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Horndeski Superconductor

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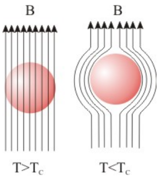
Superconductors



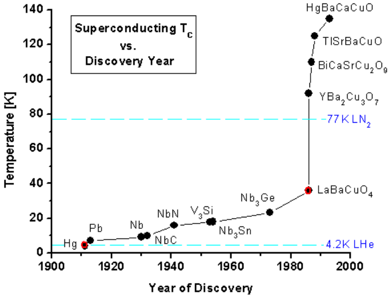
Superconductors are materials with the following two important properties:

1. Infinite DC conductivity
2. The ability to expell any magnetic field from their inside - Meissner Effect

These properties can be described by a phase transition at some critical temperature T_c

- Appearance of electron pairs with opposite spin $\rightarrow$ Cooper Pairs
- For $T < T_c$ these pairs forms a condensation $\rightarrow$ Spontaneous Breaking of the U(1) symmetry $\rightarrow$ Creation of a gap, Δ, on the energy spectrum
- Macroscopic theories like Ginzburg - Landau description of the Helmholtz free energy
- Microscopic theories like the Bardeen-Cooper-Schrieffer theory
- London Equation $\nabla^2 \vec{B} = \frac{1}{\lambda^2} \vec{B}$, for Meissner Effect

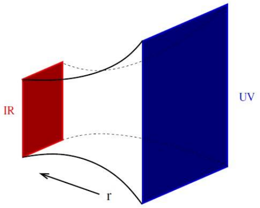
- In the context of these theories, T_c is usually small
- 1986: Discovery of the first superconductor with high T_c
- Until 2015 more High T_c have been discovered: Cuprates, Hydrogen sulfide, Iron based superconductors etc.
- The known theories about superconductivity cannot describe them. It seems that the condensation mechanics cannot be explained with the Cooper pairs.
- Appearance of strong interactions in these materials



Here Comes Holography!

Holography is the statement of equivalence between two different theories:

1. Strongly interacting quantum field theories in d spacetime dimensions (boundary)
2. Theories of gravity in $d+1$ spacetime dimensions (bulk)



Key relation [Witten, 98]

$$Z_{QFT}[\phi_0] = \int DA \exp\left(iS_{QFT} + i\int \phi_0 \mathcal{O}(A)\right) = e^{iS_{\text{bulk}}}|_{\phi\to\phi_0} \qquad (1)$$

But Why Holography?

- Need for a microscopic theory of superconductivity
- Since the CFT that governs the Superconductor is strongly coupled, perturbation theory and almost free particle descriptions are not valid
- The condensation that arises from holography it isn't a Cooper condensation. It's something more general
- One can use a holographic description in such cases since strongly coupled d- dimensional field theories are dual to weakly coupled (classical) gravitational theories asymptoticaly AdS in $D = d + 1$ dimensions.

AdS/CFT Dictionary

What are the minimal ingredients for a superconductor model?

∂AdS: CFT	Bulk AdS: Gravity
Global Symmetry	Gauge Symmetry
Temperature	Hawking Temperature
Chemical potential/charge density	Boundary values of the gauge field
Scalar operator $\mathcal{O}(x)$	scalar field $\phi(r, x)$
Energy - Momentum Tensor T_{ab}	Metric tensor $g_{\mu\nu}$
Global internal symmetry current J_a	Maxwell field A_μ

Holographic Superconductor

We will study the Einstein-Maxwell action with one scalar field minimally coupled in $3+1$ dimensions [Hartnoll-Herzog-Horowitz, 2008]

$$
\begin{aligned}
S_{Bulk} &= S_{gravity} + S_{matter} \\
&= \int d^4x\sqrt{-g}\left[\overbrace{\frac{R+\Lambda}{16\pi G}}^{\text{Einstein}} - \overbrace{\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}^{\text{Maxwell}} - \overbrace{|\nabla\psi - iqA\psi|^2 - m^2|\psi|^2}^{\text{Scalar}}\right]
\end{aligned}
\tag{2}
$$

- Minimization of S with respect to $g_{\mu\nu}$ $\Rightarrow$ Einstein field equations

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}, \quad T_{\mu\nu} = T^{(\psi)}_{\mu\nu} + T^{(EM)}_{\mu\nu}$$

where

$$
\begin{aligned}
T^{(\psi)}_{\mu\nu} &= D_\mu\psi(D_\nu\psi)^* + D_\nu\psi(D_\mu\psi)^* - g_{\mu\nu}(g^{ab}D_a\psi(D_b\psi)^* + m^2|\psi|^2) \\
T^{(EM)}_{\mu\nu} &= F_\mu{}^a F_{\nu a} - \frac{1}{4}g_{\mu\nu}F_{ab}F^{ab}
\end{aligned}
$$

