_b^2}{b^2}\right] = k^2\rho$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k_a^2}{a^2} + n\frac{\ddot{b}}{b} + 2n\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{n(n-1)}{2}\left[\left(\frac{\dot{b}}{b}\right)^2 + \frac{k_b^2}{b^2}\right] = -k^2 p_a$$

$$3\frac{\ddot{a}}{a} + 3\left(\frac{\dot{a}}{a}\right)^2 + 3\frac{k_a^2}{a^2} + (n-1)\frac{\ddot{b}}{b} + 3(n-1)\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{(n-1)(n-2)}{2}\left[\left(\frac{\dot{b}}{b}\right)^2 + \frac{k_b^2}{b^2}\right] = -k^2 p_b$$

while from conservation of energy, ${T^A}_{0;A} = 0$, we have:

$$\frac{\dot{\rho}}{\rho} = -3(1+w_a)\frac{\dot{a}}{a} - n(1+w_b)\frac{\dot{b}}{b}$$

Equations of the Hubble Parameters

We will consider the case $k_a = k_b = 0$, with simple equations of state:

$$p_{a,b} = w_{a,b}\rho$$

Moreover we will work with the equations of the Hubble parameters instead, by using:

$$A = \frac{\dot{a}}{a}, B = \frac{\dot{b}}{b} \quad \longrightarrow \quad \frac{\ddot{a}}{a} = \dot{A} + A^2,\ \frac{\ddot{b}}{b} = \dot{B} + B^2$$

$$= \frac{3\big((n-1)w_a - nw_b - n - 1\big)}{2+n}A^2 + \frac{n\big((n-1)(3w_a - 1) - 3nw_b\big)}{2+n}AB + \frac{n(n-1)(1+(n-1)w_a - nw_b)}{2(2+n)}B^2$$

$$\dot{B} = \frac{-3(3w_a - 2w_b - 1)}{2+n}A^2 + \frac{-3(2 + 3nw_a - 2nw_b)}{2+n}AB + \frac{-n(3(n-1)w_a - 2(n-1)w_b + n + 5)}{2(2+n)}B^2$$

We can immediately read an exact stabilization constraint (Bringmann et al. 2003):

$$3w_a - 2w_b - 1 = 0$$

By eliminating time we get a single diff. equation that is always integrable for constant w_a, w_b.

Solutions

- Its solution is

$$const. = \left|6\frac{A}{B} + \left(3n + \sqrt{3n(2+n)}\right)\right|^{\sqrt{2+n}(3+n-3w_a-nw_b)+\sqrt{3n}(2+n)(w_a-w_b)} \cdot$$

$$\left|6\frac{A}{B} + \left(3n - \sqrt{3n(2+n)}\right)\right|^{\sqrt{2+n}(3+n-3w_a-nw_b)-\sqrt{3n}(2+n)(w_a-w_b)} \cdot$$

