Testing General Relativity with Gravitational Waves from Coalescing Compact Binaries in the Advanced Detector Era

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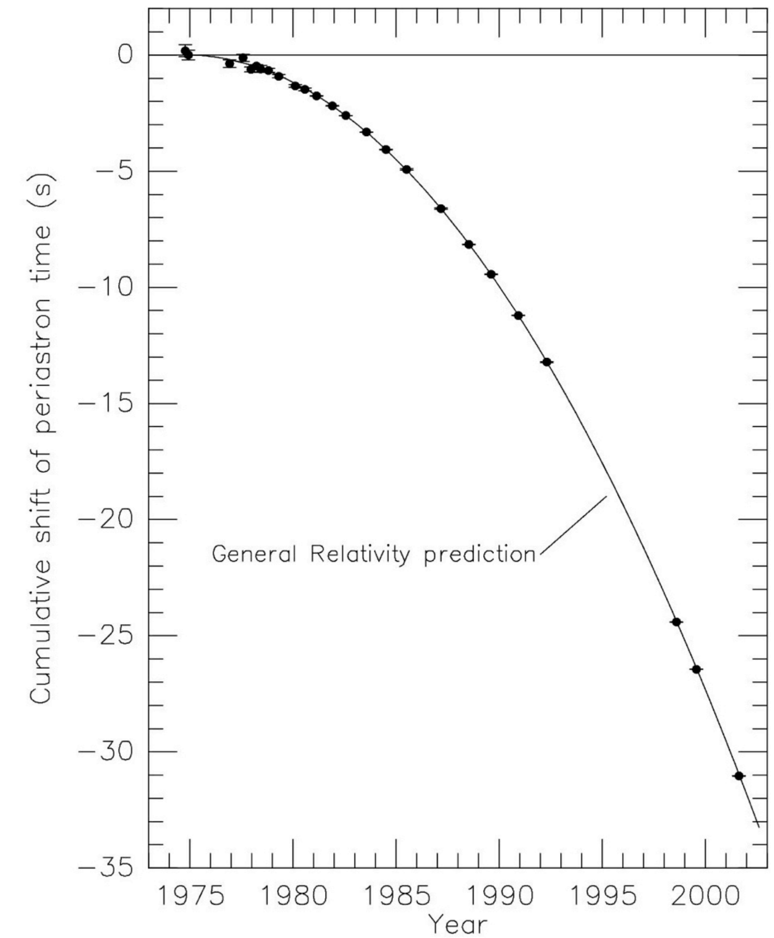
Phys. Rev. D 85 (2012), 082003 [arXiv:1110.0530]
J.Phys.Conf.Ser. 363 (2012) 012028 [arXiv:1111.5274]
MG13 Proceedings (2012) [arXiv:1305.2963]
Phys. Rev. D 89 (2014), 082001 [arXiv:1311.0420]

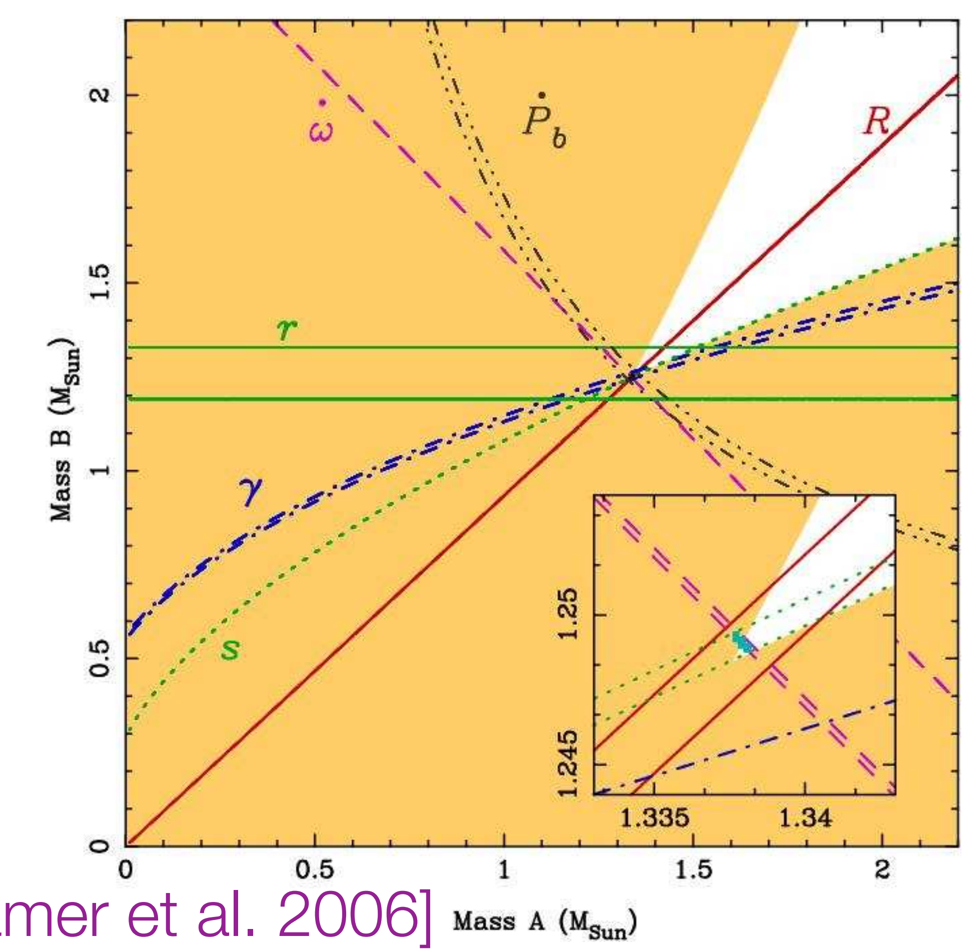
8th Aegean summer school
Rethymno, 30 June 2015

Testing GR: Motivation

- GR has passed all tests to date
- **But:** up to now, all tests were in weak field regime and/or involving slow-moving sources
- Binary Pulsar: $M/R \sim 10^{-6}$, $v/c \sim 10^{-3}$
- Gravitational waves from CBC will probe strong field gravity ($M/R \sim 0.2$) and relativistic sources ($v/c \sim 0.4$)
- These sources are perfect candidates for detection with aLigo/AdVirgo

[Weisberg & Taylor 2003]





[Kramer et al. 2006]

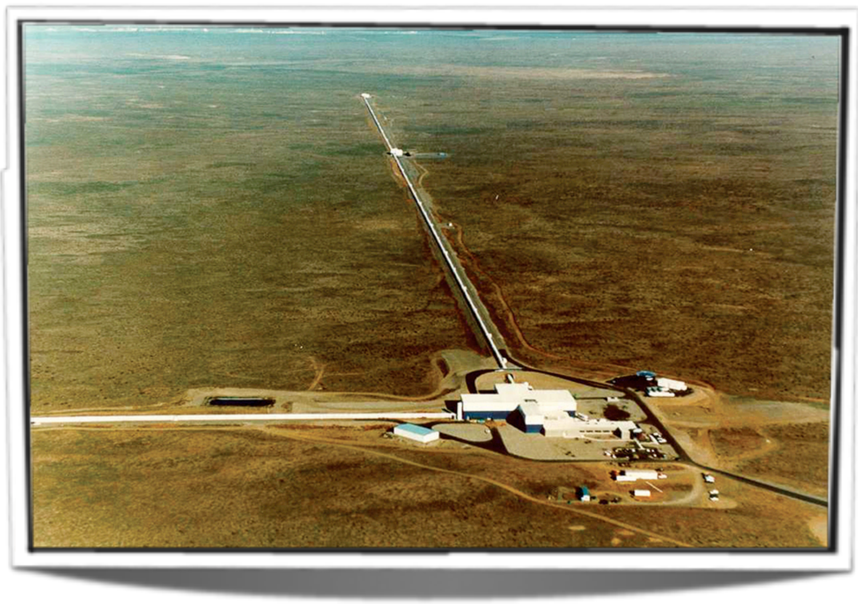
Advanced GW detectors

- The transition to the era of Advanced Interferometers (mid 2015) is expected to provide us with the first detections of GW signals

- Neutron Star binaries of M ~ a few Msun radiate within the *frequency bucket* of Adv LIGO/VIRGO

$$f_{lso} \sim 1600 \left(\frac{2.8\, M_\odot}{M_{\rm tot}} \right) \mathrm{Hz}$$

Advanced GW detectors



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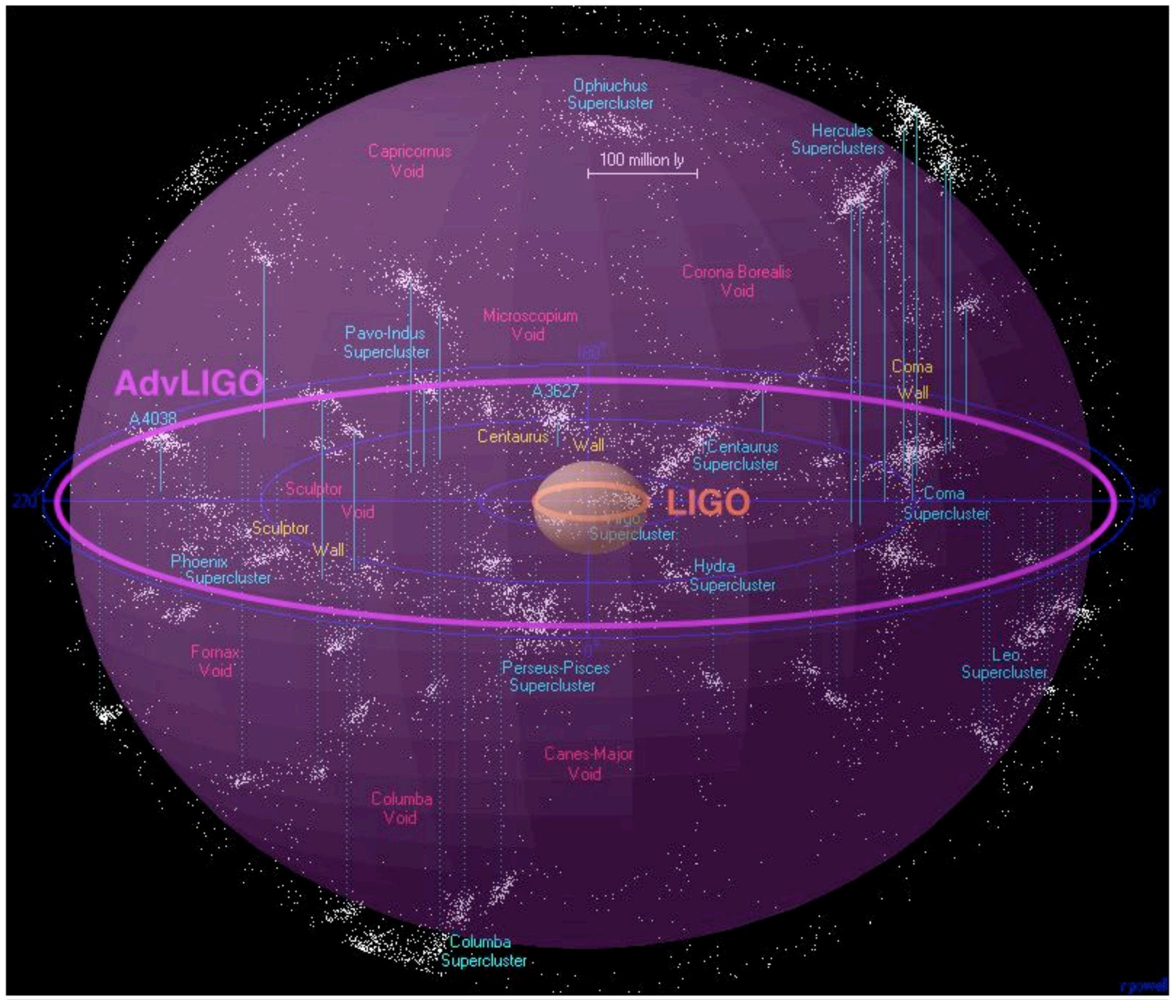
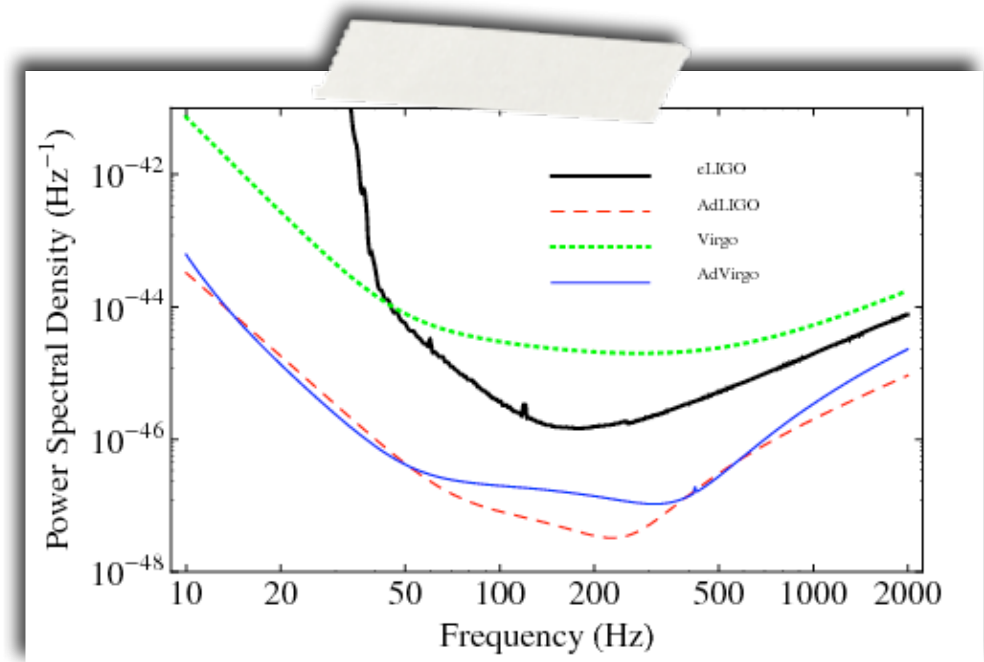
$$f_{lso} \sim 1600 \left(\frac{2.8\, M_{\odot}}{M_{\rm tot}} \right) \mathrm{Hz}$$

