Non-Minimal Derivative Coupling Cosmology in Palatini Formalism

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arXiv:1608.04014 [gr-qc]

THE TAH POE INSTITUTE IF THEORETICAL PHYSICS for FUNDAMENTAL STUDY

9th Aegean Summer School – Island of Sifnos, Greece

Outlines

- NMDC
- Palatini NMDC Cosmology
- Inflationary Predictions (Chaotic Inflation)

Introduction to

- Present cosmic acceleration supported by independent observational data: SN Ia, CMB temperature anisotropies, Baryon Acoustic Oscillations
- Dark energy (DE) responsible for the acceleration has mysterious origin. Cosmological constant: DE simplest candidates for dark energy but suffering from the cosmological constant problem.
- DE might be dynamical. The dynamical models can be distinguished with the evolution of $w_{\rm DE}$.
- DE scalar field e.g. quintessence and k-essence predict a wide range of $w_{\rm DE}$. Observations can not distinguish scalar field model from ΛCDM model. To fit a viable scalar-field model into particle physics theory is difficult due to mass required by the acceleration is very tiny, i.e. $m_\phi \lesssim 10^{-33}\,{\rm eV}$

NMDC

Non-Minimal Derivative Coupling (NMDC) term

NMDC coupling to curvature first proposed by (Amendola, Phys. Lett. B **301, 175 (1993)**. The coupling function comes in term of

$$f(\phi, \phi_{,\mu}, \phi_{,\mu\nu}, \ldots)$$

- required in scalar quantum electrodynamics to satisfy U(1) invariance
- required in models of which the gravitational constant is function of the mass density
- commonly found as lower energy limits of higher dimensional theories

- Derivative coupling term as $R\phi_{,\mu}\phi^{,\mu}$ can host larger class of inflationary attractors and have nearly scale invariant spectrum. However it can not be related to GR by conformal transformation (Magnano, Ferraris, Francaviglia, Gen. Relativ. Gravit. **19, 465 (1987)**).
- The theory has Ostorogradski instabilities.

NMDC

NMDC model

(Sushkov: Phys. Rev. D 80, 103505 (2009)) (C. Germani and A. Kehagias PRL 106, 161302 (2011)

$$\kappa_1 R \phi_{,\mu} \phi^{,\mu} \qquad \kappa_2 R^{\mu\nu} \phi_{,\mu} \phi_{,\nu}$$

With $\kappa \equiv \kappa_2 = -2\kappa_1$ Hence combined term become $\kappa G_{\mu\nu} \phi^{,\mu} \phi^{,\nu}$

Good dynamical theory: the field equations contain terms with second-order derivative of metric and the field ϕ at most.
Hence Lagrangian contains only divergence free tensors.
Cosmologically allows transit from de-Sitter phase to other evolutions.

$$S_g = \int d^4x \sqrt{-g} \left\{ R(g) - \left[\varepsilon g_{\mu\nu} + \kappa \left(R_{\mu\nu}(g) - \frac{1}{2} g_{\mu\nu} R(g) \right) \right] \phi^{,\mu} \phi^{,\nu} - 2V(\phi) \right\} + S_m[g_{\mu\nu}, \Psi]$$

$$G_{\mu\nu}(g) = R_{\mu\nu}(g) - \tfrac{1}{2} g_{\mu\nu} R(g)$$

Found as a special case of the Horndeski action (with $G_5 = \frac{\phi}{2}\kappa$) without Ostorogradski instabilities (with at most 2nd order derivative: generalized case) (Horndeski : Int. J. Theo. Phys. **10**, (1974) 363.)

NMDC-Palatini

NMDC in Palatini Formalism

- Connection and the metric are treated independently
- Let $8\pi G = 1$
- The action is:

$$S_{\text{Palatini}} = \int \mathrm{d}^4x\sqrt{-g}\left\{\tilde{R}(\Gamma) - \left[\varepsilon g_{\mu\nu} + \kappa_1 g_{\mu\nu}\tilde{R}(\Gamma) + \kappa_2 \tilde{R}_{\mu\nu}(\Gamma)\right]\phi^{;\mu}\phi^{;\nu} - 2V(\phi)\right\} + S_{\text{m}}[g_{\mu\nu},\Psi]\,.$$

(X. Luo, P. Wu and H. Yu, H.: Astrophys. Space. Sci. 350, (2014) 831)

With $\kappa \equiv \kappa_2 = -2\kappa_1$ and let $\tilde{G}_{\mu\nu}(\Gamma) = \tilde{R}_{\mu\nu}(\Gamma) - \frac{1}{2}g_{\mu\nu}\tilde{R}(\Gamma)$.

