Braneworld Scenarios from Deformed Defect Chains

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Topics

- Cyclic deformation procedure
  - Free parameter *n*
  - Kink and lump-like solutions
  - Some derived lump solutions
- Branewolrd models
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  - Lump as an ansatz
  - First order formalism
  - Brane scalar field
  - Metric perturbation
  - Configurational entropy as a tool to fix n
- Brane scalar real solutions

Cyclic deformation procedure

- Topological defect known *a priori*: $\chi(\mathrm{s})$ → scalar field in one spatial dimension.
- Calculate it's derivative: $\frac{d\chi}{ds} = w_\chi$
- We want to find, for example, two novel defects, named ϕ and ψ.
- Suppose they can be written in terms of the original defect χ, such that:
- $\frac{d\phi}{ds} = \frac{d\phi}{d\chi}\frac{d\chi}{ds} \stackrel{\text{def}}{=} y_\phi$
- $\frac{d\psi}{ds} = \frac{d\psi}{d\chi}\frac{d\chi}{ds} \stackrel{\text{def}}{=} z_\psi$

Cyclic deformation procedure

Free parameter n [1]

Chain rule

- $\dfrac{d\chi}{ds} \overset{\text{def}}{\equiv} w_\chi$
- $\dfrac{d\phi}{ds} = \dfrac{d\phi}{d\chi}\dfrac{d\chi}{ds} \overset{\text{def}}{\equiv} y_\phi$
- $\dfrac{d\psi}{ds} = \dfrac{d\psi}{d\chi}\dfrac{d\chi}{ds} \overset{\text{def}}{\equiv} z_\psi$

Ex. of deformation functions:

$$\frac{d\phi}{d\chi} = \tanh(n\chi)$$

$$\frac{d\psi}{d\chi} = \mathrm{sech}(n\chi)$$

Scalar field mass:

- $M_\chi = \int_{-\infty}^{\infty} \left(\dfrac{d\chi}{ds}\right)^2 ds$
- $M_\phi = \int_{-\infty}^{\infty} \left(\dfrac{d\phi}{ds}\right)^2 ds$
- $M_\psi = \int_{-\infty}^{\infty} \left(\dfrac{d\psi}{ds}\right)^2 ds$

Relation:

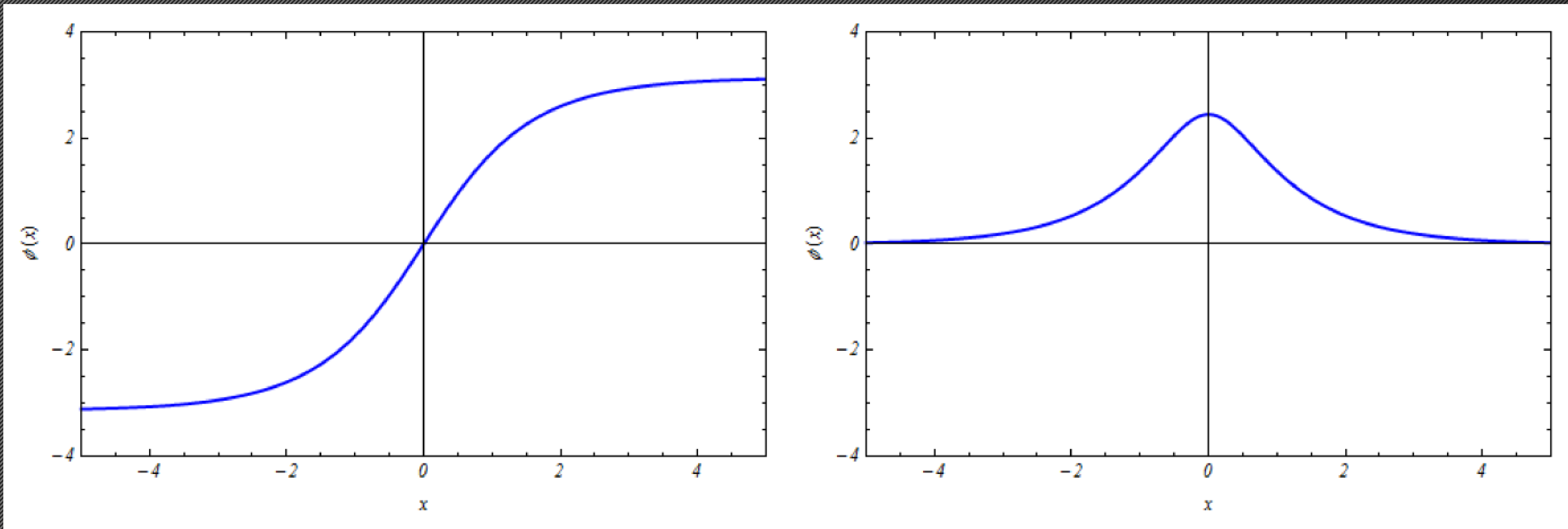
$$\tanh^2(n\chi) + \mathrm{sech}^2(n\chi) = 1$$

