Kerr/CFT correspondence for accelerating and magnetised extremal black holes

Marco Astorino

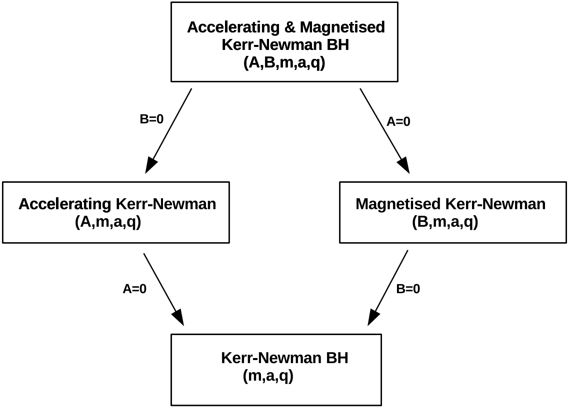
UNIVERSIDAD ADOLFO IBÁÑEZ

Universidad Adolfo Ibañez,
Viña del Mar, Chile

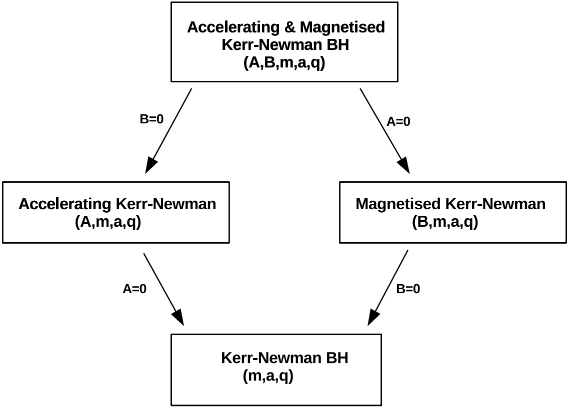
October 1, 2017

Phys.Lett. B760 393-405 , Phys.Lett. B751 96-106 , JHEP 1510 016

We are interested in exact and analytic deformations of Kerr-Newman (KN) black holes, for instance accelerating KN black hole or KN black hole immersed in an external electromagnetic field.



We are interested in exact and analytic deformations of Kerr-Newman (KN) black holes, for instance accelerating KN black hole or KN black hole immersed in an external electromagnetic field.



Example: Schwarzschild black hole immersed in Melvin magnetic universe

Schwarzschild Black Holes embedded in an external magnetic field

Example: Schwarzschild black hole immersed in Melvin magnetic universe

$$ds^2 = \Lambda^2(r,\theta)\left[-\left(1-\frac{2m}{r}\right)\,d\tau^2 + \frac{dr^2}{1-\frac{2m}{r}} + r^2 d\theta^2\right] + \frac{r^2\sin^2\theta}{\Lambda^2(r,\theta)}\,d\varphi^2$$

Example: Schwarzschild black hole immersed in Melvin magnetic universe

$$ds^2 = \Lambda^2(r,\theta)\left[-\left(1-\frac{2m}{r}\right)\, d\tau^2 + \frac{dr^2}{1-\frac{2m}{r}} + r^2 d\theta^2\right] + \frac{r^2\sin^2\theta}{\Lambda^2(r,\theta)}\, d\varphi^2$$

$$\Lambda(r,\theta) \;=\; 1 + \frac{B^2}{4} r^2 \sin^2\theta$$

Example: Schwarzschild black hole immersed in Melvin magnetic universe

$$ds^2 = \Lambda^2(r,\theta)\left[-\left(1-\frac{2m}{r}\right)\ d\tau^2 + \frac{dr^2}{1-\frac{2m}{r}} + r^2 d\theta^2\right] + \frac{r^2\sin^2\theta}{\Lambda^2(r,\theta)}\ d\varphi^2$$

$$\Lambda(r,\theta) \ = \ 1 + \frac{B^2}{4} r^2 \sin^2\theta$$

External magnetic field: $A_\mu = \left(0,0,0,-\frac{B}{2}\frac{r^2\sin^2\theta}{1+\frac{B^2}{4}r^2\sin^2\theta}\right)$

Example: Schwarzschild black hole immersed in Melvin magnetic universe

$$ds^2 = \Lambda^2(r,\theta)\left[-\left(1-\frac{2m}{r}\right)\ d\tau^2 + \frac{dr^2}{1-\frac{2m}{r}} + r^2 d\theta^2\right] + \frac{r^2\sin^2\theta}{\Lambda^2(r,\theta)}\ d\varphi^2$$

$$\Lambda(r,\theta) \ = \ 1 + \frac{B^2}{4} r^2 \sin^2\theta$$

External magnetic field: $A_\mu = \left(0,0,0,-\frac{B}{2}\frac{r^2\sin^2\theta}{1+\frac{B^2}{4}r^2\sin^2\theta}\right)$

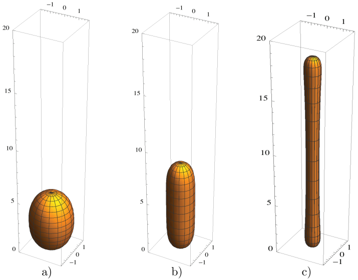


Figure: Three Melvin-Schwarzschild horizons embedded in $\mathbb{R}^3$. The magnetic field parameters are: a) $\mathcal{B}=0.1$, b) $\mathcal{B}=2.0$ and c) $\mathcal{B}=4.0$. Axes are in units of $m=r_+/2$.

1. Equatorial C_{eqt} and polar C_{pol} circumferences of the horizon

$$C_{eqt} = \int_0^{2\pi} \sqrt{g_{\phi\phi}}\, d\phi \Big|_{r=r_+} = \frac{2\pi\ r_+}{1+\frac{B^2}{4}r_+^2} < 2\pi r_+ \quad ,$$

$$C_{pol} = \int_0^{2\pi} \sqrt{g_{\theta\theta}}\, d\theta \Big|_{r=r_+} = 2\pi r_+ \left(1+\frac{B^2}{4}r_+^2\right) > 2\pi r_+ \quad .$$

2. The horizon area $\mathcal{A}$ is unchanged by the presence of the external magnetic field:

$$\mathcal{A} = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sqrt{g_{\theta\theta}}\sqrt{g_{\phi\phi}} = 4\pi r_+^2$$

3. Melvin Magnetic Universe Background for $m \to 0$ $\quad (\rho := r\sin\theta,\ \ z := r\cos\theta)$

$$ds^2 = \left(1+\frac{B^2}{4}\rho^2\right)^2 \left[-d\tau^2 + d\rho^2 + dz^2\right] + \frac{\rho^2\ d\phi^2}{\left(1+\frac{B^2}{4}\rho^2\right)^2}$$

4. The Melvin magnetic universe is a static, non-singular, cylindrical symmetric space-time in which there exists an axial magnetic field aligned with the z-axis. It describes a universe containing a parallel bundle of electromagnetic flux held together by its own gravitational field.

1. Equatorial C_{eqt} and polar C_{pol} circumferences of the horizon

$$C_{eqt} = \int_0^{2\pi} \sqrt{g_{\phi\phi}}\ d\phi\Big|_{r=r_+} = \frac{2\pi\ r_+}{1+\frac{B^2}{4}r_+^2} < 2\pi r_+ \quad ,$$

$$C_{pol} = \int_0^{2\pi} \sqrt{g_{\theta\theta}}\ d\theta\ \Big|_{r=r_+} = 2\pi r_+\left(1+\frac{B^2}{4}r_+^2\right) > 2\pi r_+ \quad .$$

2. The horizon area $\mathcal{A}$ is unchanged by the presence of the external magnetic field:

$$\mathcal{A} = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sqrt{g_{\theta\theta}}\sqrt{g_{\phi\phi}} = 4\pi r_+^2$$

3. Melvin Magnetic Universe Background for $m \to 0$ $\quad (\rho := r\sin\theta,\ \ z := r\cos\theta)$

$$ds^2 = \left(1+\frac{B^2}{4}\rho^2\right)^2\left[-d\tau^2+d\rho^2+dz^2\right]+\frac{\rho^2\ d\phi^2}{\left(1+\frac{B^2}{4}\rho^2\right)^2}$$

4. The Melvin magnetic universe is a static, non-singular, cylindrical symmetric space-time in which there exists an axial magnetic field aligned with the z-axis. It describes a universe containing a parallel bundle of electromagnetic flux held together by its own gravitational field.

1. Equatorial C_{eqt} and polar C_{pol} circumferences of the horizon

$$C_{eqt} = \int_0^{2\pi} \sqrt{g_{\phi\phi}}\ d\phi\Big|_{r=r_+} = \frac{2\pi\ r_+}{1+\frac{B^2}{4}r_+^2} < 2\pi r_+ \quad ,$$

$$C_{pol} = \int_0^{2\pi} \sqrt{g_{\theta\theta}}\ d\theta\ \Big|_{r=r_+} = 2\pi r_+ \left(1+\frac{B^2}{4}r_+^2\right) > 2\pi r_+ \quad .$$

2. The horizon area $\mathcal{A}$ is unchanged by the presence of the external magnetic field:

$$\mathcal{A} = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sqrt{g_{\theta\theta}}\sqrt{g_{\phi\phi}} = 4\pi r_+^2$$

3. Melvin Magnetic Universe Background for $m \to 0$ $\quad (\rho := r\sin\theta,\ \ z := r\cos\theta)$

$$ds^2 = \left(1+\frac{B^2}{4}\rho^2\right)^2 \left[-d\tau^2 + d\rho^2 + dz^2\right] + \frac{\rho^2\ d\phi^2}{\left(1+\frac{B^2}{4}\rho^2\right)^2}$$

4. The Melvin magnetic universe is a static, non-singular, cylindrical symmetric space-time in which there exists an axial magnetic field aligned with the z-axis. It describes a universe containing a parallel bundle of electromagnetic flux held together by its own gravitational field.

1. Equatorial C_{eqt} and polar C_{pol} circumferences of the horizon

$$C_{eqt} = \int_0^{2\pi} \sqrt{g_{\phi\phi}}\, d\phi\Big|_{r=r_+} = \frac{2\pi\ r_+}{1+\frac{B^2}{4}r_+^2} < 2\pi r_+ \quad ,$$

$$C_{pol} = \int_0^{2\pi} \sqrt{g_{\theta\theta}}\, d\theta\ \Big|_{r=r_+} = 2\pi r_+ \left(1+\frac{B^2}{4}r_+^2\right) > 2\pi r_+ \quad .$$

2. The horizon area $\mathcal{A}$ is unchanged by the presence of the external magnetic field:

$$\mathcal{A} = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sqrt{g_{\theta\theta}}\sqrt{g_{\phi\phi}} = 4\pi r_+^2$$

3. Melvin Magnetic Universe Background for $m \to 0$ $\quad (\rho := r\sin\theta,\ \ z := r\cos\theta)$

$$ds^2 = \left(1+\frac{B^2}{4}\rho^2\right)^2 \left[-d\tau^2 + d\rho^2 + dz^2\right] + \frac{\rho^2\ d\phi^2}{\left(1+\frac{B^2}{4}\rho^2\right)^2}$$

4. The Melvin magnetic universe is a static, non-singular, cylindrical symmetric space-time in which there exists an axial magnetic field aligned with the z-axis. It describes a universe containing a parallel bundle of electromagnetic flux held together by its own gravitational field.

