Dynamical space-time and gravitational waves

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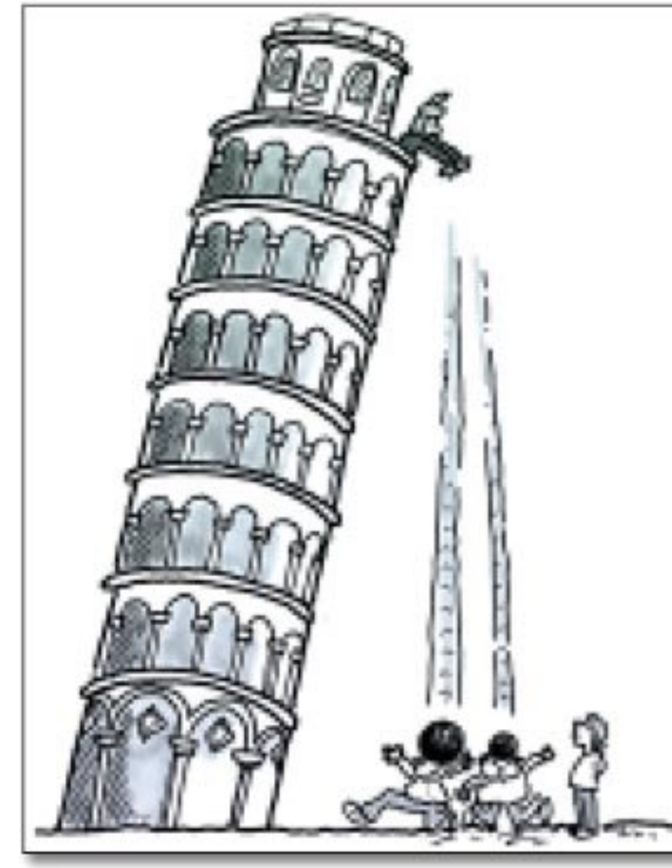
Gravity

- Universality:
  all bodies are subject to gravity independent of their nature or composition

- Equivalence principle:
  all bodies fall with the same gravitational acceleration



Experiment by Simon Stevin and Jan de Groot (1586)

DE
BEGHINSELEN
DER WEEGHCONST
BESCHREVEN DVER
SIMON STEVIN
van Brugghe.

Tot Leyden,
Inde Druckerye van Christoffel Plantijn,
By François van Raphelinghen.
clↃ. IↃ. LXXXVI.

DISCORSI
E
DIMOSTRAZIONI
MATEMATICHE,
intorno à due nuoue scienze
Attenenti alla
MECANICA & I MOVIMENTI LOCALI,
del Signor
GALILEO GALILEI LINCEO,
Filosofo e Matematico primario del Serenissimo
Grand Duca di Toscana.
Con vna Appendice del centro di grauità d'alcuni Solidi.

IN LEIDA,
Appresso gli Elsevirii. M. D. C. XXXVIII.

Discussion in Discorsi by Galilei (1638)

Newton: gravitational acceleration by
instantaneous action at a distance
absolute space and time

Einstein: geometrical interpretation of gravity
gravitational effects propagate at
finite speed (c in vacuum)
dynamical space and time

Dynamical geometry:

- cosmology: expansion of the universe
- astrophysics: gravitational collapse
  $\longrightarrow$ black holes/horizons
- cosmic messengers: gravitational waves

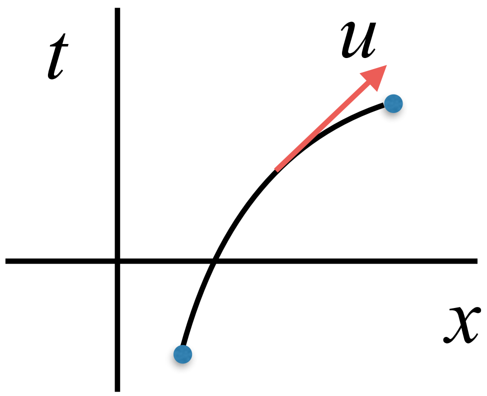
Gravity as space-time geometry

fundamental ingredients:

metric $g_{\mu\nu}$, connection $\Gamma_{\mu\nu}{}^{\lambda}$, curvature $R_{\mu\nu\kappa}{}^{\lambda}$

- The metric converts co-ordinate intervals into space-time intervals (c = 1):

$$-d\tau^2 = g_{\mu\nu}(x)\, dx^\mu dx^\nu, \qquad u^\mu = \dot{x}^\mu = \frac{dx^\mu}{d\tau}$$

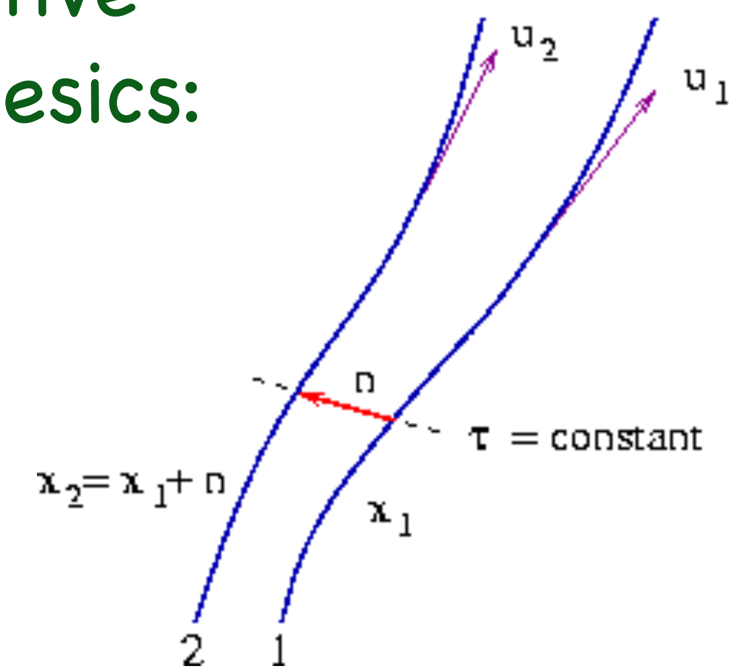


- The connection defines geodesics (free fall) by parallel transport of 4-velocity:

$$D_\tau u^\mu = \dot{u}^\mu + \Gamma_{\lambda\nu}{}^{\mu}(x)\, u^\lambda u^\nu = 0$$

- The curvature gives the relative covariant acceleration of geodesics:

$$D_\tau^2\, n^\mu = R_{\kappa\nu\lambda}{}^{\mu}(x)\, u^\kappa u^\lambda n^\nu$$



Mathematical relations

- inverse metric $g^{\mu\lambda} g_{\lambda\nu} = \delta^\mu_\nu$
- connection $\Gamma_{\mu\nu}{}^{\lambda} = \frac{1}{2} g^{\lambda\kappa}\left(\partial_\mu g_{\kappa\nu} + \partial_\nu g_{\kappa\mu} - \partial_\kappa g_{\mu\nu}\right)$
- Riemann tensor $R_{\mu\nu\kappa}{}^{\lambda} = \partial_\mu \Gamma_{\nu\kappa}{}^{\lambda} - \partial_\nu \Gamma_{\mu\kappa}{}^{\lambda} - \Gamma_{\mu\kappa}{}^{\sigma}\Gamma_{\nu\sigma}{}^{\lambda} + \Gamma_{\nu\kappa}{}^{\sigma}\Gamma_{\mu\sigma}{}^{\lambda}$
- Ricci tensor $R_{\mu\nu} = R_{\mu\lambda\nu}{}^{\lambda}$
- curvature scalar $R = R_\mu^\mu = g^{\mu\nu} R_{\mu\nu}$

Einstein equations

$$\boxed{G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\kappa^2 T_{\mu\nu}}$$

where

$$\kappa^2 = \frac{8\pi G}{c^4} = 2.1 \times 10^{-41} \ \mathrm{kg}^{-1} \ \mathrm{m}^{-1} \ \mathrm{s}^2$$

and $T_{\mu\nu}$ is the energy-momentum tensor of matter and radiation

The dynamics of matter and radiation follows from the consistency condition

$$\boxed{\nabla^\mu T_{\mu\nu} = 0}$$

Geometry of empty space-time

Large number of vacuum solutions:

$$R_{\mu\nu} = 0$$

A. Minkowski space-time: no curvature $R_{\mu\nu\kappa\lambda} = 0$

B. Vacuum solutions with curvature: decompose Riemann curvature in traceless components (Weyl tensor)

$$R_{\mu\nu\kappa\lambda} \;=\; W_{\mu\nu\kappa\lambda} + \frac{1}{(d-2)}\Big(g_{\mu\kappa}W_{\nu\lambda} - g_{\nu\kappa}W_{\mu\lambda} - g_{\mu\lambda}W_{\nu\kappa} + g_{\kappa\lambda}W_{\mu\nu}\Big)$$

$$+\frac{1}{d(d-1)}\Big(g_{\mu\kappa}g_{\nu\lambda} - g_{\mu\lambda}g_{\nu\kappa}\Big)\,R$$

with $\quad W_{\mu\nu} = R_{\mu\nu} - \frac{1}{d}\,g_{\mu\nu}R$

properties: $\quad W_{\mu\lambda\nu}{}^{\lambda} = 0\,,\quad W_{\mu}{}^{\mu} = 0$

$$W_{\mu\nu\kappa\lambda} = -W_{\mu\nu\lambda\kappa} = W_{\kappa\lambda\mu\nu}\,,\quad W_{[\mu\nu\kappa]\lambda} = 0$$

Number of components

$$
\begin{aligned}
N\left[R_{\mu\nu\kappa\lambda}\right] &= \frac{d^2(d^2-1)}{12} \;\rightarrow\; 20 \\
&= \frac{1}{12}d(d+1)(d+2)(d-3) + \frac{1}{2}d(d+1) \\
&= N\left[W_{\mu\nu\kappa\lambda}\right] + N\left[W_{\mu\nu}\right] + 1 \;\rightarrow\; 10+9+1
\end{aligned}
$$

Non-vanishing Weyl tensor implies non-vanishing curvature:

$$W_{\mu\nu\kappa\lambda} \neq 0 \quad \rightarrow \quad R_{\mu\nu\kappa\lambda} \neq 0$$

is consistent with $R_{\mu\nu} = W_{\mu\nu} = 0$

Curvature in the absence of matter

Example: PP-waves

Exact gravitational-wave solution;
in light-cone co-ordinates $u = t - z$, $v = t + z$

$$d\tau^2 = dudv - \Phi(u,x,y)du^2 - dx^2 - dy^2$$

Non-vanishing components of Riemann and Ricci tensors

$$R_{uiuj} = \frac{1}{2}\frac{\partial^2\Phi}{\partial x^i \partial x^j} \qquad R_{uu} = \frac{1}{2}\left(\partial_x^2 + \partial_y^2\right)\Phi$$