$$\left(\frac{d\chi}{ds}\right)^2 . 1 = \left(\frac{d\chi}{ds}\right)^2 \left(\left(\frac{d\psi}{d\chi}\right)^2 + \left(\frac{d\phi}{d\chi}\right)^2\right) =$$

$$= \left(\frac{d\phi}{ds}\right)^2 + \left(\frac{d\psi}{ds}\right)^2$$ → Mass constraint.

Cyclic deformation procedure

Kink and lump solutions



Cyclic deformation procedure

Some derived lump solutions

Derived lump solutions

n

$$\psi_1 = \frac{-\ln[\cosh[n\tanh(y)]\,\mathrm{sech}(n)]}{n},$$

$$\psi_2 = \frac{\cos[n\tanh(y)] - \cos(n)}{n},$$

$$\psi_3 = \frac{(\mathrm{sech}[n\tanh(y)] - \mathrm{sech}(n))}{n}$$

$$\psi_4 = \frac{(2n\,\mathrm{sech}(y) + \sin[2n\,\mathrm{sech}(y)])}{4n}$$

Braneworld models in terms of A

Lump as an ansatz [2]

$$S = \int d^4x dy \sqrt{|\, g \,|}\left[-\frac{1}{4}R + \frac{1}{2}\partial_a\phi\partial^a\phi - V(\phi)\right]$$

$$ds^2 = e^{2A}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2.$$

Ansatz → $e^{2A} = \psi$ → $A = \frac{1}{2}\ln\psi$

$$T_{00} = T^0_0 g_{00} = \left[\left(\frac{d\phi}{dy}\right)^2 - 3\left(\frac{dA}{dy}\right)^2\right]e^{2A(y)}$$

Braneworld models in terms of A

Models 1, 2, 3 all have a similar behavior

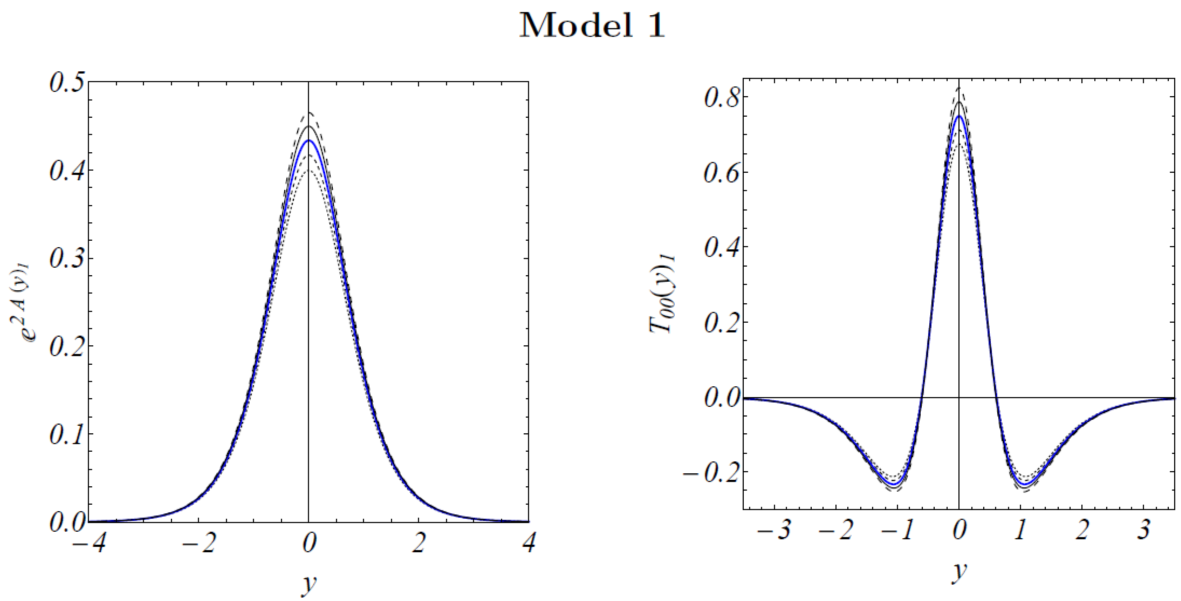


FIG. 1: Brane warp factor and energy-momentum tensor for model 1 derived from ψ_1. n ranges from $n = 1 - 2k; 1 - k, 1, 1 + k, 1 + 2kek = 0.05$. Blue line represents the solution for $n = 1$ while black lines ranges from the dotted to the dashed as n increases.

