# Inflation and reheating predictions for gravity theories with derivative coupling

**Ioannis Dalianis**
(Natl. Tech. U. Athens)
Work in collaboration with
**G. Koutsoumbas**, **K. Ntrekis** and **E. Papantonopoulos**

JCAP 1702 no.02, 027

# Non-Minimal Derivative Coupling to $G_{\mu\nu}$

## Description

- *It is a* ***scalar-tensor*** *theory*
- *It is a gravity theory which includes in the action, apart from the minimal coupling of the scalar field to gravity, a non minimal* ***a derivative coupling*** *(NMDC) of the scalar to Einstein tensor*
- *It is subset of the* ***Horndeski*** *theories*

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}M_P^2 R - \frac{1}{2}\left(g^{\mu\nu} - \frac{G^{\mu\nu}}{\tilde{M}^2}\right)\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

## Applications

- *The derivative coupling of the scalar field to Einstein tensor introduces a new scale $\tilde{M}$ in the theory which has interesting implications on*
  ***Short distances*** *for the black hole physics [Kolyvaris+, Rinaldi+, Koutsoumbas+, Vaso Zanni, Ntrekis(Talks)],*
  ***Late universe*** *cosmology [Saridakis+] ,*
  ***Inflation*** *[Germani & Kehagias '10, ID & Farakos '14, Review: Tsujikawa '12]*

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# Inflation and the NMDC

## The attractive feature of NMDC

- *The derivative coupling (NMDC) acts as a friction mechanism assisting in the realization of the inflationary slow-roll phase*

## New (viable) inflaton candidates

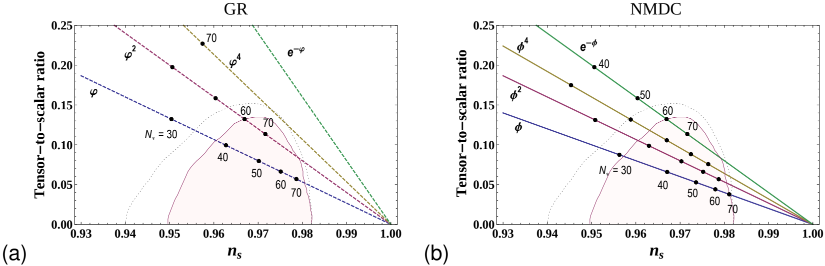
- Observationally ruled out prototype inflationary models such as the
  - $\varphi^2$ , $\varphi^4$

  can successfully fit the last observational data thanks to the NMDC
- *Also potentials too steep to drive an accelerating expansion in General Relativity (GR)*
- Some very interesting non-inflating examples:
  - *the Standard Model Higgs potential,* $V(\varphi) = \lambda\varphi^4$,
  - *the pseudo-Nambu-Goldstone potential,* $V \propto (1 - \cos(\phi/f))$ *with* $f < M_{Pl}$
  - *the exponential potential,* $V \propto e^{-\lambda_e \phi/M_{Pl}}$ *with* $\lambda_e > 1$

*Observational tests on inflationary models with NMDC [Tsujikawa '12, '13]*

# Inflation with NMDC

The theoretical predictions $r = r(n_s)$ for a canonical scalar inflaton with Einstein gravity and $V(\varphi) \propto \varphi,\ \varphi^2,\ \varphi^4,\ e^{-\varphi}$ and the theoretical prediction $r = r(n_s)$ for an inflaton with NMDC coupling and $V(\phi) \propto \phi,\ \phi^2,\ \phi^4,\ e^{-\phi}$. Apparently, the models with NMDC fit better the data.



# Theoretical consolidation of the NMDC

## The new mass scale $\tilde{M}$

- When $H \gg \tilde{M}$ the NMDC rules the evolution of the non-minimally coupled scalar and modifies the inflationary dynamics
- When $H \ll \tilde{M}$, the non-minimal coupled-scalar behaves approximately as a standard canonical field.