Regular vacuum solutions $R_{uu} = 0$:

$$\Phi = h_+(u)(x^2 - y^2) + 2h_\times(u)\, xy$$

$$R_{uxux} = -R_{uyuy} = h_+(u) \quad R_{uxuy} = h_\times(u)$$

Small-curvature waves

reduction of GR to field theory of massless spin-2 fields

- symmetric tensor field $h_{\mu\nu} = h_{\nu\mu}$
- free field equation in Minkowski space-time

$$\Box h_{\mu\nu} - \partial_\mu \partial^\lambda h_{\lambda\nu} - \partial_\nu \partial^\lambda h_{\lambda\mu} + \partial_\mu \partial_\nu h_\lambda^\lambda = 0$$

- local gauge invariance

$$h'_{\mu\nu} = h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

- unique consistent extension with self-interactions (no ghosts, no tachyons)

$$g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu} \quad \text{with} \quad R_{\mu\nu}[g] = 0$$

[Feynman, Fronsdal, Veltman]

Gauge fixing

Use local gauge invariance to impose*

$$\partial^\lambda h'_{\lambda\mu} - \frac{1}{2}\partial_\mu {h'_\lambda}^\lambda = 0 \quad \Rightarrow \quad \Box h'_{\mu\nu} = 0$$

massless wave equation; (1+1)-dimensional reduction

$$\left(\partial_z^2 - \partial_t^2\right) h = 0 \quad \Rightarrow \quad h = h_+(z+t) + h_-(z-t)$$

waves propagating at the speed of light (c = 1)
in the +z or -z direction

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* by taking $-\Box\xi_\mu = \partial^\lambda h_{\lambda\mu} - \frac{1}{2}\partial_\mu h_\lambda^\lambda$

Reformulation

Field redefinition $$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\, h_\lambda{}^\lambda$$

field equation and gauge condition for the redefined field:

$$\Box\,\bar{h}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\,\Box\,\bar{h}_\lambda{}^\lambda = 0 \qquad \partial^\lambda \bar{h}_{\lambda\mu} = 0$$

$$\Box\,\bar{h}_{\mu\nu} = 0$$

redefined gauge transformations:

$$\bar{h}'_{\mu\nu} = \bar{h}_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu - \eta_{\mu\nu}\,\partial\cdot\xi$$

gauge condition respected if $\Box \xi_\mu = 0$

Momentum representation

$$\bar{h}_{\mu\nu}(x) = \int \frac{d^4k}{(2\pi)^2}\, \varepsilon_{\mu\nu}(k) e^{-ik\cdot x} \qquad \varepsilon^*_{\mu\nu}(k) = \varepsilon_{\mu\nu}(-k)$$

Field equations:

$$k^2 \varepsilon_{\mu\nu}(k) = 0 \qquad k^\lambda \varepsilon_{\lambda\mu} = 0$$

solution:

$$\varepsilon_{\mu\nu}(k) = e_{\mu\nu}(\mathbf{k})\, \delta(k^2)$$

implying $k^\mu = (\omega_k, \mathbf{k})$ with $\omega_k = \sqrt{\mathbf{k}^2}$

$$\rightarrow \quad h_{\mu\nu}(x) = \int \frac{d^3k}{8\pi^2 \omega_k} \left(e_{\mu\nu}(\mathbf{k}) e^{-i(\mathbf{k}\cdot\mathbf{x} - \omega_k t)} + e^*_{\mu\nu}(\mathbf{k}) e^{i(\mathbf{k}\cdot\mathbf{x} - \omega_k t)} \right)$$

with $\quad \omega_k e_{0\mu} = k_i e_{i\mu}$

The amplitudes $e_{\mu\nu}(\mathbf{k})$ are defined up to a final gauge transformation with $\Box\xi_\mu = 0$ ($k^2 = 0$):

$$\xi_\mu = i \int \frac{d^3k}{8\pi^2\omega_k} \left(a_\mu(\mathbf{k}) e^{-i(\mathbf{k}\cdot\mathbf{x}-\omega_k t)} - a^*_\mu(\mathbf{k}) e^{i(\mathbf{k}\cdot\mathbf{x}-\omega_k t)} \right)$$

These gauge transformations take the form

$$e'_{00} = e_{00} + \omega_k a_0 + \mathbf{k}\cdot\mathbf{a}$$

$$e'_{0i} = e_{0i} + \omega_k a_i + k_i a_0$$

$$e'_{ij} = e_{ij} + k_i a_j + k_j a_i - \delta_{ij}\left(\mathbf{k}\cdot\mathbf{a} - \omega_k a_0\right)$$

with the following choice:

$$a_0 = -\tfrac{1}{4\omega_k}\left(e_{00} + \textstyle\sum_j e_{jj}\right) \qquad a_i = -\tfrac{1}{\omega_k} e_{0i} + \tfrac{k_i}{\omega_k}\left(e_{00} + \textstyle\sum_j e_{jj}\right)$$

this results in $e'_{00} = e'_{0i} = 0$, $\sum_j e'_{jj} = 0$, $\sum_j k_j e'_{ji} = 0$

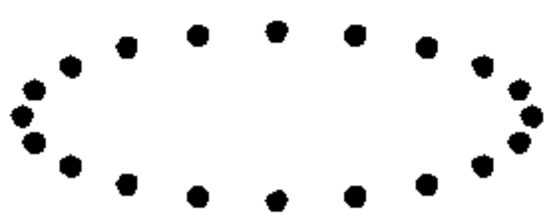
Example:

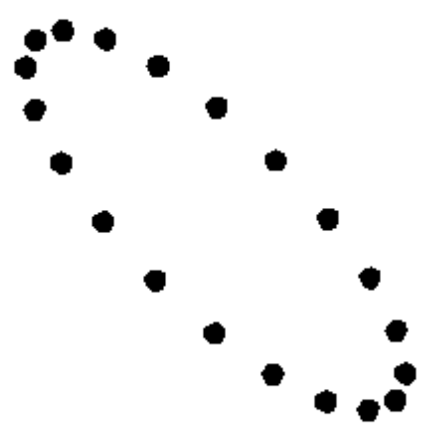
Monochromatic wave with wave vector $k^\mu = (k, 0, 0, k)$ travelling in the z-direction implies

$$e_{\mu\nu}(\mathbf{k}) = A_+(k)\varepsilon^+_{\mu\nu} + A_\times(k)\varepsilon^\times_{\mu\nu}$$

with transversely polarised amplitudes

$$\varepsilon^+_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \varepsilon^\times_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$





Space-time representation

TT-gauge:

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & h_{ij} \end{pmatrix} \qquad \sum_j h_{jj} = 0 \qquad \sum_j \nabla_j h_{ji} = 0$$

Energy and momentum density

$$\mathcal{E} = \frac{1}{2}\left(\frac{1}{c}\frac{\partial h_{ij}}{\partial t}\right)^2 + \frac{1}{2}\left(\nabla h_{ij}\right)^2 \qquad \mathbf{\Pi} = \frac{\partial h_{ij}}{\partial t}\nabla h_{ij}$$

Equation of continuity:
$$\frac{\partial \mathcal{E}}{\partial t} = \nabla \cdot \mathbf{\Pi}$$

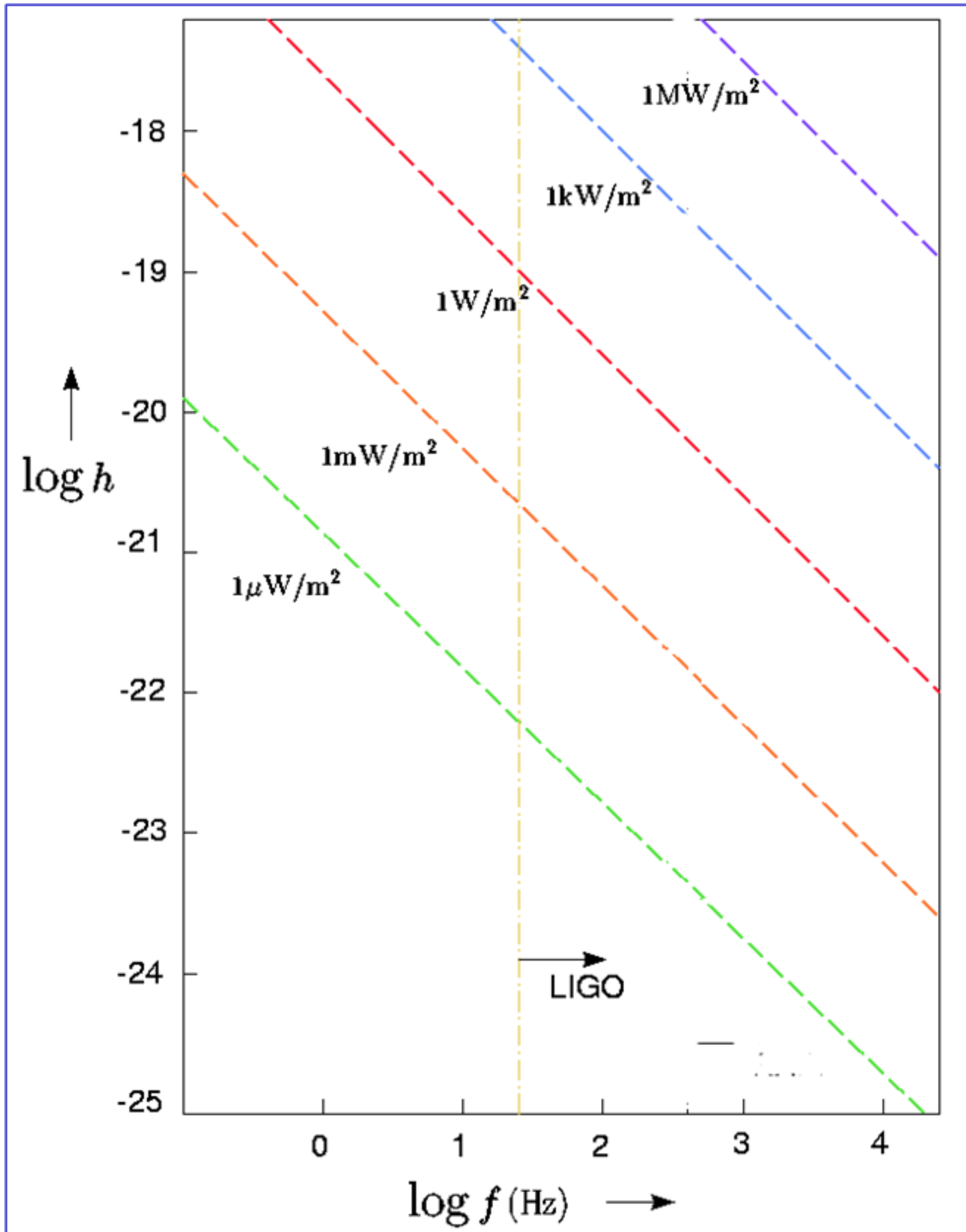
Integral form:
$$\frac{dE}{dt} = \oiint d^2\sigma\, \Pi_n$$

energy flux through spherical surface with radius D:

$$\Phi = \frac{dE}{4\pi D^2\, dt} = \frac{1}{4\pi}\iint d^2\Omega \left(\frac{\partial h_{ij}}{\partial t}\frac{\partial h_{ij}}{\partial r}\right)_D$$

