Massive and Newton-Cartan Gravity in Action

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Einstein's Theory of Gravity and its Modifications: From Theory to Observations

based on work with Jan Rosseel and Paul Townsend

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Motivation

special feature FQH Effect: existence of a gapped collective non-relativistic massive spin-2 excitation, known as the GMP mode

Girvin, MacDonald and Platzman (1985)

recent proposal for a non-relativistic spatially covariant bimetric EFT describing non-linear dynamics of this massive spin-2 GMP mode

Gromov and Son (2017)

proposal is reminiscent to non-relativistic and massive gravity

Cartan (1923); de Rham, Gabadadze, Tolley (2011)

see also lectures by Hassan and De Felice

Can EFT be obtained by some limit of Relativistic Massive Graviy?

Outline

Newton-Cartan Gravity

Outline

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Newton-Cartan Gravity

Massive Gravity

Non-relativistic Limits

Conclusions

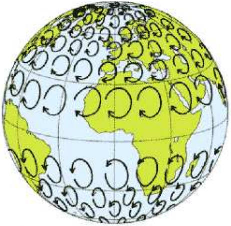
Outline

Condensed Matter

Effective Field Theory (EFT) coupled to NC background fields

serve as response functions and lead to restrictions on EFT

compare to



Coriolis force

Luttinger (1964), Greiter, Wilczek, Witten (1989), Son (2005, 2012), Can, Laskin, Wiegmann (2014)

Jensen (2014), Gromov, Abanov (2015), Gromov, Bradlyn (2017)

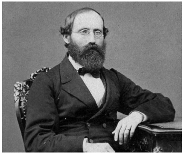
NC Gravity in a Nutshell

- Inertial frames: Galilean symmetries

- Constant acceleration: Newtonian gravity/Newton potential $\Phi(x)$

- no frame-independent formulation

  (needs geometry!)



Riemann (1867)

Galilei Symmetries

- time translations: $\delta t = \xi^0$ <u>but not</u> $\delta t = \lambda^i x^i$!
- space translations: $\delta x^i = \xi^i \qquad i = 1, 2, 3$
- spatial rotations: $\delta x^i = \lambda^i{}_j \, x^j$
- Galilean boosts: $\delta x^i = \lambda^i t$

$$[J_{ab}, P_c] = -2\delta_{c[a} P_{b]} \,, \qquad [J_{ab}, G_c] = -2\delta_{c[a} G_{b]} \,,$$

$$[G_a, H] = -P_a \,, \qquad [J_{ab}, J_{cd}] = \delta_{c[a} J_{b]d} - \delta_{a[c} J_{d]b} \,, \qquad a = 1, 2, 3$$

'Gauging' Galilei

symmetry	generators	gauge field	curvatures
time translations	H	τ_μ	$\mathcal{R}_{\mu\nu}(H)$
space translations	P^a	$e_\mu{}^a$	$\mathcal{R}_{\mu\nu}{}^a(P)$
Galilean boosts	G^a	$\omega_\mu{}^a$	$\mathcal{R}_{\mu\nu}{}^a(G)$
spatial rotations	J^{ab}	$\omega_\mu{}^{ab}$	$\mathcal{R}_{\mu\nu}{}^{ab}(J)$

Imposing Constraints

$\mathcal{R}_{\mu\nu}{}^a(P) = 0$: does only solve for part of $\omega_\mu{}^{ab}$

$\mathcal{R}_{\mu\nu}(H) = \partial_{[\mu}\tau_{\nu]} = 0 \quad \rightarrow$ absolute time $T = \int_{\mathcal{C}} \mathrm{d}x^\mu \tau_\mu = \int_{\mathcal{C}} \mathrm{d}\tau$

