Modelling binary neutron stars: pure hydrodynamics

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GOETHE UNIVERSITÄT FRANKFURT AM MAIN

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Rethymno
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Plan of the lectures

- brief introduction to relativistic hydrodynamics
- what we understand about BNSs
- characteristic frequencies and quasi-universality
  - inspiral: frequency at amplitude peak
  - merger/post-merger: EOS information from PSD peaks
- MHD simulations and EM counterparts
  - HMNS: MRI and magnetically driven winds
  - IMHD vs RMHD
  - extended x-ray emission

No good/bad questions. There are only questions: ask them!

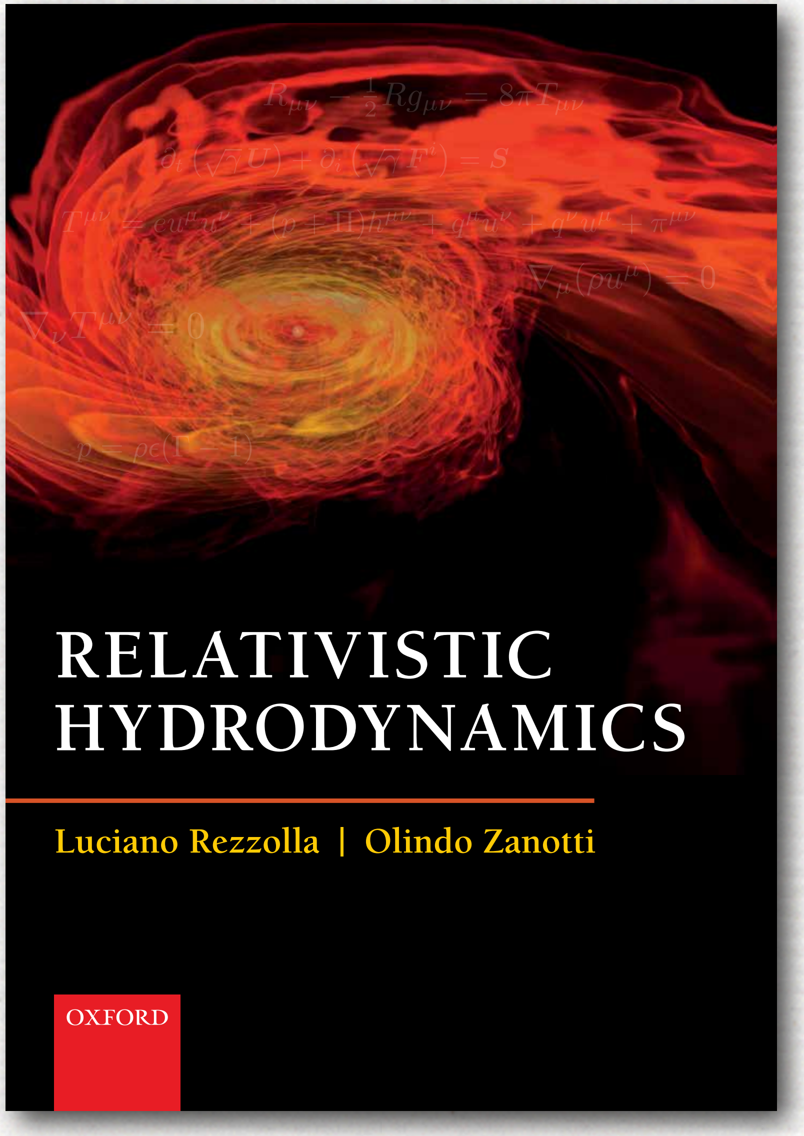
Bibliography and lecture notes

Lecture notes online on courses at University of Frankfurt:

- hydrodynamics and MHD
- advanced GR

http://astro.uni-frankfurt.de/rezzolla/teaching

(password needed; just ask me)



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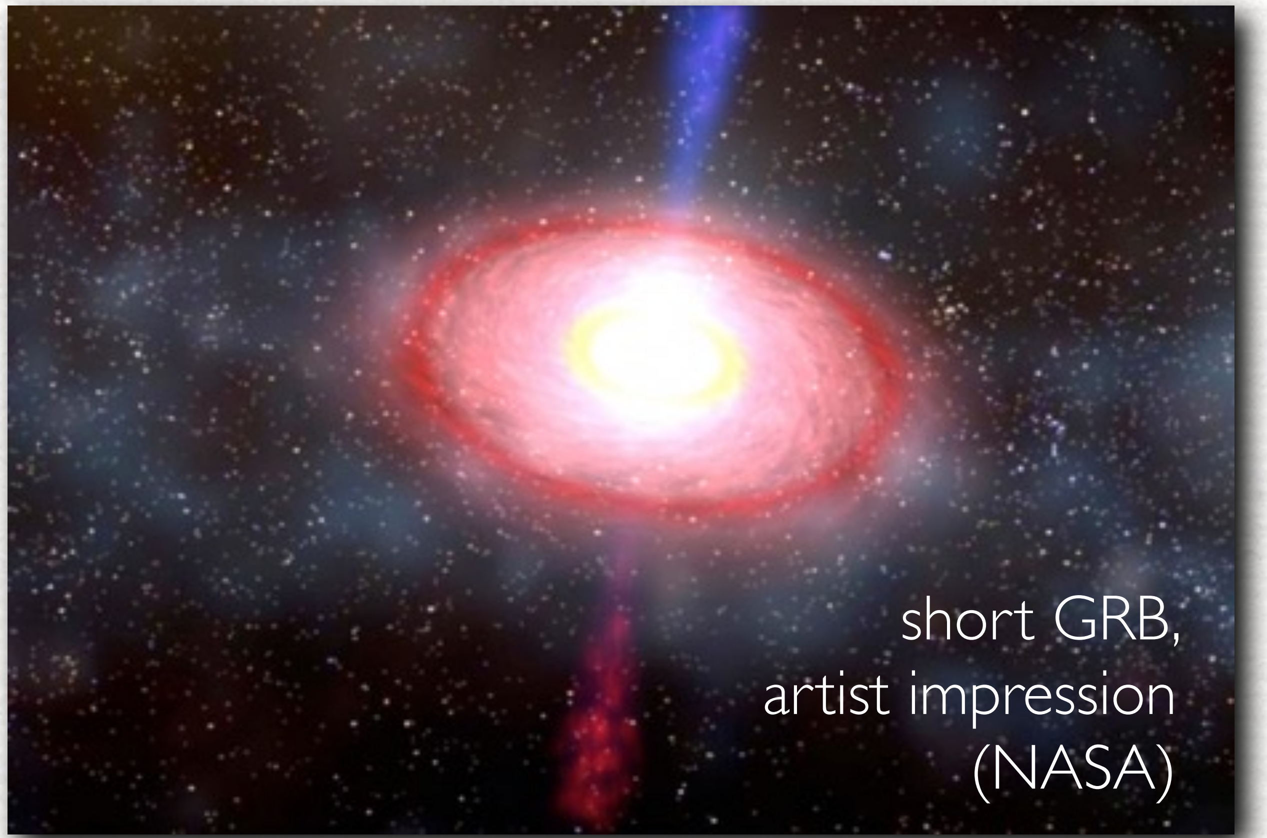
Why study binary neutron stars?

- We know they exist (as opposed to binary BHs) and are among the strongest sources of GWs

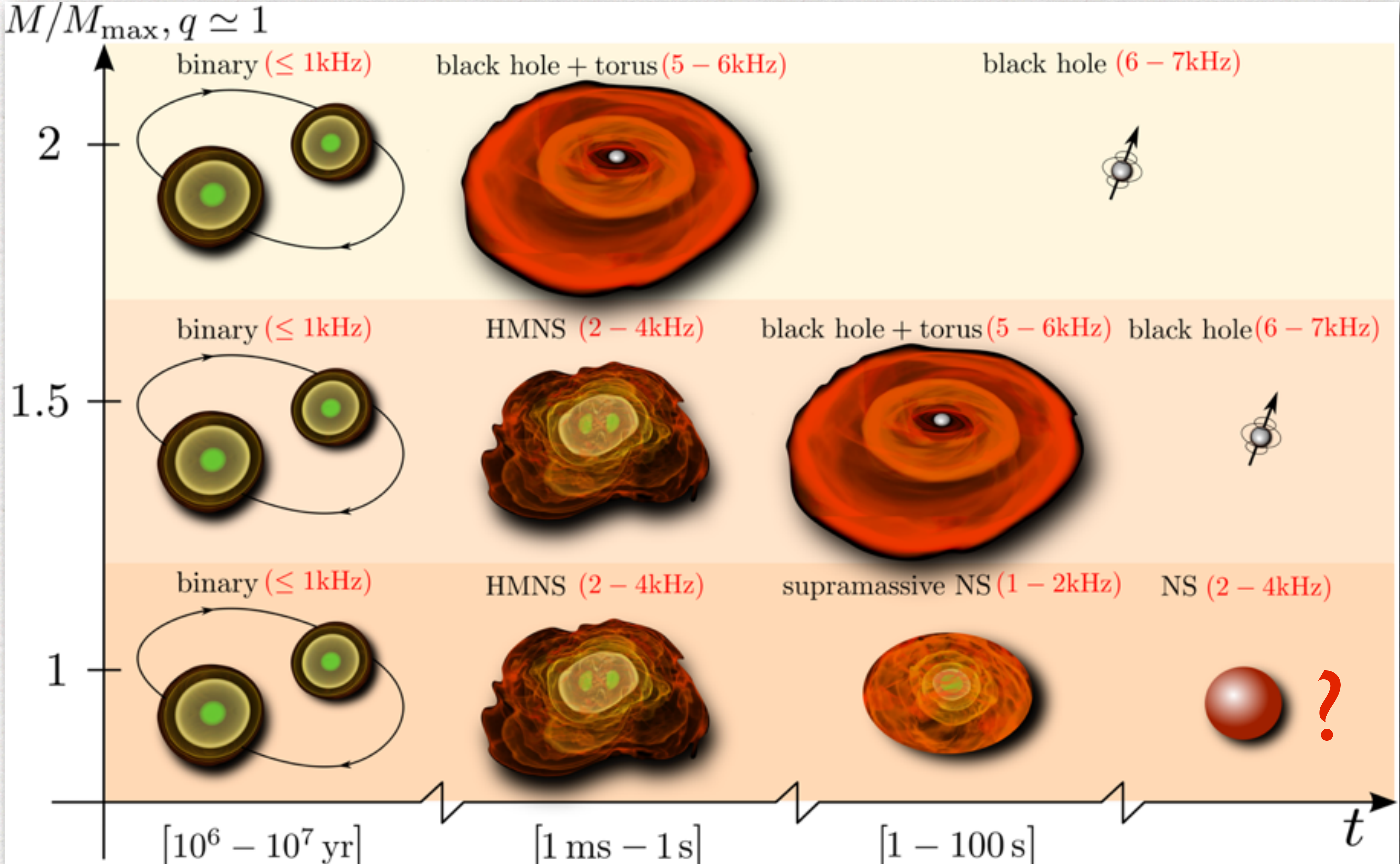
- We expect them related to short gamma-ray bursts; energies released are huge: $10^{48\text{-}50}$ erg.


short GRB,
artist impression
(NASA)

- No self-consistent model has yet been produced to explain them.

- Theoretical modelling has now reached level of maturity to shed light on central engine of SGRBs

Broadbrush picture



The equations of numerical relativity

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \cancel{8\pi T_{\mu\nu}}\,, \qquad \text{(field equations)}$$

$$\cancel{\nabla_\mu T^{\mu\nu} = 0}\,, \qquad \text{(cons. energy/momentum)}$$

$$\cancel{\nabla_\mu (\rho u^\mu) = 0}\,, \qquad \text{(cons. rest mass)}$$

$$\cancel{p = p(\rho, \epsilon, Y_e, \ldots)}\,, \qquad \text{(equation of state)}$$

$$\cancel{\nabla_\nu F^{\mu\nu} = I^\mu\,, \quad \nabla_\nu^* F^{\mu\nu} = 0}\,, \qquad \text{(Maxwell equations)}$$

$$\cancel{T_{\mu\nu} = T_{\mu\nu}^{\text{fluid}} + T_{\mu\nu}^{\text{EM}} + \ldots} \qquad \text{(energy} - \text{momentum tensor)}$$

In vacuum space times the theory is complete and the truncation error is the only error: **"CALCULATION"**

The equations of numerical relativity

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$$T_{\mu\nu} = T^{\text{fluid}}_{\mu\nu} + T^{\text{EM}}_{\mu\nu} + \ldots \qquad \text{(energy} - \text{momentum tensor)}$$

In non-vacuum space times the truncation error is the only **measurable** error: **"SIMULATION"**

It's our approximation to *"reality"*: improvable via microphysics, magnetic fields, viscosity, radiation transport, ...