$$h_{ij} = \mathrm{Re}\left[\left(A_+\varepsilon_{ij}^{+} + A_\times\varepsilon_{ij}^{\times}\right)e^{i2\pi f(z-t)}\right]$$

$$\kappa h = \sqrt{A_+^2 + A_\times^2}$$



$$\boxed{h_{ij} = e_{ij}e^{i\left(\vec{k}\cdot\vec{r}-2\pi f t\right)}}$$

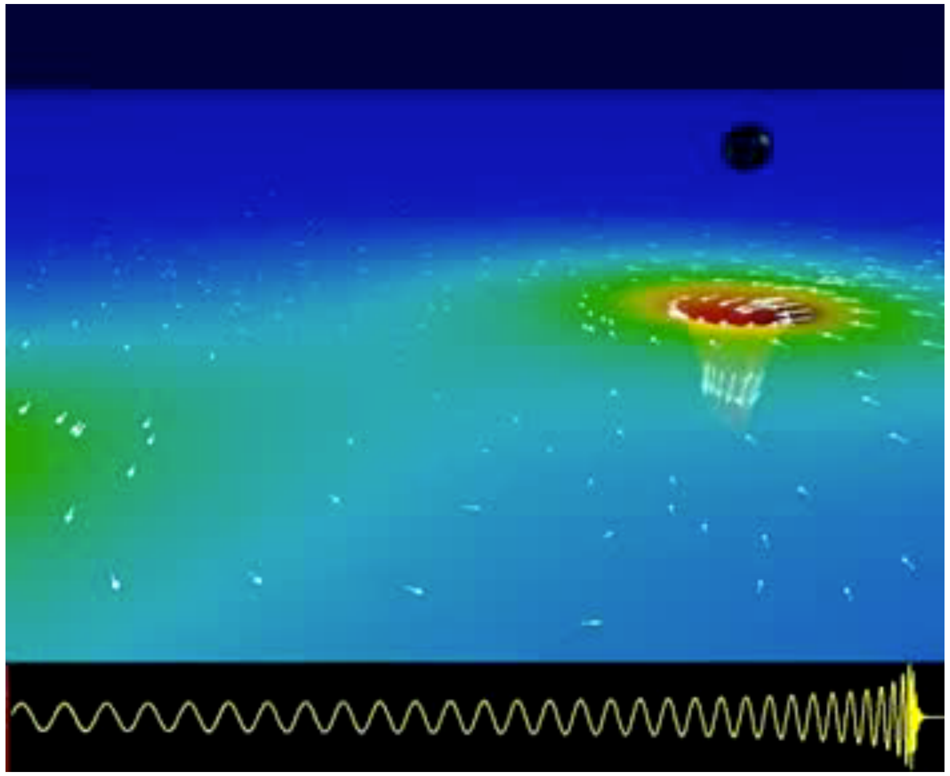
$$\boxed{\Phi = \frac{d^2E}{dAdt} = \frac{\pi c^3 f^2}{8G}\,|e_{ij}|^2}$$

Sources of gravitational waves

- There are no laboratory sources of gravitational waves
- The only plausible sources are astrophysical or cosmological

They can be classified in 3 broad categories

a. periodic: millisecond pulsars, compact binaries
b. transient: supernovae, stellar collisions
c. stochastic: background of galactic binaries,
early universe



Generation of gravitational waves

Small curvature: inhomogeneous wave equation

$$\Box h_{\mu\nu} = -\kappa T_{\mu\nu} \qquad \partial^\lambda T_{\lambda\mu} = 0 \mod \mathcal{O}(\kappa)$$

Propagating physical components

$$h_{ij} = -\frac{\kappa}{4\pi}\int d^3x' \, \frac{T_{ij}(\mathbf{x}', t - |\mathbf{x}-\mathbf{x}'|)}{|\mathbf{x}-\mathbf{x}'|}$$

$$\simeq -\frac{\kappa}{4\pi r}\int d^3x' \, T_{ij}(\mathbf{x}', t-r)$$

(compact sources)

$$\left(\partial_t T_{00} = \partial'_k T_{k0}\right)$$

$$= -\frac{\kappa}{8\pi r}\frac{\partial^2}{\partial t^2}\int d^3x' \, x'_i x'_j \, T_{00}(\mathbf{x}', t-r) \equiv \ddot{Q}_{ij}(t-r)$$

implying $\dfrac{\partial h_{ij}}{\partial r} = -\dfrac{\partial h_{ij}}{\partial t} = -\dddot{Q}_{ij}$

Non-relativistic limit:

$$\frac{dE}{dt} = -\langle \dddot{Q}_{ij} \dddot{Q}_{ij} \rangle \to -\frac{G}{5c^5} \left| \frac{d^3 I_{ij}}{dt^3} \right|^2$$

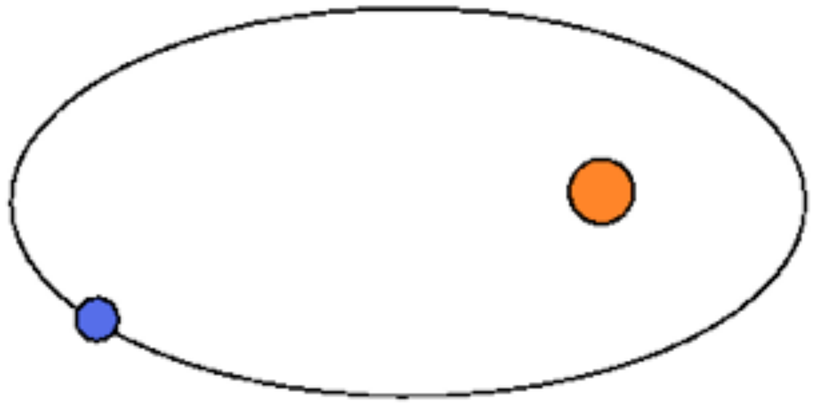
with quadrupole moment

$$I_{ij}(t) = \int d^3x \left(x_i x_j - \frac{1}{3} \delta_{ij} \mathbf{x}^2 \right) \rho(\mathbf{x}, t)$$

Observe that in mks units

$$\frac{G}{5c^5} = 0.55 \times 10^{-53} \, \mathrm{kg}^{-1} \ \mathrm{m}^{-2} \ \mathrm{s}^3$$

Newtonian binaries



$$\ddot{\mathbf{r}} = -\frac{GM}{r^3}\mathbf{r}$$
$$= -\omega^2 \mathbf{r} \quad \text{for constant radius}$$

Circular orbits:

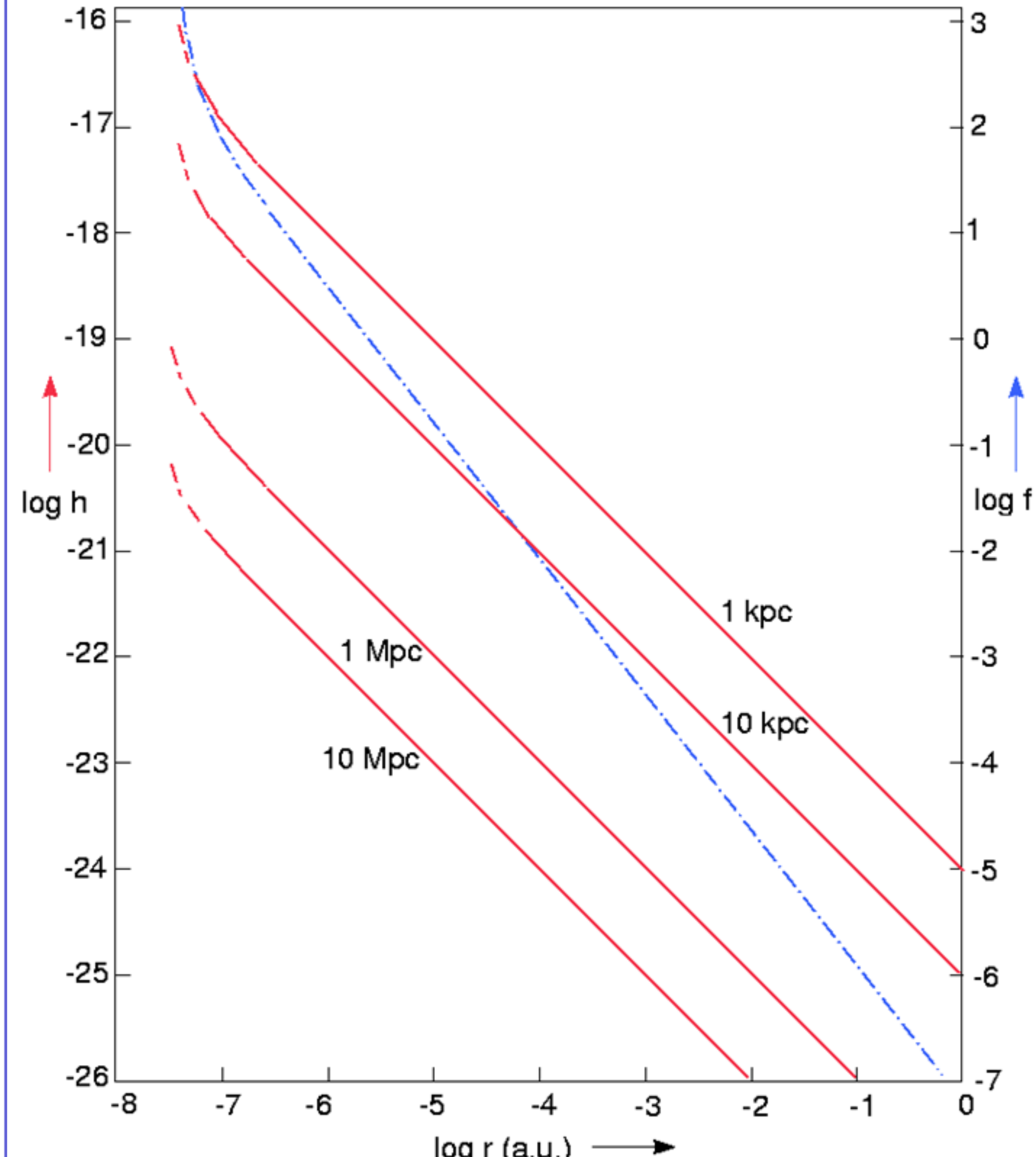
$$\mathbf{r}_1 = \frac{m_2 r}{M}(\cos\omega t, \sin\omega t, 0) \qquad \mathbf{r}_2 = -\frac{m_1 r}{M}(\cos\omega t, \sin\omega t, 0)$$

Quadrupole moment:

$$I_{ij} = \frac{\mu r^2}{2}\begin{pmatrix} \cos 2\omega t + \frac{1}{3} & \sin 2\omega t & 0 \\ \sin 2\omega t & -\cos 2\omega t + \frac{1}{3} & 0 \\ 0 & 0 & -\frac{2}{3} \end{pmatrix} \rightarrow \left|\frac{d^3 I_{ij}}{dt^3}\right|^2 = 32\mu^2 r^4 \omega^6$$

