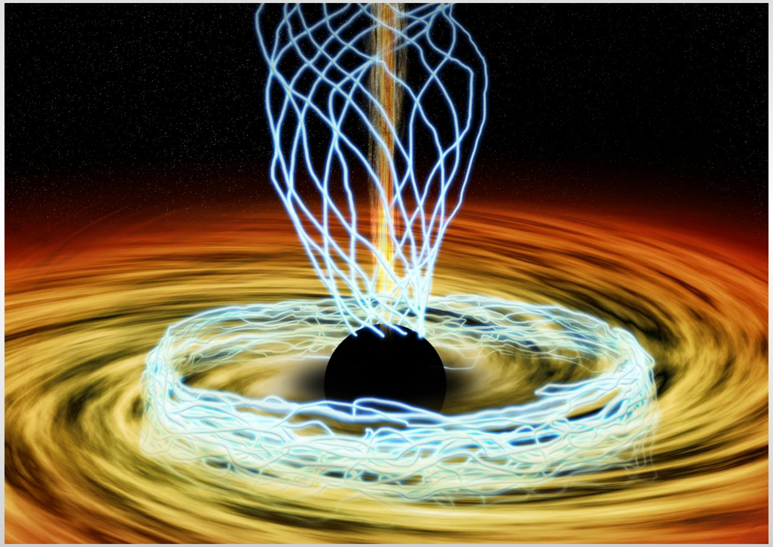
The Emperor's New Hair



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Motivation

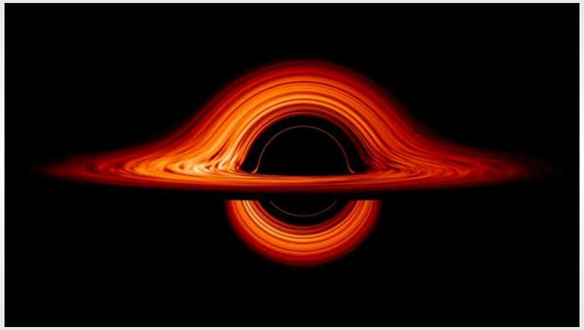
Black holes are fascinating test grounds for GR.

They represent regions of very strong gravity, and constrain any candidate to challenge GR.

Dark Energy in cosmology is a challenge for particle physics – and unless Λ, a challenge to mix with a black hole.

Outline

- Black hole no hair
- Black hole hair
- Stealth hair theorem!
- Summary



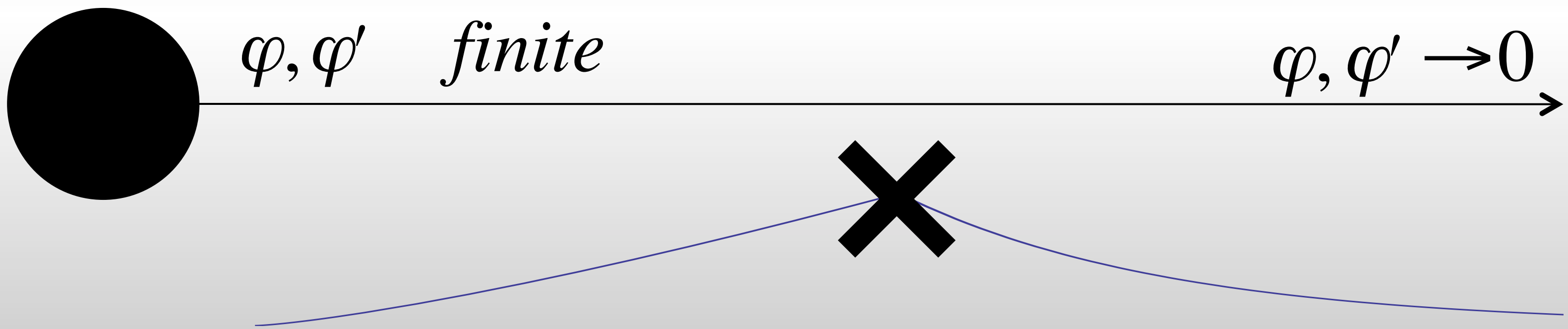
Black Hole Theorems

Black holes in 4D obey a set of theorems: We know they are spherical, that they obey laws of thermodynamics, and that they are characterized by relatively few "numbers" – or "Black Holes Have No Hair".

