On a time-dependent version of the Schwarzschild metric

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1. Introduction

- the measurement problem - it is not rigurously defined within the QM
- solution - QG (still lacking - E. Okon and D. Sudarsky, arXiv: 1701.02963).
- Connections between quantum-foundational issues and QG have been pointed out by:
  - R. Penrose, GRG 28, 581 (1996)
  - L. Diosi, PLA 120, 377 (1987), arXiv: 0703140; 0004067; 1312.6404.
  - P. Facchi and M. Ligabo: 1702.04284.
  - P. Pearle and E. Squires, Found. Phys. 26, 291 (1996).
  - D. Bedingham, Found. Phys. 41, 686 (2011).

- A link between quantum collapse of the wave function and gravity, when macroscopic objects are placed in quantum superposition at different locations
- the curvature scalar of the spacetime is responsable for the collapse.

Diosi added a gravitational term

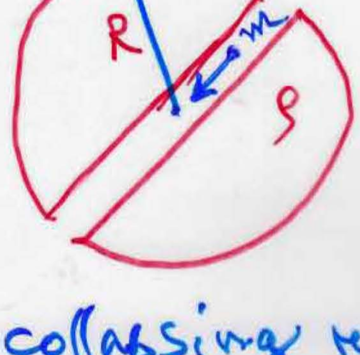
$$\left(\int U(\bar{x}-\bar{x}')\,|\psi(\bar{x}')|^2\,d\bar{x}'\right)\psi(\bar{x})$$

to the standard QM terms from the Schrödinger equation, for an object of mass m and radius R

When $|\bar{x}-\bar{x}'| \ll R$,

$$U(\bar{x}-\bar{x}') \sim U(0) + \frac{1}{2} m\omega^2 |\bar{x}-\bar{x}'|^2,$$

where $\omega \simeq m/R^3$ (the frequency of the Newtonian oscillator



$$\omega \sim \frac{4\pi\rho}{3}$$

(geodesic deviation).

the collapsing rate $= \frac{1}{\tau} \simeq \frac{m\omega^2}{\hbar}(\Delta\bar{x})^2$

- The standard kinetic term tends to spread the wave function, competing with the Diosi-Penrose spontaneous collapse which tends to shrink the wave function:

$$\underbrace{\frac{\hbar}{m(\Delta\bar{x})^2}}_{\text{spreading rate}} \approx \underbrace{\frac{m\omega^2(\Delta\bar{x})^2}{\hbar}}_{\text{collapsing rate}}$$

$$\Longrightarrow \frac{1}{\tau} \simeq \omega$$

(τ – the decoherence time needed to collapse the macroscopic superposition), or the quantum Zenon time

$$\tau_Z = \frac{\hbar}{\sqrt{\langle\psi|H^2|\psi\rangle - \langle\psi|H\psi\rangle^2}}$$

- Newtonian gravity - no experimental check in the range below 0.1 mm.

Proposal: the strength of the gravitational field is modified when a measurement is performed in a very short time interval.

2. Schwarzschild geometry with time-dependent mass.

As we know, the source giving the metric

$$ds^2 = -\left(1-\frac{2m(t)}{r}\right)dt^2 + \frac{dr^2}{1-\frac{2m(t)}{r}} + r^2 d\Omega^2$$

represents an anisotropic fluid with $\rho = p_r = 0$ but nonzero tangential pressures

$$p_t = \frac{2m(t)\ddot{m}(t) - 4\dot{m}^2(t) - r\ddot{m}(t)}{8\pi r^2\left(1-\frac{2m(t)}{r}\right)^3}$$

and energy flux

$$T^r{}_t = \frac{\dot{m}(t)}{4\pi r^2}.$$

- there is an apparent horizon

$$r_{AH}(t) = 2m(t)$$

Conjecture: $m(t) = e^{-\frac{\kappa}{t}}$, $\kappa > 0$.

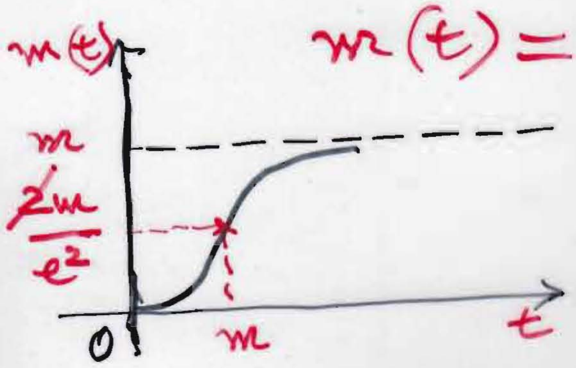
To find a suitable expression for κ, we relate it to the Zenon time, taking the mass m as the object were a BH (with the same outer field).

Therefore, with $R = 2m$:

$$\omega^2 \sim g \sim \frac{1}{m^2}, \text{ or } \tau \sim m$$

Hence, $\kappa \sim m$. Our choice: $\kappa = 2m$, and

$$m(t) = e^{-\frac{2m}{t}} \cdot m$$



t – the duration of the measurement.

For the Earth: $2GM/c^2 \simeq 1\,\mathrm{mm}$ and $\kappa = 3 \cdot 10^{-12}\,\mathrm{s}$, a reasonable decoherence time. For $t = 10^{-12}\,\mathrm{s}$, $M \to M/e^3$.

– a possible explanation of the fact that the zero point energy does not gravitate.