## Several good features:

- *The presence of the NMDC does not enhance the non-Gaussian fluctuations of the scalar field [German 2011, +]*
- *The non-minimally coupled graviton-inflaton system remains in the weak coupling regime during inflation [Germani & Watanabe '11]*
- *The quantum corrections are also found to be suppressed [Germani+, Tsujikawa+]*
- *The NMDC has been supersymmetrized in the framework of the new-minimal supergravity [Farakos+ '12 ]*
- *Inflationary potentials in supergravity can be consistently generated via the gauge-kinematic function [ID & Farakos '14 ]*

# NMDC inflationary observables

- The equation of motion (EOM) for the NMDC $\phi$ field is

$$\ddot{\phi}\left(1+3\frac{H^2}{\tilde{M}^2}\right)+3H\dot{\phi}\left(1+3\frac{H^2}{\tilde{M}^2}+\frac{2\dot{H}}{\tilde{M}^2}\right)+V'(\phi)=0\,.$$

- The expressions for the slow-roll parameters get modified

$$\epsilon=\frac{\epsilon_V}{1+3H^2\tilde{M}^{-2}}\,,\qquad \eta=\frac{\eta_V}{1+3H^2\tilde{M}^{-2}}\,,$$

- The the spectral tilt of the scalar power spectrum written in terms of the slow roll parameters

$$1-n_s\equiv\frac{d\ln\mathcal{P}_\zeta}{d\ln k}\simeq\frac{1}{1+3H^2\tilde{M}^{-2}}\left[2\epsilon_V\left(4-\frac{1}{1+3H^2\tilde{M}^{-2}}\right)-2\eta_V\right]$$

- When $H\tilde{M}^{-1}\gg 1$, we have the so called *high friction limit* and we get the simplified result

$$1-n_s\simeq 8\epsilon-2\eta\,,\qquad r=16\epsilon$$

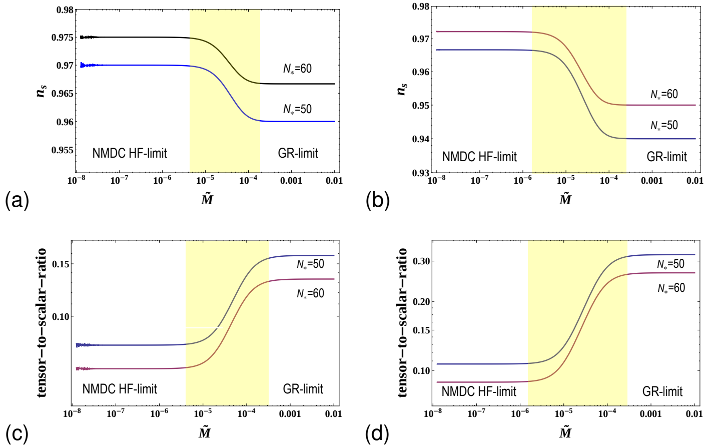
# The $\tilde{M}$ scale and the NMDC observables

For the large field models where the characteristic energy scale for inflation is $H_{\rm inf} \sim 10^{-5} M_{\rm Pl}$ **we can distinguish three cases according to the $\tilde{M}$ value.**

- $10^{-3} M_{\rm Pl} \lesssim \tilde{M}$ (**GR-limit**)
  *The NMDC plays essentially no role during the observed inflationary period.*
- $10^{-6} M_{\rm Pl} < \tilde{M} \lesssim 10^{-3} M_{\rm Pl}$ (**Intermediate region**)
  The NMDC modifies the inflationary dynamics and predictions but becomes negligible during the reheating period.
- $10^{-\beta} M_{\rm Pl} < \tilde{M} \lesssim 10^{-6} M_{\rm Pl}$ (**High Friction-limit**)
  *The NMDC modifies both the inflationary and reheating dynamics. The limit values for the power $\beta$, estimated by the CMB normalization, depend on the model. It is roughly $\beta = 14, 9, 8$ for the linear, quadratic and quartic potential respectively.*