i.e. electrovac solutions are uniquely specified by 3 parameters: M, Q, and J

No Scalar Hair

The essence of "no hair" is that the scalar field must have finite energy, and fall off at infinity. Integrating the equation of motion gives a simple relation, only satisfied for $\varphi = \varphi' \equiv 0$

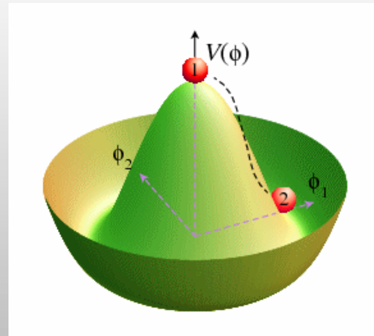


$$\int_{2GM}^{\infty} \left\{ r^2 V_{,\phi}^2 + r(r-2GM)\phi' \frac{d}{dr}\left(V_{,\phi}\right) \right\} = \left[V_{,\phi} r (r-2GM)\phi' \right]_{2GM}^{\infty} = 0.$$

No-No Hair!

But this is highly idealised:

- Static
- Vacuum
- Convex potential
- Λ not negative

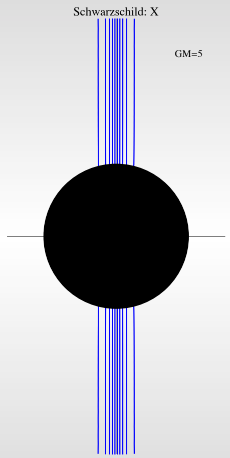


And no hair came to mean the much stronger "no field profiles".

Hair!

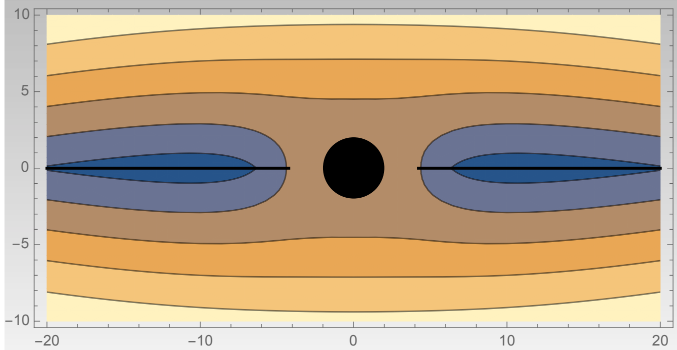
Many examples of black holes with nontrivial scalar profiles have been constructed:

- Coloured black holes (unstable)
- Black holes with strings – or walls (stable!)
- Monopole black holes (stable)

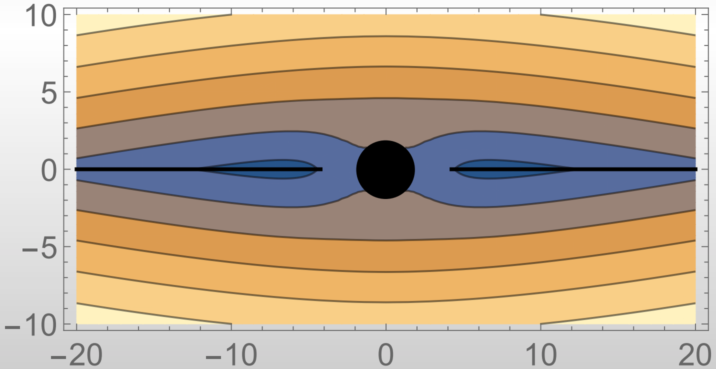


And that isn't even from this century!

