Gravitational domain walls, black holes and the dynamics of G

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Based on collaboration with C. Bunster
arXiv:1704.03372

Introduction

- Introduce a dynamical gravitational constant: integration constant, not fundamental constant.

BRUARY 20, 1937 NATURE 323

Dirac's large number hypothesis

Letters to the Editor

NOTES ON POINTS IN SOME OF THIS WEEK'S LETTERS APPEAR ON P. 332.

The Cosmological Constants

fundamental constants of physics, such as c city of light, h Planck's constant, e the charge mass of the electron, and so on, provide for of absolute units for measurement of distance, ass, etc. There are, however, more of these ts than are necessary for this purpose, with lt that certain dimensionless numbers can be cted from them. The significance of these s has excited much interest in recent times, dington has set up a theory for calculating f them purely deductively. Eddington's nts are not always rigorous, and, while they e the feeling that they are probably sub-ly correct in the case of the smaller numbers iprocal fine-structure constant hc/e^2 and the the mass of the proton to that of the electron), er numbers, namely the ratio of the electric gravitational force between electron and which is about 10^{39}, and the ratio of the mass universe to the mass of the proton, which is 10^{78}, are so enormous as to make one think me entirely different type of explanation is for them.

ding to current cosmological theories, the e had a beginning about 2×10^9 years ago, ll the spiral nebulæ were shot out from a gion of space, or perhaps from a point. If ...

constants, the gravitational 'constant' must decrease with time, proportionally to t^{-1}. Let us define the gravitational power of a piece of matter to be its mass multiplied by the gravitational constant. We then have that the gravitational power of the universe, and presumably of each spiral nebula, is increasing proportionally to t. This is to some extent equivalent to Milne's cosmology[1], in which the mass remains constant and the gravitational constant increases proportionally to t. Following Milne, we may introduce a new time variable, $\tau = \log t$, and arrange for the laws of mechanics to take their usual form referred to this new time.

To understand the present theory from the point of view of general relativity, we must suppose the element of distance defined by $ds^2 = g_{\mu\nu} dx_\mu dx_\nu$ in the Riemannian geometry to be, not the same as the element of distance in terms of atomic units, but to differ from this by a certain factor. (The former corresponds to Milne's $d\tau$ and the latter to Milne's dt.) This factor must be a scalar function of position, and its gradient must determine the direction of average motion of the matter at any point.

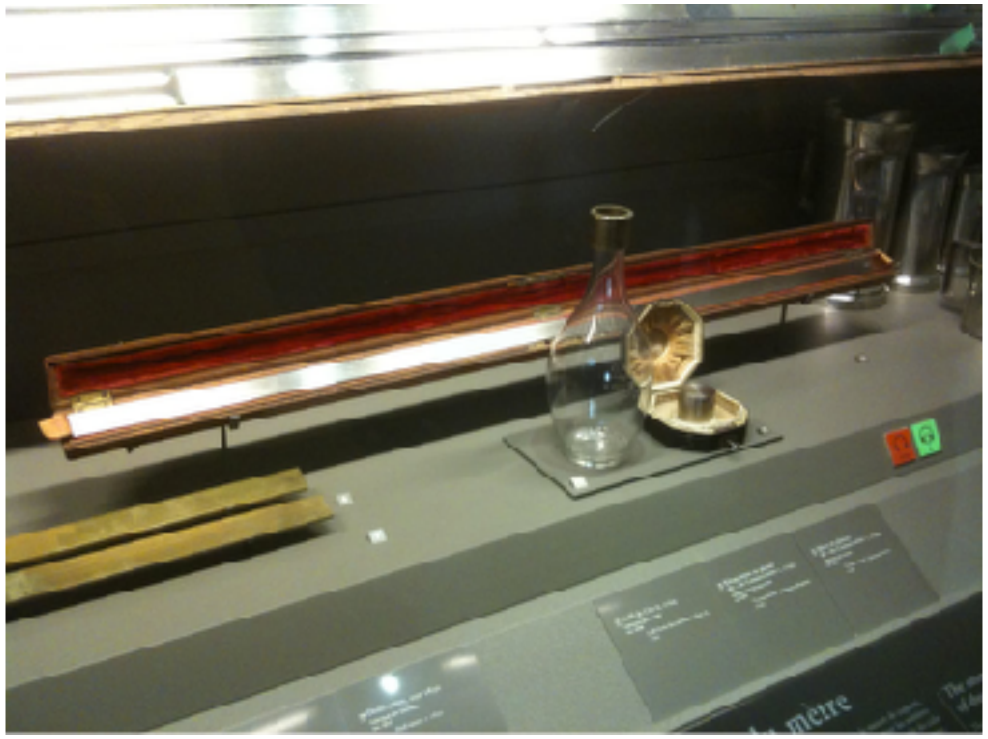
P. A. M. DIRAC.

St. John's College,
Cambridge.
Feb. 5.

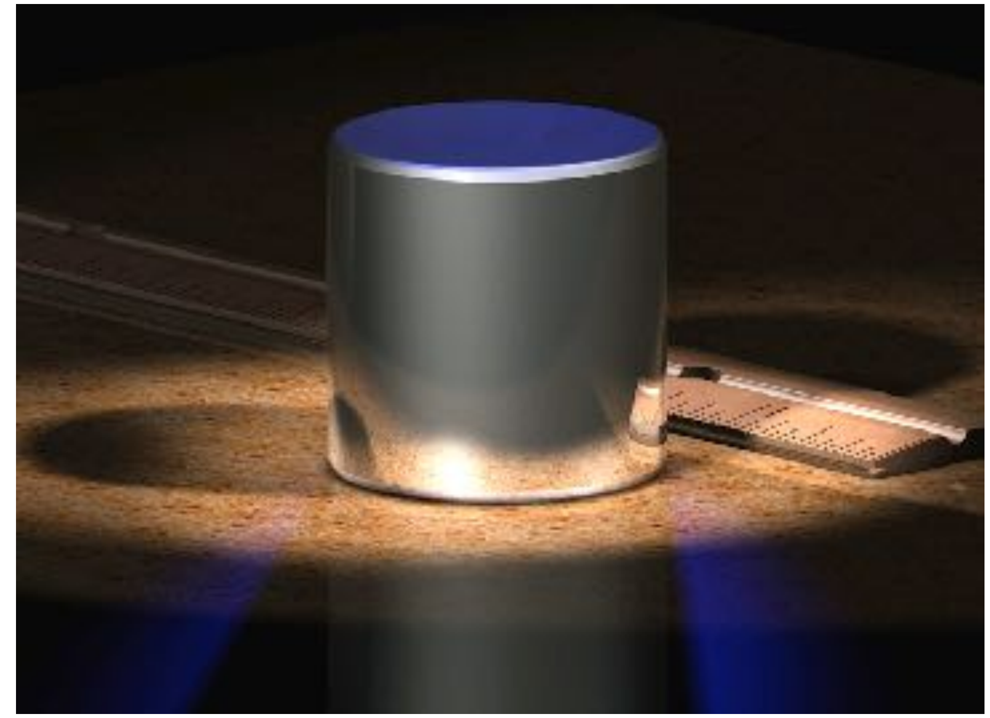
[1] Milne, *Proc. Roy. Soc.*, A, **158**, 324 (1937).

$$\frac{m_p}{m_h} = \frac{1}{m_h}\sqrt{\frac{\hbar c}{G}} \sim 10^{17}$$

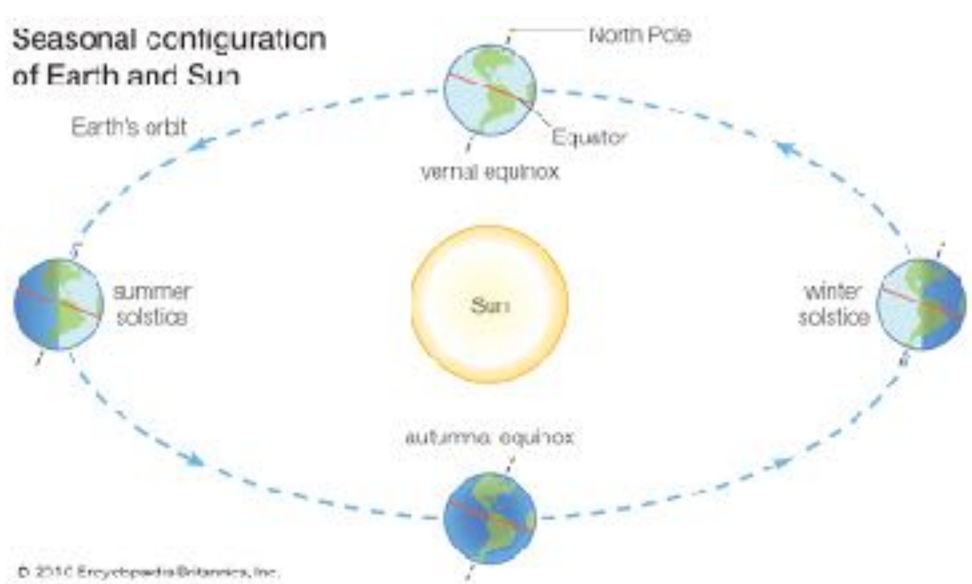
Units (International system until 1960)



