

Modelling binary neutron stars: beyond pure hydrodynamics

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Plan of the lectures

- * brief introduction to relativistic hydrodynamics
- * what we understand about BNSs
- * characteristic frequencies and quasi-universality
 - inspiral: frequency at amplitude peak
 - merger/post-merger: EOS information from PSD peaks
- * MHD simulations and EM counterparts
 - HMNS: MRI and magnetically driven winds
 - IMHD vs RMHD
 - extended x-ray emission

No good/bad questions. There are only questions: ask them!

Extending the work to **ideal** MHD

NSs have large magnetic fields and it is natural to ask:

- can B-fields be detected during the inspiral?

- can B-fields be detected in the HMNS?

- can B-fields grow after BH formation?

Extending the work to **ideal MHD**

Answering these questions requires extending the equations to **magnetohydrodynamics (MHD)**

This is far from simple as new equations and new numerical methods are needed.

Simplest approximation (which is a good one before merger) is that of **ideal-MHD (IMHD)**: infinite electrical conductivity

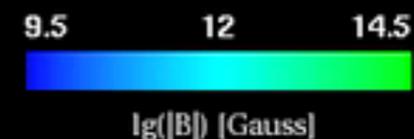
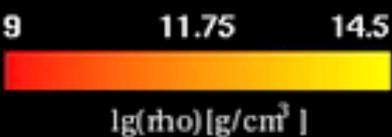
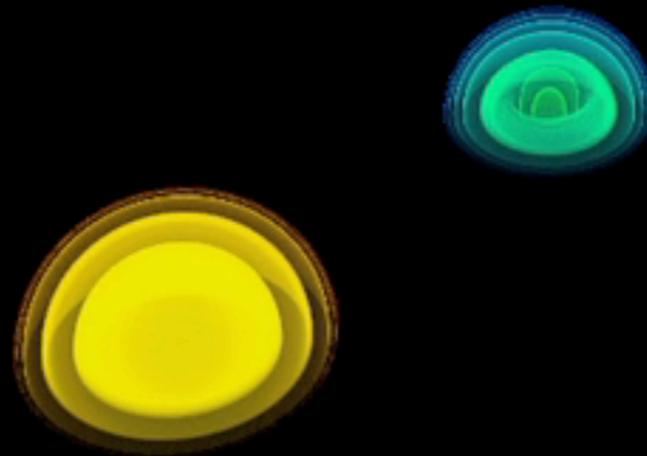
$$T_{\mu\nu} = (e + p) u_\mu u_\nu + p g_{\mu\nu} + F_\mu^\lambda F_{\nu\lambda} - \frac{1}{4} g_{\mu\nu} F^{\lambda\alpha} F_{\lambda\alpha},$$

$$\nabla^\nu T_{\mu\nu} = 0 \quad E^i = -\epsilon^{ijk} v_j B_k$$

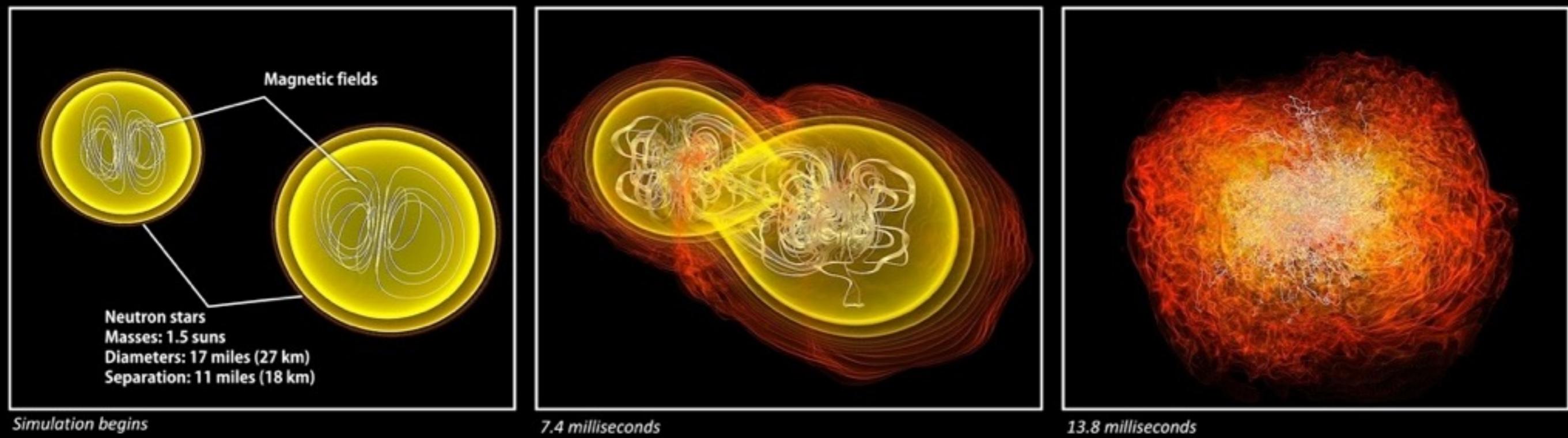
$$\nabla_\nu (F^{\mu\nu} + g^{\mu\nu} \psi) = I^\mu - \kappa n^\mu \psi, \quad \nabla_\nu (*F^{\mu\nu} + g^{\mu\nu} \phi) = -\kappa n^\mu \phi,$$

B-fields during inspiral phase

Typical evolution for a magnetized binary
(hot EOS) $M = 1.5 M_{\odot}$, $B_0 = 10^{12}$ G

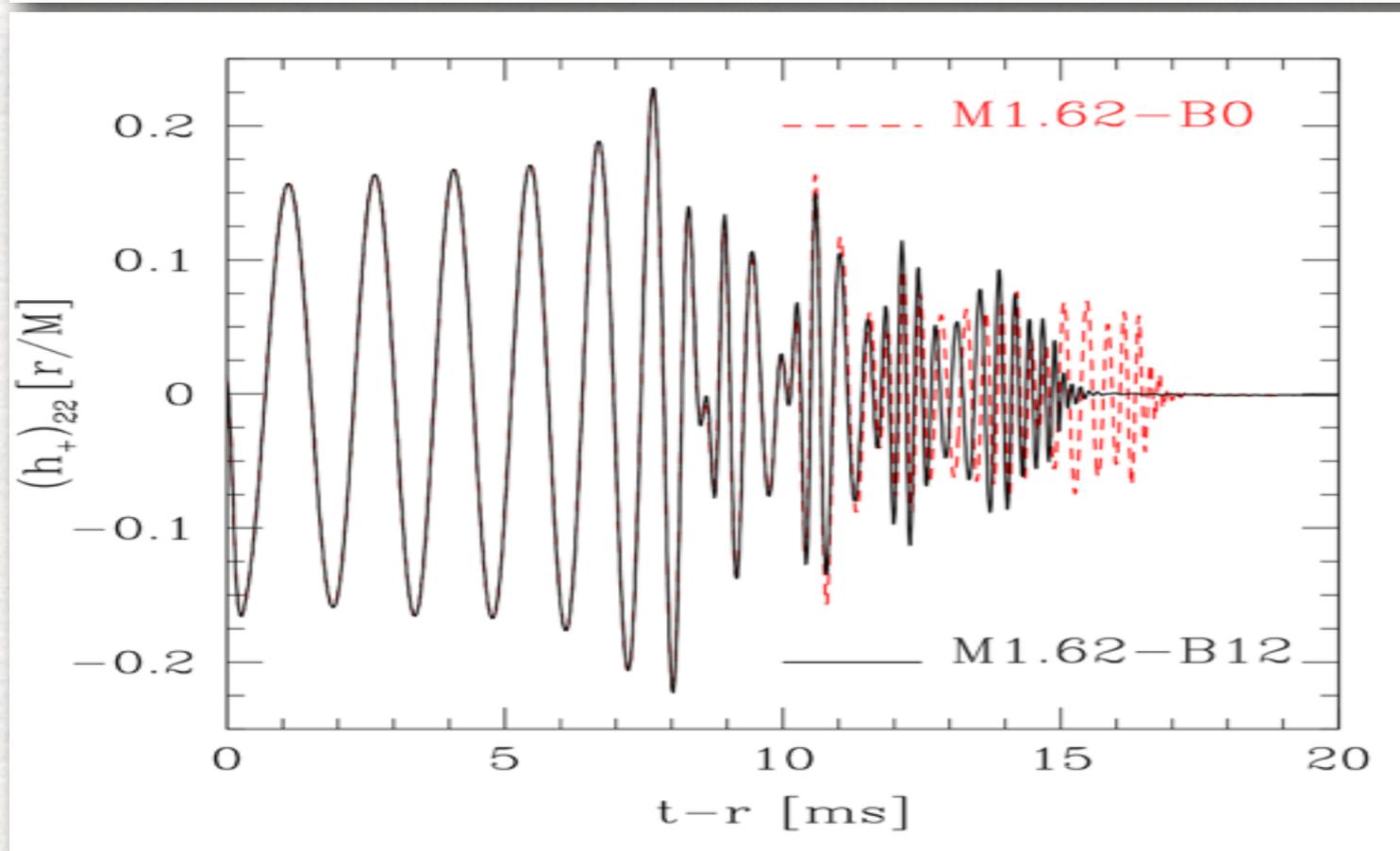
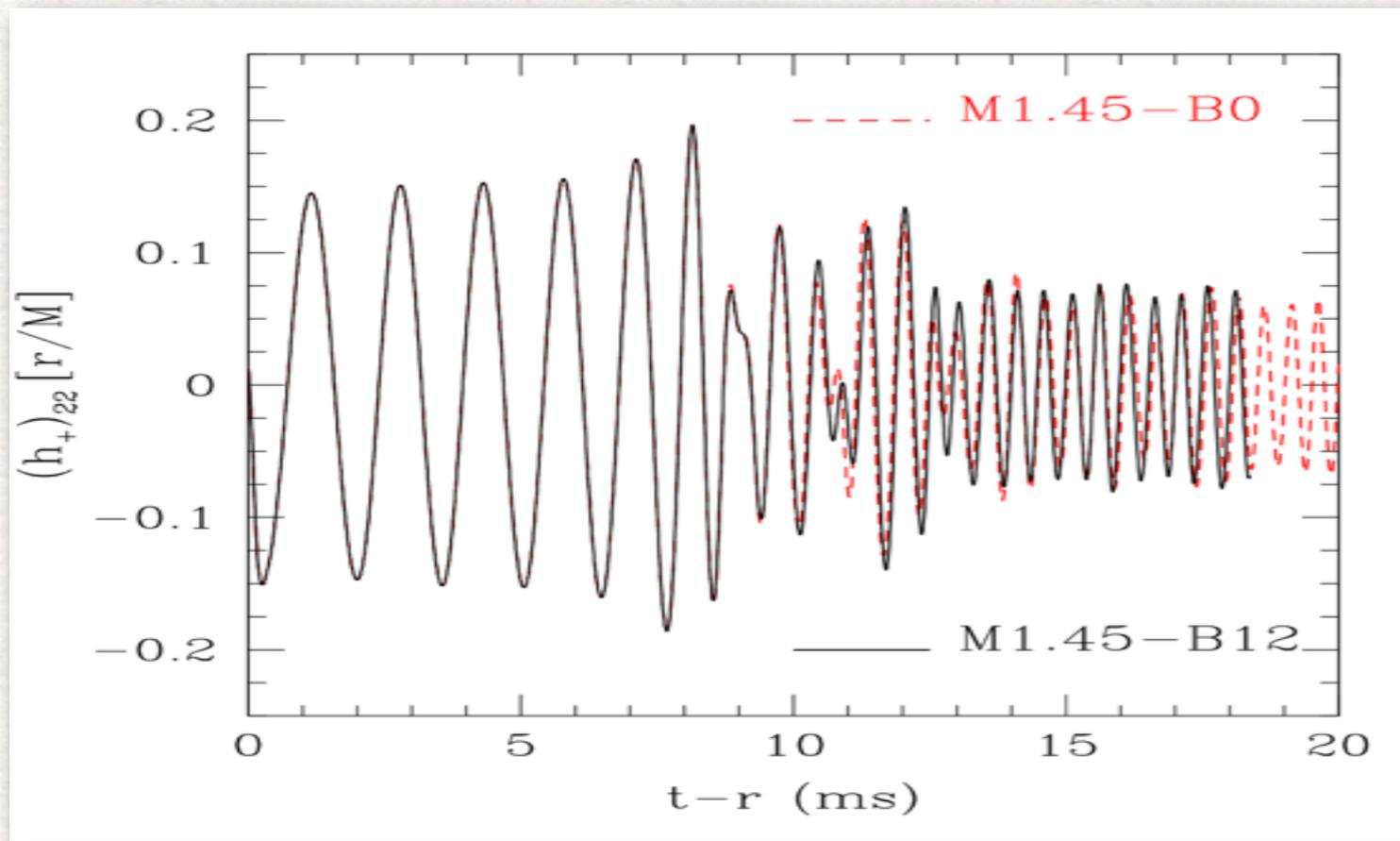


Animations: LR, Koppitz



Magnetic fields in the HMNS have complex topology: dipolar fields are destroyed.

Waveforms: comparing against magnetic fields



Compare B/no-B field:

- the evolution in the **inspiral** is different but only for ultra large B-fields (i.e. $B \sim 10^{17}$ G). For realistic fields the difference is not significant.

- the **post-merger** evolution is different for all masses; strong B-fields delay the collapse to BH

Can we detect B-fields in the inspiral?

To quantify the differences and determine whether detectors will see a difference in the inspiral, we calculate the **overlap**

$$\mathcal{O}[h_{B1}, h_{B2}] \equiv \frac{\langle h_{B1} | h_{B2} \rangle}{\sqrt{\langle h_{B1} | h_{B1} \rangle \langle h_{B2} | h_{B2} \rangle}}$$

where the scalar product is

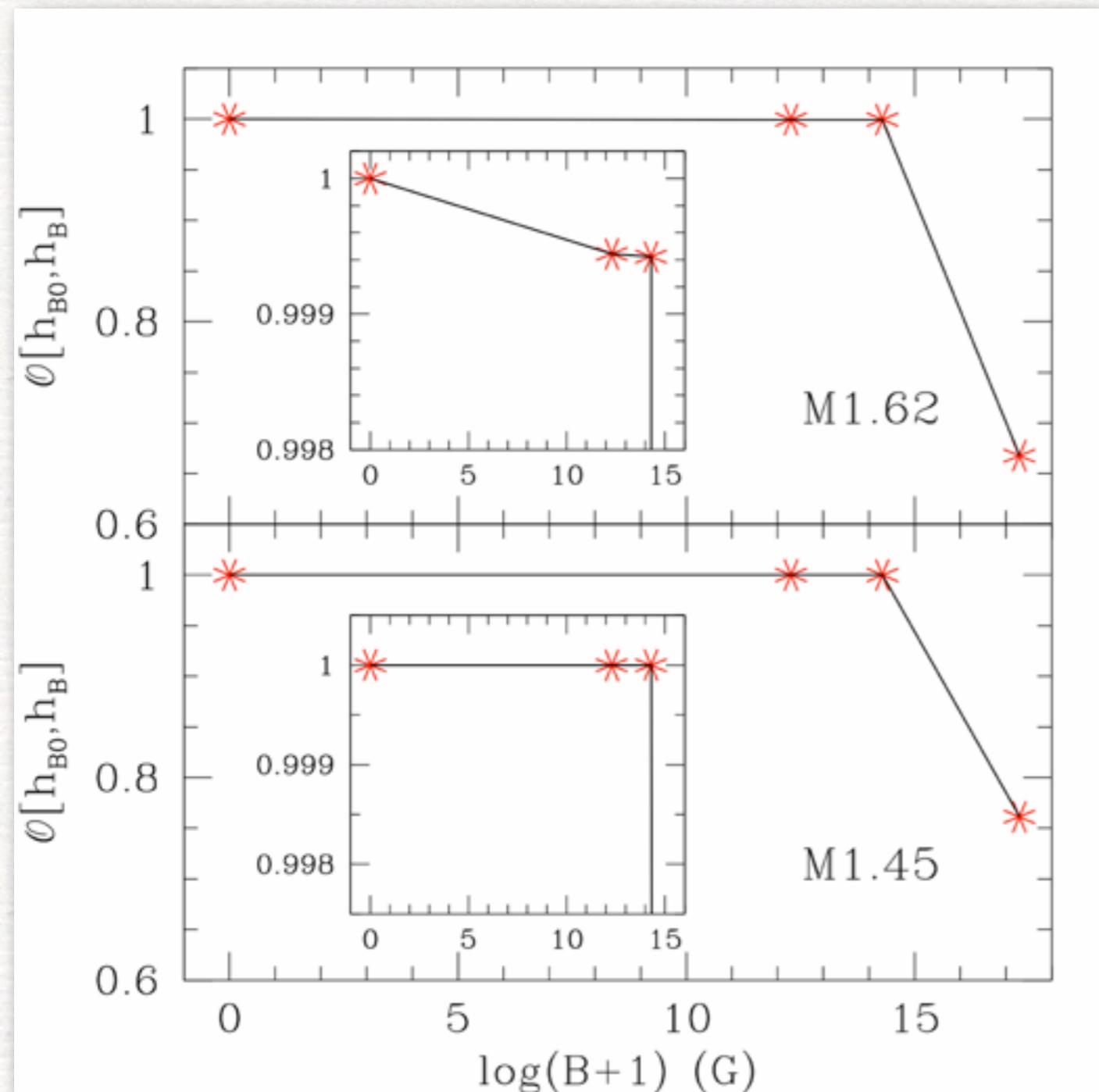
$$\langle h_{B1} | h_{B2} \rangle \equiv 4\Re \int_0^\infty df \frac{\tilde{h}_{B1}(f) \tilde{h}_{B2}^*(f)}{S_h(f)}$$

In essence, at these res:

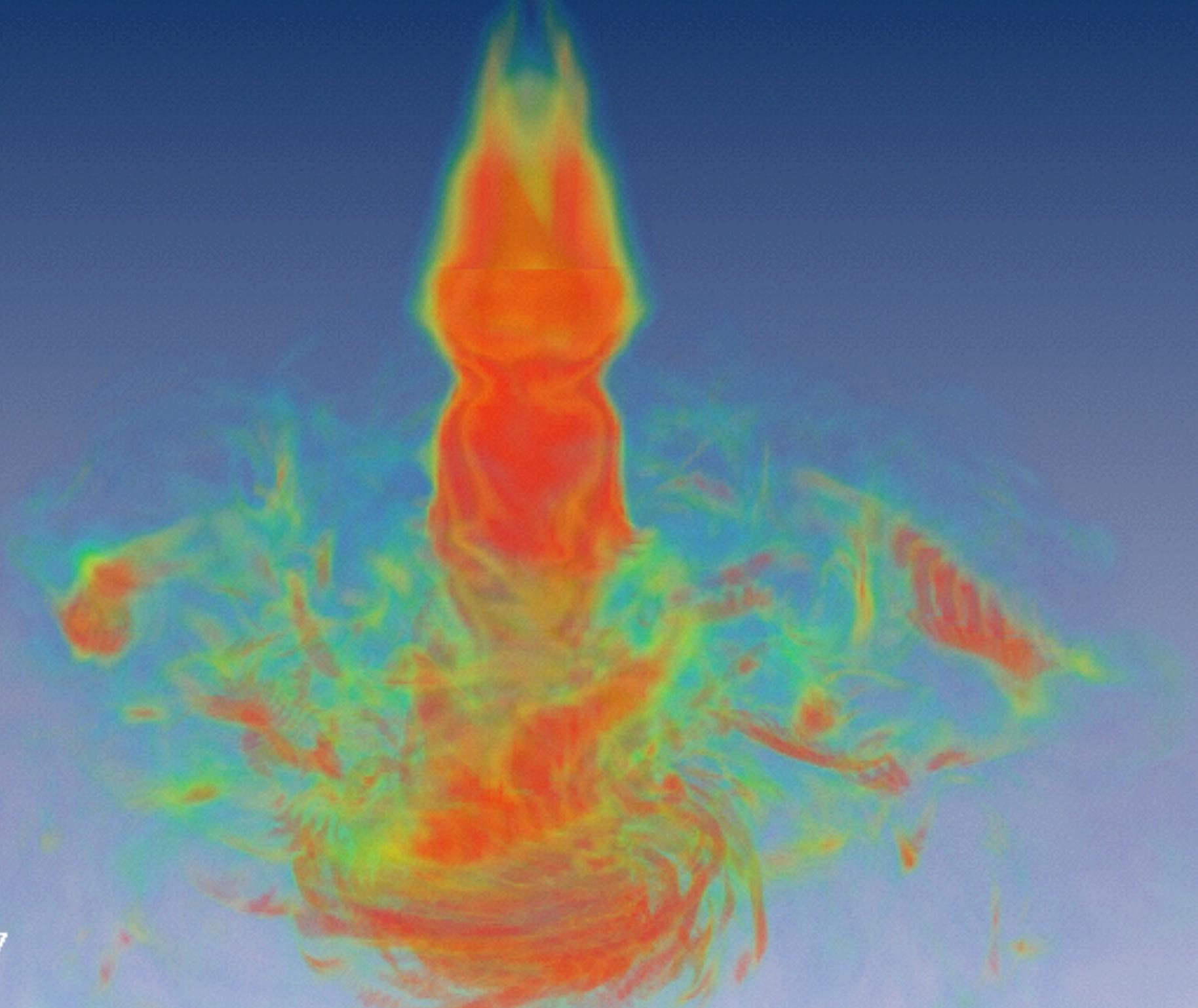
$$\mathcal{O}[h_{B0}, h_B] \gtrsim 0.999$$

$$\text{for } B \lesssim 10^{17} \text{ G}$$

Influence of B-fields on inspiral is **unlikely to be detected**



MHD instabilities and B-field amplifications



Extending the work to **ideal** MHD

NSs have large magnetic fields and it is natural to ask:

- can B-fields be detected during the inspiral?
 - ***NO**: present and future GW detectors will not be sensitive enough to measure the small differences

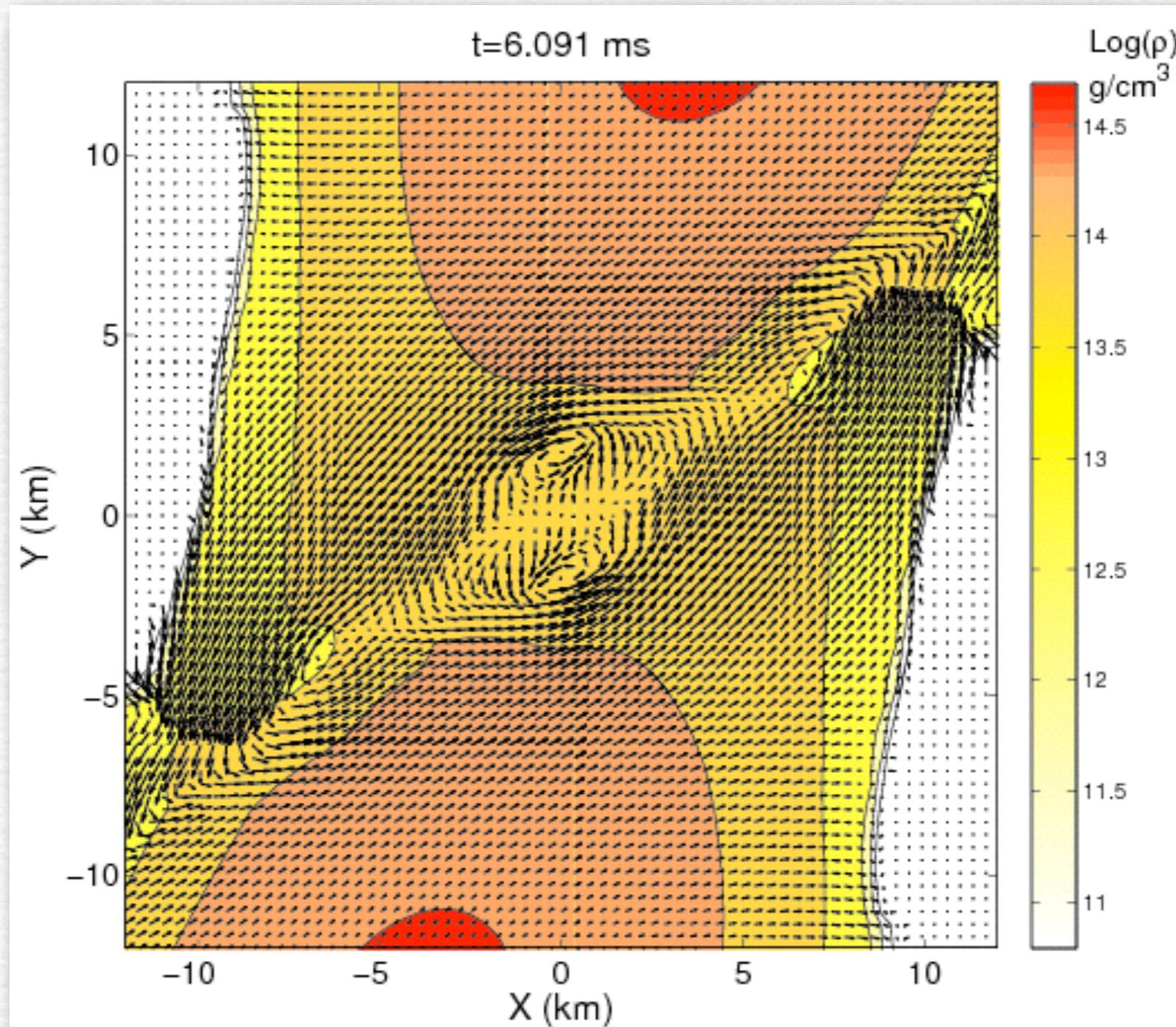
Giacomazzo, LR, Baiotti (2009)

- can B-fields be detected in the HMNS?