60 Mpc!

Expected rates for advanced detectors

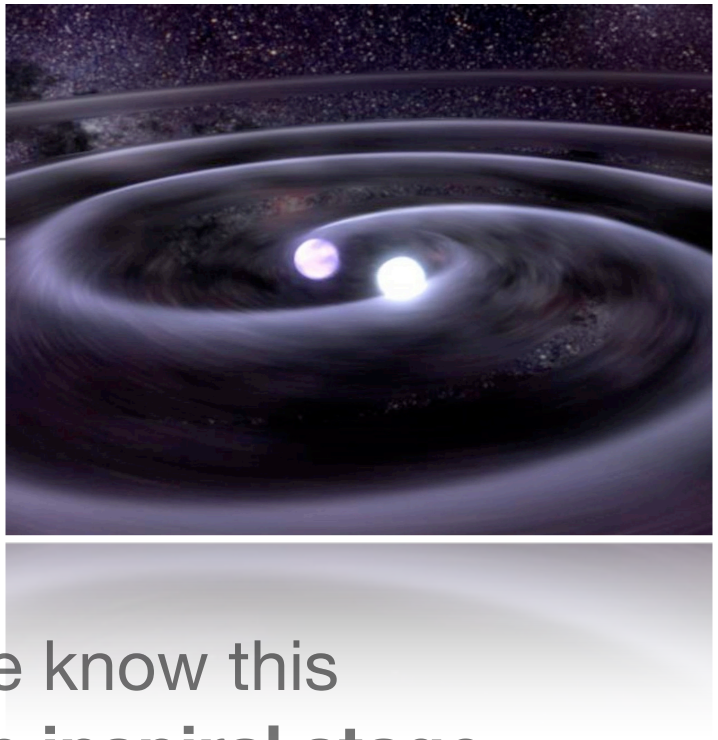


Network	Source	$\dot{N}_{\mathrm{low}}$ (yr^{-1})	$\dot{N}_{\mathrm{re}}$ (yr^{-1})	$\dot{N}_{\mathrm{high}}$ (yr^{-1})
	NS-NS	2×10^{-4}	0.02	0.2
Initial	NS-BH	7×10^{-5}	0.0004	0.1
	BH-BH	2×10^{-4}	0.007	0.5
	NS-NS	0.4	40	400
Advanced	NS-BH	0.2	10	300
	BH-BH	0.4	20	1000



Advanced LIGO/Virgo upgrade improves sensitivity by O(10)

The Post-Newtonian approximation



- The PN expansion gives approximate solutions to the 2-body problem in GR
- We consider binary neutron star (BNS) systems where we know this approximation to be very accurate for the best part of the **inspiral stage**.
- Simple frequency-domain waveform (TaylorF2) for systems with no spins:

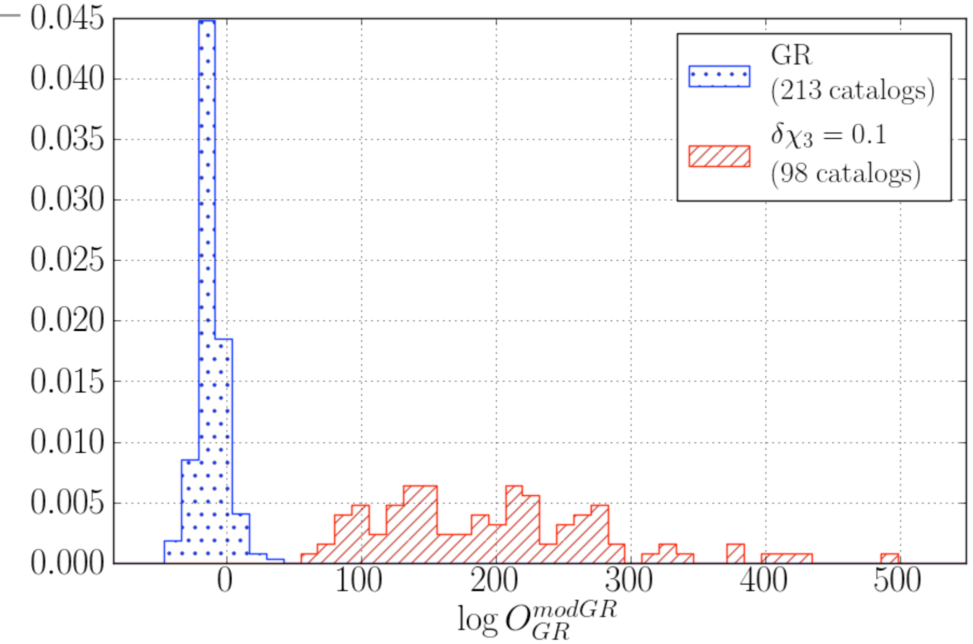
$$\tilde{h}(f) = \mathcal{A} f^{-7/6} \cos(2\Phi(f; m_1, m_2) + \phi_0) \quad v = (\pi M f)^{1/3}$$

$$\Phi = \left(\frac{v}{c}\right)^{-5} \sum_{i=0}^{N} \left[\psi_i + \psi_i^{(l)} \ln \frac{v}{c}\right] \left(\frac{v}{c}\right)^i$$

- Deviation from GR should give a different functional dependence of the PN phase coefficients $\psi_i (m_1, m_2, \mathbf{S}_1, \mathbf{S}_2)$, $\psi_i^{(l)} (m_1, m_2, \mathbf{S}_1, \mathbf{S}_2)$ on masses & spins

Testing General Relativity with GW

- If GR is violated the orbital evolution will be different than what GR predicts.



- With a few realistically quiet detections we will be able to test GR like never before!

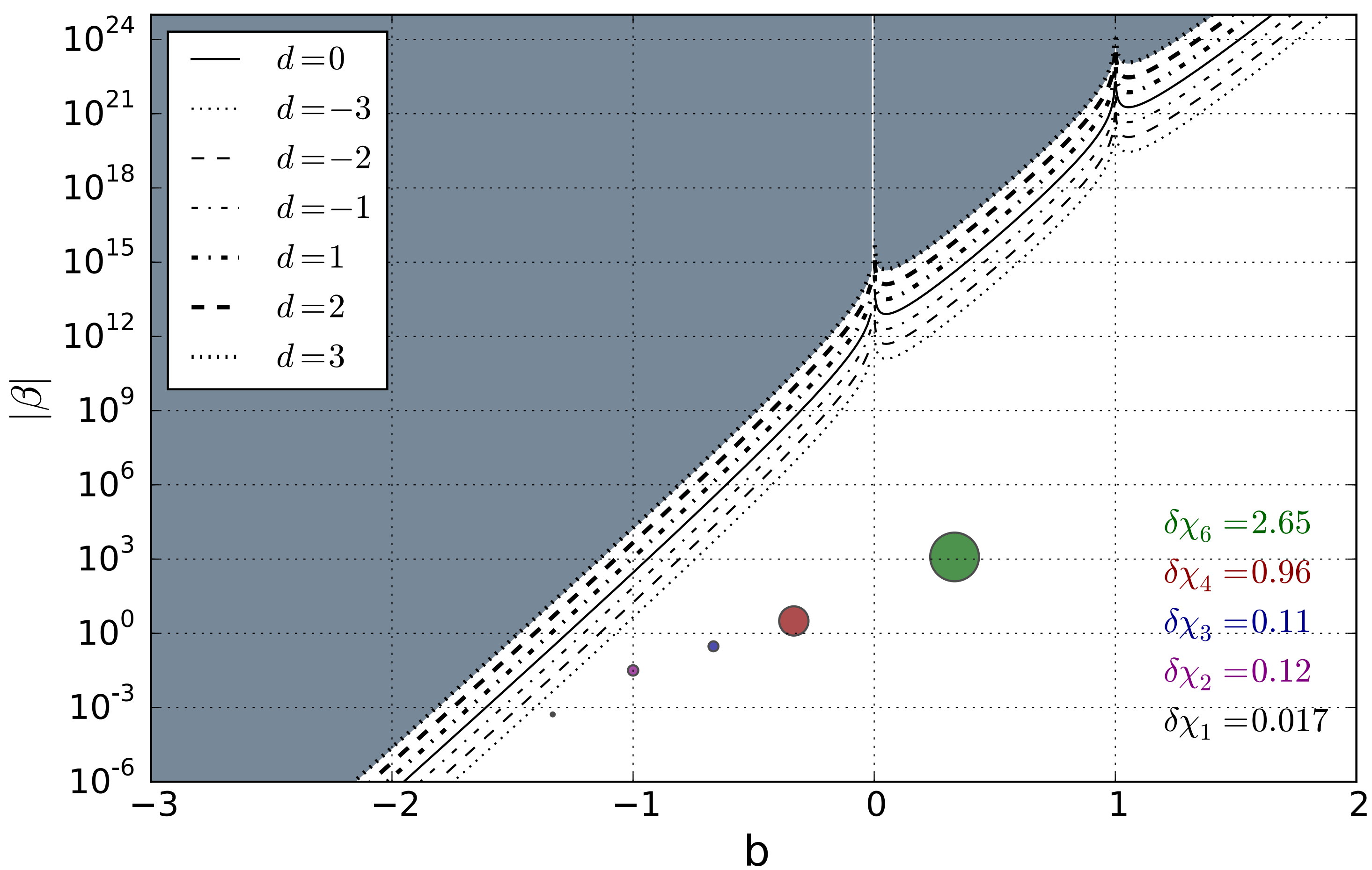
- e.g. consider a heuristic phase modification

$$\beta\eta^d(\pi\mathcal{M}_c f)^b$$

[Yunes & Hughes 2010]

- With single GW detection we can beat binary pulsar constraints by many orders of magnitude

  Li et al 2011 [arXiv:1110.0530]
  Li et al 2011 [arXiv:1111.5274]
  Agathos et al 2013 [arXiv:1311.0420]
  Agathos et al 2013 [arXiv:1305.2963]

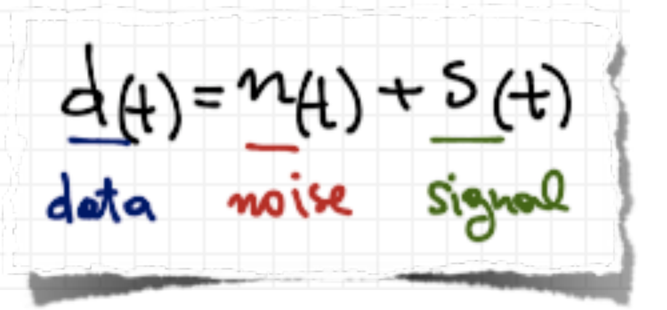


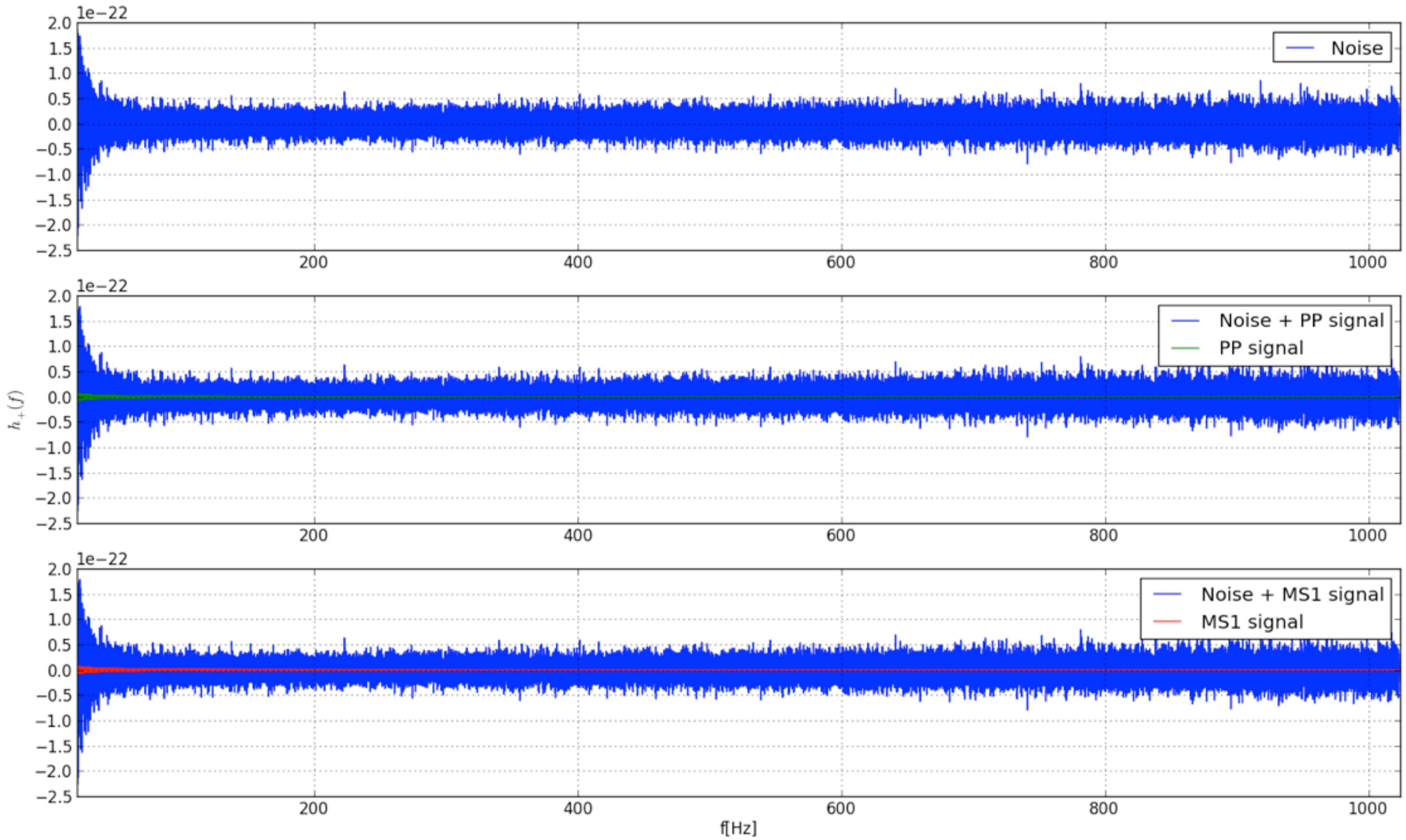
Desiderata

- Need a “theory independent” test for GR, not based on a particular alternative. If GR is violated, it may be in a way that has not yet been envisaged.
- Should be as generic as possible, parametrizable and computationally feasible.
- Has to be reliable for “quiet” sources.
- Should have ability to combine information from multiple sources, so as to arrive at a more stringent test of GR.
- Should not be tied to a particular waveform approximant.