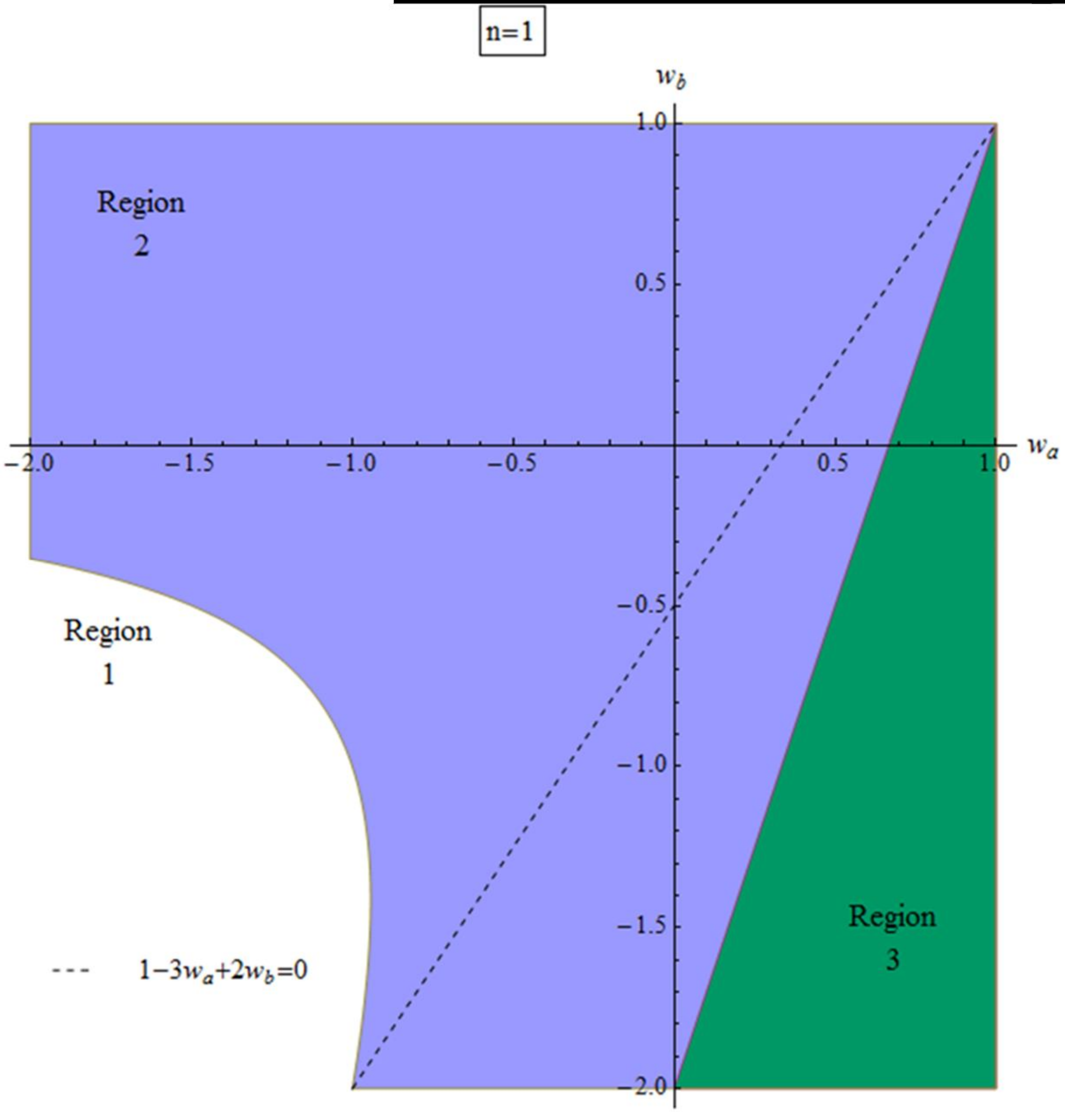
$$\left|(3w_a - 2w_b - 1)\frac{A}{B} + \left((n-1)w_a - nw_b + 1\right)\right|^{-\sqrt{2+n}\left(3-3w_a^2+n(1+3w_a^2-6w_aw_b+2w_b^2)\right)} \cdot$$

$$|B|^{\sqrt{2+n}\,[3(w_a^2-1)+n\left(1-3w_a^2+6w_aw_b-2w_b(1+w_b)\right)]}$$

- To study this it is important to know the sign off the exponents.
- So if for example we wanted to study an "equilibrium" case where $A, B \to 0$ but ${}^{B}/_{A} \to \tilde{c}$ we can see that the only way possible is if the third factor goes to zero, i.e.

$$\tilde{c} \to \frac{3w_a-2w_b-1}{(n-1)w_a-nw_b+1}$$

Consistency of solution



- Region 1: all positive
- Region 2: K3 part negative
- Region 3: only K1 part positive