Kerr black holes embedded in an external magnetic field

Electrovacuum, axisymmetric and stationary Kerr (Newman) black hole immersed in an external magnetic field

$$ds^2 = -\frac{f(\tilde{r},x)}{|\Lambda(\tilde{r},x)|^2}[\Delta_\varphi d\tilde{\varphi} - \omega(\tilde{r},x)d\tilde{t}]^2 + \frac{|\Lambda(\tilde{r},x)|^2}{f(\tilde{r},x)}\left[\rho^2 d\tilde{t}^2 - e^{-2\gamma(\tilde{r},x)}\left(\frac{d\tilde{r}^2}{\Delta_r} + \frac{dx^2}{\Delta_x}\right)\right]$$

$$A = A_{\tilde{t}}(\tilde{r},x)d\tilde{t} + A_{\tilde{\varphi}}(\tilde{r},x)d\tilde{\varphi}$$

1. When the external magnetic field vanish $\mathbf{B=0}$, the above solution recovers the **Kerr black hole**
2. For $\mathbf{m=a=0}$ the metric recovers the **Melvin Magnetic Universe**.
3. Exactly as Kerr black hole, the magnetised solution posses an inner $\tilde{r}_-$ and an outer (event) **horizon** $\tilde{\mathbf{r}}_+$ located at

$$\tilde{r}_\pm = m \pm \sqrt{m^2 - a^2} \quad .$$

4. Magnetised Black hole **area**

$$\mathcal{A} = \int_0^{2\pi} d\tilde{\varphi} \int_{-1}^{1} dx \sqrt{g_{\tilde{\varphi}\tilde{\varphi}} g_{xx}}\Big|_{\tilde{r}=\tilde{r}_+} = 4\pi\Delta_\varphi(\tilde{r}_+^2 + a^2)$$

Electrovacuum, axisymmetric and stationary Kerr (Newman) black hole immersed in an external magnetic field

$$ds^2 = -\frac{f(\tilde{r},x)}{|\Lambda(\tilde{r},x)|^2}\left[\Delta_\varphi d\tilde{\varphi} - \omega(\tilde{r},x)d\tilde{t}\right]^2 + \frac{|\Lambda(\tilde{r},x)|^2}{f(\tilde{r},x)}\left[\rho^2 d\tilde{t}^2 - e^{-2\gamma(\tilde{r},x)}\left(\frac{d\tilde{r}^2}{\Delta_r} + \frac{dx^2}{\Delta_x}\right)\right]$$

$$A = A_{\tilde{t}}(\tilde{r},x)d\tilde{t} + A_{\tilde{\varphi}}(\tilde{r},x)d\tilde{\varphi}$$

1. When the external magnetic field vanish $\mathbf{B}=\mathbf{0}$, the above solution recovers the **Kerr black hole**
2. For $\mathbf{m}=\mathbf{a}=\mathbf{0}$ the metric recovers the **Melvin Magnetic Universe**.
3. Exactly as Kerr black hole, the magnetised solution posses an inner $\tilde{r}_-$ and an outer (event) **horizon** $\tilde{\mathbf{r}}_+$ located at

$$\tilde{r}_\pm = m \pm \sqrt{m^2 - a^2} \quad .$$

4. Magnetised Black hole **area**

$$\mathcal{A} = \int_0^{2\pi} d\tilde{\varphi} \int_{-1}^{1} dx \sqrt{g_{\tilde{\varphi}\tilde{\varphi}} g_{xx}}\Big|_{\tilde{r}=\tilde{r}_+} = 4\pi\Delta_\varphi(\tilde{r}_+^2 + a^2)$$

Kerr black holes embedded in an external magnetic field

Electrovacuum, axisymmetric and stationary Kerr (Newman) black hole immersed in an external magnetic field

$$ds^2 = -\frac{f(\tilde{r},x)}{|\Lambda(\tilde{r},x)|^2}\left[\Delta_\varphi d\tilde{\varphi} - \omega(\tilde{r},x)d\tilde{t}\right]^2 + \frac{|\Lambda(\tilde{r},x)|^2}{f(\tilde{r},x)}\left[\rho^2 d\tilde{t}^2 - e^{-2\gamma(\tilde{r},x)}\left(\frac{d\tilde{r}^2}{\Delta_r} + \frac{dx^2}{\Delta_x}\right)\right]$$

$$A = A_{\tilde{t}}(\tilde{r},x)d\tilde{t} + A_{\tilde{\varphi}}(\tilde{r},x)d\tilde{\varphi}$$

1. When the external magnetic field vanish $\mathbf{B}=\mathbf{0}$, the above solution recovers the **Kerr black hole**
2. For $\mathbf{m}=\mathbf{a}=\mathbf{0}$ the metric recovers the **Melvin Magnetic Universe**.
3. Exactly as Kerr black hole, the magnetised solution posses an inner $\tilde{r}_-$ and an outer (event) **horizon** $\tilde{\mathbf{r}}_+$ located at

$$\tilde{r}_\pm = m \pm \sqrt{m^2 - a^2} \quad .$$

4. Magnetised Black hole **area**

$$\mathcal{A} = \int_0^{2\pi} d\tilde{\varphi} \int_{-1}^{1} dx \sqrt{g_{\tilde{\varphi}\tilde{\varphi}}\, g_{xx}}\Big|_{\tilde{r}=\tilde{r}_+} = 4\pi\Delta_\varphi(\tilde{r}_+^2 + a^2)$$

Electrovacuum, axisymmetric and stationary Kerr (Newman) black hole immersed in an external magnetic field

$$ds^2 = -\frac{f(\tilde{r},x)}{|\Lambda(\tilde{r},x)|^2}[\Delta_\varphi d\tilde{\varphi} - \omega(\tilde{r},x)d\tilde{t}]^2 + \frac{|\Lambda(\tilde{r},x)|^2}{f(\tilde{r},x)}\left[\rho^2 d\tilde{t}^2 - e^{-2\gamma(\tilde{r},x)}\left(\frac{d\tilde{r}^2}{\Delta_r} + \frac{dx^2}{\Delta_x}\right)\right]$$

$$A = A_{\tilde{t}}(\tilde{r},x)d\tilde{t} + A_{\tilde{\varphi}}(\tilde{r},x)d\tilde{\varphi}$$

1. When the external magnetic field vanish $\mathbf{B} = \mathbf{0}$, the above solution recovers the **Kerr black hole**
2. For $\mathbf{m} = \mathbf{a} = \mathbf{0}$ the metric recovers the **Melvin Magnetic Universe**.
3. Exactly as Kerr black hole, the magnetised solution posses an inner $\tilde{r}_-$ and an outer (event) **horizon** $\tilde{\mathbf{r}}_+$ located at

$$\tilde{r}_\pm = m \pm \sqrt{m^2 - a^2} \quad .$$

4. Magnetised Black hole **area**

$$\mathcal{A} = \int_0^{2\pi} d\tilde{\varphi} \int_{-1}^{1} dx \sqrt{g_{\tilde{\varphi}\tilde{\varphi}} g_{xx}}\Big|_{\tilde{r}=\tilde{r}_+} = 4\pi\Delta_\varphi(\tilde{r}_+^2 + a^2)$$

Kerr black holes embedded in an external magnetic field

Electrovacuum, axisymmetric and stationary Kerr (Newman) black hole immersed in an external magnetic field

$$ds^2 = -\frac{f(\tilde{r},x)}{|\Lambda(\tilde{r},x)|^2}[\Delta_\varphi d\tilde{\varphi} - \omega(\tilde{r},x)d\tilde{t}]^2 + \frac{|\Lambda(\tilde{r},x)|^2}{f(\tilde{r},x)}\left[\rho^2 d\tilde{t}^2 - e^{-2\gamma(\tilde{r},x)}\left(\frac{d\tilde{r}^2}{\Delta_r} + \frac{dx^2}{\Delta_x}\right)\right]$$

$$A = A_{\tilde{t}}(\tilde{r},x)d\tilde{t} + A_{\tilde{\varphi}}(\tilde{r},x)d\tilde{\varphi}$$

1. When the external magnetic field vanish $\mathbf{B} = \mathbf{0}$, the above solution recovers the **Kerr black hole**
2. For $\mathbf{m} = \mathbf{a} = \mathbf{0}$ the metric recovers the **Melvin Magnetic Universe**.
3. Exactly as Kerr black hole, the magnetised solution posses an inner $\tilde{r}_-$ and an outer (event) **horizon** $\tilde{\mathbf{r}}_+$ located at

$$\tilde{r}_\pm = m \pm \sqrt{m^2 - a^2} \qquad .$$

4. Magnetised Black hole **area**

$$\mathcal{A} = \int_0^{2\pi} d\tilde{\varphi} \int_{-1}^{1} dx \sqrt{g_{\tilde{\varphi}\tilde{\varphi}} g_{xx}}\Big|_{\tilde{r}=\tilde{r}_+} = 4\pi\Delta_\varphi(\tilde{r}_+^2 + a^2)$$

$$\Delta_r(\tilde{r}) := \tilde{r}^2 + a^2 - 2m\tilde{r} \quad , \qquad \Delta_x(x) := 1 - x^2 \qquad , \qquad \Delta_\varphi := 1 + a^2 m^2 B^4 \ ,$$

$$f(\tilde{r},x) := -\frac{h\ \Delta_x}{\Sigma} \qquad , \qquad \Sigma(\tilde{r},x) := r^2 + a^2 x^2 \qquad , \qquad \rho^2(\tilde{r},x) := \Delta_r \Delta_x \ ,$$

$$h(\tilde{r},x) := (\tilde{r}^2 + a^2)^2 - a^2 \Delta_r \Delta_x \quad , \quad \omega(\tilde{r},x) := \frac{a(1 - a^2 m^2 B^4) - \beta \Delta_r}{\tilde{r}^2 + a^2} + \frac{3}{4} a m^2 B^4,$$

$$\Lambda(\tilde{r},x) := 1 + \frac{B^2}{4}\left[(\tilde{r}^2 + a^2)\Delta_x - 2iamx(3 - x^2) + \frac{2ma^2\Delta_x^2}{\tilde{r} + iax}\right] \ , \ e^{-2\gamma}(\tilde{r},x) := h\ \Delta_x$$

$$\beta(\tilde{r},x) := \frac{a\Sigma}{h} + \frac{B^4}{16}\left\{-8m\tilde{r}ax^2(3 - x^2) - 6m\tilde{r}a(1 - x^2)^2 + \frac{2ma^3(1 - x^2)^3}{h}\left[(\tilde{r}^2 + a^2)\tilde{r} + 2ma^2\right]\right.$$
$$\left. + \frac{4m^2a^3x^2}{h}\left[(\tilde{r}^2 + a^2)(3 - x^2)^2 - 4a^2(1 - x^2)\right]\right\}$$

$$A_{\tilde{\varphi}}(\tilde{r},x) := A_{\tilde{\varphi}_0} + \frac{B\Delta_\varphi}{8\Sigma|\Lambda|^2}\left\{a^6B^2x^2(1 - x^2)^2 + \tilde{r}^4(1 - x^2)[4 + B^2\tilde{r}^2(1 - x^2)]\right.$$
$$+ a^2\tilde{r}\Big[4B^2m^2\tilde{r}x^2(3 - x^2)^2 + 4m(1 - x^2)^2[2 + B^2\tilde{r}^2(1 - x^2)] + \tilde{r}[4 - 4x^4 + B^2\tilde{r}^2(2 - 3x^2 + x^6)]\Big]$$
$$\left. + a^4\Big[4x^2(1 - x^2) + B^2[4m\tilde{r}(1 - x^2)^3 + 4m^2(1 + x^2)^2 + \tilde{r}^2(1 - 3x^4 + 2x^6)]\Big]\right\}$$