1 meter

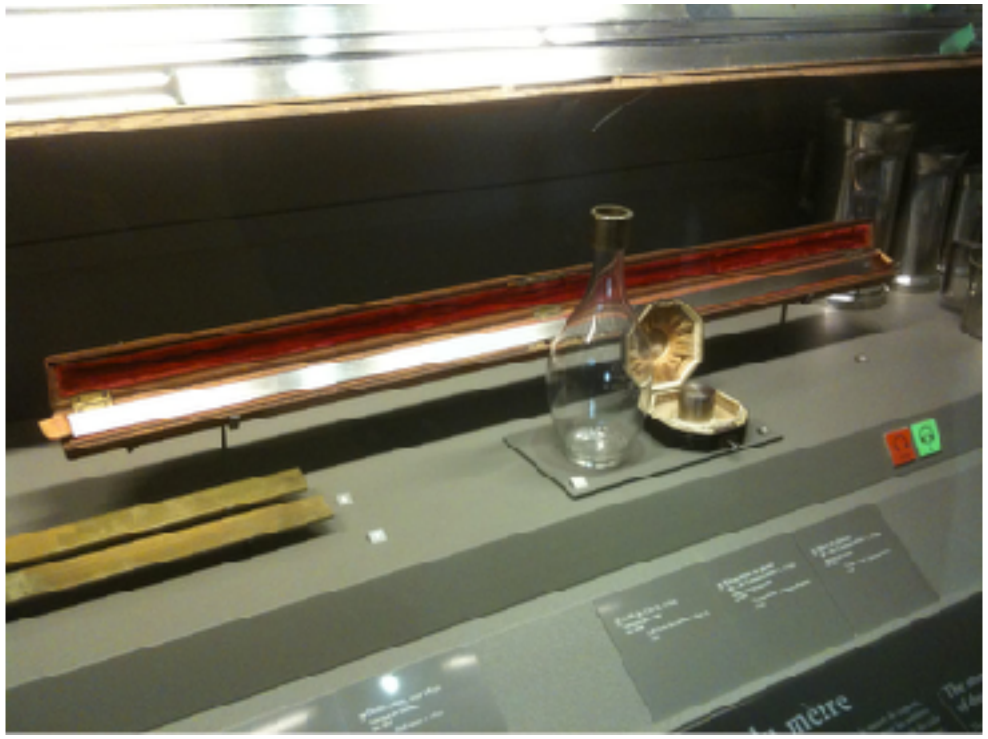


1 kilogram

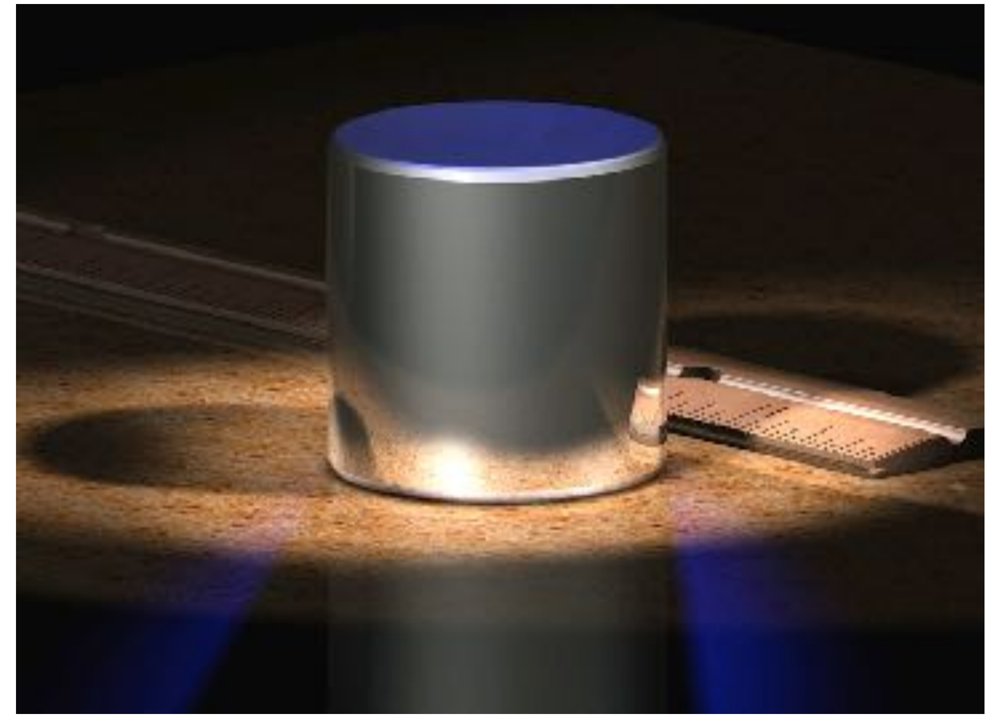


1 second= 1/31.556.925,9747 tropical year for 1900

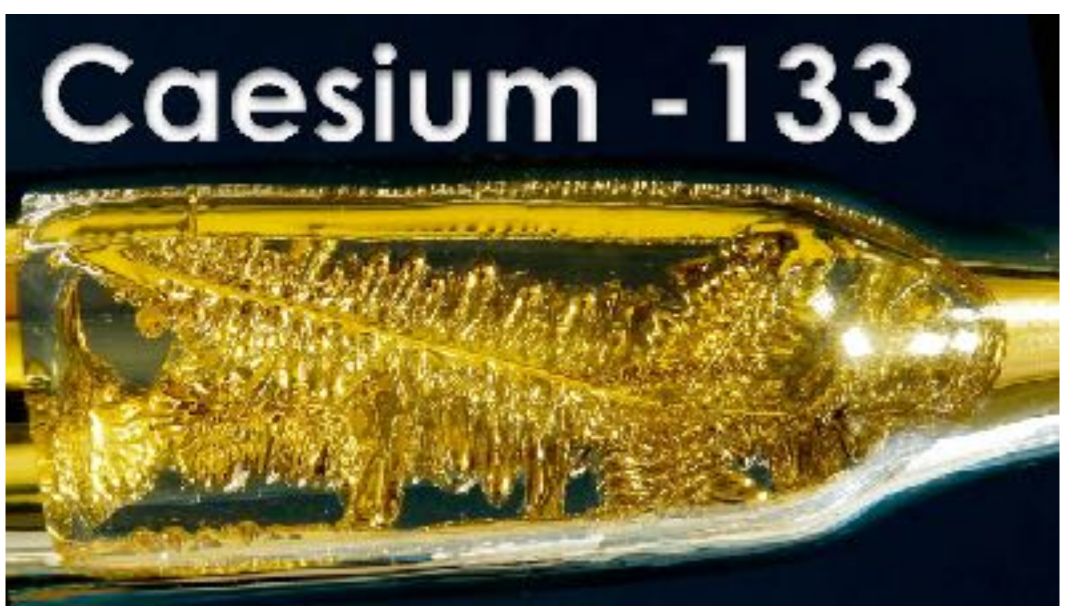
Units (International system today)



distance light travels in 1/(299.792.458) sec.

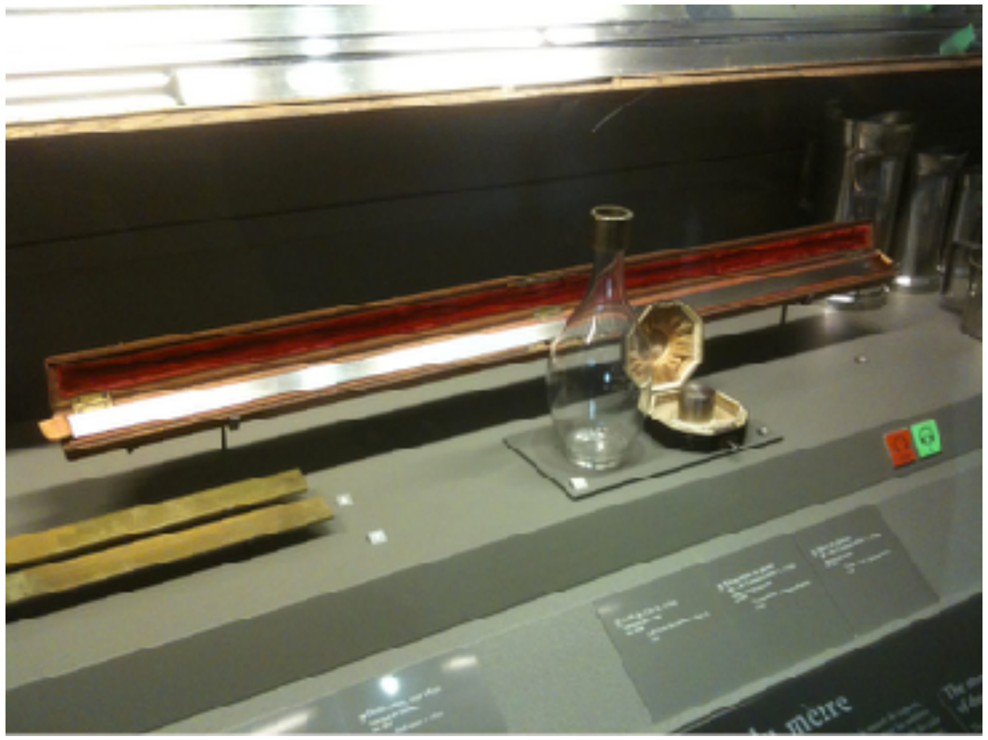


1 kilo



1 second= time duration of 9.192.631.770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium-133 atom.