'Gauging' Bargmann

symmetry	generators	gauge field	curvatures
time translations	H	τ_μ	$\mathcal{R}_{\mu\nu}(H)$
space translations	P^a	${e_\mu}^a$	${\mathcal{R}_{\mu\nu}}^a(P)$
Galilean boosts	G^a	${\omega_\mu}^a$	${\mathcal{R}_{\mu\nu}}^a(G)$
spatial rotations	J^{ab}	${\omega_\mu}^{ab}$	${\mathcal{R}_{\mu\nu}}^{ab}(J)$
central charge transf.	Z	m_μ	$\mathcal{R}_{\mu\nu}(Z)$

Imposing Constraints

$${\mathcal{R}_{\mu\nu}}^a(P) = 0\,, \qquad \mathcal{R}_{\mu\nu}(Z) = 0\ :$$ solve for spin-connection fields

$$\mathcal{R}_{\mu\nu}(H) = \partial_{[\mu}\tau_{\nu]} = 0 \quad \rightarrow$$ absolute time ('zero torsion')

The NC Transformation Rules

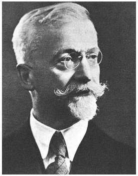
The independent NC fields $\{\tau_\mu, e_\mu{}^a, m_\mu\}$ transform as follows:

$$\delta\tau_\mu = 0\,,$$
$$\delta e_\mu{}^a = \lambda^a{}_b\, e_\mu{}^b + \lambda^a\tau_\mu\,,$$
$$\delta m_\mu = \partial_\mu\sigma + \lambda_a\, e_\mu{}^a$$

The spin-connection fields $\omega_\mu{}^{ab}$ and $\omega_\mu{}^a$ are functions of e, τ and m

The NC Equations of Motion

The NC equations of motion are given by



Élie Cartan 1923

$$\tau^\mu e^\nu{}_a \mathcal{R}_{\mu\nu}{}^a(G) = 0 \qquad \mathbf{1}$$

$$e^\nu{}_a \mathcal{R}_{\mu\nu}{}^{ab}(J) = 0 \qquad \mathbf{a} + (\mathbf{ab})$$

- after gauge-fixing and assuming flat space the first NC e.o.m. becomes $\triangle\Phi = 0$
- there is no known action that gives rise to these equations of motion

Outline

Newton-Cartan Gravity

Massive Gravity

Non-relativistic Limits

Conclusions

What is Massive Gravity ?

Massive Gravity is the name we have given to the attempt to understand what the gravitational force would be like if the graviton, the carrier of the gravitational force, has a small, but non-zero, mass

1939: Fierz-Pauli

- the free massive graviton, with mass m, is a spin-2 particle described by a symmetric tensor field $h_{\mu\nu}(x)$

- for $m = 0$ this tensor can be viewed as the linearized approximation to a metric tensor $g_{\mu\nu}(x)$:

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x) + O(h^2)$$

- the Fierz-Pauli kinetic term is the linearization of the Einstein-Hilbert term of general relativity

The Fierz-Pauli mass term

$$\mathcal{L}_{\mathrm{FP}}(\text{mass}) \sim m^2 \left(h^{\mu\nu} h_{\mu\nu} - h^2 \right) \qquad h \equiv \eta^{\mu\nu} h_{\mu\nu}$$

The Fierz-Pauli mass term

- breaks the linearized g.c.t. of the kinetic term:

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

- contains a reference metric $\bar{g}_{\mu\nu} = \eta_{\mu\nu}$

- requires fine-tuning

New Developments

First proposals for a non-linear mass term date back to

Salam, Strathdee (1969); Isham, Salam, Strathdee (1970); Zumino (1970)

- recent proposal in 4D by

de Rham, Gabadadze, Tolley (2011), Hassan, Rosen (2012)

- use Vierbeins $e_\mu{}^a$ instead of metric $g_{\mu\nu}$

Hinterbichler, Rosen (2010)

- 3D: Chern-Simons formulation in terms of $(e_\mu{}^a, \omega_\mu{}^a)$

Deser, Jackiw, 't Hooft (1984)

Achucarro, Townsend (1986); Witten (1988)