$$A_{\tilde{t}}(\tilde{r},x) := A_{\tilde{t}_0} - \omega\left(\frac{A_{\tilde{\varphi}} - A_{\tilde{\varphi}_0}}{\Delta_\varphi}\right) +$$
$$-amB^3\left[2\tilde{r} + \frac{1}{h}\left(4a^2m^2\tilde{r} - \tilde{r}(\tilde{r}^2 + a^2)\Delta_r\Delta_x - \frac{1}{4}(\tilde{r}^3 - 3a^2\tilde{r} + 2ma^2)\Delta_r\Delta_x^2\right)\right],$$

$$\Delta_r(\tilde{r}) := \tilde{r}^2 + a^2 - 2m\tilde{r} \quad , \qquad \Delta_x(x) := 1 - x^2 \quad , \qquad \Delta_\varphi := 1 + a^2 m^2 B^4 \ ,$$

$$f(\tilde{r}, x) := -\frac{h\ \Delta_x}{\Sigma} \quad , \qquad \Sigma(\tilde{r}, x) := r^2 + a^2 x^2 \quad , \qquad \rho^2(\tilde{r}, x) := \Delta_r \Delta_x \ ,$$

$$h(\tilde{r}, x) := (\tilde{r}^2 + a^2)^2 - a^2 \Delta_r \Delta_x \quad , \quad \omega(\tilde{r}, x) := \frac{a(1 - a^2 m^2 B^4) - \beta \Delta_r}{\tilde{r}^2 + a^2} + \frac{3}{4} a m^2 B^4,$$

$$\Lambda(\tilde{r}, x) := 1 + \frac{B^2}{4} \left[(\tilde{r}^2 + a^2)\Delta_x - 2iamx(3 - x^2) + \frac{2ma^2 \Delta_x^2}{\tilde{r} + iax} \right] \ , \ e^{-2\gamma}(\tilde{r}, x) := h\ \Delta_x$$

$$\beta(\tilde{r}, x) := \frac{a\Sigma}{h} + \frac{B^4}{16} \left\{ -8m\tilde{r}ax^2(3 - x^2) - 6m\tilde{r}a(1 - x^2)^2 + \frac{2ma^3(1 - x^2)^3}{h} \left[(\tilde{r}^2 + a^2)\tilde{r} + 2ma^2 \right] \right.$$
$$\left. + \frac{4m^2 a^3 x^2}{h} \left[(\tilde{r}^2 + a^2)(3 - x^2)^2 - 4a^2(1 - x^2) \right] \right\}$$

$$A_{\tilde{\varphi}}(\tilde{r}, x) := A_{\tilde{\varphi}_0} + \frac{B\Delta\varphi}{8\Sigma|\Lambda|^2} \left\{ a^6 B^2 x^2 (1 - x^2)^2 + \tilde{r}^4 (1 - x^2)[4 + B^2 \tilde{r}^2 (1 - x^2)] \right.$$
$$+ a^2 \tilde{r} \Big[4B^2 m^2 \tilde{r} x^2 (3 - x^2)^2 + 4m(1 - x^2)^2 [2 + B^2 \tilde{r}^2 (1 - x^2)] + \tilde{r}[4 - 4x^4 + B^2 \tilde{r}^2 (2 - 3x^2 + x^6)] \Big]$$
$$\left. + a^4 \Big[4x^2(1 - x^2) + B^2[4m\tilde{r}(1 - x^2)^3 + 4m^2(1 + x^2)^2 + \tilde{r}^2(1 - 3x^4 + 2x^6)] \Big] \right\}$$

$$A_{\tilde{t}}(\tilde{r}, x) := A_{\tilde{t}_0} - \omega \left(\frac{A_{\tilde{\varphi}} - A_{\tilde{\varphi}_0}}{\Delta_\varphi} \right) +$$
$$- amB^3 \left[2\tilde{r} + \frac{1}{h} \left(4a^2 m^2 \tilde{r} - \tilde{r}(\tilde{r}^2 + a^2)\Delta_r \Delta_x - \frac{1}{4}(\tilde{r}^3 - 3a^2 \tilde{r} + 2ma^2)\Delta_r \Delta_x^2 \right) \right],$$

$$\Delta_r(\tilde{r}) := \tilde{r}^2 + a^2 - 2m\tilde{r} \quad , \qquad \Delta_x(x) := 1 - x^2 \quad , \qquad \Delta_\varphi := 1 + a^2 m^2 B^4 \ ,$$

$$f(\tilde{r}, x) := -\frac{h \ \Delta_x}{\Sigma} \quad , \qquad \Sigma(\tilde{r}, x) := r^2 + a^2 x^2 \quad , \qquad \rho^2(\tilde{r}, x) := \Delta_r \Delta_x \ ,$$

$$h(\tilde{r}, x) := (\tilde{r}^2 + a^2)^2 - a^2 \Delta_r \Delta_x \quad , \quad \omega(\tilde{r}, x) := \frac{a(1 - a^2 m^2 B^4) - \beta \Delta_r}{\tilde{r}^2 + a^2} + \frac{3}{4} a m^2 B^4,$$

$$\Lambda(\tilde{r}, x) := 1 + \frac{B^2}{4} \left[(\tilde{r}^2 + a^2)\Delta_x - 2iamx(3 - x^2) + \frac{2ma^2 \Delta_x^2}{\tilde{r} + iax} \right] \ , \ e^{-2\gamma}(\tilde{r}, x) := h \ \Delta_x$$

$$\beta(\tilde{r}, x) := \frac{a\Sigma}{h} + \frac{B^4}{16} \left\{ -8m\tilde{r}ax^2(3 - x^2) - 6m\tilde{r}a(1 - x^2)^2 + \frac{2ma^3(1 - x^2)^3}{h} \left[(\tilde{r}^2 + a^2)\tilde{r} + 2ma^2 \right] \right.$$

$$\left. + \frac{4m^2 a^3 x^2}{h} \left[(\tilde{r}^2 + a^2)(3 - x^2)^2 - 4a^2(1 - x^2) \right] \right\}$$

$$A_{\tilde{\varphi}}(\tilde{r}, x) := A_{\tilde{\varphi}_0} + \frac{B\Delta_\varphi}{8\Sigma|\Lambda|^2} \left\{ a^6 B^2 x^2 (1 - x^2)^2 + \tilde{r}^4 (1 - x^2)[4 + B^2 \tilde{r}^2 (1 - x^2)] \right.$$

$$+ a^2 \tilde{r} \left[4B^2 m^2 \tilde{r} x^2 (3 - x^2)^2 + 4m(1 - x^2)^2 [2 + B^2 \tilde{r}^2 (1 - x^2)] + \tilde{r}[4 - 4x^4 + B^2 \tilde{r}^2 (2 - 3x^2 + x^6)] \right]$$

$$\left. + a^4 \left[4x^2 (1 - x^2) + B^2 [4m\tilde{r}(1 - x^2)^3 + 4m^2 (1 + x^2)^2 + \tilde{r}^2 (1 - 3x^4 + 2x^6)] \right] \right\}$$

$$A_{\tilde{t}}(\tilde{r}, x) := A_{\tilde{t}_0} - \omega \left(\frac{A_{\tilde{\varphi}} - A_{\tilde{\varphi}_0}}{\Delta_\varphi} \right) +$$

$$-amB^3 \left[2\tilde{r} + \frac{1}{h} \left(4a^2 m^2 \tilde{r} - \tilde{r}(\tilde{r}^2 + a^2)\Delta_r \Delta_x - \frac{1}{4}(\tilde{r}^3 - 3a^2 \tilde{r} + 2ma^2)\Delta_r \Delta_x^2 \right) \right],$$

$$ds^2 = \frac{1}{(1 + Ar\cos\theta)^2}\left[-Q(r)\,dt^2 + \frac{dr^2}{Q(r)} + \frac{r^2\,d\theta^2}{P(\theta)} + \frac{P(\theta)\,r^2\sin^2\theta}{(1+2Am)^2}\,d\phi^2\right]$$

where

$$P(\theta) = 1 + 2Am\cos\theta \qquad , \qquad Q(r) = (1 - A^2 r^2)\left(1 - \frac{2m}{r}\right).$$

The event and accelerating horizon are respectively located at

$$r_h = 2m \qquad , \qquad r_A = \frac{1}{A}$$

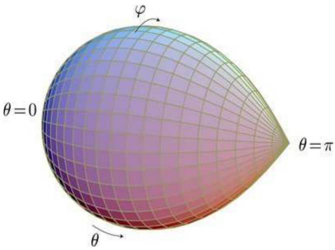


Figure: The surface of constant t and r embedded into $\mathbb{E}^3$. This is regular at $\theta = 0$, but there is a conical singularity at $\theta = \pi$ corresponding to the deficit angle $\delta_\pi = \frac{8\pi Am}{1+2Am}$.

$$ds^2 = \frac{1}{(1 + Ar\cos\theta)^2}\left[-Q(r)\,dt^2 + \frac{dr^2}{Q(r)} + \frac{r^2\,d\theta^2}{P(\theta)} + \frac{P(\theta)\,r^2\sin^2\theta}{(1+2Am)^2}\,d\phi^2\right]$$

where

$$P(\theta) = 1 + 2Am\cos\theta \qquad , \qquad Q(r) = (1 - A^2r^2)\Big(1 - \frac{2m}{r}\Big).$$

The event and accelerating horizon are respectively located at

$$r_h = 2m \qquad , \qquad r_A = \frac{1}{A}$$

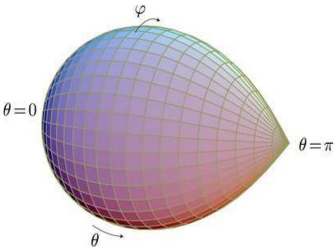


Figure: The surface of constant t and r embedded into $\mathbb{E}^3$. This is regular at $\theta = 0$, but there is a conical singularity at $\theta = \pi$ corresponding to the deficit angle $\delta_\pi = \frac{8\pi Am}{1+2Am}$.

Accelerating Schwarzschild Black Holes (C-metrics)

$$ds^2 = \frac{1}{(1 + Ar\cos\theta)^2}\left[-Q(r)\,dt^2 + \frac{dr^2}{Q(r)} + \frac{r^2\,d\theta^2}{P(\theta)} + \frac{P(\theta)\,r^2\sin^2\theta}{(1+2Am)^2}\,d\phi^2\right]$$

where

$$P(\theta) = 1 + 2Am\cos\theta \qquad , \qquad Q(r) = (1 - A^2r^2)\left(1 - \frac{2m}{r}\right).$$

The event and accelerating horizon are respectively located at

$$r_h = 2m \qquad , \qquad r_A = \frac{1}{A}$$

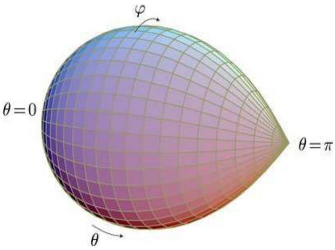


Figure: The surface of constant t and r embedded into $\mathbb{E}^3$. This is regular at $\theta = 0$, but there is a conical singularity at $\theta = \pi$ corresponding to the deficit angle $\delta_\pi = \frac{8\pi Am}{1+2Am}$.

In the weak field limit, $m = 0$, the black hole can be considered as a test particle and cease to deform the spacetime and inertial frames around it.

The timelike worldlines

$$x^\mu(\lambda) = \left(\frac{(1 + Ar\cos\theta)}{\sqrt{1 - A^2r^2}} \lambda, r, 0, 0 \right)$$

of an observer with $r =$ constant and $\theta, \phi = 0$ can be obtained by the property $u^\mu u_\mu = -1$ of the four-velocity $u^\mu = dx^\mu/d\lambda$.