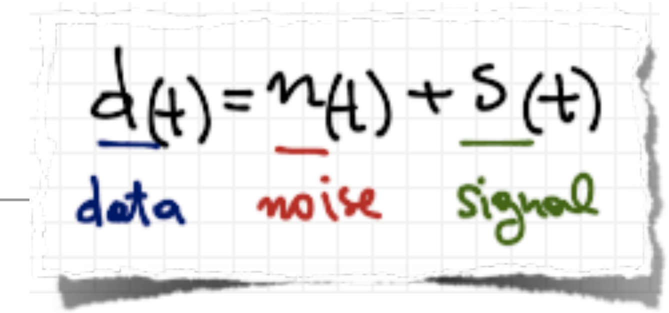
GW data analysis

- Data is dominated by noise
- Need to use stochastic properties of noise to dig out signal





GW data analysis



- Signal recovery from noisy data is possible because:
  - we know how the stochastic properties of our noise

$$\langle \tilde{n}^*(f)\tilde{n}(f')\rangle = \frac{1}{2}\delta(f-f')S_n(f)$$

  - we know how a signal should look like
  - each signal gives many data points

- Signal recovery is terribly complicated because:
  - there are many (~15) free parameters that define each source
  - noise can often mimic a GW (false alarm)
  - GR equations are hard to solve accurately

- Parameters for compact binary sources: $\{m_1, m_2, \mathbf{S}_1, \mathbf{S}_2, D, \theta, \phi, \iota, \psi, t_c, \phi_c\}$

- Noise-weighted inner product for matched filtering: $\langle a, b\rangle \equiv \Re\left\{\int_{-\infty}^{\infty} \mathrm{d}f\, \frac{\tilde{a}^*(f)\tilde{b}(f)}{\frac{1}{2}S_n(f)}\right\}$

$$P(A|B)P(B) = P(B|A)P(A)$$

Bayesian inference in GW data analysis

- CBC waveforms encompass a high-dimensional parameter space

- We want to calculate the *evidence* of the model hypothesis by marginalizing the *likelihood* over the parameter space:

$$P(d|H,I) = \int \mathrm{d}\vec{\theta}\, p(\vec{\theta}|H,I)\, p(d|\vec{\theta},H,I)$$

- Also interested in posterior PDF for parameter estimation: $p(\vec{\theta}|d,H,I)$

- Efficiently sample parameter space and obtain both evidence and posterior using the *Nested Sampling* algorithm [Skilling 2006] [Veitch & Vecchio 2009]

- Combine information from independent events:

$$p(d_1,d_2|H,I) = p(d_1|H,I)p(d_2|H,I)$$

Bayesian inference in GW data analysis

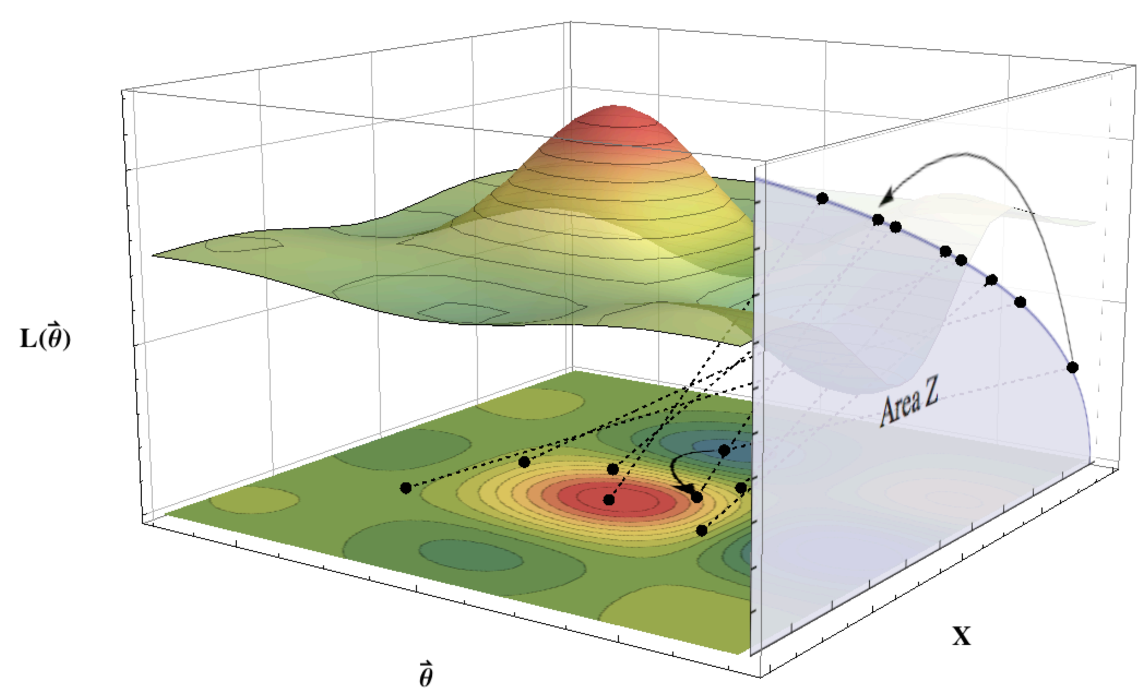
- Likelihood:

$$p(d|\vec{\theta}, H, I) = p(d - h_{\vec{\theta}}|H_{noise}, I) = \mathcal{N}\ \exp\left[-\int_{-\infty}^{\infty} \mathrm{d}f\ \frac{|\tilde{d}(f) - \tilde{h}(f;\vec{\theta})|^2}{S_n(f)}\right]$$

$$= \mathcal{N}\ e^{-\frac{\langle d - h_{\vec{\theta}}, d - h_{\vec{\theta}}\rangle}{2}}$$

- Nested sampling explores the parameter space by climbing up the likelihood function

- Evidence is numerically accumulated



Bayesian Model Selection

$$P(A|B)\,P(B) = P(B|A)\,P(A)$$

- For competing hypotheses, given a set of data d, calculate the posterior probability for each hypothesis:

$$P(H_i|d,\mathrm{I}) = \frac{\overbrace{P(d|H_i,\mathrm{I})}^{\text{evidence}}\,\overbrace{P(H_i|\mathrm{I})}^{\text{prior}}}{P(d|\mathrm{I})}$$

- Define the **odds ratio** between a pair of hypotheses:

$$\mathcal{O}_2^1 = \frac{P(H_1|d,\mathrm{I})}{P(H_2|d,\mathrm{I})} = \frac{P(H_1|\mathrm{I})}{P(H_2|\mathrm{I})}\,\frac{P(d|H_1,\mathrm{I})}{P(d|H_2,\mathrm{I})}$$

- Combine information from multiple sources:

$$\mathcal{O}_2^1 = \frac{P(H_1|d_1,\ldots,d_n,\mathrm{I})}{P(H_2|d_1,\ldots,d_n,\mathrm{I})} = \frac{P(H_1|\mathrm{I})}{P(H_2|\mathrm{I})}\prod_{i=1}^{n}\frac{P(d_i|H_1,\mathrm{I})}{P(d_i|H_2,\mathrm{I})}$$

TIGER
(**T**est **I**nfrastructure for **GE**neral **R**elativity)

- Define the GR and modGR hypotheses for the phase evolution:

> **H$_{GR}$** : All PN phase coefficients have the functional dependence on the masses (and spins) that is predicted by GR
>
> **H$_{modGR}$** : One or more of the phase coefficients are **not** as predicted by GR, without specifying which $\psi_i = \psi_i^{GR}(1 + \delta\chi_i)$

- Reformulate H$_{modGR}$ as union of testable disjoint sub-hypotheses:

$$H_{modGR} = \bigvee \mathcal{H}_{i_1 \dots i_k} \qquad P(H_{modGR}|d, \mathrm{I}) = \sum_{k; i_1 < \dots < i_k} P(\mathcal{H}_{i_1 \dots i_k}|d, \mathrm{I})$$

$\mathcal{H}_{i_1 \dots i_k}$ is the model where $\{\psi_{i_1}, \cdots, \psi_{i_k}\}$ are free to deviate away from GR, but not the others

- Combine information from multiple sources to catalog's odds ratio

$$\mathcal{O}_{GR}^{modGR} = \frac{\alpha}{2^{N_T} - 1} \sum_{i_1 < \ldots < i_k; k \leq N_T} \prod_{A=1}^{\mathcal{N}} \frac{P(\mathrm{d}_A|H_{i_1 \ldots i_k}, \mathrm{I})}{P(\mathrm{d}|\mathcal{H}_{GR}, \mathrm{I})}$$

Simulations

1. **Simulated signals**:

    Generate simulated GR/modGR signals from **BNS** inspiral and simulate detector response and noise for 3 advanced detectors (HLV)

2. **Recovery**:

    Estimate **evidence** for the GR and modGR hypotheses for each source, by integrating the likelihood over the parameter space using the *nested sampling* algorithm

3. **Post-process**:

    Combine evidence from multiple sources to cumulative odds ratio between GR and modGR for **catalogues of sources**

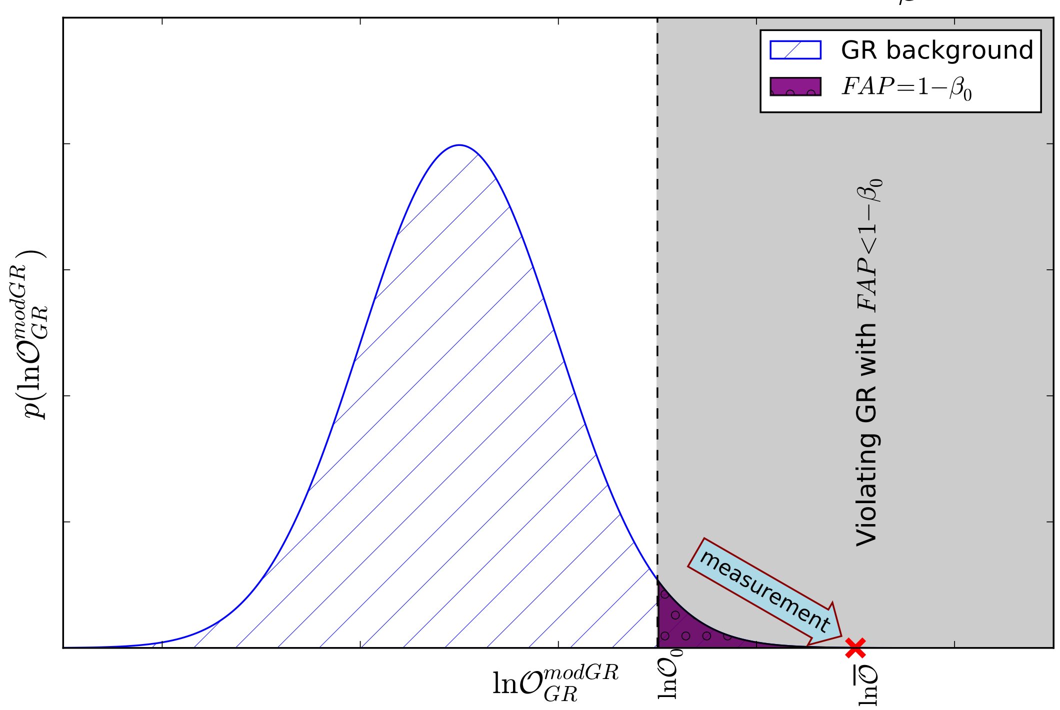
    The set of GR injections is used to form the *background* distribution of the statistic, within which deviations from GR are not measurable

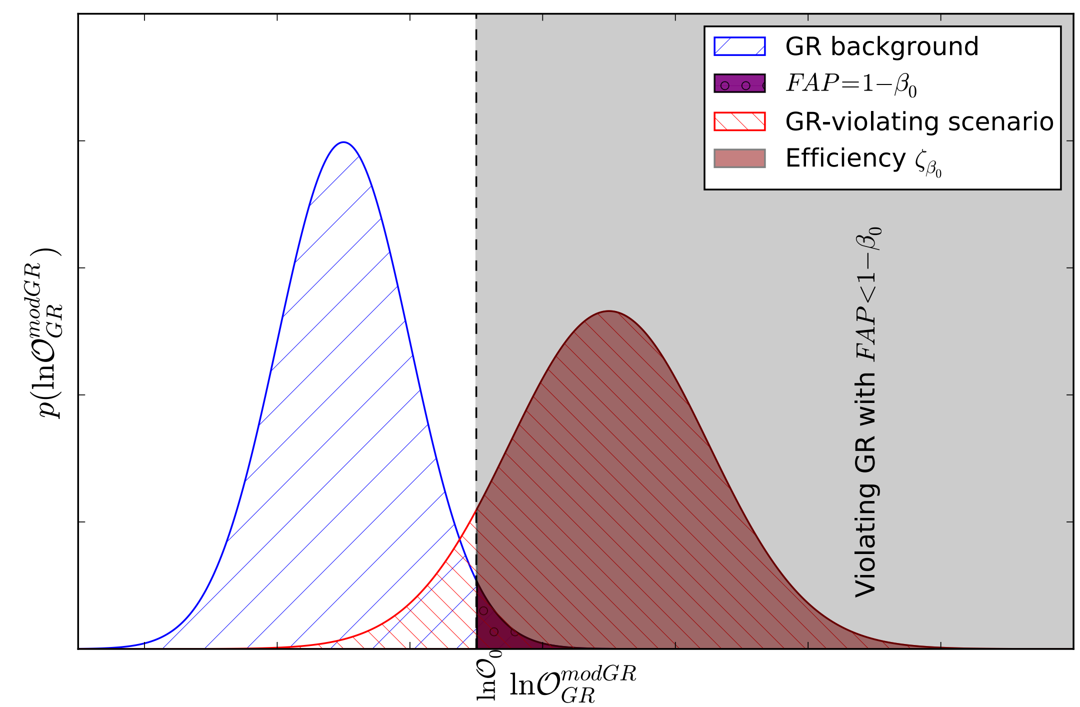
Background, foreground and efficiency

- The threshold for concluding that GR is violated is **not** at $O_{GR}^{modGR} = 1$!