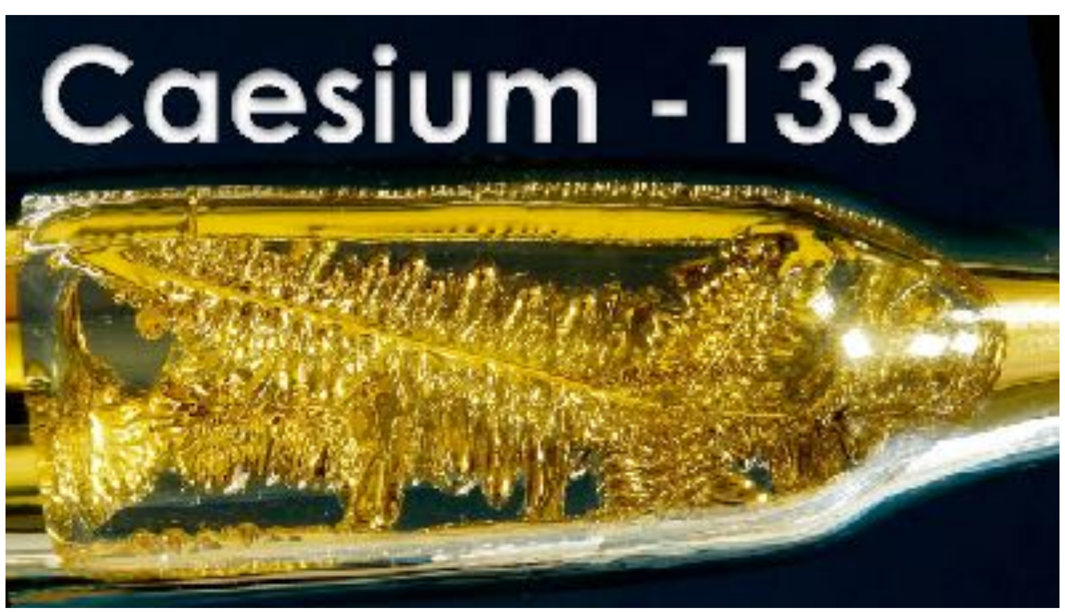
Units (International system 2018)



distance light travels in 1/(299.792.458) sec.

$\hbar$

1 kilogram



1 second= time duration of 9.192.631.770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium-133 atom.

Introduction

✻ Introduce a dynamical gravitational constant: Integration constant, not fundamental constant.

- ✦ No scalar field (Jordan-Brans-Dicke). Only one global degree of freedom (a 3-form field).
- ✦ Different regions of space with different gravitational constants are separated by charges domain walls.
- ✦ Domain walls may be spontaneously nucleated, changing the gravitational constant.

✻ Discuss, in general, how fundamental constants may be uplifted to constants of integration.

of the choice of coordinates (an *integral* invariant, which does not correspond to a *differential* invariant).[11] The energy of the whole world, in the case where the latter is closed with an even distribution of matter, is determined just by this matter alone; gravitational field energy and the energy contribution of the λ-term cancel each other out. Furthermore, it is not entirely uninteresting that the theory can easily be formulated so that λ appears not as a universal constant in the usual sense but as an integration constant, or as a Lagrangian factor. One only needs to say

$$\delta\{\int \mathfrak{H} d\tau\} = 0$$

must be satisfied for all variations that leave the volume, in natural measure,

$$\int \sqrt{-g}\, d\tau,$$

invariant. Such a formulation seems all the more natural since the vanishing of the variation of the Hamilton integral certainly cannot be postulated reasonably for *such* variations corresponding to a transition of the world to a neighboring smaller one that is also at equilibrium. There is nothing analogous to this consideration in standard mechanics, because there the masses and volume are not varied. I think I should publish this sometime when the occasion arises, because it cleanses the theory of a blemish.[12] I wonder whether one day other universal constants won't lose their painful character this way as well?[13]

A. Einstein to M. Besso, July 29th, 1918

Introduction

- Introduce a dynamical gravitational constant: Integration constant, not fundamental constant.

  - No scalar field (Jordan-Brans-Dicke). Only one global degree of freedom (a 3-form field).
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  - Domain walls may be spontaneously nucleated, changing the gravitational constant.

- Discuss, in general, how fundamental constants may be uplifted to constants of integration.

  - Cosmological constant is a know example (A. Aurilia, Nicolai, Townsend 1980, Duff, van Nieewenhuzen 1980, Freund and Rubin 1980)
  - It has been used to discuss the comological constant problem (Brown and Teitelboim 1987).

- For the gravitational constant, black hole thermodynamics mandates how domain walls (G-walls) are coupled.

Turning fundamental constant into integration constants

Example: Non- relativistic Free Particle

$$I[x] = \frac{1}{2} m \int \dot{x}^2 dt$$

$x \longrightarrow m\ddot{x} = 0$

Example: Non- relativistic Free Particle

Replace by

$$I[A, D, x] = \int dt \left(D\dot{A} + \frac{1}{2} D^2 \dot{x}^2 \right)$$

$A \longrightarrow \dot{D} = 0$

$x \longrightarrow D^2 \ddot{x} = 0 \qquad (\; m \equiv D^2 \;)$

$D \longrightarrow \dot{A} + D\dot{x}^2 = 0$

$$D = -\frac{\dot{A}}{\dot{x}^2}$$

Example: Non- relativistic Free Particle

$$I = -\frac{1}{2}\int \frac{\dot{A}^2}{\dot{x}^2}$$

Actually, in general,

$$I[A, D, x] = \int dt \left(D\dot{A} + \frac{1}{2} m(D)\dot{x}^2 \right)$$

arbitrary function

Generalization to Field theory

$$I = \int d^4x \, \mathcal{L}\left(\phi^A, \partial_\mu \phi^A, \kappa(D)\right)$$

$$I = \int d^4x \left\{ \partial_\alpha \overset{*}{\mathcal{A}}{}^\alpha D + \mathcal{L}\left(\phi^A, \partial_\mu \phi^A, \kappa(D)\right) \right\}$$

$\kappa(D)$ arbitrary function

$$\overset{*}{\mathcal{A}}{}^\alpha = \frac{1}{3!} \epsilon^{\alpha\beta\gamma\delta} A_{\beta\gamma\delta}$$

$$I = \int d^4x \left\{ \partial_\alpha \overset{*}{\mathcal{A}}{}^\alpha D + \mathcal{L}\left(\phi^A, \partial_\mu \phi^A, \kappa(D)\right) \right\}$$

$A_{\beta\gamma\delta}$ → $\partial_\alpha D = 0$

Φ^A → Same as $\mathcal{L}$, with $\kappa \equiv \kappa(D) =$ constant.

D → Allows, in general, elimination of the auxiliar variable D

Example: Dynamical cosmological constant

$$I = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(R - 2\Lambda\right)$$

$$I[g_{\mu\nu}, A_{\alpha\beta\gamma}, D] = \int d^4x \left\{ D\partial_\alpha {}^*\!\mathcal{A}^\alpha + \frac{1}{16\pi G}\sqrt{-g}\left(R - 2\Lambda(D)\right)\right\}$$

$${}^*\mathcal{F} = \partial_\alpha {}^*\!\mathcal{A}^\alpha = \frac{1}{8\pi G}\frac{d}{dD}\Lambda(D)$$

$$I[g_{\mu\nu}, A_{\alpha\beta\gamma}, D] = \int d^4x \left\{ D\partial_\alpha \overset{*}{\mathcal{A}}{}^\alpha + \frac{1}{16\pi G}\sqrt{-g}\left(R - 2\Lambda(D)\right)\right\}$$

A simple choice

$$\Lambda(D) = 4\pi G D^2 + \lambda \longrightarrow {}^*\mathcal{F} = \partial_\alpha \overset{*}{\mathcal{A}}{}^\alpha = D$$

leads to Maxwell's theory after eliminating D

$$I = \int d^4x \sqrt{-g}\left\{\frac{1}{16\pi G}(R - 2\lambda) - \frac{1}{48}F^{\alpha\beta\gamma\delta}F_{\alpha\beta\gamma\delta}\right\}$$

(Brown and Teitelboim, 1987)