# NMDC and the inflationary observables

*The CMB observables $n_s$ and tensor-to-scalar-ratio $r$ change with respect to the $\tilde{M}$ value for the potentials $V = m_\phi^2\phi^2/2$ (left panels) and $\lambda_\phi\phi^4$ (right panels). There are three distinct regions according to the $\tilde{M}$ value where the NMDC is dominant (NMDC HF-limit), subdominant (GR-limit), and the intermediate region (light-yellow region).*



# The CMB normalization

For $V(\phi) = \lambda_p M_{\rm Pl}^4 \left(\frac{\phi}{M_{\rm Pl}}\right)^p$

The power spectrum, measured at the scale $k_*$ by the CMB observational probes, reads in the NMDC case

$$\lambda_p(N_*, \tilde{M}) = \frac{24\pi^2 c_s\, p}{2(p+2)N_* + p}\, \frac{M_{\rm Pl}^p}{\left[\frac{2p(p+2)N_* + p^2}{2\lambda_p} \tilde{M}^2 M_{\rm Pl}^p\right]^{p/(p+2)}}\, \mathcal{P}_\zeta\,,$$

Examples

- Quadratic Potential

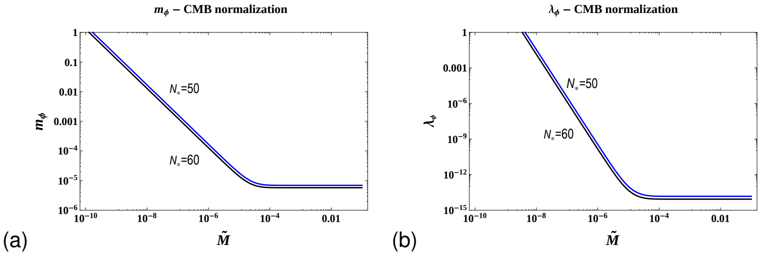
$$m_\phi(N_*, \tilde{M}) = 1.7 \times 10^{-10} \left(\frac{50}{N_*}\right)^{3/2} \left(\frac{\mathcal{P}_\zeta}{2 \times 10^{-9}}\right) \frac{M_{\rm Pl}^2}{\tilde{M}}\,,$$

- Quartic Potential

$$\lambda_\phi(N_*, \tilde{M}) \simeq 2.3 \times 10^{-32} \left(\frac{50}{N_*}\right)^5 \left(\frac{\mathcal{P}_\zeta}{2 \times 10^{-9}}\right) \left(\frac{M_{\rm Pl}}{\tilde{M}}\right)^4\,,$$

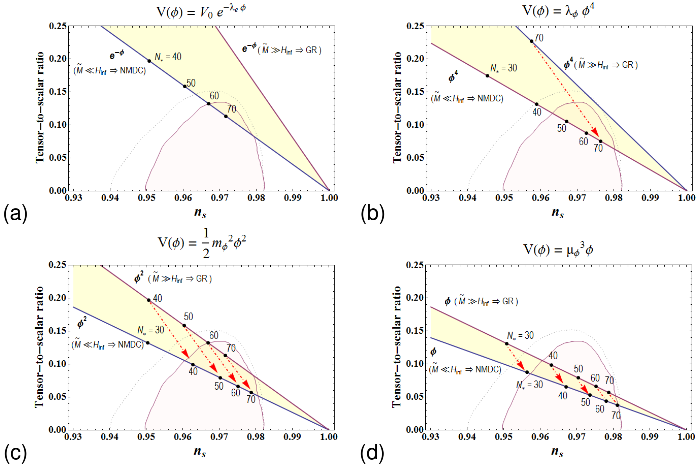
# The CMB normalization and the $\tilde{M}$ mass scale

*The CMB normalized values for the $m_\phi$ (left panel) and $\lambda_\phi$ (right panel), with respect to the $\tilde{M}$ value for the potentials $V = m_\phi^2 \phi^2/2$ and $\lambda_\phi \phi^4$ and inflaton with NMDC. Remarkably large values for the $m_\phi$ and $\lambda_\phi$ become compatible with observations. The plots indicate a minimum value for the $\tilde{M}$ where the inflationary models are reliable and a maximum value for the $\tilde{M}$, where the NMDC effects become negligible.*