- can B-fields grow after BH formation?

MHD instabilities and B-field amplifications

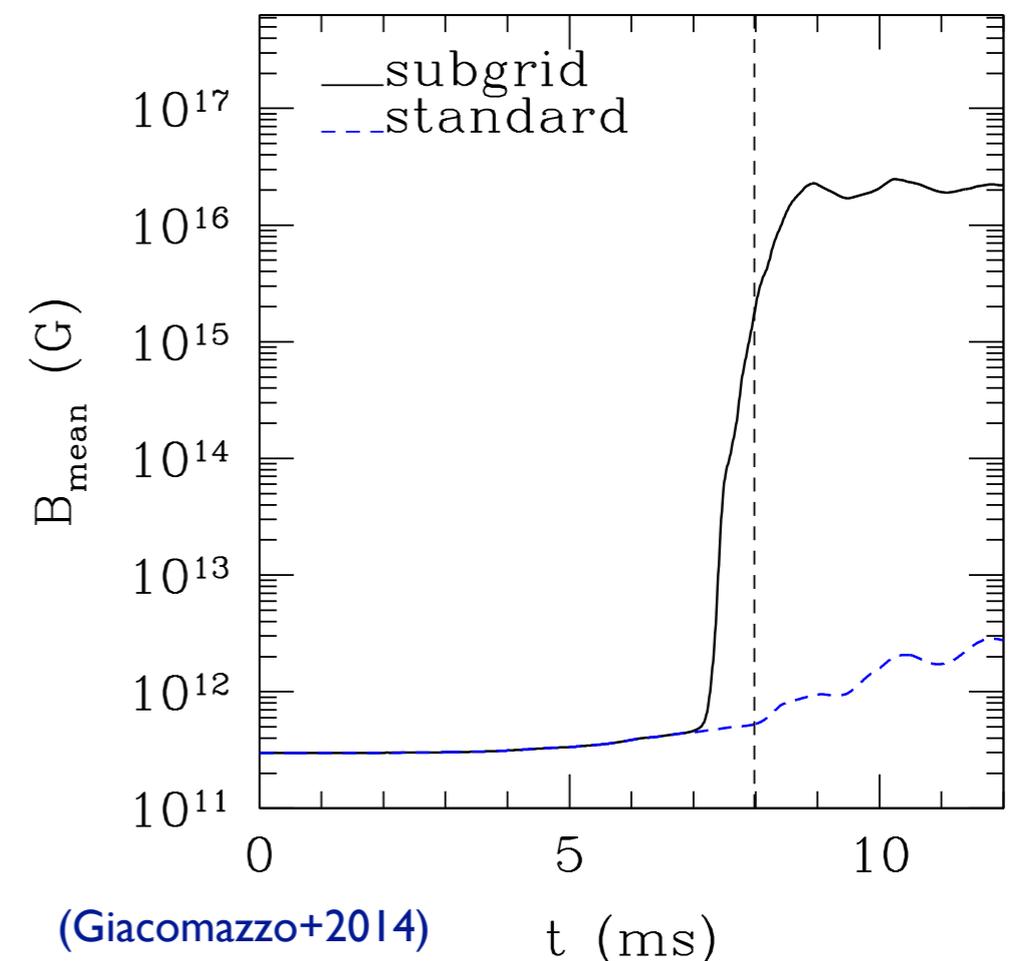
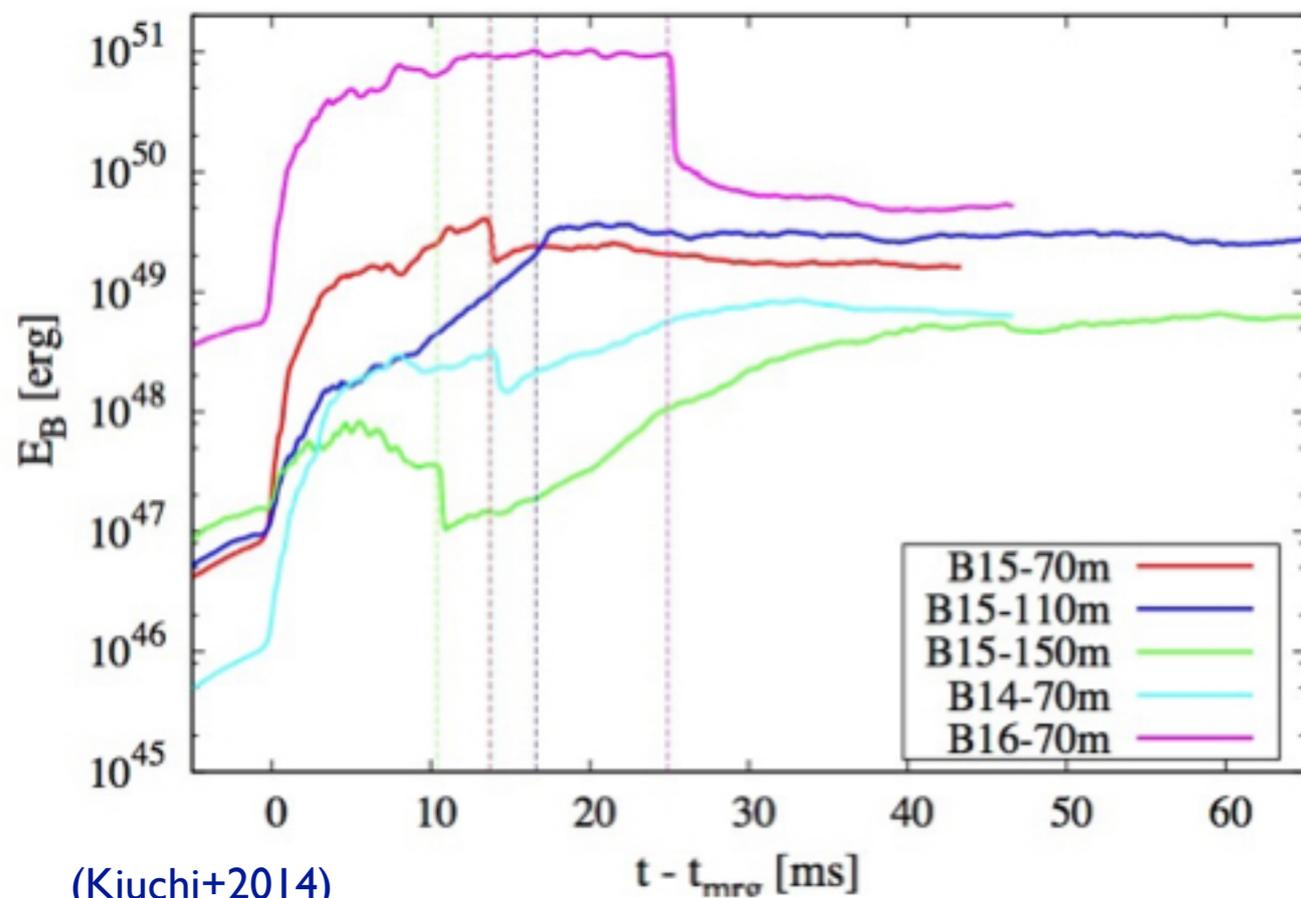
- at the merger, the NS create a strong shear layer which could lead to a **Kelvin-Helmholtz instability**; magnetic field can be amplified



(Baiotti+2008)

MHD instabilities and B-field amplifications

- at the merger, the NS create a strong shear layer which could lead to a **Kelvin-Helmholtz instability**; magnetic field can be amplified
- direct simulations don't show significant exponential growth (Giacomazzo+2011, Kiuchi+2014). Timescale too short? Resolution too poor?
- sub-grid models suggest B-field grows to 10^{16} G (Giacomazzo+2014)

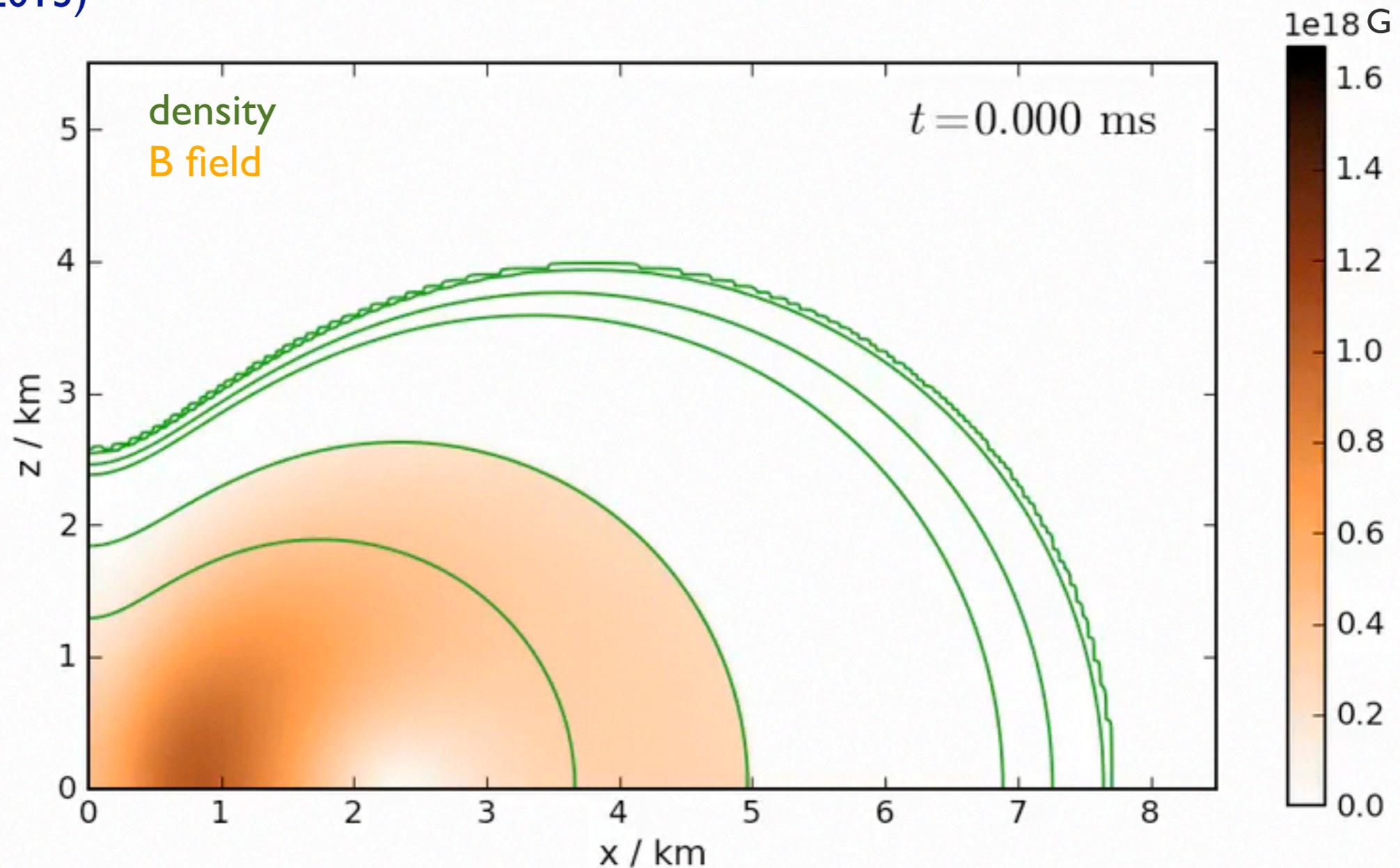


MHD instabilities and B-field amplifications

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- sub-grid models suggest B-field grows to 10^{16} G (Giacomazzo+2014)
- differentially rotating magnetized fluids develop the **MRI** (**magnetorotational instability**; Velikhov 1959, Chandrasekhar 1960)
- the MRI leads to exponential growth of B-field and to an outward transfer of angular momentum: responsible for accretion in discs

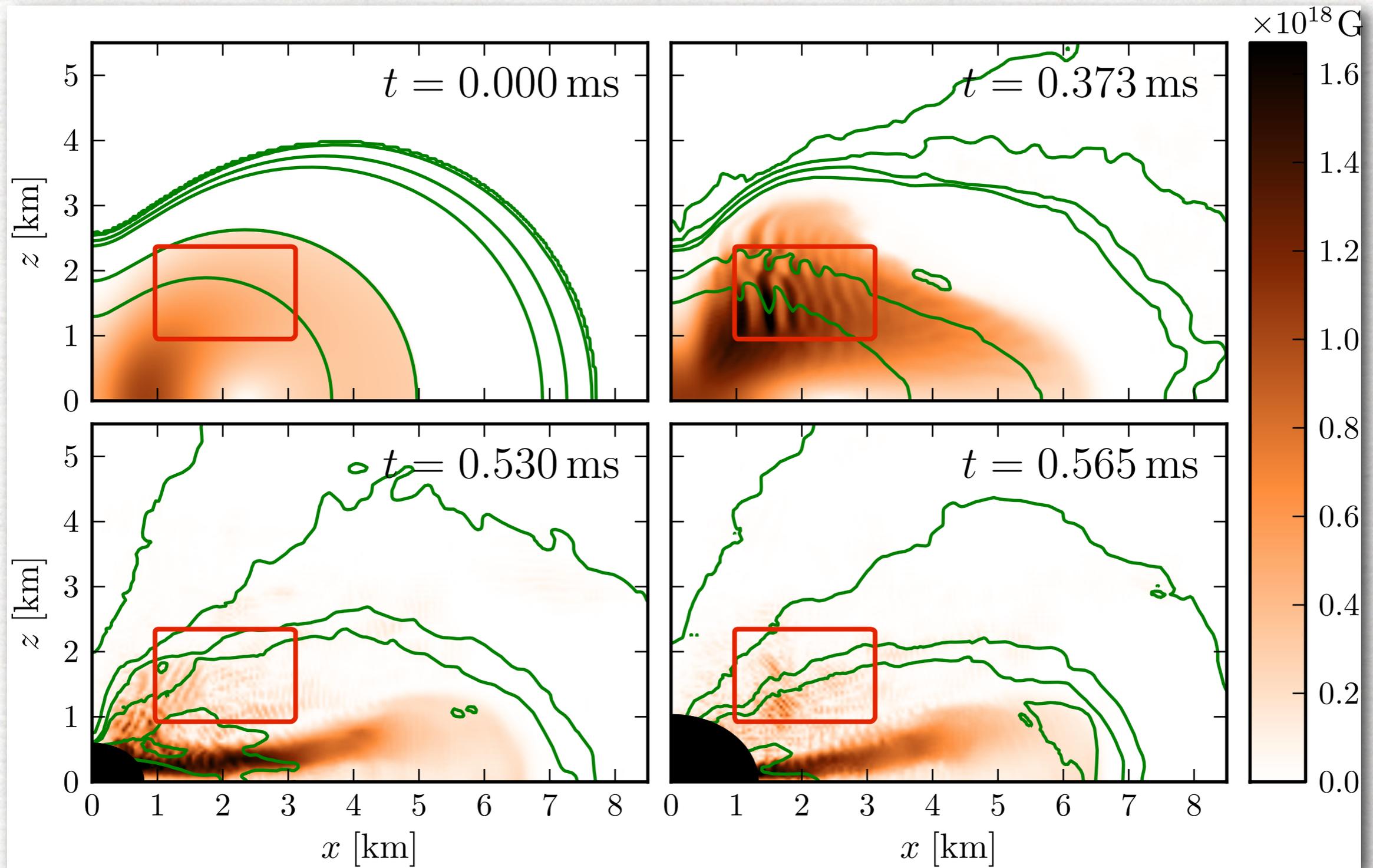
First global simulations in full GR

Siegel + (2013)



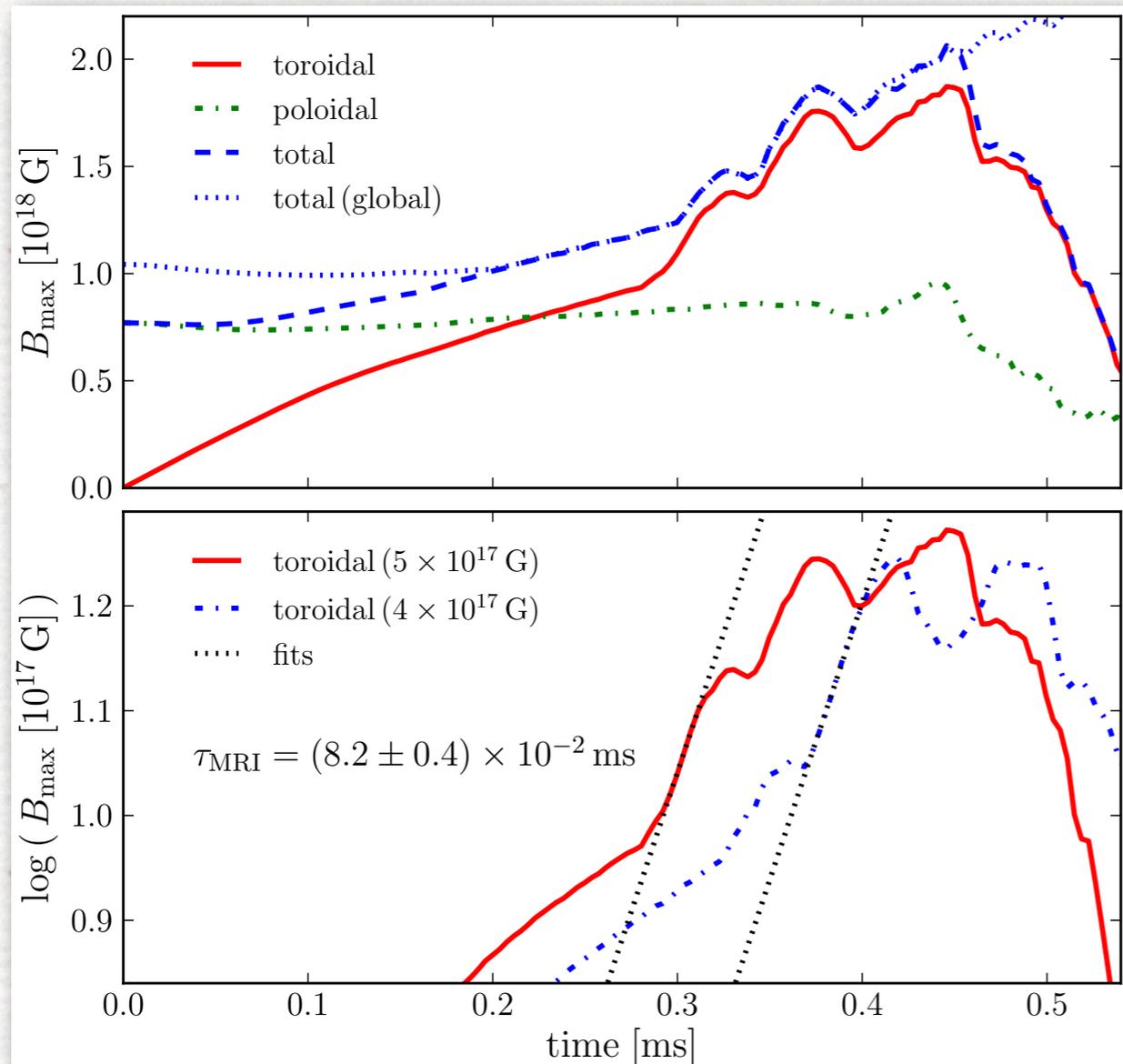
- ideal MHD (WhiskyMHD code)
- ideal-fluid EOS, $p = (\Gamma - 1)\rho\epsilon$
- spacetime evolution (I+log slicing, Gamma-driver)
- axisymmetric initial model ($M = 2.23M_{\odot}$)
 - purely poloidal B field ($B_c^{\text{in}} = 5\text{e}17$ G)
 - differential rotation: j -constant law
- cartesian grid
[0, 94.6] \times [0, 94.6] \times [0, 53.9] km
- 4 refinement levels,
finest gridspacing $h = 44$ m
- $\pi/2$ and z-reflection symmetry

A local view in a global simulation



highest resolution ever used in 3D MHD calculations: 44 m

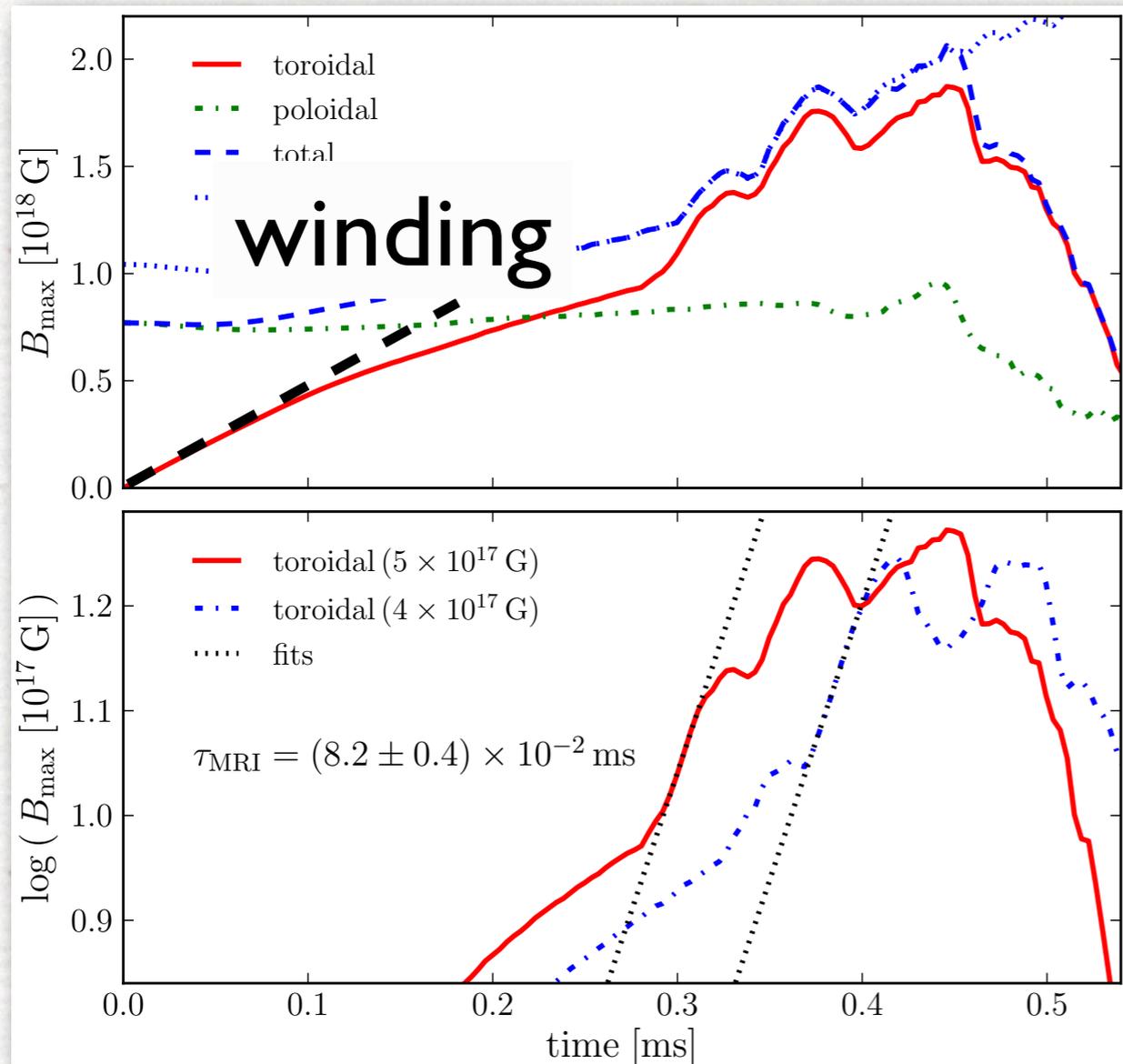
Magnetic field growth: linear and exponential