- Minimization of S with respect to $A_{\mu\nu}$ $\Rightarrow$ Maxwell Equations

$$\nabla_\nu F^{\mu\nu} + g^{\mu\nu}[2q^2A_\nu|\psi|^2 + iq(\psi^*\nabla_\nu\psi - \psi\nabla_\nu\psi^*)] = 0$$

- Minimization of S with respect to ψ $\Rightarrow$ Klein-Gordon Equation

$$(\partial_\mu - iqA_\mu)[\sqrt{-g}g^{\mu\nu}(\partial_\nu - iqA_\nu)\psi] = \sqrt{-g}m^2\psi$$

The Probe Limit

Rescaling the fields:

$$A_\mu = \frac{\tilde{A}_\mu}{q}, \quad \psi = \frac{\tilde{\psi}}{q}$$

The action (2) becomes:

$$S = S_{gravity} + \frac{1}{q^2}\tilde{S}_{matter}$$

- In the limit $q \to \infty$ the second term becomes small. Then we can neglect that term and solve the field equations without the presence of matter. This is called the Probe limit.
- However, the interesting physics are captured by the interaction of the scalar field $\psi(r)$ with the gauge field A_μ.

We consider a planar Schwarzschild ADS black hole:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(dx^2 + dy^2), \quad f(r) = r^2 - \frac{r_h^3}{r}$$

Taking the symmetric ansatz $\psi = |\psi(r)|$, $A_x = A_y = A_r = 0$ and $A_t = \phi(r)$ equations Maxwell and Klein-Gordon become:

$$\psi'' + \left(\frac{f'}{f} + \frac{2}{r}\right)\psi' + \left(\frac{\phi^2}{f^2} - \frac{m^2}{f}\right)\psi = 0$$

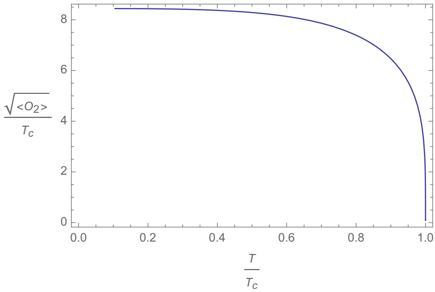
$$\phi'' + \frac{2}{r}\phi' - \frac{2|\psi|^2}{f}\phi = 0$$

It is important to assume the correct boundary conditions at infinity and the horizon, in order to get the correct correspondence. In the horizon we impose $\phi(r) = 0$. The asymptotic behaviour of the fields for $r \to \infty$ is:

$$\psi = \frac{\psi^{(1)}}{r} + \frac{\psi^{(2)}}{r^2}, \quad \phi = \mu - \frac{\rho}{r}$$

We choose $\psi^{(1)} = 0$. For the condensating operator: $< \mathcal{O}_2 >= \sqrt{2}\psi^{(2)}$.

Condensate



- We observe the condensation of the operator for a critical value of the temperature T_c.
- Same results with BCS theory.
- The approximation doesnt't work at low temperatures where the condesate is large.
- After fitting the curve at temperatures close to the critical temperature we find for the condensate, $< \mathcal{O}_+ > \approx 144 T_c^2 (1 - \frac{T}{T_c})^{\frac{1}{2}}$.
- The square root behaviour is typical for a second order transition.

Optical Conductivity

The next step is to compute the optical conductivity $\sigma(\omega)$ of the CFT. We need to solve for fluctuations of the vector potential A_x in the bulk. Taking $\delta A_x = e^{-i\omega t} A_x(r)$ with zero spatial momentum, the x component of the Maxwell equations gives:

$$A_x'' + \frac{f'}{f} A_x' + \left(\frac{\omega^2}{f^2} - \frac{2\psi^2}{f} \right) A_x = 0$$

The asymptotic behaviour of the Maxwell field is:

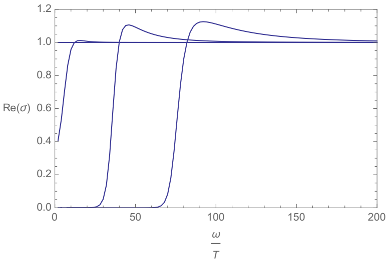
$$A_x = A_x^{(0)} + \frac{A_x^{(1)}}{r} + \dots$$