Solutions

- Moreover, each one of these factors represents a special case solution of the form

$$B = cA$$

$$c_1 = -\frac{6}{3n+\sqrt{3n(2+n)}}, \qquad c_2 = -\frac{6}{3n-\sqrt{3n(2+n)}}, \qquad c_3 = \frac{3w_a - 2w_b - 1}{(n-1)w_a - nw_b + 1}$$

- The first two correspond to the Kasner solutions (Kasner 1922)

$$A(t) = \frac{A_0(n-1)}{n-1+A_0 t\left(-3+\sqrt{3n(2+n)}\right)}, B(t) = -\frac{6A_0}{3n+\sqrt{3n(2+n)}+\left(3n+3\sqrt{3n(2+n)}\right)A_0 t}$$

$$A(t) = \frac{A_0(n-1)}{n-1-A_0 t\left(3+\sqrt{3n(2+n)}\right)}, B(t) = \frac{6A_0}{-3n+\sqrt{3n(2+n)}+(-3n+3\sqrt{3n(2+n)})A_0 t}$$

They both give:

$\rho = 0$ and $q = const$

Solutions

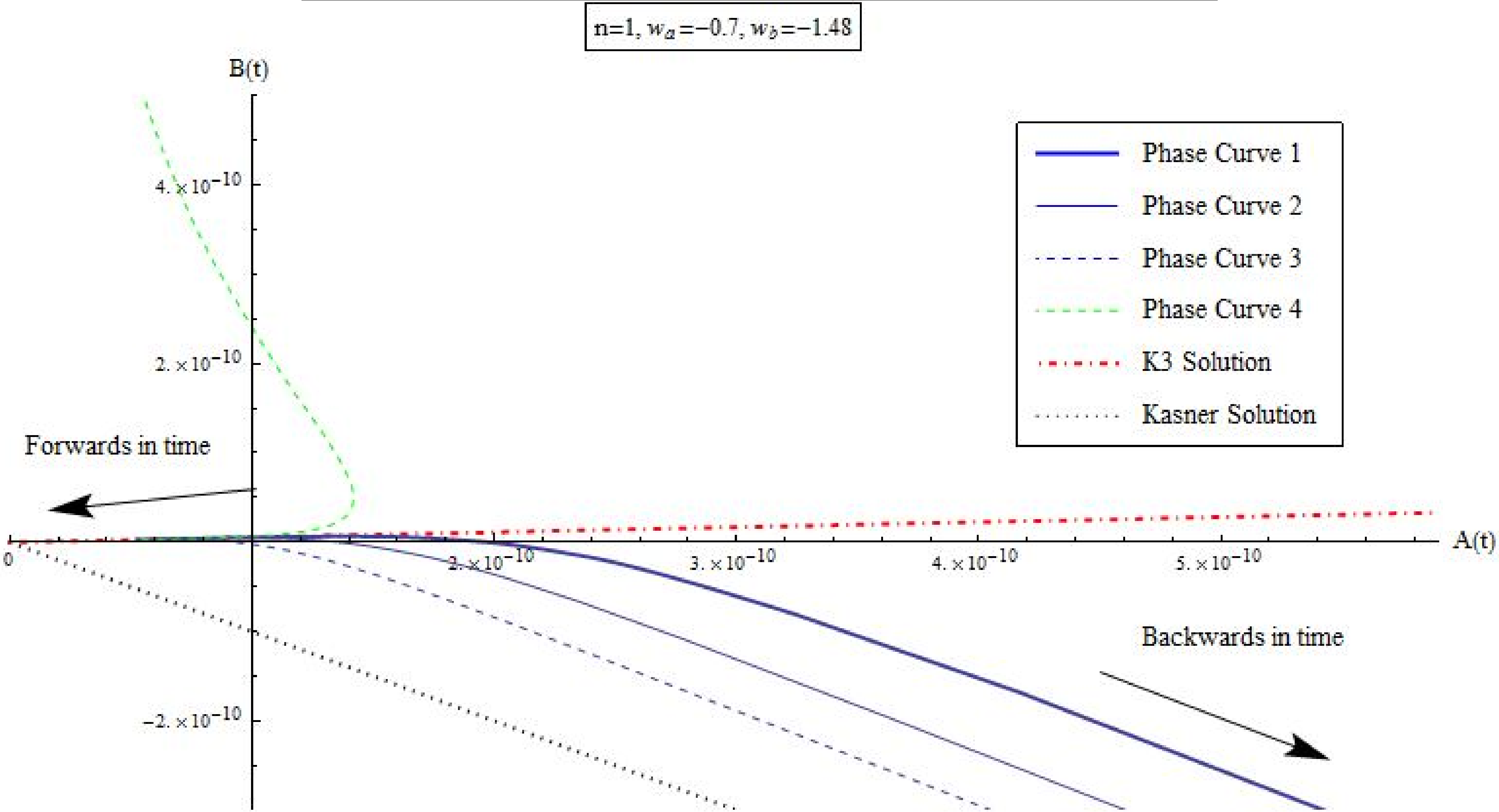
- The third one (K3) is another Kasner-type solution with much better properties.