- **poloidal field** is not amplified during the evolution
- **toroidal field** initially generated by magnetic winding:

$$B_{\text{tor}} \approx (r B^i \partial_i \Omega) t = a_w t$$

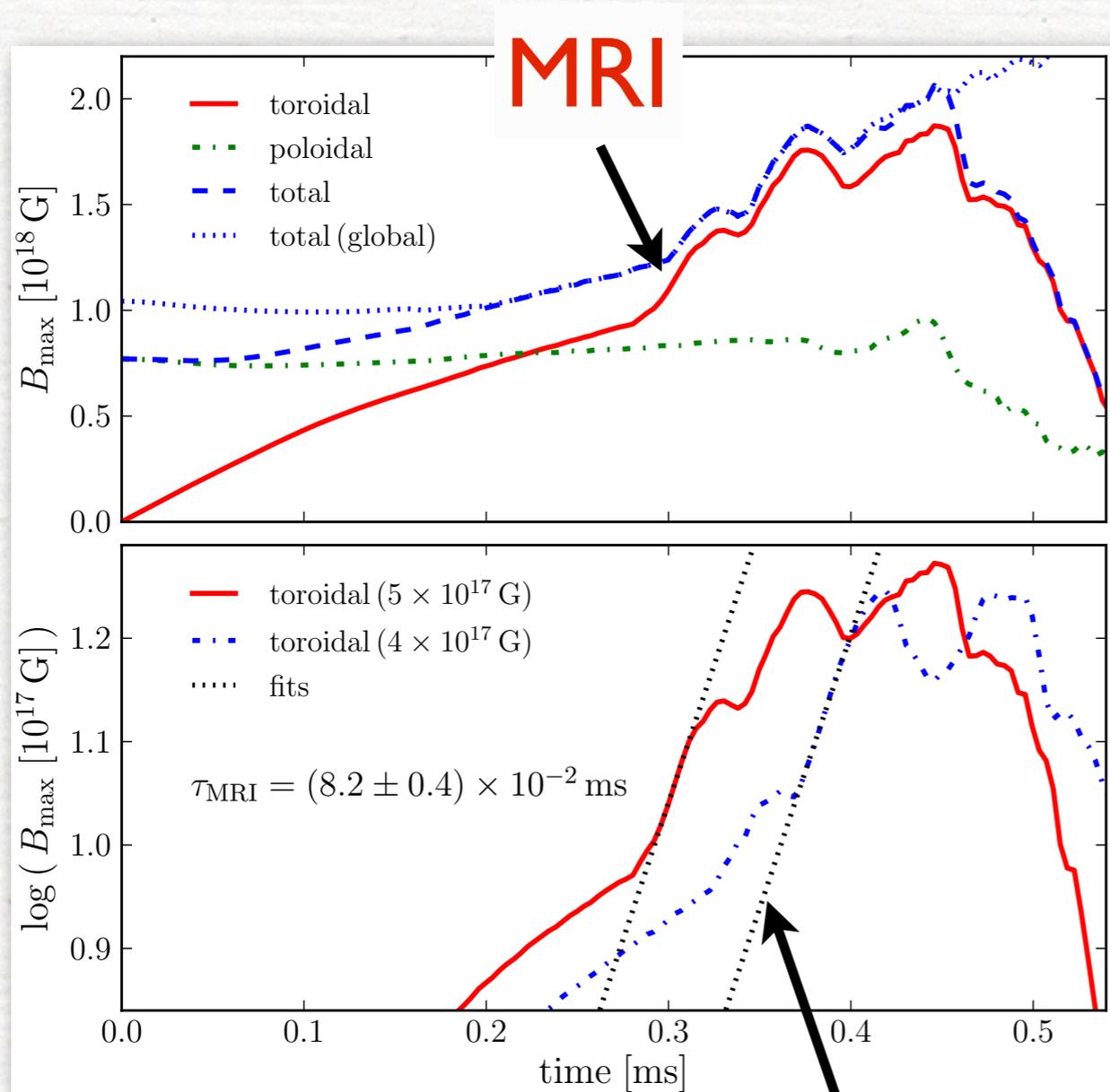
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- at 0.28 ms MRI sets in with growth time:

measured

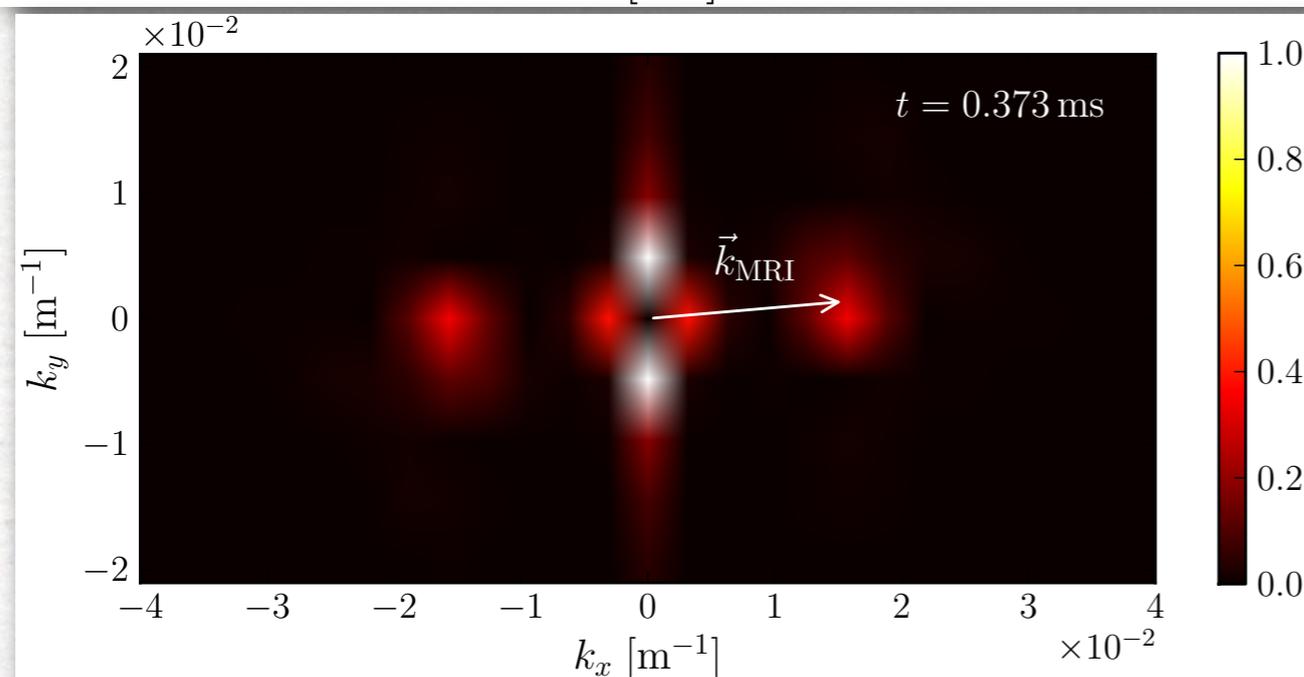
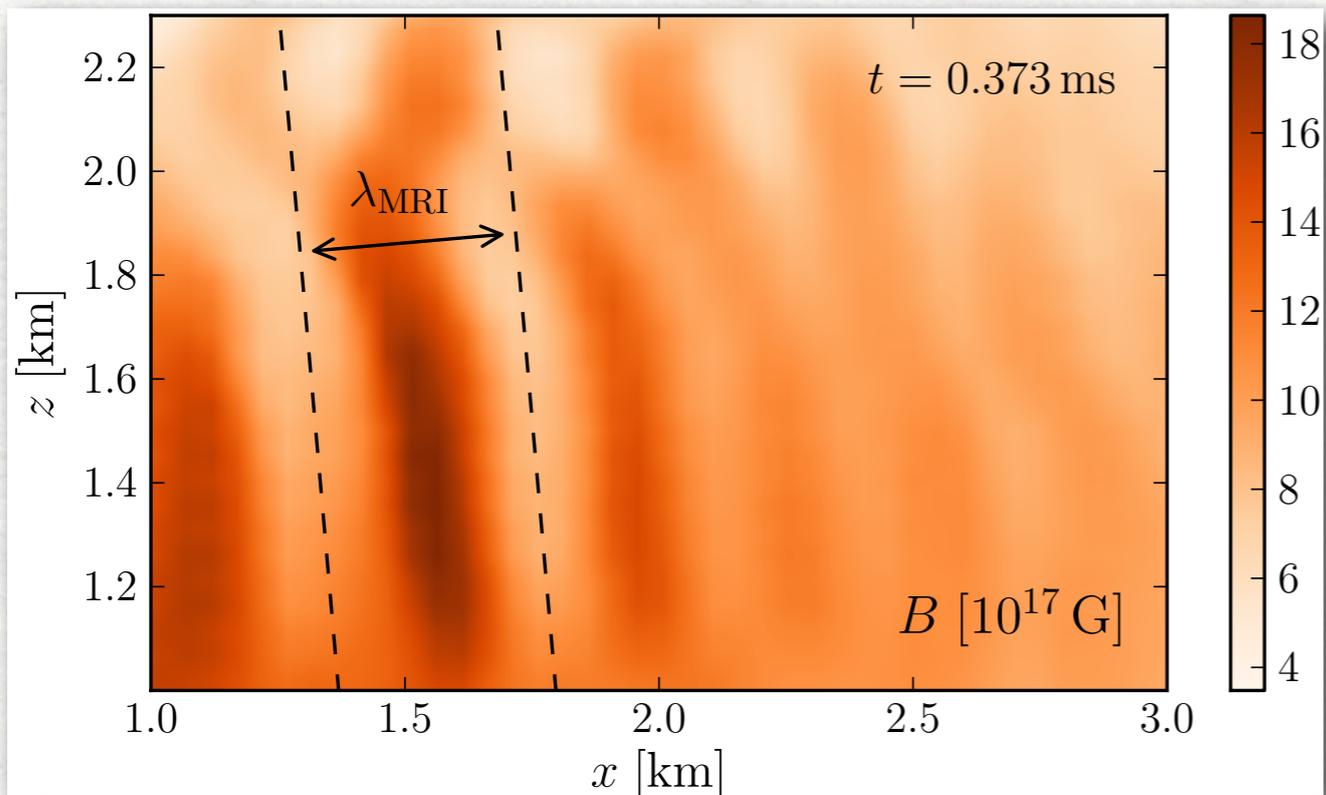
$$\tau_{\text{MRI,fit}} = (8.2 \pm 0.4) \times 10^{-2} \text{ ms}$$

order-of-mag. prediction

$$\tau_{\text{MRI}} \sim \Omega^{-1} \approx (4 - 5) \times 10^{-2} \text{ ms}$$

τ_{MRI} does not depend on magnetic field strength

An important signature: channel flows



- onset of channel-flow merging visible in upper part
- power spectrum reveals a **single dominant mode** k_{MRI} (apart from contributions from large-scale gradients)
- wavelength consistent with channel flows

measured

$$\lambda_{\text{MRI}} \approx 0.4 \text{ km}$$

order-of-mag. prediction

$$\lambda_{\text{MRI}} \sim (0.5 - 1.0) \text{ km}$$

Altogether: first evidence for development of MRI in HMNSs!

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***NO**: present and future GW detectors will not be sensitive enough to measure the small differences

Giacomazzo, LR, Baiotti (2009)

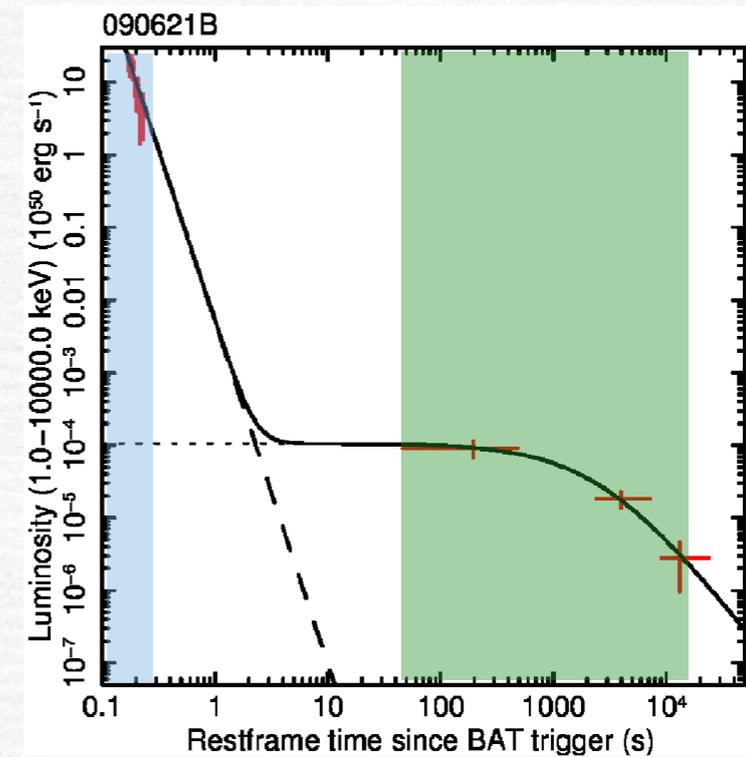
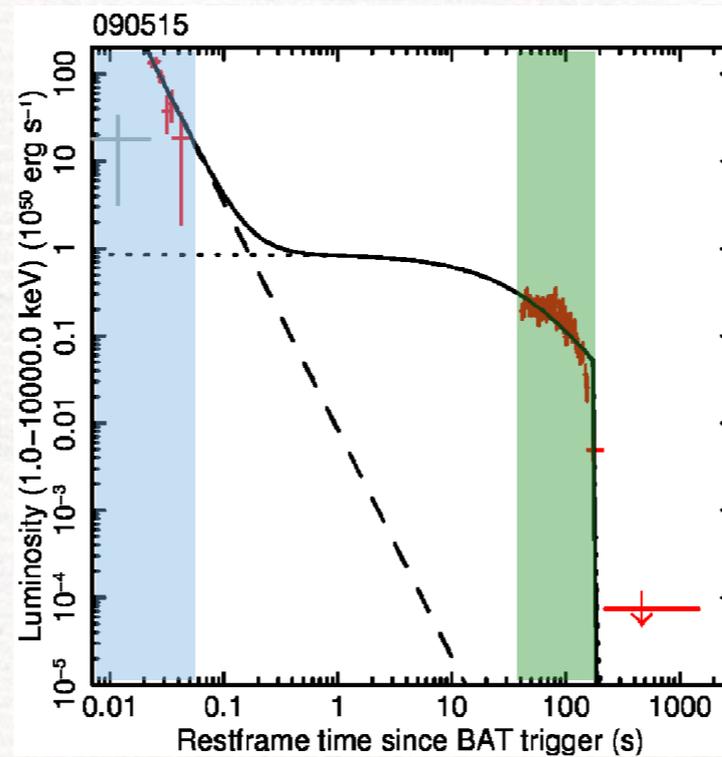
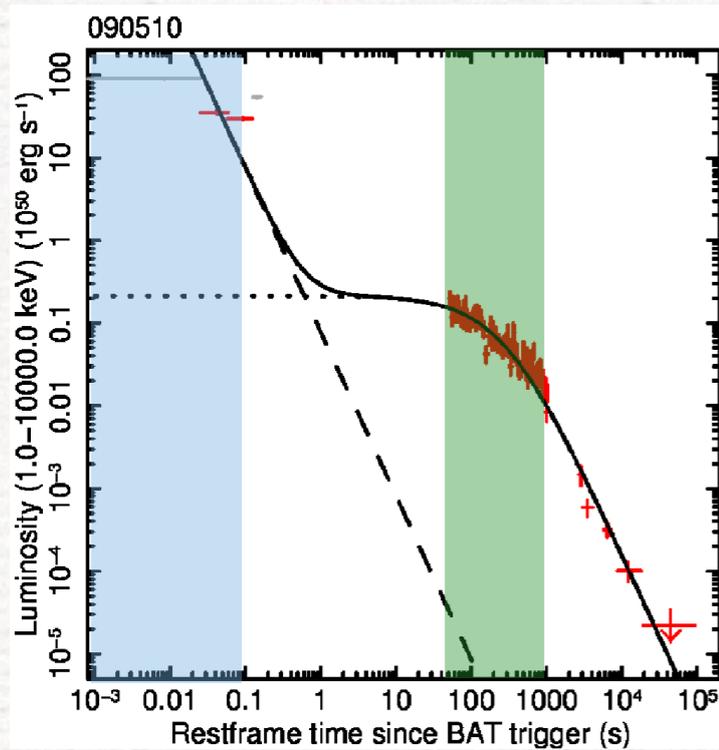
- can B-fields be detected in the HMNS?

***YES** (in principle): different B-fields change the survival time of the HMNS and can grow via MRI

Giacomazzo, LR, Baiotti (2010), Siegel, LR+ (2013)

- can B-fields grow after BH formation?

The puzzle of X-ray extended emission



- X-ray afterglows have been observed by Swift lasting as long as 10^2 – 10^4 s (Rowlinson+ 13; Gompertz+ 13)
- The x-ray afterglow could be produced by “proto-magnetar wind” with $L_x \sim 10^{49}$ erg s^{-1} (Zhang & Mezsaros 01, Metzger+ 11, Zhang 13).

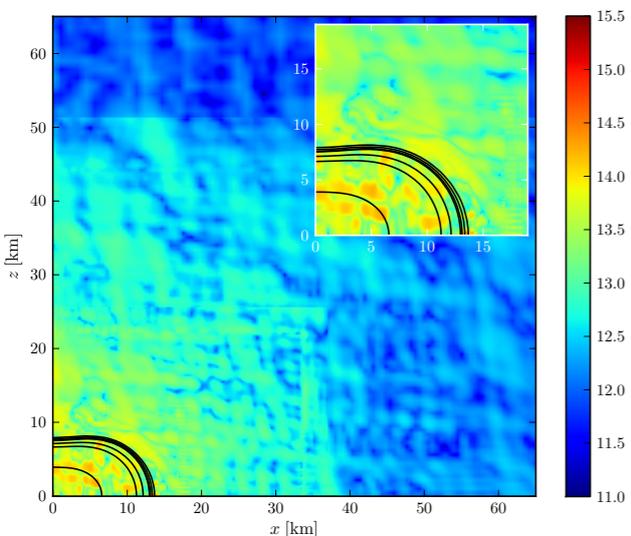
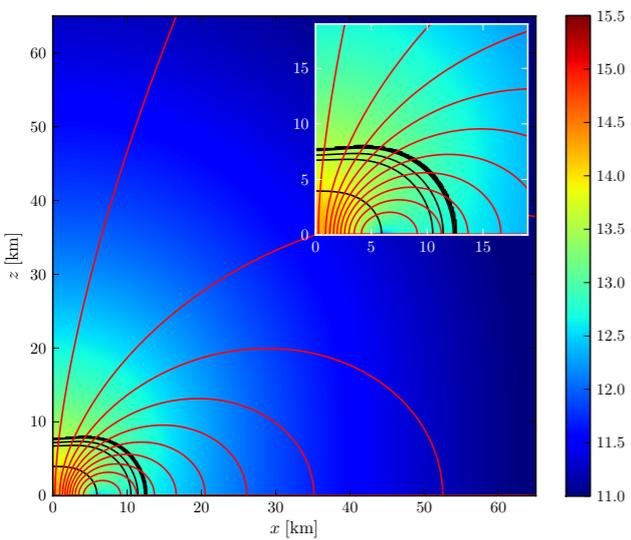
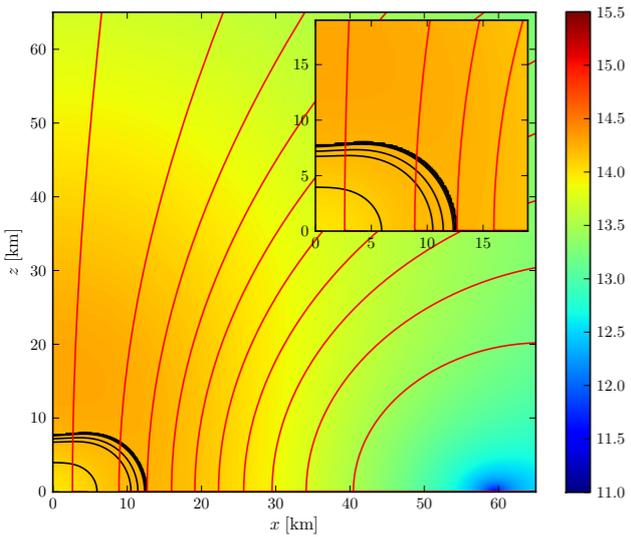
Magnetically driven winds

Inevitable in HMNSs with strong magnetisation (Shibata+11; Bucciantini+12; Kiuchi+12, Siegel+13); important to establish correlations of field topology with:

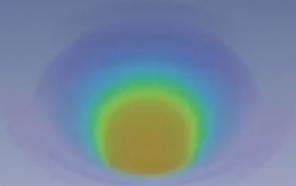
- efficiency of the emission
- geometry of the outflow
- physical properties of the outflow

Considered 3 field topologies that covering the ranges of possible behaviours.

Used simplified initial data (axisymmetric) but evolutions in 3D with very high resolutions.



poloidal magnetic field,
neutral line at 60 km



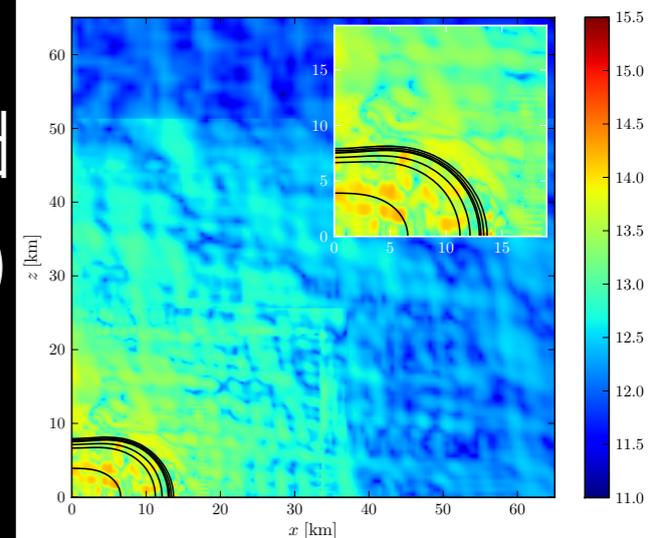
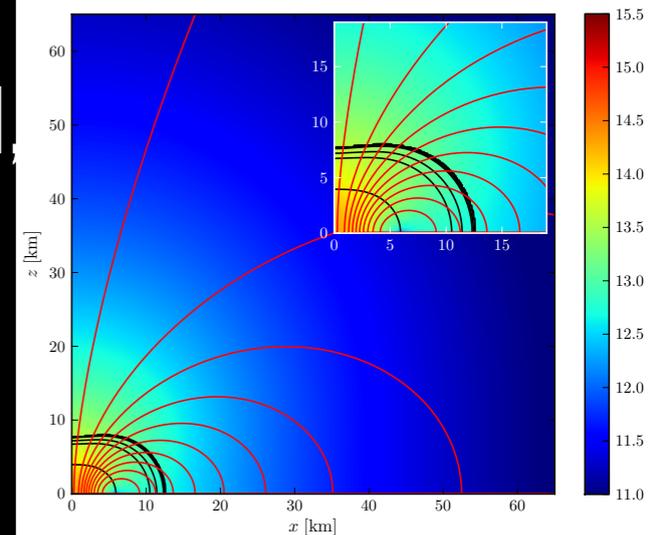
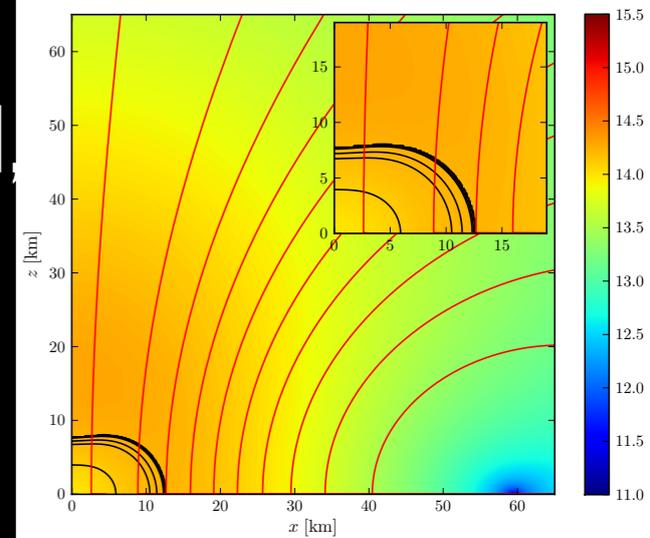
poloidal magnetic field,
neutral line at 6 km



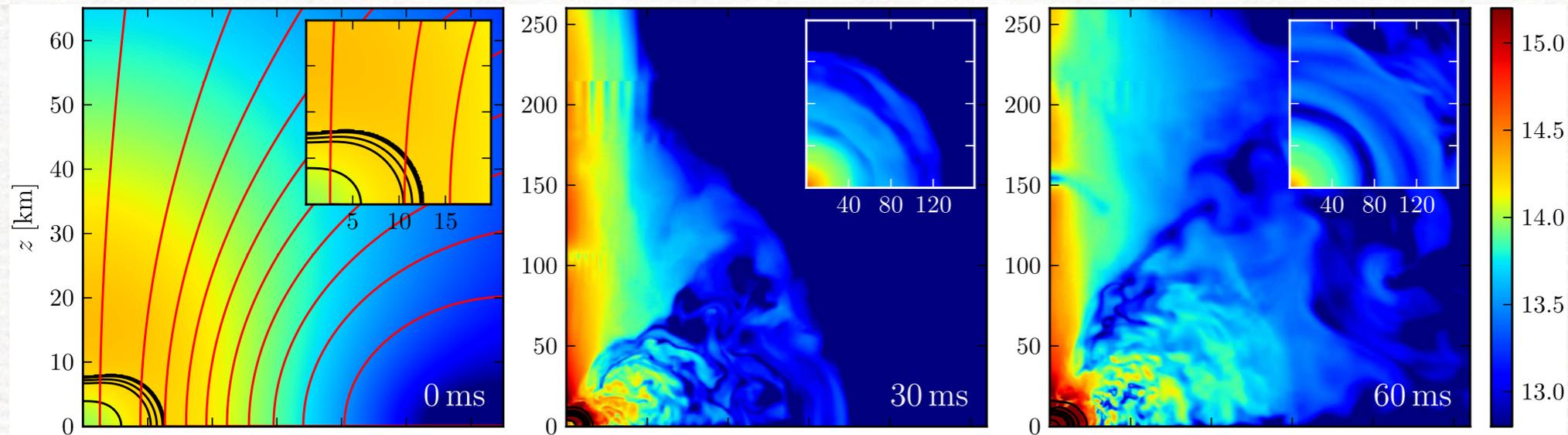
random magnetic field
(poloidal and toroidal)