The two-body problem in GR

- For BHs we know what to **expect**:

BH + BH → BH + gravitational waves (GWs)

- For NSs the question is more **subtle:** the merger leads to an hyper-massive neutron star (HMNS), ie a metastable equilibrium:

NS + NS → HMNS + **... ?** → BH + torus + **... ?** → BH

All complications are in the intermediate stages; the rewards high:

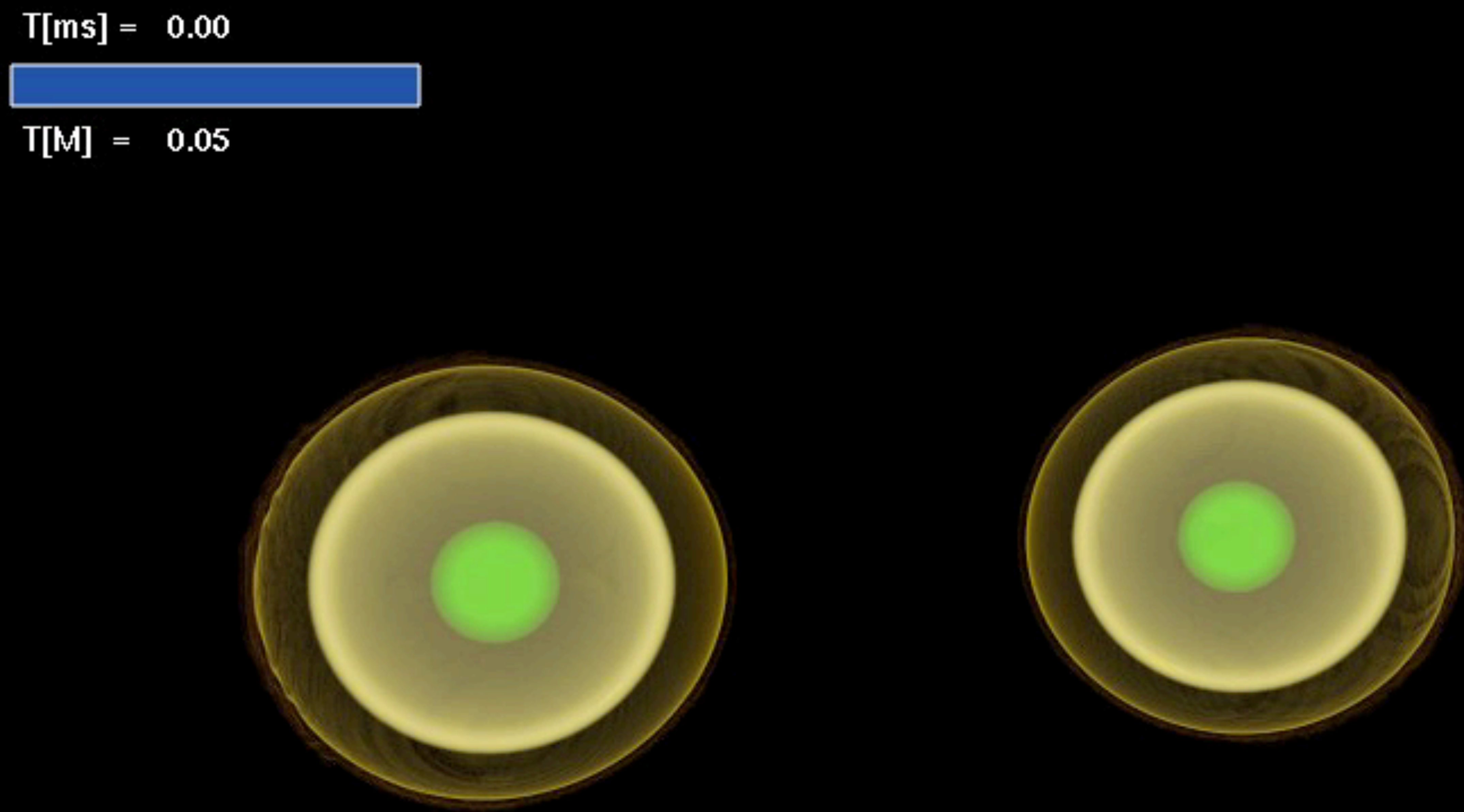
- studying the HMNS will show strong and precise imprint on the EOS
- studying the BH+torus will tell us on the central engine of GRBs

NOTE: with advanced detectors we expect to have a **realistic** rate of ~**40 BNSs** inspirals a year, ie ~ 1 a week (Abadie+ 2010) *“merger → HMNS → BH + torus”*

Quantitative differences are produced by:

- **differences induced by the gravitational MASS:**
  a binary with smaller mass will produce a HMNS further away from the stability threshold and will collapse at a later time
- **differences induced by the EOS:**
  a binary with an EOS with large thermal capacity (ie hotter after merger) will have more pressure support and collapse later

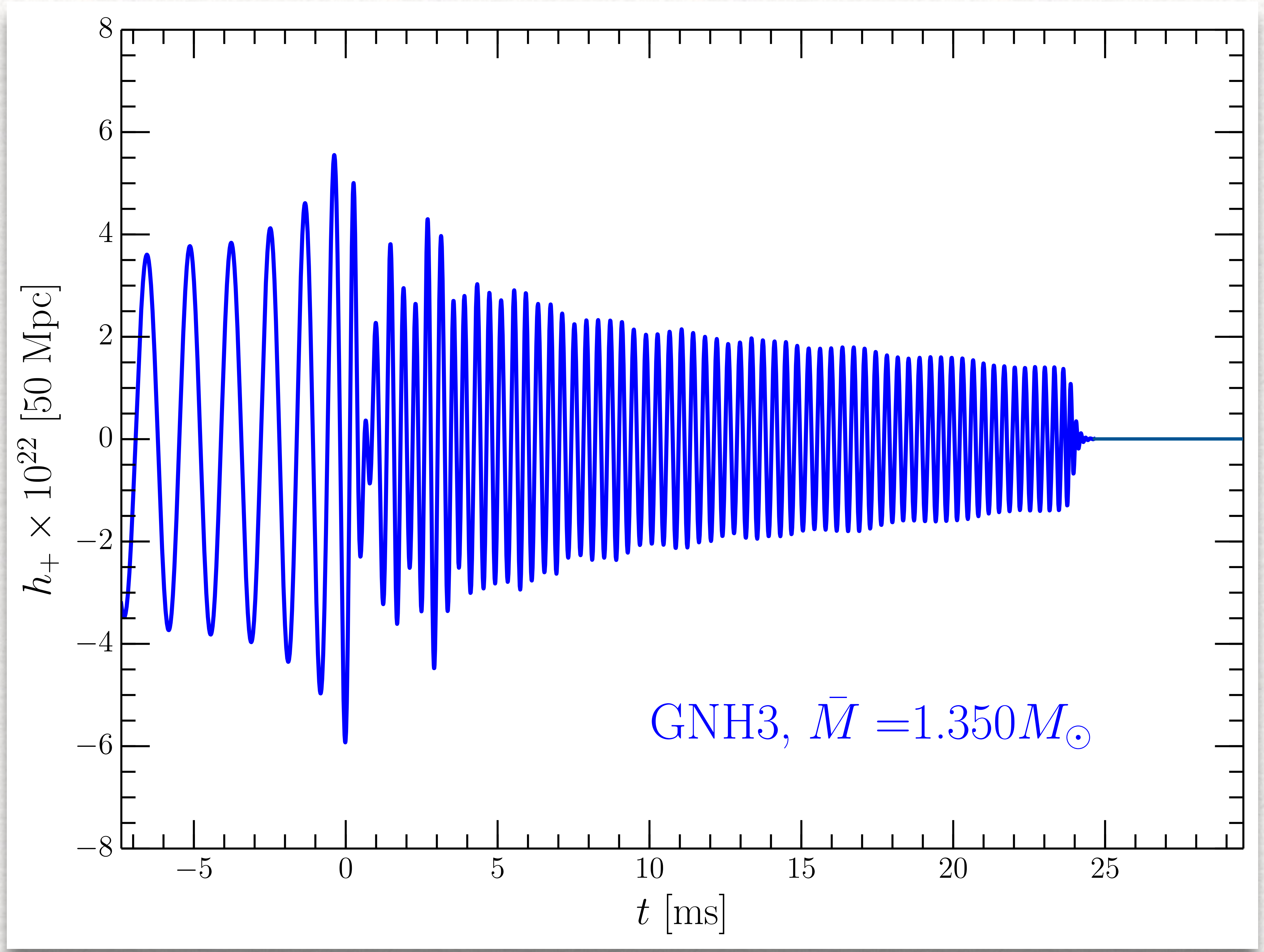
Animations: Kaehler, Giacomazzo, Rezzolla