The magnitude a of the 4-acceleration, $a^\mu = (\nabla_\nu u^\mu)u^\nu$, for this kind of observer is

$$|a| = \sqrt{a_\mu a^\mu}\,\Big|_{r=0} = A \tag{1}$$

Since $a_\mu u^\mu = 0$, the value $|a|$ is also the magnitude of the 3-acceleration in the rest frame of the observer. From eq. (1) we conclude that the origin of the weak-metric, $r = 0$, is being accelerated with an uniform acceleration whose value is given by the parameter A.

In the weak field limit, $m = 0$, the black hole can be considered as a test particle and cease to deform the spacetime and inertial frames around it.

The timelike worldlines

$$x^\mu(\lambda) = \left(\frac{(1 + Ar\cos\theta)}{\sqrt{1 - A^2 r^2}} \lambda, r, 0, 0 \right)$$

of an observer with $r =$ constant and $\theta, \phi = 0$ can be obtained by the property $u^\mu u_\mu = -1$ of the four-velocity $u^\mu = dx^\mu / d\lambda$.

The magnitude a of the 4-acceleration, $a^\mu = (\nabla_\nu u^\mu) u^\nu$, for this kind of observer is

$$|a| = \sqrt{a_\mu a^\mu} \Big|_{r=0} = A \quad (1)$$

Since $a_\mu u^\mu = 0$, the value $|a|$ is also the magnitude of the 3-acceleration in the rest frame of the observer. From eq. (1) we conclude that the origin of the weak-metric, $r = 0$, is being accelerated with an uniform acceleration whose value is given by the parameter A.

In the weak field limit, $m = 0$, the black hole can be considered as a test particle and cease to deform the spacetime and inertial frames around it.

The timelike worldlines

$$x^\mu(\lambda) = \left(\frac{(1 + Ar\cos\theta)}{\sqrt{1 - A^2 r^2}} \lambda, r, 0, 0 \right)$$

of an observer with $r =$ constant and $\theta, \phi = 0$ can be obtained by the property $u^\mu u_\mu = -1$ of the four-velocity $u^\mu = dx^\mu / d\lambda$.

The magnitude a of the 4-acceleration, $a^\mu = (\nabla_\nu u^\mu) u^\nu$, for this kind of observer is

$$|a| = \sqrt{a_\mu a^\mu} \Big|_{r=0} = A \tag{1}$$

Since $a_\mu u^\mu = 0$, the value $|a|$ is also the magnitude of the 3-acceleration in the rest frame of the observer. From eq. (1) we conclude that the origin of the weak-metric, $r = 0$, is being accelerated with an uniform acceleration whose value is given by the parameter A.

In the weak field limit, $m = 0$, the black hole can be considered as a test particle and cease to deform the spacetime and inertial frames around it.

The timelike worldlines

$$x^\mu(\lambda) = \left(\frac{(1 + Ar\cos\theta)}{\sqrt{1 - A^2 r^2}} \lambda, r, 0, 0 \right)$$

of an observer with $r =$ constant and $\theta, \phi = 0$ can be obtained by the property $u^\mu u_\mu = -1$ of the four-velocity $u^\mu = dx^\mu / d\lambda$.

The magnitude a of the 4-acceleration, $a^\mu = (\nabla_\nu u^\mu) u^\nu$, for this kind of observer is

$$|a| = \sqrt{a_\mu a^\mu}\ \Big|_{r=0} = A \tag{1}$$

Since $a_\mu u^\mu = 0$, the value $|a|$ is also the magnitude of the 3-acceleration in the rest frame of the observer. From eq. (1) we conclude that the origin of the weak-metric, $r = 0$, is being accelerated with an uniform acceleration whose value is given by the parameter A.

Accelerating Kerr-Newman black hole metric:

$$ds^2 = \frac{1}{(1+\tilde{r}xA)^2}\left\{\frac{G(\tilde{r})}{\tilde{r}^2+a^2x^2}\left[d\tilde{t}+a(1-x^2)\Delta_\varphi d\tilde{\varphi}\right]^2 - \frac{\tilde{r}^2+a^2x^2}{G(\tilde{r})}d\tilde{r}^2 + \frac{H(x)}{\tilde{r}^2+a^2x^2}\left[(\tilde{r}^2+a^2)\Delta_\varphi d\tilde{\varphi}+ad\tilde{t}\right]^2 + \frac{\tilde{r}^2+a^2x^2}{H(x)}dx^2\right\}$$

where

$$G(\tilde{r}) := \left(A^2\tilde{r}^2-1\right)\left(\tilde{r}-r_+\right)\left(\tilde{r}-r_-\right) ,$$

$$H(x) := \left(1-x^2\right)\left(1+Axr_+\right)\left(1+Axr_-\right) .$$

The horizon are $\tilde{r}_\pm = m \pm \sqrt{m^2-a^2-q^2-p^2}$, $r_A = \frac{1}{A}$.

While the electromagnetic potential remains basically the same of the (non-accelerating) Kerr-Newman solution

$$A_\mu = \left\{-\frac{q\tilde{r}+pax}{\tilde{r}^2+a^2x^2},\ 0,\ 0,\ -\frac{aq\tilde{r}(1-x^2)-px(\tilde{r}^2+a^2)}{\tilde{r}^2+a^2x^2}\Delta_\varphi\right\} .$$

The accelerating KN black hole area

$$\mathcal{A} = \int_0^{2\pi} d\tilde{\varphi}\int_{-1}^{1} dx\sqrt{g_{\tilde{\varphi}\tilde{\varphi}}g_{xx}}\ \Big|_{\tilde{r}=r_+} = 4\pi\Delta_\varphi\frac{r_+^2+a^2}{1-A^2r_+^2} .$$

Accelerating Kerr-Newman black hole metric:

$$ds^2 = \frac{1}{(1+\tilde{r}xA)^2}\left\{\frac{G(\tilde{r})}{\tilde{r}^2+a^2x^2}\left[d\tilde{t}+a(1-x^2)\Delta_\varphi d\tilde{\varphi}\right]^2 - \frac{\tilde{r}^2+a^2x^2}{G(\tilde{r})}d\tilde{r}^2 + \frac{H(x)}{\tilde{r}^2+a^2x^2}\left[(\tilde{r}^2+a^2)\Delta_\varphi d\tilde{\varphi}+ad\tilde{t}\right]^2 + \frac{\tilde{r}^2+a^2x^2}{H(x)}dx^2\right\}$$

where

$$G(\tilde{r}) := \left(A^2\tilde{r}^2-1\right)\left(\tilde{r}-r_+\right)\left(\tilde{r}-r_-\right) \quad ,$$
$$H(x) := \left(1-x^2\right)\left(1+Axr_+\right)\left(1+Axr_-\right) \quad .$$

The horizon are $\tilde{r}_\pm = m \pm \sqrt{m^2-a^2-q^2-p^2} \quad , \qquad r_A = \frac{1}{A}$.

While the electromagnetic potential remains basically the same of the (non-accelerating) Kerr-Newman solution

$$A_\mu = \left\{-\frac{q\tilde{r}+pax}{\tilde{r}^2+a^2x^2},\ 0,\ 0,\ -\frac{aq\tilde{r}(1-x^2)-px(\tilde{r}^2+a^2)}{\tilde{r}^2+a^2x^2}\Delta_\varphi\right\} \quad .$$

The accelerating KN black hole area

$$\mathcal{A} = \int_0^{2\pi} d\tilde{\varphi}\int_{-1}^{1} dx\sqrt{g_{\tilde{\varphi}\tilde{\varphi}}g_{xx}}\,\Big|_{\tilde{r}=r_+} = 4\pi\Delta_\varphi\frac{r_+^2+a^2}{1-A^2r_+^2} \quad .$$

Accelerating Kerr-Newman black hole metric:

$$ds^2 = \frac{1}{(1+\tilde{r}xA)^2}\left\{\frac{G(\tilde{r})}{\tilde{r}^2+a^2x^2}\left[d\tilde{t}+a(1-x^2)\Delta_\varphi d\tilde{\varphi}\right]^2 - \frac{\tilde{r}^2+a^2x^2}{G(\tilde{r})}d\tilde{r}^2 + \frac{H(x)}{\tilde{r}^2+a^2x^2}\left[(\tilde{r}^2+a^2)\Delta_\varphi d\tilde{\varphi}+ad\tilde{t}\right]^2 + \frac{\tilde{r}^2+a^2x^2}{H(x)}dx^2\right\}$$

where

$$G(\tilde{r}) := \left(A^2\tilde{r}^2-1\right)\left(\tilde{r}-r_+\right)\left(\tilde{r}-r_-\right) \quad ,$$
$$H(x) := \left(1-x^2\right)\left(1+Axr_+\right)\left(1+Axr_-\right) \quad .$$

The horizon are $\tilde{r}_\pm = m \pm \sqrt{m^2-a^2-q^2-p^2}$, $r_A = \frac{1}{A}$.

While the electromagnetic potential remains basically the same of the (non-accelerating) Kerr-Newman solution

$$A_\mu = \left\{-\frac{q\tilde{r}+pax}{\tilde{r}^2+a^2x^2},\ 0,\ 0,\ -\frac{aq\tilde{r}(1-x^2)-px(\tilde{r}^2+a^2)}{\tilde{r}^2+a^2x^2}\Delta_\varphi\right\} \quad .$$

The accelerating KN black hole area

$$\mathcal{A} = \int_0^{2\pi} d\tilde{\varphi}\int_{-1}^{1} dx\sqrt{g_{\tilde{\varphi}\tilde{\varphi}}g_{xx}}\ \Big|_{\tilde{r}=r_+} = 4\pi\Delta_\varphi\frac{r_+^2+a^2}{1-A^2r_+^2} \quad .$$

Accelerating Kerr-Newman black hole metric:

$$ds^2 = \frac{1}{(1+\tilde{r}xA)^2}\left\{\frac{G(\tilde{r})}{\tilde{r}^2+a^2x^2}\left[d\tilde{t}+a(1-x^2)\Delta_\varphi d\tilde{\varphi}\right]^2 - \frac{\tilde{r}^2+a^2x^2}{G(\tilde{r})}d\tilde{r}^2 + \frac{H(x)}{\tilde{r}^2+a^2x^2}\left[(\tilde{r}^2+a^2)\Delta_\varphi d\tilde{\varphi}+ad\tilde{t}\right]^2 + \frac{\tilde{r}^2+a^2x^2}{H(x)}dx^2\right\}$$

where

$$G(\tilde{r}) := \left(A^2\tilde{r}^2-1\right)\left(\tilde{r}-r_+\right)\left(\tilde{r}-r_-\right) \quad ,$$
$$H(x) := \left(1-x^2\right)\left(1+Axr_+\right)\left(1+Axr_-\right) \quad .$$

The horizon are $\tilde{r}_\pm = m \pm \sqrt{m^2-a^2-q^2-p^2}$, $r_A = \frac{1}{A}$.