Hence .

$$S_{\text{Palatini}} = \int \mathrm{d}^4x\sqrt{-g}\left\{\tilde{R}(\Gamma) - \left[\varepsilon g_{\mu\nu} + \kappa_1 g_{\mu\nu}\tilde{R}(\Gamma) + \kappa_2 \tilde{R}_{\mu\nu}(\Gamma)\right]\phi^{;\mu}\phi^{;\nu} - 2V(\phi)\right\} + S_{\text{m}}[g_{\mu\nu},\Psi]$$

NMDC-Palatini

The Ricci tensor is

$$\tilde{R}_{\mu\nu}(\Gamma) = \tilde{R}^{\lambda}_{\mu\lambda\nu}(\Gamma) = \partial_\lambda \Gamma^{\lambda}{}_{\mu\nu} - \partial_\nu \Gamma^{\lambda}{}_{\mu\lambda} + \Gamma^{\lambda}{}_{\sigma\lambda}\Gamma^{\sigma}{}_{\mu\nu} - \Gamma^{\lambda}{}_{\sigma\nu}\Gamma^{\sigma}{}_{\mu\lambda},$$

The field equations are .

$$T_{\mu\nu} = \tilde{G}_{\mu\nu}(\Gamma) + \Big[\frac{\kappa}{2}\tilde{G}_{\mu\nu}(\Gamma)\phi_{,\lambda}\phi^{,\lambda} + \frac{\kappa}{2}\tilde{R}_{\alpha\beta}(\Gamma)g_{\mu\nu}\phi^{,\alpha}\phi^{,\beta} - \kappa\tilde{R}_{\nu\lambda}(\Gamma)\phi_{,\mu}\phi^{,\lambda} + \frac{\kappa}{2}\tilde{R}(\Gamma)\phi_{,\mu}\phi_{,\nu}$$
$$-2\kappa\tilde{R}_{\mu\lambda}(\Gamma)\phi_{,\nu}\phi^{,\lambda} + \frac{\varepsilon}{2}g_{\mu\nu}\phi_{,\alpha}\phi^{,\alpha} - \varepsilon\phi_{,\mu}\phi_{,\nu} + g_{\mu\nu}V(\phi)\Big],$$

(varying $g_{\mu\nu}$)

$$\nabla^{\Gamma}_{\lambda}\left(\sqrt{-g}g^{\mu\nu}f\right) = 0,$$

(varying Γ)

NMDC-Palatini

There is a factor $f = 1 - \frac{1}{2}\kappa\phi^{;\alpha}\phi_{,\alpha}$ relating the two metrics.

$h_{\mu\nu}$ is the effective metric of the connection field

$$h_{\mu\nu} = fg_{\mu\nu} = (1 - \frac{1}{2}\kappa\phi^{;\alpha}\phi_{,\alpha})g_{\mu\nu}$$

and we have $\sqrt{-h} = \sqrt{-g}f^2$

Hence the action written in term of effective metric $h_{\mu\nu}$ and f is,

$$S_{\text{Palatini}} = \int \mathrm{d}^4x\sqrt{-h}\left\{\frac{\tilde{R}(h)}{f^2} - \left[\frac{\varepsilon h_{\mu\nu}}{f^3} + \kappa\frac{\tilde{G}_{\mu\nu}(h)}{f^2}\right]\phi^{;\mu}\phi^{;\nu} - \frac{2V(\phi)}{f^2}\right\} + S_{\text{m}}\left(\frac{h_{\mu\nu}}{f}, \Psi\right)$$

Cosmology of NMDC-Palatini

If the $g_{\mu\nu}$ metric is FLRW with homogeneous scalar field: .

$$f(\dot{\phi}) = 1 - \frac{\kappa}{2} g^{00} \frac{\mathrm{d}\phi}{\mathrm{d}t}\frac{\mathrm{d}\phi}{\mathrm{d}t} = 1 + \frac{\kappa}{2}\dot{\phi}^2$$

$$h_{\mu\nu} = \begin{pmatrix} -1 - \frac{\kappa}{2}\dot{\phi}^2 & 0 & 0 & 0 \\ 0 & a^2(1+\frac{\kappa}{2}\dot{\phi}^2) & 0 & 0 \\ 0 & 0 & a^2(1+\frac{\kappa}{2}\dot{\phi}^2) & 0 \\ 0 & 0 & 0 & a^2(1+\frac{\kappa}{2}\dot{\phi}^2) \end{pmatrix}$$

- To preserve Lorentz signature (-,+,+,+) , it requires $-2/\dot{\phi}^2 < \kappa$

- For fast-rolling field, the coupling is allowed in positive region or in very small negative region. For slowly-rolling field, the coupling is permitted in vast negative region.