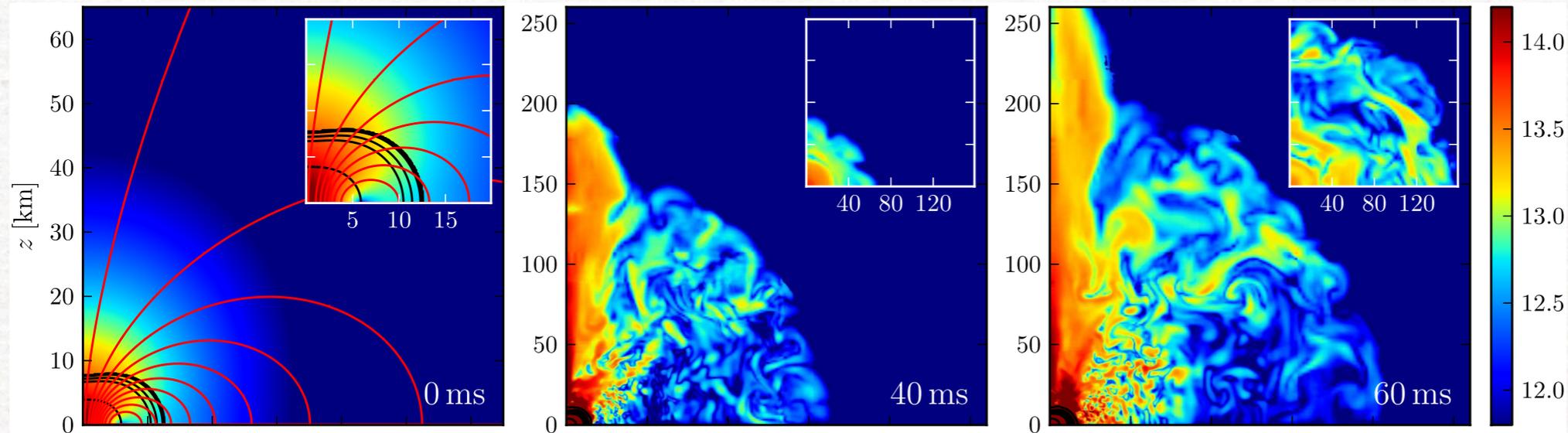
- **ideal MHD:**
poloidal magnetic field,
 $B_{\max} \approx 2 \times 10^{14} \text{ G}$
neutral line at 60 km
- **ideal-fluid EOS,**
- **standard gauges**
- **max extents:**
[800 × 800 × 553] M_{\odot}
[1160 × 1160 × 800] field
- **neutral line at 6 km**
 $0.096 M_{\odot}$, $\sim 140 \text{ m}$
- **7 refinement lev.**
- **z-reflection and rotation symmetry**
- **initial model**
- **axisymmetric**
- **differential rotation:**
j-constant law



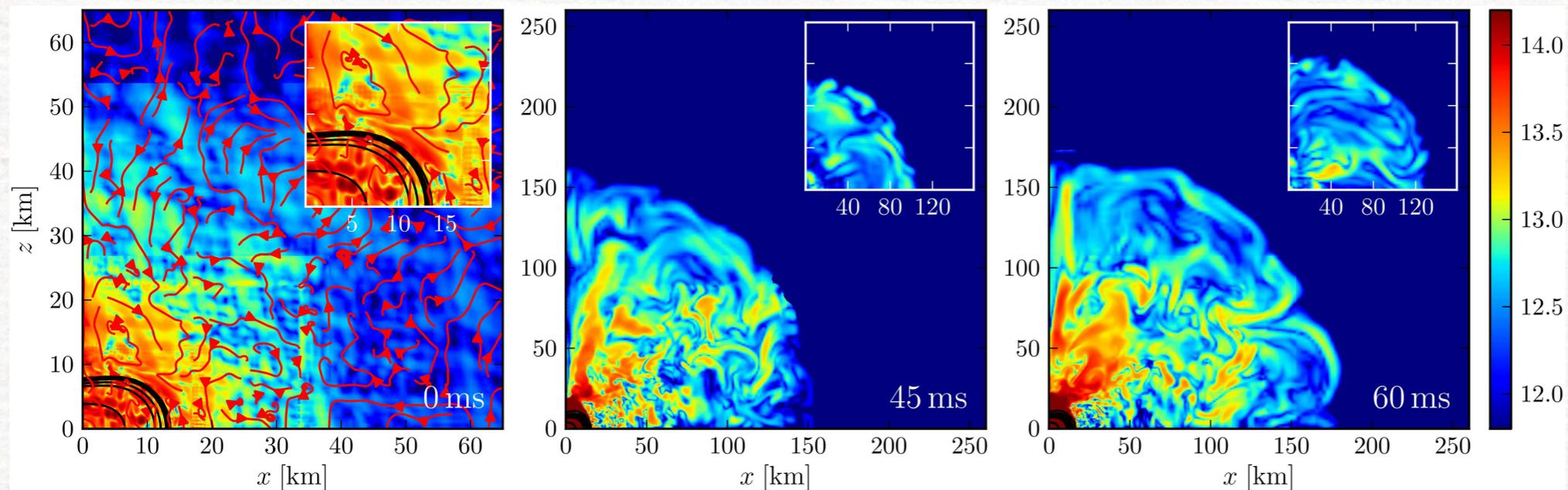
Comparative table



Uniform B-field inside the HMNS leads to highly collimated flow and modest isotropic wind.

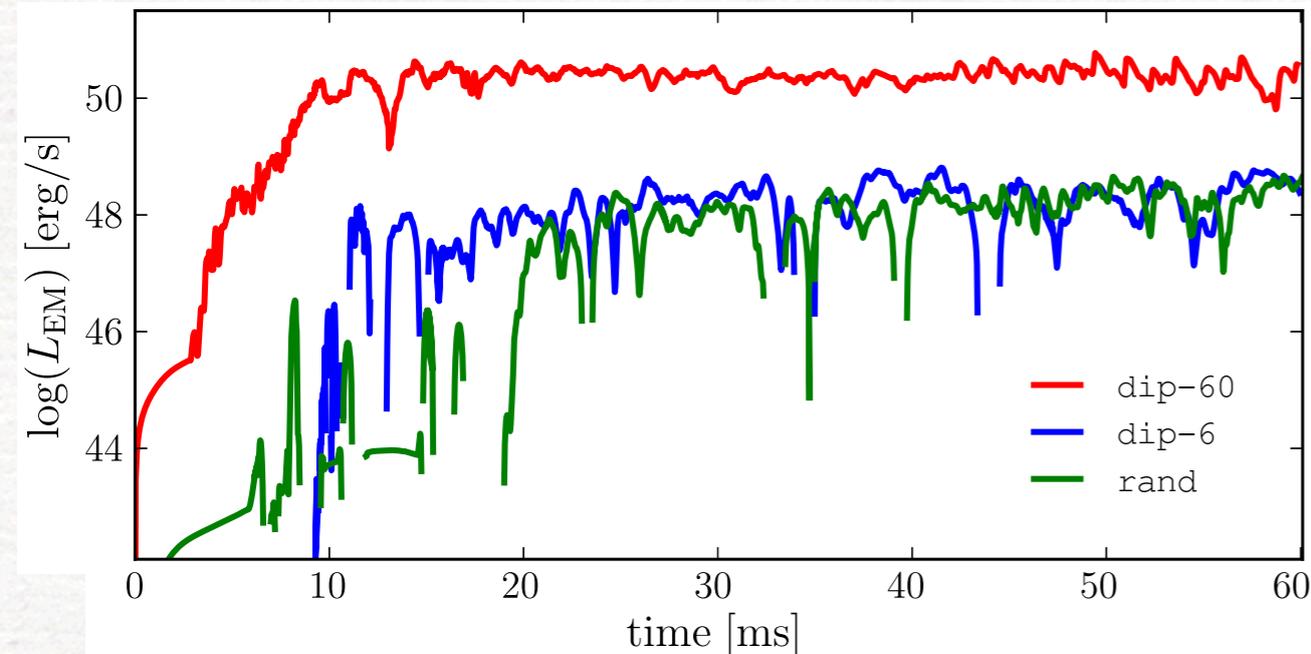


Dipolar B-field inside the HMNS leads to collimated flow and isotropic wind.



Random B-field inside the HMNS leads to absence of collimated flow and highly isotropic wind.

Electromagnetic luminosities

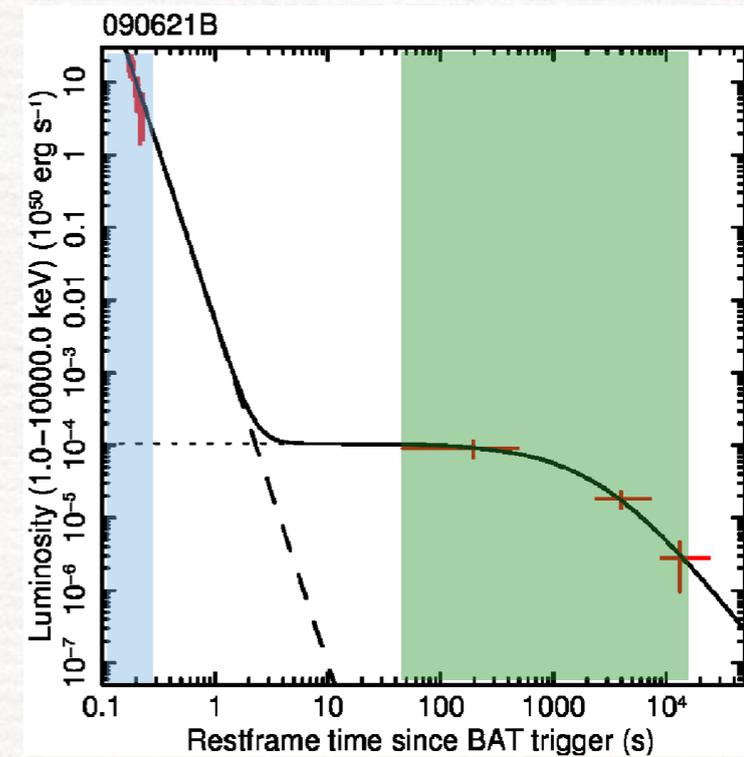
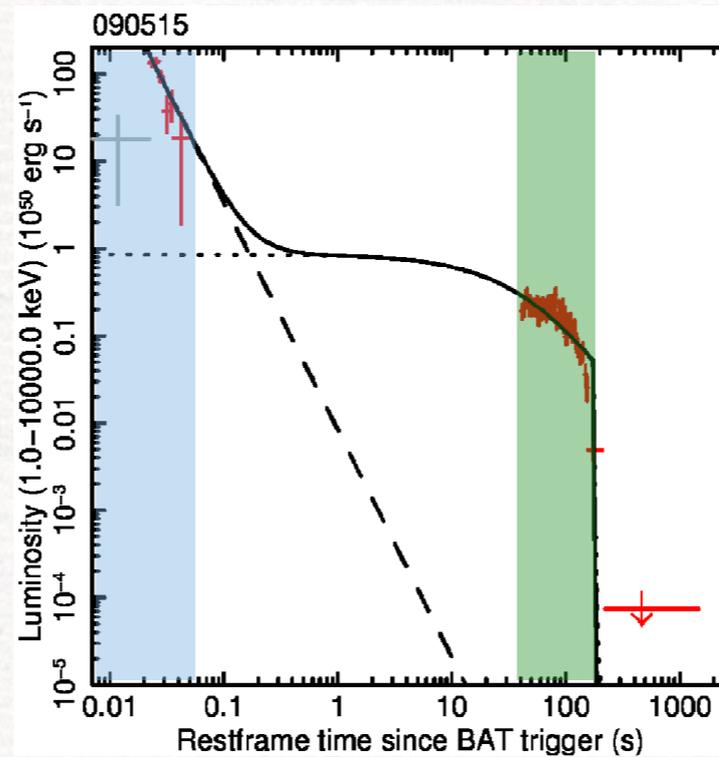
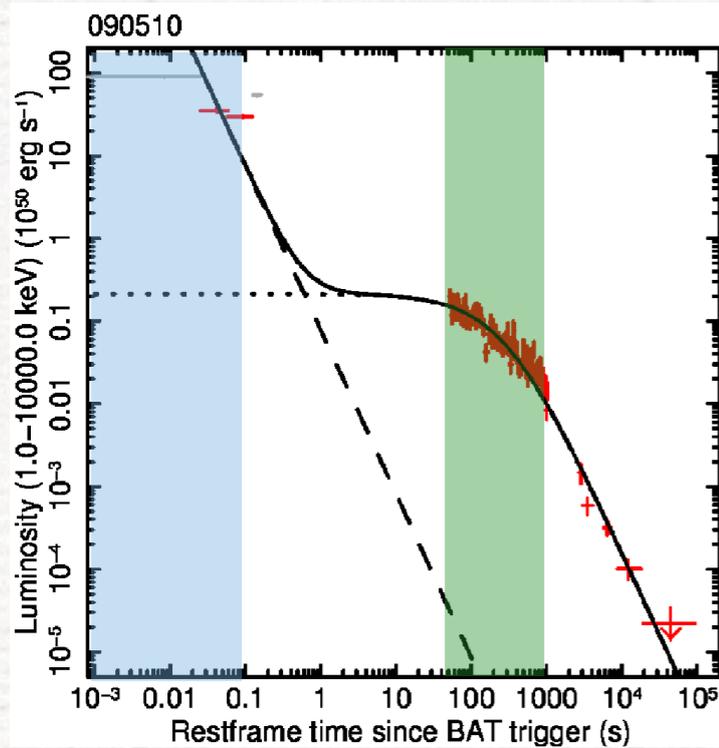


- luminosities compatible with observations for random B-field.
- the geometry does make a difference in terms of luminosity
- poloidal B-field at 60 km yields luminosity ~ 100 times larger.

- other topologies yield comparable luminosities.
- what matters is the energy in the system;
- when rescaled, B-field at 60 km yields same luminosity;
- simple scaling formula

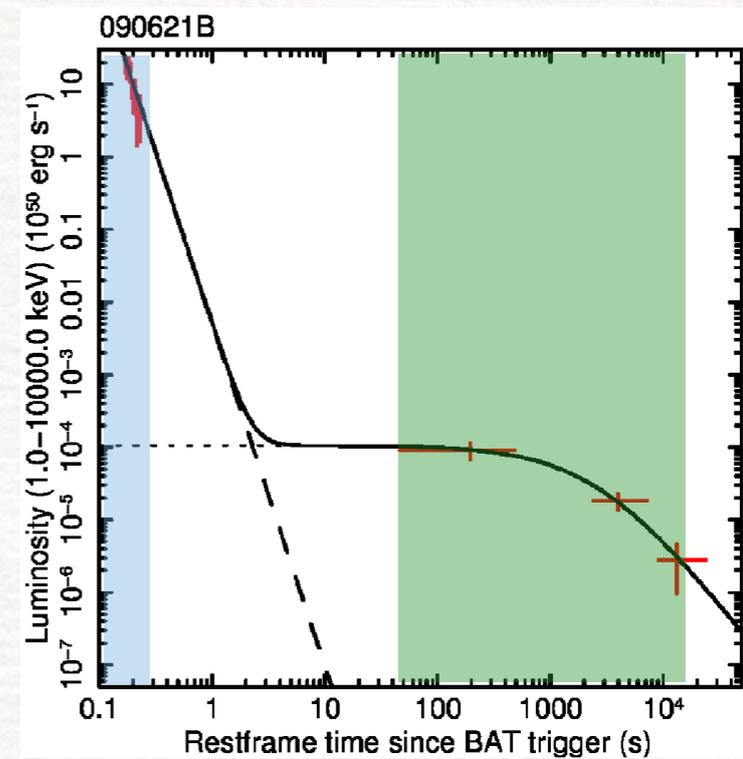
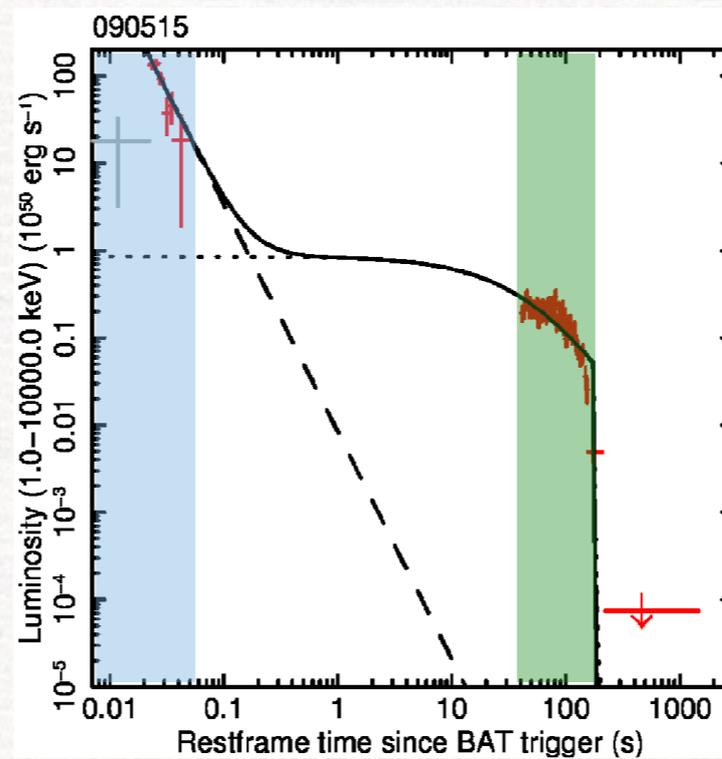
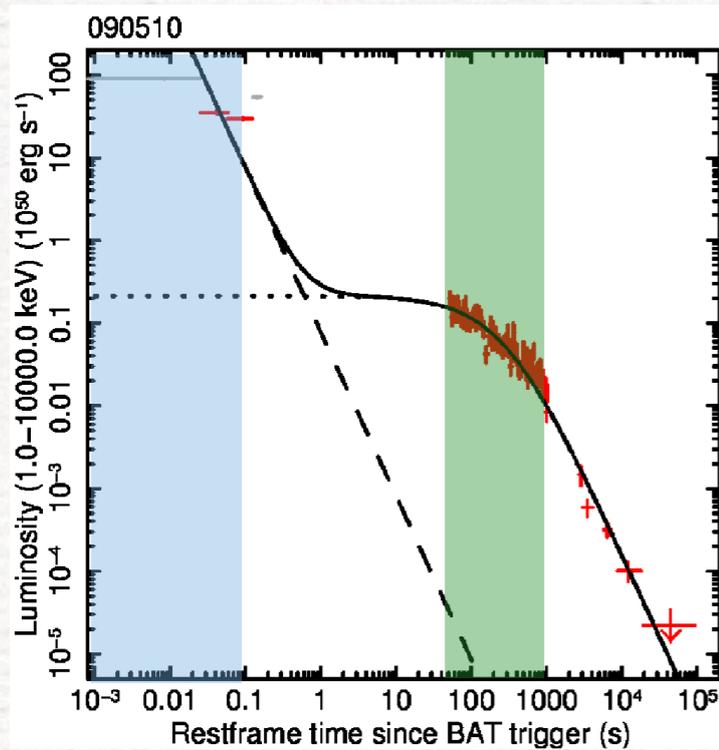
$$L_{\text{EM}} \simeq 10^{48} \chi \left(\frac{B_0}{10^{14} \text{ G}} \right)^2 \left(\frac{R_e}{10^6 \text{ cm}} \right)^3 \left(\frac{P}{10^{-4} \text{ s}} \right)^{-1} \text{ erg s}^{-1},$$

The puzzle of X-ray extended emission



- X-ray afterglows have been observed by Swift lasting as long as 10^2 - 10^4 s (Rowlinson+ 13; Gompertz+ 13)
- The x-ray afterglow could be produced by “proto-magnetar wind” with $L_x \sim 10^{49}$ erg s $^{-1}$ (Zhang & Mezsaros 01, Metzger+ 11, Zhang 13).
 - ★ is dipolar emission really taking place?
 - ★ what is the geometry of the wind?
 - ★ how large is the luminosity?
 - ★ do results depend on field topology?
- **NO**
- **essentially spherical**
- **$\sim 10^{48}$ erg/s**
- **very sensitively**

The puzzle of X-ray extended emission



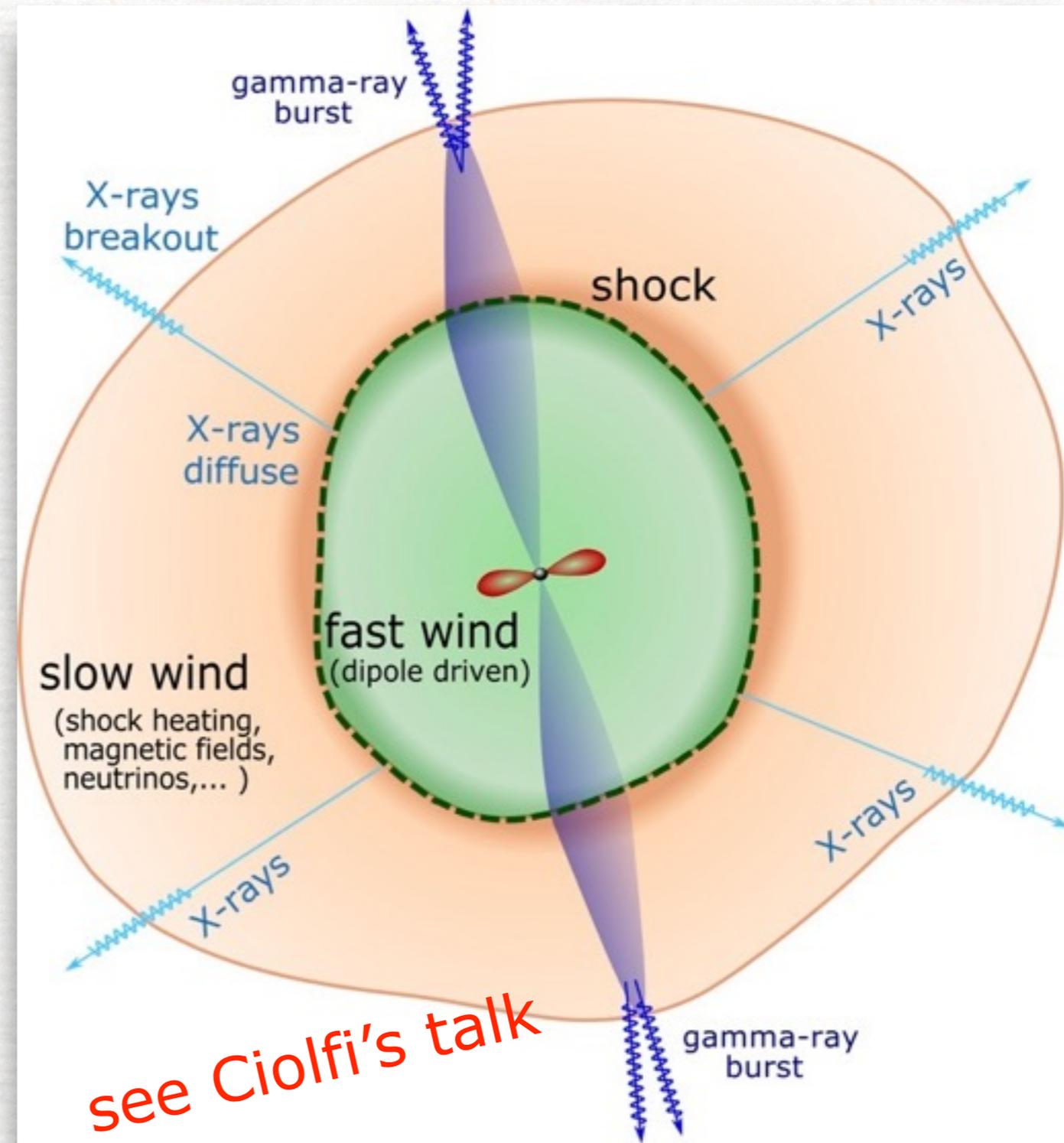
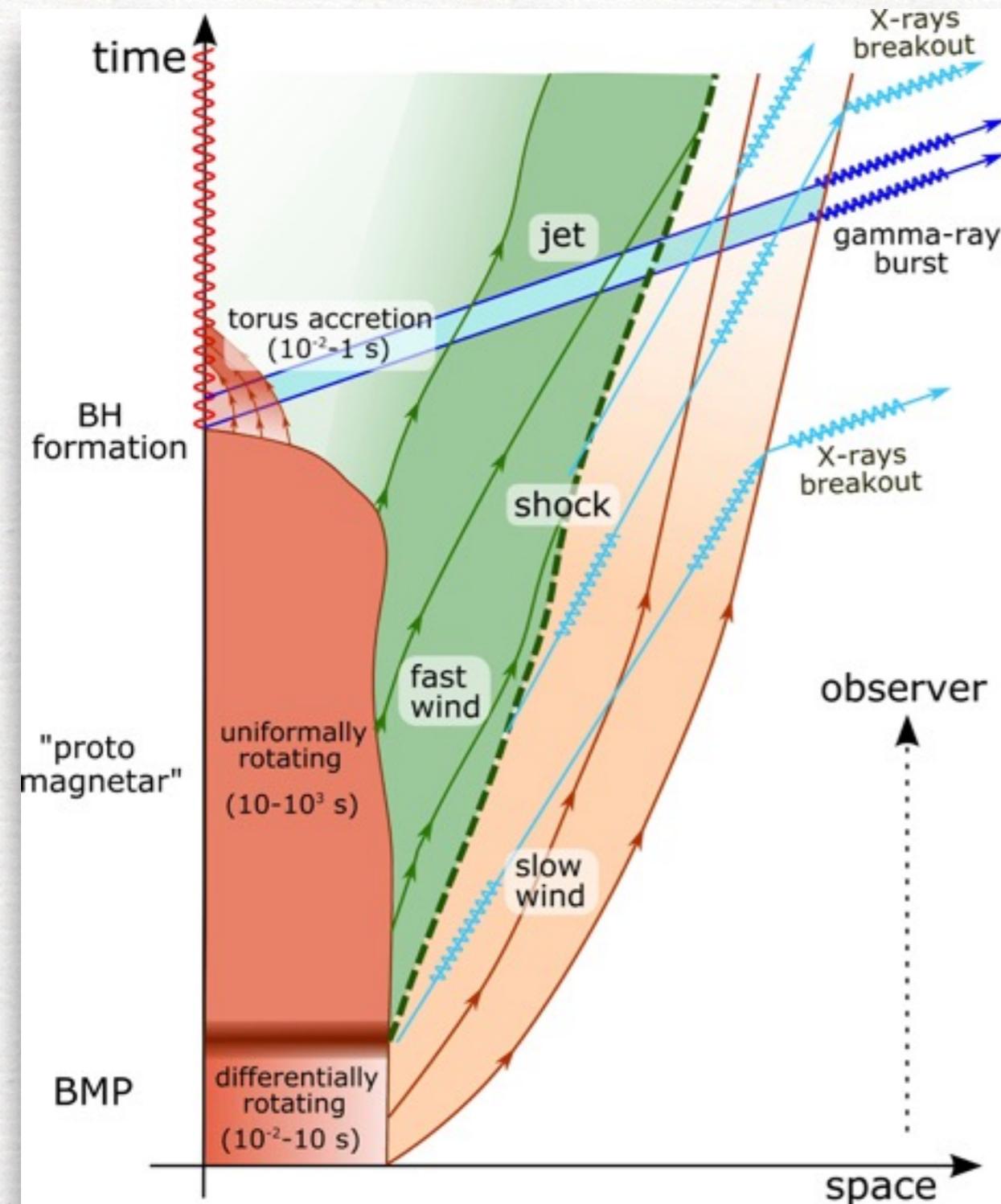
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Even so, plateaus remain a **riddle**:

- differential rotation lost over 10s: what can operate for >1000 s?
- if gamma rays produced by jet, and X-rays by HMNS, how can X-rays be an afterglow? (BH formed after HMNS!)