# Inflation with NMDC VS Planck data

The upper straight line in he panels is the prediction for $r = r(n_s)$ with the NMDC effect negligible and the lower straight line is the prediction for $r = r(n_s)$ with the NMDC effect dominant. The lightyellow region between the two theoretical straight lines corresponds intermediate $\tilde{M}$ values, $\tilde{M} \sim H_{\rm inf}$



# Slow-Roll Phase Duality

A remarkable observation is that in the high friction limit, $\tilde{M} \ll H_{\rm inf}$, and during the slow-roll phase the evolution of the field $\phi$ with potential $V(\phi)$ the evolution resembles the evolution of a minimally coupled field $\varphi$ with potential $V_{\rm GR}(\varphi)$ where [Germani+, ID & Farakos]

$$\varphi = \int \frac{V^{1/2}}{M_{\rm Pl}\tilde{M}}\, d\phi\,.$$

For the monomial potentials

$$V(\phi) = \lambda_p\, M_{\rm Pl}^4 \left(\frac{\phi}{M_{\rm Pl}}\right)^p$$

it is

$$V_{\rm GR}(\varphi) = \lambda_p M_{\rm Pl}^{4-p} \left(\frac{p+2}{2}\lambda_p^{-1/2} M_{\rm Pl}^{p/2-1}\,\tilde{M}\,\varphi\right)^{2p/(p+2)}$$

Hence

$$V \propto \phi^p \quad \longleftrightarrow \quad V_{\rm GR} \propto \varphi^{\frac{2p}{p+2}}$$

# Observational challenges

- The correspondence between models during the slow-roll phase

| Inflaton with NMDC | Minimally coupled inflaton |
|---|---|
| $\frac{1}{2}M_P^2 R - \frac{1}{2}\left(g^{\mu\nu} - \frac{G^{\mu\nu}}{\tilde{M}^2}\right)\partial_\mu\phi\partial_\nu\phi - m_\phi^2\phi^2/2$ | $\frac{1}{2}M_P^2 R - \frac{1}{2}\partial^\nu\varphi\partial_\nu\varphi - \mu_\varphi^3\varphi$ |
| $\frac{1}{2}M_P^2 R - \frac{1}{2}\left(g^{\mu\nu} - \frac{G^{\mu\nu}}{\tilde{M}^2}\right)\partial_\mu\phi\partial_\nu\phi - \lambda_\phi\phi^4$ | $\frac{1}{2}M_P^2 R - \frac{1}{2}\partial^\nu\varphi\partial_\nu\varphi - \xi_\varphi^{8/3}\varphi^{4/3}$ |
| $\frac{1}{2}M_P^2 R - \frac{1}{2}\left(g^{\mu\nu} - \frac{G^{\mu\nu}}{\tilde{M}^2}\right)\partial_\mu\phi\partial_\nu\phi - V_0 e^{-\lambda_e\phi/M_P}$ | $\frac{1}{2}M_P^2 R - \frac{1}{2}\partial^\nu\varphi\partial_\nu\varphi - m_\varphi^2\varphi^2/2$ |

## Observational degeneracy

- *At first sight it seems that cosmological models with dominant NMDC for the scalar field to Einstein tensor cannot give a distinct feature compared to cosmological models with a canonical coupling and GR gravity*

# Breaking the theoretical degeneracy

## Reheating

- The energy stored in the inflaton has to be converted to a plasma of relativistic particles (= **Reheating**)
- The presence of the NMDC can modify the standard picture of the reheating phase
- When the NMDC is active the inflaton oscillates rapidly without significant damping [Sadjadi+, Ghalee Sadjad+ '13, Gumjudpai+, Jinno+, Ema+ '15, Myung+, Ema+ '16].
- The Hubble parameter also oscillates rapidly

