Cosmological Implications in Massive Gravity

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[with S. Mukohyama, J.P. Uzan, M. Oliosi, arXiv: 1506.01594/1512.04008/1607.03368/1701.01581/1702.04490/1709.03108/1702.04490]

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Introduction

- Three conditions for good alternative theories of gravity
- Theoretically consistent: e.g. no ghost instability
- Experimentally viable: solar system / table top experiments
- Predictable: e.g. no strong coupling

IR modifications?

- Gravity at long distances
- Dark matter?
- Dark energy?
- New phenomenology: late-time acceleration, clustering, flattening galaxy rotation curves, GW‘s…

Massive gravity/bigravity

- Change gravity at IR
- Give the graviton a mass (Fierz-Pauli , 1939)
- Theory found in 2010: dRGT model

de Rham, Gabadadze, Tolley 2010
Hassan, Rosen, 2012; Kugo, Ohta 2014

- dRGT → MTMG → MQD...

ADF, Mukohyama
arXiv: 1506.01594, 1512.04008

- HR bigravity...

Hassan, Rosen 2012

[Hassen, Rosen: 2012; Kugo, Ohta:]

Problems for cosmology

- FLRW is unstable for dGRT: no stable homogeneous and isotropic cosmology ADF, Gumrukcuoglu, Mukohyama, 2012
- Higuchi bound for bigravity → Comelli, Crisostomi, Nesti, Pilo,2012
  ADF, Nakamura, Tanaka, 2014
  ADF, Gumrukcuoglu, Mukohyama, Tanahashi, Tanaka 2014
- Trying to avoid problems by changing the theory
- Trying to go beyond FLRW

  D'Amico, de Rham, Dubovsky, Gabadadze, Pirtskhalava, 2012
  Gumrukcuoglu, Lin, Mukohyama, 2012

Beyond dRGT/bigravity

- How to solve dRGT-bigravity problems?
- Many attemps
- Quasi-dilatonic theories (making fiducial metric dynamical)

  D'Amico, Gabadadze, Hui, Pirtskhalava, 2012
- Exploring composite metrics

  De Rham, Heisenberg, Ribeiro, 2014

  Gumrukcuoglu, Heisenberg, Mukohyama, Tanahashi, 2014
- Exploring partially constrained formalism

  ADF, Gumrukcuoglu, Heisenberg, Mukohyama, 2015

Bigravity → Higuchi bound: an obstacle

- Fierz-Pauli theory on de Sitter (Higuchi 1987):
  if $H^2 > m_T^2$ → helicity-0 ghost
- Same for dRGT massive gravity & bigravity on de Sitter
- Generic FLRW: if $H^2 > O(1) \times m_T^2$ → helicity-0 ghost
- If $m_T^2 \sim H_0^2$ today → need a UV completion to describe the early universe

Chameleon

(Khoury & Weltman 2004)

- Non-minimal coupling

$$\mathcal{L} = \sqrt{-g}\left\{-\frac{M_{\rm Pl}^2\mathcal{R}}{2} + \frac{(\partial\phi)^2}{2} + V(\phi)\right\} + \mathcal{L}_m(\psi^{(i)}, g_{\mu\nu}^{(i)})$$

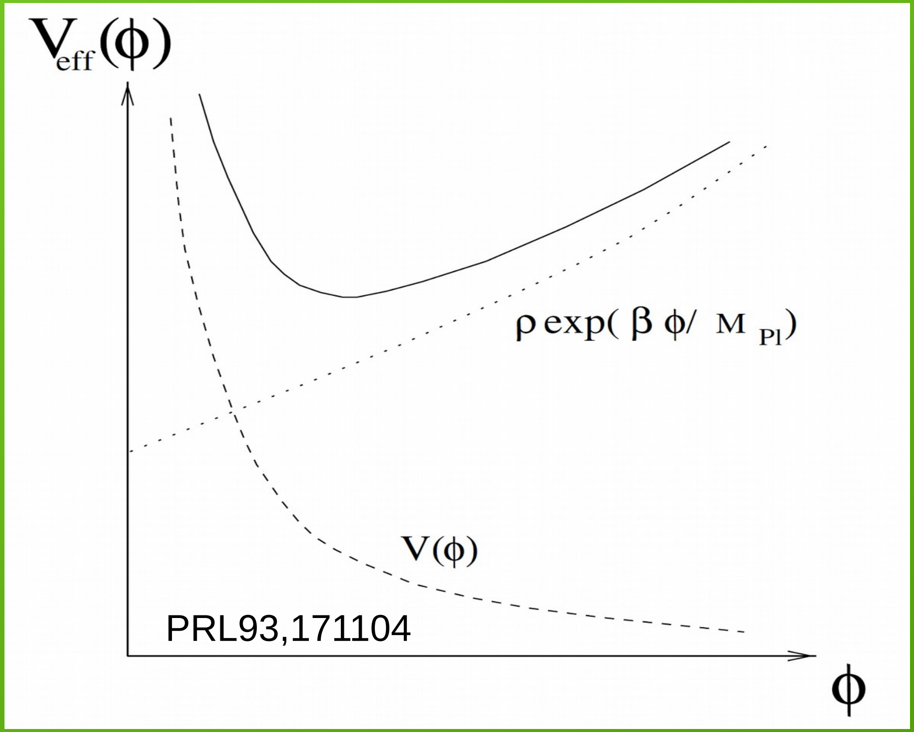
$$g_{\mu\nu}^{(i)} = \exp(2\beta_i\phi/M_{\rm Pl})g_{\mu\nu}$$