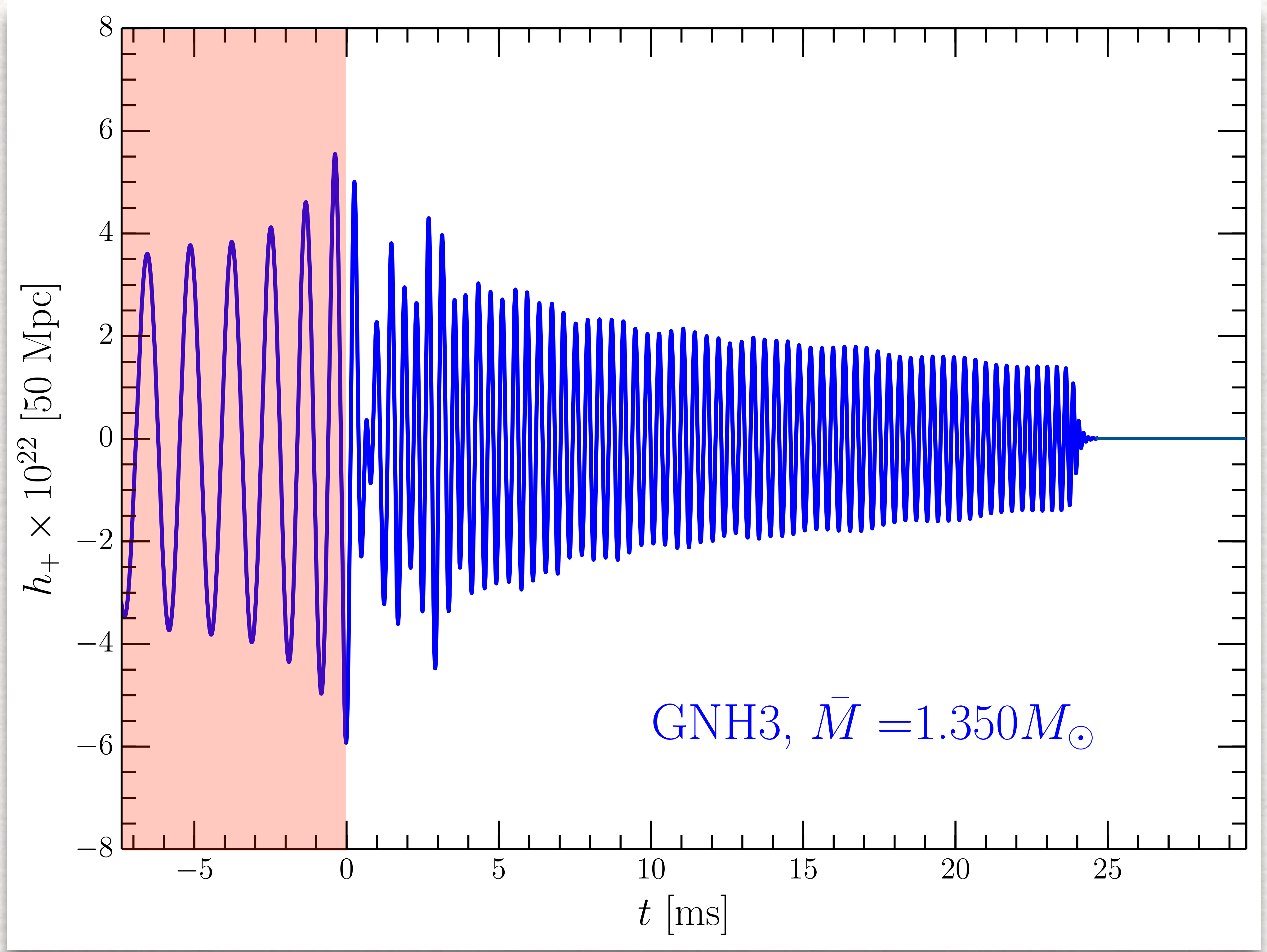
Hot EOS: **high-mass** binary

$$M = 1.6\,M_\odot$$

Anatomy of the GW signal

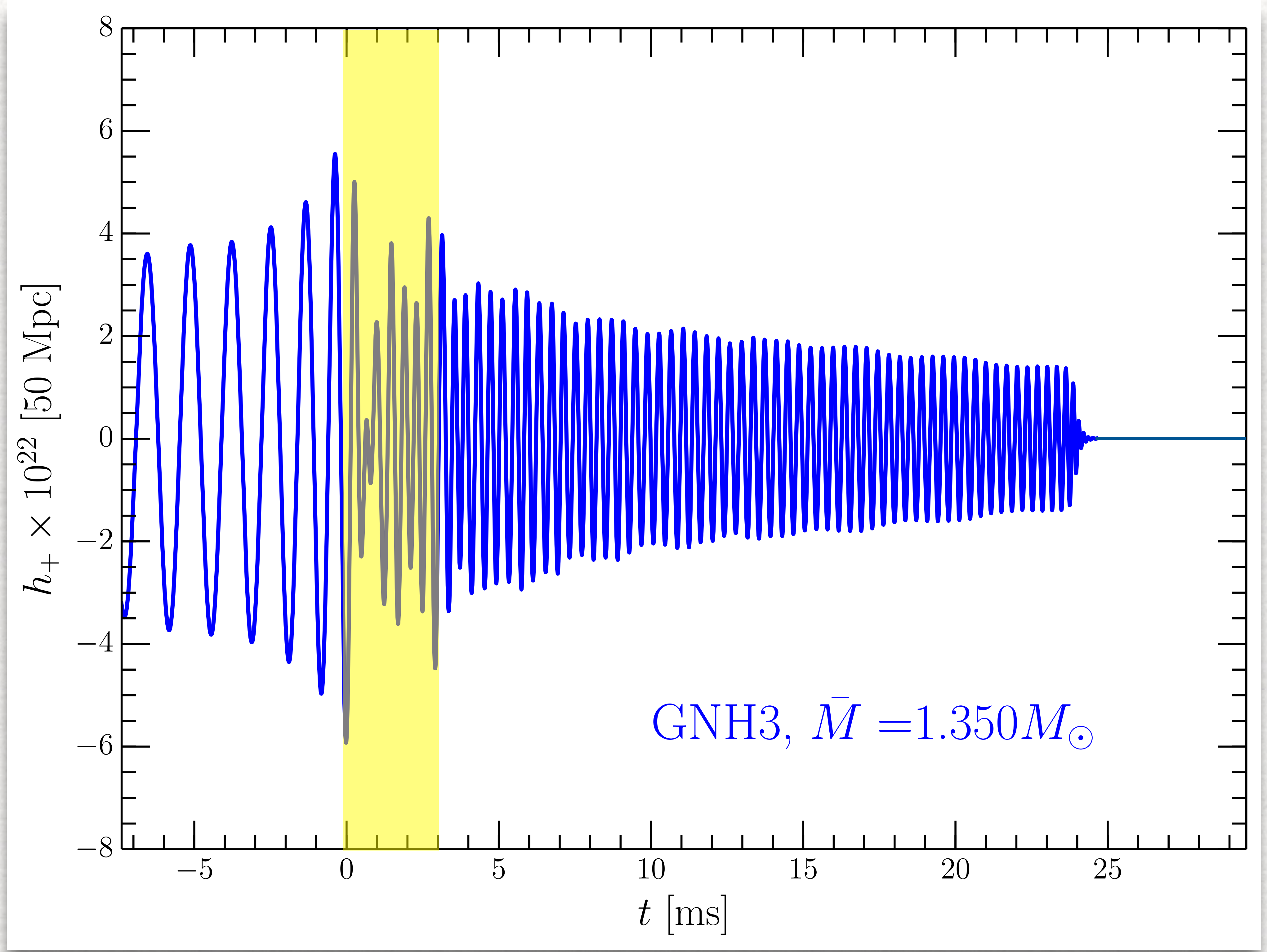


Anatomy of the GW signal



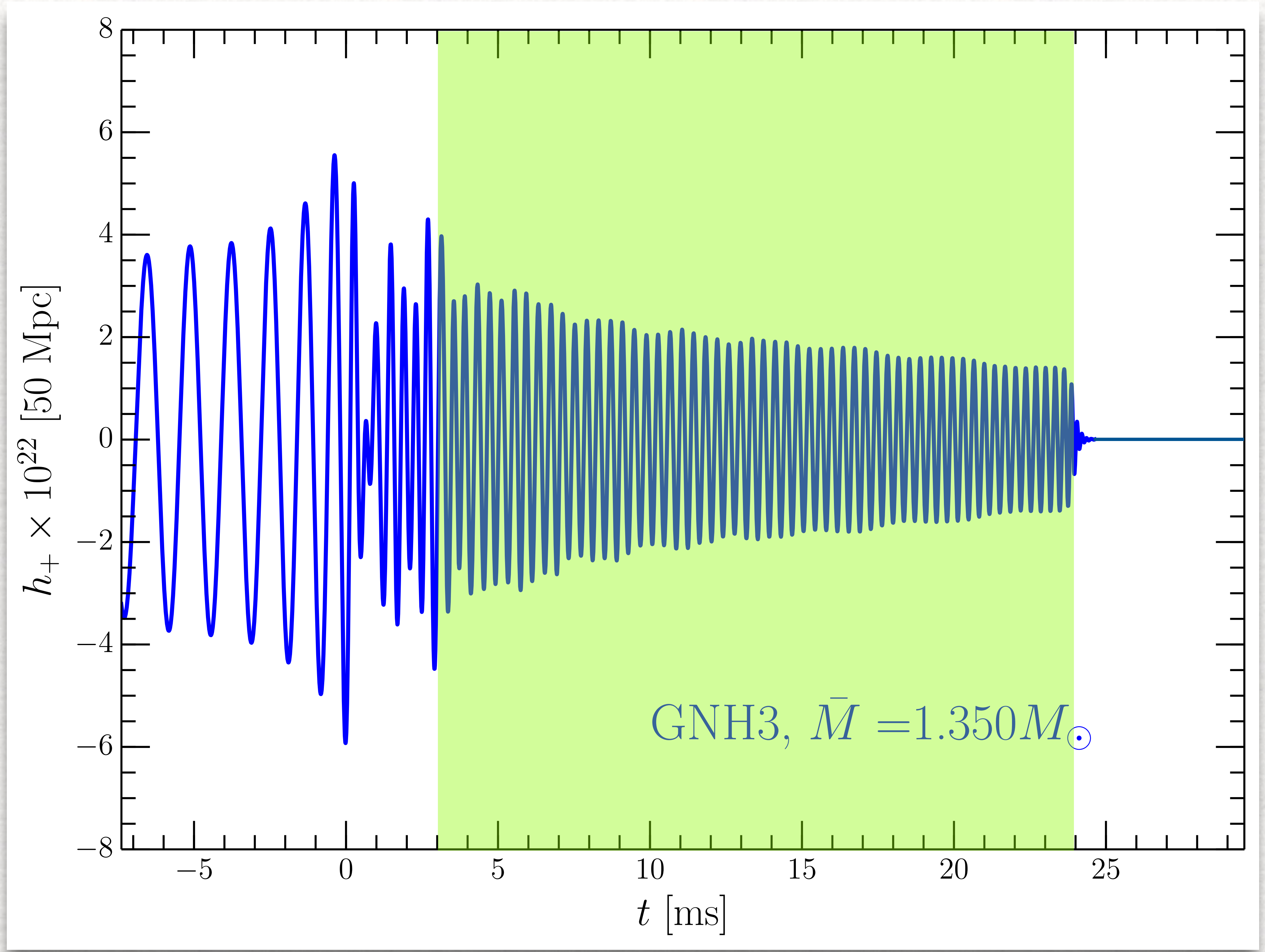
Inspiral: well approximated by PN/EOB; tidal effects important

Anatomy of the GW signal



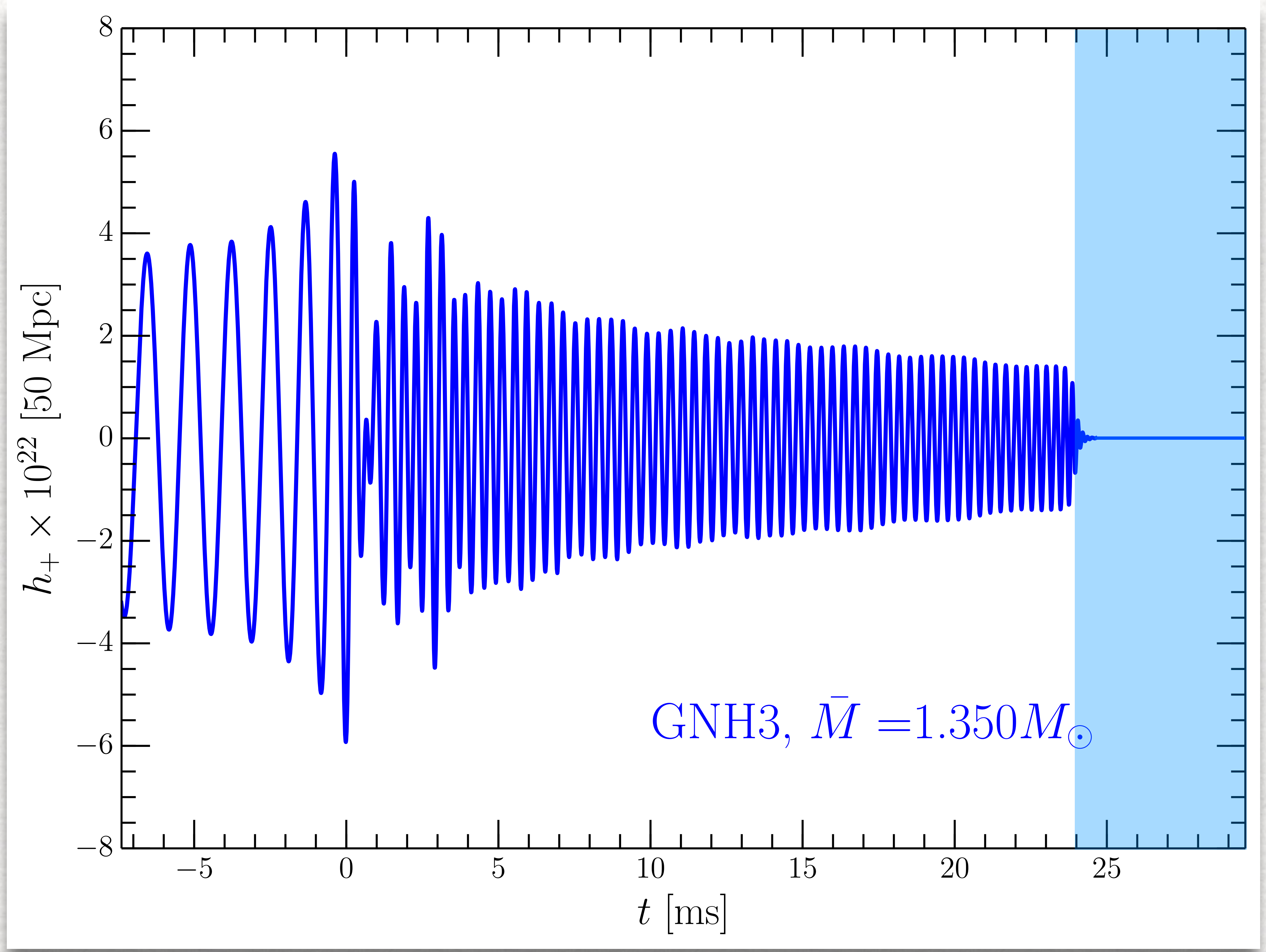
Merger: highly nonlinear but analytic description possible