- ***Efficiency***: how much of the foreground is above a given fraction of the background?

$$\zeta = \int_{\ln \mathcal{O}_\beta}^{\infty} P(ln\mathcal{O}|\kappa, \mathcal{H}_{non-GR}, I)\mathrm{d}\ln\mathcal{O}$$



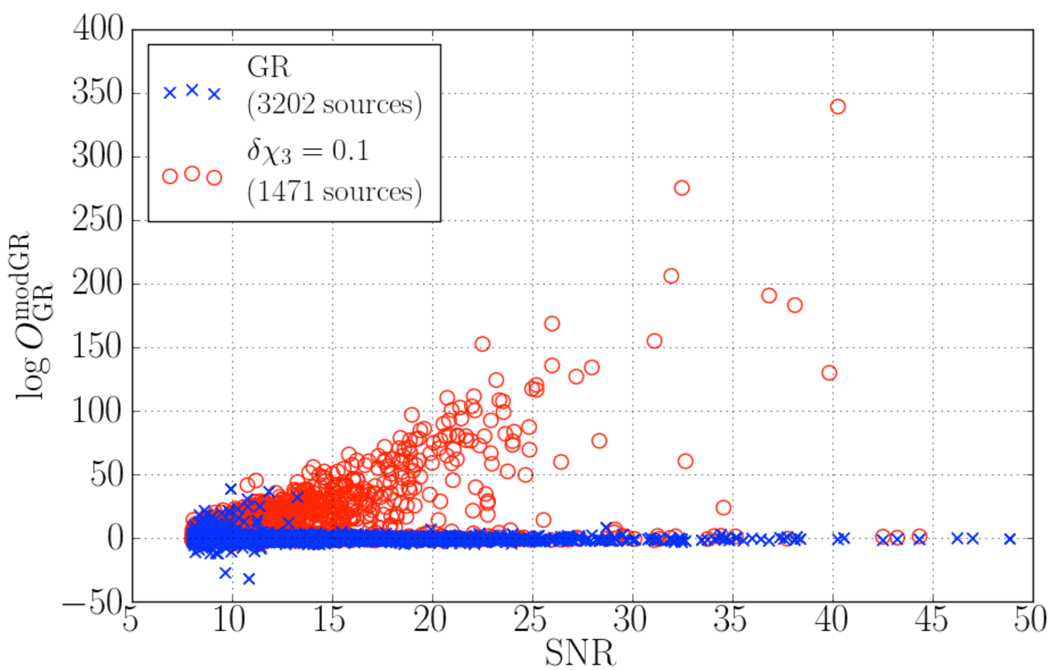


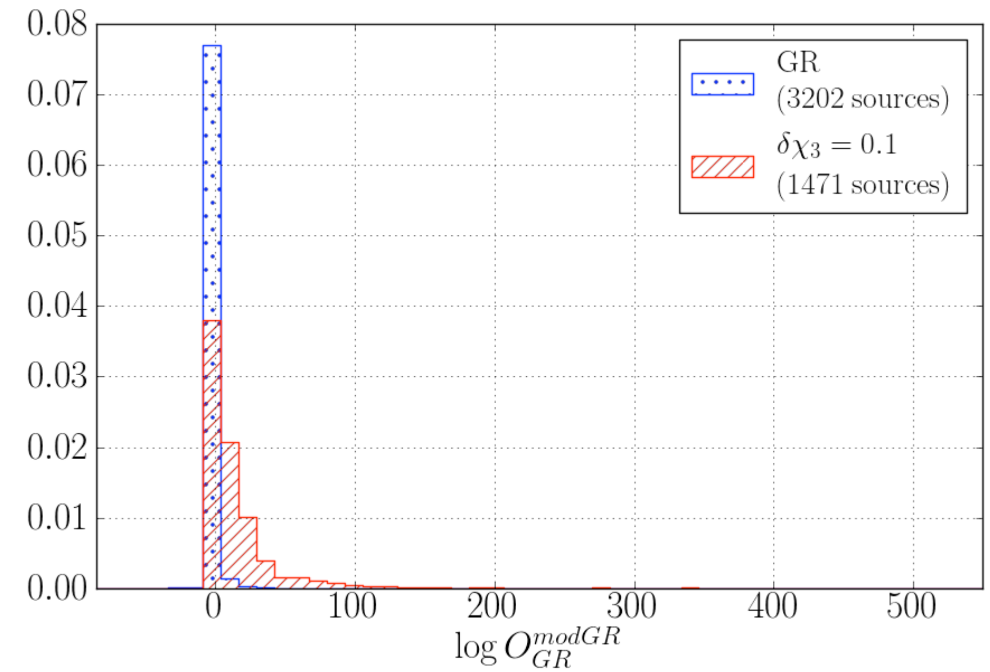
- where **β** is a given FAP

$$\beta = \int_{\ln \mathcal{O}_\beta}^{\infty} P(ln\mathcal{O}|\kappa, \mathcal{H}_{GR}, I)\mathrm{d}\ln\mathcal{O}$$

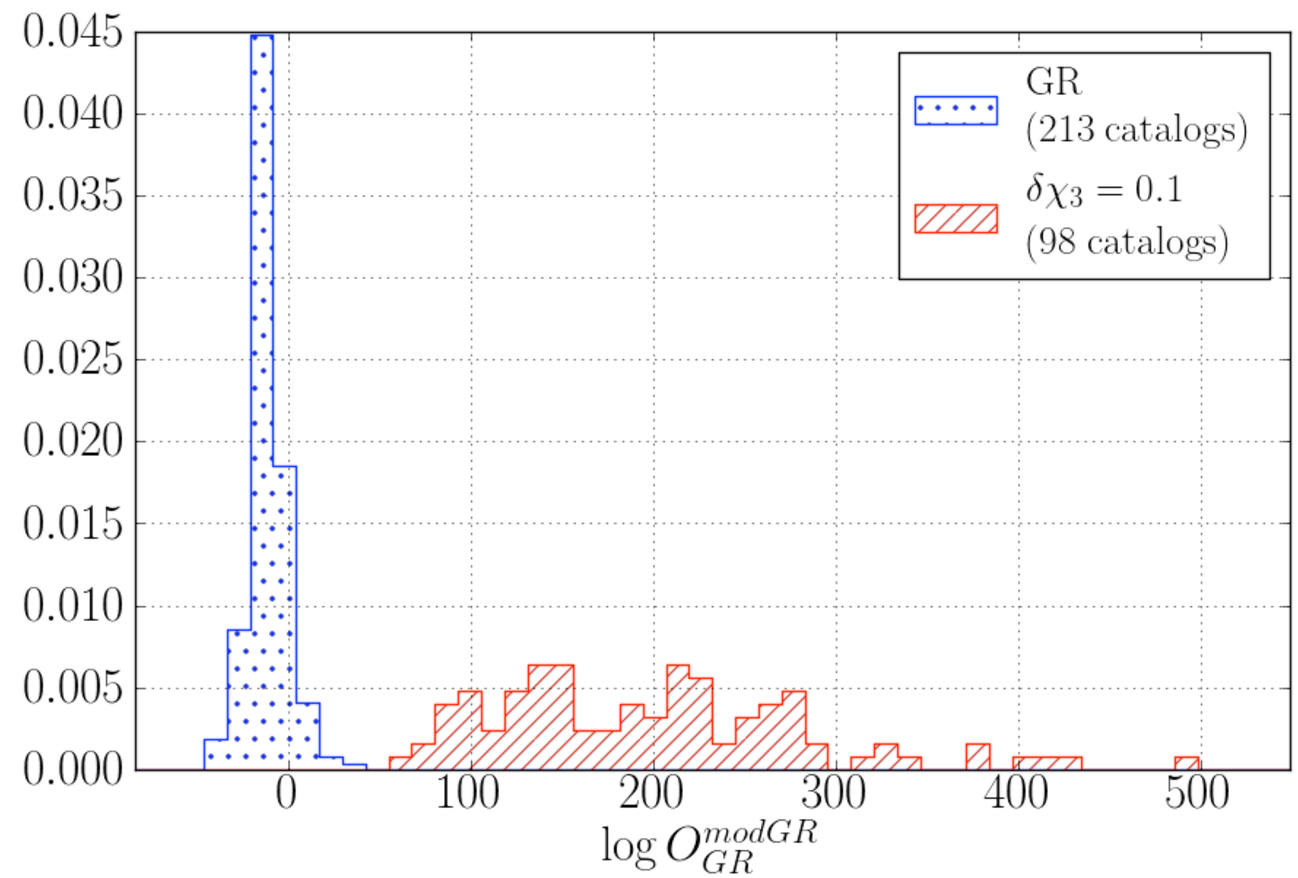
Li et al 2011 [arXiv:1110.0530]

First results : 10% shift at 1.5PN



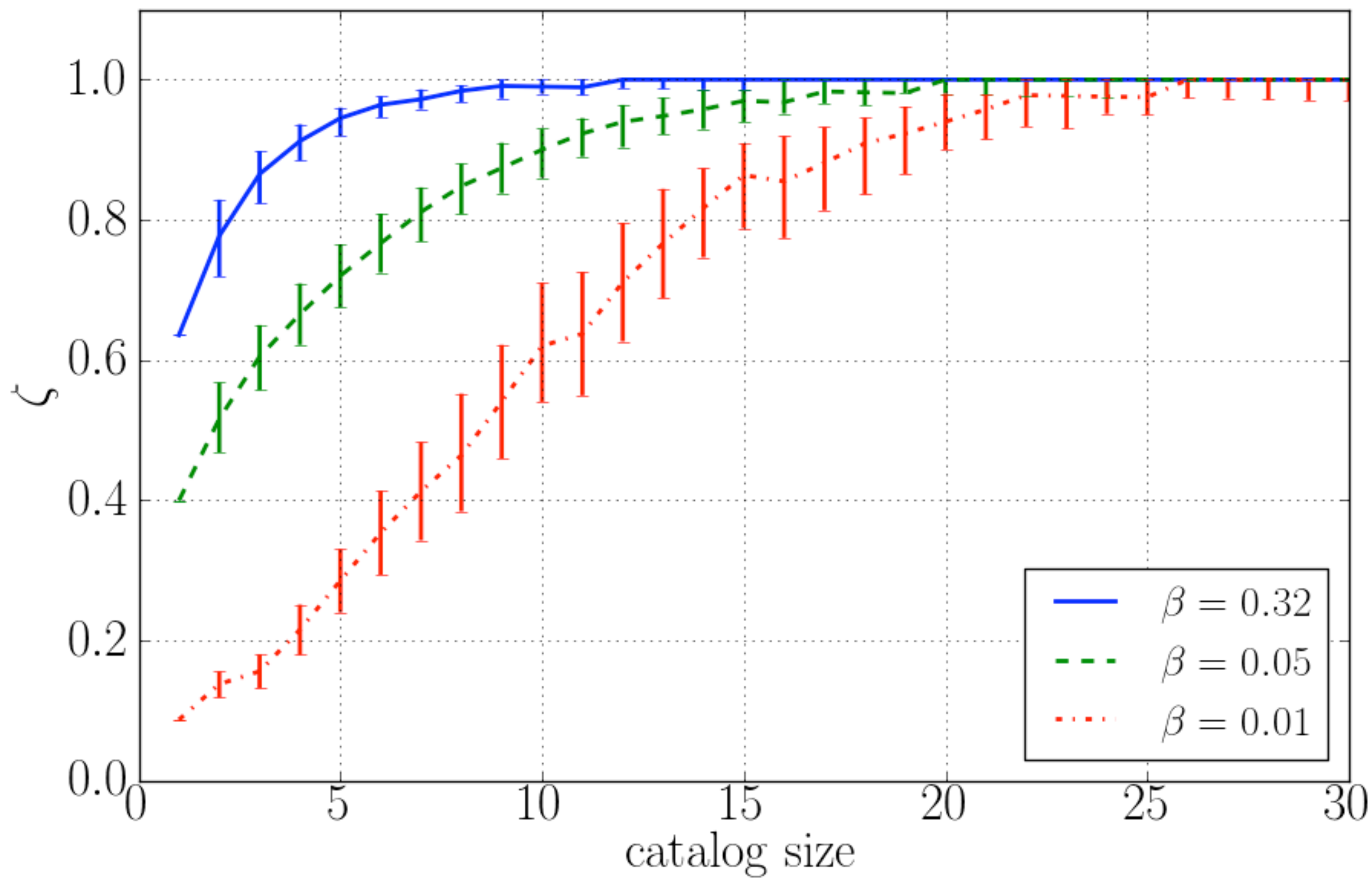


For a constant shift of 10% at 1.5PN, the efficiency is close to 100% for catalogs of 15 sources each