- Effective potential

$$V_{\rm eff}(\phi) \equiv V(\phi) + \rho e^{\beta\phi/M_{\rm Pl}}$$

- Screening 5th force

ϕ gets heavy in dense environment



From Mukohyama san

Chameleon bigravity

- Bigravity action

ADF, Mukohyama, Uzan 1702.04490

$$S=\frac{M_g^2}{2}\int R[g]\sqrt{-g}\mathrm{d}^4x+\frac{M_f^2}{2}\int R[f]\sqrt{-f}\mathrm{d}^4x+M_g^2m^2\int\sum_{i=0}^{4}\beta_i U_i[s]\sqrt{-g}\mathrm{d}^4x$$

$$s^\mu_\alpha s^\alpha_\nu=g^{\mu\alpha}f_{\alpha\nu}$$

- Promoting β_i to functions of ϕ

$$\beta_i=\beta_i(\phi)$$

$$\beta_i=-c_i e^{-\lambda\phi/M_g}$$

- Non-minimal coupling of matter

$$\tilde{g}_{\mu\nu}=A^2(\phi)g_{\mu\nu}$$

$$S_{\mathrm{mat}}=\int\mathcal{L}_{\mathrm{mat}}(\psi,\tilde{g}_{\mu\nu})\sqrt{-\tilde{g}}\,\mathrm{d}^4x$$

$$A=e^{\beta\phi/M_g}$$

- Adding kinetic term of ϕ

$$S_{\mathrm{kin}}=-\frac{1}{2}\int g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\sqrt{-g}\mathrm{d}^4x$$

From Mukohyama san

Working on de Sitter

- On de Sitter $\xi = \tilde{a}/a$ is a function of params: e.g. c_i, $\kappa = M_f^2/M_g^2$
- The mass of the tensor modes $\frac{m_T^2}{H^2} = \frac{3(1+\kappa\xi^2)(c_1+2c_2\xi+c_3\xi^2)}{c_1+3c_2\xi+3c_3\xi^2+c_4\xi^3}$
- But $H^2 \propto \rho \rightarrow m_T^2 \propto \rho$
- It is possible to satisfy the Higuchi bound.
- Found stable de Sitter with nice property.

On FLRW

- Found stable RD scaling solution with same property.
- It becomes possible to satisfy the Higuchi bound
- New possible phenomenology for bigravity

Intro to MTMG

[ADF, S. Mukohyama: arXiv:1506.01594; PLB752 2016]

- 2 physical dof only = massive gravitational waves
- exactly same FLRW background as in dRGT
- no BD ghost, no Higuchi ghost, no nonlinear ghost
- Three steps:
- 1. Fix local Lorentz to realize ADM vielbein in dRGT
- 2. Switch to Hamiltonian
- 3. Add 2 additional constraints

Cosmology of MTMG I

- Background constraint $(c_3+2c_2X+c_1X^2)(\dot{X}+NHX-MH)=0, \quad X=\tilde{a}/a$

$$X=\frac{-c_2\pm\sqrt{c_2^2-c_1c_3}}{c_1},$$

$$3M_P^2H^2=\frac{m^2M_P^2}{2}(c_4+3c_3X+3c_2X^2+c_1X^3)+\rho$$

- Λ_{eff} from graviton mass term (even when c_4 = 0)
- Scalar/vector equal to ΛCDM
- Time-dependent mass for the gravity waves