- The conservation (EoM of the connection field effect) is:

$$\nabla^{\Gamma}_{\lambda}(\sqrt{-h}h^{\mu\nu}) = \nabla^{\Gamma}_{\lambda}(\sqrt{-g}g^{\mu\nu} f) = 0$$

Cosmology of NMDC-Palatini

- The connection field is written as

$$\Gamma^{\lambda}_{\mu\nu}(h) = \frac{1}{2}h^{\lambda\sigma}\left(\partial_\mu h_{\sigma\nu} + \partial_\nu h_{\sigma\mu} - \partial_\sigma h_{\mu\nu}\right)$$

- Effective Newton's constant

$$G_{\text{eff}} = \frac{f^2}{8\pi} = \frac{1}{8\pi}\left(1 + \frac{\kappa}{2}\dot{\phi}^2\right)^2$$

$$\frac{\dot{G}_{\text{eff}}}{G_{\text{eff}}} = \frac{2\kappa\dot{\phi}\ddot{\phi}}{(1 + \frac{\kappa}{2}\dot{\phi}^2)}$$

$$\dot{G}_{\text{eff}}/G_{\text{eff}} \simeq 4\ddot{\phi}/\dot{\phi}$$ (fast roll)

$$\dot{G}_{\text{eff}}/G_{\text{eff}} \simeq 2\kappa\dot{\phi}\ddot{\phi}$$ (slow roll)

Cosmology of NMDC-Palatini

- standard result with conformal transformation (math. valid in our case)

$$\tilde{R}_{\sigma\nu}(h) = R_{\sigma\nu}(g) - \left[(n-2)\delta^{\alpha}_{\sigma}\delta^{\beta}_{\nu} + g_{\sigma\nu}g^{\alpha\beta}\right]\frac{1}{\sqrt{f}}(\nabla^{g}_{\alpha}\nabla^{g}_{\beta}\sqrt{f}) + \left[2(n-2)\delta^{\alpha}_{\sigma}\delta^{\beta}_{\nu} - (n-3)g_{\sigma\nu}g^{\alpha\beta}\right]\frac{1}{f}(\nabla^{g}_{\alpha}\sqrt{f})(\nabla^{g}_{\beta}\sqrt{f}),$$

$$T_{00} = \tilde{G}_{00}(h) - \frac{\kappa}{2}\tilde{G}_{00}(h)\dot{\phi}^2 + \frac{5\kappa}{2}\tilde{R}_{00}(h)\dot{\phi}^2 + \frac{\kappa}{2}\tilde{R}(h)\dot{\phi}^2 - \left(\frac{\varepsilon}{2}\dot{\phi}^2 + V(\phi)\right)$$

- Ricci curvature (FLRW)

$$R_{00}(g) = -3(\dot{H} + H^2), \qquad R_{ii}(g) = a^2(\dot{H} + 3H^2), \qquad R(g) = 6(\dot{H} + 2H^2)$$

$$\tilde{R}_{00}(h) = -3(\dot{H} + H^2) - \frac{3}{2}\left(\frac{\ddot{f}}{f} - \frac{2\dot{f}^2}{f^2}\right), \qquad \tilde{R}_{ii}(h) = R_{ii}(g) + \frac{a^2\ddot{f}}{2f}$$