$$A(t) = \frac{(2 + 2\,(n-1)w_a - nw_b)A_0}{2 + 2\,(n-1)w_a - 2nw_b + (3 - 3w_a^2 + n(1 + 3w_a^2 + 2w_b^2 - 6w_a w_b))A_0 t},$$

$$B(t) = \frac{2(2w_b - 3w_a + 1)A_0}{2 + 2\,(n-1)w_a - 2nw_b + (3 - 3w_a^2 + n(1 + 3w_a^2 + 2w_b^2 - 6w_a w_b))A_0 t}$$

- It has a singularity that is defined by the values of the w parameters, as is its deceleration parameter and ρ.
- More importantly it acts as an attractor for the general solution for most cosmologically relevant cases.

Phase Space Diagram

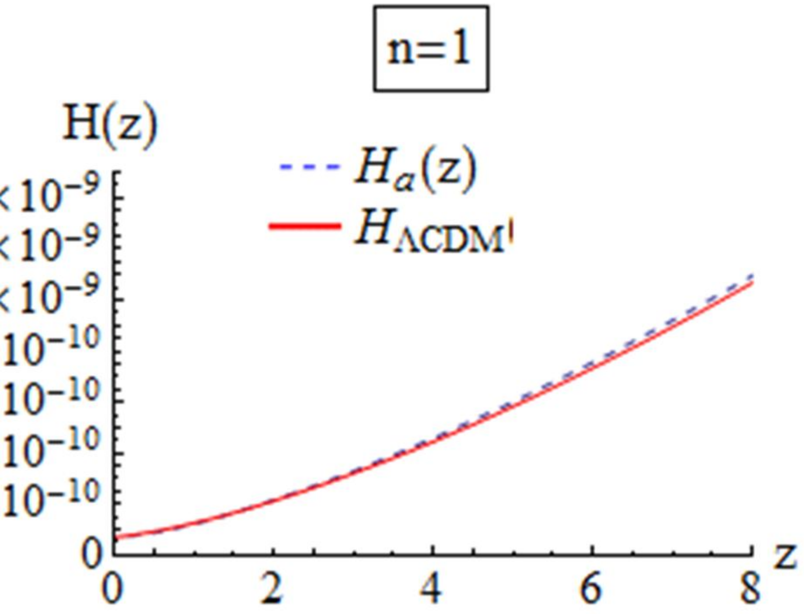


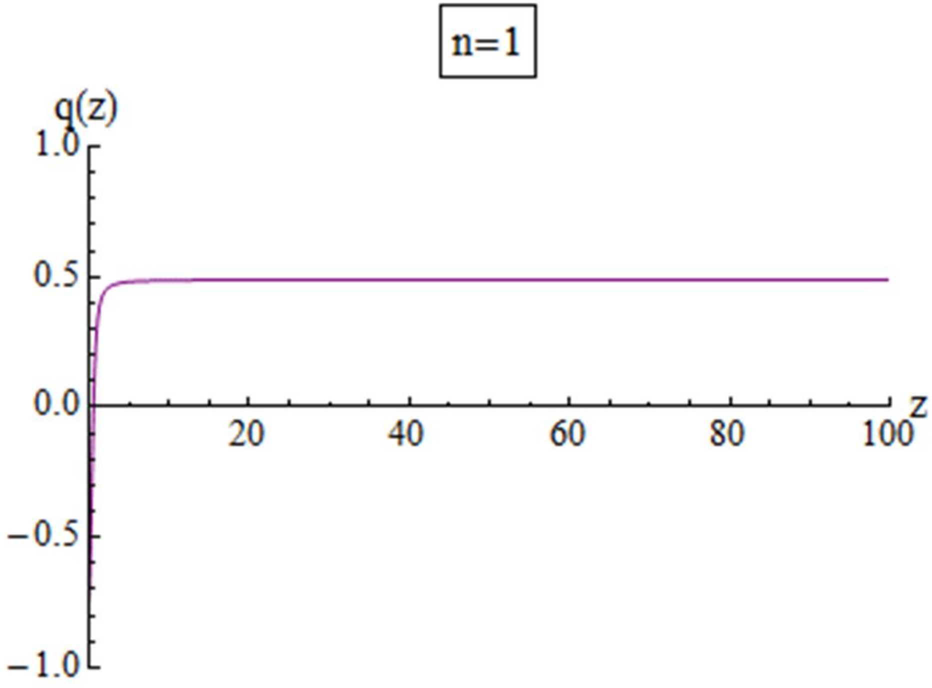
A specific model

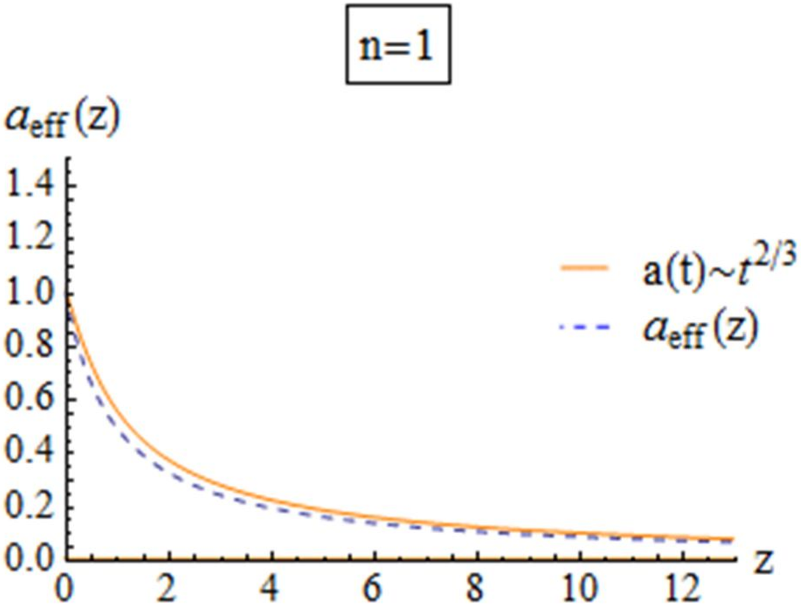
- We will use this to our advantage to quantify the behavior of the general solution by using the analytical expressions of the K3 solution to achieve:

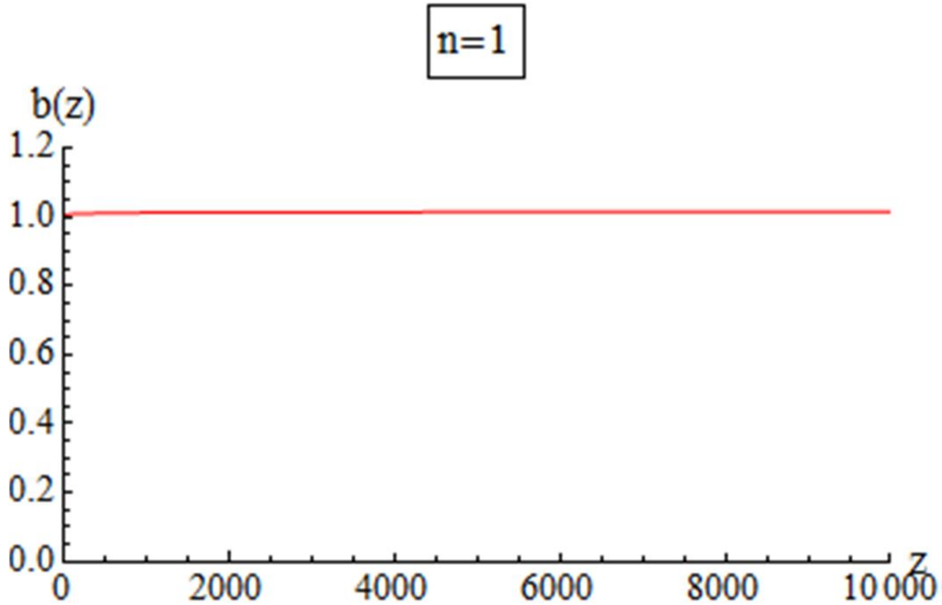
1. Stabilization ($^{\Delta b}/_{b} \approx 1\%$) of the extra space from as early as radiation domination until today. (Bergstrom et al, 1999)
2. A transition to an accelerating expanding era.