Braneworld models in terms of A

$$A(y)_4 = \frac{1}{2}\ln\left[\frac{2n\ \mathrm{sech}(y) + \sin[2n\ \mathrm{sech}(y)]}{4n}\right]$$

$$T_{00}(y)_4 = \frac{3}{4}\mathrm{sech}(y)[\cos(n\,\mathrm{sech}(y))^2\,\mathrm{sech}(y)^2 + \tanh(y)^2(-\cos[n\,\mathrm{sech}(y)]^2 + n\,\mathrm{sech}(y)\sin[2n\,\mathrm{sech}(y)])].$$

Model 4

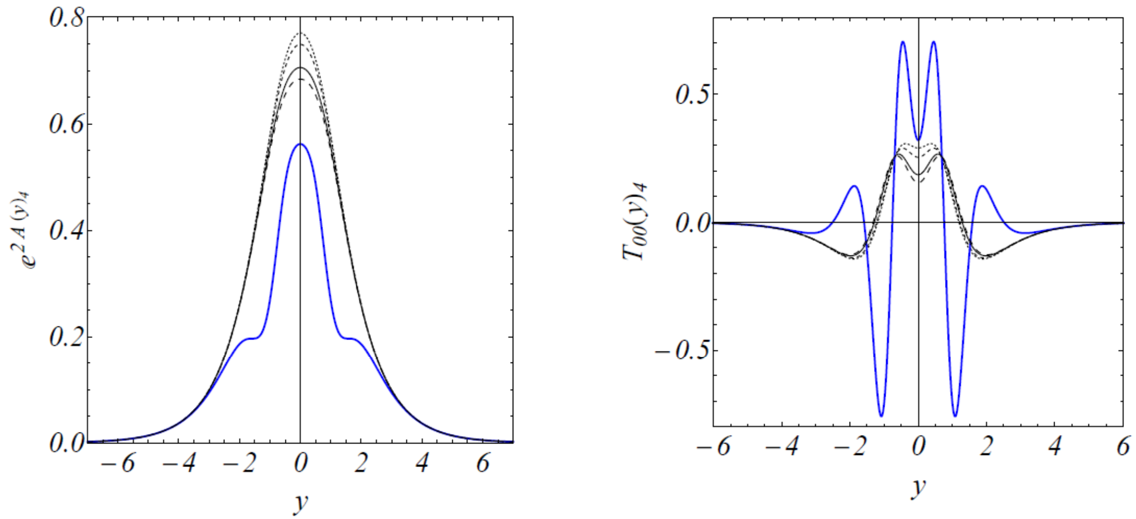


FIG. 4: Brane warp factor and energy-momentum tensor for model 4 derived from ψ_4. n ranges from $n = 1-2k; 1-k, 4, 1+k, 1+2kek = 0.05$. Blue line represents the solution for $n = 4$ while black lines ranges from the dotted to the dashed as n increases.

Braneworld models in terms of A

Field equations and first order formalism

$$\phi'' + 4A'\phi' = \frac{dV}{d\phi},$$

$$A'' = -\frac{2}{3}\phi'^2,$$

$$A''^2 = \frac{1}{6}\phi'^2 - \frac{1}{3}V(\phi).$$

→

$$V(\phi) = \frac{1}{8}\left(\frac{dW(\phi)}{d\phi}\right)^2 - \frac{1}{3}W(\phi)^2$$

$$\phi' = \frac{1}{2}\frac{dW(\phi)}{d\phi},$$

$$A' = -\frac{1}{3}W(\phi).$$

Braneworld models in terms of A

Brane scalar field

- Ansatz $$e^{2A} = \psi$$
- Known solution $$\psi_4 = \frac{(2n\,\mathrm{sech}(y) + \sin[2n\,\mathrm{sech}(y)])}{4n}$$
- First order formalism $$A' = -\frac{1}{3}W(\phi) \qquad \phi' = \frac{1}{2}\frac{dW(\phi)}{d\phi}$$
- Scalar field solution $$\phi = \int \sqrt{-\frac{3}{2}A''}\; dy$$

Configurational entropy as a tool to fix n

Analysis

- Metric perturbation
- Configurational Entropy (CE)
- Newtonian limit
- Hierarchy problem