The 3D dRGT Chern Simons Model

$$\mathcal{L}_{\mathrm{dRGT}}(e,\omega;\bar{e}) = -M_{\mathrm{P}}\Big\{ e_a \wedge R^a(\omega) + \epsilon^{abc}\big(\alpha_1\, e_a \wedge e_b \wedge e_c + \beta_1\, e_a \wedge e_b \wedge \bar{e}_c + \beta_2\, e_a \wedge \bar{e}_b \wedge \bar{e}_c\big)\Big\}$$

$$R^a(\omega) = d\omega^a + \frac{1}{2}\epsilon^{abc}\omega_b \wedge \omega_c \; : \qquad \text{curvature tensor}$$

- reduces to Fierz-Pauli in linearized approximation

The 3D dRGT CS model can be obtained as a scaling limit of an underlying 3D "ZDG model" containing zwei Dreibeine

Zwei-Dreibein Gravity

4D : Hassan, Rosen (2012); de Haan, Hohm, Merbis, Townsend (2013)

we introduce two ("zwei") Dreibeine ${e_I}_\mu{}^a$ and two independent spin-connections ${\omega_I}_\mu{}^a$ $(I = 1, 2)$

$$\begin{aligned}\mathcal{L}_{\text{ZDG}}(e_I{}^a, \omega_I{}^a) \;=\; -M_{\mathrm{P}}\Big\{ & e_{1a} \wedge R_1^a + e_{2a} \wedge R_2^a \\ & +\epsilon^{abc}\Big(\alpha_1\, e_{1a} \wedge e_{1b} \wedge e_{1c} + \alpha_2\, e_{2a} \wedge e_{2b} \wedge e_{2c} + \\ & +\beta_1\, e_{1a} \wedge e_{1b} \wedge e_{2c} + \beta_2\, e_{1a} \wedge e_{2b} \wedge e_{2c}\Big)\Big\}\end{aligned}$$

Outline

From General Relativity to NC gravity

$$
\begin{array}{ccc}
\text{Poincare} \otimes \text{U(1)} & \overset{\text{`gauging'}}{\Longrightarrow} & \text{GR plus } \partial_\mu M_\nu - \partial_\nu M_\mu = 0 \\
\text{contraction} \quad \Downarrow & & \Downarrow \quad \text{the NC limit} \\
\text{Bargmann} & \overset{\text{`gauging'}}{\Longrightarrow} & \text{Newton-Cartan gravity}
\end{array}
$$

Contraction Poincare

$$\left[P_A, M_{BC}\right] = 2\,\eta_{A[B}\,P_{C]}\,, \qquad \left[M_{AB}, M_{CD}\right] = 4\,\eta_{[A[C}\,M_{D]B]}$$

$$P_0 = \frac{1}{2\omega}\,H\,, \qquad P_a = P_a\,, \qquad A = (0, a)$$

$$M_{ab} = J_{ab}\,, \qquad M_{a0} = \omega\,G_a$$

Taking the limit $\omega \rightarrow \infty$ gives the Galilei algebra:

$$\left[P_a, G_b\right] = 0$$

Contraction Poincare $\otimes$ U(1)

$$\left[P_A, M_{BC}\right] = 2\,\eta_{A[B}\,P_{C]}\,, \qquad \left[M_{AB}, M_{CD}\right] = 4\,\eta_{[A[C}\,M_{D]B]} \quad \text{plus} \quad \mathcal{Z}$$

$$P_0 = \frac{1}{2\omega}\,H + \omega\,Z\,, \qquad \mathcal{Z} = \frac{1}{2\omega}\,H - \omega\,Z\,, \qquad A = (0, a)$$

$$P_a = P_a\,, \qquad M_{ab} = J_{ab}\,, \qquad M_{a0} = \omega\,G_a$$

Taking the limit $\omega \to \infty$ gives the Bargmann algebra including Z:

$$\left[P_a, G_b\right] = \delta_{ab}\,Z$$

The NC Limit I

Dautcourt (1964); Rosseel, Zojer + E.B. (2015)