Anatomy of the GW signal



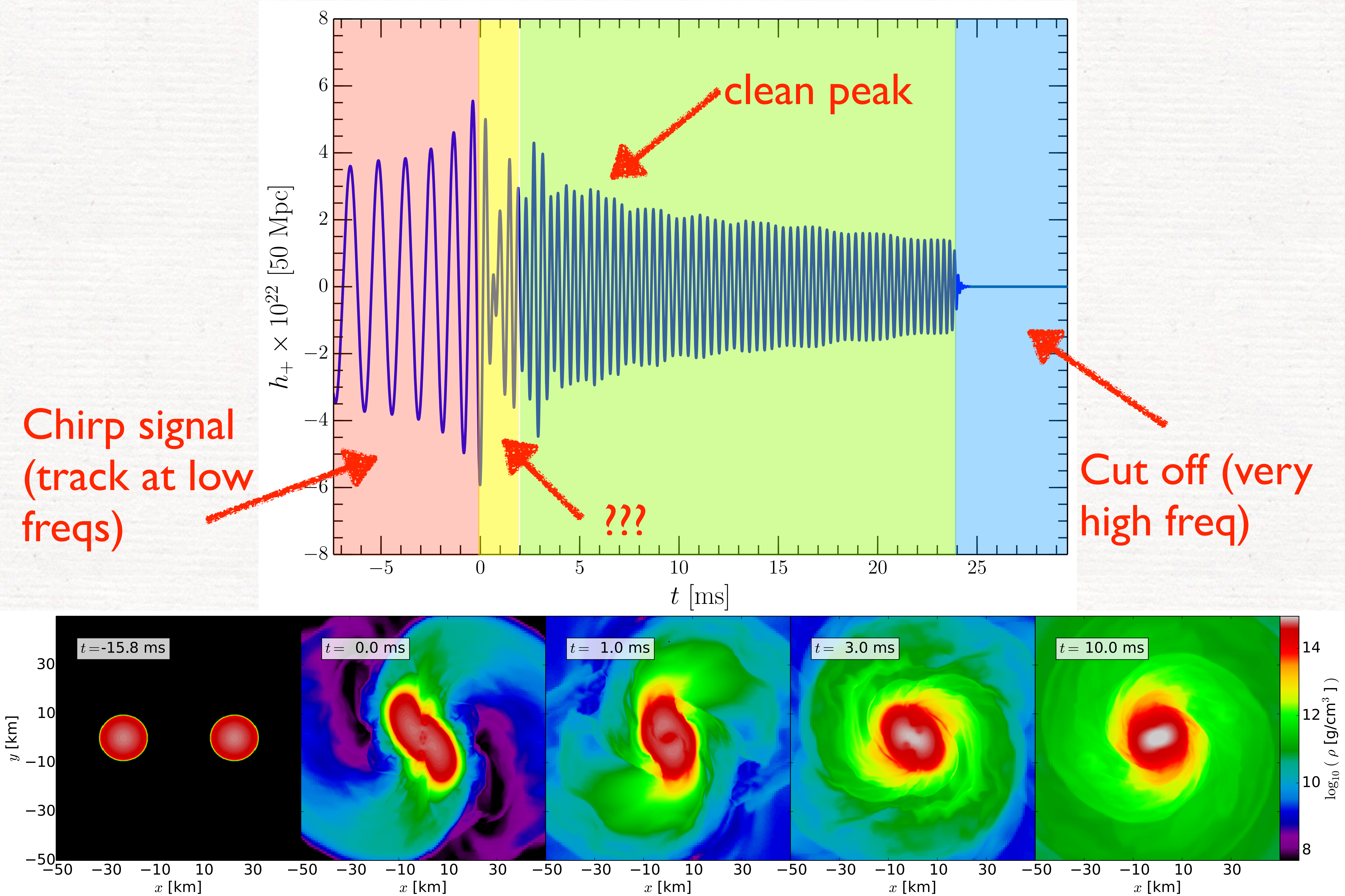
post-merger: quasi-periodic emission of bar-deformed HMNS

Anatomy of the GW signal



Collapse-ringdown: signal essentially shuts off.

Anatomy of the GW signal



"merger → HMNS → BH + torus"

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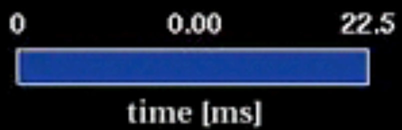
- **differences induced by MASS ASYMMETRIES:**
  tidal disruption before merger; may lead to prompt BH



Animations: Giacomazzo, Koppitz, LR

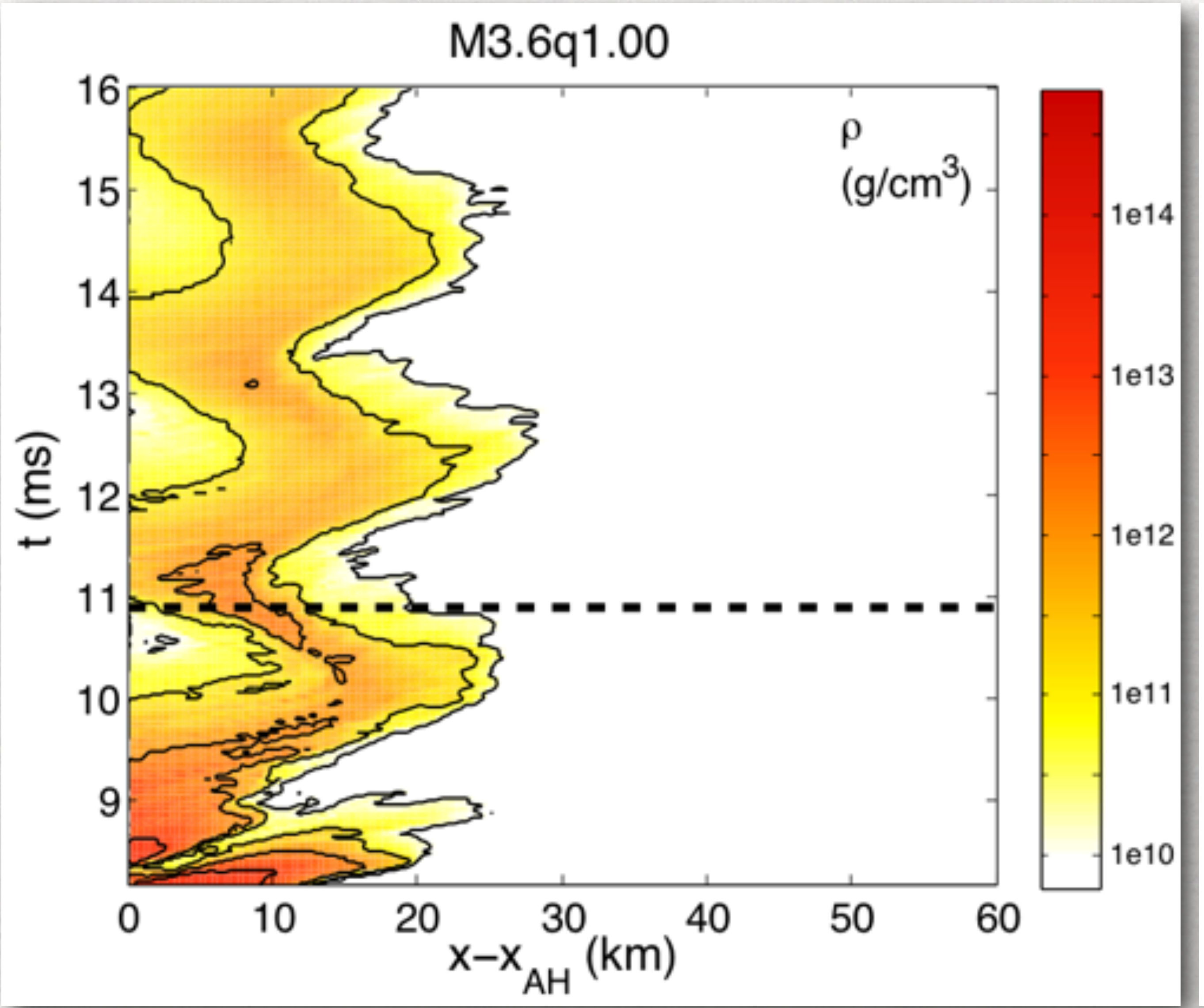
Total mass : $3.37\,M_\odot$;   mass ratio :0.80;

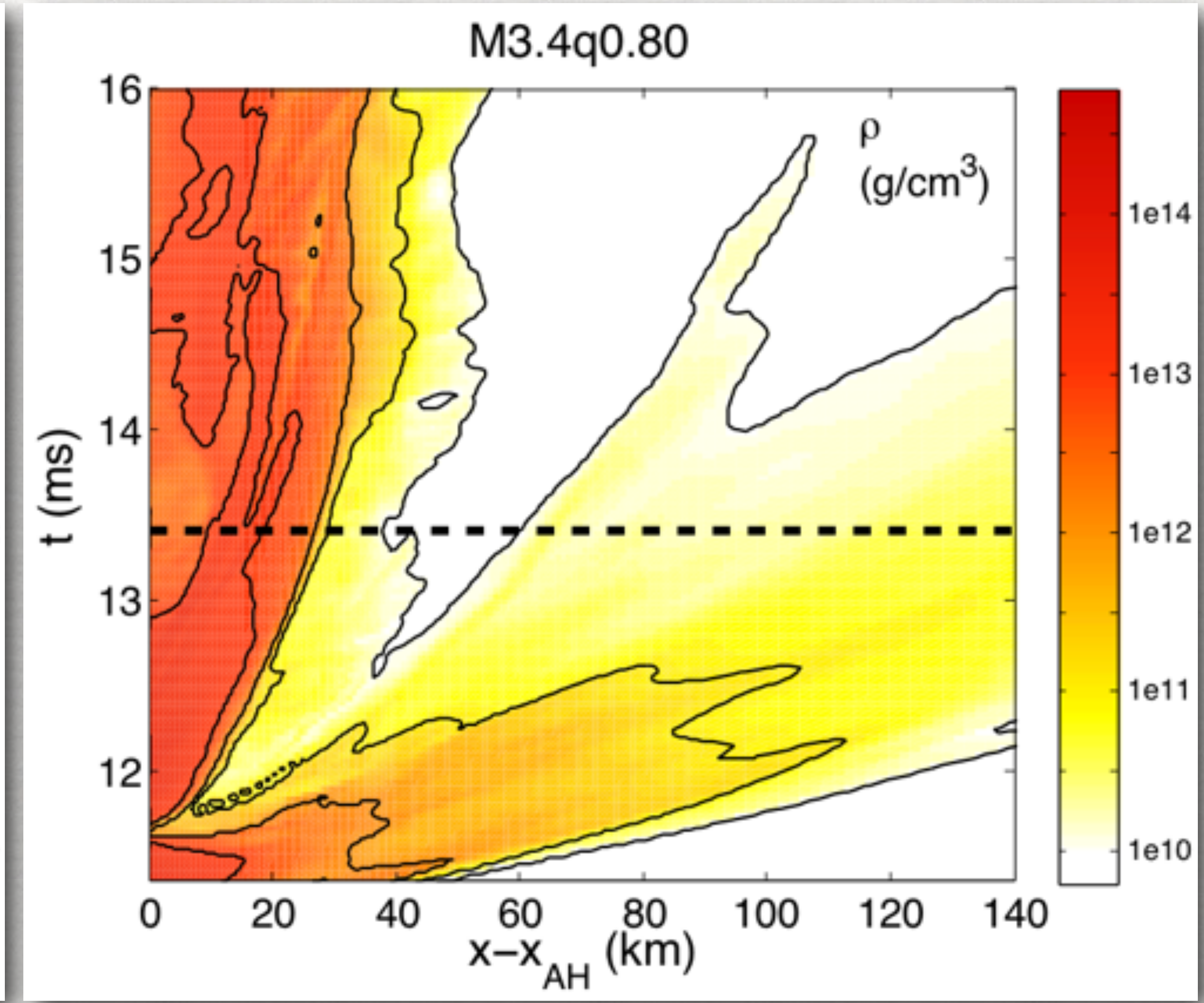
- the torii are generically more massive
- the torii are generically more extended
- the torii tend to stable quasi-Keplerian configurations
- overall unequal-mass systems have all the ingredients needed to create a GRB