Metric perturbation

$$ds^2 = e^{2A(y)} \left(\eta_{\mu\nu} + h_{\mu\nu}\right) dx^\mu \, dx^\nu - dy^2$$

$$\left(\frac{d^2}{dy^2} + 4\frac{dA}{dy}\frac{d}{dy} - e^{-2A(y)}\partial_\mu \partial^\mu\right) h_{\mu\nu}(x, y) = 0$$

Considering $h_{\mu\nu} \sim e^{ik^\nu x_\nu} e^{-3A/2} H_{\mu\nu}(z)$ and $dz = e^{-A(y)} dy$ one has

$$-\frac{d^2 H_{\mu\nu}}{dz^2} + U(z) H_{\mu\nu} = k^2 \, H_{\mu\nu}$$

with

$$U(z) = \frac{3}{2}A''(z) + \frac{9}{4}A'^2(z)$$

$$H_{\mu\nu}(z) = N_{\mu\nu} e^{3A(z)/2}$$

All the models have presented zero-mode normalized solution.

Configurational entropy

- Gleiser and Stamatopoulos [3] propose an entropic measure of ordering in field configuration space for nonlinear models with spatially localized energy solutions.

$$S_C[f] = -\sum f_n \ln(f_n).$$

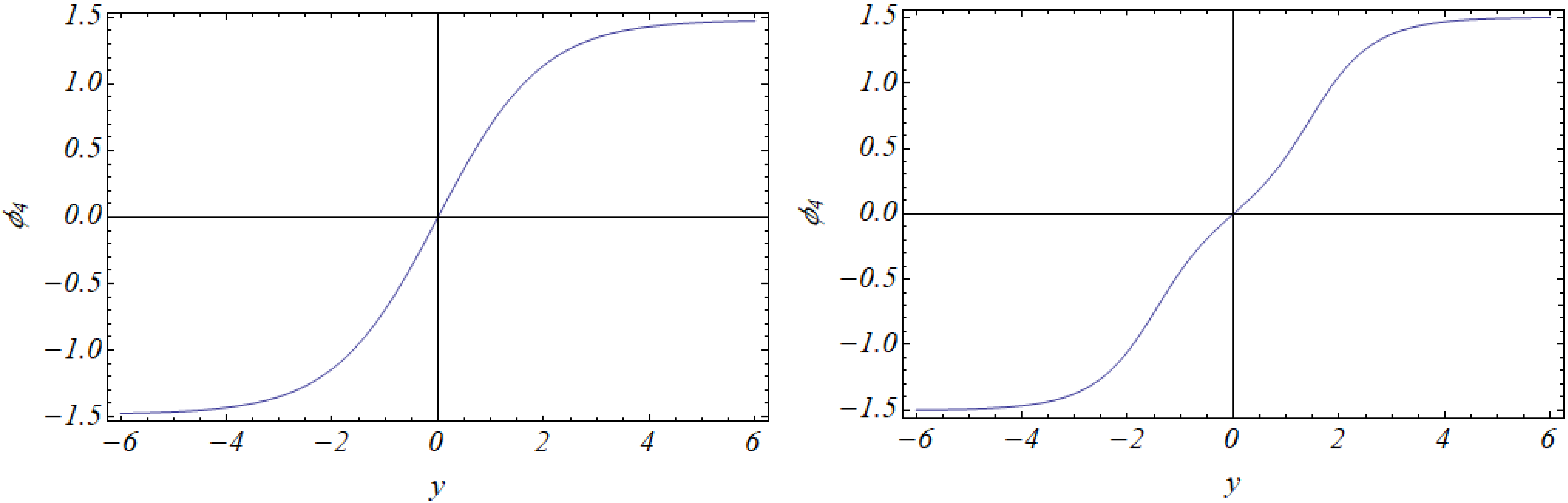
$$f(k) \to f_n = |A_n|^2 / \sum |A_n|^2$$

- where A_n is the coefficient of the n-th Fourier mode.

- Solutions to the eom tend to be the most ordered, given their specific dynamic constraints.