A novel paradigm for GRBs?

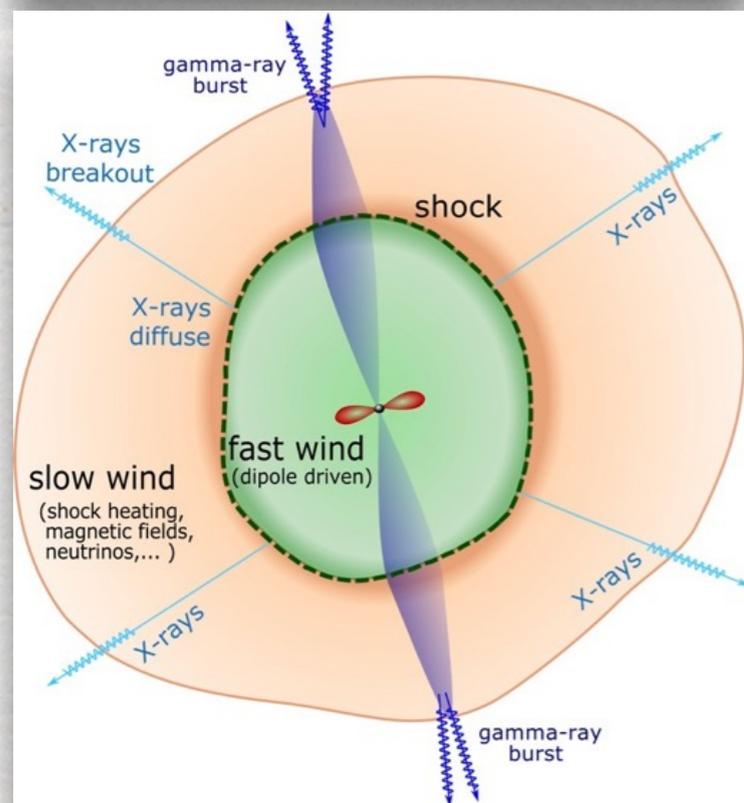
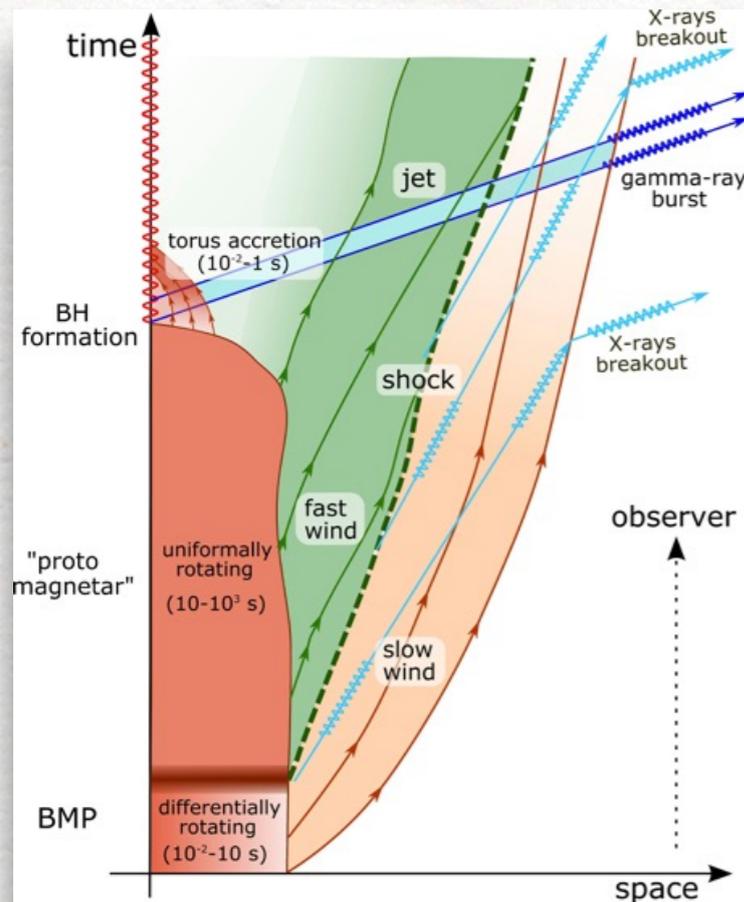
LR, Kumar (2014) (also Ciolfi, Siegel 2014)



see Ciolfi's talk

A novel paradigm for GRBs?

LR, Kumar (2014)



- *solves the timescale riddle*: X-ray luminosity is produced by BMP and can last up to 10^4 s
- *solves the timing riddle*: X-ray emission is produced before gamma emission but propagates more slowly.
- *consistent with simulations*: slow wind is produced by a number of effects.
- *proposes unifying view with long GRBs*: here too a jet has to propagate in confining medium
- *predictions*: X-ray emission possible before gamma; IC of thermal photons at break out.
- GW signal peak *earlier* than thought before.
- *potential problem*: need to produce a disk at collapse and could be difficult (Margalit+15).

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Giacomazzo, LR, Baiotti (2009)

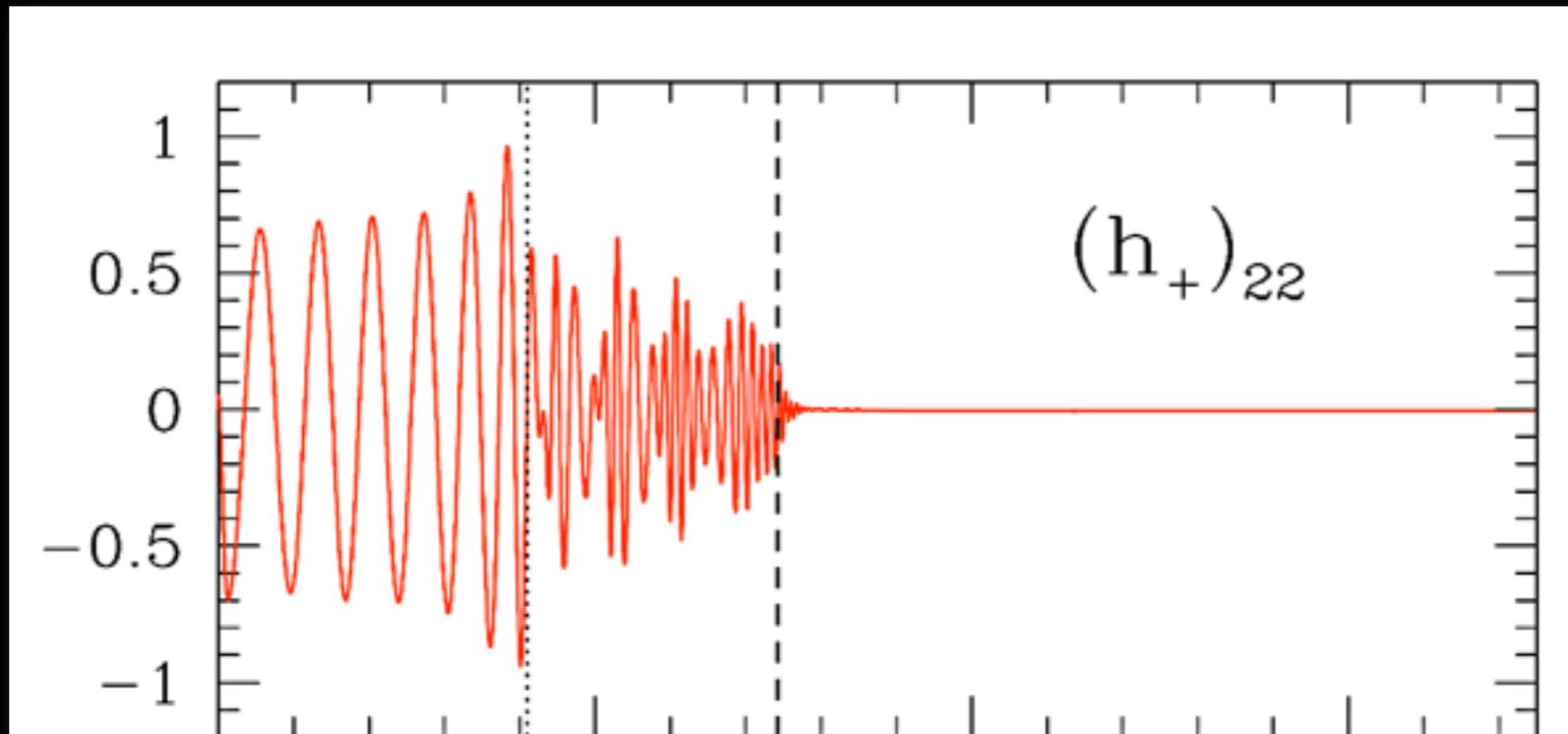
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Giacomazzo, LR, Baiotti (2010), Siegel, LR+ (2013)

- can B-fields grow after BH formation?

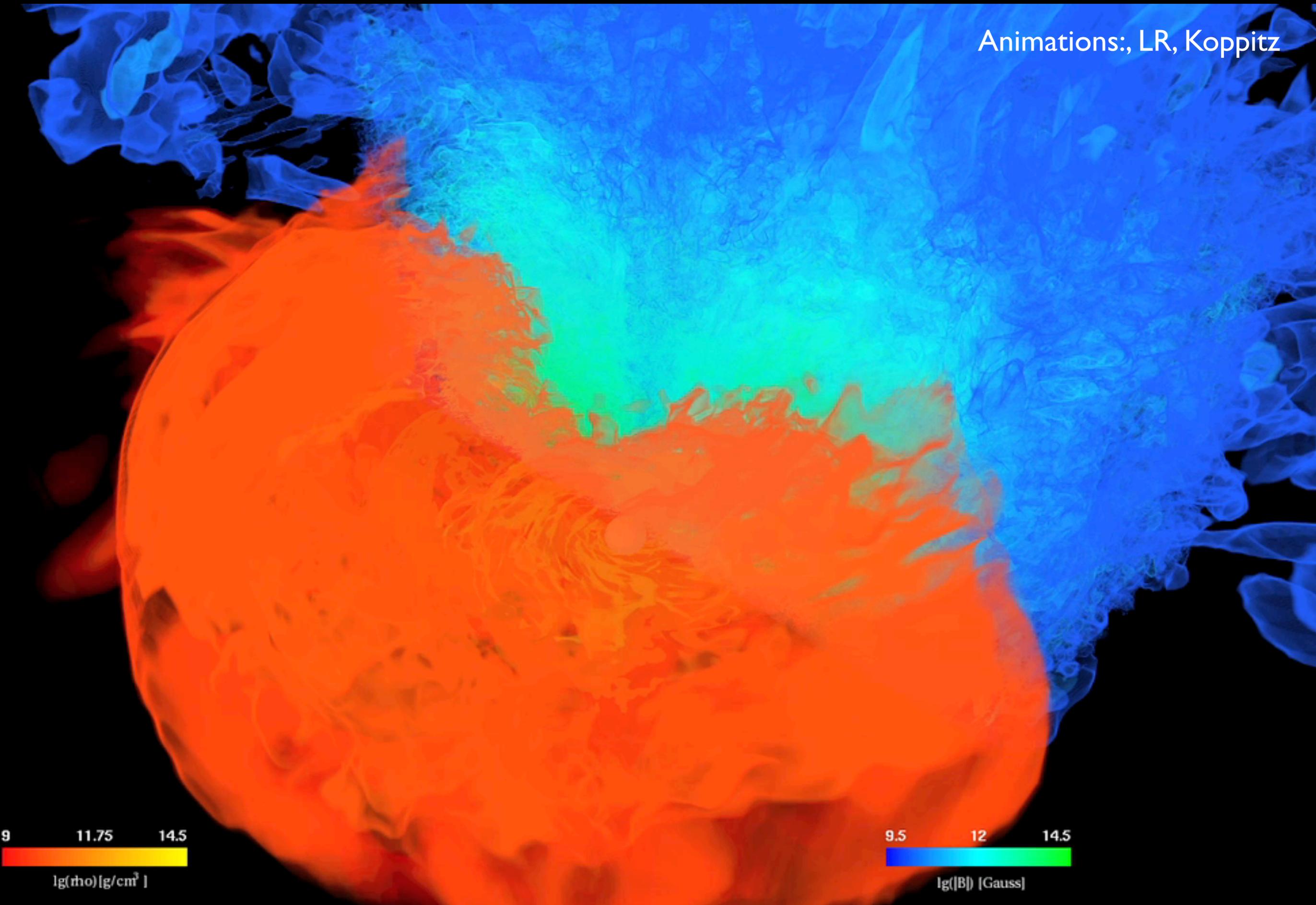
B-fields after BH formation



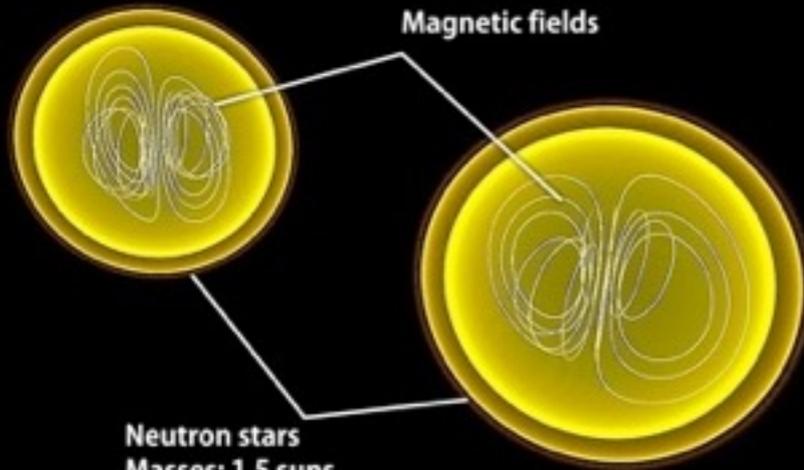
From a GW point of view, the binary becomes silent after BH formation and ringdown.

Is this really the end of the story?

Animations:, LR, Koppitz

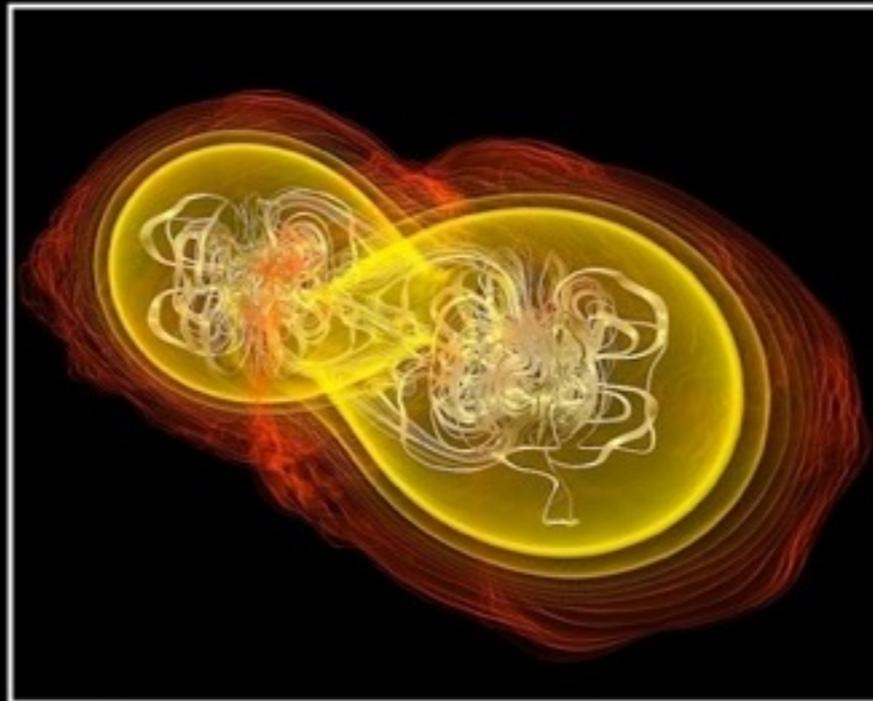


Crashing neutron stars can make gamma-ray burst jets

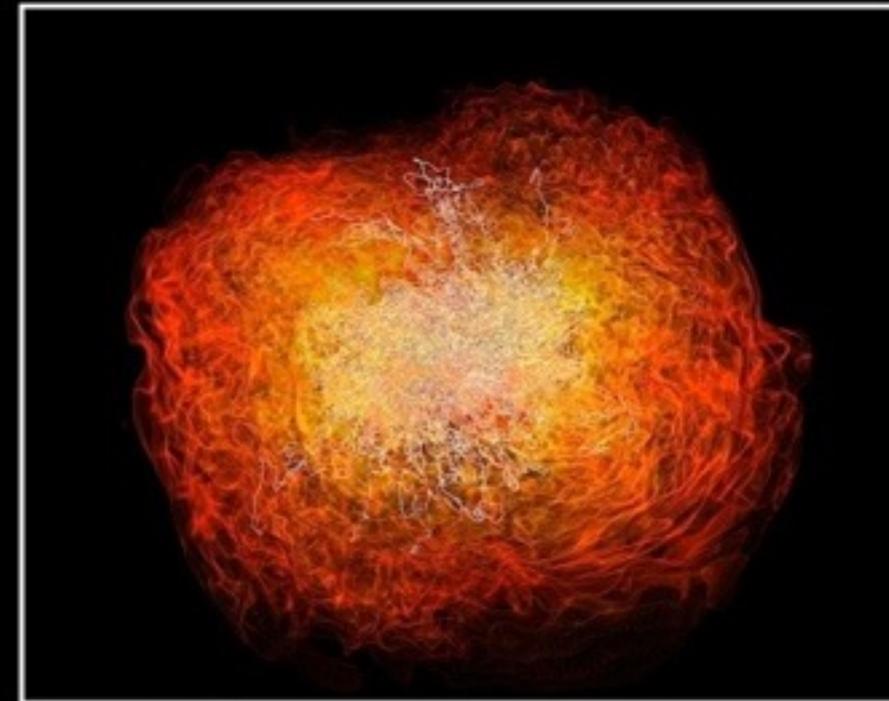


Neutron stars
Masses: 1.5 suns
Diameters: 17 miles (27 km)
Separation: 11 miles (18 km)