Efficiency for 10% shift at 1.5PN

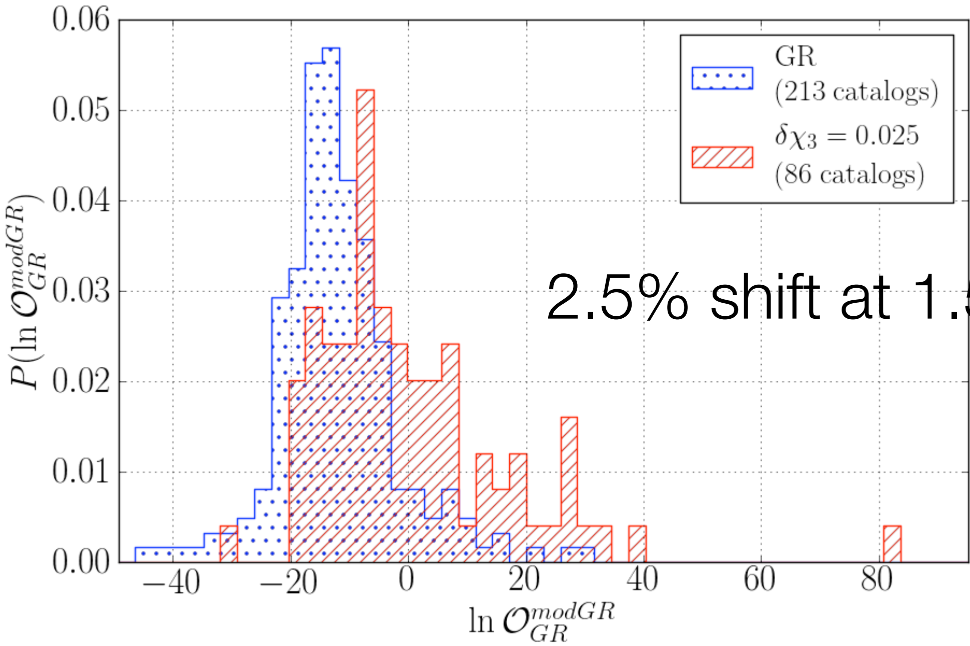
- How does our performance go with an increasing number of sources per catalog?



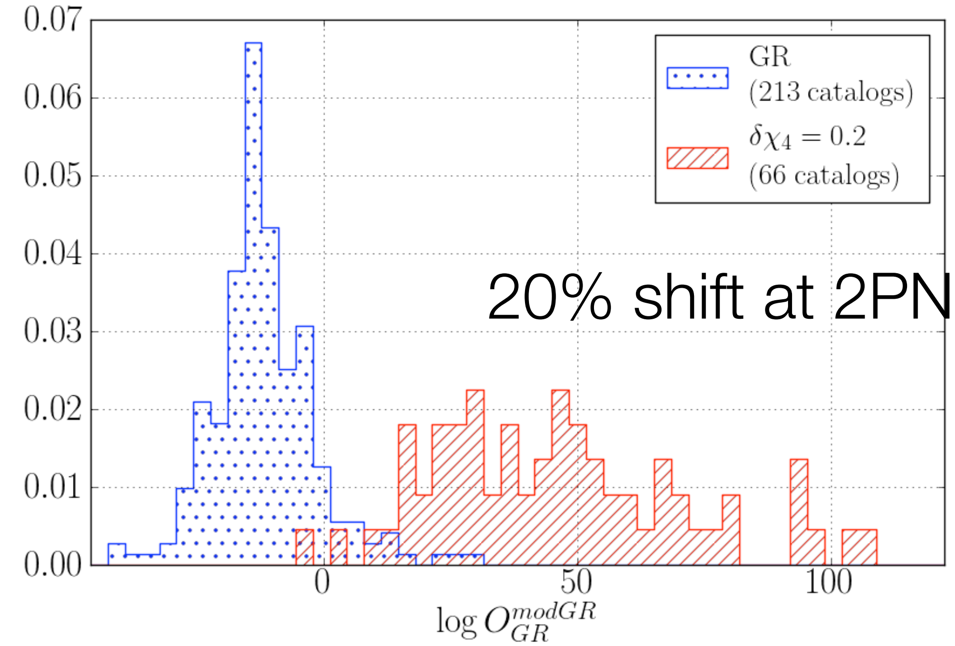
Testing different types of GR-violations

Li et al 2011 [arXiv:1110.0530]

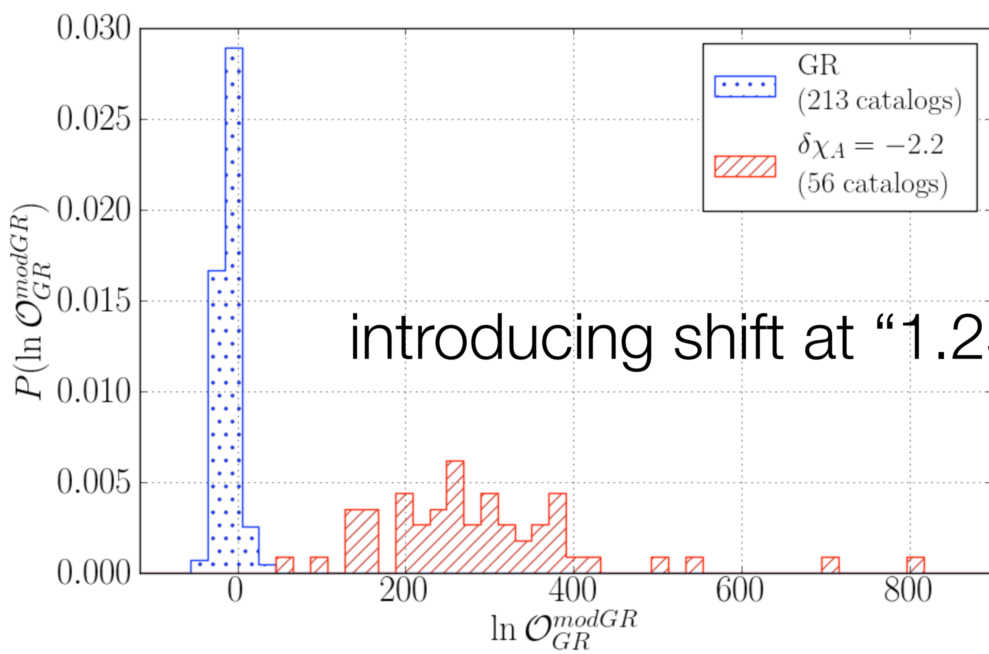
Li et al 2011 [arXiv:1111.5274]



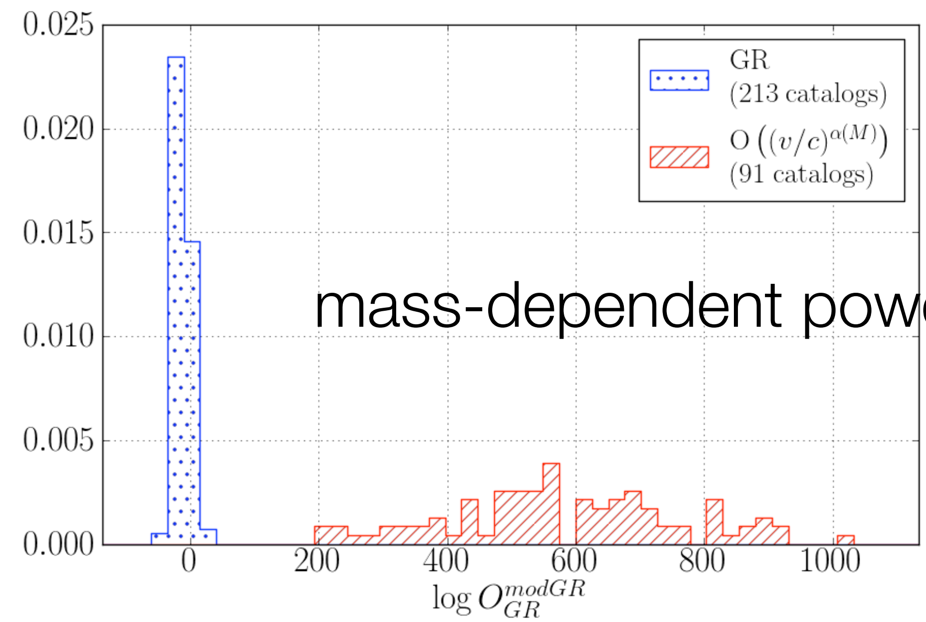
2.5% shift at 1.5PN



20% shift at 2PN



introducing shift at “1.25PN”



mass-dependent power of v/c

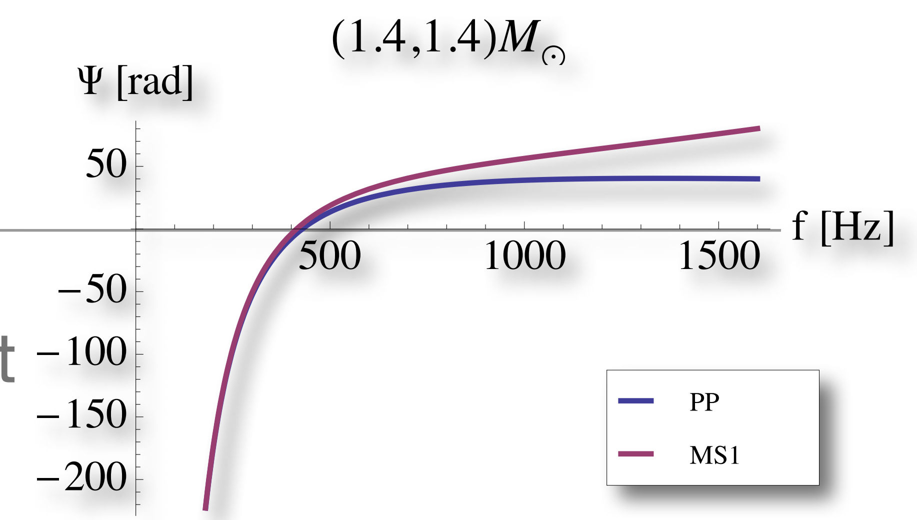
$$\Psi^{\rm GR}(\mathcal{M},\eta;f) \rightarrow \Psi^{\rm GR}(\mathcal{M},\eta;f) + \frac{3}{128\eta}(\pi M f)^{-2+M/(3M_\odot)}$$

Robustness

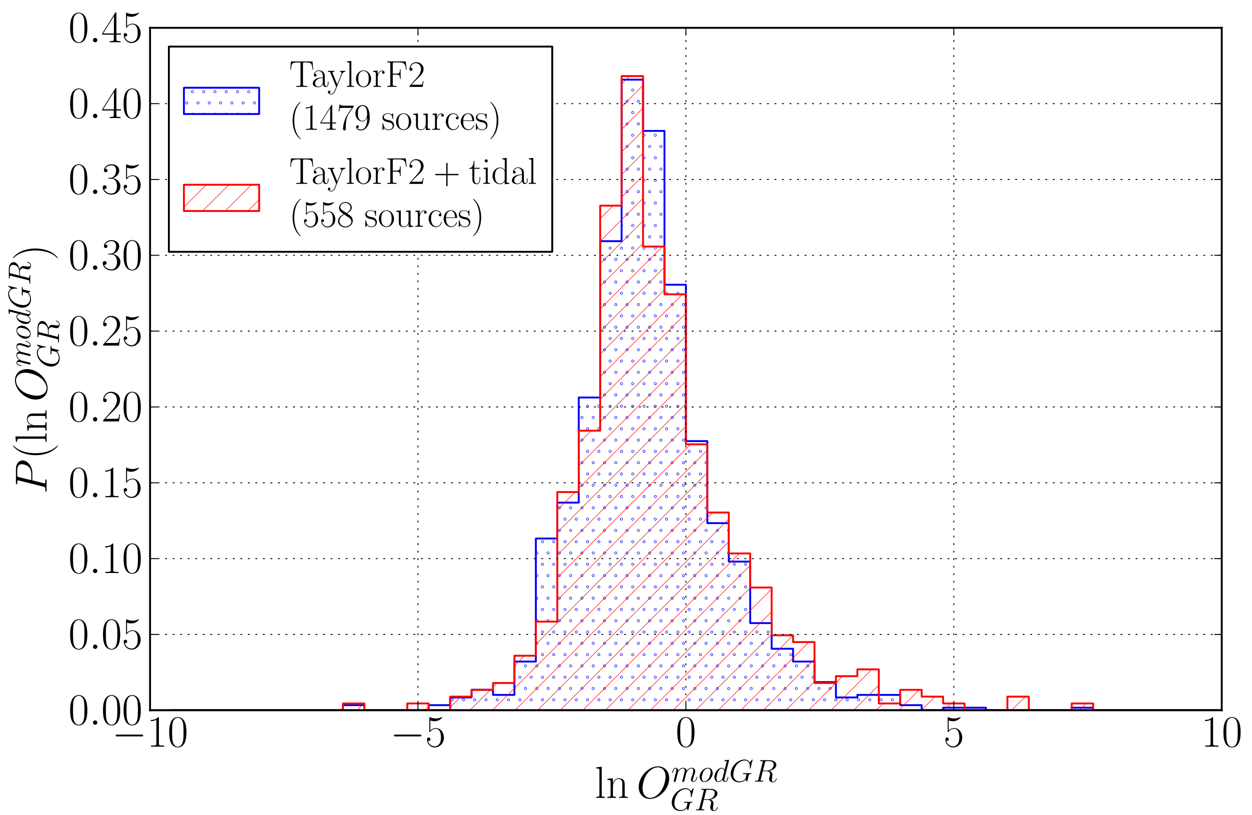
Agathos et al 2013 [arXiv:1311.0420]