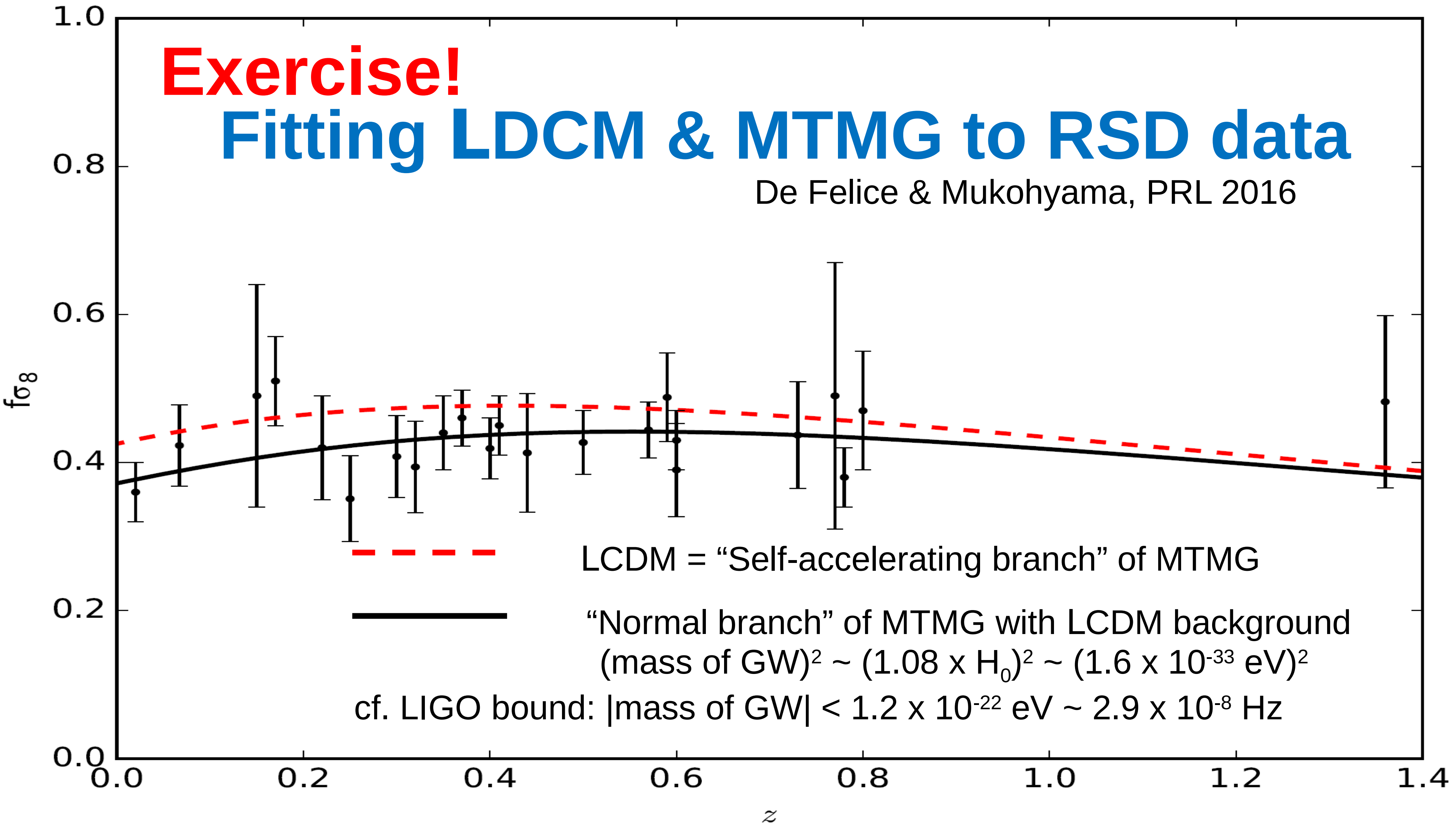
Cosmology of MTMG II

- Background constraint $\left(c_3+2c_2X+c_1X^2\right)\left(\dot{X}+NHX-MH\right)=0, \quad X=\tilde{a}/a$

$$H=XH_f, \quad H_f=M^{-1}\dot{\tilde{a}}/\tilde{a},$$

$$3M_P^2H^2=\frac{m^2M_P^2}{2}\left(c_4+3c_3X+3c_2X^2+c_1X^3\right)+\rho$$

- Dark component without extra dof
- Scalar part recovers GR in UV (L << 1/m) but not GR when L >> 1/m
- Non-zero mass for the gravity waves