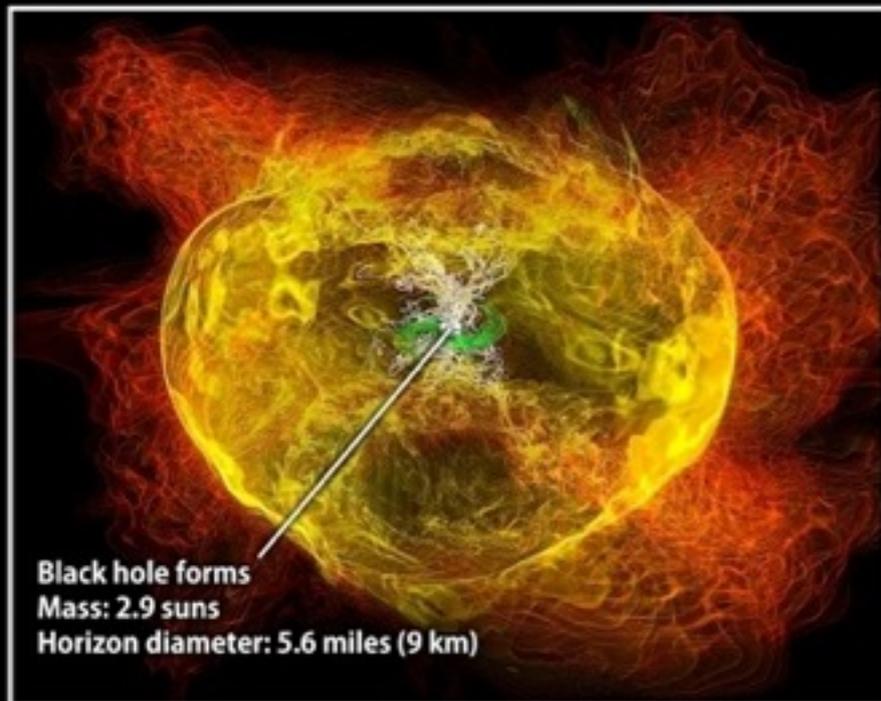
Simulation begins



7.4 milliseconds



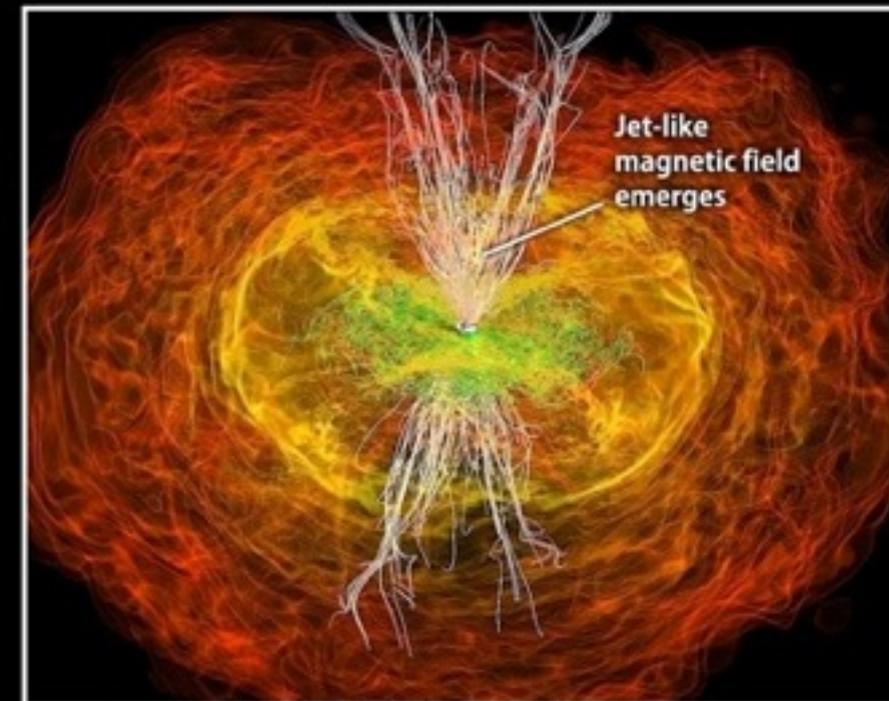
13.8 milliseconds



15.3 milliseconds



21.2 milliseconds



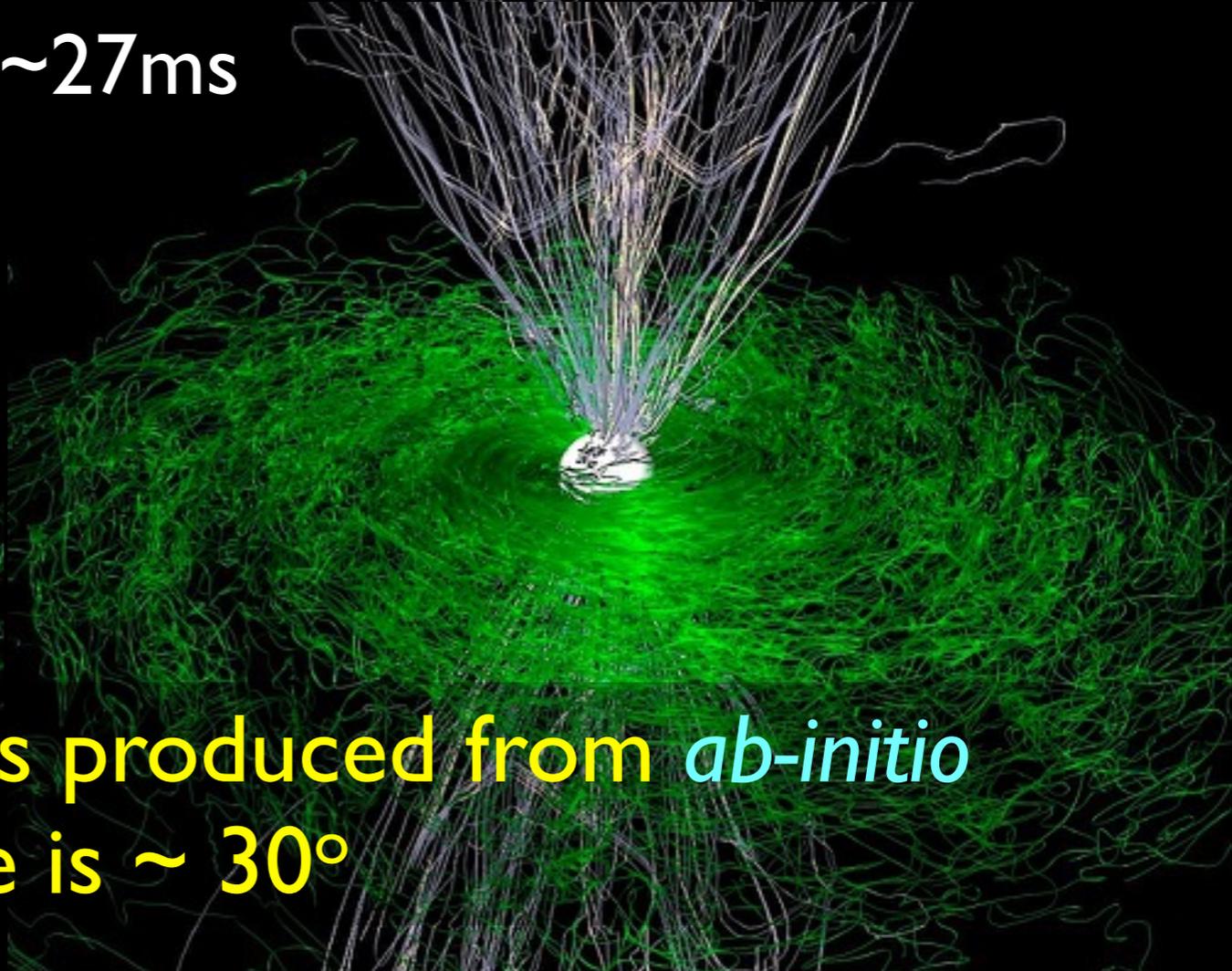
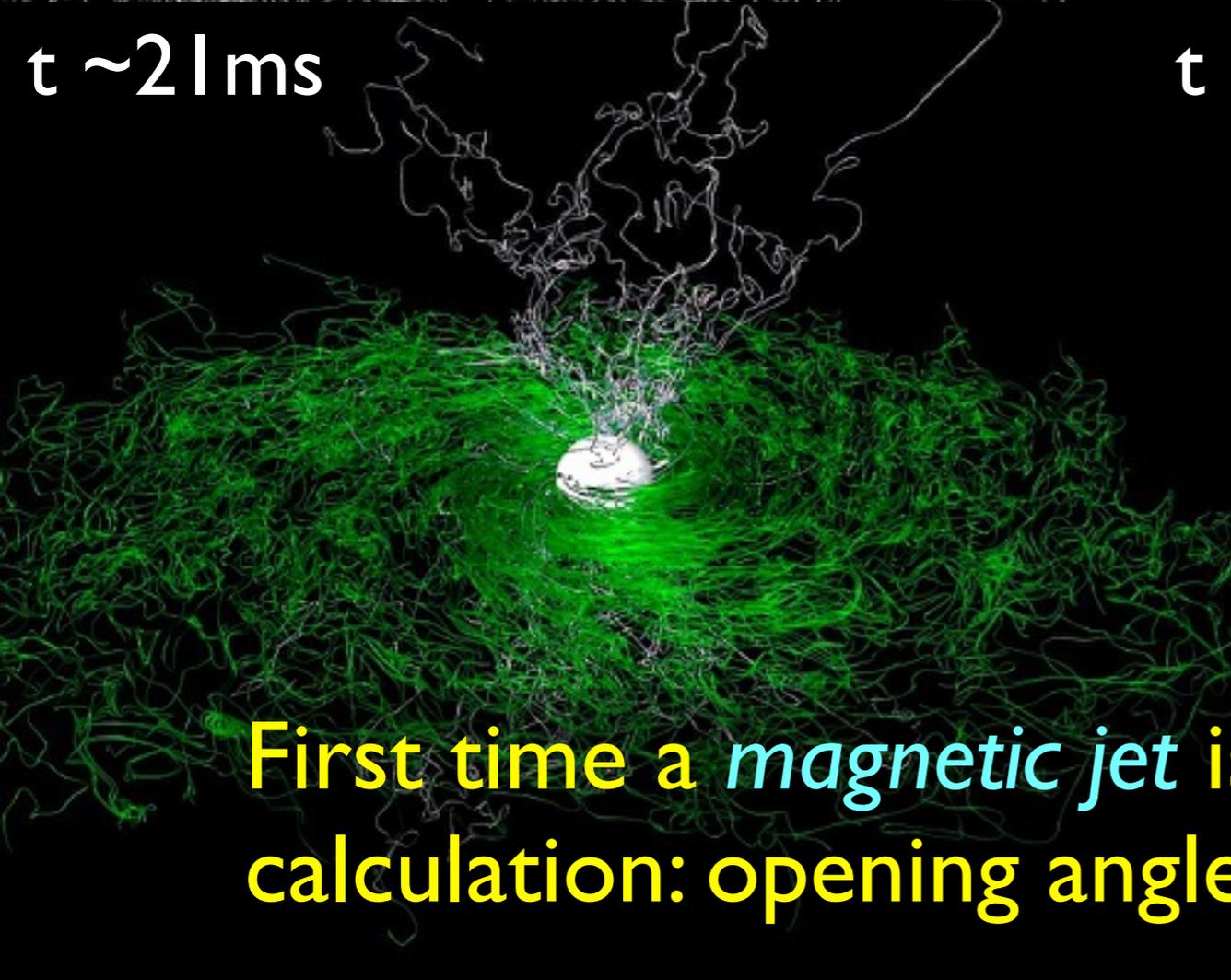
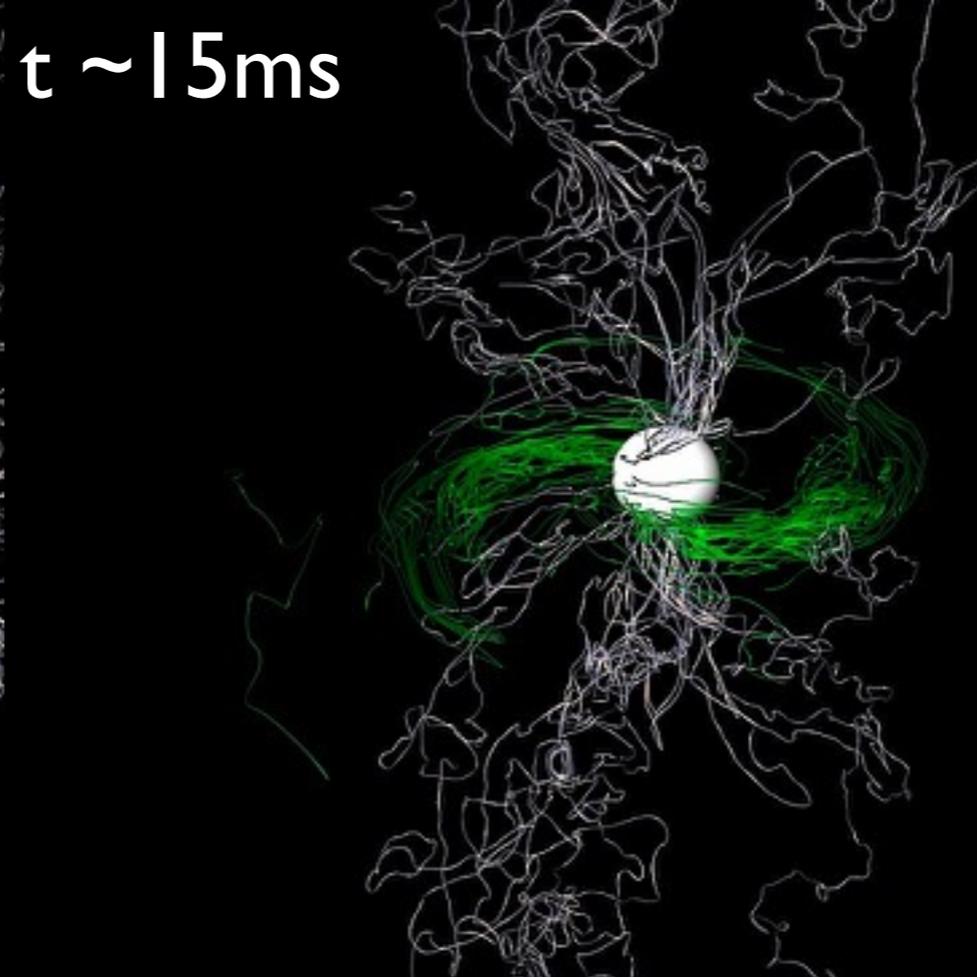
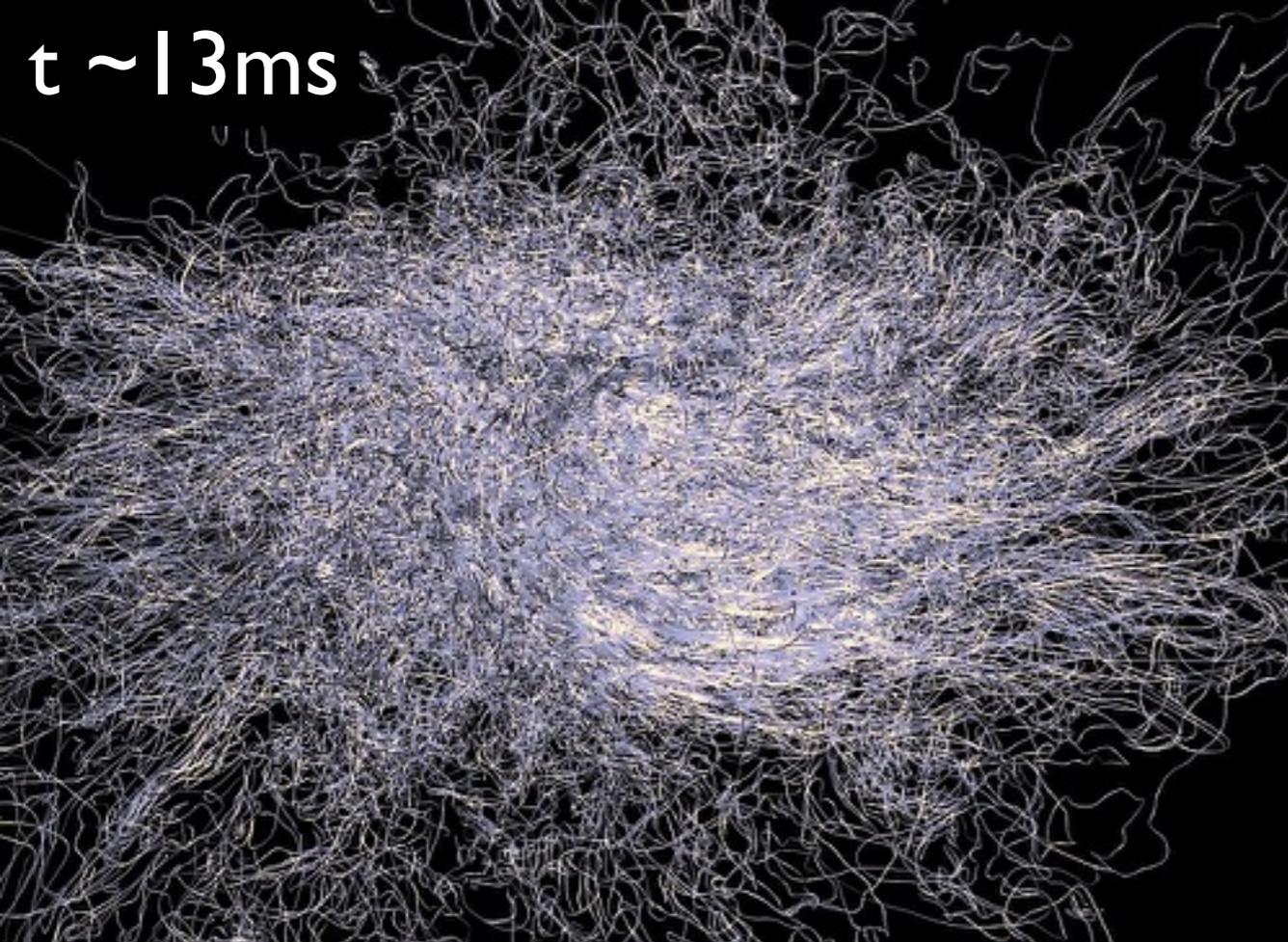
26.5 milliseconds

Credit: NASA/AEI/ZIB/M. Koppitz and L. Rezzolla

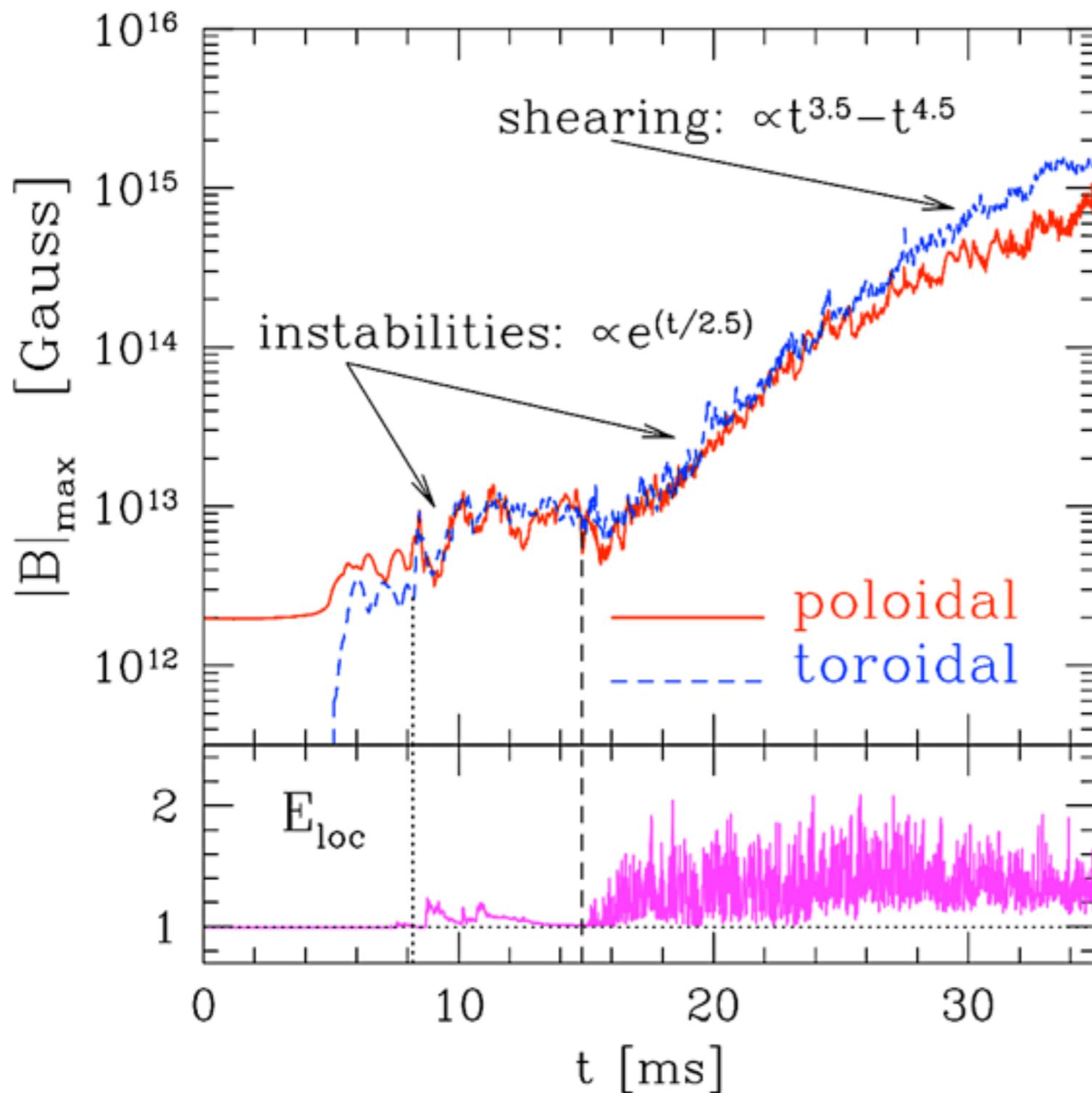
$$J/M^2 = 0.83$$

$$M_{\text{tor}} = 0.063 M_{\odot}$$

$$t_{\text{accr}} \simeq M_{\text{tor}} / \dot{M} \simeq 0.3 \text{ s}$$



First time a *magnetic jet* is produced from *ab-initio* calculation: opening angle is $\sim 30^\circ$



*B-field grows exponentially first because of the magnetorotational instability:

$$\tau_{\text{MRI}} = 2 \left(\frac{\partial \Omega}{\partial \varpi} \right)^{-1} \simeq 1 \Omega_3^{-1} \text{ ms}$$

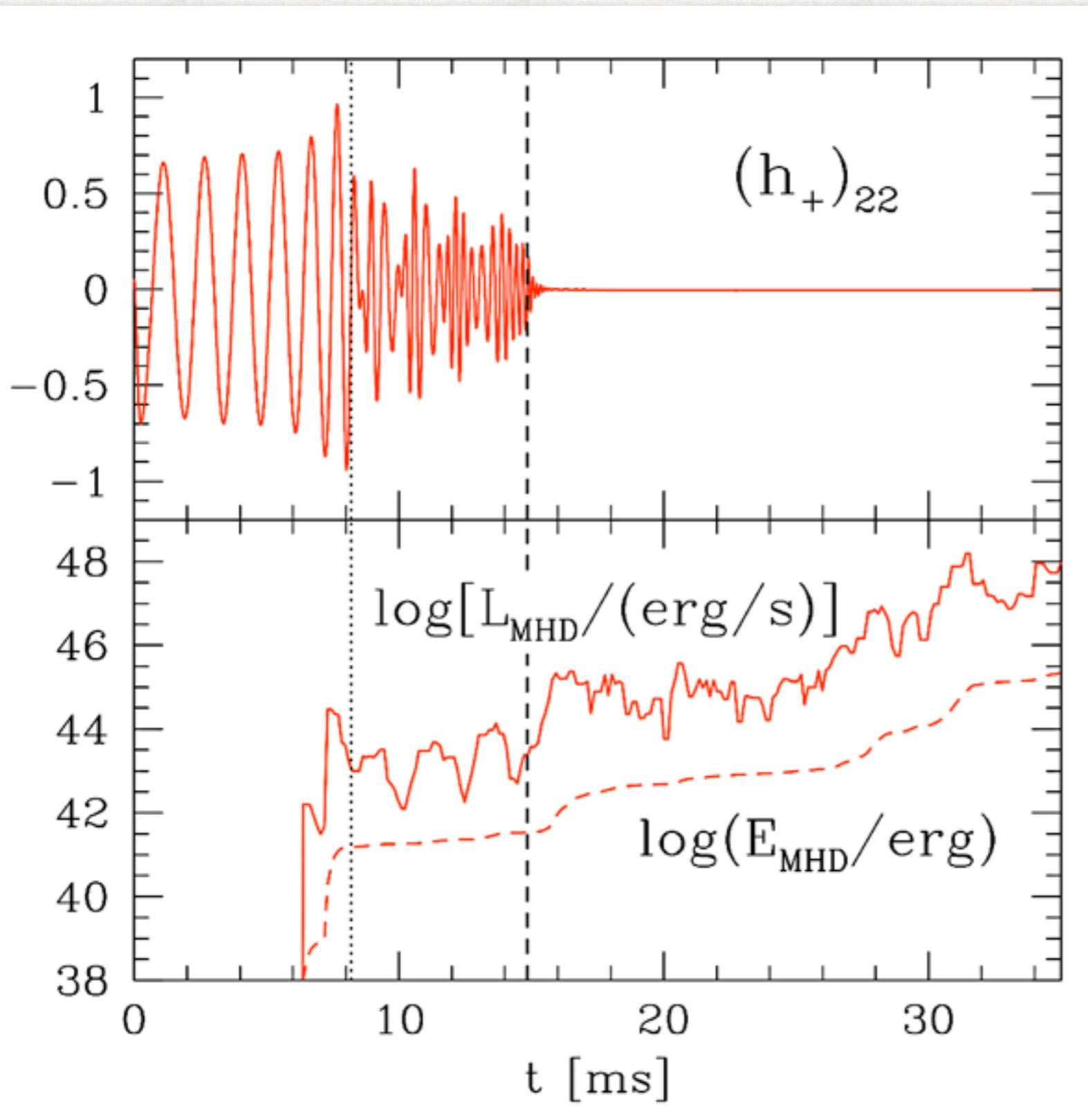
$$\lambda_{\text{max}} \simeq 2\pi v_A / \Omega \sim 10^4 \Omega_3^{-1} B_{15} \text{ cm}$$

*Later on the growth is only a power law as the B-field reaches equipartition

*B-field is mostly toroidal in the torus and $\sim 10^{15}$ G. A poloidal component dominant along the BH spin axis.

*Note that material becomes unbound soon after the BH is formed indicating that an outflow can be produced; mildly relativistic: $\gamma \lesssim 4$

Multimessenger signal



*Note that the GW signal is essentially *shuts-off* after BH formation.

*After the merger the EM signal *starts* but is essentially constant during the HMNS phase

*After the BH formation, the EM signal starts to *grow exponentially*

*At the end of the simulation the system has released a total EM energy of $\sim 10^{46}$ erg and reached an EM luminosity of $\sim 10^{48}$ erg/s

*Despite the crudeness of the physics, the ball-park numbers match observations.

The GR-Resistive MHD formalism

Dionysopoulou, Alic, LR (2015)

Ideal MHD is a good approximation in the inspiral, but not a after the merger (high temp, low densities).

Main difference in resistive regime is the current

$$\begin{aligned}\partial_t(\sqrt{\gamma}B^i) + \partial_k(-\beta^k\sqrt{\gamma}B^i + \alpha\epsilon^{ikj}\sqrt{\gamma}E_j) &= -\sqrt{\gamma}B^k(\partial_k\beta^i) - \alpha\sqrt{\gamma}\gamma^{ij}\partial_j\phi, \\ \partial_t(\sqrt{\gamma}E^i) + \partial_k(-\beta^k\sqrt{\gamma}E^i - \alpha\epsilon^{ikj}\sqrt{\gamma}B_j) &= -\sqrt{\gamma}E^k(\partial_k\beta^i) - \alpha\sqrt{\gamma}\gamma^{ij}\partial_j\psi - \alpha\sqrt{\gamma}J^i,\end{aligned}$$

The current is dictated by poorly known microphysics.