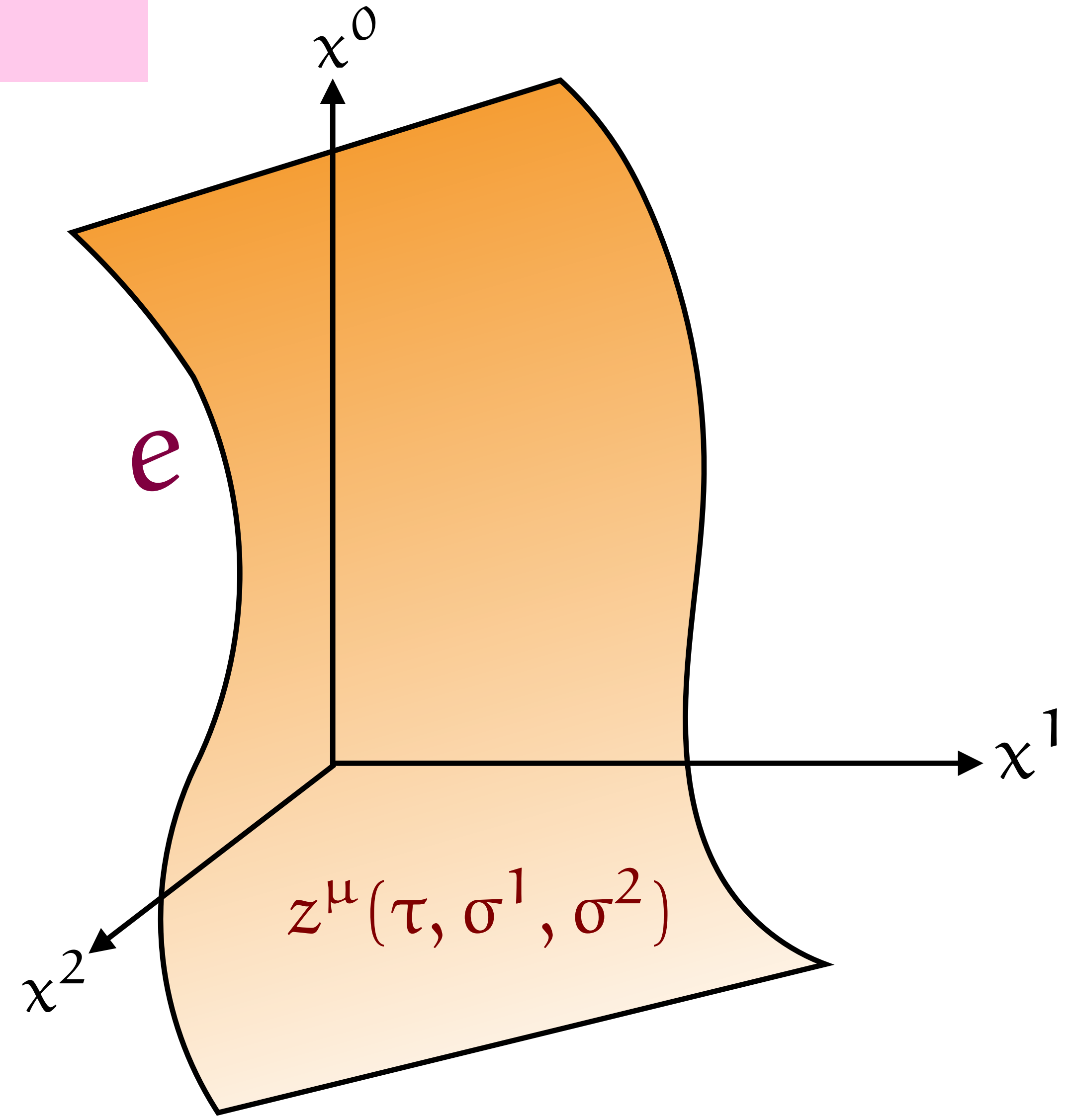
Adding charged sources

All this is boring unless we may vary D.
To make that posible we introduce 2D membranes (domain walls in 4D).

$$e \int A = \; e \int A_{\alpha\beta\gamma} \frac{\partial z^\alpha}{\partial \tau} \frac{\partial z^\beta}{\partial \sigma_1} \frac{\partial z^\gamma}{\partial \sigma_2} d\tau d^2\sigma$$

And a kinematical term

$$\frac{\mu}{4\pi} \int \sqrt{-\det \gamma}\; d^3\sigma$$



Dynamics of the membranes

$$I = I_0 + I_1$$

$$I_0 = \int d^4x \left\{ \mathcal{L}\left((\phi^A, \partial_\mu \phi^A, \kappa(D)\right) + D\partial_\alpha {}^*\!\mathcal{A}^\alpha \right\}$$ Bulk

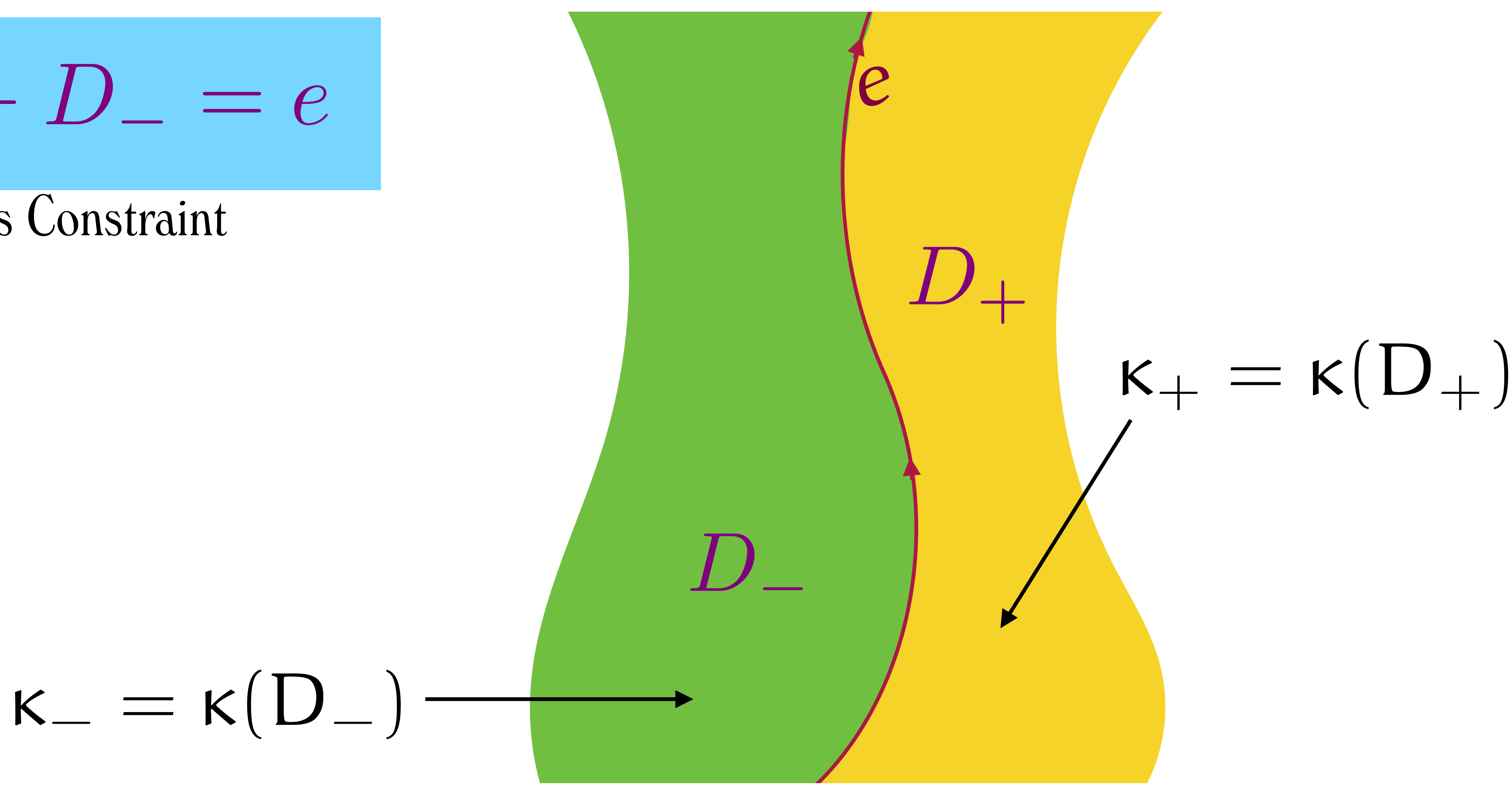
$$I_1 = e \int A - \frac{\mu}{4\pi} \int \sqrt{-\det\gamma}\, d^3\sigma$$ Membrane

Now, when varying $A_{\alpha\beta\gamma}$ we get a delta source in the gauss constraint:

- Domain walls divide spacetime in regions with different fundamental constants.
- The dynamics depend on the function $\kappa(D)$.
- Closed walls may be spontaneously nucleated by quantum or thermal fluctuations.