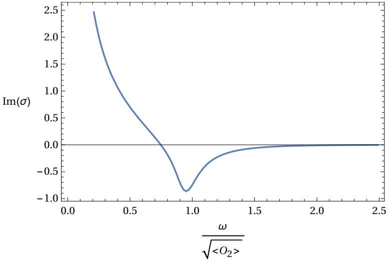
To compute causal behaviour, we will take ingoing wave boundary conditions at the horizon, $f^{-i\omega/3r_0}$. The AdS/CFT dictionary tells us that the dual sourse and expectation value for the current are given by:

$$A_{x,CFT} = A_x^{(0)}, \quad < J_{x,CFT} >= A_x^{(1)}$$

Now from Ohm's law we can obtain the conductivity:

$$\sigma(\omega) = \frac{< J_{x,CFT} >}{E_{x,CFT}} = -\frac{< J_{x,CFT} >}{\dot{A}_{x,CFT}} = -\frac{i < J_{x,CFT} >}{\omega A_{x,CFT}} = -\frac{i A_x^{(1)}}{\omega A_x^{(0)}}$$





- The subsequent curves describe successively lower values of the temperature (for fixed charge density).
- From the real part of the conductivity we observe the creation of the energy gap Δ.
- The real part must have a delta function in the limit $\omega \to 0$, which doesn't appear in our numerical results. Although, due to the Kramers-Kronig relations:

$$Im[\sigma(\omega)] = -\frac{1}{\pi} P \int_{\infty}^{\infty} \frac{Re[\sigma(\omega')]}{\omega' - \omega} d\omega'$$

  The pole of the imaginary part $\frac{1}{\omega}$ is equivalent with a delta function for the real part.
- Fitting the curve we can obtain for the energy gap, $2\Delta \approx 8.4T_c$
- The theoretical estimation of the BCS theory is $2\Delta \approx 3.54T_c$

Remarks

- This simple model for the holographic superconductor gave good results, which agree with the experimental data.
- However, the value for the energy gap is almost two times that which is obtained from the BCS theory.
- In real materials, the energy gap is decreased with the introduction of paramagnetic impurities.
- There is need for a dual description of these impurities with Ads/CFT correspondence.

Non Minimal Model

Now we consider the action [Papantonopoulos-Xiao Mei, 2016]

$$S_{bulk} = S_{gravity} + S_{matter} =$$
$$\int d^4x\sqrt{-g}\left[\frac{R+\Lambda}{16\pi G} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - (g^{\mu\nu} + kG^{\mu\nu})D_\mu\psi(D_\nu\psi)^* - m^2|\psi|^2\right]$$

In this case the scalar field is coupled with the Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$. Puting:

$$\begin{aligned} \Phi_{\mu\nu} &\equiv D_\mu\psi(D_\nu\psi)^* \\ \Phi &\equiv g^{\mu\nu}\Phi_{\mu\nu} \\ C^{\mu\nu} &\equiv g^{\mu\nu} + kG^{\mu\nu} \end{aligned}$$

The Einstein field equations are:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}, \quad T_{\mu\nu} = T^{(\psi)}_{\mu\nu} + T^{(EM)}_{\mu\nu} + k\Theta_{\mu\nu}$$

where:

$$\Theta_{\mu\nu} = -g_{\mu\nu}R^{ab}\Phi_{ab} + R_\nu{}^a(\Phi_{\mu a} + \Phi_{a\mu}) + R_\mu{}^a(\Phi_{a\nu} + \Phi_{\nu a}) - \frac{1}{2}R(\Phi_{\mu\nu} + \Phi_{\nu\mu})$$
$$- G_{\mu\nu}\Phi - \frac{1}{2}\nabla^a\nabla_\mu(\Phi_{a\nu} + \Phi_{\nu a}) - \frac{1}{2}\nabla^a\nabla_\nu(\Phi_{\mu a} + \Phi_{a\mu}) + \frac{1}{2}\Box(\Phi_{\mu\nu} + \Phi_{\mu\nu})$$
$$+ \frac{1}{2}g_{\mu\nu}\nabla_a\nabla_b(\Phi^{ab} + \Phi^{ba}) + \frac{1}{2}(\nabla_\mu\nabla_\nu + \nabla_\nu\nabla_\mu)\Phi - g_{\mu\nu}\Box\Phi$$

Maxwell Equations now have the following form:

$$\nabla_\nu F^{\mu\nu} + C^{\mu\nu}[2q^2 A_\nu |\psi|^2 + iq(\psi^* \nabla_\nu \psi - \psi \nabla_\nu \psi^*)] = 0$$

and the Klein-Gordon equation

$$(\partial_\mu - iqA_\mu)[\sqrt{-g}C^{\mu\nu}(\partial_\nu - iqA_\nu)\psi] = \sqrt{-g}m^2\psi$$