Torus properties: density

Rezzolla+ (2010)

spacetime diagram of rest-mass density along x-direction



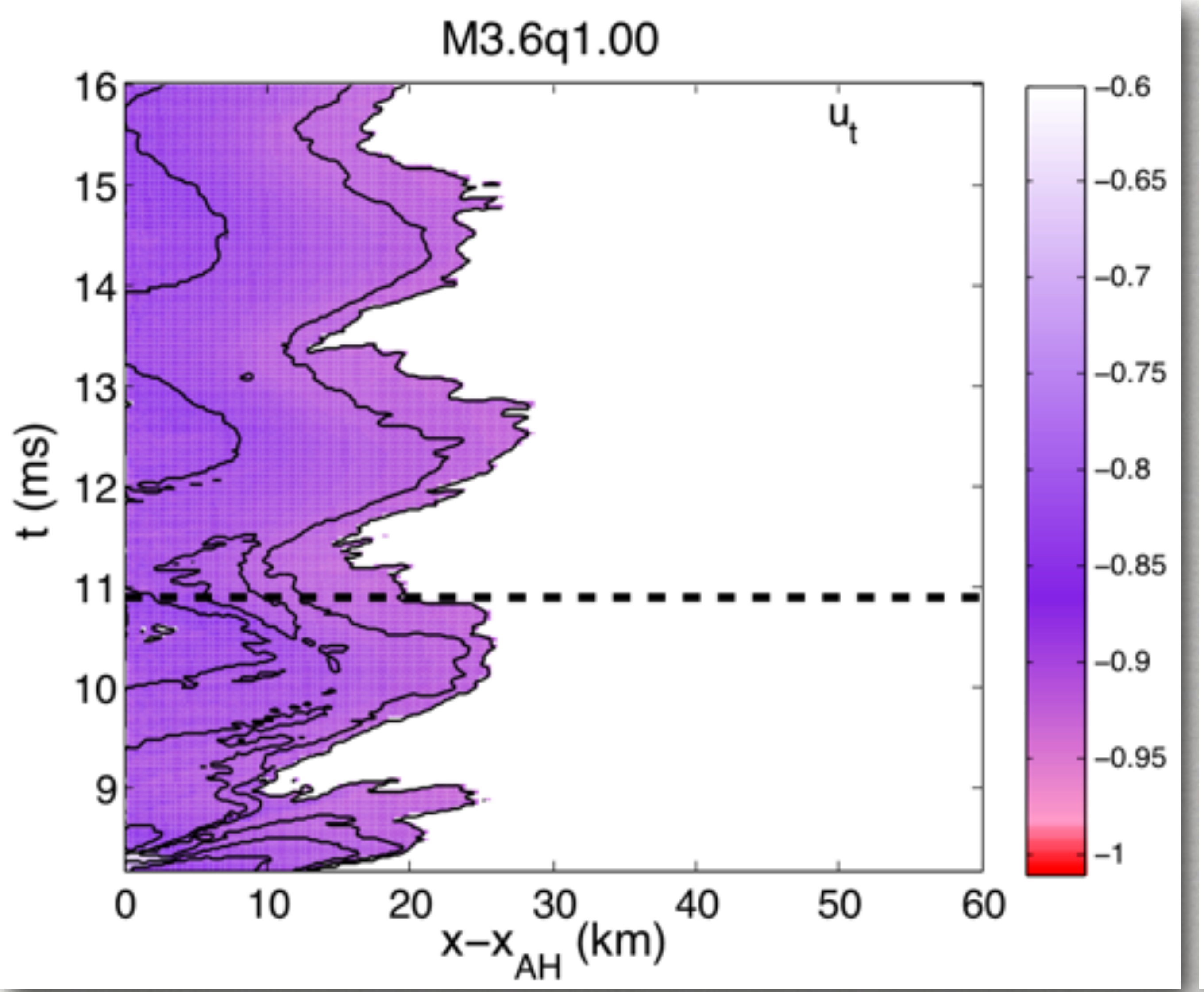


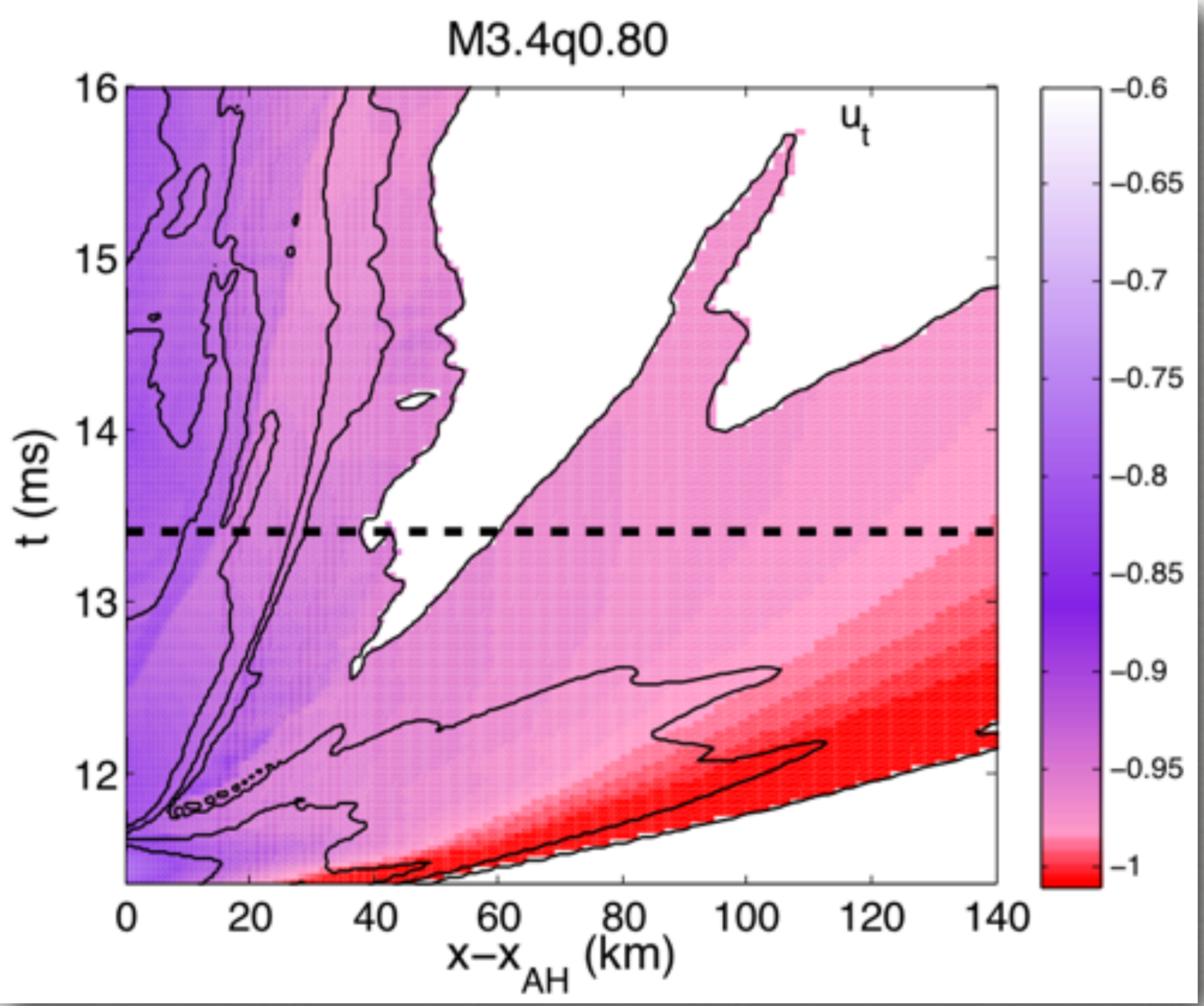
equal mass binary: note the periodic accretion and the compact size; densities are not very high

unequal mass binary: note the continuous accretion and the very large size and densities (temperatures)

