Higher Derivative Theories

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Based on [arXiv:1601.04658], [arXiv:1602.03119], [arXiv:1608.08135], [arXiv:1703.01623] and [arXiv:1710.soon]

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Historical Overview

- 1971 – **Lovelock**

  the most general metric theory of gravity yielding conserved second order equations of motion in arbitrary number of dimensions

- 1974 – **Horndeski**

  the most general scalar-tensor theory of gravity yielding conserved second order equations of motion in 4 dimensions

- 2008 – **Galileons** [Nicolis, Rattazzi, Trincherini]

  a set of terms within 4-dimensional EFT obeying the symmetry $\phi \to \phi + c + b_\mu x^\mu$ (in a non-trivial way)

- 2009 – **Covariant / Generalized Galileons** [Deffayet et al.]

  rediscovery of Horndeski

Second order field equations

What's wrong with higher order field equations?

Ostrogradsky theorem – 1850

Assumptions: 1) single variable 2) non-degenerate (nd)

- Newton, i.e. $L = L(q,\, \dot q)$

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot q} = 0 \qquad \overset{\text{nd}}{\Longrightarrow} \qquad \ddot q = F(q,\, \dot q)$$

$$p \equiv \frac{\partial L}{\partial \dot q} \qquad \text{nd} \Longrightarrow \dot q = f(q,\, p) \qquad\qquad H(q,\, p) = p\, f - L(q,\, f)$$

- Higher derivative $L = L(q,\, \dot q,\, \ddot q)$

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot q} + \frac{d^2}{dt^2}\frac{\partial L}{\partial \ddot q} = 0 \qquad \overset{\text{nd}}{\Longrightarrow} \qquad \ddddot q = F(q,\, \dot q,\, \ddot q,\, \dddot q)$$

$$Q \equiv \dot q\,, \quad p \equiv \frac{\partial L}{\partial \dot q} - \frac{d}{dt}\frac{\partial L}{\partial \ddot q}\,, \quad P \equiv \frac{\partial L}{\partial \ddot q}\,, \qquad \text{nd} \Longrightarrow \ddot q = f(q,\, Q,\, P)$$

$$H(q,\, Q,\, p,\, P) = p\,Q + P f - L(q,\, Q,\, f)$$

H linear in p $\Rightarrow$ unbounded energy

WAY OUT: break the assumptions

Evading the Ostrogradsky instability

Assumptions: 1) two variables 2) degenerate Langlois & Noui [arXiv:1510.06930]

- $L = L(q_1,\, \dot q_1,\, \ddot q_1,\, q_2,\, \dot q_2) \quad \longrightarrow \quad L(q_1,\, Q,\, \dot Q,\, q_2,\, \dot q_2) + \lambda(\dot q_1 - Q)$

$$p_1 \equiv \frac{\partial L}{\partial \dot q_1} = \lambda \qquad P \equiv \frac{\partial L}{\partial \dot Q} \qquad p_2 \equiv \frac{\partial L}{\partial \dot q_2}$$

Primary constraint $\psi(P,\, p_2) \approx 0 \iff \boxed{\det \mathbb{H} = 0}$ $\qquad \mathbb{H} = \begin{pmatrix} \dfrac{\partial^2 L}{\partial \dot Q^2} & \dfrac{\partial^2 L}{\partial \dot Q \partial \dot q_2} \\ \dfrac{\partial^2 L}{\partial \dot q_2 \partial \dot Q} & \dfrac{\partial^2 L}{\partial \dot q_2^2} \end{pmatrix}$

Generalization: Motohashi et al. [arXiv:1603.09355]; Klein & Roest [arXiv:1604.01719]

- $L = L(\ddot\phi_m, \dot\phi_m, \phi_m, \dot q_\alpha, q_\alpha) \qquad v^A_m = (\delta^n_m, V^\alpha_m) \qquad V^\alpha_m \equiv -L_{\ddot\phi_m \dot q_\beta} L^{-1}_{\dot q_\beta \dot q_\alpha}$

  $\psi_A \equiv (\dot\phi_m, q_\alpha)$

Primary constraints $\iff 0 = P_{(mn)} \equiv v^A_m L_{\dot\psi_A \dot\psi_B} v^B_n$