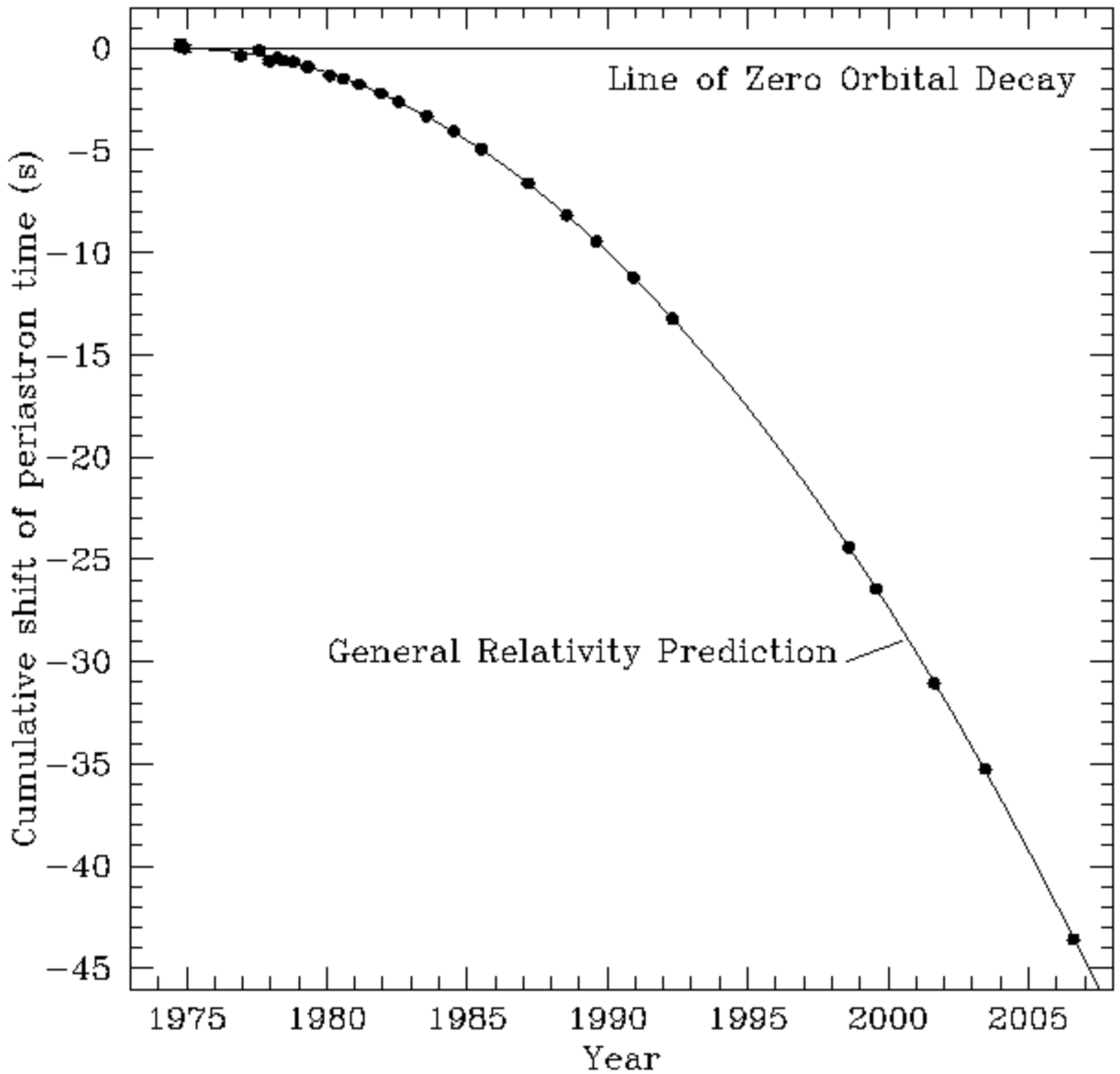
$$f = \frac{\omega}{\pi} = \frac{1}{\pi}\sqrt{\frac{GM}{r^3}}$$

$$h \simeq \frac{R_S^2}{rD} = \frac{G^2\mu M}{c^4 r D}$$



$$f = 0.6 \times 10^{-7}\,\frac{\sqrt{(M_1 + M_2)/M_\odot}}{(r/[1\,\mathrm{a.u.}])^{3/2}}\ \mathrm{Hz}$$

Binary pulsar 1913+16



$$m_1 = 1.44\, M_\odot \qquad m_2 = 1.39\, M_\odot$$

$$P = 2.79 \times 10^4\,\mathrm{s}$$

$$r = \sqrt{r_+ r_-} = 1.53 \times 10^6\,\mathrm{km}$$

$$D = 6.4\,\mathrm{kpc} = 2 \times 10^{17}\,\mathrm{km}$$

Expect frequency $f = 0.7 \times 10^{-4}$ Hz
and dimensionless amplitude $h = 0.6 \times 10^{-22}$

Beyond flat space-time: waves in a curved background

Background metric $\bar{g}_{\mu\nu}$

local true metric $g_{\mu\nu} = \bar{g}_{\mu\lambda}\left(\delta_\nu^\lambda + 2\kappa h_\nu^\lambda\right)$

Use background metric for raising/lowering indices; expand Einstein tensor in powers of κ :

$$
\begin{aligned}
G_{\mu\nu} \;\; = \;\; & \bar{G}_{\mu\nu} - \kappa \bar{R} h_{\mu\nu} + \kappa\left(\bar{R}_{\mu\lambda} h_\nu^\lambda + \bar{R}_{\nu\lambda} h_\mu^\lambda\right) - 2\kappa \bar{R}_{\mu\kappa\nu\lambda} h^{\kappa\lambda} + \kappa \bar{g}_{\mu\nu} \bar{R}_{\kappa\lambda} h^{\kappa\lambda} \\
& + \kappa\left[\bar{D}^\lambda \bar{D}_\lambda h_{\mu\nu} - \bar{D}_\mu \bar{D}_\lambda h_\nu^\lambda - \bar{D}_\nu \bar{D}_\lambda h_\mu^\lambda + \bar{D}_\mu \bar{D}_\nu h_\lambda^\lambda\right. \\
& \qquad \left. + \, \bar{g}_{\mu\nu}\left(\bar{D}_\kappa \bar{D}_\lambda h^{\kappa\lambda} - \bar{D}^\lambda \bar{D}_\lambda h_\kappa^\kappa\right)\right]
\end{aligned}
$$

In vacuum background space-time: $\bar{G}_{\mu\nu} = \bar{R}_{\mu\nu} = 0$

$$
-\bar{D}^\lambda \bar{D}_\lambda h_{\mu\nu} + \bar{D}_\mu \bar{D}_\lambda h_\nu^\lambda + \bar{D}_\nu \bar{D}_\lambda h_\mu^\lambda - \bar{D}_\mu \bar{D}_\nu h_\lambda^\lambda + 2\bar{R}_{\mu\kappa\nu\lambda} h^{\kappa\lambda} = \kappa T_{\mu\nu}
$$

Gauge transformations

$$\delta g_{\mu\nu} = \bar{D}_\mu \xi_\nu + \bar{D}_\nu \xi_\mu$$

$$\delta h_{\mu\nu} = \xi^\lambda \bar{D}_\lambda h_{\mu\nu} + h_{\mu\lambda} \bar{D}_\nu \xi^\lambda + h_{\nu\lambda} \bar{D}_\mu \xi^\lambda$$

After the gauge choice

$$\bar{D}_\mu \bar{D}_\lambda h^\lambda_\nu + \bar{D}_\nu \bar{D}_\lambda h^\lambda_\nu - \bar{D}_\mu \bar{D}_\nu h^\lambda_\lambda = 0$$

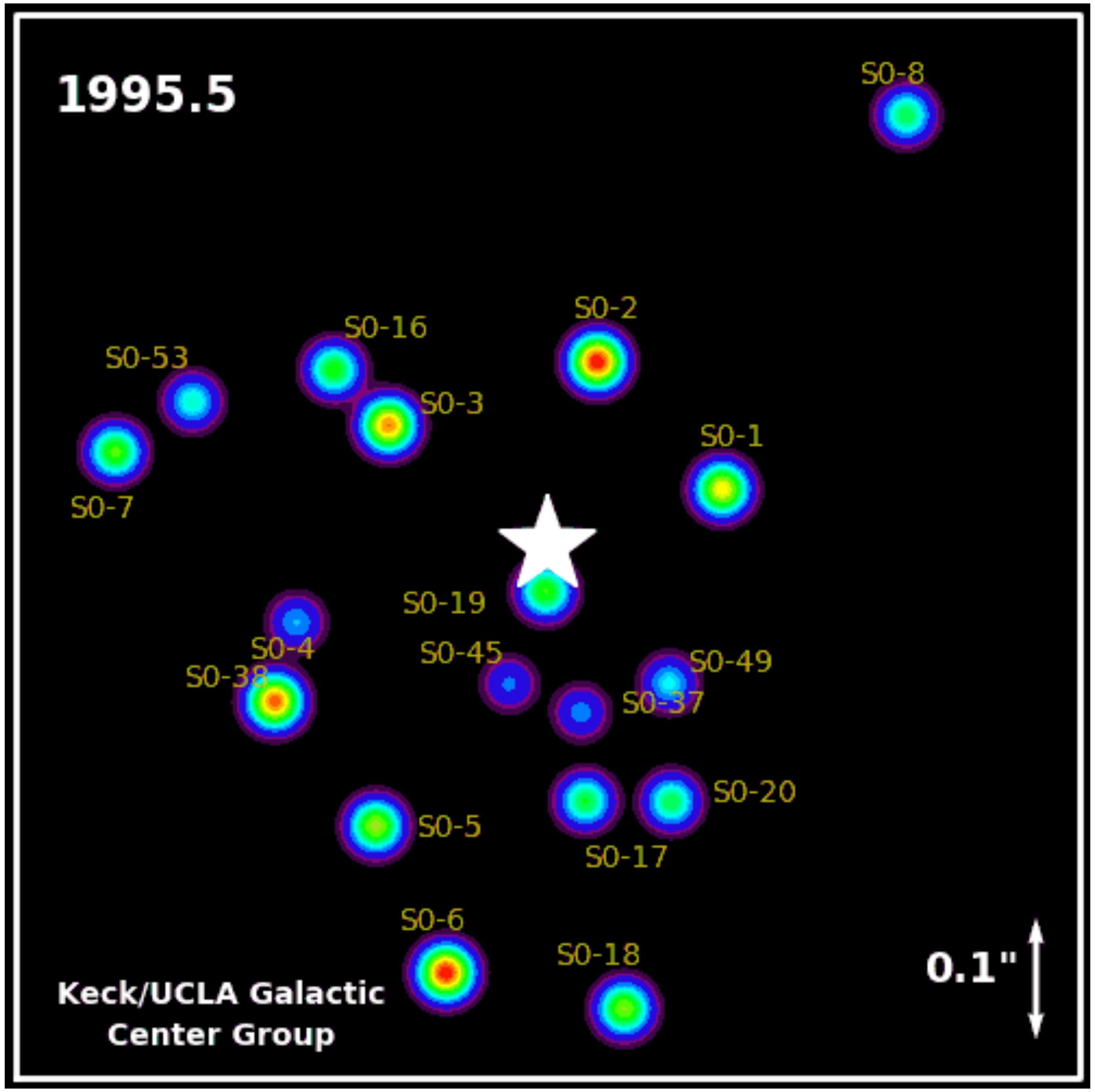
the wave equation becomes

$$-\bar{D}^\lambda \bar{D}_\lambda h_{\mu\nu} + 2\bar{R}_{\mu\kappa\nu\lambda}\, h^{\kappa\lambda} = \kappa T_{\mu\nu}$$

the curvature acts as an (anisotropic) index of refraction
e.g. lensing of gravitational waves; e.g.