Torus properties: bound matter

spacetime diagram of local fluid energy: u_t



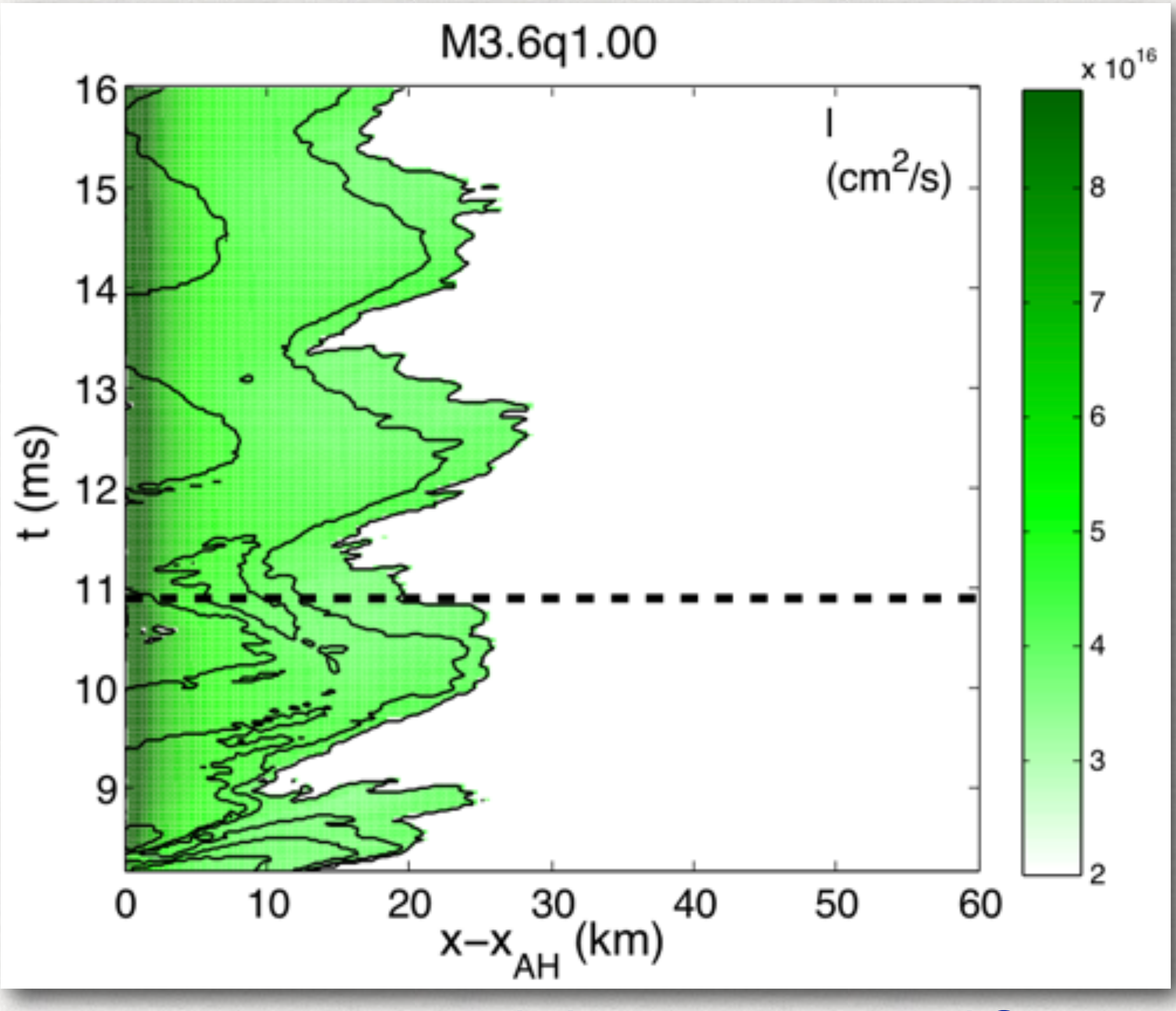


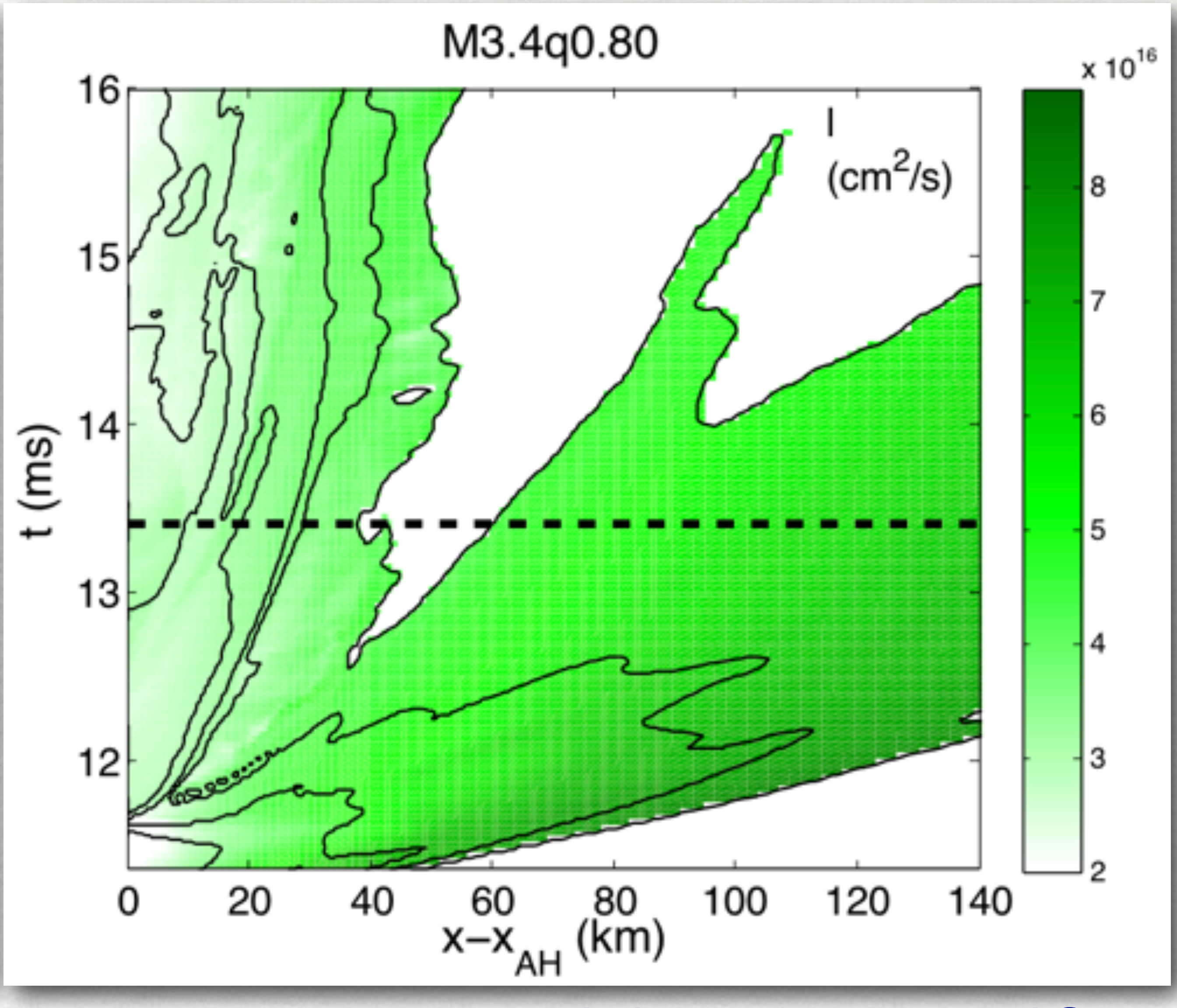
equal mass : all matter is clearly bound, i.e. $u_t < -1$
Note the accretion is quasi-periodic

unequal mass: some matter is unbound while other is ejected at large distances (cf. scale). In these regions r-processes can take place

Torus properties: specific ang. momentum

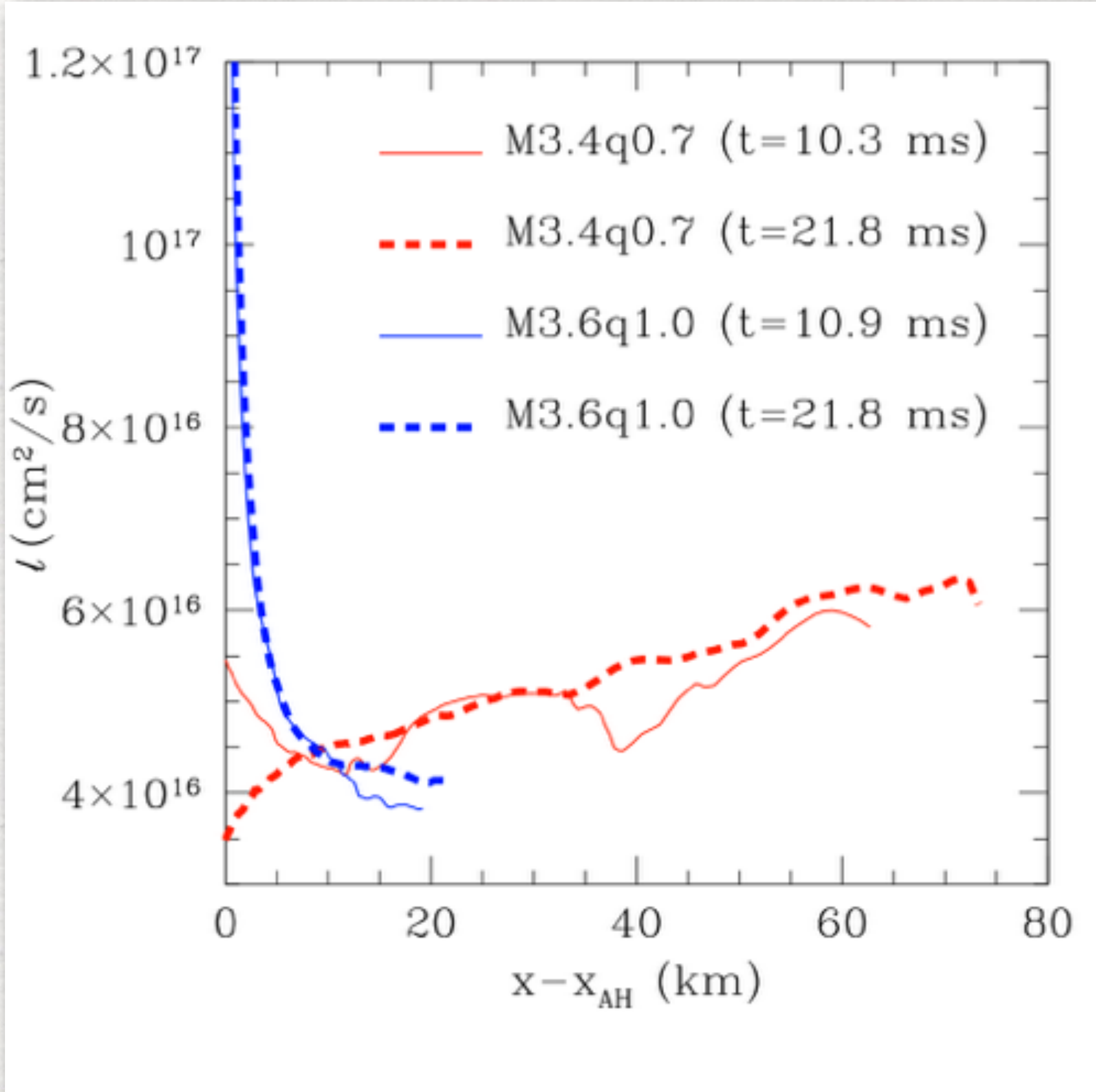
spacetime diagram of specific angular mom. : $\ell \equiv -u_\phi / u_t$

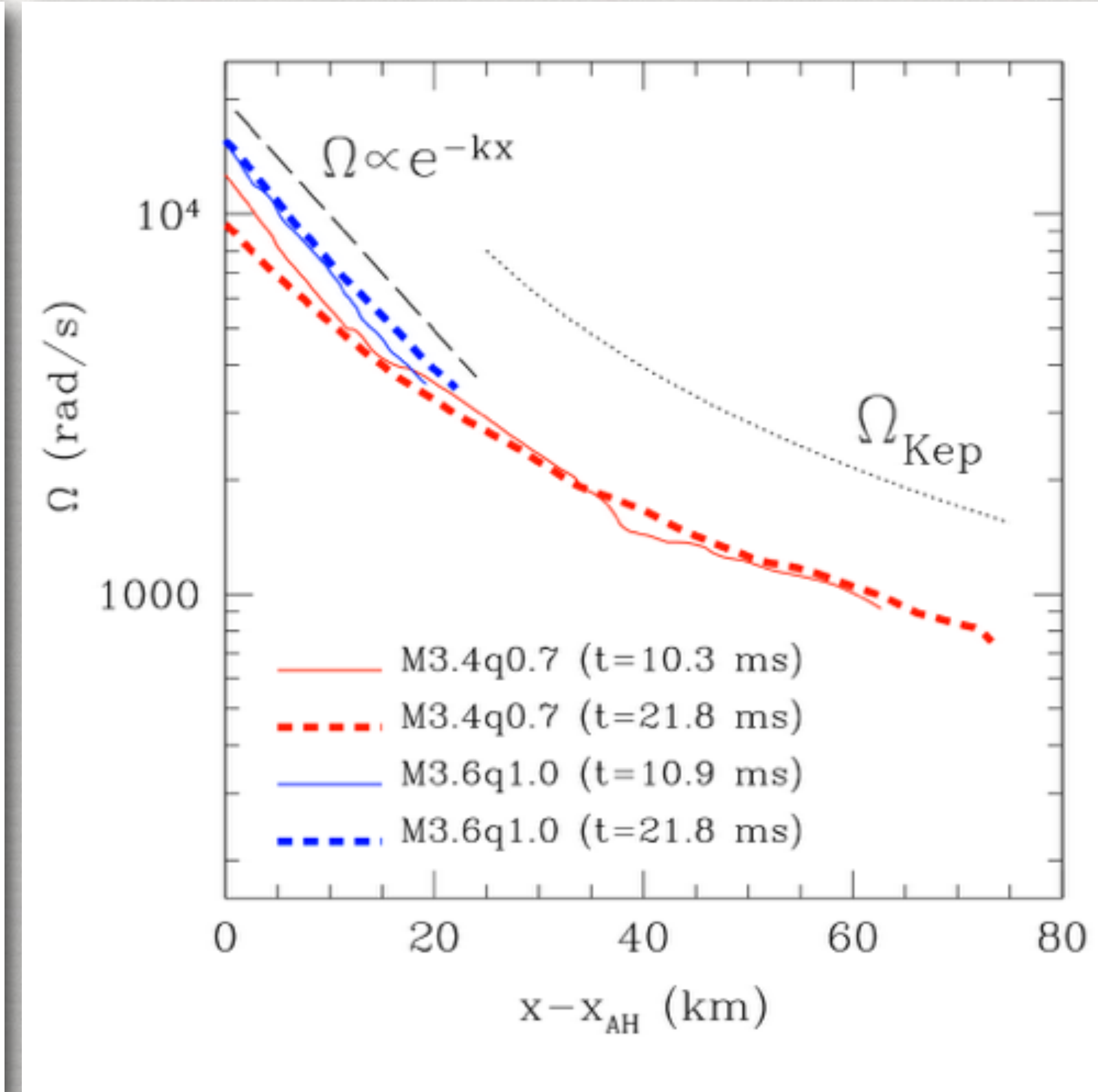




equal mass binary: specific angular momentum is larger at the inner edge and decreases outwards

unequal mass binary: specific angular momentum is smaller at inner edge and increases outwards