Secondary constraints $\iff 0 = S_{[mn]} \equiv 2\, v^A_m L_{\dot\psi_{[A} \psi_{B]}} v^B_n$

Evading the Ostrogradsky instability

Field theories: [arXiv:1703.01623]

- $L = L(\partial_\mu \partial_\nu \phi_m, \partial_\mu \phi_m, \phi_m, \partial_\mu q_\alpha, q_\alpha)$ $\qquad v_m^A = (\delta_m^n, V_m^\alpha) \qquad V_m^\alpha \equiv -L_{\ddot{\phi}_m \dot{q}_\beta} L^{-1}_{\dot{q}_\beta \dot{q}_\alpha}$

  $\psi_A \equiv (\dot{\phi}_m, q_\alpha)$

Primary constraints $\iff$ $0 = P_{(mn)} \equiv v_m^A L_{\dot{\psi}_A \dot{\psi}_B} v_n^B$

Secondary constraints $\iff$ $0 = (S_i)_{(mn)} \equiv 2\, v_m^A L_{\dot{\psi}_{(A} \partial_i \psi_{B)}} v_n^B$

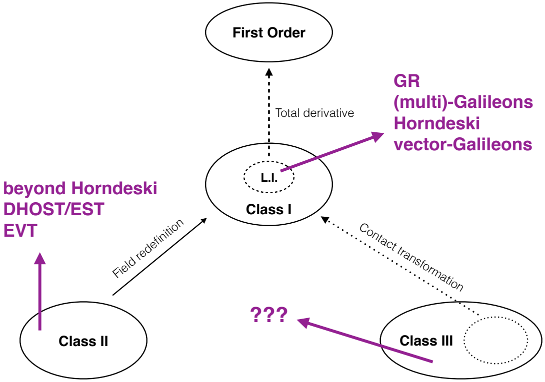
$$0 = S_{[mn]} \equiv 2\, v_m^A L_{\dot{\psi}_{[A} \psi_{B]}} v_n^B + 2\, v_{[m}^A L_{\dot{\psi}_A \partial_i \psi_B} \partial_i v_{n]}^B - \partial_i \left(v_m^A L_{\dot{\psi}_{[A} \partial_i \psi_{B]}} v_n^B \right)$$

Lorentz invariance $\implies$ $(S_i)_{(mn)} = 0$ if $P_{(mn)} = 0$

1 primary constraint $\implies$ $\exists$ **1** secondary constraint

Kind of theories

- $L = L(\partial_\mu\partial_\nu\phi_m, \partial_\mu\phi_m, \phi_m, \partial_\mu q_\alpha, q_\alpha)$ $\quad v^A_m = (\delta^n_m, V^\alpha_m)$ $\quad V^\alpha_m \equiv -L_{\ddot{\phi}_m \dot{q}_\beta} L^{-1}_{\dot{q}_\beta \dot{q}_\alpha}$
  - Class I: $V^\alpha_m = 0 \quad \longrightarrow$ trivial primary constraints
  - Class II: $V^\alpha_m \neq V^\alpha_m(\partial_\mu\partial_\nu\phi_n, \partial_\mu q_\beta) \quad \longrightarrow$ linear primary constraints
  - Class III: $V^\alpha_m = V^\alpha_m(\partial_\mu\partial_\nu\phi_n, \partial_\mu q_\beta) \quad \longrightarrow$ nonlinear primary constraints



Scalar-Tensor Theories

two fields $(g_{\mu\nu},\, \phi)$

Second derivative of ϕ: $\quad L = L(\nabla_\mu \partial_\nu \phi) \quad \longrightarrow \quad L(\nabla_\mu A_\nu) + \lambda^\mu(\partial_\mu \phi - A_\mu)$