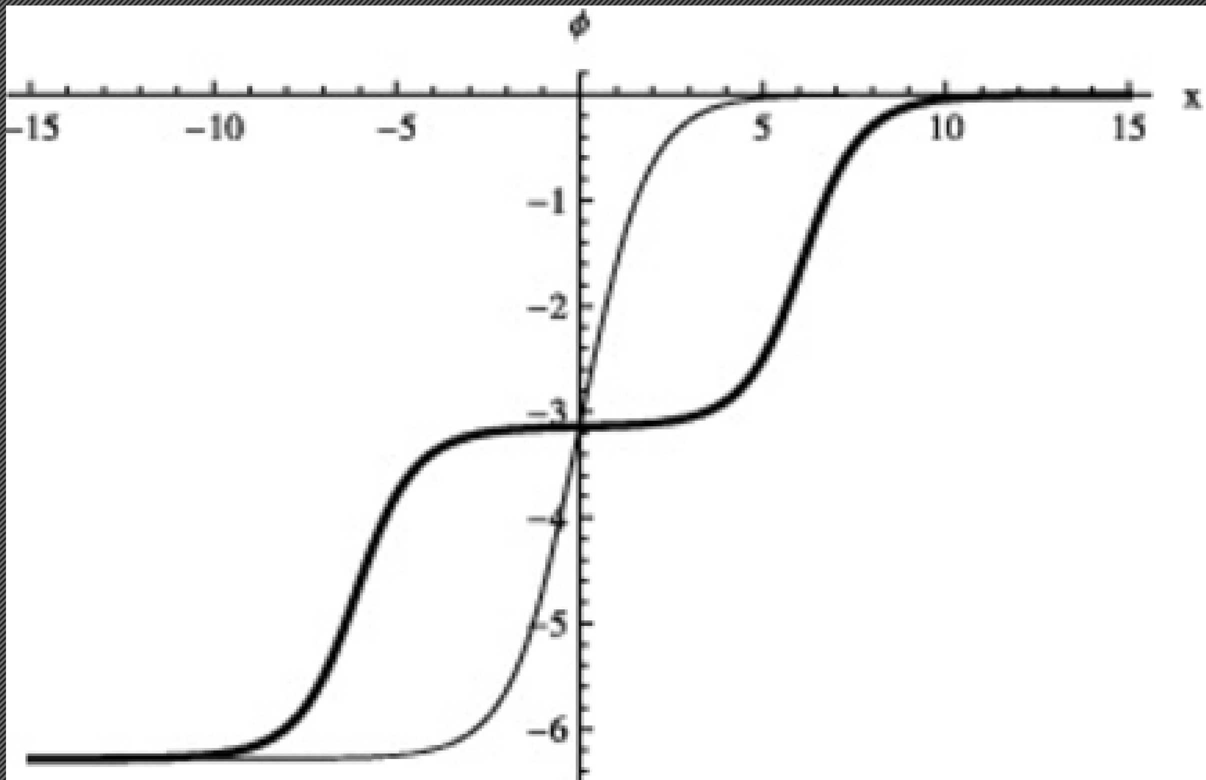
Brane scalar real solutions

CE - Model 4 for n = 0.85 and for n = 1.5



Brane scalar real solutions

Comparison between single and double kink



Configurational entropy as a tool to fix n

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Thank you

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[1] Bernardini, A. E.; Da Rocha, Roldão, Advances in High Energy Physics 2013, 1 (2013).
[2] Chinaglia, M. and Bernardini, A. E. and da Rocha, R., International Journal of Theoretical Physics 55, 4605 (2016).
[3] Marcelo Gleiser, Nikitas Stamatopoulos, Phys. Lett. B713, 304 (2012).

Braneworld models in terms of A

$$A(y)_1 = \frac{1}{2}\ln\left[\frac{-\ln[\cosh[n\tanh(y)]\,\mathrm{sech}(n)]}{n}\right],$$

$$T_{00}(y)_1 = \frac{3}{4}\left[n\,\mathrm{sech}(y)^4\,\mathrm{sech}[n\tanh(y)]^2 - 2\,\mathrm{sech}(y)^2\tanh(y)\tanh[n\tanh(y)]\right].$$

$$A(y)_2 = \frac{1}{2}\ln[\frac{-\cos(n)+\cos[n\tanh(y)]}{n}],$$

$$T_{00}(y)_2 = \frac{3}{4}\left[n\cos[n\tanh(y)]\,\mathrm{sech}(y)^4 - 2\,\mathrm{sech}(y)^2\sin[n\tanh(y)]\tanh(y)\right].$$

$$A(y)_3 = \frac{1}{2}\ln\left[\frac{\mathrm{sech}[n\tanh(y)] - \mathrm{sech}(n)}{n}\right],$$

$$T_{00}(y)_3 = -\frac{3}{8}\,\mathrm{sech}(y)^3\,\mathrm{sech}[n\tanh(y)]^3\times$$

$$[n(-3+\cosh(2n\tanh(y)))\,\mathrm{sech}(y) + 2\sinh(y)\sinh(2n\tanh(y))]$$