$$\bar{R}_{\mu\nu\kappa\lambda} = \frac{1}{12}\left(g_{\mu\kappa} g_{\nu\lambda} - g_{\mu\lambda} g_{\nu\kappa}\right) R \quad \rightarrow \quad \left(-\bar{D}^\lambda \bar{D}_\lambda + \frac{1}{3} R\right) h_{\mu\nu} = \kappa T_{\mu\nu}$$

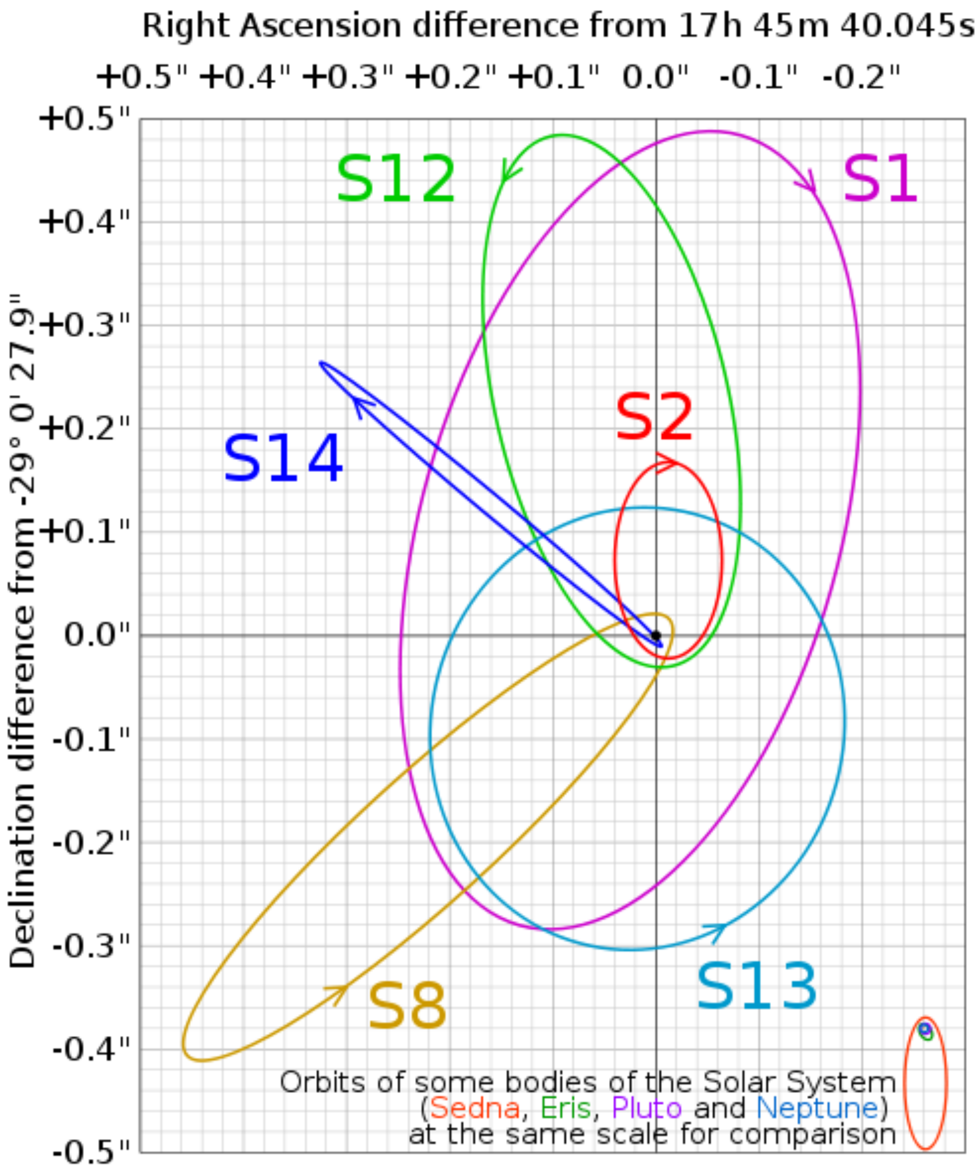
Extreme mass-ratio binaries

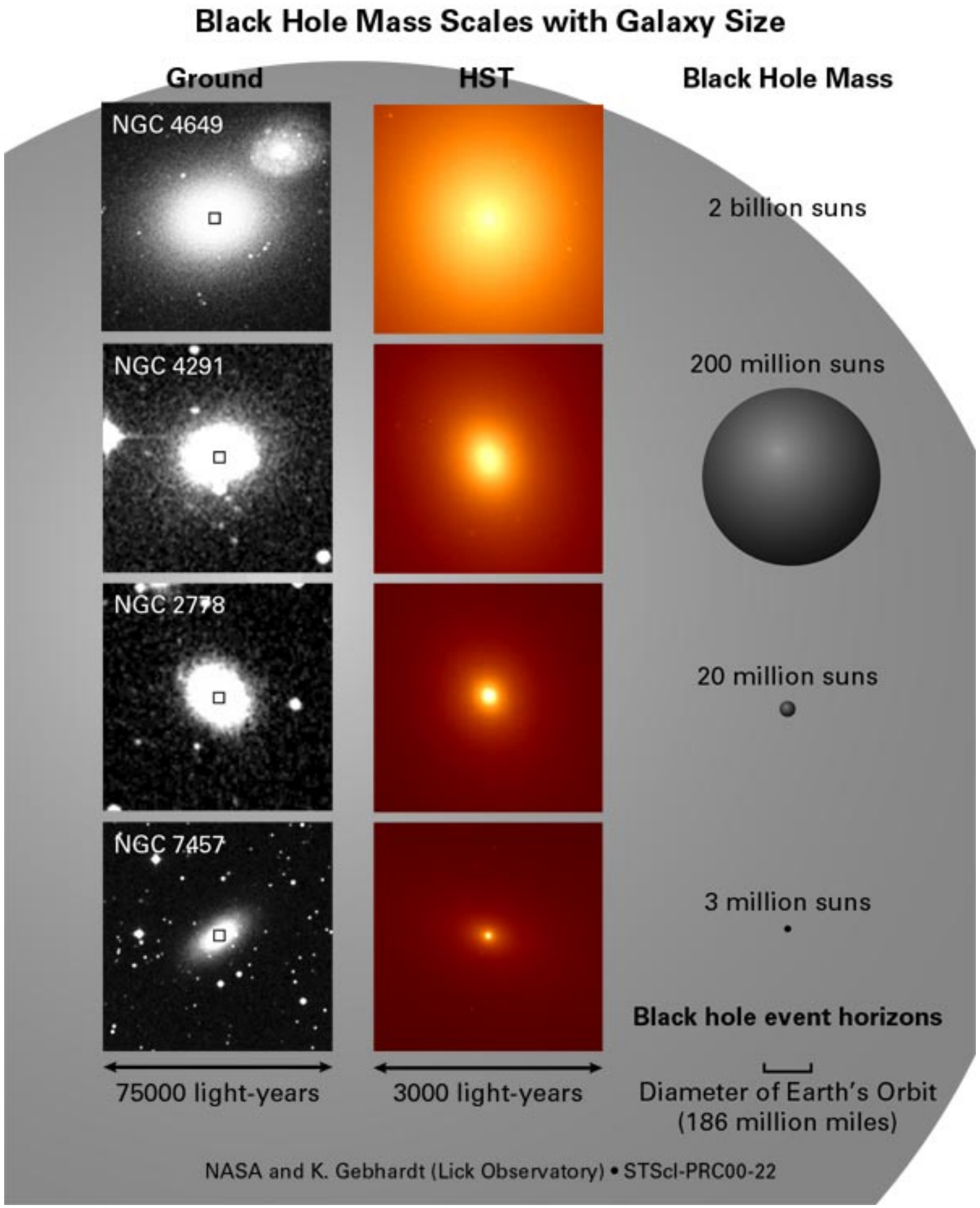


These images/animations were created by Prof. Andrea Ghez and her research team at UCLA and are from data sets obtained with the W. M. Keck Telescopes.

Supermassive black hole in galactic center

$$M_{BH} \simeq 4 \times 10^6\, M_\odot$$


Right Ascension difference from 17h 45m 40.045s
+0.5" +0.4" +0.3" +0.2" +0.1" 0.0" -0.1" -0.2"
+0.5" +0.4" +0.3" +0.2" +0.1" 0.0" -0.1" -0.2" -0.3" -0.4" -0.5"
Declination difference from -29° 0' 27.9"
S12
S1
S2
S14
S13
S8
Orbits of some bodies of the Solar System
(Sedna, Eris, Pluto and Neptune)
at the same scale for comparison


Black Hole Mass Scales with Galaxy Size
Ground
HST
Black Hole Mass
NGC 4649
2 billion suns
NGC 4291
200 million suns
NGC 2778
20 million suns
NGC 7457
3 million suns
Black hole event horizons
75000 light-years
3000 light-years
Diameter of Earth's Orbit
(186 million miles)
NASA and K. Gebhardt (Lick Observatory) • STScI-PRC00-22

Background space-time: Schwarzschild metric

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\varphi^2$$

Perturbation by orbiting compact companion: white dwarf, neutron star, stellar-mass black hole