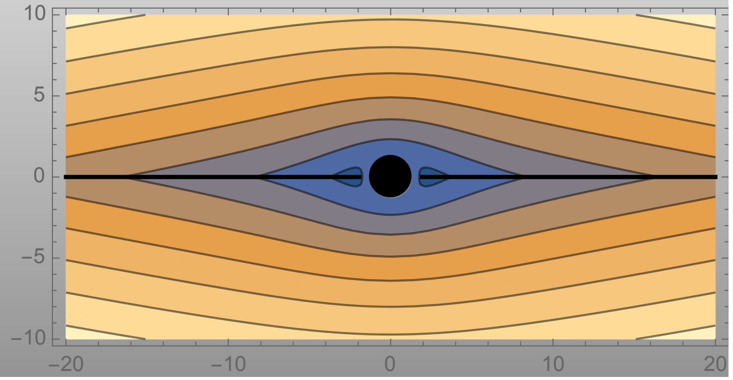
E.G. Chameleon Kerr Hair



The chameleon mass and VEV strongly dependent on density, accretion disc provides an anchor for hair.



(With Anne Davis and Rahul Jha)



Other Hair?

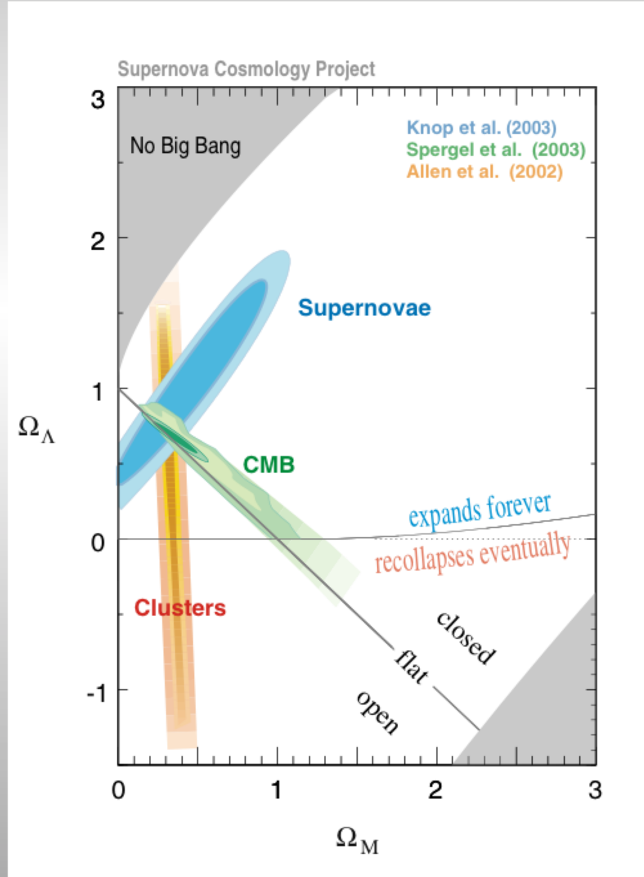
These examples use topology to construct the nontrivial configuration – and scalar condensates in string theory use negative Λ, so can time-dependence get around no hair?

Q-balls: scalar solutions with internal time dependence, but externally looking “static”

Time-dependent scalars with black holes found in the context of cosmologically rolling scalars.