3. $q_0 \approx -0.6$
4. $H_0 \approx 70\ km/s \cdot Mpc$

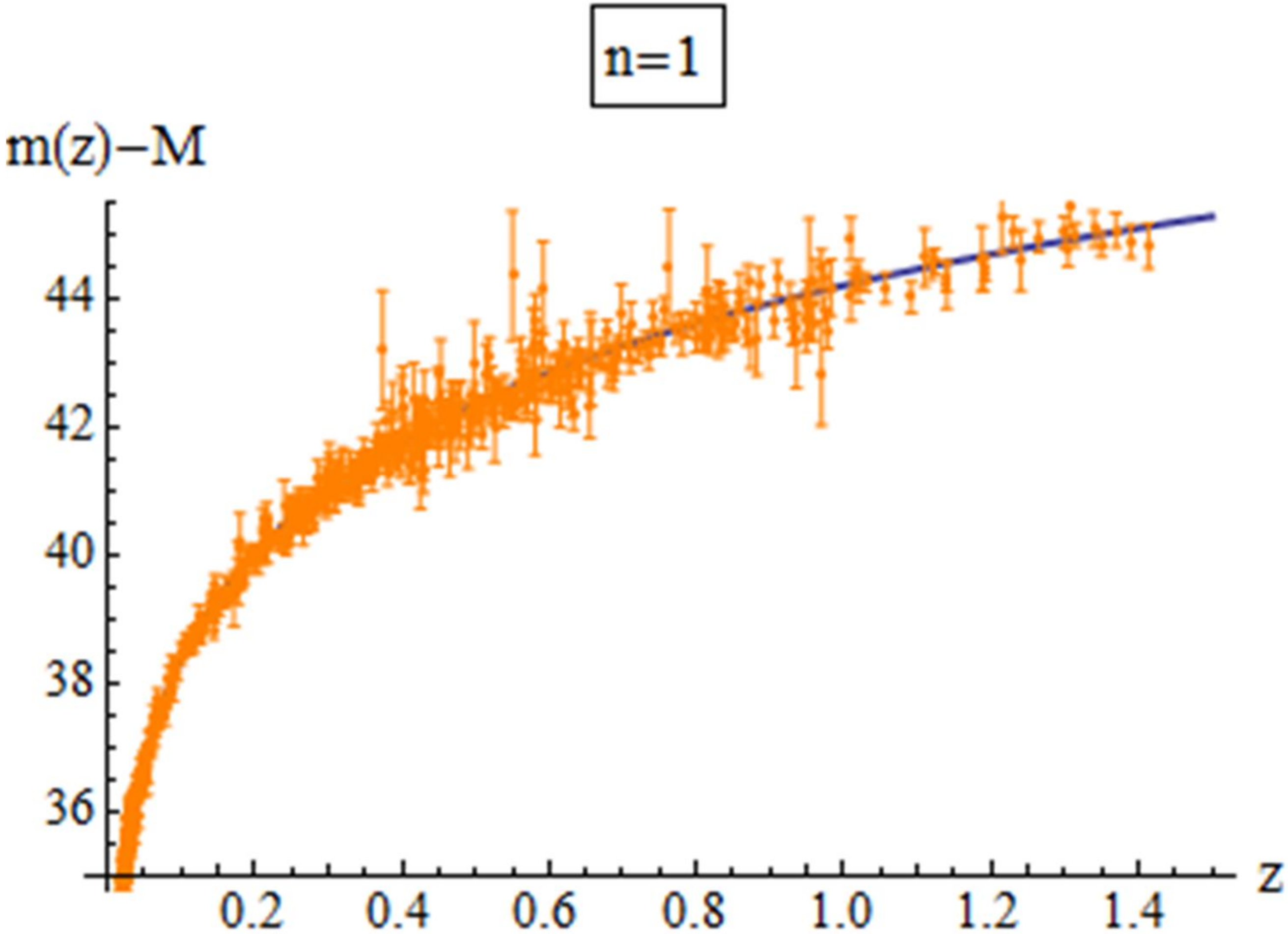
	Radiation Era	Matter Era	Dark Energy Era
w_a	$\approx 1/3$	≈ 0	$\approx -7/10$
w_b	≈ 0	$\approx -1/2$	$\approx -3/2$