$$D_+ - D_- = e$$

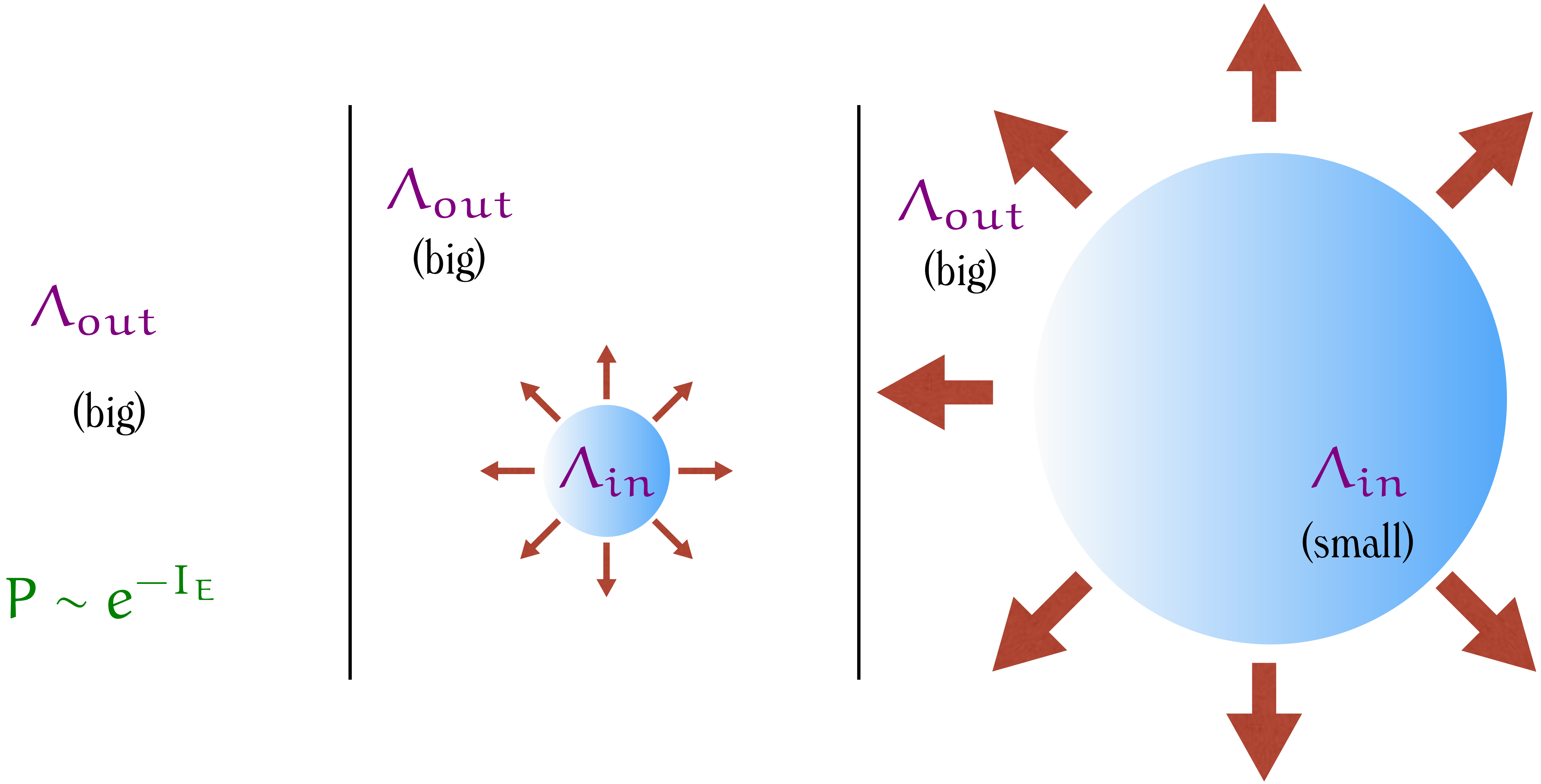
Gauss Constraint



Example: Cosmological constant

✻ Mechanism for relaxation of the cosmological constant by nucleation of membranes

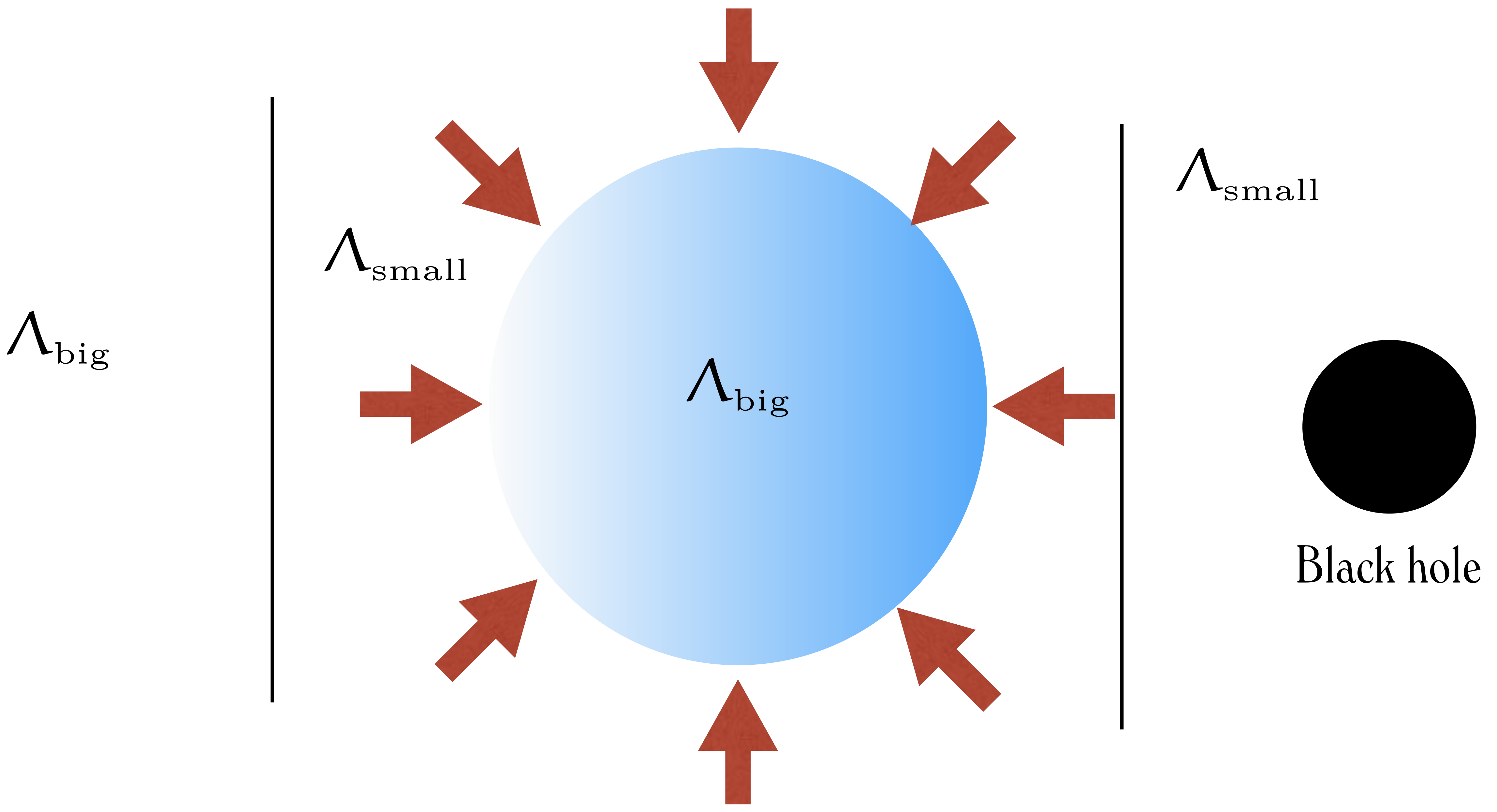
Instantons: quantum tunneling at zero temperature, (D. Brown and C. Teitelboim, 1987)



Example: Cosmological constant

✻ Mechanism for relaxation of the cosmological constant by nucleation of membranes

"Thermalons": Thermal activation, (A.G., M. Henneaux, C. Teitelboim, F. Wilczek, 2004)



Example: Cosmological constant

- ✻ Mechanism for relaxation of the cosmological constant by nucleation of membranes
- ✻ In Lovelock theories there are degenerate vacua with different cosmological constants.

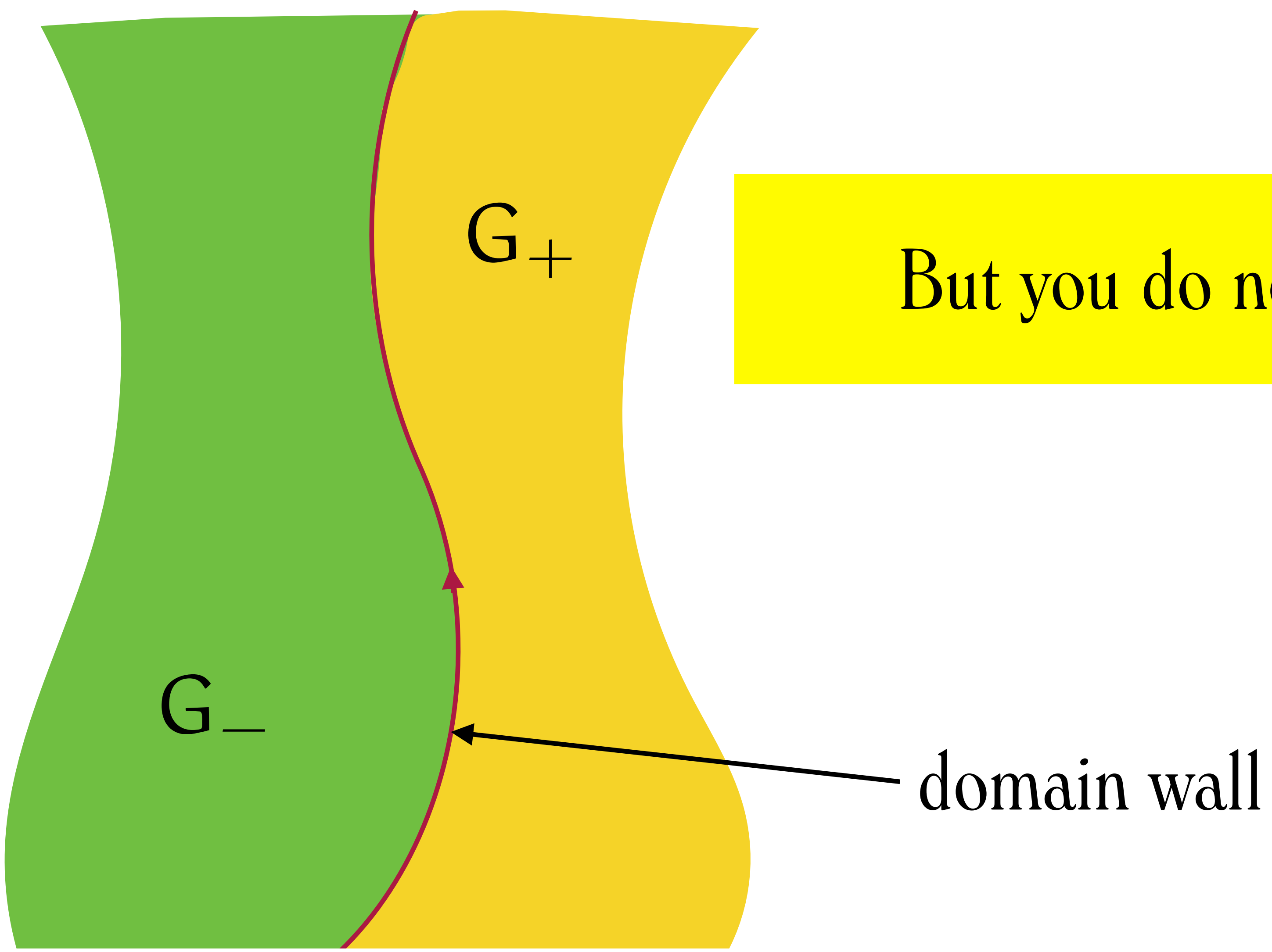
There exists an analog mechanism, but where the membranes are made out by the gravitational field itself:

C. Charmousis, A. Padilla 2008,
X.O. Camanho, J.D.Edelstein, G. Giribet and A. Gomberoff, 2012.

The gravitational constant and G-walls

Gravitational case

$$I[g_{\mu\nu}, A_{\alpha\beta\gamma}, D] = \int d^4x \left\{ D\partial_\alpha {}^*\mathcal{A}^\alpha + \frac{1}{16\pi G(D)}\sqrt{-g}R \right\}$$



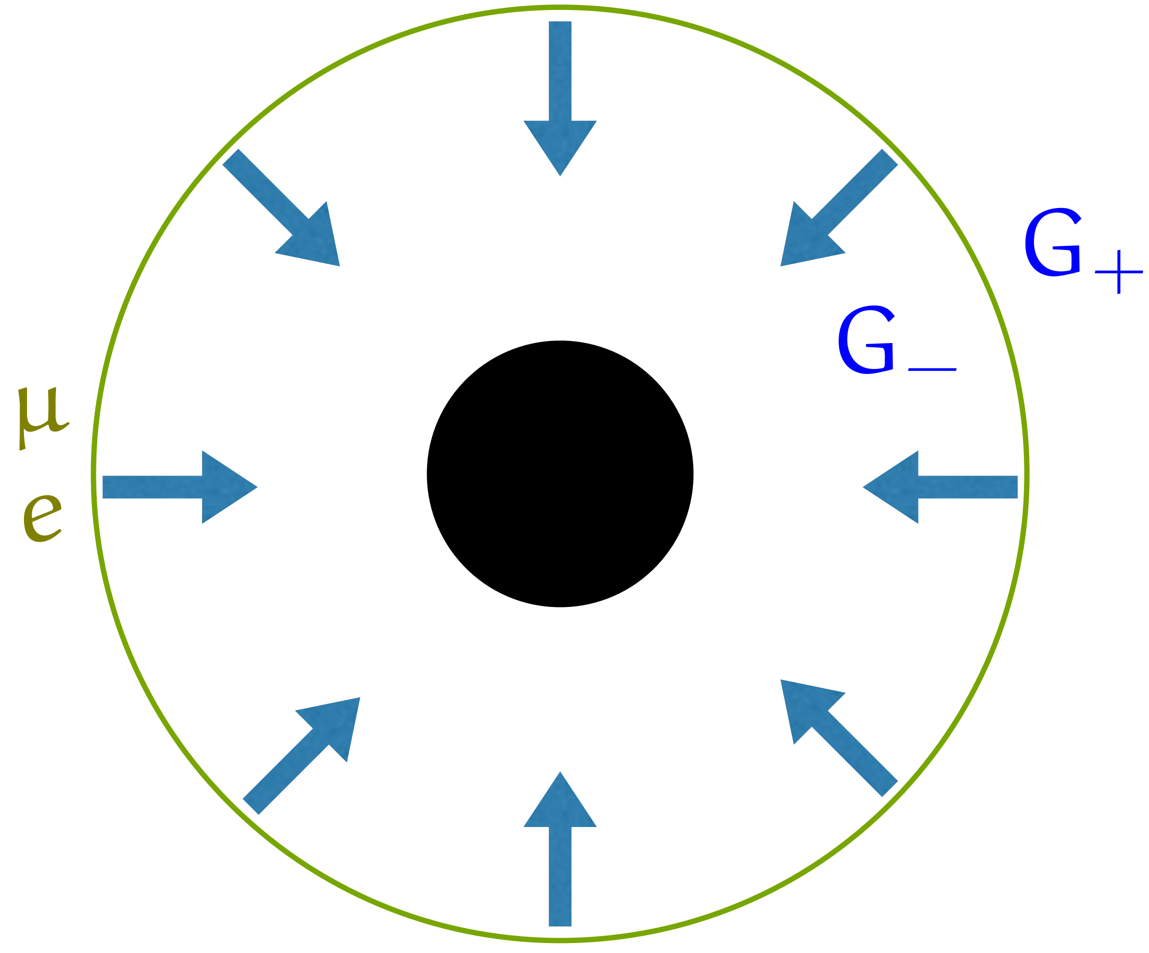
But you do not mess with gravity that easy!

Second law of thermodynamics is violated!

$$S = \frac{k_B c^3}{4 G \hbar} \text{ x (Area of horizon)}$$

Schwarzschild case

$$S = \frac{4\pi k_B G}{\hbar c} M^2$$



Way out: treat G as a scale factor

$$I = \int dt \left(\frac{m_1}{2} \vec{v}_1^2 + \frac{m_2}{2} \vec{v}_2^2 + \frac{m_1 m_2 G}{r_{12}} \right)$$

$$t = \lambda^{1/2} \tilde{t}, \qquad \vec{r} = \lambda^{1/2} \tilde{\vec{r}} \qquad m = \lambda^{-1/2} \tilde{m}$$

$$I = \int d\tilde{t} \left(\frac{\lambda^{1/2} m_1}{2} \vec{v}_1^2 + \frac{\lambda^{1/2} m_2}{2} \vec{v}_2^2 + \frac{m_1 m_2 G}{\tilde{r}_{12}} \right)$$

$$I = \int d\tilde{t} \left(\frac{\tilde{m}_1}{2} \vec{v}_1^2 + \frac{\tilde{m}_2}{2} \vec{v}_2^2 + \frac{\tilde{m}_1 \tilde{m}_2 G}{\lambda \tilde{r}_{12}} \right)$$

$$I = \int d\tilde{t} \left(\frac{\tilde{m}_1}{2}\vec{v}_1^2 + \frac{\tilde{m}_2}{2}\vec{v}_2^2 + \frac{\tilde{m}_1 \tilde{m}_2 G}{\lambda \tilde{r}_{12}} \right)$$

$$t = \lambda^{1/2}\tilde{t}, \qquad \vec{r} = \lambda^{1/2}\tilde{\vec{r}} \qquad m = \lambda^{-1/2}\tilde{m}$$

$$G \longrightarrow G/\lambda$$
$$c \longrightarrow c$$
$$\hbar \longrightarrow \hbar$$

$$\lambda = G \qquad I = \int d\tilde{t} \left(\frac{\tilde{m}_1}{2}\vec{v}_1^2 + \frac{\tilde{m}_2}{2}\vec{v}_2^2 + \frac{\tilde{m}_1 \tilde{m}_2}{\tilde{r}_{12}} \right)$$

Gravitational units

$$g_{\mu\nu} = G(D)\tilde{g}_{\mu\nu}$$

$$I_{grav} = \frac{1}{16\pi}\int d^4x \, \sqrt{-\tilde{g}}\tilde{R} - \int (\partial_\alpha D) \, {}^*\mathcal{A}^\alpha \, d^4x$$

$$I_{wall} = \int A - \frac{\tilde{\mu}}{4\pi}\int \sqrt{-\det\tilde{\gamma}} \, d^3\sigma$$

$$I_{matter} = -\int d^4x \, u\sqrt{-g} = -\int d^4x \, uG^2(D)\sqrt{-\tilde{g}}$$