- Tidal effects of unknown magnitude
- Effect of instrumental calibration errors
- Waveform mismatch, truncated PN expansion
- Effect of spins (aligned or generic)
- Real (non-gaussian, non-stationary) noise

Tidal effects

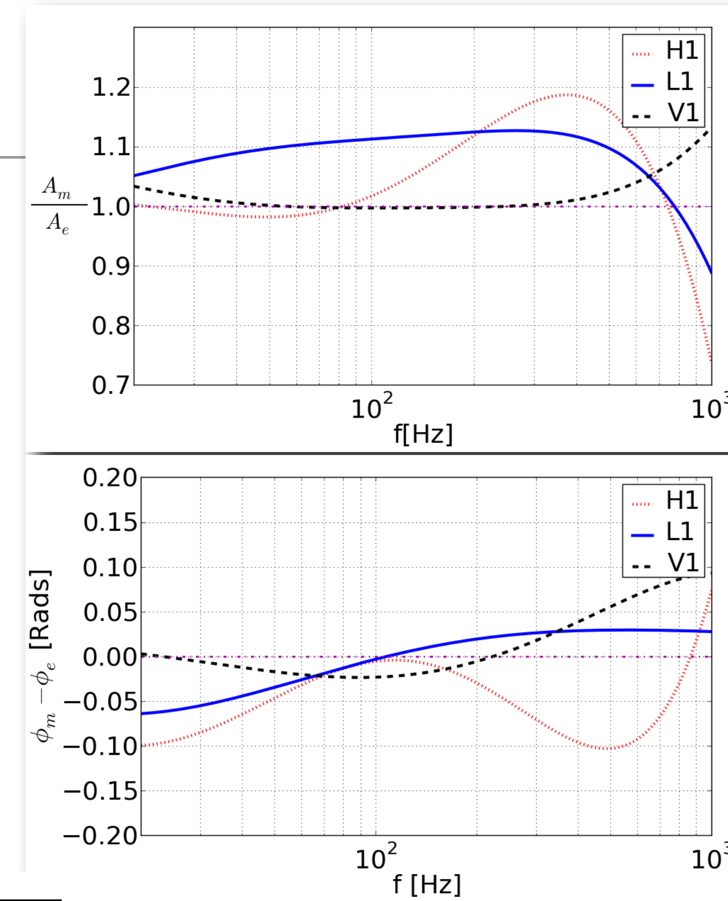


- For BNS systems, tidal effects will become important at high frequencies (>450Hz) $\Psi = \Psi_{PP} + \Psi_{tidal}$
  [Hinderer et al 2010]
- We **cut off** our analysis at 400Hz with a minor SNR loss ~1%
- Turn on tidal effects and see how performance is affected

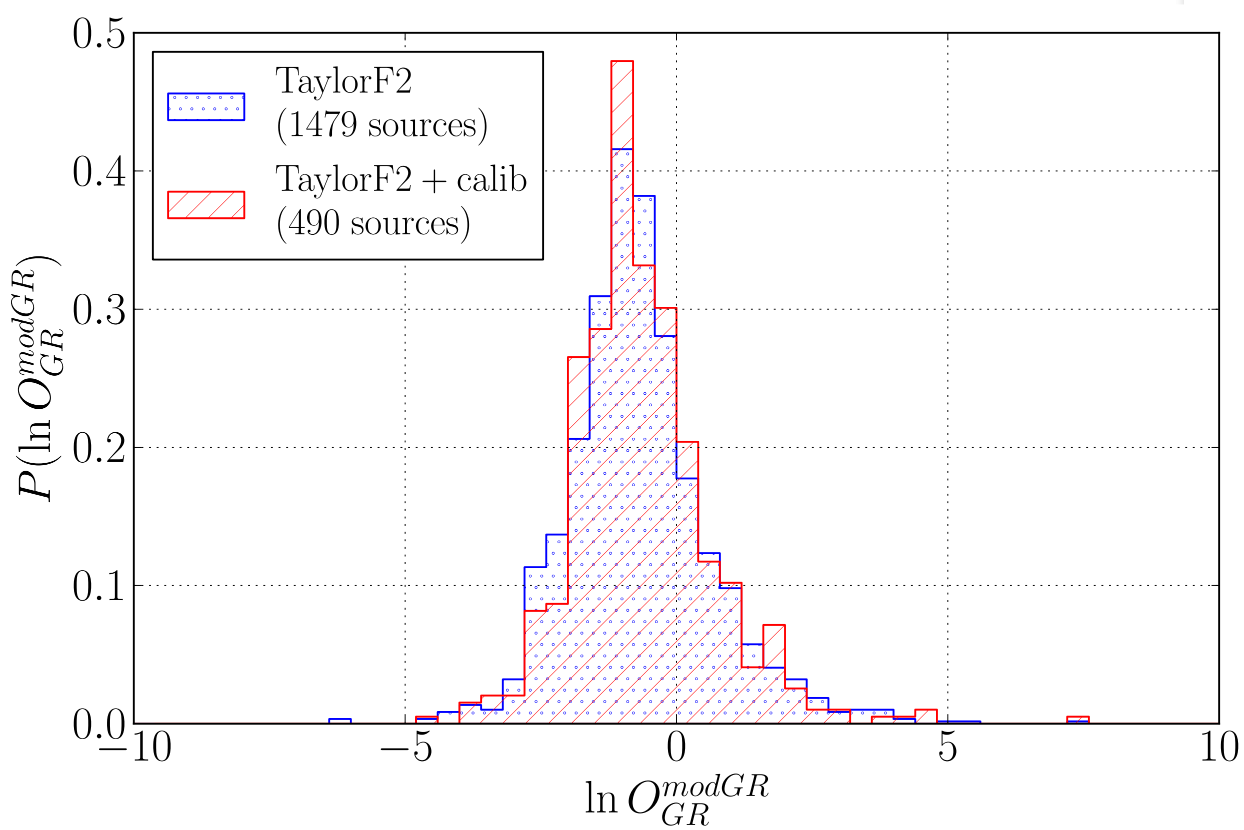


Vitale et al 2011 [arXiv:1111.3044]

Instrumental calibration errors



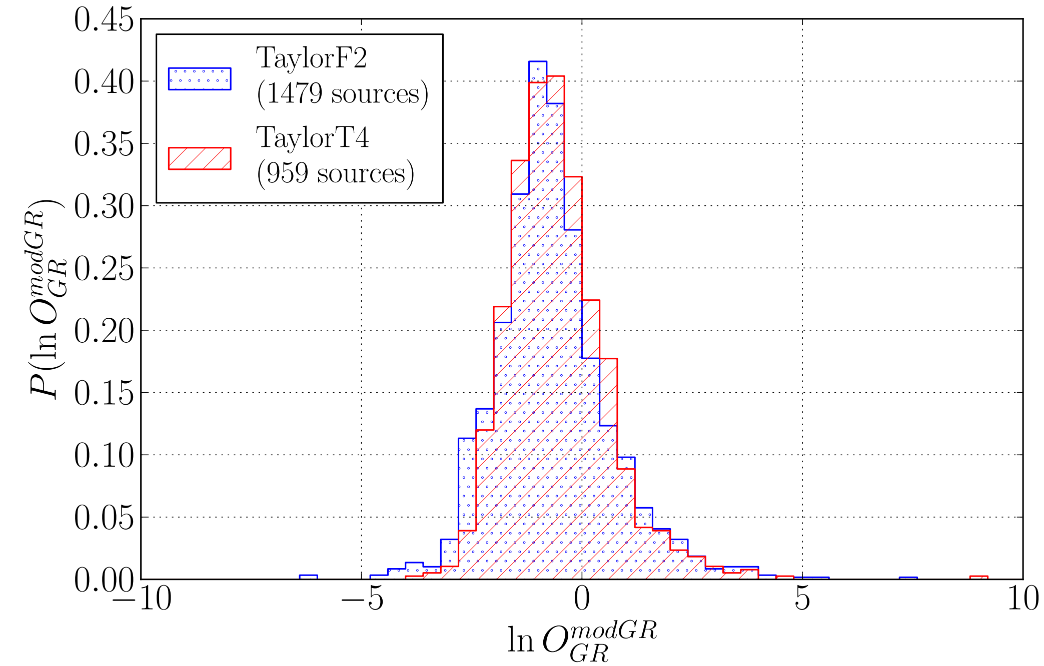
- Errors in the calibration of one or more parts of the detector may lead to “misinterpretation” of the data.
- No severe impact on parameter estimation
- What is the impact on TIGER?



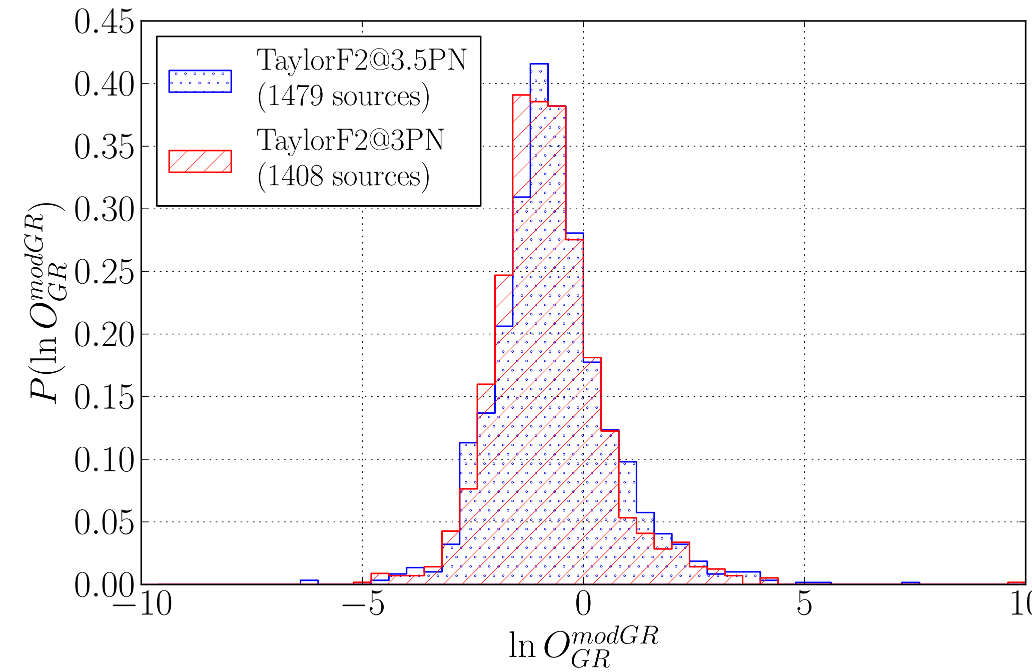
Waveform mismatch/missing PN terms

- For the BNS part of the parameter space (low masses) our waveform models are reliable and their mismatch is tiny [Buonanno et al. 2009]

1) Pick two different waveform approximants for the **signals**, but in both cases stick to TaylorF2 for recovery



2) Use the same waveform approximant for the **signals** (TaylorF2) but with the highest available PN term missing in the recovery waveform



Fully precessing spins

- Spin contributions enter the phase at 1.5PN order and beyond

$$\beta = \frac{1}{12}\sum_{i=1}^{2}\left[113\left(\frac{m_i}{M}\right)^2 + 75\eta\right]\hat{L}\cdot\vec{S}_i$$

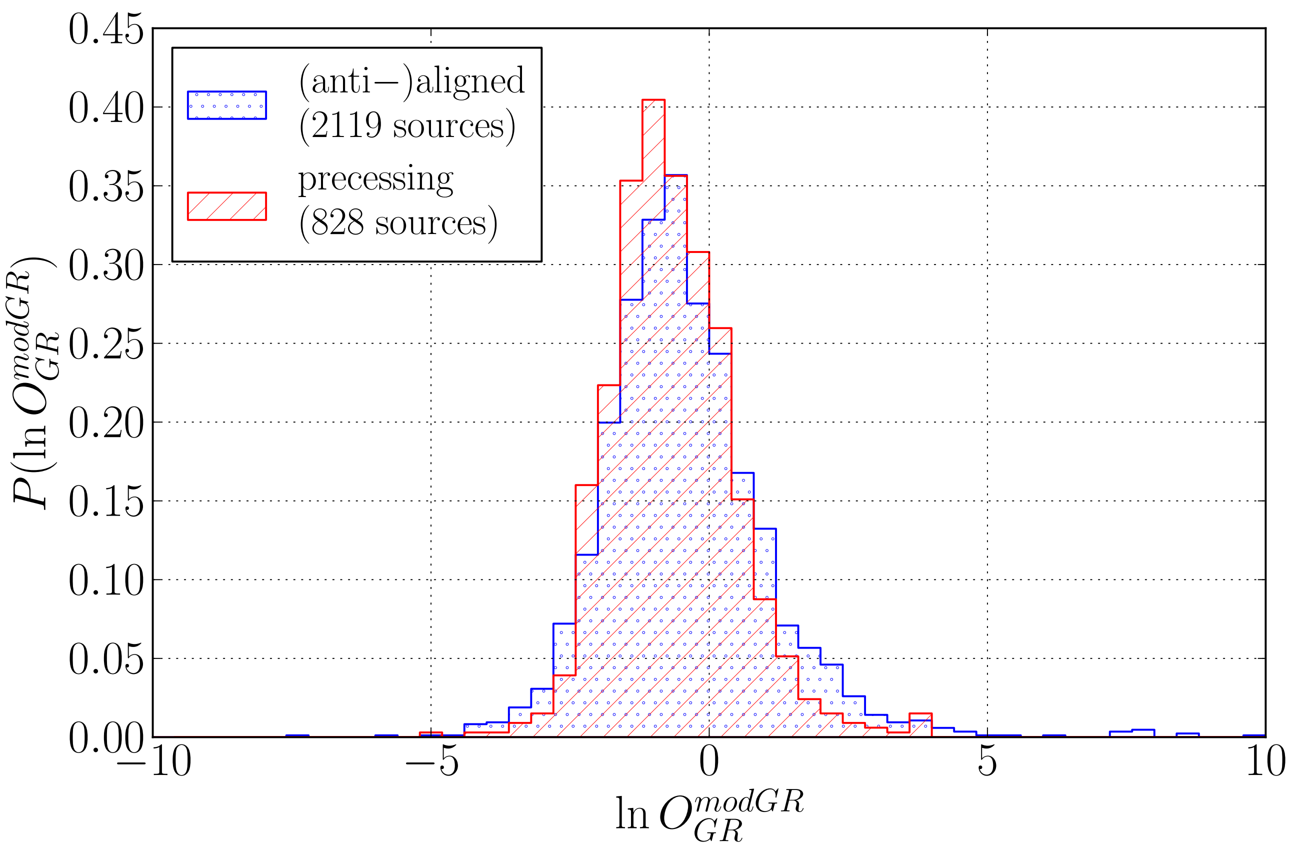
- From the known NS binaries [O'Shaughnessy & Kim 2009] we expect spins to be small

$$\psi_3 = [-16\pi + 4\beta](\pi M)^{-2/3}$$

[Kidder et al. 1993]