STEP I: express relativistic fields $\{E_\mu{}^A, M_\mu\}$ in terms of non-relativistic fields $\{\tau_\mu, e_\mu{}^a, m_\mu\}$

$$E_\mu{}^0 = \omega\,\tau_\mu + \frac{1}{2\omega}\,m_\mu\,, \quad M_\mu = \omega\,\tau_\mu - \frac{1}{2\omega}\,m_\mu\,, \quad E_\mu{}^a = e_\mu{}^a \qquad \Rightarrow$$

$$E^\mu{}_a = e^\mu{}_a - \frac{1}{2\omega^2}\,\tau^\mu e^\rho{}_a m_\rho + \mathcal{O}\left(\omega^{-4}\right) \text{ and similar for } E^\mu{}_0$$

The NC Limit II

STEP II: take the limit $\omega \to \infty$ in e.o.m. $\Rightarrow$

- the NC transformation rules are obtained
- the NC equations of motion are obtained (but no action!)

Note: the standard textbook limit gives Newton gravity

What about Matter?

Real Representations

consider a real scalar field with mass M

$$E^{-1}\,\mathcal{L}_{\rm rel} = -\frac{1}{2}\,g^{\mu\nu}\,\partial_\mu\Phi\partial_\nu\Phi - \frac{M^2}{2}\,\Phi^2$$

Rescale $\Phi = \frac{1}{\sqrt{\omega}}\phi$ and take non-relativistic limit $\omega \to \infty \quad \to$

$$e^{-1}\mathcal{L}_{\rm non-rel.} = -\frac{1}{2}\left(\partial_a\phi\right)^2 - \frac{M^2}{2}\,\phi^2$$

Complex Representations: from KG to Schrödinger

Jensen, Karch (2014), Rosseel, Zojer + E.B. (2015), Fuini, Karch, Uhlemann (2015)

$$\begin{array}{ccc} \text{Klein-Gordon} + \text{GR} & \overset{\text{‘limit’}}{\Longrightarrow} & \text{Schrödinger} + \text{NC} \\ \text{general frames} \quad \Uparrow & & \Downarrow \quad \text{inertial frames} \\ \text{Klein-Gordon} & \Longrightarrow & \text{Schrödinger} \end{array}$$

The Schrödinger Limit I

consider a complex scalar field with mass M

Lévy Leblond (1963,1967)

$$E^{-1}\,\mathcal{L}_{\rm rel} = -\frac{1}{2}\,g^{\mu\nu}\,D_\mu\Phi^*D_\nu\Phi - \frac{M^2}{2}\,\Phi^*\Phi \quad \text{with}$$

$$D_\mu\Phi = \partial_\mu\Phi - \mathrm{i}\,M\,M_\mu\,\Phi\,, \qquad \delta\Phi = \mathrm{i}\,M\,\Lambda\,\Phi$$

- M_μ is not an electromagnetic field ($M \neq q$)!
- M_μ couples to the current that expresses conservation of # particles − # antiparticles

The Schrödinger Limit II

Take non-relativistic limit extended with $M = \omega m\,, \Phi = \sqrt{\frac{\omega}{m}}\phi \quad \rightarrow$

$$e^{-1}\mathcal{L}_{\text{Schroedinger}} = \Big[\,\frac{\mathrm{i}}{2}\Big(\phi^*\mathcal{D}_0\phi - \phi\mathcal{D}_0\phi^*\Big) - \frac{1}{2m}\,\big|\mathcal{D}_a\phi\big|^2\,\Big] \quad \text{with}$$

$$\mathcal{D}_\mu\phi = \partial_\mu\phi + \mathrm{i}\,m\,m_\mu\,\phi\,, \qquad \delta\phi = \xi^\mu\partial_\mu\phi - \mathrm{i}\,m\,\sigma\,\phi$$

- m_μ couples to the current that expresses conservation of # particles
- going to inertial frames gives Schrödinger equation