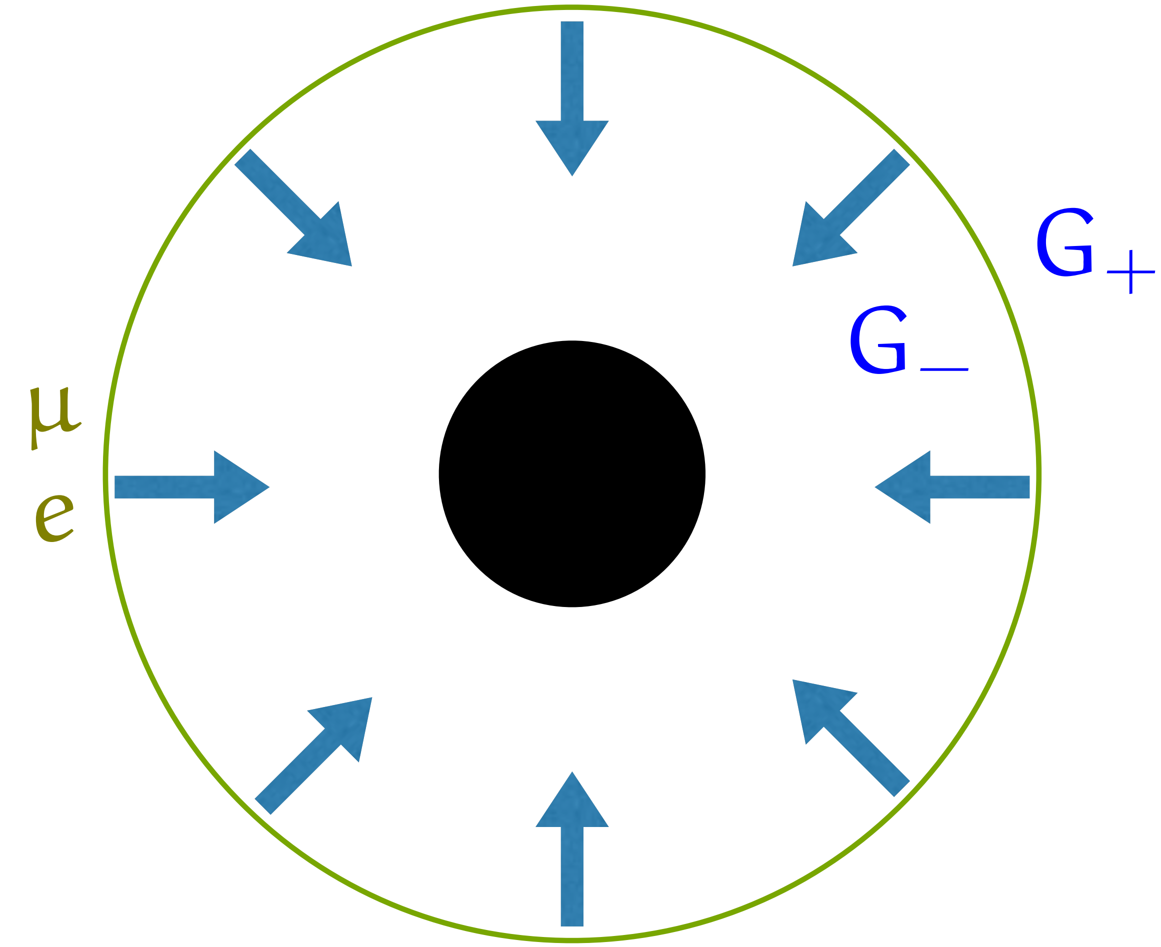
Second law of thermodynamics is fullfiled!