- Resulting

$$\rho_{\rm m} = \dot{H}\left[12f + \frac{6}{f} - 18\right] + H^2\left[12f + \frac{12}{f} - 21\right] - \frac{3}{2}(1-f)\left(\frac{4\ddot{f}}{f} - \frac{8\dot{f}^2}{f^2}\right) - \frac{3\ddot{f}}{2f} + \frac{3\ddot{f}}{f^2} + \frac{3\dot{f}^2}{f^2} - \frac{3\dot{f}^2}{2f^3} - \rho_\phi$$

$$p_{\rm m} = \dot{H}\left(4f - 6\right) + H^2\left(6f - 9\right) - \frac{3}{2}(1-f)\left(\frac{\ddot{f}}{f} - \frac{2\dot{f}^2}{f^2}\right) + \frac{\ddot{f}}{f} - \frac{3\ddot{f}}{2f} + \frac{3\dot{f}^2}{4f^2} - p_\phi$$

With $\rho_{\rm tot} \equiv \rho_{\rm m} + \rho_\phi$ and $\rho_\phi = \varepsilon\dot{\phi}^2/2 + V(\phi)$

$p_{\rm tot} \equiv p_{\rm m} + p_\phi$ $\qquad p_\phi = \varepsilon\dot{\phi}^2/2 - V(\phi)$

Cosmology of NMDC-Palatini

- Let

$$A \equiv 4f - 6,$$
$$B \equiv 6f - 9,$$
$$C \equiv -\frac{3}{2}(1-f)(\frac{\ddot{f}}{f} - \frac{2\dot{f}^2}{f^2}) + \frac{\ddot{f}}{f} - \frac{3\ddot{f}}{2f} + \frac{3\dot{f}^2}{4f^2},$$
$$D \equiv 12f + \frac{6}{f} - 18,$$
$$E \equiv 12f + \frac{12}{f} - 21,$$
$$F \equiv -\frac{3}{2}(1-f)(\frac{4\ddot{f}}{f} - \frac{8\dot{f}^2}{f^2}) - \frac{3\ddot{f}}{2f} + \frac{3\ddot{f}}{f^2} + \frac{3\dot{f}^2}{f^2} - \frac{3\dot{f}^2}{2f^3}$$

- hence

$$w_{\rm eff} \equiv \frac{p_{\rm tot}}{\rho_{\rm tot}} = \frac{A\dot{H} + BH^2 + C}{D\dot{H} + EH^2 + F}$$

Modified Friedmann equations:

$$\dot{H} = \frac{[(B - Ew_{\rm eff})H^2 - Fw_{\rm eff} + C]}{Dw_{\rm eff} - A}, \qquad H^2 = \frac{\rho_{\rm tot}}{3}\frac{\left[1 - \frac{(C - Fw_{\rm eff})D}{(Dw_{\rm eff} - A)\rho_{\rm tot}} - \frac{F}{\rho_{\rm tot}}\right]}{\left[\frac{(B - Ew_{\rm eff})D}{3(Dw_{\rm eff} - A)} + \frac{E}{3}\right]}$$

Cosmology of NMDC-Palatini

- standard result with conformal transformation (valid in our case)

$$\ddot{\phi}\left[-\varepsilon+\frac{\kappa}{2}\left(\tilde{R}(h)-\tilde{R}_{00}(h)\right)\right]-\kappa\dot{\phi}\nabla_0^h\tilde{R}_{00}(h)+\frac{\kappa}{2}\dot{\phi}\nabla_0^h\tilde{R}(h)-3\varepsilon H\dot{\phi}-V'=0$$

$$\nabla_\mu^h\nabla_\nu^h\phi=\nabla_\mu^g\nabla_\nu^g\phi-\left(\delta_\mu^\alpha\delta_\nu^\beta+\delta_\mu^\beta\delta_\nu^\alpha-g_{\mu\nu}g^{\alpha\beta}\right)\frac{1}{\sqrt{f}}\left(\nabla_\alpha^g\sqrt{f}\right)\left(\nabla_\beta^g\phi\right)$$

- hence $\nabla_0^h\nabla_0^h\phi=\ddot{\tilde{\phi}}=\ddot{\phi}/f$. The mod. KG equation:

$$\ddot{\phi}\left\{-\varepsilon+\frac{\kappa}{2}\left[\left(\frac{6\dot{H}+12H^2}{f}+3\left(\frac{\ddot{f}}{f^2}-\frac{\dot{f}^2}{2f^3}\right)\right)-\left(-3\left(\dot{H}+H^2\right)-\frac{3}{2}\left(\frac{\ddot{f}}{f}-\frac{2\dot{f}^2}{f^2}\right)\right)\right]\right\}$$
$$+\frac{\kappa}{2}\dot{\phi}\nabla_0^g\left[\frac{6\dot{H}+12H^2}{f}+3\left(\frac{\ddot{f}}{f^2}-\frac{\dot{f}^2}{2f^3}\right)\right]-3\varepsilon H\dot{\phi}-V'=0.$$