Massive Particle

complex helicity mode $\Leftrightarrow$ Schrödinger

The Massive Spin 1 Case

J. Rosseel, P. Townsend + E.B., in preparation

consider a complex Proca field in D dimensions with mass M:

$$E^{-1}\mathcal{L}_{\rm rel} = -\frac{1}{4}\, g^{\mu\rho} g^{\nu\sigma} F^{*}_{\mu\nu} F_{\rho\sigma} - \frac{1}{2}\, M^2\, g^{\mu\nu} A^{*}_{\mu} A_{\nu} \quad \text{with}$$

$$F_{\mu\nu} = 2\, D_{[\mu} A_{\nu]} = 2\, \partial_{[\mu} A_{\nu]} - 2\,\mathrm{i}\, M\, M_{[\mu} A_{\nu]}\,, \qquad \delta A_{\mu} = \mathrm{i}\, M\, \Lambda\, A_{\mu}$$

Take non-rel. limit extended with $M = \omega m$ and go to inertial frames $\rightarrow$

$$e^{-1}\mathcal{L}_{\rm non-rel} = -\frac{1}{4}\, F^{*}_{ab} F^{ab} - \frac{1}{2}\,\mathrm{i}\, m\, A^{*}_{a} F_{a0} + \frac{1}{2}\,\mathrm{i}\, m\, A_{a}\, F^{*}_{a0} + \frac{1}{2}\, m^2\, \left|A_0\right|^2$$

Three Dimensions

flat background : $\quad \mathrm{Q} = \mathrm{A}_1 + \mathrm{i}\mathrm{A}_2\,, \qquad \tilde{\mathrm{Q}} = \mathrm{A}_1 - \mathrm{i}\mathrm{A}_2$

- Q and $\tilde{Q}$ transform under spatial rotations and $Q \overset{\mathrm{P}}{\Leftrightarrow} \tilde{Q}$

solve for A_0 : $\quad mA_0 = i\partial Q + i\bar{\partial}\tilde{Q} \qquad \Rightarrow$

$$\dot{Q} + \tfrac{i}{2m}\bar{\partial}\partial Q = 0 \quad \text{and} \quad \dot{\tilde{Q}} + \tfrac{i}{2m}\bar{\partial}\partial\tilde{Q} = 0$$

Alternative Limit Real Proca

2 complex helicity modes $\Leftrightarrow$ 2 x Schrödinger

but also

2 real P even helicity modes $\Leftrightarrow$ P odd Schrödinger

$$Q = A_1 + iA_2 \,, \qquad A_1, A_2 \text{ real}$$

Proca $\rightarrow$ $\sqrt{\text{Proca}}$

$$\mathcal{L}_{\sqrt{\text{Proca}}} = \epsilon^{\mu\nu\rho} A^*_\mu \partial_\nu A_\rho + M A^\star_\mu A^\mu$$

$$\begin{array}{ccc}
\text{Proca} & \xrightarrow{\text{non rel. limit}} & \text{Q} + \tilde{Q} \\
\text{'truncation'} \Downarrow & & \Downarrow \text{truncation} \\
\sqrt{\text{Proca}} & \xrightarrow{\text{'special' limit}} & \text{Q}
\end{array}$$

Proca $\rightarrow$ $\sqrt{\text{Proca}}$

$$\mathcal{L}_{\sqrt{\mathrm{Proca}}} = \epsilon^{\mu\nu\rho} A^*_\mu \partial_\nu A_\rho + M A^\star_\mu A^\mu - \frac{M}{2} B^\star_\mu B^\mu$$

$$\begin{array}{ccc}
\text{Proca} & \overset{\text{non rel. limit}}{\Longrightarrow} & Q \; + \; \tilde{Q} \\
\text{‘truncation’} \Downarrow & & \Downarrow \text{truncation} \\
\sqrt{\text{Proca}} & \overset{\textbf{‘special’ limit}}{\Longrightarrow} & Q
\end{array}$$

- there is an interesting relation with Scherk-Schwarz null reductions and 4D complex self-dual Maxwell

- consider NR limit of $\sqrt{\text{Proca}} \quad \rightarrow \quad \text{TME}$

- spinning article

A Road Map for Non-relativistic Massive Spin-2

- extend Proca to Fierz-Pauli (FP) and $\sqrt{\text{Proca}}$ to $\sqrt{\text{FP}}$

- Take non-relativistic limit of Zwei-Dreibein Gravity
  - two NC metrics $\rightarrow$ bimetic NC Gravity
  - one NC metric and one complex 'matter' metric

    $\rightarrow$ 2 x Schrödinger
  - one NC metric and one real 'matter' metric

    $\rightarrow$ P odd Schrödinger

- no action!