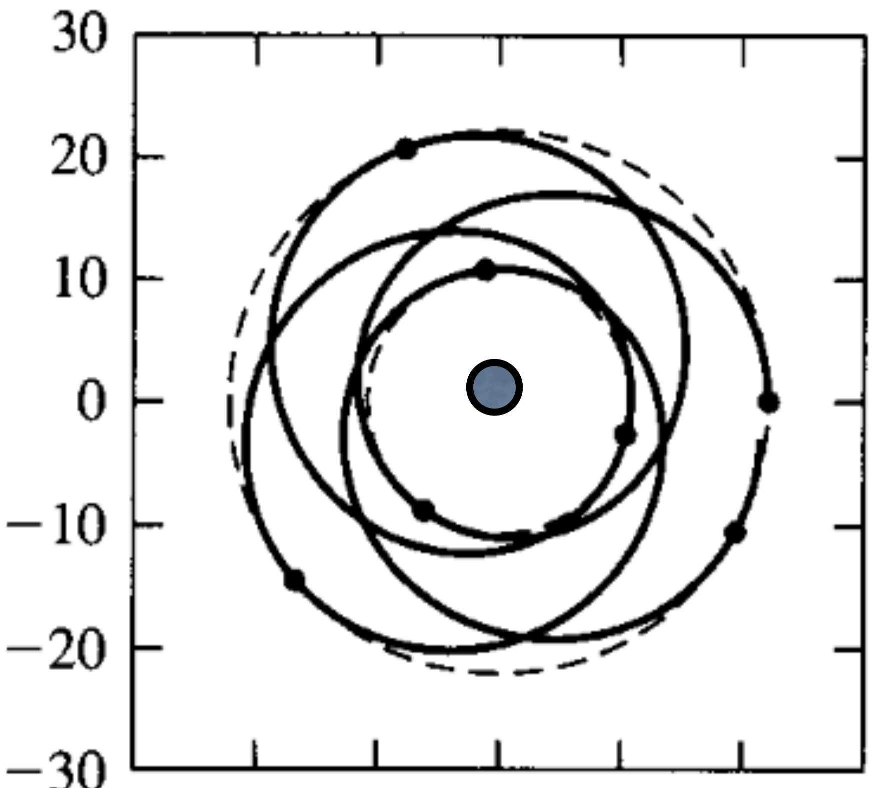
$$T_{\mu\nu}(x) = m \int d\tau \, u_\mu u_\nu \frac{1}{\sqrt{-g}} \delta^4 \left(x - X(\tau)\right)$$

As before $u^\mu(\tau)$ is the 4-velocity $u^\mu = \dfrac{dX^\mu}{d\tau}$

Observe: $\nabla_\mu T^{\mu\nu} = m \int d\tau \, \dfrac{Du^\nu}{D\tau} \dfrac{1}{\sqrt{-g}} \delta^4 \left(x - X(\tau)\right) = 0$

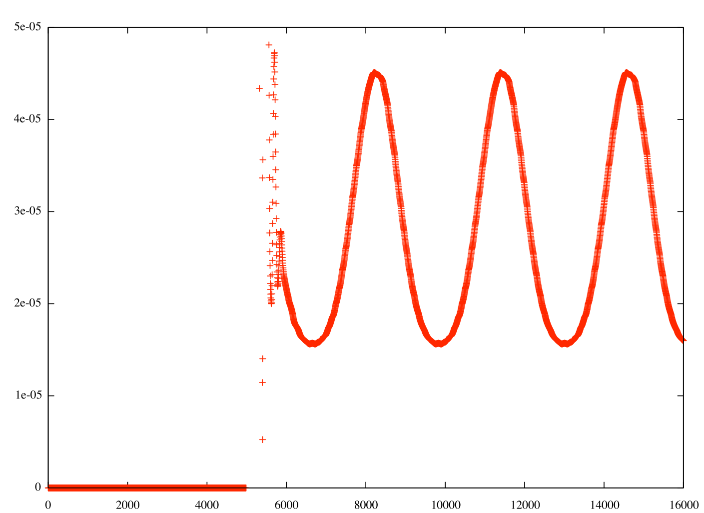
by equation of motion

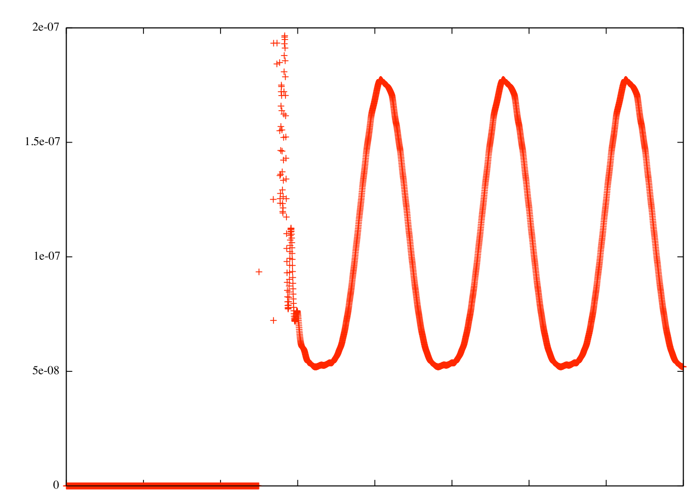
Inspiral phase: quasi-stable orbits



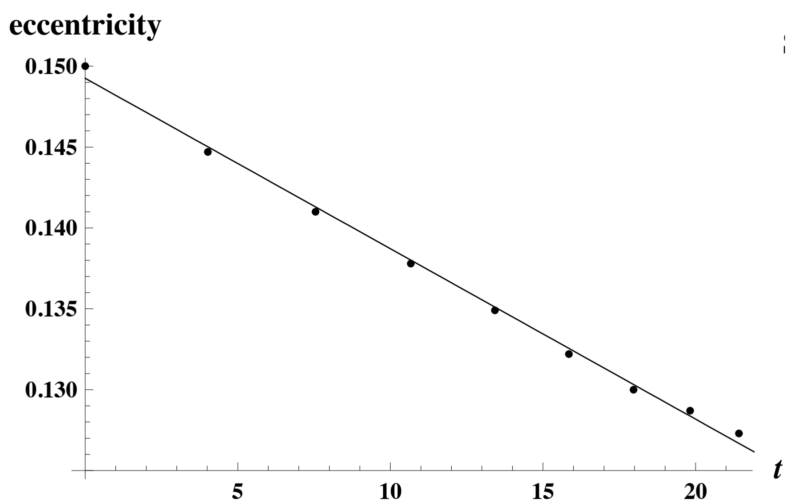
precession of apastron/periastron determined by eccentricity

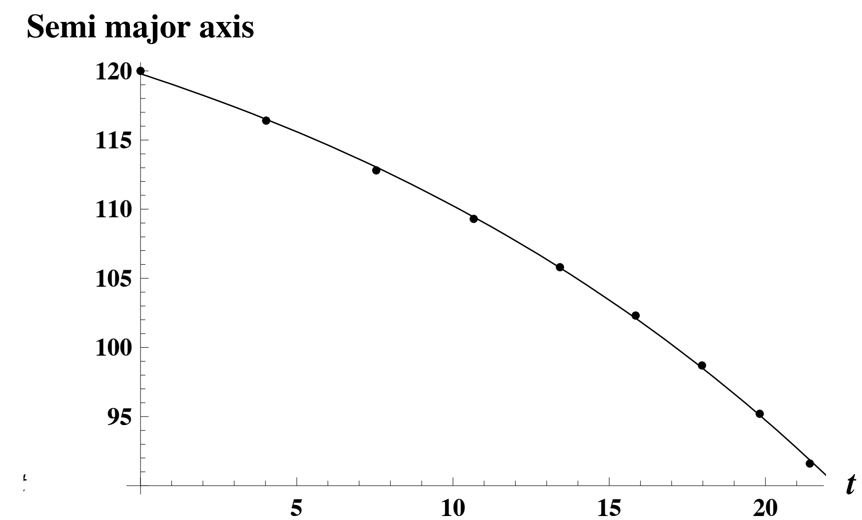
Power of gravitational waves in two polarisation modes

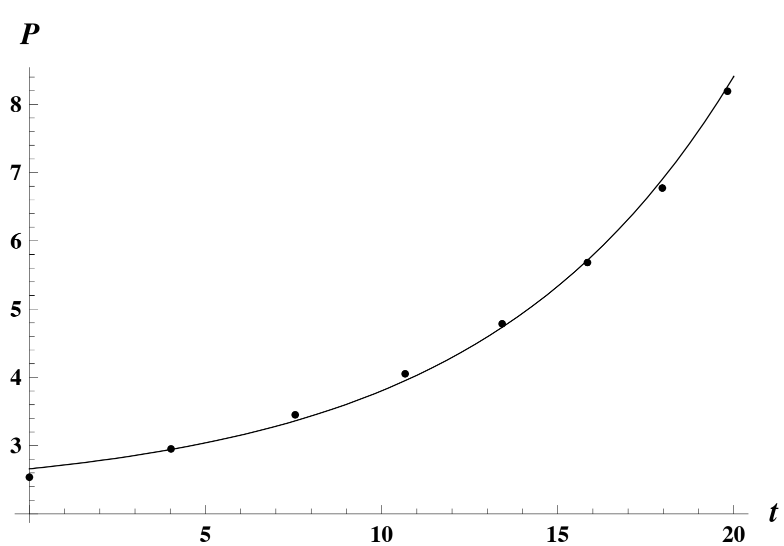


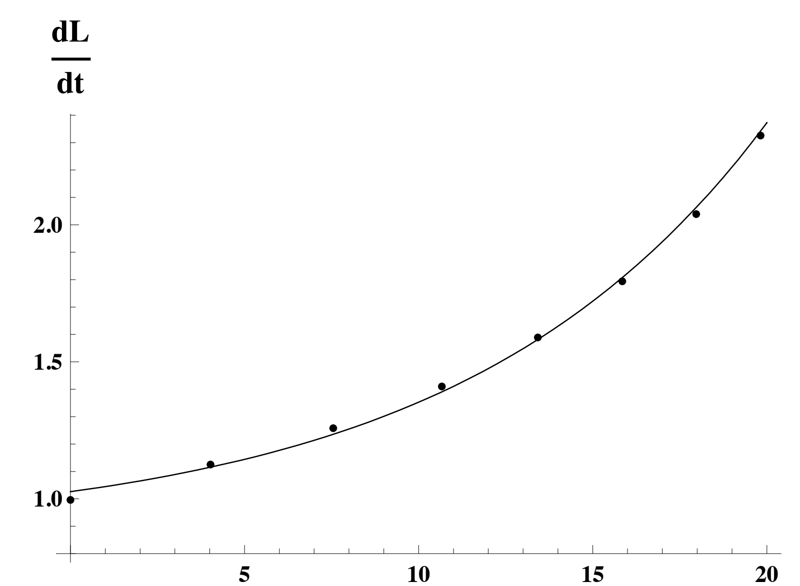


Evolution of inspiralling orbit (e = 0.15)





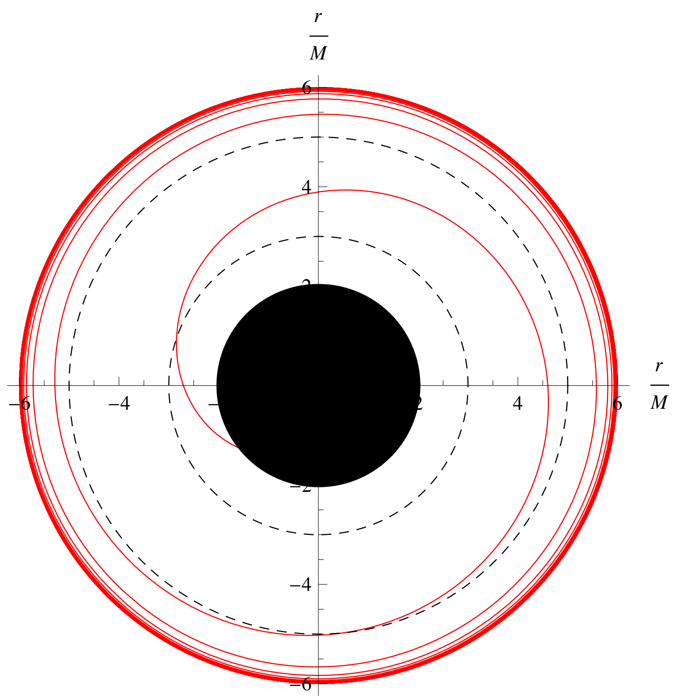




Merger phase:

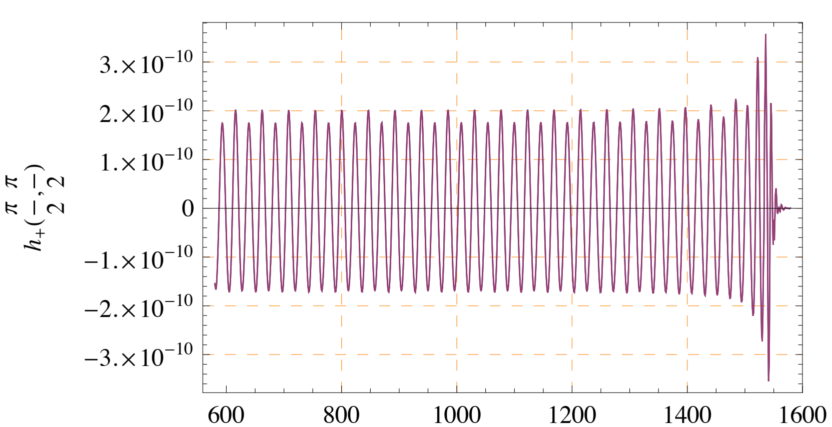
fall towards the horizon $R = 2M$

from the ISCO $R = 6M$

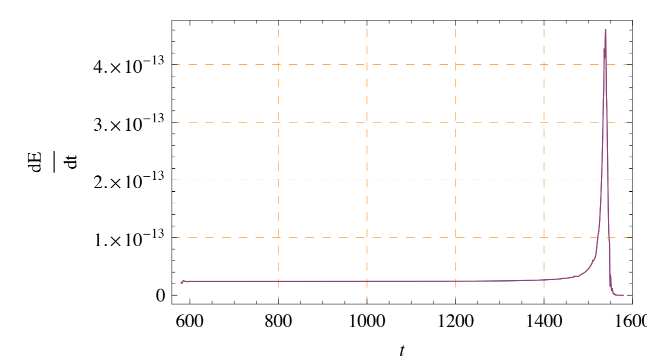


$$r = \frac{R}{1 + e\, ctg^2(A\varphi/2)}$$

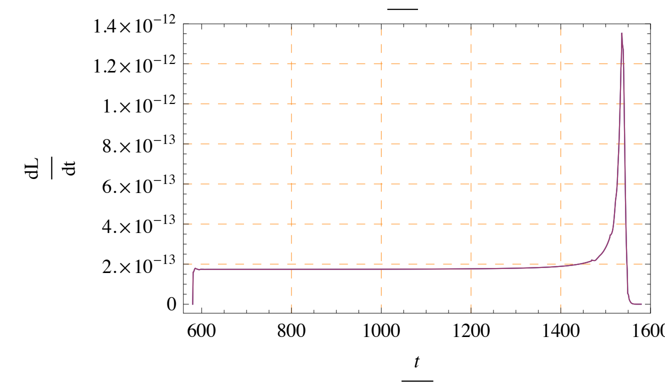
$$e = \frac{3}{2}\left(1 - \frac{R}{6M}\right) \qquad A = \sqrt{\frac{1}{2}\left(\frac{6M}{R} - 1\right)}$$



amplitude

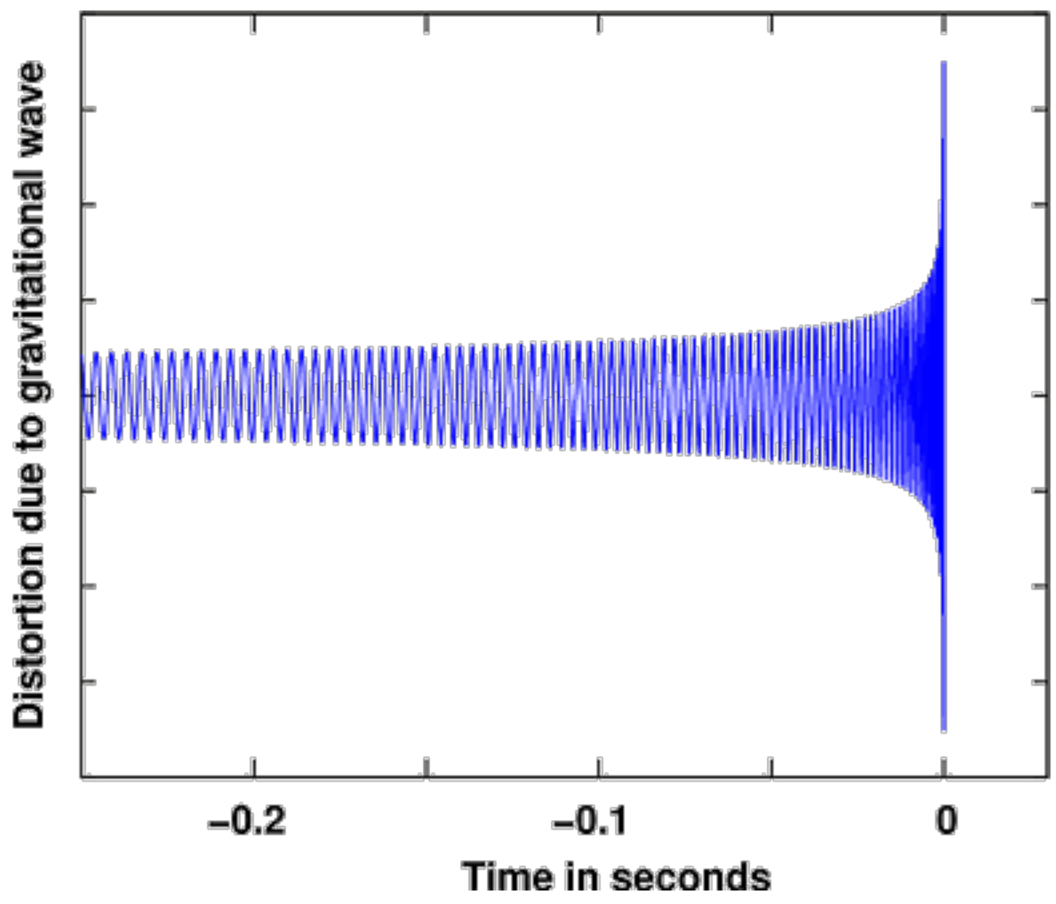


power



loss of angular momentum

Chirp signal of equal-mass binary neutron star

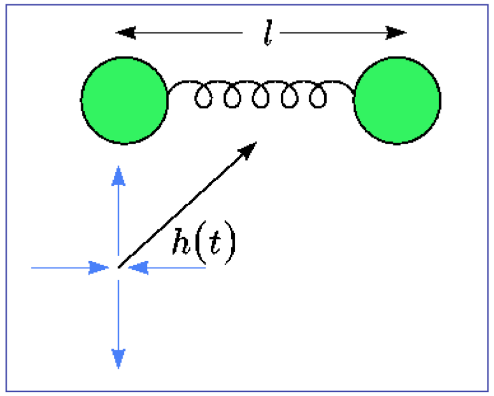


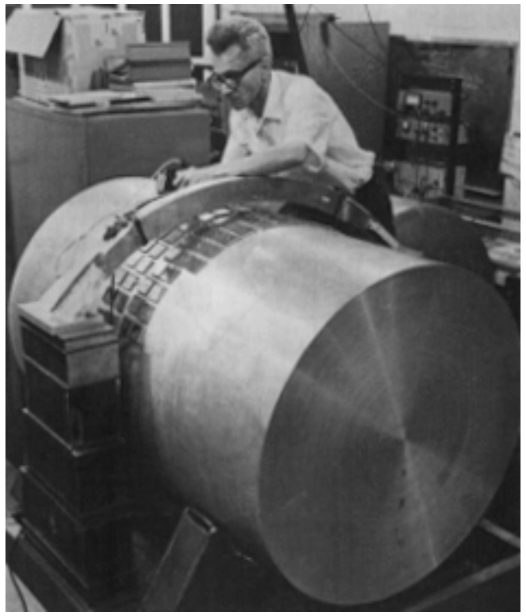
Chirp

Detection

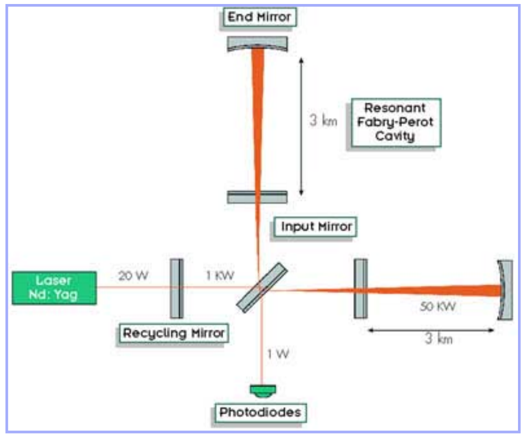
Energy transfer between wave and detector