Slow-roll

- Slow-Roll approximation $|\dddot{\phi}| \ll |\ddot{\phi}| \ll |\dot{\phi}|$

$$0 \sim |\ddot{f}| \ll |\dot{f}| \ll |f|$$

Hence acceleration eq.:

$$\frac{\ddot{a}}{a} \simeq -\frac{1}{6}\rho_{\rm tot}\left[1 + \frac{7}{2}\kappa\dot{\phi}^2 + 3w_{\rm eff}\left(1 + \frac{3}{2}\kappa\dot{\phi}^2\right)\right]$$

Acceleration condition: $w_{\rm eff} \lesssim -\frac{1}{3}\left(1 + 2\kappa\dot{\phi}^2\right)$

$$H^2 \simeq \frac{1}{3}\rho_{\rm tot}\left[\frac{3(Dw_{\rm eff} - A)}{BD - EA}\right] \simeq \frac{\rho_{\rm tot}}{3}\left[1 + \frac{3}{2}\kappa\dot{\phi}^2(1 + w_{\rm eff})\right]$$

$$\ddot{\phi}\left\{\varepsilon - \frac{9\kappa}{2}\dot{H}\left(1 - \kappa\dot{\phi}^2\right) - \frac{3\kappa}{2}H^2\left(5 - 6\kappa\dot{\phi}^2\right)\right\} + 3H\dot{\phi}\left[\varepsilon - \left(\frac{\ddot{H}}{H} + 4\dot{H}\right)\kappa\left(1 - \frac{\kappa\dot{\phi}^2}{2}\right)\right] + V' \simeq 0$$

Slow-roll

- Further slow-roll approx.: $|\ddot{H}| \ll |H\dot{H}| \ll |H^3|$

$$\dot{\phi} \simeq -\frac{V'(\phi)}{3H(\varepsilon - 4\kappa\dot{H})} \qquad \dot{H} \simeq \frac{V'\dot{\phi}}{6HM_{\rm P}^2} \simeq \frac{\sqrt{3}V'\dot{\phi}}{6\sqrt{V}M_{\rm P}}$$

$$H^2 \simeq \frac{\rho_\phi}{3M_{\rm P}^2}\left[1 + \frac{3}{2}\kappa\dot{\phi}^2\left(1 + \frac{p_\phi}{\rho_\phi}\right)\right] \simeq \frac{1}{3}\frac{V(\phi)}{M_{\rm P}^2}$$

$$\dot{H} \simeq -\frac{(V'(\phi))^2}{18H^2M_{\rm P}^2(\varepsilon - 4\kappa\,\dot{H})}$$

- Slow-roll parameters

$$\epsilon_{\rm V} \equiv -\frac{\dot{H}}{H^2} \simeq \frac{M_{\rm P}^2}{2(\varepsilon - 4\kappa\,\dot{H})}\left(\frac{V'}{V}\right)^2$$

Slow-roll Parameters

$$\delta \equiv \frac{\ddot{\phi}}{H\dot{\phi}} \simeq -\frac{V''(\phi)}{3H^2(\varepsilon - 4\kappa\dot{H})} + \frac{V'(\phi)\dot{H}}{3H^3(\varepsilon - 4\kappa\dot{H})\dot{\phi}} - \frac{4\kappa\ddot{H}V'(\phi)}{3M_{\rm P}^2H^2(\varepsilon - 4\kappa\dot{H})^2\dot{\phi}}$$

RHS first term is

$$\eta_{\rm V} \equiv \frac{M_{\rm P}^2}{(\varepsilon - 4\kappa\dot{H})}\frac{V''(\phi)}{V(\phi)} \simeq \frac{V''(\phi)}{3H^2(\varepsilon - 4\kappa\dot{H})}$$

RHS last term is

$$\eta_\kappa \equiv \frac{4\kappa\ddot{H}}{HM_{\rm P}^2(\varepsilon - 4\kappa\dot{H})} \simeq -\frac{4\kappa\ddot{H}V'(\phi)}{3H^2M_{\rm P}^2(\varepsilon - 4\kappa\dot{H})^2\dot{\phi}} \simeq \frac{4\kappa}{M_{\rm P}^2(\varepsilon - 4\kappa\dot{H})^3}\left[\frac{V''(V')^2}{18V} - \frac{V'^4}{36V^2}\right]$$