Covariant 3+1 decomposition: $\quad t^\mu = \partial/\partial t = N n^\mu + N^\mu \quad \begin{cases} g_{\mu\nu} = h_{\mu\nu} - n_\mu n_\nu \\ A_\mu = \hat{A}_\nu h^\nu_\mu - A_* n_\mu \end{cases}$

$$\nabla_\mu A_\nu = \lambda_{\mu\nu} V + \Lambda_{\mu\nu}^{\ \ \rho\sigma} K_{\rho\sigma} + \cdots \qquad V \equiv n^\mu \nabla_\mu A_* \sim \ddot{\phi}$$

$$\mathbb{H} = \begin{pmatrix} \mathcal{A} & \mathcal{B}^{ij} \\ \mathcal{B}^{kl} & \mathcal{K}^{ij,kl} \end{pmatrix}, \qquad \mathcal{A} \equiv \frac{\partial^2 L}{\partial V^2}, \qquad \mathcal{B}^{ij} \equiv \frac{\partial^2 L}{\partial V \partial K_{ij}}, \qquad \mathcal{K}^{ij,kl} \equiv \frac{\partial^2 L}{\partial K_{ij} \partial K_{kl}}$$

$\det \mathbb{H} = 0 \quad \Longrightarrow \quad$ ghost free

Degenerate Scalar-Tensor Theories

$$S = \int d^4x \sqrt{-g} \left(f_2 \, R + C_{(2)}^{\mu\nu\rho\sigma} \, \phi_{\mu\nu} \, \phi_{\rho\sigma} + f_3 \, G_{\mu\nu} \phi^{\mu\nu} + C_{(3)}^{\mu\nu\rho\sigma\alpha\beta} \, \phi_{\mu\nu} \, \phi_{\rho\sigma} \, \phi_{\alpha\beta} \right)$$

- 24 classes of theories with 17 free functions [arXiv:1608.08135]

Phenomenological applications:

- ET of DE
  Langlois et al. [arXiv:1703.03797]
- Stable bouncing cosmology
  Creminelli et al. [arXiv:1610.04207]
- Breaking of Vainshtein mechanism
  Kobayashi et al. [arXiv:1411.4130] + many others (including Eugeny)

Pure Metric Theories

$$S[g_{\mu\nu}] = \int d^4x\,\sqrt{-g}\,L\,(g_{\mu\nu},\,\partial_\rho g_{\mu\nu},\,\partial_\rho\partial_\sigma g_{\mu\nu}) \qquad \supseteq \qquad \dot{K}_{ij} \qquad \supseteq \qquad \ddot{\gamma}_{ij}$$

FULLY DEGENERATE THEORIES

6 primary constraints $\iff \quad \mathcal{A}^{ij,\lambda m}(x,y) \equiv \dfrac{\partial^2 L}{\partial\ddot{\gamma}_{ij}(x)\partial\ddot{\gamma}_{\lambda m}(y)} = 0$

$$GB \equiv (\star R^{\mu\nu}{}_{\alpha\beta})(\star R^{\alpha\beta}{}_{\mu\nu})\,, \quad P \equiv (\star R^{\mu\nu}{}_{\alpha\beta}) R^{\alpha\beta}{}_{\mu\nu}\,, \quad C \equiv (\star R^{\mu\nu}{}_{\rho\sigma})(\star R^{\rho\sigma}{}_{\alpha\beta})(\star R^{\alpha\beta}{}_{\mu\nu})$$

$$\star R^{\mu\nu}{}_{\rho\sigma} \equiv \varepsilon^{\mu\nu\alpha\beta} R_{\alpha\beta\rho\sigma}$$

6 secondary constraints $\implies$ EH

C has NO secondary constraints $\longrightarrow$ 5 dof (3 Ostrogradsky modes)

Pure Metric Theories

$$S[g_{\mu\nu}] = \int d^4x \sqrt{-g}\, L\left(g_{\mu\nu},\, \partial_\rho g_{\mu\nu},\, \partial_\rho \partial_\sigma g_{\mu\nu}\right) \quad \supseteq \quad \dot{K}_{ij} \quad \supseteq \quad \ddot{\gamma}_{ij}$$

PARTIALLY DEGENERATE THEORIES

5 primary constraints $\iff$ Rank $[\mathcal{A}^{ij,\lambda m}(x,y)] = 1$

$f(R)$ $f(GB)$ $f(P) \longrightarrow$ Chern-Simons gravity Jackiw & Pi [gr-qc/0308071]

5 secondary constraints $\Longrightarrow$ f(R) , f(GB) + X

CS is conformal invariant $\longrightarrow$ 4 dof

4 secondary constraints missing :(

In the Unitary gauge is OK ! :)