Exercise!
Fitting LDCM & MTMG to RSD data
De Felice & Mukohyama, PRL 2016
LCDM = “Self-accelerating branch” of MTMG
“Normal branch” of MTMG with LCDM background
(mass of GW)^2 ~ (1.08 x H_0)^2 ~ (1.6 x 10^{-33} eV)^2
cf. LIGO bound: |mass of GW| < 1.2 x 10^{-22} eV ~ 2.9 x 10^{-8} Hz
$f\sigma_8$
z
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

Part III: Minimal Quasidilaton

- Normal branch of MTMG had interesting phenomenology
- Make fiducial metric dynamical by introducing a field
- We want the theory to have only 2 tensor and 1 scalar-QD
- In order to get rid of dRGT strong coupling / ghosts
- To make phenomenology simpler

MQD – step 1: precursor theory

[ADF, S. Mukohyama, M. Oliosi 2017]

- Total Lagrangian

$$\mathscr{L}_{pre} = \mathscr{L}_{EH} - \frac{1}{2} M_P^2 m^2 \sum_{n=0}^{4} c_n \mathscr{L}_n + \mathscr{L}_\sigma$$

$$\mathscr{L}_\sigma = -\omega N \sqrt{\gamma} \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma$$

- Mass term in ADM-vielbein form
- Precursor because MQD massive term is present
- Needs to be extended to give viable phenomenology

Precursor action

- Satisfies symmetry $\sigma \to \sigma + \sigma_{0,}\ \ \phi^0 \to e^{-(1+\alpha)\sigma_0/M_P}\phi^0,\ \ \phi^p \to e^{-\sigma_0/M_P}\phi^0,$
- Build 3D metric, lapse, shift $\gamma_{ij}, N, N^i, \quad \tilde{\gamma}_{ij}, M$
- Mass term in unitary gauge

$$\mathcal{L}_0 = \sqrt{\tilde{\gamma}}\, M e^{(4+\alpha)\sigma/M_p},$$

$$\mathcal{L}_1 = \sqrt{\tilde{\gamma}}\, e^{3\sigma/M_p}\left(N + M e^{\alpha\sigma/M_p} K\right), \quad K^i{}_l K^l{}_j = \tilde{\gamma}^{il}\gamma_{lj}, \quad K = K^i{}_i,$$

$$\mathcal{L}_2 = \sqrt{\tilde{\gamma}}\, e^{2\sigma/M_p}\left[N K + \frac{M}{2} e^{\alpha\sigma/M_p}\left(K^2 - K^i{}_j K^j{}_i\right)\right],$$

$$\mathcal{L}_3 = \sqrt{\gamma}\, e^{\sigma/M_p}\left(N k + M e^{\alpha\sigma/M_p}\right), \quad k^i{}_l K^l{}_j = \delta^i{}_j, \quad k = k^i{}_i,$$

$$\mathcal{L}_4 = N\sqrt{\gamma}.$$

Degrees of freedom for precursor action

- We consider γ_{ij}, σ as the dynamical variables
- N, N^i Lagrange multipliers
- Unitary gauge. Fiducial metric elements: $M=1, \tilde{\gamma}_{ij}=\delta_{ij}, \tilde{\gamma}^{ij}=\delta^{ij}$
- In phase space there are 2*6 + 2 = 14 variables
- Hamiltonian formalism

Precursor Hamiltonian

- Primary Hamiltonian

$$H_{pre}^{(p)} = \int d^3x \left[-N R_0 - N^i R_i + \frac{M_p^2}{2} m^2 M H_1 \right]$$

where

$$R_0 = R_0^{GR} - \frac{M_p^2}{2} m^2 H_{0,} \quad R_i = 2\gamma_{ik} D_j \pi^{kj} - \pi_\sigma \partial_i \sigma,$$

$$R_0^{GR} = R_0^{EH} - \sqrt{\gamma} \left[\frac{\tilde{\pi}_\sigma^2}{2\omega} + \frac{\omega}{2} \sigma^{;i} \sigma_{;i} \right], \quad \tilde{\pi}_\sigma = \frac{\pi_\sigma}{\sqrt{\gamma}},$$

$$H_0 = \sqrt{\tilde{\gamma}} \left(c_1 e^{3\sigma/M_p} + c_2 e^{2\sigma/M_p} K \right) + \sqrt{\gamma} \left(c_3 e^{\sigma/M_p} k + c_4 \right),$$

$$H_1 = e^{\alpha\sigma/M_p} \sqrt{\tilde{\gamma}} \left[c_0 e^{4\sigma/M_p} + c_1 e^{3\sigma/M_p} K + \frac{1}{2} c_2 e^{2\sigma/M_p} \left(K^2 - K^i{}_j K^j{}_i \right) \right] + c_3 \sqrt{\gamma}\, e^{(1+\alpha)\sigma/M_p}.$$