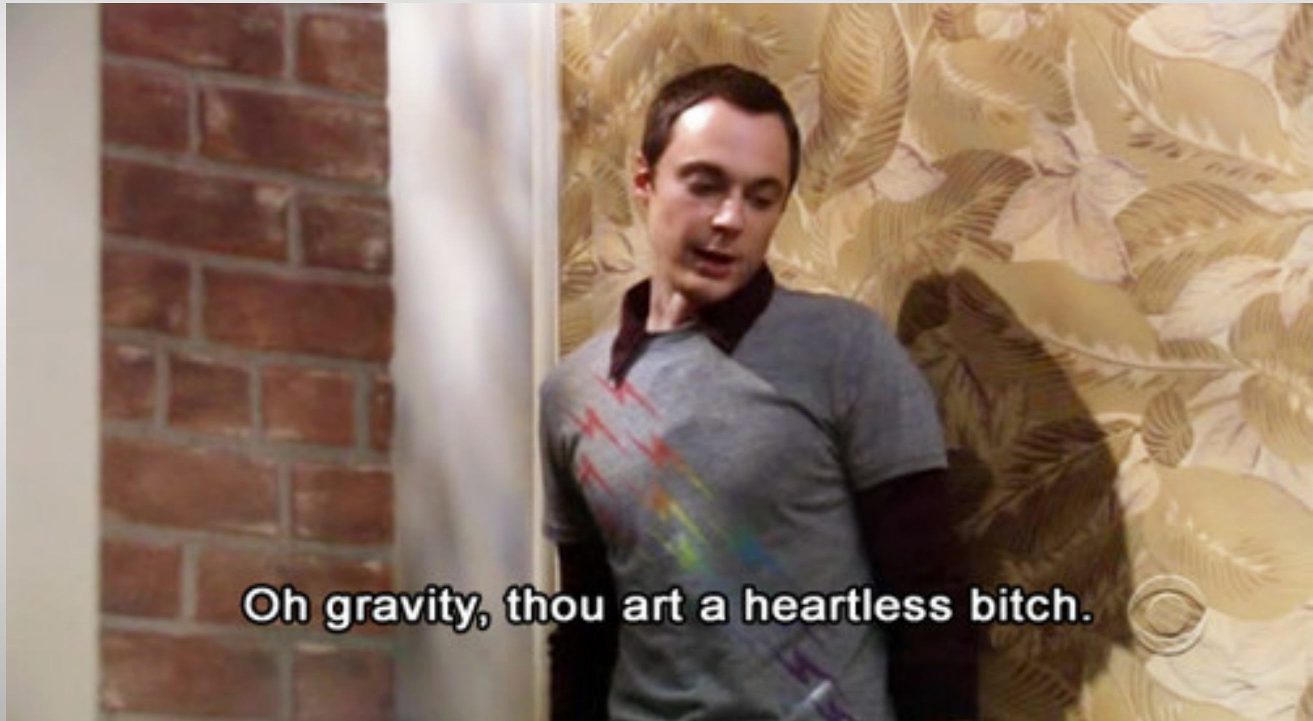
Acceleration

Cosmological rolling scalars arose from the challenge of explaining the late time acceleration of our Universe, which is gently accelerating with an effective cosmological constant of 10^{-30} g cm^{-3}.



Other Gravity

Other attempts to explain acceleration modify gravity – braneworlds, massive gravity, galileons…



So what can we find here?

Scalar Hair

$$S = \int d^4x\sqrt{-g}\left[\zeta R - \eta(\partial\phi)^2 + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 2\Lambda\right]$$

Babichev-Charmousis & Sotiriou-Zhou found hair in a Horndeski gravity with inherent time dependence in the scalar; (for SCH):

$$\phi = q\left[v - r + 2\sqrt{2GMr} - 4GM\log\left(\sqrt{\frac{r}{2GM}} + 1\right)\right]$$

The scalar rolls on the horizon, and throughout the geometry, BUT, does not backreact, since we have explicitly assumed the Einstein equations.

Stealth Hair?

It’s hair (Jim) –

Stealth Hair?

– but not as we know it!

Scalar Hair

How generic is this? For simplicity, consider the plain John + Einstein

$$\mathcal{L} = \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \zeta R$$

THEOREM:

A solution of the Einstein equations can support scalar stealth hair iff the metric can be written as a 3+1 decomposition of a flat 3-space with a time-like vector field

Proof

To demonstrate this, we will first use equations of motion to derive a set of constraints on any solution for the scalar, assuming the background Einstein equations.

We then find the most general solution of these algebraic constraints, and show that this implies a 3+1 foliation with flat spatial sections.

Preliminaries

Equations of motion:

$$\zeta G_{ab} = -\frac{\beta}{2}\Big[G_{ab}(\partial\phi)^2 + 2P_{acbd}\nabla^c\phi\nabla^d\phi - 2\Box\phi\nabla_a\nabla_b\phi$$

$$+ 2\nabla_a\nabla_c\phi\nabla_b\nabla^c\phi + \left((\Box\phi)^2 - \phi_{;cd})^2\right)g_{ab}\Big]$$

$$0 = \beta\nabla_a\left(G^{ab}\nabla_b\phi\right)$$

Where P is the double dual of Riemann, and write

$$\nabla_a\phi \leftrightarrow \phi_a \ , \quad \nabla_a\nabla_b\phi \leftrightarrow \phi_{ab}$$