## Reheating relics

- *Perturbative and non-perturbative decay of the inflaton [Koutsoumbas+ '13, Ema+, '15]*

## Expansion History

- *The expansion law during reheating is significantly different than the standard case of a canonically coupled inflaton [Jinno+ '14, Ema+, '15]*

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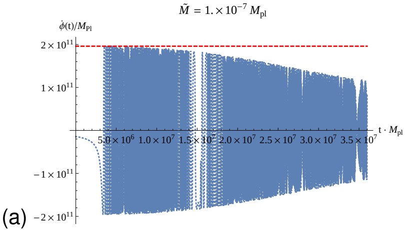
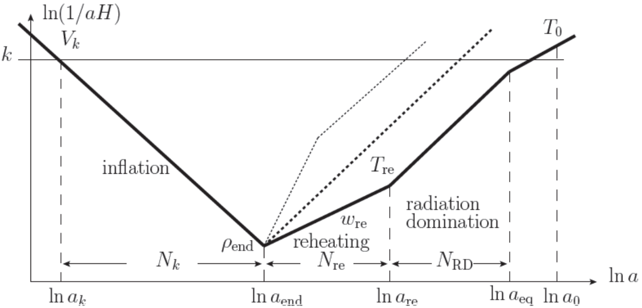
# The NMDC and the inflaton oscillations

The rapid oscillations of the inflaton field velocity for quadratic (left panel) and quartic (right panel) potentials when the NMDC dominates, $\tilde{M} = 10^{-7} M_{\rm Pl}$. The red-dashed line shows the maximum allowed value for the $\dot{\phi}$



# Expansion history effects

The reheating-thermal-history does not change the evolution of the adiabatic curvature perturbations, it changes the mapping of observed scales in the CMB to horizon exit during inflation. Uncertainty between the end of inflation and thermalization leads to an uncertainty in the number of e-folds $N_*$ [Liddle, Leach 2003]



# Reheating Expansion History

## The GR case

- The size of given scale $k^{-1}$ that exited the Hubble radius $H_k^{-1}$ during inflation can be related to the size of the present Hubble radius $H_0^{-1}$ via the relation

$$\frac{k}{a_0 H_0} = \frac{a_k}{a_{\rm end}} \frac{a_{\rm end}}{a_{\rm reh}} \frac{a_{\rm reh}}{a_{\rm eq}} \frac{a_{\rm eq} H_{\rm eq}}{a_0 H_0} \frac{H_k}{H_{\rm eq}} ,$$

- The number of e-folds during the reheating period, $N_{\rm reh}$, reads

$$N_{\rm reh}(n_s, q, \bar{w}_{\rm reh}) \simeq \frac{4}{1-3\bar{w}_{\rm reh}} \left[57.4 - N_*(n_s) + \frac{1}{4}\ln\gamma + \frac{1-q/2}{4} \ln \frac{q}{4N_*(n_s)}\right] ,$$

- The reheating temperature can be written in terms of the model dependent parameters $q$, $\bar{w}_{\rm reh} \equiv \frac{\langle p \rangle}{\langle \rho \rangle} = \frac{q-2}{q+2}$, and the observable quantity $n_s$,

$$T_{\rm reh}(n_s, q, \bar{w}_{\rm reh}) = \left(\frac{1}{\gamma}\right)^{1/4} \lambda_q^{1/4} \left(\frac{q}{\sqrt{2}}\right)^{q/4} \left(\frac{30}{\pi^2 g_*}\right)^{1/4} M_{\rm Pl}\, e^{-\frac{3}{4}(1+\bar{w}_{\rm reh})N_{\rm reh}(n_s)}$$

# Reheating Expansion History

## The NMDC case

- In the NMDC scenarios with monomial potentials the number of e-folds that take place during the reheating phase are given by the expression

$$N_{\rm reh}(n_s, p, \bar{w}_{\rm reh}) \simeq \frac{4}{1-3\bar{w}_{\rm reh}}\left[57.4 - N_*(n_s) - \frac{1}{2(p+2)}\ln\left(1+\frac{2(p+2)}{\gamma\, p}N_*(n_s)\right)\right], \quad (1)$$