Secondary precursor Hamiltonian

- Time derivative of the primary constraints
- Look for secondary constraints
- Only two other secondary contraints: $\tilde{C}^{\sigma}, \quad \sigma=1,2$

$$\dot{R}_0=\{R_0, H^{(p)}\}\approx 0, \quad \dot{R}_i=\{R_i, H^{(p)}\}\approx 0,$$

$$\{R_0, R_0\}\approx 0, \quad \{R_i, R_j\}\approx 0, \quad \{R_0, R_i\}\neq 0$$

$$H_{pre}^{(s)}=\int d^3x\left[-N R_0-N^i R_i+\frac{M_p^2}{2}m^2 M H_1+\tilde{\lambda}^{\sigma}\tilde{C}_{\sigma}\right]$$

Dof for precursor theory

- No more (tertiary) constraints
- All constraints are of second class
- Therefore 6 s.c. constraints: $R_0 , \; R_i , \; \tilde{C}^{\sigma}$
- Therefore (7 * 2 – 6) / 2 = 4 modes remaining
- In fact, Gws + 2 scalar modes, on normal branch
- 1 scalar possibly strongly coupled/ghost

Removing one more scalar dof

- Consider that $$H_{pre}^{(s)} \approx \bar{H}_1 \equiv \frac{M_p^2}{2} \int d^3x\, m^2 M H_1$$
- Consider now

$$\frac{M_p^2}{2} C_0 \equiv \{R_0, \bar{H}_1\},$$

$$\frac{M_p^2}{2} C_i \equiv \{R_i, \bar{H}_1\}$$

- $\tilde{C}_\sigma \approx$ two linear combinations of C_i

Theory of minimal quasidilaton

- The theory is defined by imposing 4 constraints (instead of 2)

$$C_0 \approx 0, \quad C_i \approx 0$$

- Hamiltonian

$$H = \int d^3x \left[-N R_0 - N^i R_i + \frac{M_p^2}{2} m^2 M H_1 + \frac{M_p^2}{2} \left(\lambda C_0 + \lambda^i C_i \right) \right]$$

- 8 second class constraints
- (7 * 2 – 8) / 2 = 3 dof

Lagrangian of the theory

- Legendre transformation
- Metric formalism

$$\mathscr{L}_{QD}=\mathscr{L}_{PRE}+\frac{M_p^2}{2}N\sqrt{\gamma}\left(\frac{m^2 M\lambda}{4N}\right)^2\left[\left(\gamma_{ik}\gamma_{jl}-\frac{1}{2}\gamma_{ij}\gamma_{kl}\right)\Theta^{ij}\Theta^{kl}+\frac{M_p^2}{2}\frac{8}{\omega}\left(\frac{1}{\sqrt{\gamma}}\frac{\partial H_1}{\partial\sigma}\right)\right]$$

$$-\frac{M_p^2}{2}\left(\lambda\bar{C}_0+\lambda^i C_i\right),\quad \bar{C}_0=C_0(\lambda=0),$$

$$\Theta^{ij}=e^{\alpha\sigma/M_p}\left[\frac{\sqrt{\tilde{\gamma}}}{\sqrt{\gamma}}\left\{c_1 e^{3\sigma/M_p}\left(\gamma^{il}K^j{}_l+\gamma^{jl}K^i{}_l\right)+c_2 e^{2\sigma/M_p}\left[K\left(\gamma^{il}K^j{}_l+\gamma^{jl}K^i{}_l\right)-2\tilde{\gamma}^{ij}\right]\right\}+2c_3 e^{\sigma/M_p}\gamma^{ij}\right]$$

$$K^i{}_j\equiv\left(\sqrt{\tilde{\gamma}^{-1}\gamma}\right)^i{}_j$$

What is the physical content?