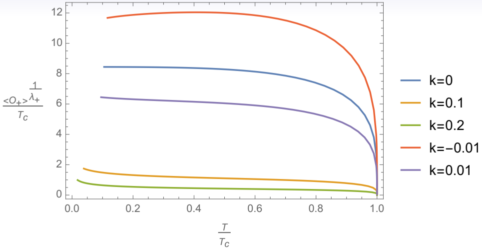
Following the same process as before, the new equations which we have to numerically solve are:

$$\left(1 + k\left(\frac{f}{r^2} + \frac{f'}{r}\right)\right)\psi'' + \left[\frac{2}{r} + \frac{f'}{f} + k\left(\frac{3f'}{r^2} + \frac{(f')^2}{rf} + \frac{f''}{r}\right)\right]\psi' + \left[\frac{\phi^2}{f^2}\left(1 + k\left(\frac{f}{r^2} + \frac{f'}{r}\right)\right) - \frac{m^2}{f}\right]\psi = 0$$

and

$$\phi'' + \frac{2}{r}\phi' - \frac{2|\psi|^2}{f}\left[1 + k\left(\frac{f'}{r} + \frac{f}{r^2}\right)\right]\phi = 0$$

Notice that now the equations depend on the second derivative of f.



k	-0.01	0	0.01	0.1	0.5
T_c	0.1218	0.1184	0.1158	0.1043	0.09053
C_1	243	140	73	2	0.01

- With the introduction of the new coupling k we observe that the strength of the condensation decreases
- The critical temperature decreases as well

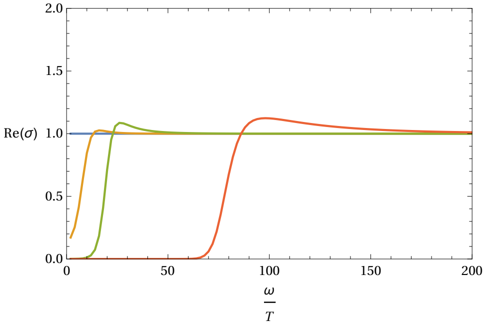
Fitting the curve for temperatures near the critical temperature gives:

$$< \mathcal{O}_+ > \approx C_1(k) T_c^2 (1 - \frac{T}{T_c})^{\frac{1}{2}} \quad (3)$$

Optical Conductivity

Again for the calculation of the optical conductivity we will find the fluctuations of the A_x component. The corresponding Maxwell equation now has the form:

$$A_x'' + \frac{f'}{f} A_x' + \left[\frac{\omega^2}{f^2} - \frac{2|\psi|^2}{f} \left(1 + k \left(\frac{f''}{2} + \frac{f'}{r} \right) \right) \right] A_x = 0$$

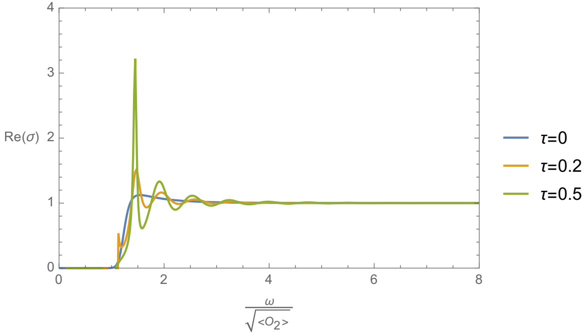


Conclusions

- By introducing the coupling k we observe a decrease in the strength of the condensation and the critical temperature. The decrease in the temperature means that the phase transition is hard to occur.
- Increasing the coupling, the condensation decreases faster than the temperature, so the energy gap tends to 0 for low temperatures.
- The order of the phase transition doesn't change in this case.
- Fitting the curve for high coupling, $k = 0.5$, we find that $2\Delta \approx 4T_c$, reproducing the same relation as the BCS theory.
- This means that the coupling k can be interpeted as the dual description of the paramagnetic impurities on real materials.

Future Study

- One characteristic of this model is it's symmetry to spatial translations.
- But real materials doesn't have this symmetry.
- So we must break that symmetry. We achieve that by giving the scalar field a spatial dependence $\phi = \phi(r, x)$, with $\nabla_x^2 \phi = -\tau^2 \phi$.



- We notice the appearance of some peaks for $\omega \neq 0$ at the real part of the conductivity.
- These peaks signal the creation of new charged degrees of freedom that contribute to the conductive current.
- The energy of these new degrees can be found from the distance between the peak and the energy gap.
- They appear even when $k = 0$. So they don't have any relation to the paramagnetic impurities.
- There is need for additional work in order to understand the nature of these peaks.
- The main idea for future work involves studying the normal phase of the superconductor with holography.