For the above metric we have; for to avoid a signature flip:

$$-g_{00} = 1 - \frac{2m}{r} e^{-\frac{2m}{t}} \equiv f(r,t) > 0$$

$f(r,t)$ – a monotonic decreasing function of t, at constant r, with

$$0 < f(r,t) < 1$$

3. Properties of the gravitational fluid

The only nonzero components of the $T^a{}_b$ are:

$$T^r{}_t = \frac{m^2 e^{-\frac{2m}{t}}}{2\pi r^2 t^2} \; ; \; T^t{}_r = -\frac{m^2 e^{-\frac{2m}{t}}}{2\pi r^2 t^2 \left(1-\frac{2m}{r}e^{-\frac{2m}{t}}\right)^2}$$

$$T^\theta{}_\theta = T^\varphi{}_\varphi = \frac{m^2 e^{-\frac{2m}{t}}}{2\pi r t^3 \left(1-\frac{2m}{r}e^{-\frac{2m}{t}}\right)}\left[1-\frac{m\left(1+\frac{2m}{r}e^{-\frac{2m}{t}}\right)}{t\left(1-\frac{2m}{r}e^{-\frac{2m}{t}}\right)}\right],$$

with $\rho = p_r = 0$.

$t \to \infty, \Rightarrow T^a{}_b \to 0$.

$t \to 0, \Rightarrow m(t) \to 0$, and the geometry becomes Minkowskian.

Note that p_t vanishes at

$$r_0 = \frac{t+m}{t-m}\cdot 2m e^{-\frac{2m}{t}} > 2m(t), \quad t > m$$

which becomes $r_0 \simeq 2m$ for $t \gg m$.

For a "static" observer ($u^i = 0$, $i = 1,2,3$), one obtains

$$a^b \equiv u^a \nabla_a u^b = \left(0, \frac{m}{r^2}e^{-\frac{2m}{t}}, 0, 0\right)$$

For $t \ll 2m$, $a^r \simeq 0$ and the gravitational effects are strongly weakened. Similarly for $r \to 0$ (taken together with $t \to 0$).

For the scalar expansion of the congruence we have:

$$\theta \equiv \nabla_a u^a = \frac{2m^2 e^{-\frac{2m}{t}}}{r t^2 \left(1 - \frac{2m}{r} e^{-\frac{2m}{t}}\right)^{3/2}},$$

which vanishes when $t \to 0$ or $t \to \infty$.

The shear tensor components are

$$\sigma^r{}_r = -2\sigma^\theta{}_\theta = -2\sigma^\varphi{}_\varphi = \frac{2}{3}\theta$$

4. Brown-York quasilocal energy

Take an observer laying at $r = $ const. The total energy flow measured by that observer will be given by (T. Padmanabhan, gr-qc/0308070 ; H.C., Progr. Theor. Exp. Phys. 123E02 (2016) (1605.04467)

$$E = \int T^a{}_b u^b n_a \sqrt{-g}\, dt\, d\theta\, d\varphi\,,$$

with $n_a = \left(0, 1/\sqrt{1 - \frac{2m(t)}{r}}, 0, 0\right)$ and $n_a u^a = 0$. Using the expression of $T^r{}_t$, one obtains

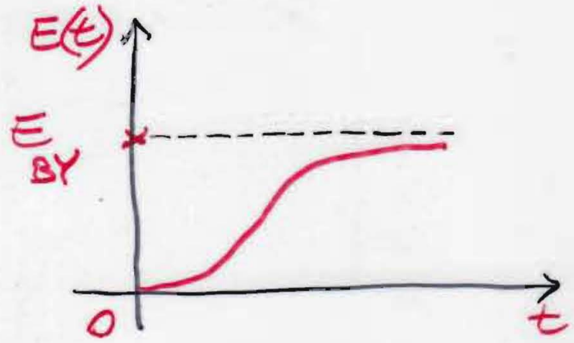
$$E = \int \frac{\dot{m}(t)}{\sqrt{1 - \frac{2m(t)}{r}}}\, dt \quad, \quad r = \text{const}$$

Hence

$$E(r,t) = \frac{1}{8\pi}\int_B (K - K_0)\sqrt{\sigma}\, d^2x$$

$$E(r,t) = r\left(1 - \sqrt{1 - \frac{2m}{r} e^{-\frac{2m}{t}}}\right)$$

where $\sigma = \det(\sigma_{ab})$ ~~and~~ and $\sigma_{ab} = g_{ab} + u_a u_b - n_a n_b$ is the induced metric on the 2-boundary B, and K is the mean extrinsic curvature of B.



$r = \text{const}$

$$E_{BY} = r\left(1 - \sqrt{1 - \frac{2m}{r}}\right)$$

(A.P. Lundgren et al, gr-qc/0610088).

$E(r, 0) = 0$.

- No energy flux when $t \to 0$.

5. Conclusions.

- We proposed that $m(t) = m e^{-\frac{2m}{t}}$, where $m \equiv m_\infty = \lim_{t \to \infty} m(t) = t_Z$ (Zenon)
- t - the duration of the measurement
- $\rho = p_r = 0$.
- $t \gg 2m \to$ the standard Schwarzschild metric is retrieved.
- $r > 2m(t)$ (no signature flip).
- there is an AH at $r = 2m(t)$, where $R^a_{\ a} \to \infty$.
- $a^r \to 0$, when $t \ll 2m$
- $\omega = 1/t_Z$ is the Newtonian oscillator frequency.