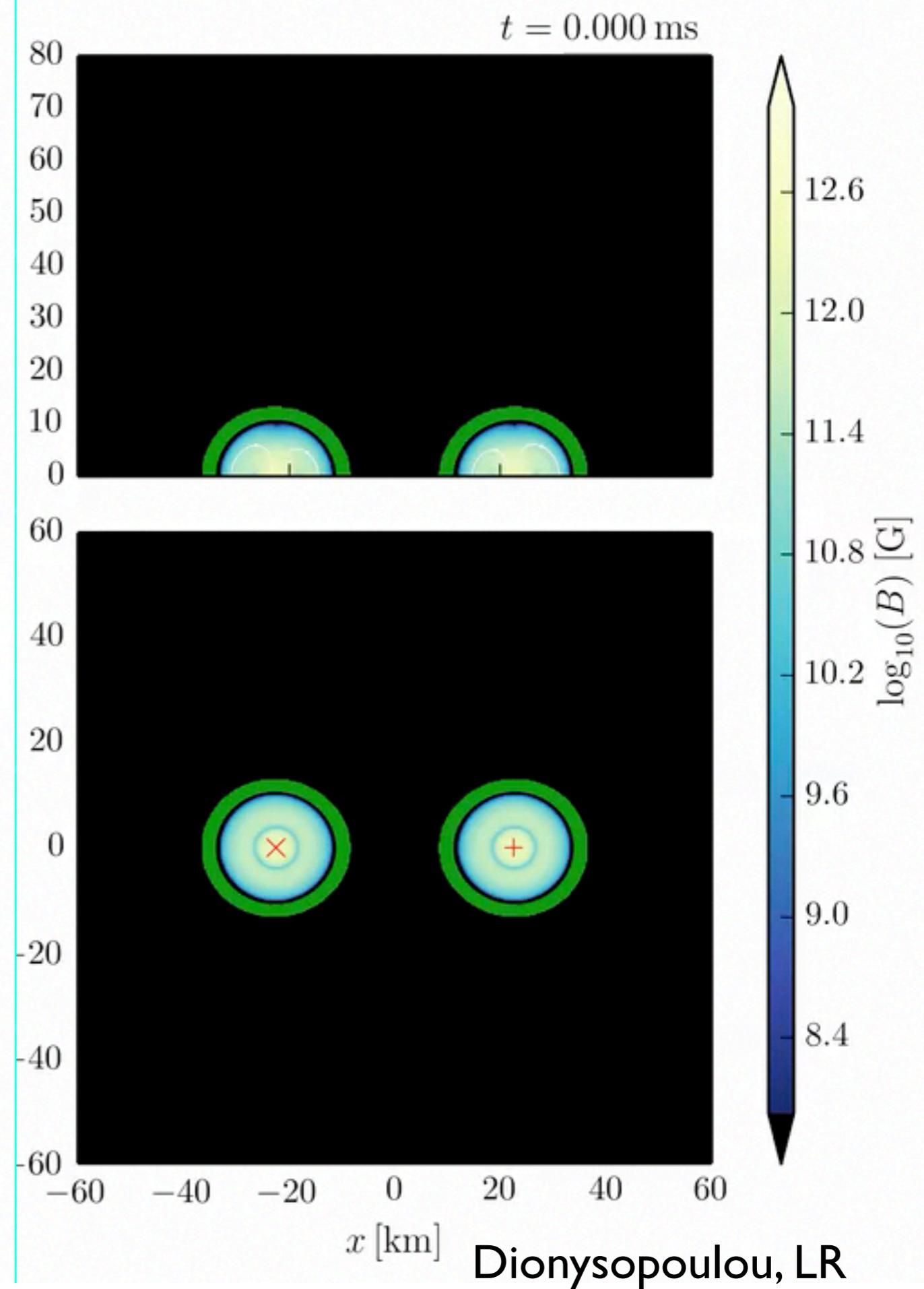
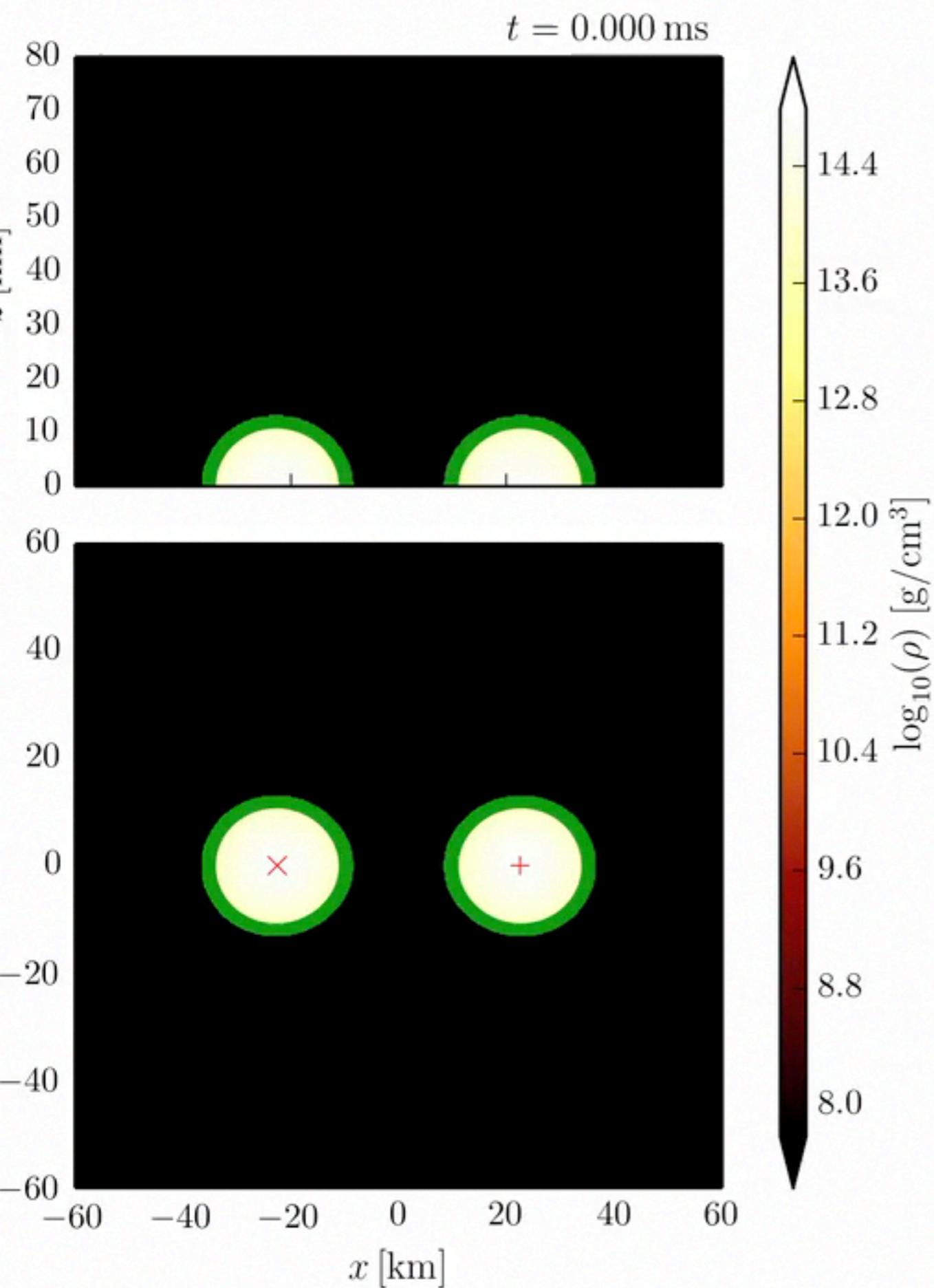
A simple prescription with scalar conductivity σ is

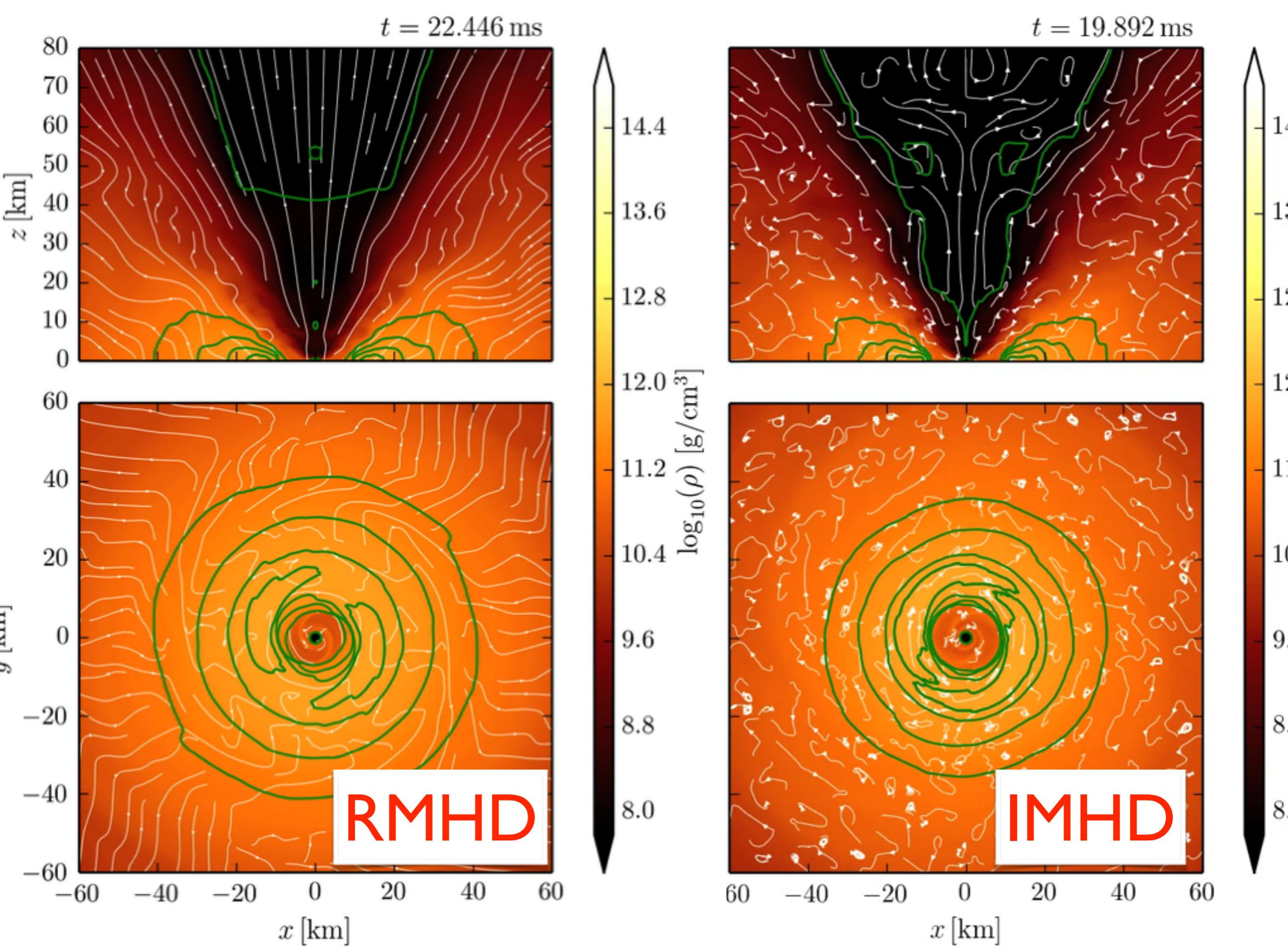
$$J^i = qv^i + W\sigma[E^i + \epsilon^{ijk}v_jB_k - (v_kE^k)v^i],$$

$\sigma \rightarrow \infty$ ideal-MHD (IMHD) regime

$\sigma \neq 0$ resistive-MHD (RMHD) regime

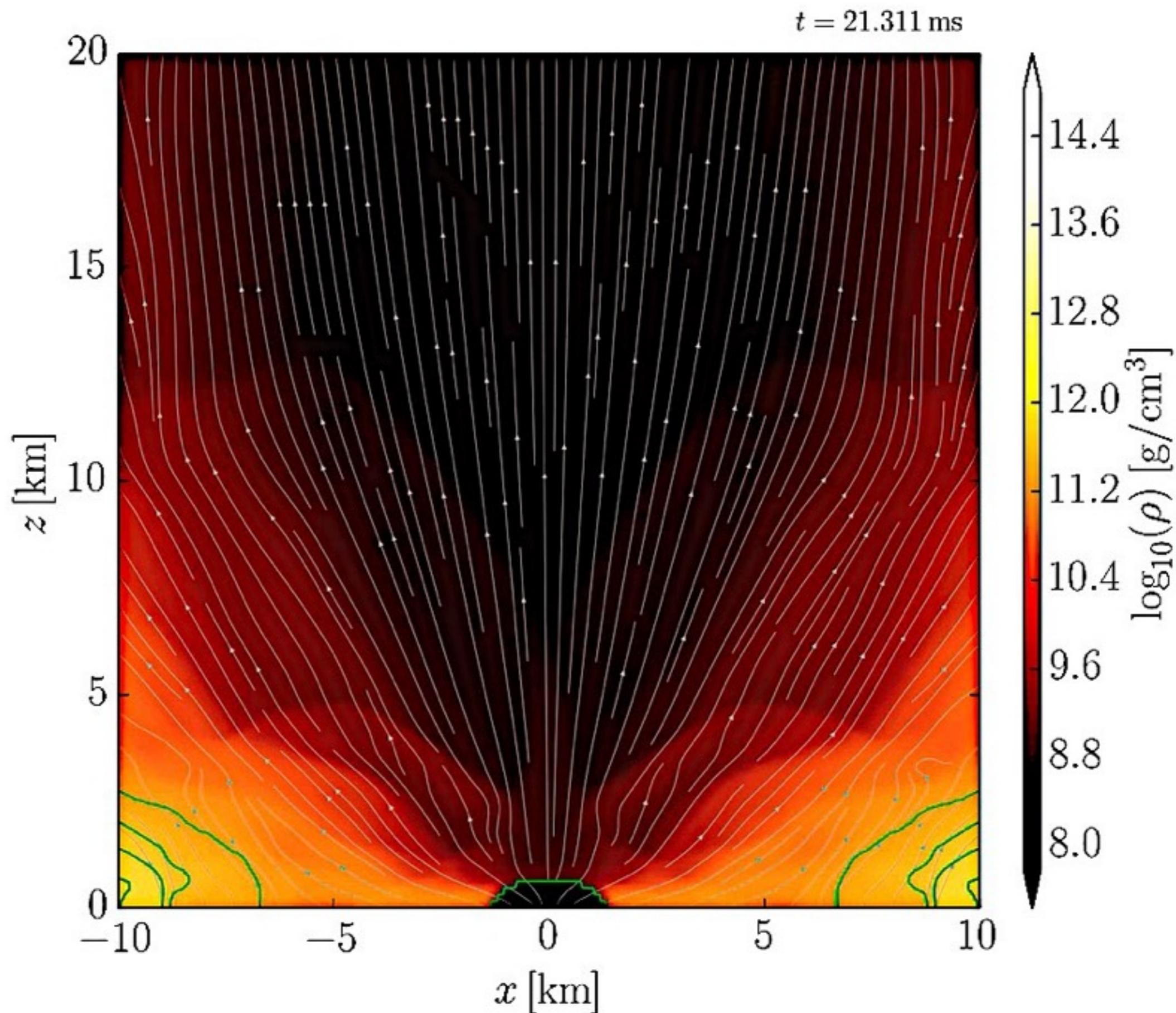
$\sigma \rightarrow 0$ electrovacuum



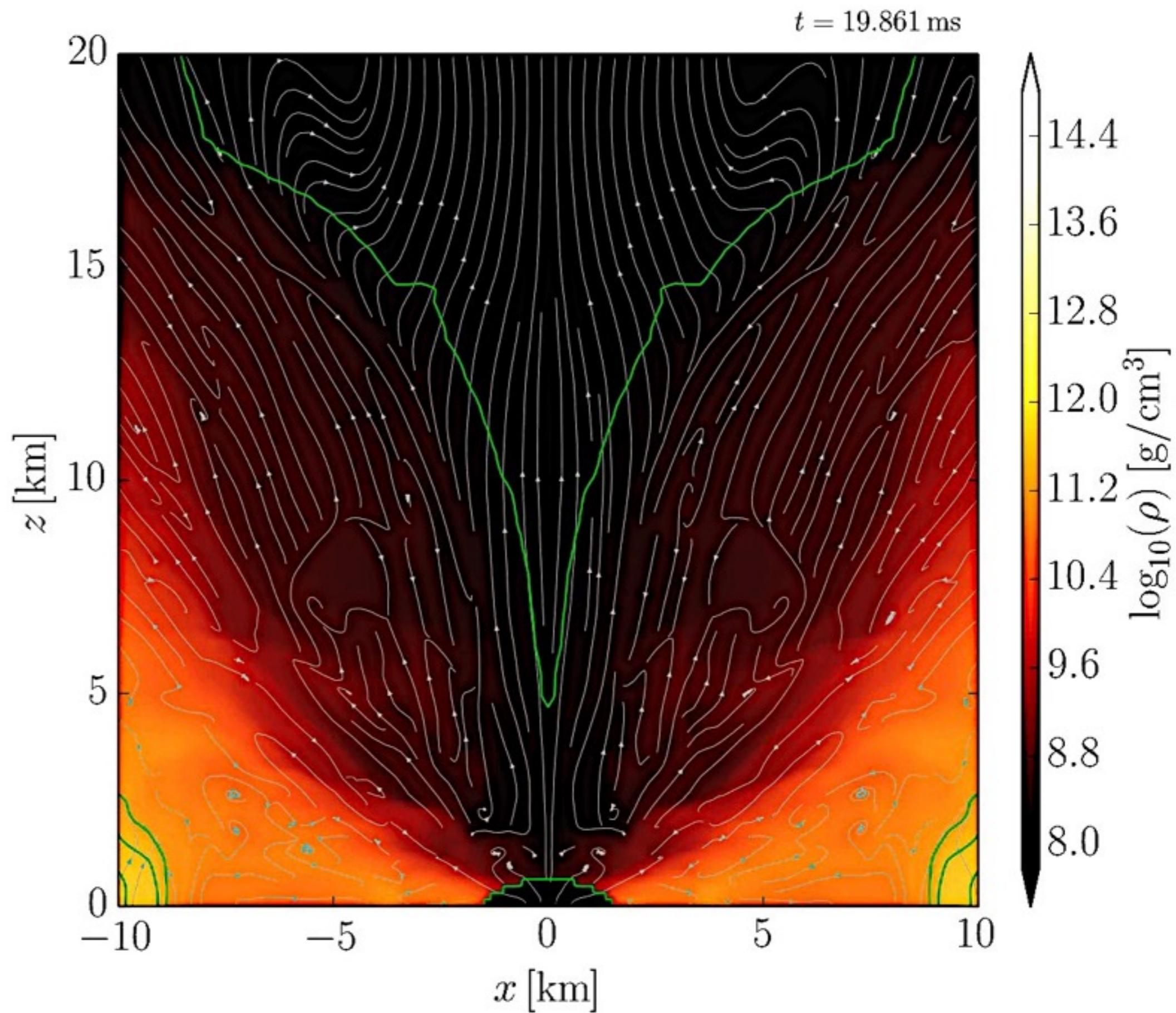


NOTE: the magnetic jet is not an outflow. It's a magnetic structure confining plasma.

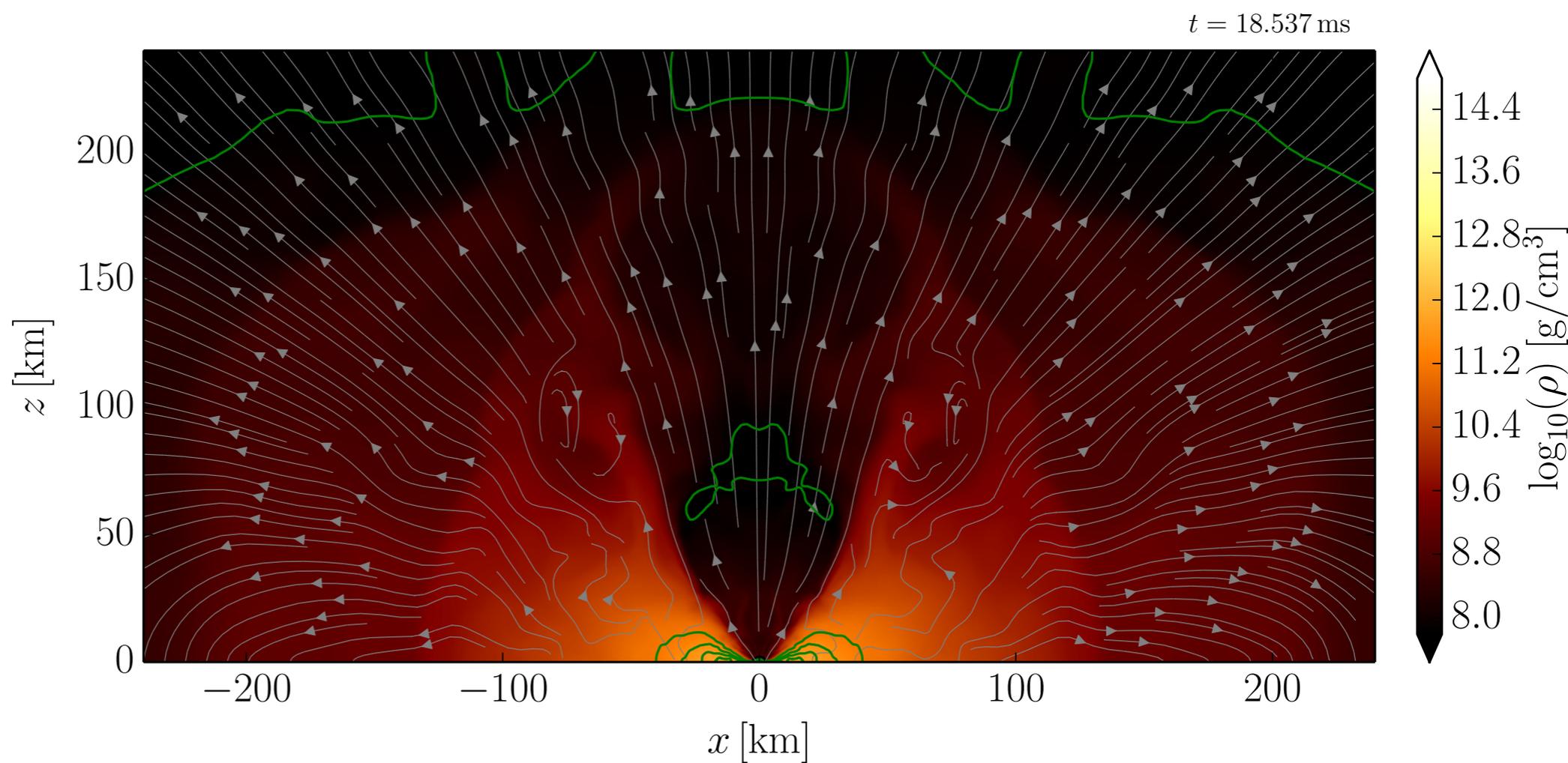
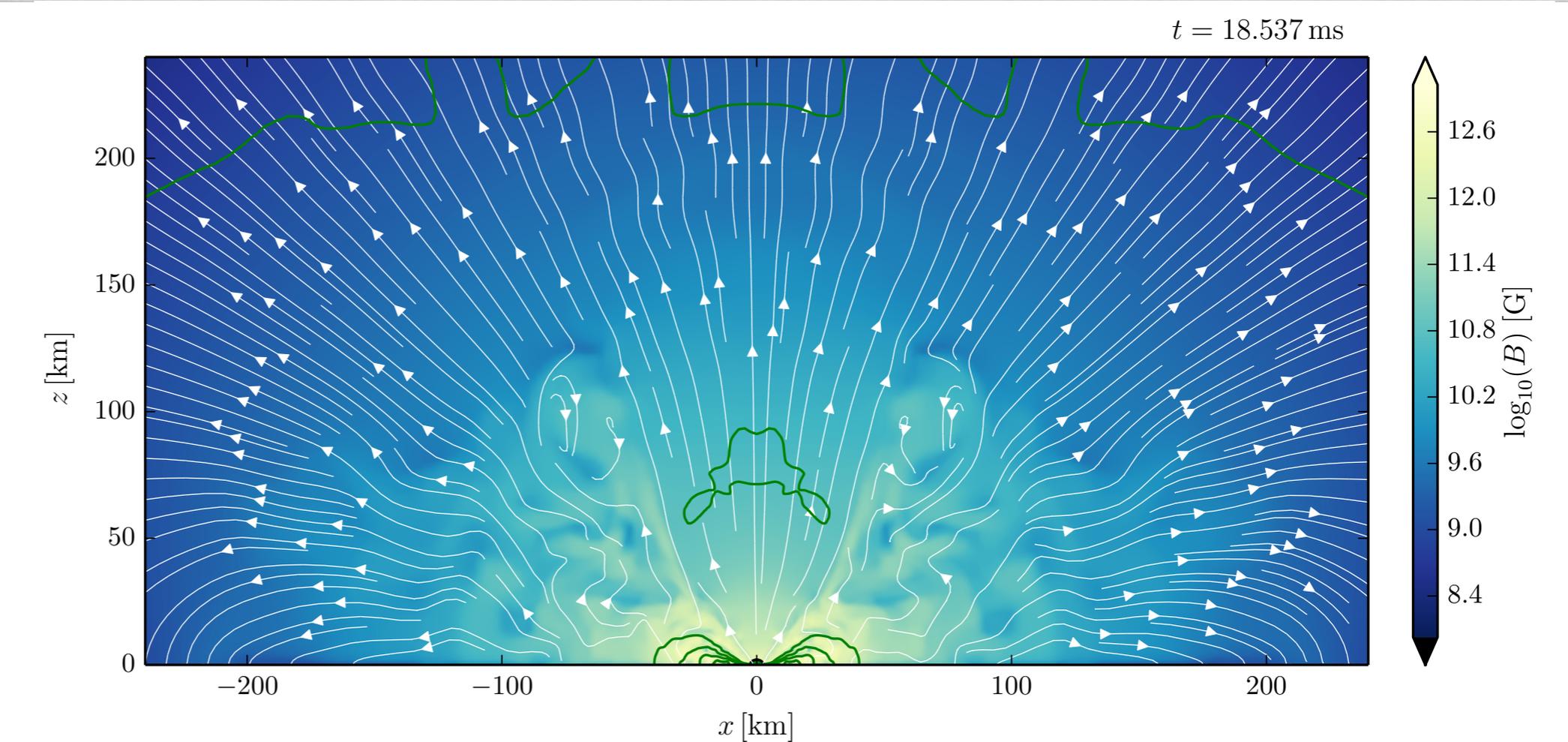
In RMHD the magnetic jet structure is present from the scale of the horizon (resolution only $\sim 150\text{m}$).