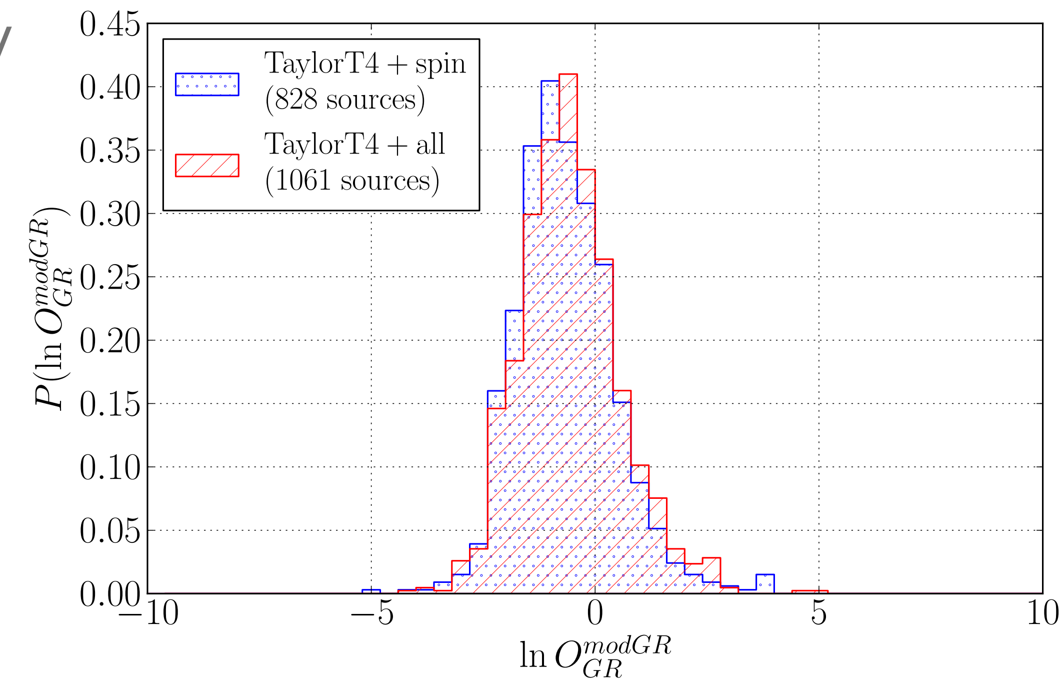
- Use **SpinTaylorT4** to simulate sources with spins of magnitude following a gaussian distribution (μ=0, σ=0.05) and of **random orientation**

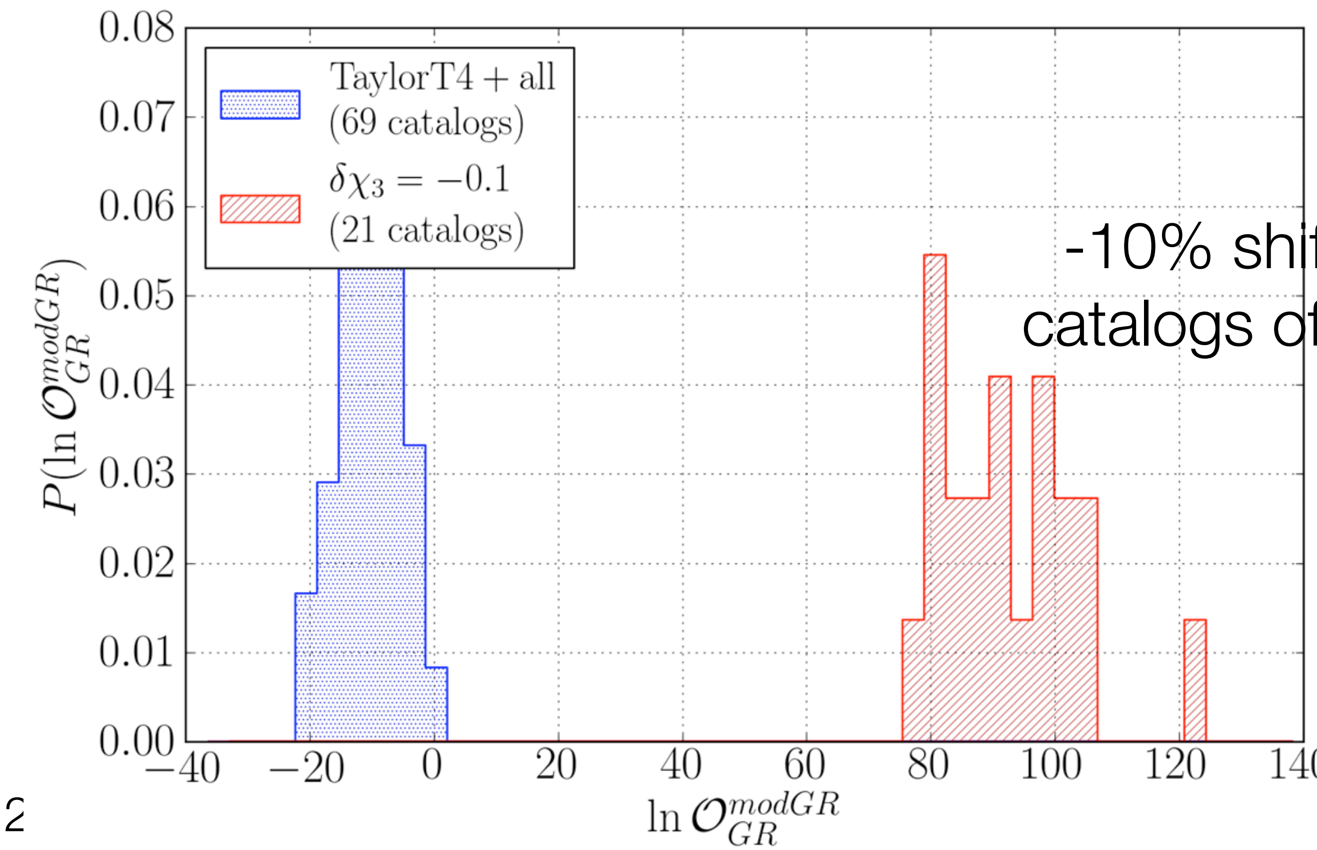
- Recover with (anti-)aligned spinning TaylorF2



Putting everything together

- ✓ SpinTaylorT4 (different waveforms for injection/recovery)
- ✓ Tidal effects, 5PN + 6PN in phase using a very hard equation of state
- ✓ Generic, precessing spins
- ✓ Calibration errors