- For the monomial potentials the reheating temperature can be written in terms of the parameters $p$, $\bar{w}$ and the observable quantity $n_s$,

$$T_{\rm reh}(n_s, p, \bar{w}_{\rm reh}) = \left(\frac{1}{\gamma}\right)^{1/4}\lambda_p^{1/4}\left(\frac{p}{\sqrt{3\lambda_p}}\tilde{M}^2 M_{\rm Pl}^p\right)^{\frac{p}{4p+8}}\left(\frac{30}{\pi^2 g_*}\right)^{1/4} M_{\rm Pl}^{\frac{4-p}{4}}\, e^{-\frac{3}{4}(1+\bar{w}_{\rm reh})N_{\rm reh}(n_s)}. \quad (2)$$

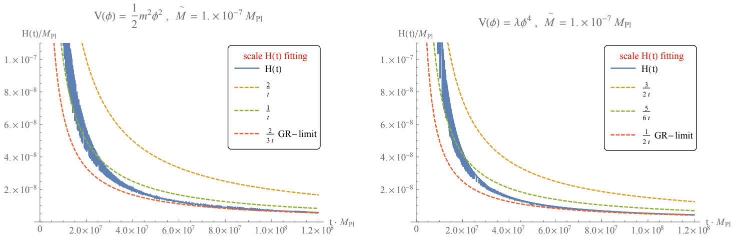
## The reheating EOS in NMDC?

- The non-standard result for $H = H(t)$ yields a much different relation for the averaged EoS, $\bar{w}_{\rm reh}$, with respect to the shape of the potential

$$\bar{w}_{\rm reh(DC)} \sim -\frac{1}{p+1}$$

# The expansion rate in theories with NMDC

*The evolution of the Hubble parameter and fitting curves, for quadratic (left panel) and quartic (right panel) potentials and for $\tilde{M} = 10^{-7} M_{Pl}$. The fitting curves indicate the effective $\bar{w}_{reh}$ value.*




$$\langle H \rangle \sim \frac{2p+2}{3p} \frac{1}{t} \, , \qquad \bar{w}_{\mathrm{reh(DC)}} \sim -\frac{1}{p+1} \, ,$$

# Reheating Expansion History in the NMDC

- As long as the NMDC dominates over the canonical term the $\bar{w}_{\rm reh}$ value is determined by the NMDC dynamics. Afterwards the $\bar{w}_{\rm reh}$ approaches its canonical GR value, $w_{\rm reh(DC)} \to w_{\rm reh(GR)}$.

$$N_{\rm reh} \, = \, N_{\rm reh(DC)}\Big|_{\tilde{M}\lesssim H} \, + \, N_{\rm reh(GR)}\Big|_{\tilde{M}>H}$$

where

$$N_{\rm reh(DC)} \equiv \frac{1}{3(1+\bar{w}_{\rm reh(DC)})} \ln \frac{\rho_{\rm end}}{\rho_{\rm reh(DC)}} \, , \qquad N_{\rm reh(GR)} \equiv \frac{1}{3(1+\bar{w}_{\rm reh(GR)})} \ln \frac{\rho_{\rm reh(DC)}}{\rho_{\rm reh(GR)}}$$