$$S = \frac{k_B c^3}{4G\hbar} \text{ x (Area of horizon)}$$

Schwarzschild case

$$S = \frac{4\pi G M^2}{\hbar} = \frac{4\pi \tilde{M}^2}{\hbar}$$

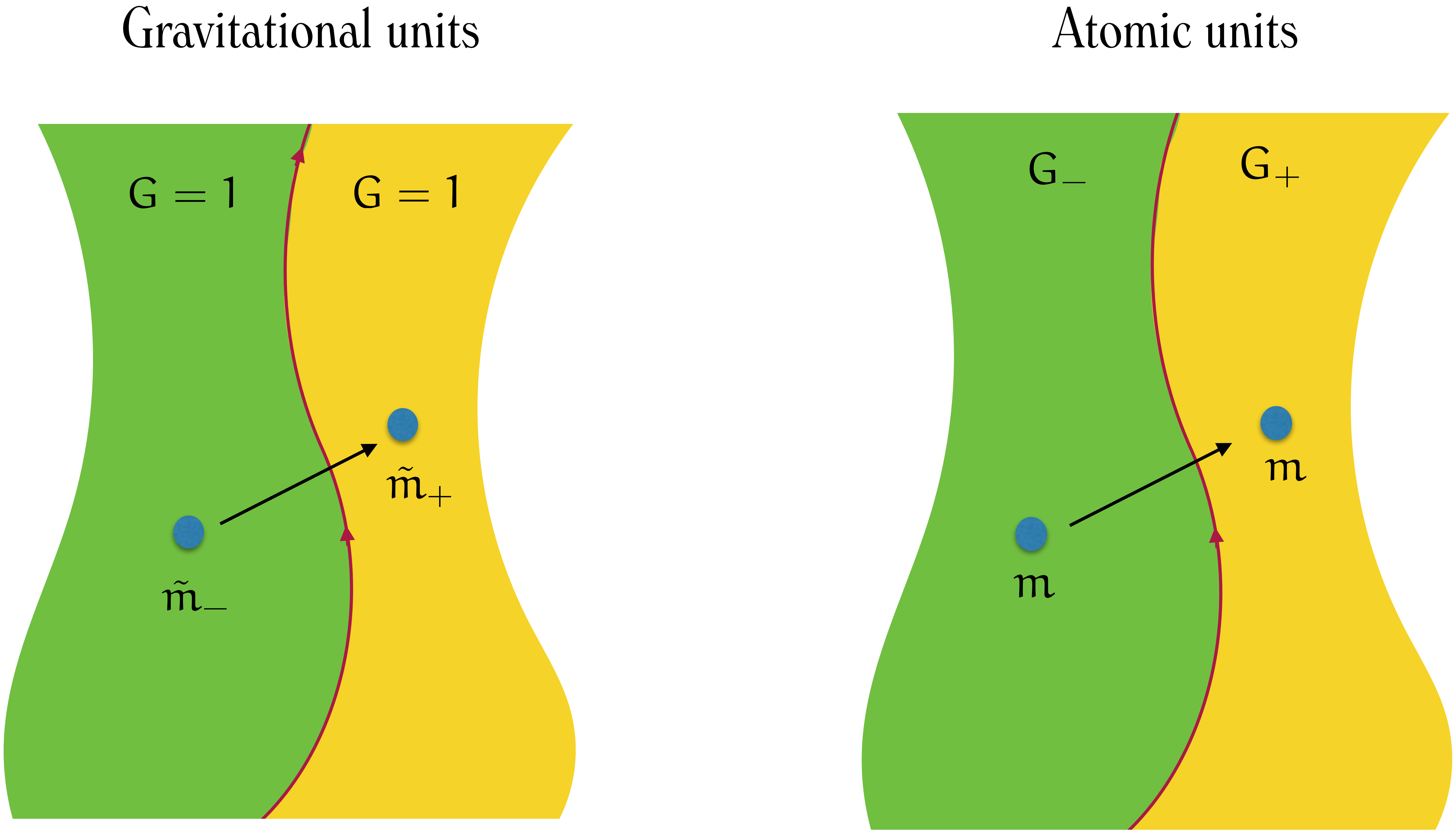
$$\tilde{M} = \sqrt{G}M$$



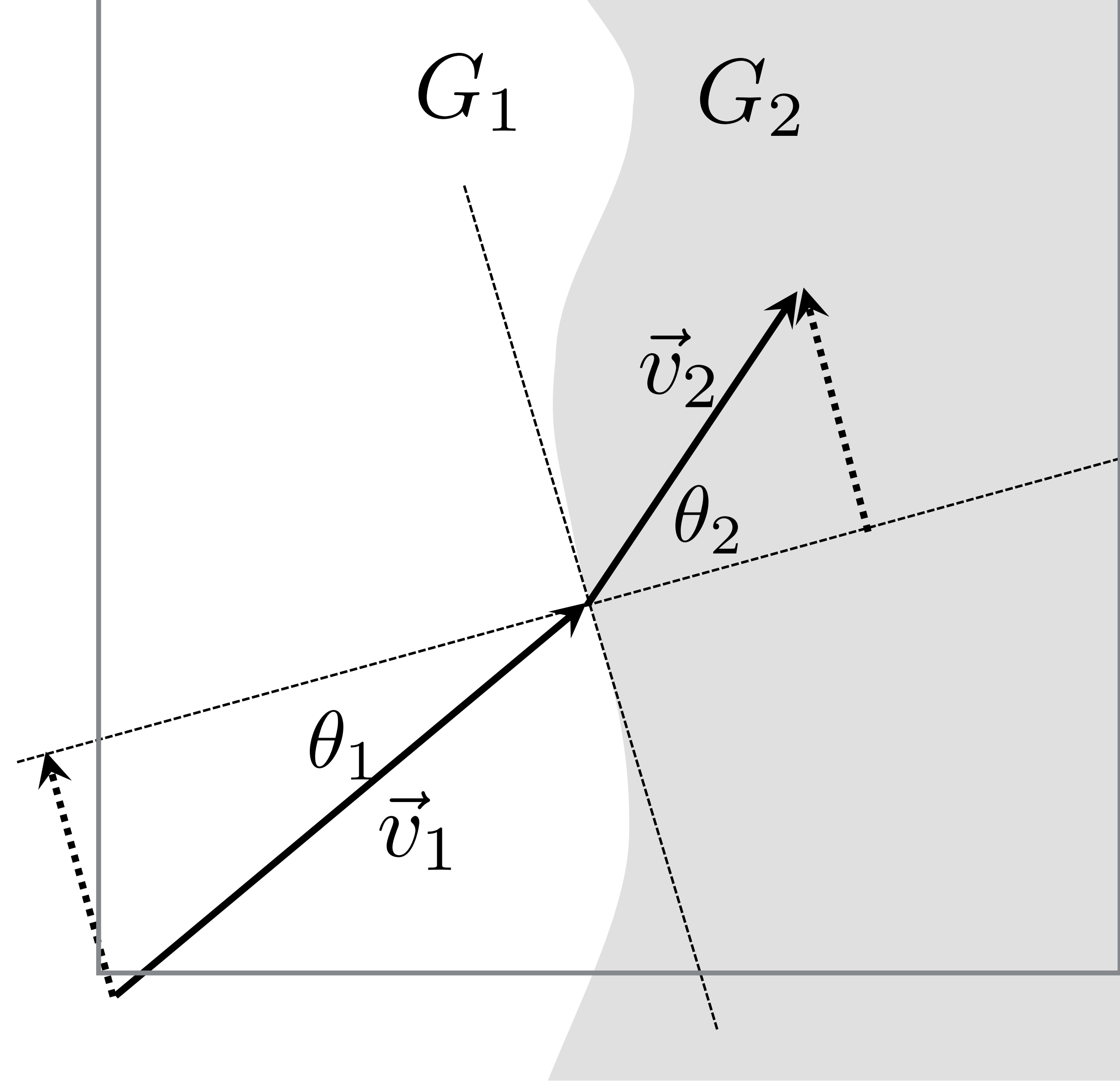
Some consequences

$$g_{\mu\nu} = G\tilde{g}_{\mu\nu}$$

G-Walls not isometrically embedded



Refraction of massive particles



$$I = -\int m \,\sqrt{-g_{\mu\nu} dx^\mu dx^\nu} = -\int \tilde{m}(x)\, \sqrt{-\tilde{g}_{\mu\nu} dx^\mu dx^\nu}$$

$$\tilde{m}(\tilde{x}) = \begin{cases} m\sqrt{G_1} & \text{left of the wall,} \\ m\sqrt{G_2} & \text{right of the wall.} \end{cases}$$

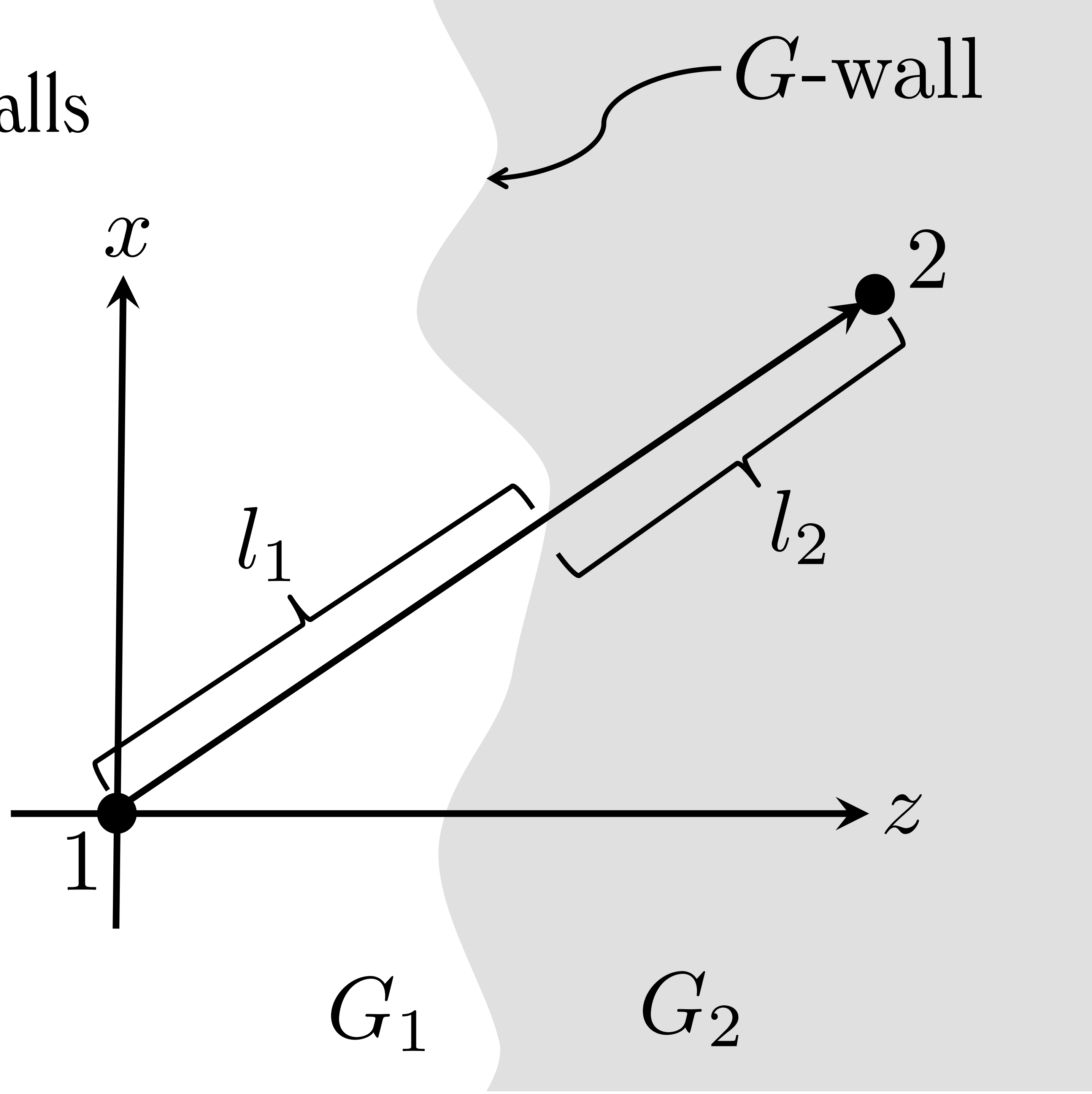
Forces in the presence of G-walls

Newton

$$F_{1\to 2} = -\frac{m_1 m_2 G_1^{1/2} G_2^{1/2}}{\left(\sqrt{\frac{G_2}{G_1}}\, l_1 + l_2\right)^2}$$

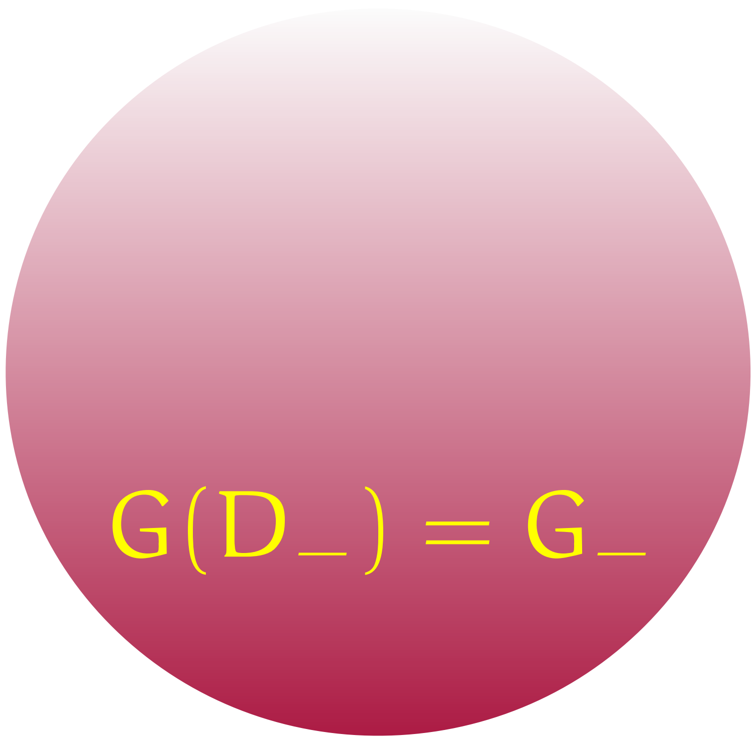
Coulomb

$$F_{1\to 2} - \frac{e_1 e_2}{\left(\sqrt{\frac{G_2}{G_1}}\, l_1 + l_2\right)^2}$$



Nucleation of G-Walls and relaxation of the gravitational constant

$$G(D_+) = G_+$$



$$I_{grav} = \frac{1}{16\pi}\int d^4x\,\sqrt{-\tilde{g}}\tilde{R} - \int (\partial_\alpha D)\,{}^*\mathcal{A}^\alpha\, d^4x$$

$$I_{wall} = \int A - \frac{\tilde{\mu}}{4\pi}\int \sqrt{-\det\tilde{\gamma}}\, d^3\sigma$$

$$I_{matter} = -\int d^4x\, u\sqrt{-g} = -\int d^4x\, uG^2(D)\sqrt{-\tilde{g}}$$

$$G(D) = \frac{G_0^2}{D^2}$$

Instanton

$$P \sim \exp\left(-\left[\frac{27\pi^2}{8^4|(\log G)'|}\right]\left[\frac{\tilde{\mu}^4}{\hbar u^3 G^6}\right]\right)$$

Conclusions

- ✦ In general, we may promote fundamental constants into constants of integration.
- ✦ One introduces jumps in the values of these constants by bringing in charged membranes.
- ✦ Doing it for gravitational constant requires a jump in the scale, that is, an non-isometric embedding, if one requires that the second law of black hole thermodynamics holds.
- ✦ Spontaneous nucleation of G-walls may relax the value of G from a value conmensurable with atomic scales to the one observed today (although we do not know how to implement this in a sensible cosmological model).