As a whole, this is $\delta = -\eta_{\rm V} + \epsilon_{\rm V} + \eta_\kappa$

- Slow-roll condition $|\delta| \ll 1$ hence $|\eta_{\rm V}| \ll 1$ and $\epsilon_{\rm V} \ll 1$
- Spectral index $n_{\rm s} - 1 = -4\epsilon_{\rm V} - 2\delta$ is hence

$$n_{\rm s} - 1 = -6\epsilon_{\rm V} + 2\eta_{\rm V} - 2\eta_\kappa$$

Amount of Inflation

- e-folding number (estimating that $\dot{H}$ is constant during inflation)

$$\mathcal{N} \simeq \frac{(\varepsilon - 4\kappa\dot{H})}{M_P^2} \int_{\phi_f}^{\phi_i} \frac{V(\phi)}{V'(\phi)} d\phi = (\varepsilon - 4\kappa\dot{H}) \int_{\phi_f}^{\phi_i} \frac{1}{\sqrt{2\epsilon_{v,GR}}} \frac{d\phi}{M_P}$$

where $\epsilon_{v,GR} \equiv (M_P^2/2)(V'/V)^2$. Need to know potential form.

Consider only non-phantom case $\varepsilon = 1$, slow-roll (hence $\dot{H} < 0$)

- $\kappa < 0$ case reduces the amount of inflation from that of the GR case.
- $\kappa > 0$ case increase the amount of inflation from that of the GR case.

NMDC metric: increases the amount of inflation from that of the GR case.
$\kappa < 0$ [S. Tsujikawa PRD 85, 083518(2012)]

Chaotic Inflation

- Potential

$$V(\phi) = V_0\phi^n \qquad V_0 \equiv \lambda(M_P^4/M_P^n)$$

$$\epsilon_v = \frac{n^2}{2(1-4\kappa\dot{H})}\frac{M_P^2}{\phi^2}, \quad \eta_v = \frac{n(n-1)}{(1-4\kappa\dot{H})}\frac{M_P^2}{\phi^2}, \quad \eta_\kappa = \frac{n^3(n-2)}{9(1-4\kappa\dot{H})^3}\kappa V_0^2\frac{\phi^{2n-4}}{M_P^2}$$

$$n_s - 1 = -\frac{M_P^2\,[n(n+2)]}{(1-4\kappa\dot{H})}\phi^{-2} - \frac{2\kappa V_0^2\,[n^3(n-2)]}{9M_P^2(1-4\kappa\dot{H})^3}\phi^{2n-4}.$$

GR case, $n > \sqrt{2}$ is super-Planckian to satisfy slow-roll condition.

NMDC Metric case, $\kappa < 0$, ϕ is smaller than that of GR (for $n = 2$)

[S. Tsujikawa PRD 85, 083518(2012)]

NMDC Palatini case, $[|n|/(\sqrt{2}\sqrt{1-4\kappa\dot{H}})]M_P < \phi$

$\kappa > 0$ super-Planckian avoidance for $\kappa < -(n^2-2)/(8\dot{H})$

$\kappa < 0$ more super-Planckian

Chaotic Inflation

$$\phi^2 \equiv \phi_{\rm i}^2(n,\mathcal{N},\dot{H}) \simeq \frac{2nM_{\rm P}^2}{(1-4\kappa\dot{H})}\mathcal{N}$$

$$\mathcal{N}_{\rm GR} = \phi^2/(2nM_{\rm P}^2) \qquad \mathcal{N} = \mathcal{N}_{\rm GR}(1-4\kappa\dot{H})$$

Therefore $\mathcal{N} > \mathcal{N}_{\rm GR}$ for $\kappa > 0$.

$$\epsilon_{\rm v} = \frac{n}{4\mathcal{N}_{\rm GR}}\left(\frac{1}{1-4\kappa\dot{H}}\right), \qquad \eta_{\rm v} = \frac{n-1}{2\mathcal{N}_{\rm GR}}\left(\frac{1}{1-4\kappa\dot{H}}\right), \qquad \eta_\kappa = \frac{\kappa V_0^2 2^{n-2} M_{\rm P}^{2n-6}}{9(1-4\kappa\dot{H})^3}(n-2)n^{n+1}\mathcal{N}_{\rm GR}^{n-2}$$