3D Gravity is not unique!

- 3D Galilei Algebra allows two central extensions. The second central extension is related to anyons

Jackiw, Nair (2000)

- limit of 3D General Relativity plus term with two auxiliary vector fields of the form $\epsilon^{\mu\nu\rho} M_\mu \partial_\nu S_\rho$ gives 3D Extended Bargmann Gravity

Rosseel + E.B. (2016)

- consider limit of Zwei-Dreibein Gravity with extra vector fields $\Rightarrow$
  - Bimetric Extended Bargmann Gravity
  - EFT describing non-linear dynamics of massive spin 2?

Outline

Newton-Cartan Gravity

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Open questions

- Does non-relativistic limit of some 3D Massive Relativistic Gravity model give the fully-covariant completion of the proposal of the EFT for the GMP mode in the FQE Effect?

Gromov and Son (2017)

- If so, can you also construct extensions involving
  - higher derivatives?
  - higher spins?

Take-Home Message

Taking non-relativistic limits is non-trivial!

Mainz Institute for Theoretical Physics

ACTIVITIES 2018

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SCIENTIFIC PROGRAMS

Probing Physics Beyond the SM with Precision
Ansgar Denner U Würzburg, Stefan Dittmaier U Freiburg, Tilman Plehn Heidelberg U
February 26 - March 9, 2018

Bridging the Standard Model to New Physics with the Parity Violation Program at MESA
Jens Erler UNAM, Mikhail Gorshteyn, Hubert Spiesberger JGU
April 23 - May 4, 2018

Modern Techniques for CFT and AdS
Bartlomiej Czech IAS Princeton, Michal P. Heller MPI for Gravitational Physics, Alessandro Vichi EPFL
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The Dawn of Gravitational Wave Science
Luis Lehner Perimeter Inst., Rafael A. Porto ICTP-SAIFR, Riccardo Sturani IIP Natal, Salvatore Vitale MIT
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The Future of BSM Physics
Gian Giudice CERN, Giulia Ricciardi U Naples Federico II, Tobias Hurth, Joachim Kopp, Matthias Neubert JGU
June 4 - 15, 2018, Capri, Italy

Probing Baryogenesis via LHC and Gravitational Wave Signatures
Germano Nardini U Bern, Carlos E.M. Wagner U Chicago / Argonne Nat. Lab., Pedro Schwaller JGU
June 18 - 29, 2018

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Fabrizio Caola IPPP Durham, Bernhard Mistlberger, Giulia Zanderighi CERN
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String Theory, Geometry and String Model Building
Philip Candelas, Xenia de la Ossa, Andre Lukas U Oxford, Daniel Waldram Imperial College London, Gabriele Honecker, Duco van Straten JGU
September 10 - 21, 2018

TOPICAL WORKSHOPS

The Evaluation of the Leading Hadronic Contribution to the Muon Anomalous Magnetic Moment
Carlo Carloni Calame INFN Pavia, Massimo Passera INFN Padua, Luca Trentadue U Parma, Graziano Venanzoni INFN Pisa
February 19 - 23, 2018

Applied Newton-Cartan Geometry
Eric Bergshoeff U Groningen, Niels Obers NBI Copenhagen, Dam Thanh Son U Chicago
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Richard Hill U Kentucky / Fermilab, Gil Paz Wayne State U, Randolf Pohl JGU
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Save the Date: March 12 - 16, 2018!