While the electromagnetic potential remains basically the same of the (non-accelerating) Kerr-Newman solution

$$A_\mu = \left\{-\frac{q\tilde{r}+pax}{\tilde{r}^2+a^2x^2},\ 0,\ 0,\ -\frac{aq\tilde{r}(1-x^2)-px(\tilde{r}^2+a^2)}{\tilde{r}^2+a^2x^2}\Delta_\varphi\right\} \quad .$$

The accelerating KN black hole area

$$\mathcal{A} = \int_0^{2\pi} d\tilde{\varphi}\int_{-1}^{1} dx\sqrt{g_{\tilde{\varphi}\tilde{\varphi}}g_{xx}}\Big|_{\tilde{r}=r_+} = 4\pi\Delta_\varphi\frac{r_+^2+a^2}{1-A^2r_+^2} \quad .$$

Accelerating Kerr-Newman black hole metric:

$$ds^2 = \frac{1}{(1+\tilde{r}xA)^2}\left\{\frac{G(\tilde{r})}{\tilde{r}^2+a^2x^2}\left[d\tilde{t}+a(1-x^2)\Delta_\varphi d\tilde{\varphi}\right]^2 - \frac{\tilde{r}^2+a^2x^2}{G(\tilde{r})}d\tilde{r}^2 + \frac{H(x)}{\tilde{r}^2+a^2x^2}\left[(\tilde{r}^2+a^2)\Delta_\varphi d\tilde{\varphi}+ad\tilde{t}\right]^2 + \frac{\tilde{r}^2+a^2x^2}{H(x)}dx^2\right\}$$

where

$$G(\tilde{r}) := \left(A^2\tilde{r}^2-1\right)\left(\tilde{r}-r_+\right)\left(\tilde{r}-r_-\right) \quad ,$$
$$H(x) := \left(1-x^2\right)\left(1+Axr_+\right)\left(1+Axr_-\right) \quad .$$

The horizon are $\tilde{r}_\pm = m \pm \sqrt{m^2-a^2-q^2-p^2}$, $r_A = \frac{1}{A}$.

While the electromagnetic potential remains basically the same of the (non-accelerating) Kerr-Newman solution

$$A_\mu = \left\{-\frac{q\tilde{r}+pax}{\tilde{r}^2+a^2x^2},\ 0,\ 0,\ -\frac{aq\tilde{r}(1-x^2)-px(\tilde{r}^2+a^2)}{\tilde{r}^2+a^2x^2}\Delta_\varphi\right\} \quad .$$

The accelerating KN black hole area

$$\mathcal{A} = \int_0^{2\pi} d\tilde{\varphi}\int_{-1}^{1} dx\sqrt{g_{\tilde{\varphi}\tilde{\varphi}}g_{xx}}\ \Big|_{\tilde{r}=r_+} = 4\pi\Delta_\varphi\frac{r_+^2+a^2}{1-A^2r_+^2} \quad .$$

Near horizon dimensionless coordinates (t, r, φ) :

$$\tilde{r}(r) := r_e + \lambda r_0 r \qquad , \qquad \tilde{t}(t) := \frac{r_0}{\lambda} t \qquad , \qquad \tilde{\varphi}(\varphi, t) := \varphi + \Omega_J^{ext} \ \frac{r_0}{\lambda} t \qquad .$$

Near horizon electromagnetic gauge fixing $\quad A_{\tilde{t}} \rightarrow A_{\tilde{t}} + \Phi_e \quad ,$
The angular velocity Ω_J and the Coulomb electromagnetic potential Φ_e

$$\Omega_J := - \frac{g_{\tilde{t}\tilde{\varphi}}}{g_{\tilde{\varphi}\tilde{\varphi}}} \bigg|_{\tilde{r}=r_+} = - \frac{a}{a^2 + r_+^2} \ \frac{1}{\Delta_\varphi} \quad , \qquad \Phi_e := -\chi^\mu A_\mu \Big|_{\tilde{r}=r_+} = \frac{q \ r_+}{a^2 + r_+^2} , \quad \chi = \partial_{\tilde{t}} + \Omega_J \partial_{\tilde{\varphi}}$$

The near horizon, extreme, accelerating Kerr-Newman geometry (NHEAKN) is obtained as the limit of the EAKN for $\lambda \rightarrow 0$. It can be cast as a warped and twisted product of $AdS_2 \times S^2$

$$ds^2 = \Gamma(x) \left[-r^2 dt^2 + \frac{dr^2}{r^2} + \alpha^2(x) \frac{dx^2}{1 - x^2} + \gamma^2(x) \ (d\varphi + \kappa r dt)^2 \right] ,$$

$$\Gamma(x) = \frac{a^2 x^2 + r_+ r_-}{\left[1 - A^2 r_+ r_- \right] \left(1 + Ax \sqrt{r_+ r_-} \right)^2} \quad , \qquad r_0 = \pm \sqrt{\frac{a^2 + r_+ r_-}{1 - A^2 r_+ r_-}} \quad ,$$

$$\gamma(x) = \pm \frac{(a^2 + r_+ r_-) \sqrt{1 - x^2} \Delta_\varphi^{ext}}{\Gamma \sqrt{1 - A^2 r_+ r_-} \left(1 + Ax \sqrt{r_+ r_-} \right)} \quad , \qquad \kappa = - \frac{2 a r_0^2 \sqrt{r_+ r_-}}{(a^2 + r_+ r_-)^2 \Delta_\varphi^{ext}} \quad ,$$

$$\alpha(x) = \pm \frac{\sqrt{1 - A^2 r_+ r_-}}{1 + x A \sqrt{r_+ r_-}} \quad .$$

Near horizon geometry at extremality (metric)

Near horizon dimensionless coordinates (t, r, φ) :

$$\tilde{r}(r) := r_e + \lambda r_0 r \qquad , \qquad \tilde{t}(t) := \frac{r_0}{\lambda} t \qquad , \qquad \tilde{\varphi}(\varphi, t) := \varphi + \Omega_J^{ext} \frac{r_0}{\lambda} t \qquad .$$

Near horizon electromagnetic gauge fixing $\qquad A_{\tilde{t}} \rightarrow A_{\tilde{t}} + \Phi_e \quad ,$

The angular velocity Ω_J and the Coulomb electromagnetic potential Φ_e

$$\Omega_J := -\frac{g_{\tilde{t}\tilde{\varphi}}}{g_{\tilde{\varphi}\tilde{\varphi}}}\bigg|_{\tilde{r}=r_+} = -\frac{a}{a^2 + r_+^2} \frac{1}{\Delta_\varphi} \quad , \qquad \Phi_e := -\chi^\mu A_\mu\Big|_{\tilde{r}=r_+} = \frac{q\ r_+}{a^2 + r_+^2}, \quad \chi = \partial_{\tilde{t}} + \Omega_J \partial_{\tilde{\varphi}}$$

The near horizon, extreme, accelerating Kerr-Newman geometry (NHEAKN) is obtained as the limit of the EAKN for $\lambda \rightarrow 0$. It can be cast as a warped and twisted product of $AdS_2 \times S^2$

$$ds^2 = \Gamma(x) \left[-r^2 dt^2 + \frac{dr^2}{r^2} + \alpha^2(x) \frac{dx^2}{1 - x^2} + \gamma^2(x) \ (d\varphi + \kappa r dt)^2 \right],$$

$$\Gamma(x) = \frac{a^2 x^2 + r_+ r_-}{\left[1 - A^2 r_+ r_-\right] \left(1 + Ax\sqrt{r_+ r_-}\right)^2} \quad , \qquad r_0 = \pm\sqrt{\frac{a^2 + r_+ r_-}{1 - A^2 r_+ r_-}} \quad ,$$

$$\gamma(x) = \pm \frac{(a^2 + r_+ r_-)\sqrt{1 - x^2}\Delta_\varphi^{ext}}{\Gamma\sqrt{1 - A^2 r_+ r_-}\left(1 + Ax\sqrt{r_+ r_-}\right)} \quad , \qquad \kappa = -\frac{2 a r_0^2 \sqrt{r_+ r_-}}{(a^2 + r_+ r_-)^2 \Delta_\varphi^{ext}} \quad ,$$

$$\alpha(x) = \pm \frac{\sqrt{1 - A^2 r_+ r_-}}{1 + xA\sqrt{r_+ r_-}} \qquad .$$

Near horizon dimensionless coordinates (t, r, φ) :

$$\tilde{r}(r) := r_e + \lambda r_0 r \quad , \quad \tilde{t}(t) := \frac{r_0}{\lambda} t \quad , \quad \tilde{\varphi}(\varphi, t) := \varphi + \Omega_J^{ext} \ \frac{r_0}{\lambda} t \quad .$$

Near horizon electromagnetic gauge fixing $\quad A_{\tilde{t}} \rightarrow A_{\tilde{t}} + \Phi_e \quad ,$
The angular velocity Ω_J and the Coulomb electromagnetic potential Φ_e

$$\Omega_J := -\frac{g_{\tilde{t}\tilde{\varphi}}}{g_{\tilde{\varphi}\tilde{\varphi}}}\bigg|_{\tilde{r}=r_+} = -\frac{a}{a^2 + r_+^2} \ \frac{1}{\Delta_\varphi} \quad , \qquad \Phi_e := -\chi^\mu A_\mu\bigg|_{\tilde{r}=r_+} = \frac{q \ r_+}{a^2 + r_+^2}, \quad \chi = \partial_{\tilde{t}} + \Omega_J \partial_{\tilde{\varphi}}$$

The near horizon, extreme, accelerating Kerr-Newman geometry (NHEAKN) is obtained as the limit of the EAKN for $\lambda \rightarrow 0$. It can be cast as a warped and twisted product of $AdS_2 \times S^2$

$$ds^2 = \Gamma(x)\left[-r^2 dt^2 + \frac{dr^2}{r^2} + \alpha^2(x)\frac{dx^2}{1 - x^2} + \gamma^2(x) \ (d\varphi + \kappa r dt)^2\right],$$

$$\Gamma(x) = \frac{a^2 x^2 + r_+ r_-}{\left[1 - A^2 r_+ r_-\right]\left(1 + Ax\sqrt{r_+ r_-}\right)^2} \quad , \qquad r_0 = \pm\sqrt{\frac{a^2 + r_+ r_-}{1 - A^2 r_+ r_-}} \quad ,$$

$$\gamma(x) = \pm\frac{(a^2 + r_+ r_-)\sqrt{1 - x^2}\,\Delta_\varphi^{ext}}{\Gamma\sqrt{1 - A^2 r_+ r_-}\left(1 + Ax\sqrt{r_+ r_-}\right)} \quad , \qquad \kappa = -\frac{2 a r_0^2 \sqrt{r_+ r_-}}{(a^2 + r_+ r_-)^2 \Delta_\varphi^{ext}} \quad ,$$

$$\alpha(x) = \pm\frac{\sqrt{1 - A^2 r_+ r_-}}{1 + xA\sqrt{r_+ r_-}} \quad .$$

Near horizon dimensionless coordinates (t, r, φ) :

$$\tilde{r}(r) := r_e + \lambda r_0 r \quad , \quad \tilde{t}(t) := \frac{r_0}{\lambda} t \quad , \quad \tilde{\varphi}(\varphi, t) := \varphi + \Omega_J^{ext} \frac{r_0}{\lambda} t \quad .$$

Near horizon electromagnetic gauge fixing $\quad A_{\tilde{t}} \to A_{\tilde{t}} + \Phi_e \quad ,$
The angular velocity Ω_J and the Coulomb electromagnetic potential Φ_e

$$\Omega_J := -\frac{g_{\tilde{t}\tilde{\varphi}}}{g_{\tilde{\varphi}\tilde{\varphi}}}\bigg|_{\tilde{r}=r_+} = -\frac{a}{a^2 + r_+^2}\frac{1}{\Delta\varphi} \quad , \quad \Phi_e := -\chi^\mu A_\mu\Big|_{\tilde{r}=r_+} = \frac{q\ r_+}{a^2 + r_+^2}, \quad \chi = \partial_{\tilde{t}} + \Omega_J \partial_{\tilde{\varphi}}$$