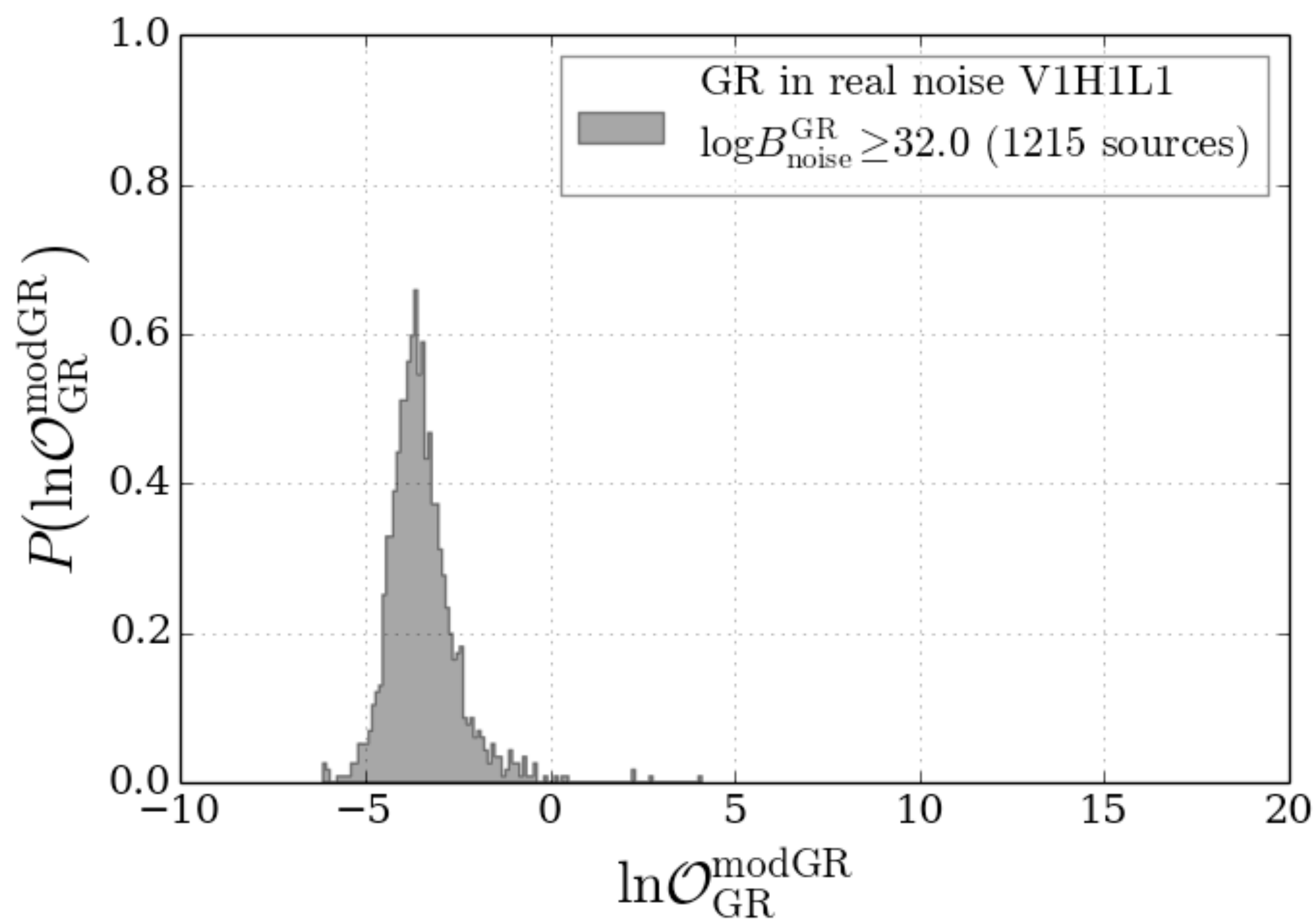




-10% shift at 1.5PN
catalogs of 15 sources

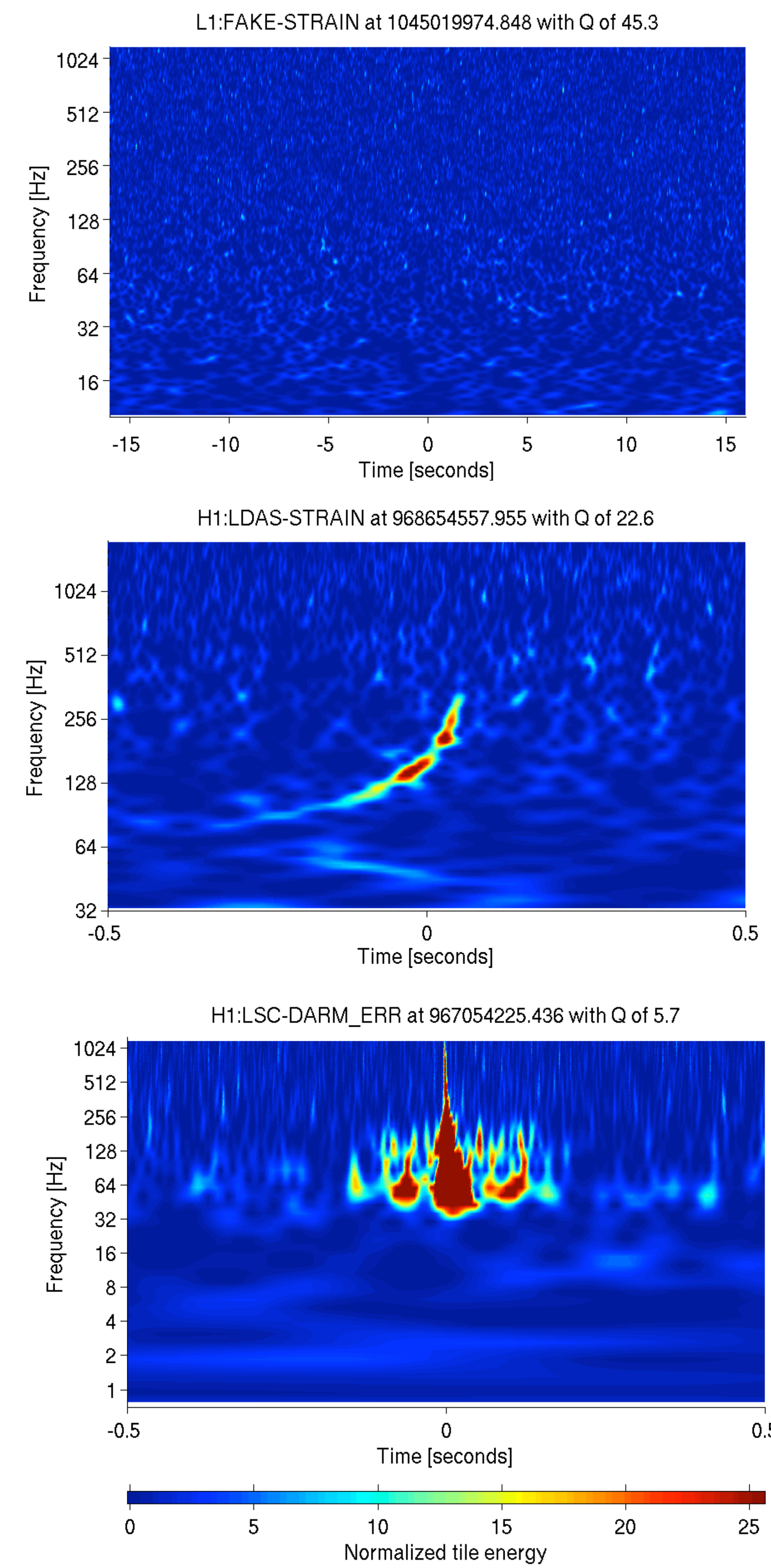
Realistic noise

- The above results used synthetic Gaussian noise based on design sensitivities
- In reality, noise will be **non-Gaussian** (glitchy) and **non-stationary** (varying PSD). Filter data by applying “vetoes”.
- Used S6,VSR2/3 data, recolored to aLIGO/AdVirgo noise curves after filtering out glitchy data segments
- Estimate PSD “on the fly” in the vicinity of the detection
- Ready for **O1**



Realistic noise (cont.)

- Non-Gaussian noise features (glitches) may compromise our GR test
- Glitches can be caused by environmental factors or temporary misbehaviour of detector component
- Rely on data quality algorithms to **veto** bad data segments and exclude glitches from analysis
- Correlate candidate glitches with data from auxiliary channels (microphones, magnetic sensors, accelerometers, etc.) to identify source



Extending TIGER to BBH/NSBH (preliminary)

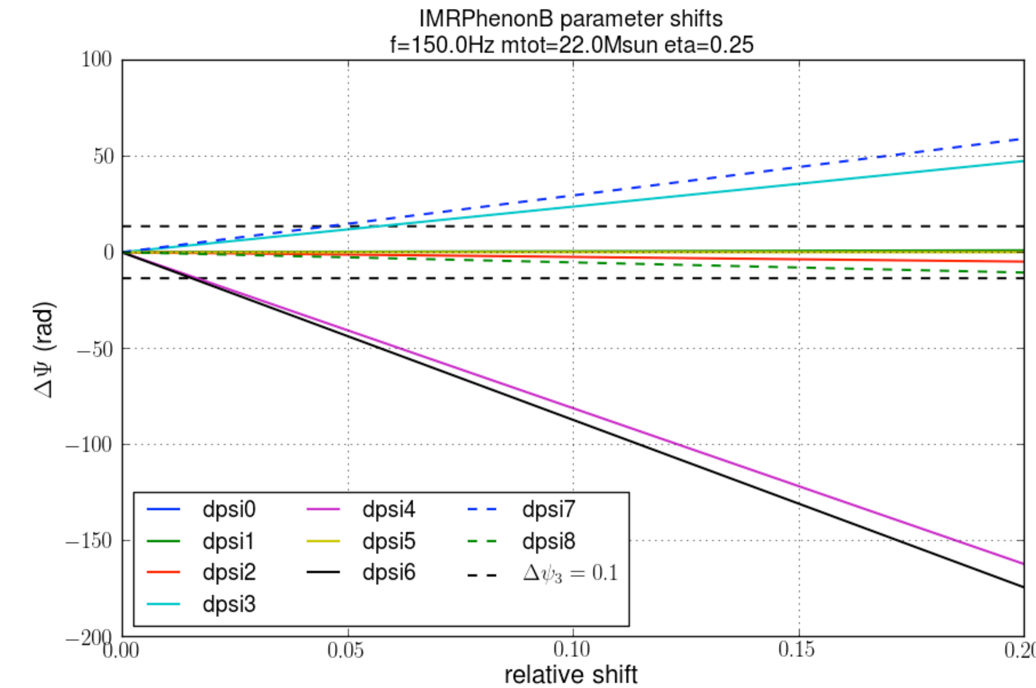
- need faithful waveforms that capture precession for background **injections** (e.g. *SEOBNR*, Pan et al. 2013 [arXiv:1307.6232])

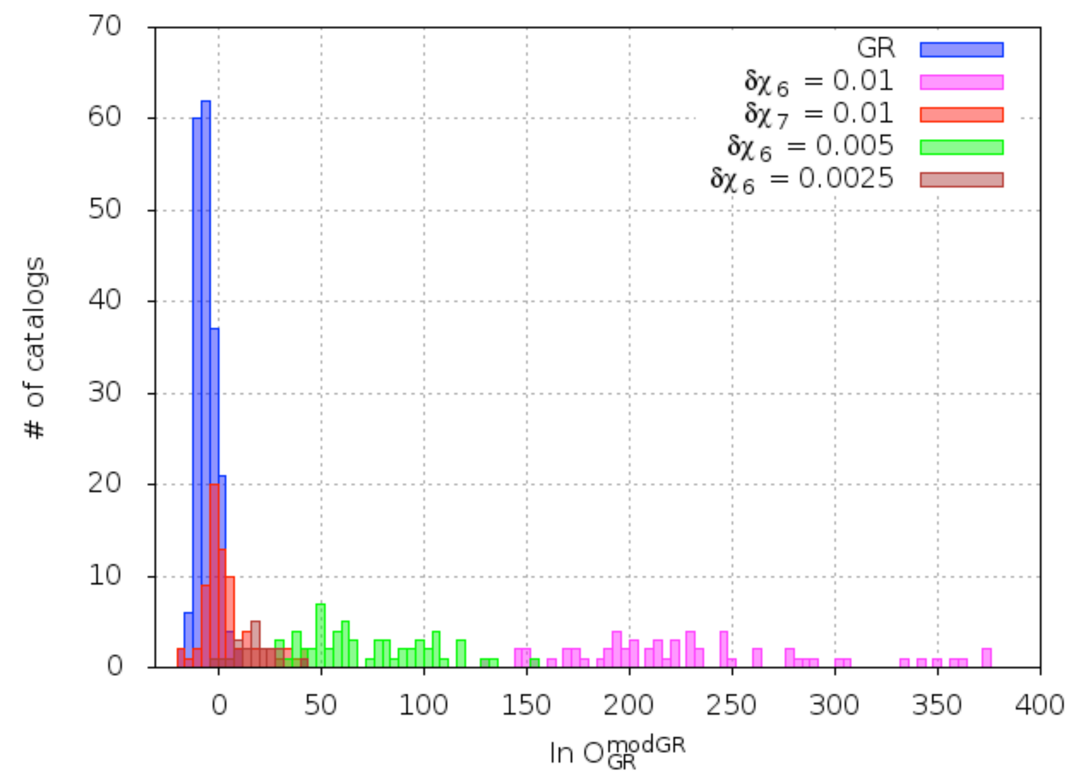
- need fast waveforms that capture precession for **recovery** (e.g. *IMRPhenomP* by Hannam et al. 2013 [arXiv:1308.3271])

- shorter waveforms but richer parameter space to be covered

- more susceptible to glitch contamination

- possibly use Reduced Order Modeling methods (Canizares et al. 2014 [arXiv:1404.6284])





What comes next?

- Possible outcomes for observed set of sources
  - GR seems to be favoured: start placing bounds on modGR parameters
  - marginal: wait for more data to arrive
  - GR seems to be violated: follow up with tailor-made tests to pinpoint the nature of violation; model selection against alternative theories, parameter estimation on their parameter space

- Alternative theories come with their own extended parameter space:
  - choose appropriate parameterization
  - identify locus not excluded by current bounds
  - identify locus that is distinguishable from GR
  - identify locus that does not modify signal "too much"

- Interesting scenarios: theories that predict no effect at weak/non-relativistic regime but may be observeable towards compact binary coalescence (phase transition *e.g.* spontaneous scalarization)

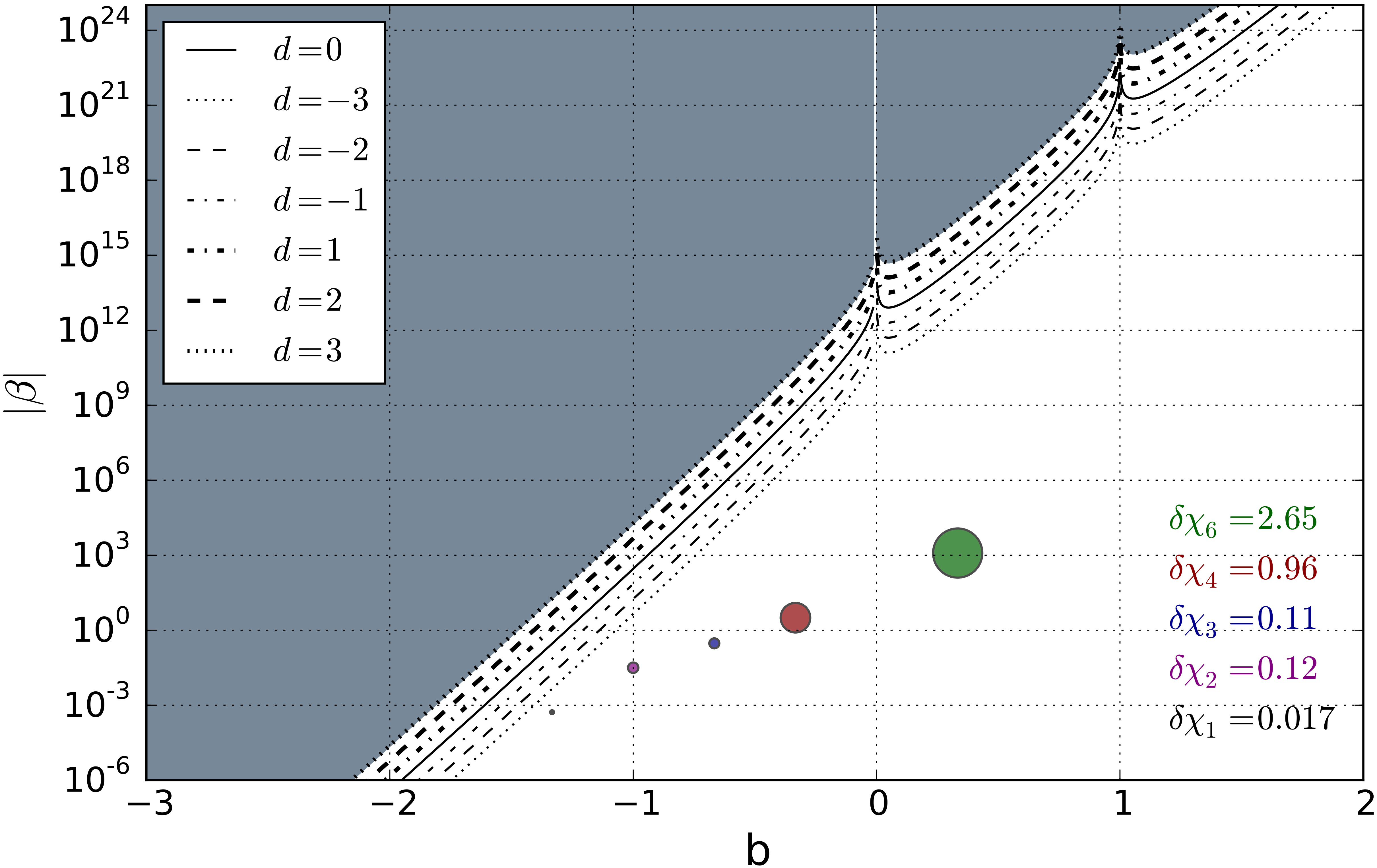
Conclusions

- TIGER is a Bayesian post-detection pipeline for BNS that **accumulates evidence** for finding possible violations of GR with unprecedented precision

- In the case of an observed GR-violation, more specific follow-up tests can be performed to narrow down the nature of the deviation (model selection & parameter estimation)

- Many different possible concerns have been addressed and the pipeline is shown to be robust

- extending TIGER to BBH/NSBH:
  - need faithful waveforms for background **injections**
  - need fast waveforms for **recovery**
  - Reduced Order Modeling methods



$d = 0$
$d = -3$
$d = -2$
$d = -1$
$d = 1$
$d = 2$
$d = 3$
$\delta\chi_6 = 2.65$
$\delta\chi_4 = 0.96$
$\delta\chi_3 = 0.11$
$\delta\chi_2 = 0.12$
$\delta\chi_1 = 0.017$
$|\beta|$
b
10^{24}
10^{21}
10^{18}
10^{15}
10^{12}
10^{9}
10^{6}
10^{3}
10^{0}
10^{-3}
10^{-6}
−3
−2
−1
0
1
2