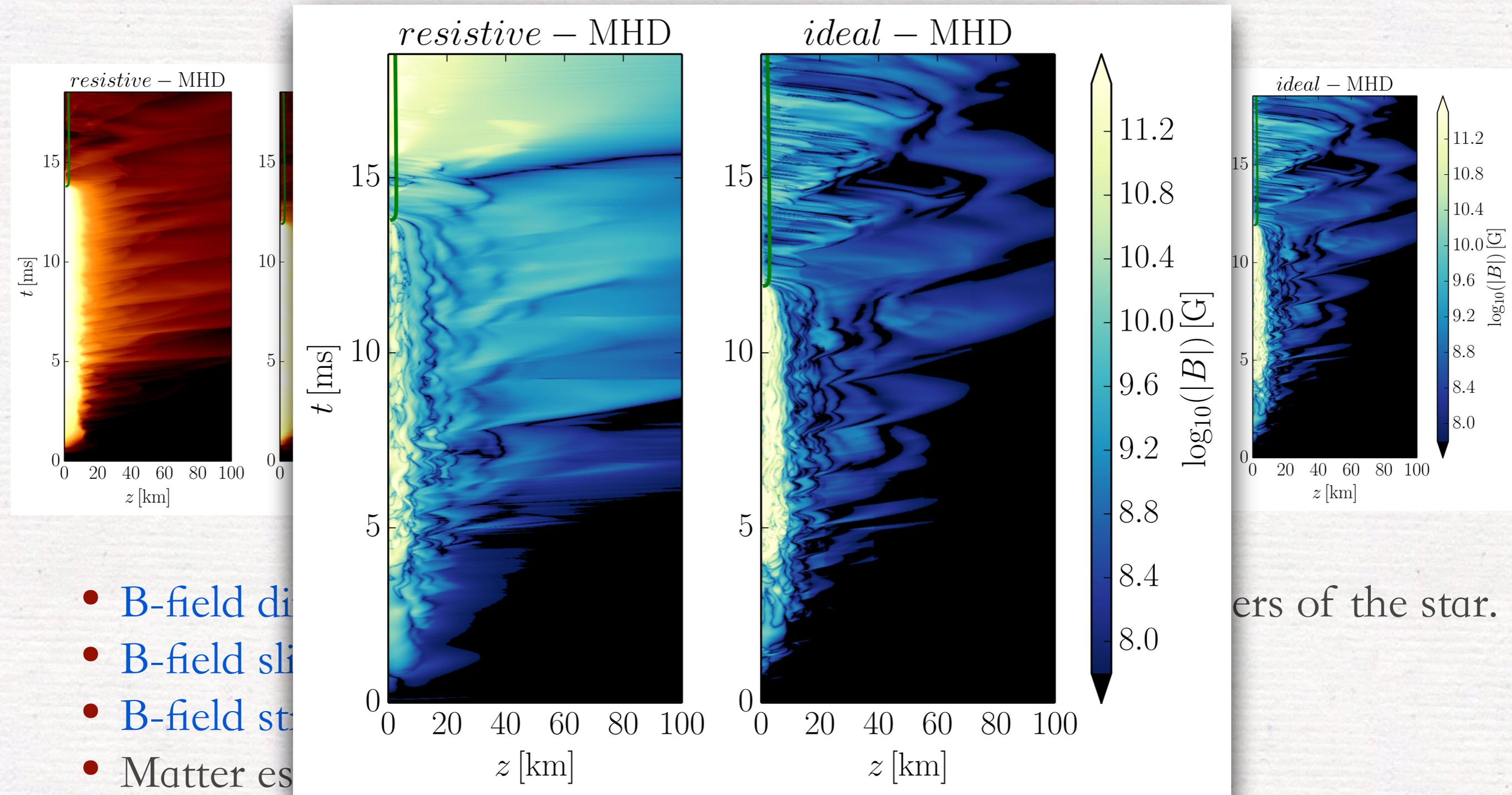
In **IMHD** the magnetic jet structure is present but less regular.



The magnetic jet structure maintains its coherence up to the largest scale of the system.



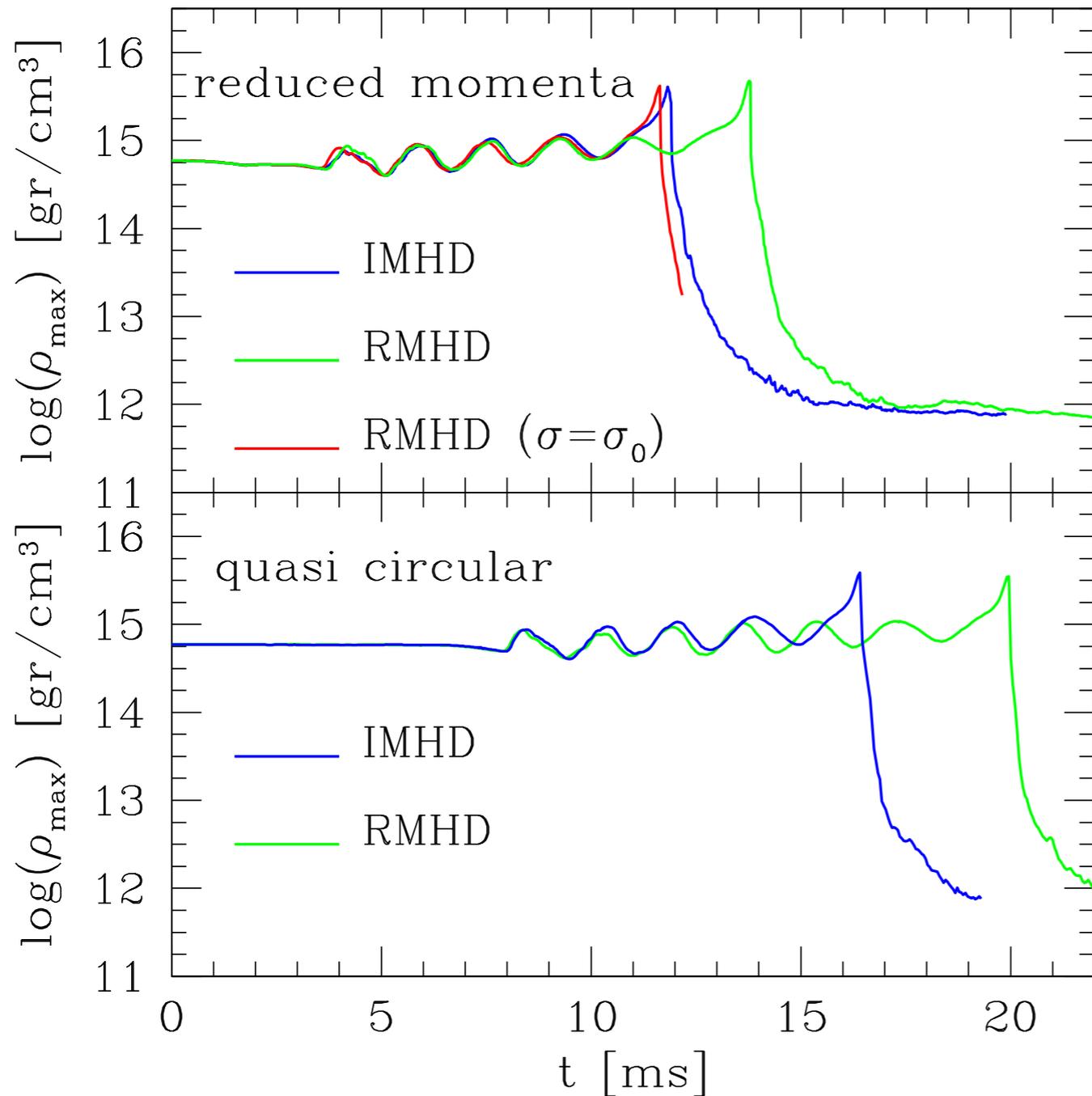
Magnetically driven winds and bursting activity



- B-field di
- B-field sli
- B-field st
- Matter es
- Wind deposits $0.009 M_{\odot}$ in the ambient medium.
- Wind has modulations in both ε and B (duration ~ 2 ms).

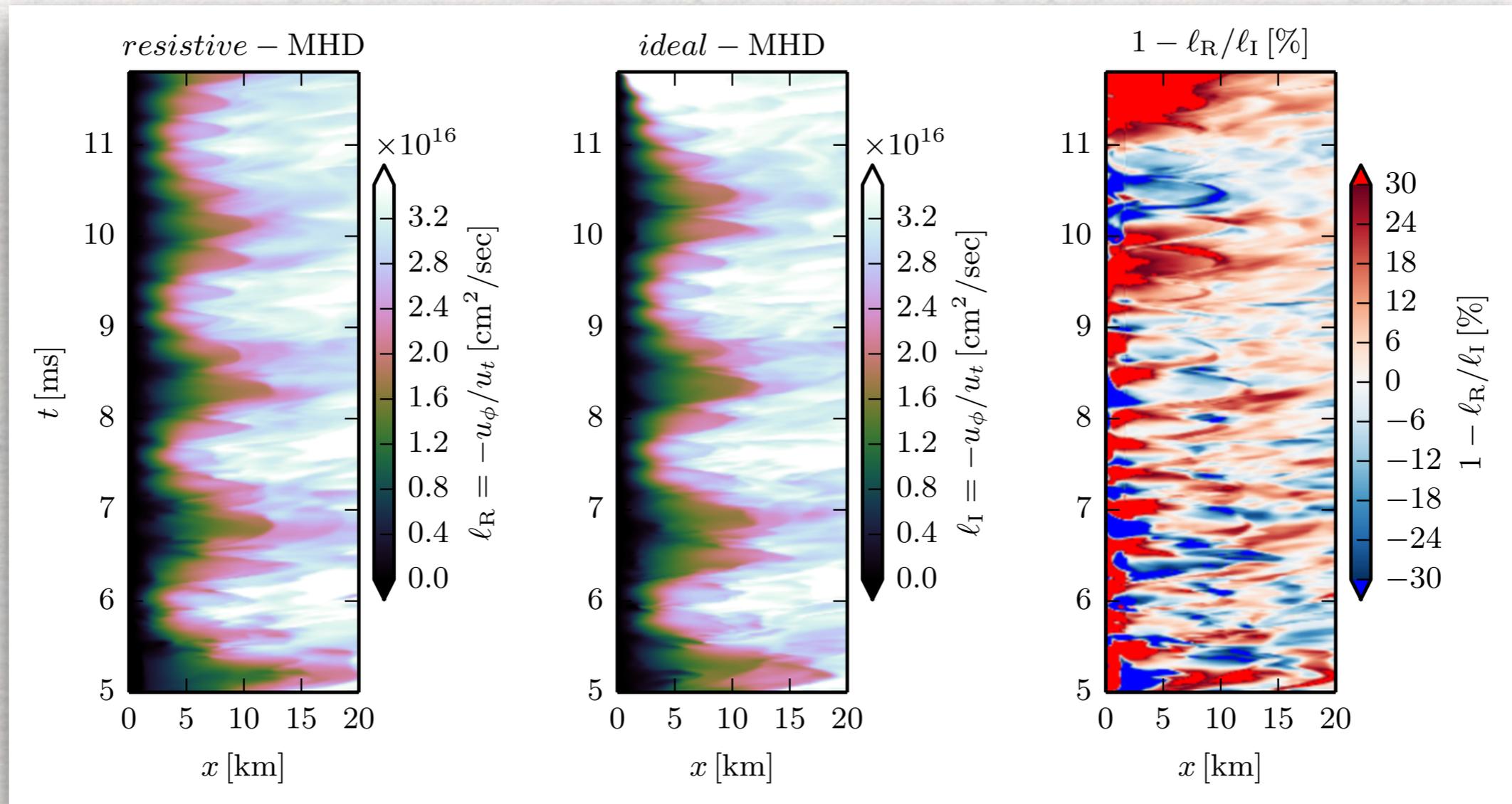
ers of the star.

HMNS: delayed collapse to BH



- Collapse takes place earlier in IMHD.
- Not entirely surprising: in RMHD the B-field is not fully advected and a certain slippage takes place between plasma and B-field.
- Angular momentum transport is less efficient and this increases lifetime.

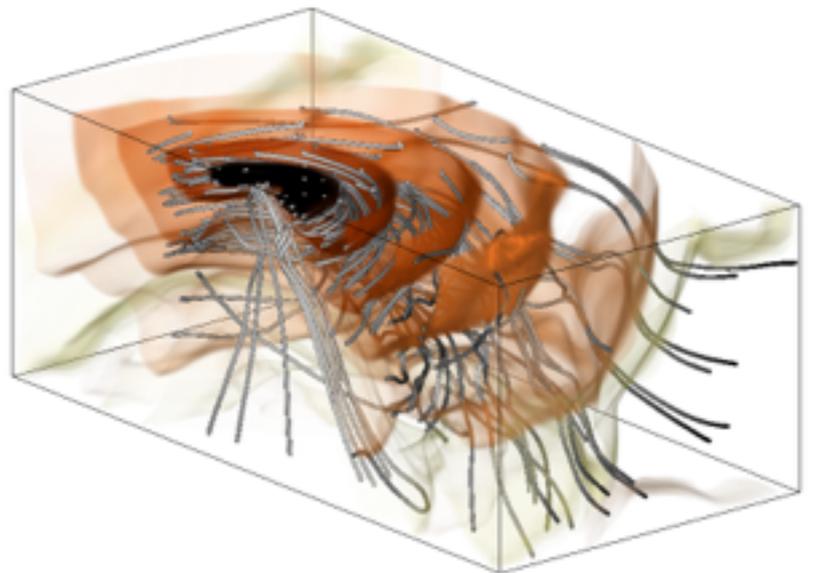
HMNS: delayed collapse to BH



- Frozen-in condition **not exactly true** if IMHD assumption is relaxed.
- B-field lines are allowed to **drift** in RMHD.
- B-field tension and B-field braking is **less efficient** in removing l_0 .

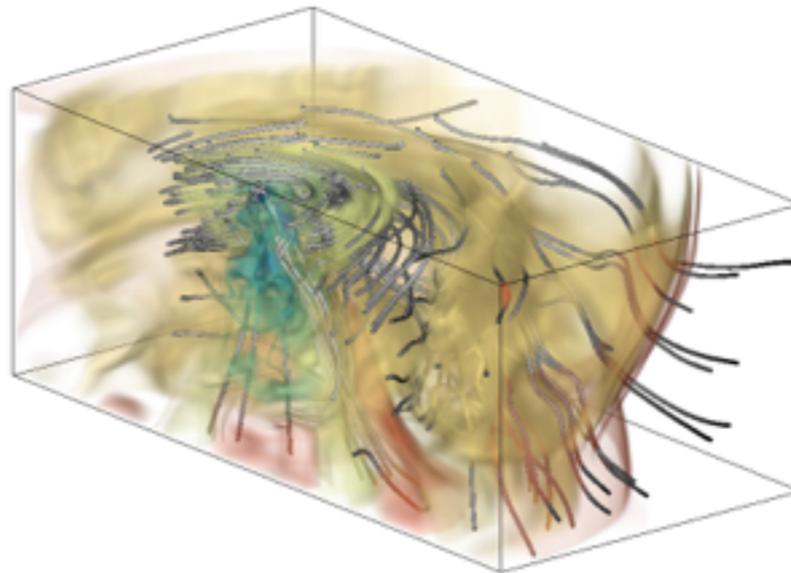
Magnetic-field topology

Time = 18.53 ms



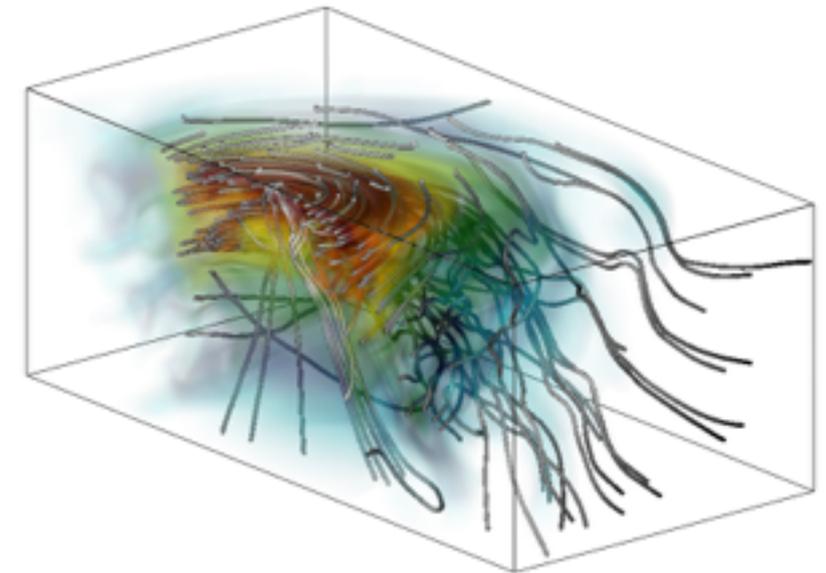
7.70 8.70 9.70 10.7 11.7
Max: 12.0
Min: 6.79
log(ρ) (gr/cm³)

Time = 18.53 ms



18.00 18.50 19.00 19.50 20.00
Max: 20.53
Min: 12.05
log(ϵ) (erg/gr)

Time = 18.53 ms



9.500 10.12 10.75 11.38 12.00
Max: 12.35
Min: 7.113
log(B) (G)

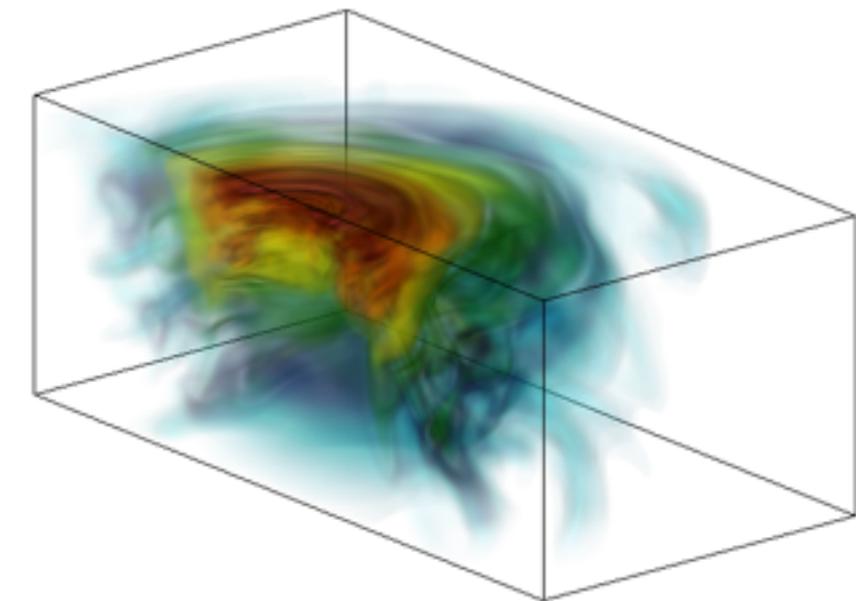
- **BH+torus+funnel** reaching a quasi-stationary configuration.
- The **B-field in the torus is twisted** because of differential rotation.
- The **magnetic field lines** could potentially **reconnect in the funnel**.

Magnetic-field topology

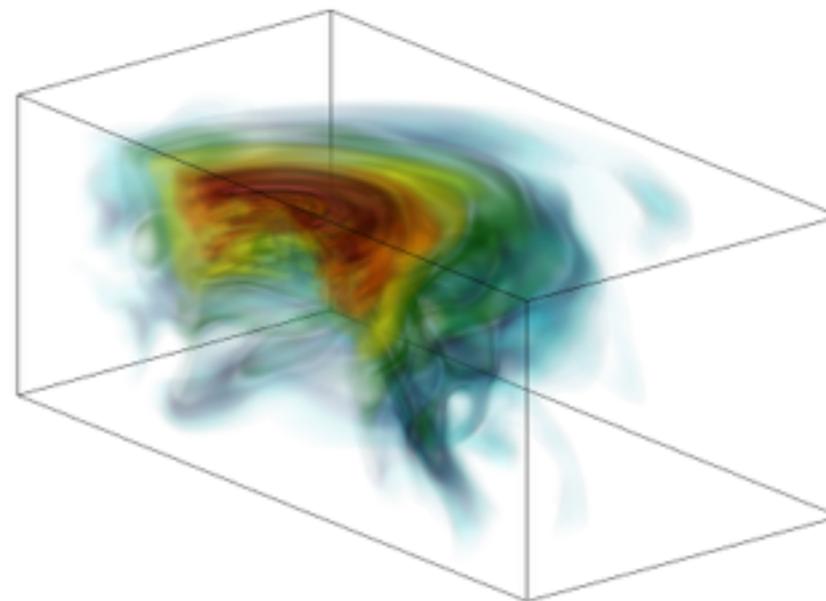
Time = 18.53 ms

Time = 18.53 ms

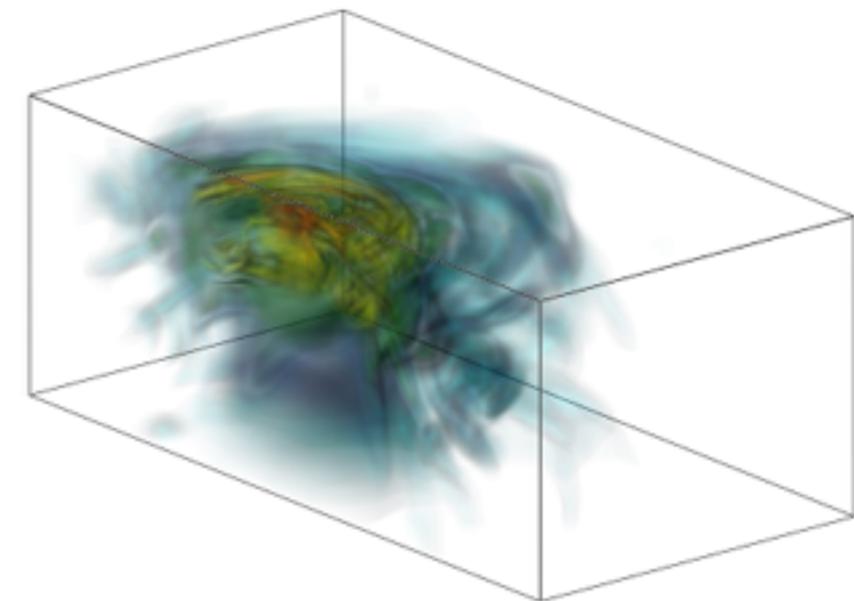
Time = 18.53 ms



9.500 10.12 10.75 11.38 12.00
Max: 12.35
Min: 7.113
log(|B|) (G)



9.500 10.12 10.75 11.38 12.00
Max: 12.34
Min: 1.519
log(|B_Tor|) (G)



9.500 10.12 10.75 11.38 12.00
Max: 12.03
Min: 7.112
log(|B_Pol|) (G)

- B-field is predominantly toroidal in the torus.
- The poloidal B-field is dominating in the nearly evacuated funnel.
- The conductivity in the funnel region is essentially zero.
- Evolution of B-fields is essentially vacuum EM waves.

Conclusions

- *Modelling of binary NSs in full GR is **mature**: GWs from the inspiral can be computed with precision of binary BHs.
- *Spectra of post-merger shows clear peaks: cf lines for stellar atmospheres. Some peaks are **"universal"**.
- *If observed, post-merger signal will set tight constraints on EOS.
- *Magnetic fields unlikely to be detected during the inspiral but **important** after the merger: instabilities and EM counterparts.
- *Is a jet created? New RMHD simulations show cleaner jet structure.
- *Mass ejecta, afterglows and neutrinos are important elements of this picture and need to be properly accounted for.

Binary neutron stars are a rich lab of physics and astrophysics.
Numerical relativity is a perfect tool to explore it.

EXTRAS:

BNISS