The near horizon, extreme, accelerating Kerr-Newman geometry (NHEAKN) is obtained as the limit of the EAKN for $\lambda \to 0$. It can be cast as a warped and twisted product of $AdS_2 \times S^2$

$$ds^2 = \Gamma(x)\left[-r^2 dt^2 + \frac{dr^2}{r^2} + \alpha^2(x)\frac{dx^2}{1-x^2} + \gamma^2(x)\ (d\varphi + \kappa r dt)^2\right],$$

$$\Gamma(x) = \frac{a^2x^2 + r_+r_-}{\left[1 - A^2 r_+ r_-\right]\left(1 + Ax\sqrt{r_+r_-}\right)^2} \quad , \quad r_0 = \pm\sqrt{\frac{a^2 + r_+r_-}{1 - A^2 r_+ r_-}} \quad ,$$

$$\gamma(x) = \pm\frac{(a^2 + r_+r_-)\sqrt{1-x^2}\,\Delta_\varphi^{ext}}{\Gamma\sqrt{1 - A^2r_+r_-}\left(1 + Ax\sqrt{r_+r_-}\right)} \quad , \quad \kappa = -\frac{2ar_0^2\sqrt{r_+r_-}}{(a^2 + r_+r_-)^2\Delta_\varphi^{ext}} \quad ,$$

$$\alpha(x) = \pm\frac{\sqrt{1 - A^2r_+r_-}}{1 + xA\sqrt{r_+r_-}} \quad .$$

Near horizon dimensionless coordinates (t, r, φ) :

$$\tilde{r}(r) := r_e + \lambda r_0 r \quad , \quad \tilde{t}(t) := \frac{r_0}{\lambda} t \quad , \quad \tilde{\varphi}(\varphi, t) := \varphi + \Omega_J^{ext} \ \frac{r_0}{\lambda} t \quad .$$

Near horizon electromagnetic gauge fixing $A_{\tilde{t}} \rightarrow A_{\tilde{t}} + \Phi_e$,
The angular velocity Ω_J and the Coulomb electromagnetic potential Φ_e

$$\Omega_J := -\frac{g_{\tilde{t}\tilde{\varphi}}}{g_{\tilde{\varphi}\tilde{\varphi}}}\bigg|_{\tilde{r}=r_+} = -\frac{a}{a^2 + r_+^2} \frac{1}{\Delta_\varphi} \quad , \quad \Phi_e := -\chi^\mu A_\mu\Big|_{\tilde{r}=r_+} = \frac{q \ r_+}{a^2 + r_+^2}, \quad \chi = \partial_{\tilde{t}} + \Omega_J \partial_{\tilde{\varphi}}$$

The near horizon, extreme, accelerating Kerr-Newman geometry (NHEAKN) is obtained as the limit of the EAKN for $\lambda \rightarrow 0$. It can be cast as a warped and twisted product of $AdS_2 \times S^2$

$$ds^2 = \Gamma(x) \left[-r^2 dt^2 + \frac{dr^2}{r^2} + \alpha^2(x) \frac{dx^2}{1 - x^2} + \gamma^2(x) \ (d\varphi + \kappa r dt)^2 \right],$$

$$\Gamma(x) = \frac{a^2 x^2 + r_+ r_-}{\left[1 - A^2 r_+ r_-\right] \left(1 + Ax\sqrt{r_+ r_-}\right)^2} \quad , \qquad r_0 = \pm\sqrt{\frac{a^2 + r_+ r_-}{1 - A^2 r_+ r_-}} \quad ,$$

$$\gamma(x) = \pm\frac{(a^2 + r_+ r_-)\sqrt{1 - x^2}\Delta_\varphi^{ext}}{\Gamma\sqrt{1 - A^2 r_+ r_-} \left(1 + Ax\sqrt{r_+ r_-}\right)} \quad , \qquad \kappa = -\frac{2ar_0^2\sqrt{r_+ r_-}}{(a^2 + r_+ r_-)^2 \Delta_\varphi^{ext}} \quad ,$$

$$\alpha(x) = \pm\frac{\sqrt{1 - A^2 r_+ r_-}}{1 + xA\sqrt{r_+ r_-}} \quad .$$

Also the electromagnetic connection fall into the same general class of near horizon gauge potential

$$A = \ell(x)(d\varphi + \kappa r dt) - \frac{e}{\kappa} d\varphi \qquad ,$$

where

$$\ell(x) = -\frac{r_0^2}{\kappa} \frac{q(r_+ r_- - a^2 x^2) + 2axp\sqrt{r_+ r_-}}{(r_+ r_- + a^2 x^2)(a^2 + r_+ r_-)} \qquad , \qquad e = q r_0^2 \frac{r_+ r_- - a^2}{\left(r_+ r_- + a^2\right)^2} \qquad .$$

The near horizon Killing vectors

$$\zeta_{-1} = \partial_t \qquad , \qquad \zeta_0 = t\partial_t - r\partial_r$$

$$\zeta_1 = \left(\frac{1}{2r^2} + \frac{t^2}{2}\right)\partial_t - t\ r\ \partial_r - \frac{\kappa}{r}\partial_\varphi \qquad , \qquad L_0 = \partial_\varphi \qquad .$$

span $SL(2,\mathbb{R}) \times U(1)$ algebra

$$[\zeta_0, \zeta_\pm] = \pm\zeta_\pm \qquad , \qquad [\zeta_{-1}, \zeta_1] = \zeta_0$$

Also the electromagnetic connection fall into the same general class of near horizon gauge potential

$$A = \ell(x)(d\varphi + \kappa r dt) - \frac{e}{\kappa} d\varphi \qquad ,$$

where

$$\ell(x) = -\frac{r_0^2}{\kappa} \frac{q(r_+ r_- - a^2 x^2) + 2axp\sqrt{r_+ r_-}}{(r_+ r_- + a^2 x^2)(a^2 + r_+ r_-)} \qquad , \qquad e = q r_0^2 \frac{r_+ r_- - a^2}{\left(r_+ r_- + a^2\right)^2} \quad .$$

The near horizon Killing vectors

$$\zeta_{-1} = \partial_t \qquad , \qquad \zeta_0 = t\partial_t - r\partial_r$$

$$\zeta_1 = \left(\frac{1}{2r^2} + \frac{t^2}{2}\right)\partial_t - t\ r\ \partial_r - \frac{\kappa}{r}\partial_\varphi \qquad , \qquad L_0 = \partial_\varphi \quad .$$

span $SL(2,\mathbb{R}) \times U(1)$ algebra

$$[\zeta_0, \zeta_\pm] = \pm\zeta_\pm \qquad , \qquad [\zeta_{-1}, \zeta_1] = \zeta_0$$

Also the electromagnetic connection fall into the same general class of near horizon gauge potential

$$A = \ell(x)(d\varphi + \kappa r dt) - \frac{e}{\kappa} d\varphi \qquad ,$$

where

$$\ell(x) = -\frac{r_0^2}{\kappa} \frac{q(r_+ r_- - a^2 x^2) + 2axp\sqrt{r_+ r_-}}{(r_+ r_- + a^2 x^2)(a^2 + r_+ r_-)} \qquad , \qquad e = q r_0^2 \frac{r_+ r_- - a^2}{\left(r_+ r_- + a^2\right)^2} \quad .$$

The near horizon Killing vectors

$$\zeta_{-1} = \partial_t \qquad , \qquad \zeta_0 = t\partial_t - r\partial_r$$

$$\zeta_1 = \left(\frac{1}{2r^2} + \frac{t^2}{2}\right)\partial_t - t\ r\ \partial_r - \frac{\kappa}{r}\partial_\varphi \qquad , \qquad L_0 = \partial_\varphi \quad .$$

span $SL(2,\mathbb{R}) \times U(1)$ algebra

$$[\zeta_0, \zeta_\pm] = \pm\zeta_\pm \qquad , \qquad [\zeta_{-1}, \zeta_1] = \zeta_0$$

Also the electromagnetic connection fall into the same general class of near horizon gauge potential

$$A = \ell(x)(d\varphi + \kappa r dt) - \frac{e}{\kappa} d\varphi \qquad ,$$

where

$$\ell(x) = -\frac{r_0^2}{\kappa} \frac{q(r_+ r_- - a^2 x^2) + 2axp\sqrt{r_+ r_-}}{(r_+ r_- + a^2 x^2)(a^2 + r_+ r_-)} \qquad , \qquad e = q r_0^2 \frac{r_+ r_- - a^2}{\left(r_+ r_- + a^2\right)^2} \quad .$$

The near horizon Killing vectors

$$\zeta_{-1} = \partial_t \qquad , \qquad \zeta_0 = t\partial_t - r\partial_r$$

$$\zeta_1 = \left(\frac{1}{2r^2} + \frac{t^2}{2}\right)\partial_t - t\ r\ \partial_r - \frac{\kappa}{r}\partial_\varphi \qquad , \qquad L_0 = \partial_\varphi \quad .$$

span $SL(2, \mathbb{R}) \times U(1)$ algebra

$$[\zeta_0, \zeta_\pm] = \pm\zeta_\pm \qquad , \qquad [\zeta_{-1}, \zeta_1] = \zeta_0$$

According to the Kerr/CFT correspondence it is possible to infer the thermodynamic properties of extremal black holes from the asymptotic symmetry of their near horizon fields. The fall-off behaviour for the metric, at large radial distance r, is taken as follows

$$
\begin{aligned}
g_{tt} &= \mathcal{O}\left(r^2\right) \quad , \qquad g_{t\varphi} = \kappa\ \Gamma(x)\ \gamma^2(x)\ r + \mathcal{O}(1)\,, \\
g_{tx} &= \mathcal{O}\left(\frac{1}{r}\right) \quad , \qquad g_{tr} = \mathcal{O}\left(\frac{1}{r^2}\right) \ , \qquad g_{\varphi\varphi} = \mathcal{O}(1)\ , \\
g_{\varphi x} &= \mathcal{O}\left(\frac{1}{r}\right) \quad , \qquad g_{\varphi r} = \mathcal{O}\left(\frac{1}{r}\right) \quad , \qquad g_{xr} = \mathcal{O}\left(\frac{1}{r^2}\right), \\
g_{xx} &= \frac{\Gamma(x)\alpha(x)^2}{1-x^2} + \mathcal{O}\left(\frac{1}{r}\right) \quad , \qquad g_{rr} = \frac{\Gamma(x)}{r^2} + \mathcal{O}\left(\frac{1}{r^3}\right),
\end{aligned}
$$

According to the Kerr/CFT correspondence it is possible to infer the thermodynamic properties of extremal black holes from the asymptotic symmetry of their near horizon fields. The fall-off behaviour for the metric, at large radial distance r, is taken as follows

$$
\begin{aligned}
g_{tt} &= \mathcal{O}\left(r^2\right) \quad , \quad g_{t\varphi} = \kappa\ \Gamma(x)\ \gamma^2(x)\ r + \mathcal{O}(1) , \\
g_{tx} &= \mathcal{O}\left(\frac{1}{r}\right) \quad , \quad g_{tr} = \mathcal{O}\left(\frac{1}{r^2}\right) \quad , \quad g_{\varphi\varphi} = \mathcal{O}(1) , \\
g_{\varphi x} &= \mathcal{O}\left(\frac{1}{r}\right) \quad , \quad g_{\varphi r} = \mathcal{O}\left(\frac{1}{r}\right) \quad , \quad g_{xr} = \mathcal{O}\left(\frac{1}{r^2}\right) , \\
g_{xx} &= \frac{\Gamma(x)\alpha(x)^2}{1-x^2} + \mathcal{O}\left(\frac{1}{r}\right) \quad , \quad g_{rr} = \frac{\Gamma(x)}{r^2} + \mathcal{O}\left(\frac{1}{r^3}\right) ,
\end{aligned}
$$

while the electromagnetic field is considered to decay in the following way

$$
\begin{aligned}
A_t &= \mathcal{O}(r) \quad , \quad A_\varphi = \ell(\theta) - \frac{e}{\kappa} + \mathcal{O}\left(\frac{1}{r}\right) , \\
A_x &= \mathcal{O}(1) \quad , \quad A_r = \mathcal{O}\left(\frac{1}{r^2}\right) \quad .
\end{aligned}
$$