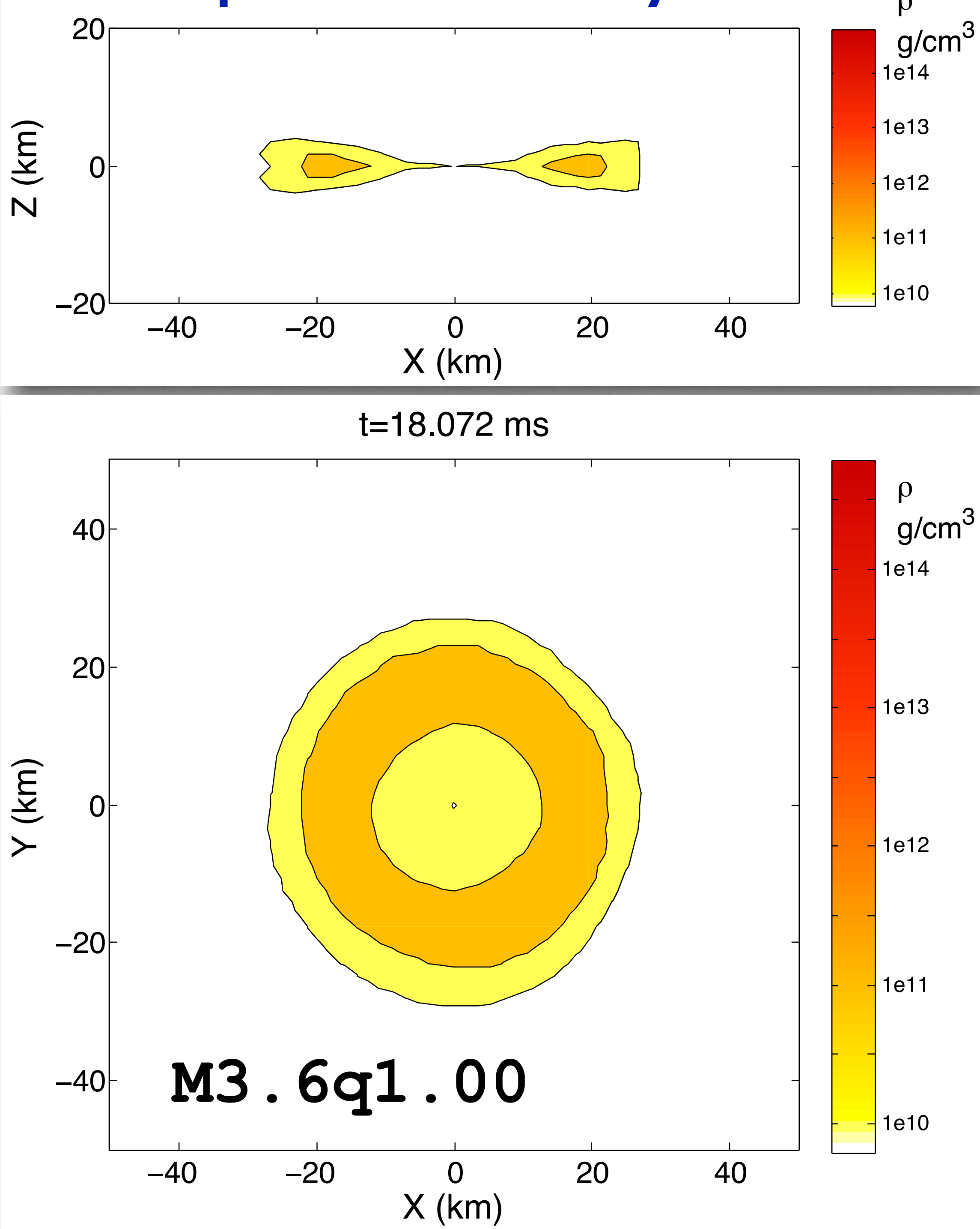




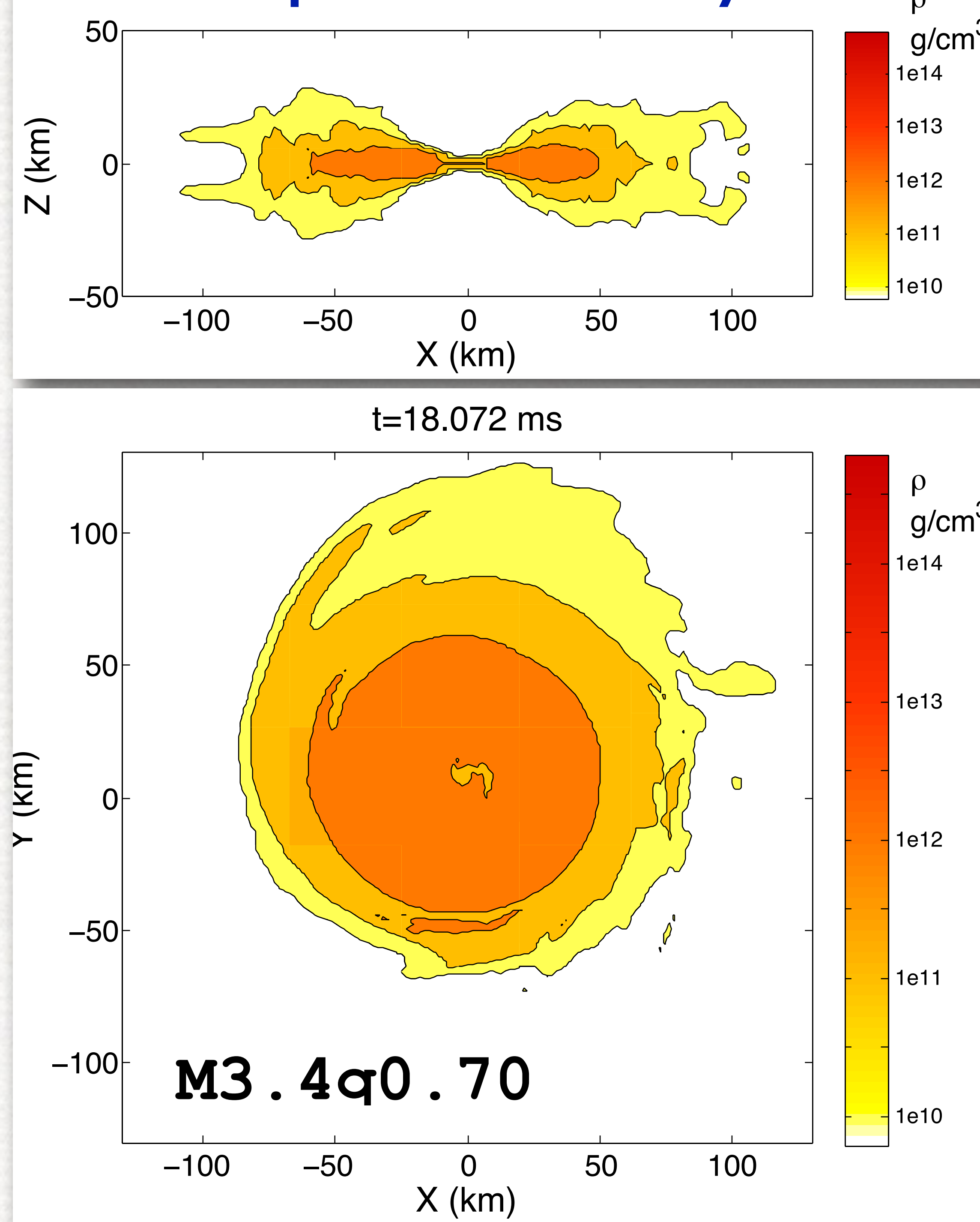
- specific angular momentum has very different behaviour in the two cases: $d\ell/dx \geq 0$ for stability
- equal-mass binary has exponential differential rotation while the unequal-mass is essentially Keplerian

Torus properties: size

equal-mass binary

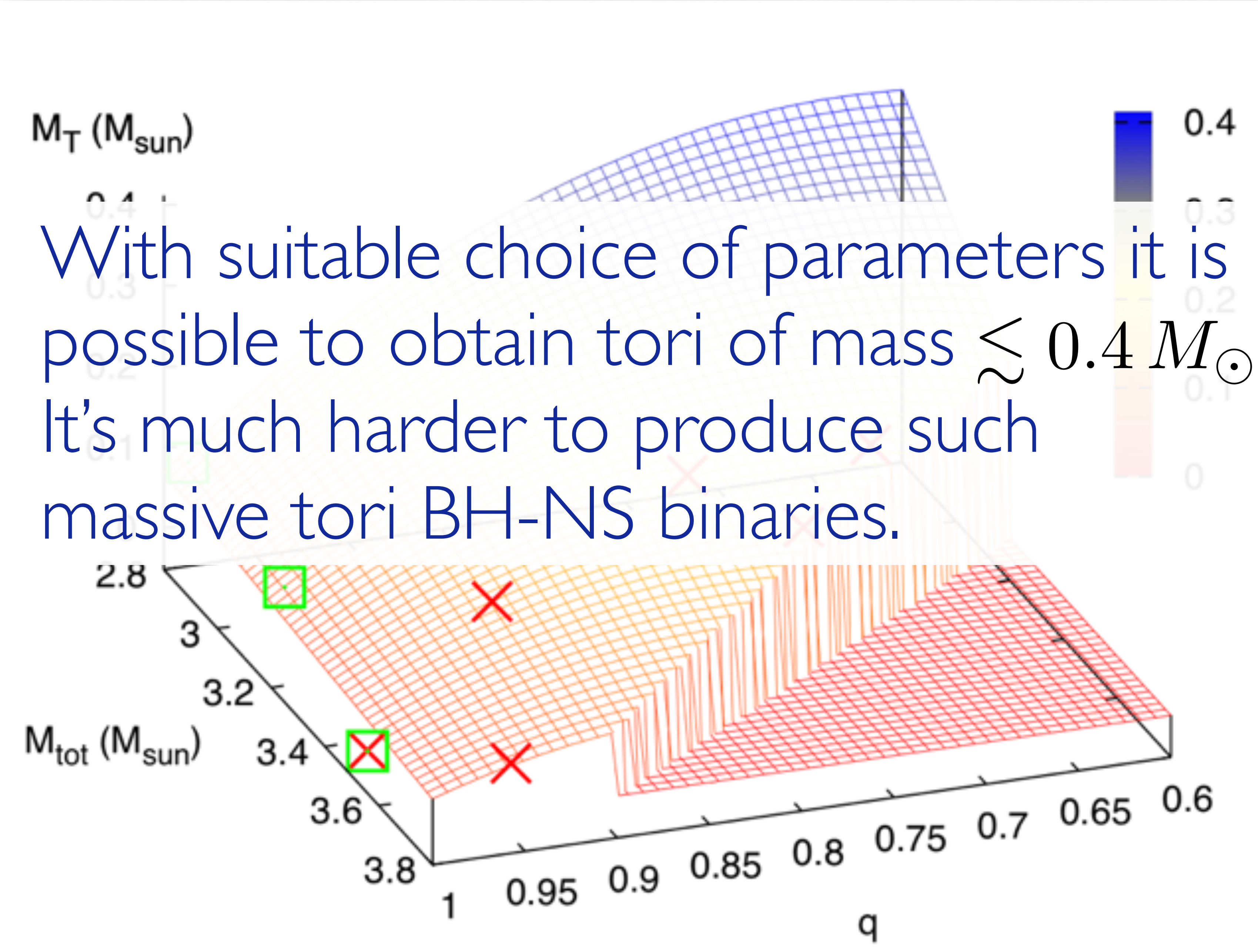


unequal-mass binary



Note that although the total mass is very similar, the unequal-mass binary yields a torus which is about ~ 4 times larger and ~ 200 times more massive

Torus properties: unequal-masses



With suitable choice of parameters it is possible to obtain tori of mass $\lesssim 0.4\, M_\odot$ It's much harder to produce such massive tori BH-NS binaries.