- Resonant detection:
  GW changes the length of a spring



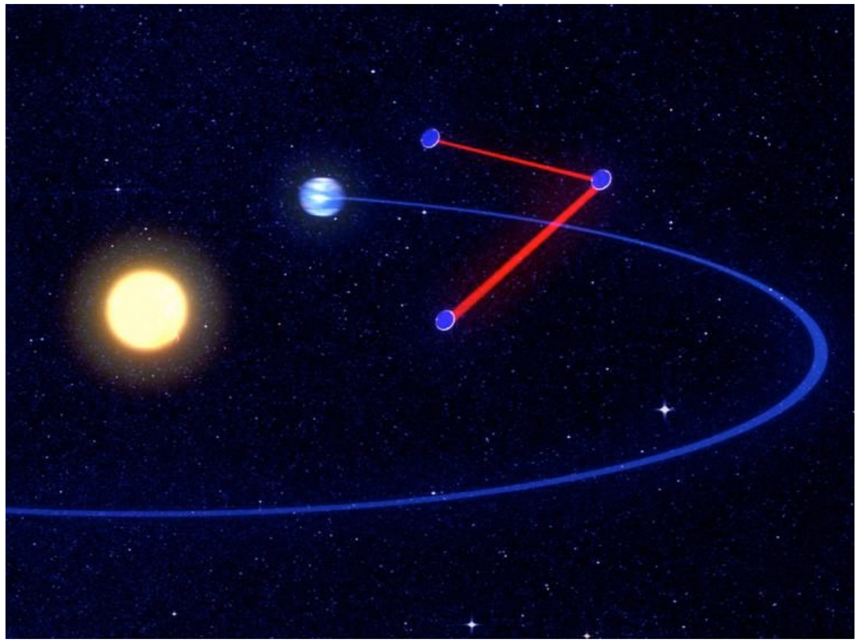


- Interferometric detection:
  GW changes the armlength of an interferometer

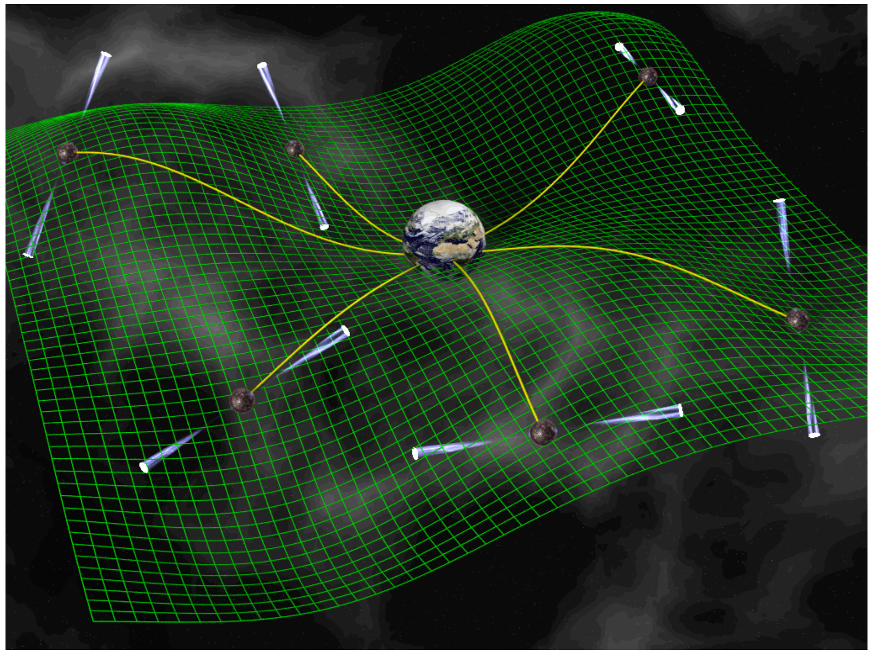


Detection in space

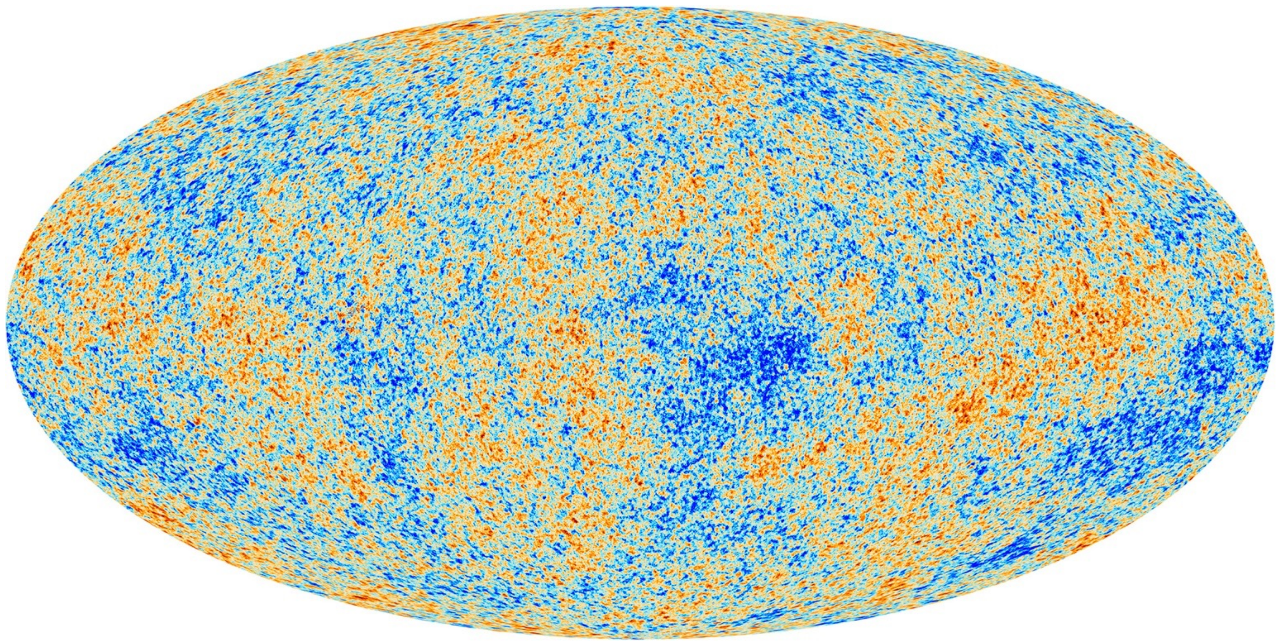
- eLISA space craft with laser tracking in solar orbit



- Pulsar timing: tracking timing variations of an array of pulsars in our region of the galaxy

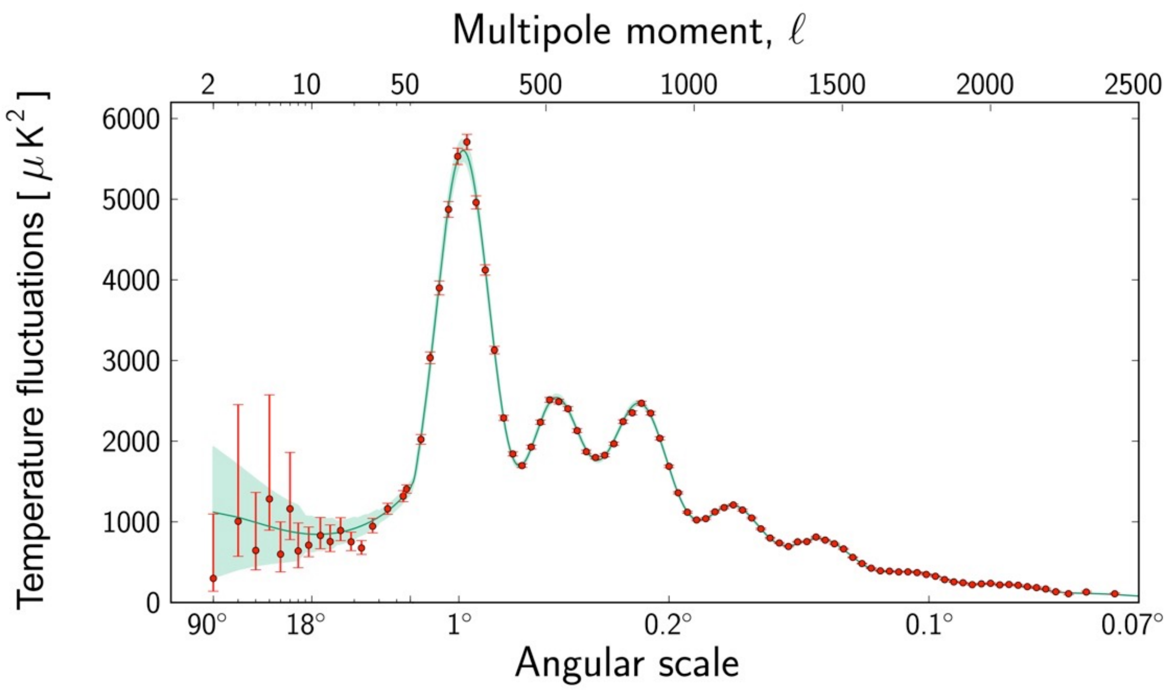


CMB



CMB temperature map measured by PLANCK

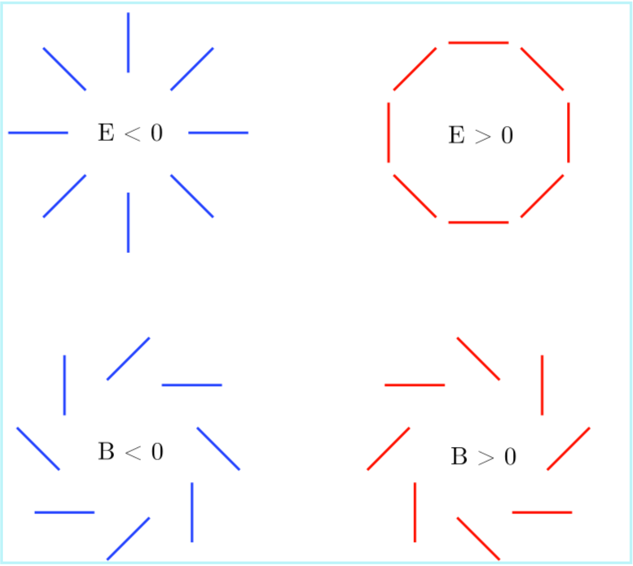
Spectrum of density fluctuations

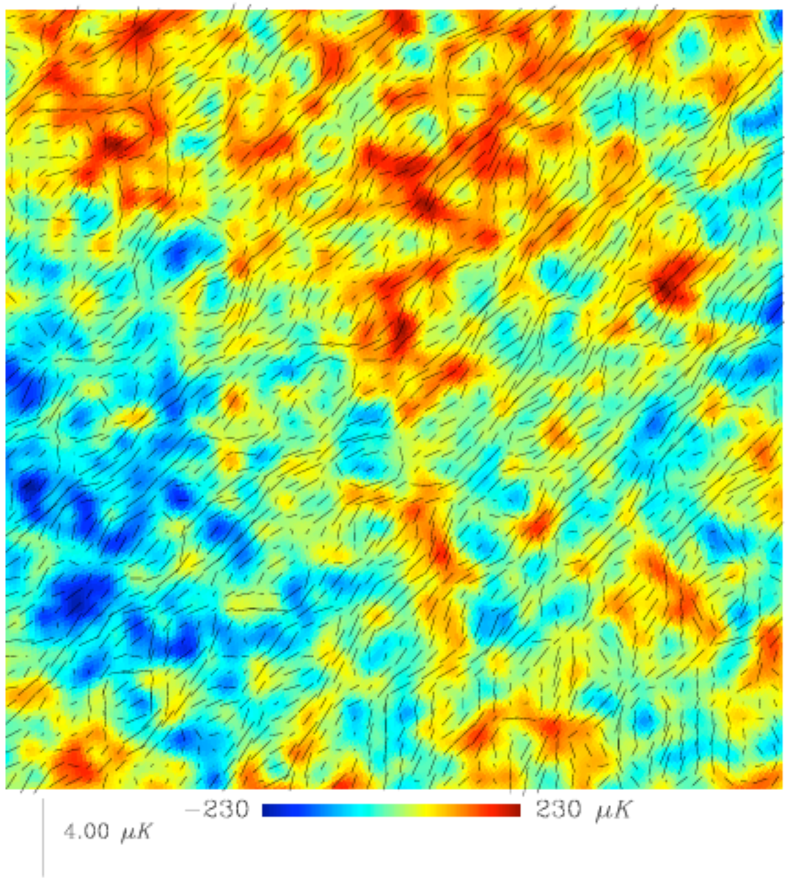


Tensor modes

Polarization of CMB

→ E,B modes





(Keating & Miller)

The Gravitational Wave Spectrum