These boundary conditions are preserved by the following asymptotic Killing vectors

$$\zeta_\epsilon = \epsilon(\varphi)\partial_\varphi - r\epsilon'(\varphi)\partial_r + \text{subleading terms} \quad ,$$
$$\xi_\epsilon = -\left[\ell(\theta) - \frac{e}{\kappa}\right]\epsilon(\varphi) + \text{subleading terms} \quad .$$

Expanding the generators in Fourier modes such that

$$\epsilon(\varphi) = -e^{-in\varphi} \quad ,$$

we can verify that each $n-$mode couple in the Fourier series expansion can be considered as a generator, $L_n = (\zeta_n, \xi_n)$, which obey the following Witt algebra (Virasoro algebra without the central extension)

$$i\ [L_m, L_n] = (m-n)\ L_{m+n} \quad .$$

The commutation bracket are defined by

$$[L_m, L_n] := [(\zeta_m, \xi_m), (\zeta_n, \xi_n)] = ([\zeta_m, \zeta_n], [\xi_m, \xi_n]_\zeta) \quad ,$$

where $[\zeta_m, \zeta_n]$ is the standard Lie commutator, while $[\xi_m, \xi_n]_\zeta := \zeta_m^\mu \partial_\mu \xi_n - \zeta_n^\mu \partial_\mu \xi_m$

These boundary conditions are preserved by the following asymptotic Killing vectors

$$\zeta_\epsilon = \epsilon(\varphi)\partial_\varphi - r\epsilon'(\varphi)\partial_r + \text{subleading terms} \quad ,$$
$$\xi_\epsilon = -\left[\ell(\theta) - \frac{e}{\kappa}\right]\epsilon(\varphi) + \text{subleading terms} \quad .$$

Expanding the generators in Fourier modes such that

$$\epsilon(\varphi) = -e^{-in\varphi} \quad ,$$

we can verify that each $n-$mode couple in the Fourier series expansion can be considered as a generator, $L_n = (\zeta_n, \xi_n)$, which obey the following Witt algebra (Virasoro algebra without the central extension)

$$i\ [L_m, L_n] = (m-n)\ L_{m+n} \quad .$$

The commutation bracket are defined by

$$[L_m, L_n] := [(\zeta_m, \xi_m), (\zeta_n, \xi_n)] = ([\zeta_m, \zeta_n], [\xi_m, \xi_n]_\zeta) \quad ,$$

where $[\zeta_m, \zeta_n]$ is the standard Lie commutator, while $[\xi_m, \xi_n]_\zeta := \zeta_m^\mu \partial_\mu \xi_n - \zeta_n^\mu \partial_\mu \xi_m$

These boundary conditions are preserved by the following asymptotic Killing vectors

$$\zeta_\epsilon = \epsilon(\varphi)\partial_\varphi - r\epsilon'(\varphi)\partial_r + \text{subleading terms} \quad ,$$
$$\xi_\epsilon = -\left[\ell(\theta) - \frac{e}{\kappa}\right]\epsilon(\varphi) + \text{subleading terms} \quad .$$

Expanding the generators in Fourier modes such that

$$\epsilon(\varphi) = -e^{-in\varphi} \quad ,$$

we can verify that each $n-$mode couple in the Fourier series expansion can be considered as a generator, $L_n = (\zeta_n, \xi_n)$, which obey the following Witt algebra (Virasoro algebra without the central extension)

$$i\ [L_m, L_n] = (m-n)\ L_{m+n} \quad .$$

The commutation bracket are defined by

$$[L_m, L_n] := [(\zeta_m, \xi_m), (\zeta_n, \xi_n)] = ([\zeta_m, \zeta_n], [\xi_m, \xi_n]_\zeta) \quad ,$$

where $[\zeta_m, \zeta_n]$ is the standard Lie commutator, while $[\xi_m, \xi_n]_\zeta := \zeta^\mu_m \partial_\mu \xi_n - \zeta^\mu_n \partial_\mu \xi_m$

These boundary conditions are preserved by the following asymptotic Killing vectors

$$\begin{aligned} \zeta_\epsilon &= \epsilon(\varphi)\partial_\varphi - r\epsilon'(\varphi)\partial_r \quad + \quad \text{subleading terms} \quad , \\ \xi_\epsilon &= -\left[\ell(\theta) - \frac{e}{\kappa}\right]\epsilon(\varphi) \quad + \quad \text{subleading terms} \quad . \end{aligned}$$

Expanding the generators in Fourier modes such that

$$\epsilon(\varphi) = -e^{-in\varphi} \quad ,$$

we can verify that each $n-$mode couple in the Fourier series expansion can be considered as a generator, $L_n = (\zeta_n, \xi_n)$, which obey the following Witt algebra (Virasoro algebra without the central extension)

$$i\ [L_m, L_n] = (m-n)\ L_{m+n} \quad .$$

The commutation bracket are defined by

$$[L_m, L_n] := [(\zeta_m, \xi_m), (\zeta_n, \xi_n)] = ([\zeta_m, \zeta_n], [\xi_m, \xi_n]_\zeta) \quad ,$$

where $[\zeta_m, \zeta_n]$ is the standard Lie commutator, while $[\xi_m, \xi_n]_\zeta := \zeta^\mu_m \partial_\mu \xi_n - \zeta^\mu_n \partial_\mu \xi_m$

Evaluating the Dirac bracket between the charges associated with the generators of the asymptotic symmetries, one can observe that the Witt algebra is enlarged into the full Virasoro algebra. The central charge can be calculated as the coefficient of the cubic factor in the m-expansion of the following asymptotic charge

$$c_J = 12\ i \lim_{r\to\infty} \mathcal{Q}^{\text{Einstein}}_{L_m}[\mathcal{L}_{L_{-m}}\bar{g};\bar{g}]\Big|_{m^3} \quad ,$$

$\mathcal{Q}^{\text{Einstein}}_{\xi}[h;\bar{g}]$ is the conserved charge associated with the Killing vector ξ^μ of the linearised metric $h_{\mu\nu}$ around the background $\bar{g}_{\mu\nu}$; for general relativity it reads

$$\mathcal{Q}^{\text{Ein}}_{\xi}[h;\bar{g}] = \frac{1}{8\pi G_N}\int_S dS_{\mu\nu}\Big(\xi^\nu\nabla^\mu h + \xi^\mu\nabla_\sigma h^{\sigma\nu} + \xi_\sigma\nabla^\nu h^{\sigma\mu} + \frac{1}{2}h\nabla^\nu\xi^\mu + \frac{1}{2}h^{\mu\sigma}\nabla_\sigma\xi^\nu + \frac{1}{2}h^{\nu\sigma}\nabla^\mu\xi_\sigma\Big)$$

From the above near horizon geometry we obtain a general expression for the central charge given by

$$c_J = 3\kappa\int_{-1}^{1}\frac{dx}{\sqrt{1-x^2}}\,\Gamma(x)\alpha(x)\gamma(x) \quad = \quad \frac{12a\sqrt{r_+r_-}}{\left[1-A^2r_+r_-\right]^2} .$$

Evaluating the Dirac bracket between the charges associated with the generators of the asymptotic symmetries, one can observe that the Witt algebra is enlarged into the full Virasoro algebra. The central charge can be calculated as the coefficient of the cubic factor in the m-expansion of the following asymptotic charge

$$c_J = 12\ i \lim_{r\to\infty} \mathcal{Q}^{\text{Einstein}}_{L_m}[\mathcal{L}_{L_{-m}}\bar{g};\bar{g}]\Big|_{m^3} \quad ,$$

$\mathcal{Q}^{\text{Einstein}}_{\xi}[h;\bar{g}]$ is the conserved charge associated with the Killing vector ξ^μ of the linearised metric $h_{\mu\nu}$ around the background $\bar{g}_{\mu\nu}$; for general relativity it reads

$$\mathcal{Q}^{\text{Ein}}_{\xi}[h;\bar{g}] = \frac{1}{8\pi G_N}\int_S dS_{\mu\nu}\Big(\xi^\nu\nabla^\mu h+\xi^\mu\nabla_\sigma h^{\sigma\nu}+\xi_\sigma\nabla^\nu h^{\sigma\mu}+\frac{1}{2}h\nabla^\nu\xi^\mu+\frac{1}{2}h^{\mu\sigma}\nabla_\sigma\xi^\nu+\frac{1}{2}h^{\nu\sigma}\nabla^\mu\xi_\sigma\Big)$$

From the above near horizon geometry we obtain a general expression for the central charge given by

$$c_J = 3\kappa\int_{-1}^{1}\frac{dx}{\sqrt{1-x^2}}\ \Gamma(x)\alpha(x)\gamma(x) \quad = \quad \frac{12a\sqrt{r_+r_-}}{\left[1-A^2r_+r_-\right]^2} .$$

Central charge

Evaluating the Dirac bracket between the charges associated with the generators of the asymptotic symmetries, one can observe that the Witt algebra is enlarged into the full Virasoro algebra. The central charge can be calculated as the coefficient of the cubic factor in the m-expansion of the following asymptotic charge

$$c_J = 12\ i \lim_{r\to\infty} \mathcal{Q}^{\text{Einstein}}_{L_m}[\mathcal{L}_{L_{-m}}\bar{g};\bar{g}]\Big|_{m^3} \quad ,$$

$\mathcal{Q}^{\text{Einstein}}_{\xi}[h;\bar{g}]$ is the conserved charge associated with the Killing vector ξ^μ of the linearised metric $h_{\mu\nu}$ around the background $\bar{g}_{\mu\nu}$; for general relativity it reads

$$\mathcal{Q}^{\text{Ein}}_{\xi}[h;\bar{g}] = \frac{1}{8\pi G_N}\int_S dS_{\mu\nu}\Big(\xi^\nu\nabla^\mu h + \xi^\mu\nabla_\sigma h^{\sigma\nu} + \xi_\sigma\nabla^\nu h^{\sigma\mu} + \frac{1}{2}h\nabla^\nu\xi^\mu + \frac{1}{2}h^{\mu\sigma}\nabla_\sigma\xi^\nu + \frac{1}{2}h^{\nu\sigma}\nabla^\mu\xi_\sigma\Big)$$

From the above near horizon geometry we obtain a general expression for the central charge given by

$$c_J = 3\kappa\int_{-1}^{1}\frac{dx}{\sqrt{1-x^2}}\ \Gamma(x)\alpha(x)\gamma(x) \qquad = \qquad \frac{12a\sqrt{r_+ r_-}}{\left[1 - A^2 r_+ r_-\right]^2} .$$

Evaluating the Dirac bracket between the charges associated with the generators of the asymptotic symmetries, one can observe that the Witt algebra is enlarged into the full Virasoro algebra. The central charge can be calculated as the coefficient of the cubic factor in the m-expansion of the following asymptotic charge

$$c_J = 12\, i \lim_{r\to\infty} \mathcal{Q}^{\text{Einstein}}_{L_m}[\mathcal{L}_{L_{-m}}\bar{g};\bar{g}]\Big|_{m^3} \quad ,$$