First step

Tracing the “Einstein” equation:

$$\cancel{-\zeta R} = \frac{\beta}{2}\left[\cancel{R\phi_a^2} + \cancel{2G_{cd}\phi^c\phi^d} + 2\phi_{cd}^2 - 2(\Box\phi)^2\right]$$

Gives

$$\phi_{cd}^2 = (\Box\phi)^2$$

Then contracting the Einstein equation with dϕ

$$\cancel{\zeta G_{ab}\phi^a} = -\frac{\beta}{2}\Big[\cancel{G_{ab}\phi^a\phi_c^2} + 2\phi^a\phi_{ac}\phi_b^c$$

$$+ \left(\cancel{(\Box\phi)^2 - \phi_{cd}^2}\right)\phi_b\Big] - 2\Box\phi\phi^a\phi_{ab}$$

Gives a constraint on derivatives of ϕ

$$0 = \phi^a \phi_{ac} \left[\phi_b^c - \Box\phi\delta_b^c\right]$$

We now use these constraints to pin down the background geometry. Either $\phi^a \phi_{ac} = 0$ or is normal to $\left[\phi_b^c - \Box\phi\delta_b^c\right]$

So, define:

$$n_a = \phi_a / |\phi_a|$$

If $\phi^a \phi_{ac} = 0$, n is tangent to a global congruence of affinely parametrised geodesics.

Foliate the spacetime with surfaces of constant ϕ, these have normal n^a and extrinsic curvature:

$$K_{ab} = (\delta_a^c - n_a n^c(\delta_b^d - n_b n^d)\nabla_{(c} n_{d)} = \frac{\phi_{ab}}{|\phi_a|}$$

Putting this in the Einstein equation gives the result

$$\begin{aligned} 0 &= P_{acbd}\nabla^c\phi\nabla^d\phi - \phi_c^c\phi_{ab} + \phi_{ac}\phi_b^c \\ &= |\phi_a|^2\left[R_{acbd}n^c n^d + K_{ac}K_b^c - KK_{ab}\right] = {}^{(3)}R_{ab} \end{aligned}$$

Otherwise – again foliate the spacetime with surfaces of constant ϕ, so the metric has the form:

$$ds^2 = Nd\phi^2 + 2N_i dx^i d\phi - \gamma_{ij} dx^i dx^j$$

Now define another unit vector orthogonal to n, l:

$$n^b \phi_{ab} = (\phi_{00} n_a + \phi_{01} l_b)$$

And complete to an o/n basis (n, l, m_1, m_2). By construction, $\phi_{02} = \phi_{03} = 0$ and the idea is to apply the algebraic constraints to the remaining components of ϕ

Doing this, surprisingly, shows that ϕ_{ab} has a very simple form:

$$\boxed{\phi_{\hat{a}\hat{b}} = \begin{bmatrix} \begin{matrix} \phi_{00} & \phi_{01} \\ \phi_{01} & \phi_{11} \end{matrix} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}}$$

With $$\phi_{01}^2 = \phi_{00}\phi_{11}$$

This in turn implies $R_{acbd}\phi^c\phi^d = 0$ & $K_{ab} = \dfrac{\phi_{11}}{|\partial\phi|} l_a l_b$

Which is enough to guarantee ${}^{(3)}R_{ab} = 0$ $\blacksquare$

Check Schwarzschild

To check Schwarzschild, recall $$\phi = t + R(r)$$

Where $$R' = V_s^{-1}\sqrt{1 - V_s} = \frac{\sqrt{2GM}}{(r - 2GM)}$$

giving the Painleve Gullstrand flat foliation:

$$V_s d\phi^2 - 2V_s R' dr d\phi - dr^2 - r^2 d\Omega_{II}^2$$

Other examples: Rindler, Milne…

Summary

$$ds^2 = \mathcal{N}^2 dt^2 - \delta_{ij}(dx^i - N_i dt)(dx^j - N_j dt)$$

Any solution of Einstein equations gives a ϕ proportional to "t" of geodesic form.

Unfortunately, according to the exact solution bible – the full set of Einstein spaces with this form is not known.

The case with $\phi_{01} = 0$ only has flat spacetime as a solution.

The remaining case is in progress.