- Let us study the de Sitter/Minkowski background
- Since $\dot{R}_0 = \{R_0, H\} = C_0 + \int dy\, \lambda \{R_0, C_0(y)\} + \dots$

  $\lambda(t) = 0$ on the background.
- Same result on combining eoms in Lagrangian formalism
- Giving FLRW ansatz

  $g_{00} = N(t)^2, \;\; g_{ij} = a(t)^2 \delta_{ij}, \;\; \sigma = \sigma(t), \;\; X \equiv e^{\sigma/M_p} a_f / a, \;\; r \equiv (M/N) e^{(1+\alpha)\sigma/M_p} / X,$
- Late time attractor

$$\frac{1}{N}\frac{d}{dt}[a^{4+\alpha} X^{1+\alpha} J] = 0, \;\; J \equiv c_0 X^3 + 3 c_1 X^2 + 3 c_2 X + c_3$$

Background

- For $\alpha \neq 4, X \neq 0$ the theory has J(X) = 0 attractor solution
- On this solution X = constant, and independent eoms

$$(6-\omega) H^2 = m^2 (c_4 + 3 c_3 X + 3 c_2 X^2 + c_1 X^3), \quad H \equiv \frac{\dot{a}}{N a}, \quad \omega < 6,$$

$$r - 1 = \frac{2\omega}{6-\omega} \frac{c_4 + 3 c_3 X + 3 c_2 X^2 + c_1 X^3}{X (c_1 X^2 + 2 c_2 X + c_3)}$$

- $c_4 = 0$ self-acceleration solution

Perturbations

- One scalar perturbation, $\delta\sigma$
- No vector perturbations
- 2 tensor perturbations do exist

$$S=\frac{M_P^2}{8}\sum_\sigma \int d^4x\, N(t)\, a(t)^3\left[\frac{\dot{h}_\sigma^2}{N^2}-\frac{(\partial h_\sigma)^2}{a^2}-\mu^2 h_\sigma^2\right],$$

$$\mu^2=\frac{[\alpha+6-(\alpha+4)r]\,\omega H^2}{2(r-1)},\ \ \alpha\neq 4$$

Scalar mode reduced action

- One single field propagating
- Imposing $c_s^2 = 1$ reduces parameter space, simplifies no-ghost condition

$$S=\frac{1}{2}\int d^4x\, N a^3 \omega \left[\left(\frac{1}{N}\frac{\partial(\delta\sigma)}{\partial t}\right)^2-\frac{1}{a^2}[\partial_i(\delta\sigma)]^2-\nu_{dS}^2 H^2(\delta\sigma)^2\right]$$

- $\nu_{dS}=\nu_{dS}(\omega,\alpha,r)$
- Look for available parameter space

Horndeski extension to MQD

- In MQD, one single field propagating (besides matter)
- We still need to screen this field in solar system
- We can implement the Vainshtein mechanism
- We need at least a G3 term

Horndeski extension to MQD: recipe

- Add dRGT-ADM term to the Horndeski G3 action
- Find resulting precursor Hamiltonian
- Add two extra consraints
- Find back the Lagrangian of EMQD

Horndeski extension to MQD

- Starting point Horndeski G3 action

$$S=\int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R+P(X)-G(X)\nabla^2\sigma\right],\quad X=-\frac{1}{2}(\partial\sigma)^2$$

- It can be rewritten as

$$S=\int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R+P(X)+\chi\left(X+\frac{1}{2}(\partial\sigma)^2\right)+S\,[G(X)-\theta]+g^{\mu\nu}\partial_\mu\theta\partial_\nu\sigma\right],$$

Constraints

- Primary constraints

$$R_0, R_i, P_X, P_\chi, P_S$$

- From time-derivative of primary constraints we also have 5 secondary constraints S_X, S_χ, S_S, C_τ
- From time-derivative of constraints one finds one more tertiary constr., T
- Only one first class constraint $P_S + G_{,X} P_\chi$
- Dof: [(6+5)*2 – (7 + 5)*1 – 1*2]/2 = [22 – 12 – 2]/2 = 8/2 = 4

Minimal theory extension

- Add two second class constraints in addition to C_τ

$$\{R_i, H_1\} = \frac{M_P^2}{2} C_i, \quad \{R_0, H_1\} = \frac{M_P^2}{2} C_0$$

- From total Hamiltonian go back to the Lagrangian
- EMQD constructed
- We checked de Sitter stability and reduction to MQD
- Only 3 dof propagating

Conclusions – II

- Three [steps] to the Minimal Theory(ies)
- 1. Fix local Lorentz by introducing precursor mass Lagrangian
- 2. Switch to Hamiltonian
- 3. Add 2 additional constraints. 3 ½: Go back to the Lagrangian
- Stable de Sitter for simplest model of quasidilaton in MTMG
- Phenomenology now accessible