$\mathcal{Q}^{\text{Einstein}}_{\xi}[h;\bar{g}]$ is the conserved charge associated with the Killing vector ξ^μ of the linearised metric $h_{\mu\nu}$ around the background $\bar{g}_{\mu\nu}$; for general relativity it reads

$$\mathcal{Q}^{\text{Ein}}_{\xi}[h;\bar{g}] = \frac{1}{8\pi G_N}\int_S dS_{\mu\nu}\left(\xi^\nu\nabla^\mu h + \xi^\mu\nabla_\sigma h^{\sigma\nu} + \xi_\sigma\nabla^\nu h^{\sigma\mu} + \frac{1}{2}h\nabla^\nu\xi^\mu + \frac{1}{2}h^{\mu\sigma}\nabla_\sigma\xi^\nu + \frac{1}{2}h^{\nu\sigma}\nabla^\mu\xi_\sigma\right)$$

From the above near horizon geometry we obtain a general expression for the central charge given by

$$c_J = 3\kappa\int_{-1}^{1}\frac{dx}{\sqrt{1-x^2}}\,\Gamma(x)\alpha(x)\gamma(x) \quad = \quad \frac{12a\sqrt{r_+r_-}}{\left[1-A^2r_+r_-\right]^2} .$$

Kerr/CFT correspondence exploits the assumption that near horizon geometry of extremal black holes can be described by the left sector of a CFT in two dimensions. For these latter theories Cardy found that the asymptotic grown of density states is given by

$$\mathcal{S}_{CFT} = 2\pi\sqrt{\frac{c_L \mathcal{L}_0}{6}} \quad .$$

For $\mathcal{L}_0 >> c_L$ and using the definition of left temperature $\frac{\partial \mathcal{S}_{CFT}}{\partial \mathcal{L}_0} = \frac{1}{T_L}$, Cardy formula become

$$\mathcal{S}_{CFT} = \frac{\pi}{3} c_L \; T_L \quad .$$

We cannot associate to the left temperature the Hawking temperature T_H because, as the surface gravity k_s, it vanishes on the event horizon since the outer and inner horizon overlap in a double degenerate horizon

$$T_H := \frac{\bar{h}\ k_s}{k_B\ 2\pi} = \frac{1}{2\pi}\sqrt{-\frac{1}{2}\nabla_\mu \chi_\nu \nabla^\mu \chi^\nu}\ \Big|_{r_+} = \frac{1-A^2 r_+^2}{2\pi}\ \frac{r_+ - r_-}{2(r_+^2 + a^2)} \quad .$$

Kerr/CFT correspondence exploits the assumption that near horizon geometry of extremal black holes can be described by the left sector of a CFT in two dimensions. For these latter theories Cardy found that the asymptotic grown of density states is given by

$$\mathcal{S}_{CFT} = 2\pi \sqrt{\frac{c_L \mathcal{L}_0}{6}} \quad .$$

For $\mathcal{L}_0 >> c_L$ and using the definition of left temperature $\frac{\partial \mathcal{S}_{CFT}}{\partial \mathcal{L}_0} = \frac{1}{T_L}$, Cardy formula become

$$\mathcal{S}_{CFT} = \frac{\pi}{3} c_L \; T_L \quad .$$

We cannot associate to the left temperature the Hawking temperature T_H because, as the surface gravity k_s, it vanishes on the event horizon since the outer and inner horizon overlap in a double degenerate horizon

$$T_H := \frac{\bar{h} \; k_s}{k_B \; 2\pi} = \frac{1}{2\pi} \sqrt{-\frac{1}{2} \nabla_\mu \chi_\nu \nabla^\mu \chi^\nu} \; \Big|_{r_+} = \frac{1 - A^2 r_+^2}{2\pi} \; \frac{r_+ - r_-}{2(r_+^2 + a^2)} \quad .$$

Kerr/CFT correspondence exploits the assumption that near horizon geometry of extremal black holes can be described by the left sector of a CFT in two dimensions. For these latter theories Cardy found that the asymptotic grown of density states is given by

$$\mathcal{S}_{CFT} = 2\pi\sqrt{\frac{c_L \mathcal{L}_0}{6}} \quad .$$

For $\mathcal{L}_0 >> c_L$ and using the definition of left temperature $\frac{\partial \mathcal{S}_{CFT}}{\partial \mathcal{L}_0} = \frac{1}{T_L}$, Cardy formula become

$$\mathcal{S}_{CFT} = \frac{\pi}{3} c_L \ T_L \quad .$$

We cannot associate to the left temperature the Hawking temperature T_H because, as the surface gravity k_s, it vanishes on the event horizon since the outer and inner horizon overlap in a double degenerate horizon

$$T_H := \frac{\bar{h} \ k_s}{k_B \ 2\pi} = \frac{1}{2\pi}\sqrt{-\frac{1}{2}\nabla_\mu \chi_\nu \nabla^\mu \chi^\nu} \ \Bigg|_{r_+} = \frac{1 - A^2 r_+^2}{2\pi} \ \frac{r_+ - r_-}{2(r_+^2 + a^2)} \quad .$$

To take into account the rotational degrees of freedom, the Frolov-Thorne vacuum is used to define a temperature. This can be considered as a generalisation of the Hartle-Hawking vacuum originally built for the static Schwarzschild black hole. The Frolov-Thorne vacuum is defined for stationary black holes, in the region where a timelike Killing vector, such as the generator of the horizon, remains timelike. At extremality it is defined as

$$T_\varphi := \lim_{\tilde{r}_+ \to \tilde{r}_e} \frac{T_H}{\Omega_J^{ext} - \Omega_J} = -\frac{\Delta_\varphi^{ext}}{4\pi} \frac{(a^2 + r_+ r_-)\left[1 - A^2 r_+ r_-\right]}{a\sqrt{r_+ r_-}} = \frac{1}{2\pi\kappa} \qquad .$$

Finally inserting the central charge and the rotational left temperature in the Cardy formula we can obtain the value of the entropy of the conformal field theory model associated to the extremal accelerating black hole

CFT Entropy

$$\mathcal{S}_{\mathcal{CFT}} = \frac{\pi^2}{3} c_L T_L = \frac{\pi(a^2 + r_+ r_-)\Delta_\varphi^{ext}}{1 - A^2 r_+ r_-} = \frac{1}{4}\mathcal{A}^{ext} \qquad .$$

This dual entropy coincides with the classical Bekenstein-Hawking entropy of the black hole, i.e. with one quarter of its event horizon area, as expected.

To take into account the rotational degrees of freedom, the Frolov-Thorne vacuum is used to define a temperature. This can be considered as a generalisation of the Hartle-Hawking vacuum originally built for the static Schwarzschild black hole. The Frolov-Thorne vacuum is defined for stationary black holes, in the region where a timelike Killing vector, such as the generator of the horizon, remains timelike. At extremality it is defined as

$$T_\varphi := \lim_{\tilde{r}_+ \to \tilde{r_e}} \frac{T_H}{\Omega_J^{ext} - \Omega_J} = -\frac{\Delta_\varphi^{ext}}{4\pi} \frac{(a^2 + r_+ r_-)\left[1 - A^2 r_+ r_-\right]}{a\sqrt{r_+ r_-}} = \frac{1}{2\pi\kappa} \qquad .$$

Finally inserting the central charge and the rotational left temperature in the Cardy formula we can obtain the value of the entropy of the conformal field theory model associated to the extremal accelerating black hole

CFT Entropy

$$\mathcal{S}_{\mathcal{CFT}} = \frac{\pi^2}{3} c_L T_L = \frac{\pi(a^2 + r_+ r_-)\Delta_\varphi^{ext}}{1 - A^2 r_+ r_-} = \frac{1}{4}\mathcal{A}^{ext} \qquad .$$

This dual entropy coincides with the classical Bekenstein-Hawking entropy of the black hole, i.e. with one quarter of its event horizon area, as expected.

To take into account the rotational degrees of freedom, the Frolov-Thorne vacuum is used to define a temperature. This can be considered as a generalisation of the Hartle-Hawking vacuum originally built for the static Schwarzschild black hole. The Frolov-Thorne vacuum is defined for stationary black holes, in the region where a timelike Killing vector, such as the generator of the horizon, remains timelike. At extremality it is defined as

$$T_\varphi := \lim_{\tilde{r}_+ \to \tilde{r_e}} \frac{T_H}{\Omega_J^{ext} - \Omega_J} = -\frac{\Delta_\varphi^{ext}}{4\pi} \frac{(a^2 + r_+ r_-)\left[1 - A^2 r_+ r_-\right]}{a\sqrt{r_+ r_-}} = \frac{1}{2\pi\kappa} \qquad .$$

Finally inserting the central charge and the rotational left temperature in the Cardy formula we can obtain the value of the entropy of the conformal field theory model associated to the extremal accelerating black hole

CFT Entropy

$$\mathcal{S}_{\mathcal{CFT}} = \frac{\pi^2}{3} c_L T_L = \frac{\pi(a^2 + r_+ r_-)\Delta_\varphi^{ext}}{1 - A^2 r_+ r_-} = \frac{1}{4}\mathcal{A}^{ext} \qquad .$$

This dual entropy coincides with the classical Bekenstein-Hawking entropy of the black hole, i.e. with one quarter of its event horizon area, as expected.

Summary & Conclusions

- At extremality the near horizon geometry of an accelerating Kerr-Newman can be written as a warped and twisted product of $AdS_2 \times S^2$, but it is different from the extremal KN near horizon geometry.

- All the methods of the Kerr/CFT can be smoothly applied in presence of the acceleration (or in presence of an external electromagnetic field).

- We confirmed that, **at extremality**, the entropy, computed with the tools provided by the CFT, matches the gravitational Bekenstein-Hawking entropy for the accelerating and rotating extremal black hole.

- However, **outside the extremal case**, it is not clear how to implement some of the ad-hoc assumptions on the nature of the central charges of the standard Kerr/CFT correspondence.

- At extremality the near horizon geometry of an accelerating Kerr-Newman can be written as a warped and twisted product of $AdS_2 \times S^2$, but it is different from the extremal KN near horizon geometry.

- All the methods of the Kerr/CFT can be smoothly applied in presence of the acceleration (or in presence of an external electromagnetic field).

- We confirmed that, **at extremality**, the entropy, computed with the tools provided by the CFT, matches the gravitational Bekenstein-Hawking entropy for the accelerating and rotating extremal black hole.

- However, **outside the extremal case**, it is not clear how to implement some of the ad-hoc assumptions on the nature of the central charges of the standard Kerr/CFT correspondence.

Summary & Conclusions

- At extremality the near horizon geometry of an accelerating Kerr-Newman can be written as a warped and twisted product of $AdS_2 \times S^2$, but it is different from the extremal KN near horizon geometry.
- All the methods of the Kerr/CFT can be smoothly applied in presence of the acceleration (or in presence of an external electromagnetic field).
- We confirmed that, **at extremality**, the entropy, computed with the tools provided by the CFT, matches the gravitational Bekenstein-Hawking entropy for the accelerating and rotating extremal black hole.
- However, **outside the extremal case**, it is not clear how to implement some of the ad-hoc assumptions on the nature of the central charges of the standard Kerr/CFT correspondence.

- At extremality the near horizon geometry of an accelerating Kerr-Newman can be written as a warped and twisted product of $AdS_2 \times S^2$, but it is different from the extremal KN near horizon geometry.
- All the methods of the Kerr/CFT can be smoothly applied in presence of the acceleration (or in presence of an external electromagnetic field).
- We confirmed that, **at extremality**, the entropy, computed with the tools provided by the CFT, matches the gravitational Bekenstein-Hawking entropy for the accelerating and rotating extremal black hole.
- However, **outside the extremal case**, it is not clear how to implement some of the ad-hoc assumptions on the nature of the central charges of the standard Kerr/CFT correspondence.