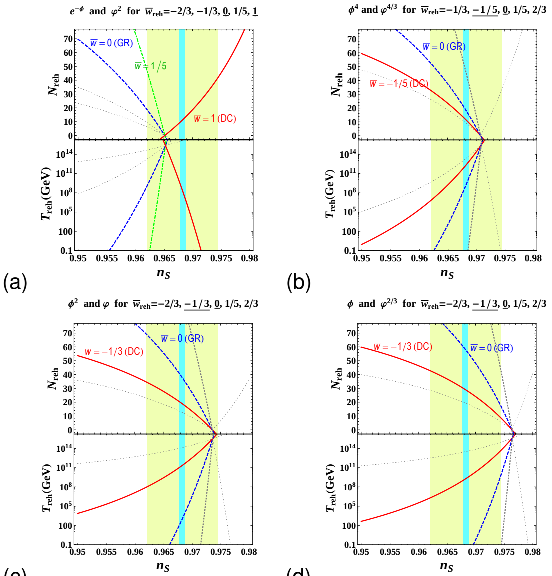
- The CMB normalization constrains the NMDC scale $\tilde{M} \gtrsim 10^{-8-14} M_{\rm Pl}$ and the evolution of the Hubble scale indicates that $-2/3 < \bar{w}_{\rm reh} < 0$, hence

$$1 \, \lesssim N_{\rm reh(DC)} \, < 20 \, .$$

- If the NMDC is effective till the time of the perturbative inflaton decay, that is $N_{\rm reh(GR)} \to 0$, a case expected for $\tilde{M} \ll H_{\rm inf}$, then the reheating temperature lies in the range

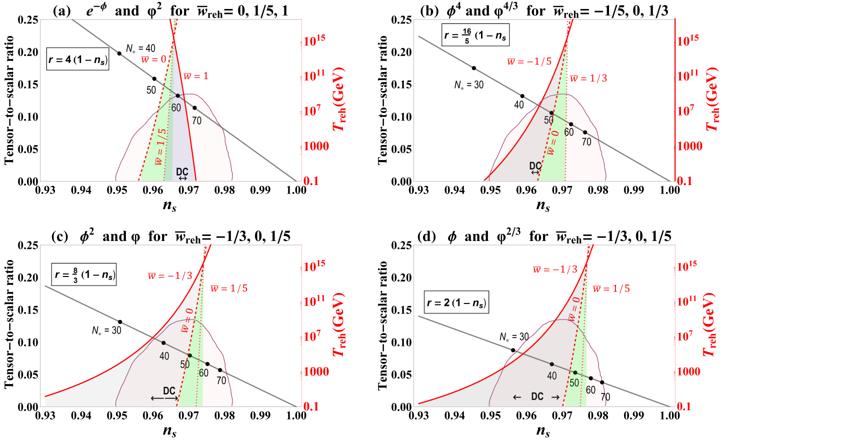
$$10^{-3} \, T_{\rm max} \, \lesssim \, T_{\rm reh} \, \lesssim \, T_{\rm max} \, .$$

## The inflationary predictions for the e-folds number, $N_{reh}$, and the reheating temperature $T_{reh}$, for the $V(\phi)$-NMDC and $V(\varphi)$-GR models



# Model Selection

*In the $(n_s, r)$ axes-system we plot the marginalized joint 95% CL observational contour for the $n_s$ and the $r_{0.002}$ and in the $(n_s, T_{reh})$ axes-system we plot the predicted reheating temperature $T_{reh}$ -in red-dashed curves for the GR models and red-solid curves for the NMDC models where benchmark EoS values are used.*



# Conclusions

## Outline

- Inflation models with NMDC extend the parameter space that implements inflation and the suppress the tensor-to-scalar ratio
- During the slow-roll phase the dynamics of models with NMDC become practically indistinguishable from GR models (de-Sitter duals)
- The degeneracy in the inflationary predictions, $r = r(n_s)$, with GR models can break when the reheating period is taken into account.
- The NMDC models predict much higher values for $T_{\rm reh}$ and are compatible with a larger part of the observationally constrained $(n_s, r)$ plane than their GR duals.

## Future Research Directions

- The $\tilde{M}$ scale is also possible to be probed by the measurement of the reheating temperature
- Complementary theoretical studies regarding the small-scale instabilities may constrain further the allowed $\tilde{M}$ values.

## Observational Prospects

- Upcoming CMB experiments promise to reduce the $\delta n_s$ and $\delta r$ uncertainty to $\mathcal{O}(10^{-2} - 10^{-3})$ level, making the observational discrimination between different inflationary mechanisms possible.

# Thank you!