Where $\epsilon_{\rm v,GR} \equiv n/(4\mathcal{N}_{\rm GR})$ and $\eta_{\rm v,GR} \equiv (n-1)/(2\mathcal{N}_{\rm GR})$

Spectral index:

$$n_{\rm s} \simeq 1 - \frac{n+2}{2\mathcal{N}_{\rm GR}}\left(\frac{1}{1-4\kappa\dot{H}}\right) - 2^{n-1}\frac{\kappa V_0^2 M_{\rm P}^{2n-6}}{9(1-4\kappa\dot{H})^3}(n-2)n^{n+1}\mathcal{N}_{\rm GR}^{n-2}$$

Chaotic Inflation

GR Case

- For $n = 2$ and $\mathcal{N}_{\rm GR} = 60$

$$r \simeq 16\epsilon_{\rm v,GR} \simeq 0.13 \qquad n_{\rm s} \simeq 0.967$$

- For $n = 4$ and $\mathcal{N}_{\rm GR} = 60$

$$r \simeq 0.27 \qquad n_{\rm s} \simeq 0.95$$

Disfavored by Planck 2015 results: $r < 0.12$ and $n_{\rm s} = 0.968 \pm 0.006$

Chaotic Inflation

NMDC Metric Formalism Case (large negative coupling $\kappa < 0$)

[S. Tsujikawa PRD 85, 083518(2012)]

- For $n = 2$ and $\mathcal{N} = 60$

$r = 0.066$ $n_s = 0.975$

- For $n = 4$ and $\mathcal{N} = 60$

$r = 0.088$ $n_s = 0.972$

Favored by Planck 2015 results: $r < 0.12$ and $n_s = 0.968 \pm 0.006$

Chaotic Inflation

NMDC Palatini Formalism Case

Using approximation:

$$\dot{H} \simeq \frac{V'\dot{\phi}}{6HM_P^2} \simeq \frac{\sqrt{3}V'\dot{\phi}}{6\sqrt{V}M_P} \longrightarrow \dot{H} \simeq \frac{\sqrt{3}\sqrt{V_0}}{6} n\phi^{(n-2)/2}\frac{\dot{\phi}}{M_P}$$

Planck 2015 results: $r < 0.12$ and $n_s = 0.968 \pm 0.006$

- For $n = 2$ the range $\kappa\dot{H} \lesssim -0.027$ satisfies Planck data $r < 0.12$

  Hence $\kappa > 0$ is favored

The range below satisfies the Planck constraint $n_s = 0.968 \pm 0.006$

$$0.071 \gtrsim \kappa|\dot{H}| \gtrsim 0.027 \longrightarrow \frac{0.174}{|\dot{\phi}|(m/M_P)} \gtrsim \kappa \gtrsim \frac{0.066}{|\dot{\phi}|(m/M_P)}$$

$$\downarrow$$

$$0.213/m^2 > \kappa > 0.081/m^2$$

where $V = V_0\phi^2$ and $V_0 \equiv (1/2)m^2 = \lambda M_P^2$

Chaotic Inflation

- For $n = 4$ the range $\kappa \dot{H} \lesssim -0.306$ satisfy Planck data $r < 0.12$