n=1
H(z)
$H_a(z)$
$H_{\Lambda CDM}$
z


n=1
q(z)
z


n=1
$a_{eff}(z)$
$a(t) \sim t^{2/3}$
$a_{eff}(z)$
z


n=1
b(z)
z



*Using data from Suzuki et al. (2012)

A large exponent for a(t)

- One last interesting property of the K3 solution: the scale factors have a particular relation too! For $n = 2$ for example:

$$a(t) = |f(w_a, w_b) + g(w_a, w_b)A_0 t|^{\frac{2+2w_a-4w_b}{5+3{w_a}^2-12\,w_a w_b+4{w_b}^2}}$$

$$b(t) = |f(w_a, w_b) + g(w_a, w_b)A_0 t|^{\frac{2(1-3w_a+2w_b)}{5+3{w_a}^2-12\,w_a w_b+4{w_b}^2}}$$

- We see that the denominator has a solution that happens to also be a solution of the exact stabilization constraint:

$$w_a = -1, \qquad w_b = -2$$

- So by a suitable pair of w parameters very close to these we can have a very positive exponent for $a(t)$ and at the same time a negative exponent for $b(t)$.
- For larger n we might have a larger variety in the values of the w's that can achieve this.

Conclusion and Remarks

- We see that for most cosmologically relevant cases, the study of the general solution of this UED scenario reduces to the study of its special solution, K3.
- That, in turn, depends on the w parameters, and can be made to follow a number of observational constraints for suitable values of the w's.
- We can thus manipulate the general solution into being stabilized very early in its evolution, by simply stabilizing the corresponding K3's for each pair of w's, enabling us to recreate a very similar picture to that of the Λ-CDM.
- Moreover, we have shown that a period of extremely fast evolution for $a(t)$ is possible in this model, with a much slower accompanying contraction of the extra space.
- However, the w parameters that achieve this are rather exotic and their nature is to be explained to give credibility to such a model. (Brandenberger 1989, Kaya-Rador 2003) or (Caldwell 1999, 2003)
- Alternatively, a different approach may be needed in terms of the equation of state for the extra space.

Thank you!