Chiral Scalar-Tensor Theories

One new ingredient: Levi-Civita tensor $\varepsilon^{\mu\nu\rho\sigma}$

First derivatives of the scalar field only:

$$L_1 \equiv \varepsilon^{\mu\nu\alpha\beta} R_{\alpha\beta\rho\sigma} R_{\mu\nu}{}^{\rho}{}_{\lambda} \phi^{\sigma} \phi^{\lambda} \,, \qquad L_2 \equiv \varepsilon^{\mu\nu\alpha\beta} R_{\alpha\beta\rho\sigma} R_{\mu\lambda}{}^{\rho\sigma} \phi_{\nu} \phi^{\lambda} \,,$$

$$L_3 \equiv \varepsilon^{\mu\nu\alpha\beta} R_{\alpha\beta\rho\sigma} R^{\sigma}{}_{\nu} \phi^{\rho} \phi_{\mu} \,, \qquad L_4 \equiv X\, P$$

Including second derivatives of the scalar field:

$$L_1 = \varepsilon^{\mu\nu\alpha\beta} R_{\alpha\beta\rho\sigma} \phi^{\rho} \phi_{\mu} \phi^{\sigma}_{\nu}$$

$$L_2 = \varepsilon^{\mu\nu\alpha\beta} R_{\alpha\beta\rho\sigma} \phi^{\rho}_{\mu} \phi^{\sigma}_{\nu} \,, \qquad L_3 = \varepsilon^{\mu\nu\alpha\beta} R_{\alpha\beta\rho\sigma} \phi^{\sigma} \phi^{\rho}_{\mu} \phi^{\lambda}_{\nu} \phi_{\lambda} \,,$$

$$L_4 = \varepsilon^{\mu\nu\alpha\beta} R_{\alpha\beta\rho\sigma} \phi_{\nu} \phi^{\rho}_{\mu} \phi^{\sigma}_{\lambda} \phi^{\lambda} \,, \qquad L_5 = \varepsilon^{\mu\nu\alpha\beta} R_{\alpha\rho\sigma\lambda} \phi^{\rho} \phi_{\beta} \phi^{\sigma}_{\mu} \phi^{\lambda}_{\nu} \,,$$

$$L_6 = \varepsilon^{\mu\nu\alpha\beta} R_{\beta\gamma} \phi_{\alpha} \phi^{\gamma}_{\mu} \phi^{\lambda}_{\nu} \phi_{\lambda} \,, \qquad L_7 = (\Box\phi) L_1$$

6 (+ 1) primary constraints $\Longrightarrow$ ✓ tuning the free functions

6 (+ 1) secondary constraints $\Longrightarrow$ X no way

Chiral Scalar-Tensor Theories

Unitary gauge:

First derivatives of the scalar field only

$$\begin{aligned} S_{UG} &= \frac{2\,\dot{\phi}^2\,\epsilon^{ij\lambda}}{N}\Big[2\,(2a_1+a_2+4a_4)\Big(K K_{mi} D_\lambda K^m_j + {}^{(3)}R_{mi} D_\lambda K^m_j - K_{mi} K^{mn} D_\lambda K_{jn}\Big) \\ &- (a_2+4a_4)\Big(2\,K_{mi} K^n_j D_n K^m_\lambda + {}^{(3)}R_{j\lambda m}{}^n D_n K^m_i\Big)\Big] \end{aligned}$$

Including second derivatives of the scalar field

$$\begin{aligned} S_{UG} &= \frac{\dot{\phi}^3}{N^4}\,\epsilon^{ij\lambda}\Big\{2\,N\Big[b_1 N K_{mi} D_\lambda K^m_j + (b_4+b_5-b_3)\,\dot{\phi} K_{mi} K^n_j D_n K^m_\lambda\Big] \\ &+ \dot{\phi}\Big[b_3\ {}^{(3)}R_{j\lambda m}{}^n K^m_i D_n N - 2\,(b_4+b_5)\ {}^{(3)}R_{m\lambda} K^m_j D_i N\Big]\Big\} \end{aligned}$$

Summary

- It is possible to avoid the Ostrogradsky instability in higher order theories in a non-trivial way

- Brand new theories

- New phenomenology – GW in parity breaking scalar-tensor theories

Thank you