  Corresponding to $\kappa > 0$ and

$$\kappa \lesssim 0.264/[\sqrt{\lambda}\,|\dot{\phi}|(\phi/M_{\mathrm{P}})]$$

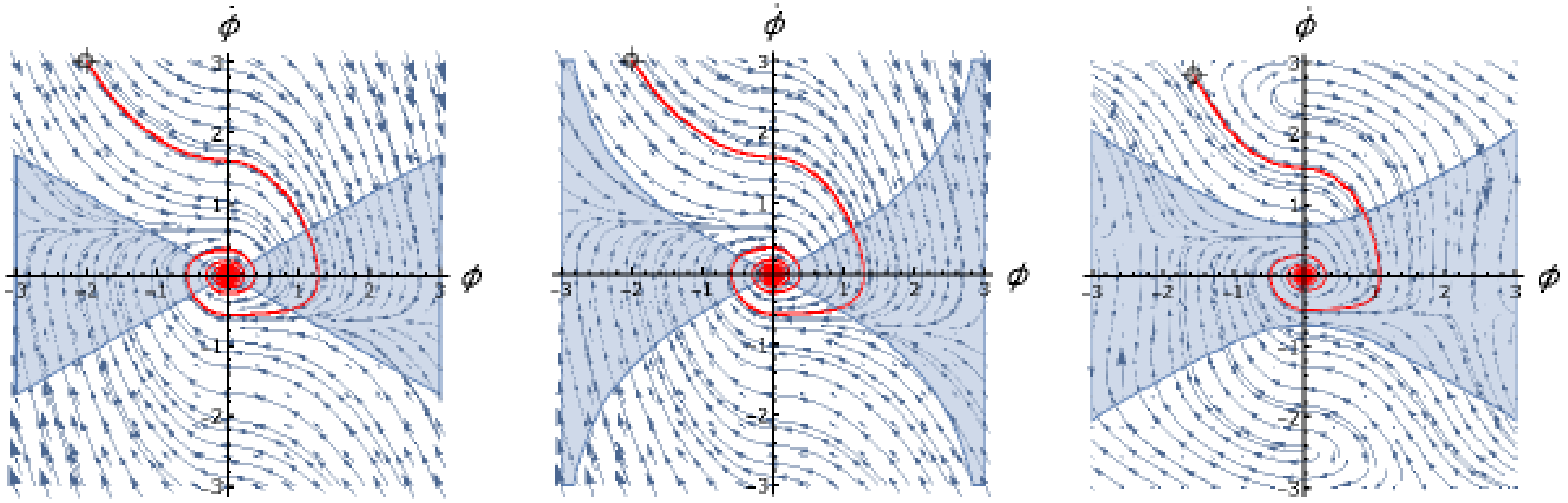
  However, spectral index needs to be much fine tuned , i.e.

$$n_{\mathrm{s}} = 1 - \frac{3}{\mathcal{N}_{\mathrm{GR}}(1-4\kappa\dot{H})} - \frac{16384}{9}\frac{\kappa V_0^2 M_{\mathrm{P}}^2}{(1-4\kappa\dot{H})^3}\mathcal{N}_{\mathrm{GR}}^2$$

$\kappa > 0$

$V = V_0\phi^2$

Phase Plots



GR NMDC-metric NMDC Palatini

Acc. Condition for the NMDC_palatini $\varepsilon\dot{\phi}^2 < V_0\phi^2\left[1 + \frac{16\kappa V_0 M_{\rm P}^2}{(\varepsilon\phi^2)} + 30\kappa^2 V_0^2\right]$

NMDC-Palatini effect enlarges acceleration region with new saddle points

Conclusions

- NMDC (Horndeski subclasss) – Palatini approach
- Chaotic Potential $V = V_0\phi^2$ passes the CMB constraint in some range of parameters with

$$\kappa > 0$$

- This allows superluminal effective metric, stronger gravitational constant (less BH Ap. Hor. entropy)
- more e-folding number , avoidanec of the Super-Planckian field initial value.
- NMDC-Palatini enlarges acceleration region with new saddle points
- Further investigation compared with metric approach (which favors negative coupling) , other potentials

Thank you

Thank you.

Thank you.

NMDC

Capozziello, Lambiase and Schmidt's result

(Capozziello, Lambiase, Schmidt: Annalen Phys. **9, 39 (2000)**)

All other possible coupling Lagrangian terms are not necessary in scalar-curvature coupling theory, leaving only $R\phi_{,\mu}\phi^{,\mu}$ and $R^{\mu\nu}\phi_{,\mu}\phi_{,\nu}$

- Two new terms modulates gravitational strength with a free canonical kinetic term without either scalar field potential or cosmological constant. Hence resulting in effective cosmological constant giving de-Sitter expansion.

- model is tightly constrained in weakly coupling regime (local gravity-solar system test) (Daniel, Caldwell: Class. Quant. Grav. **24, 5573 (2007)**)

NMDC

Granda's two coupling constant model

(Granda: JCAP **1007, 006 (2010)**)

$$-(1/2)\kappa R\phi^{-2}g_{\mu\nu}\phi^{,\mu}\phi^{,\nu} \qquad -(1/2)\eta\phi^{-2}R_{\mu\nu}\phi^{\mu}\phi^{\nu}$$

- Dynamics is rescaled by inverse field square.
- Two coupling constants
- NMDC plays role of DM at early time but to have late acceleration, the potential is needed.