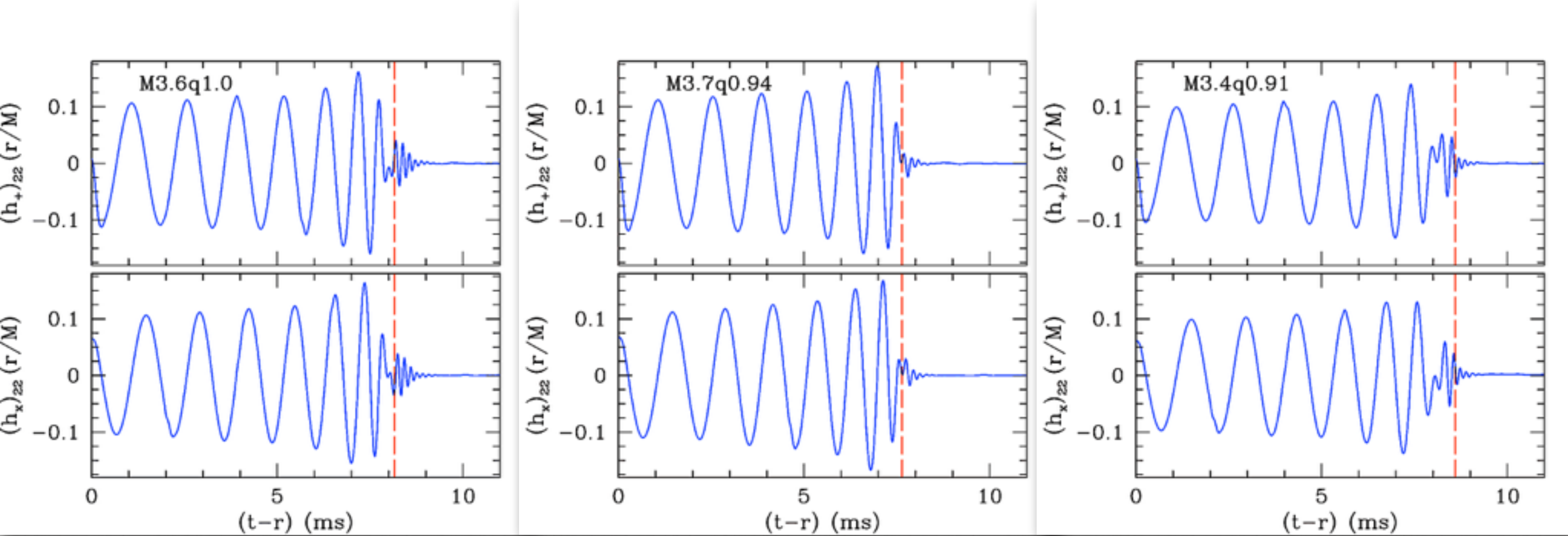
Model	$M_{\rm total}$ $(M_\odot)$	q	$M_{\rm torus}$ $(M_\odot)$
M3.6q1.00	3.558	1	0.0010
M3.7q0.94	3.680	0.94	0.0100
M3.4q0.91	3.404	0.91	0.0994
M3.4q0.80	3.375	0.80	0.2088
M3.5q0.75	3.464	0.75	0.0802
M3.4q0.70	3.371	0.70	0.2116

The torus mass decreases with the mass ratio and with the total mass; at lowest order:

$$\widetilde{M}_{\rm tor}(q, M_{\rm tot}) = (M_{\rm max} - M_{\rm tot})\,[c_1(1-q) + c_2]$$

where $M_{\rm max}$ is the maximum (baryonic) mass of the binary and c_1, c_2 are coefficients computed from the simulations.

Gravitational waveforms



Note the waveforms are very simple with moderate modulation induced by mass asymmetry.

Furthermore, no HMNS is produced and the QNM ringing (shown by dashed vertical line) is choked by the intense mass accretion rate (the BH cannot ringdown...)

“merger → HMNS → BH + torus”

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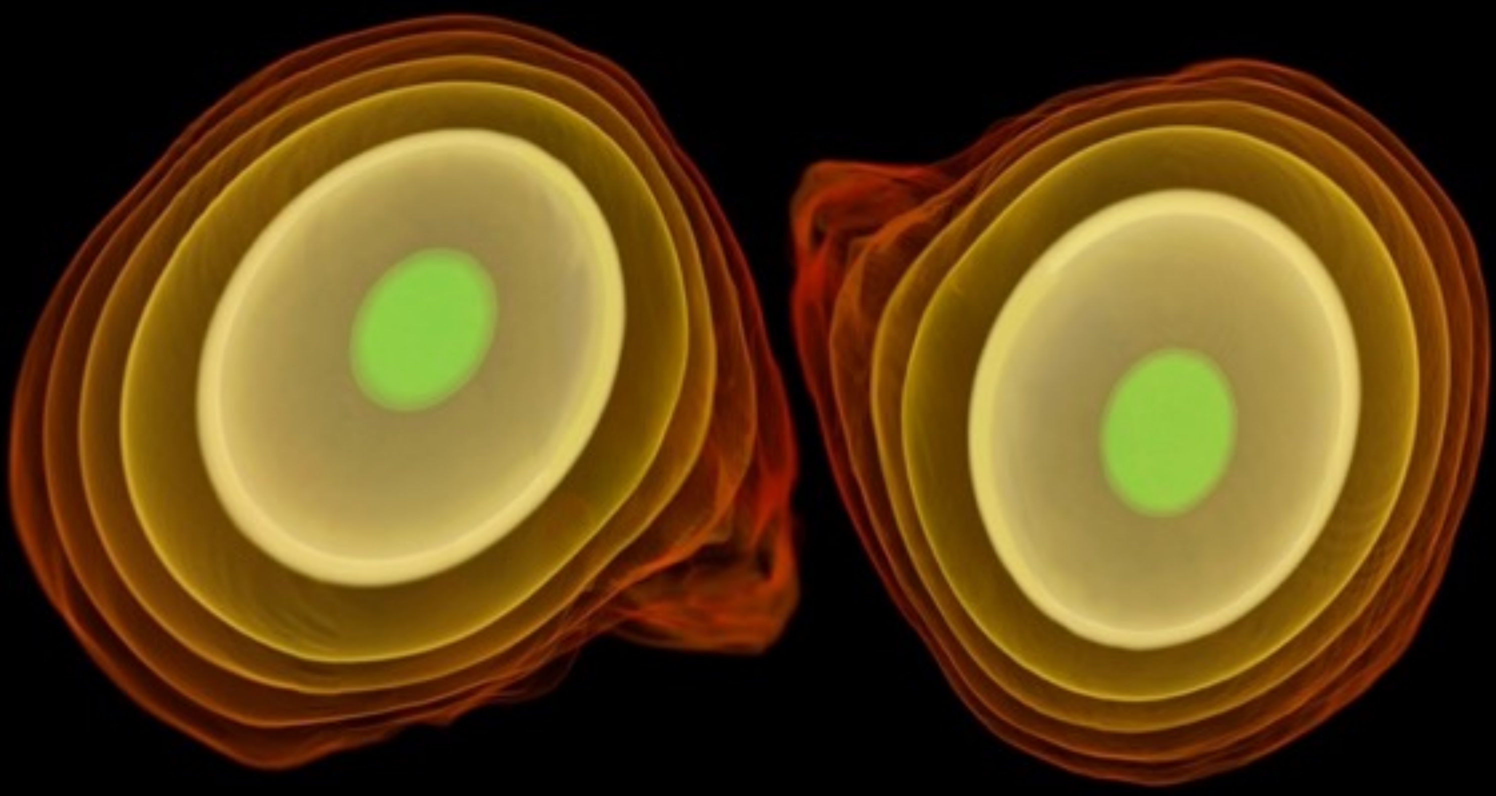
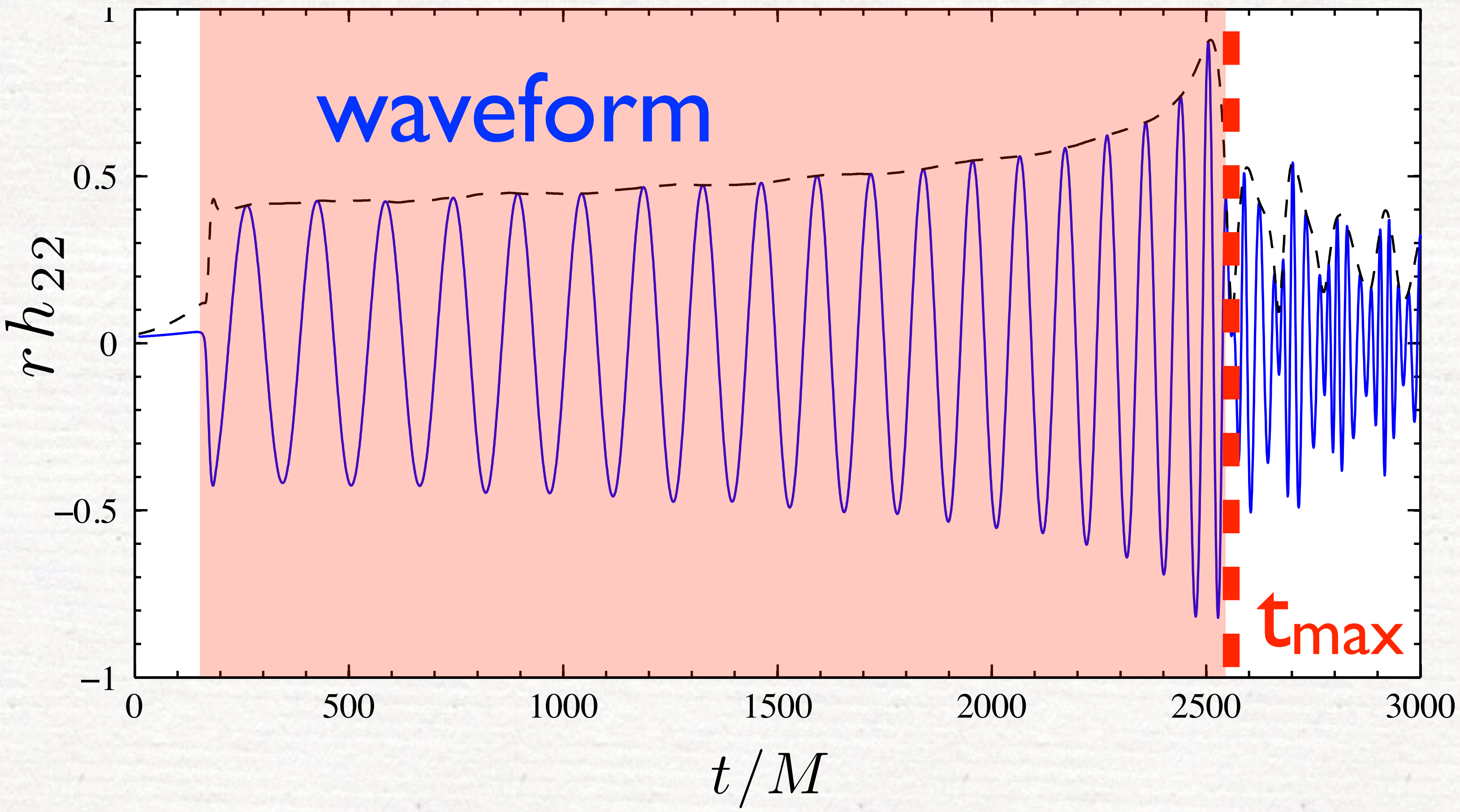
- **differences induced by MASS ASYMMETRIES:**
  tidal disruption before merger; may lead to prompt BH
- **differences induced by MAGNETIC FIELDS:**
  the angular momentum redistribution via magnetic braking or MRI can increase/decrease time to collapse; EM counterparts!
- **differences induced by RADIATIVE PROCESSES:**
  radiative losses will alter the equilibrium of the HMNS

How to constrain the EOS

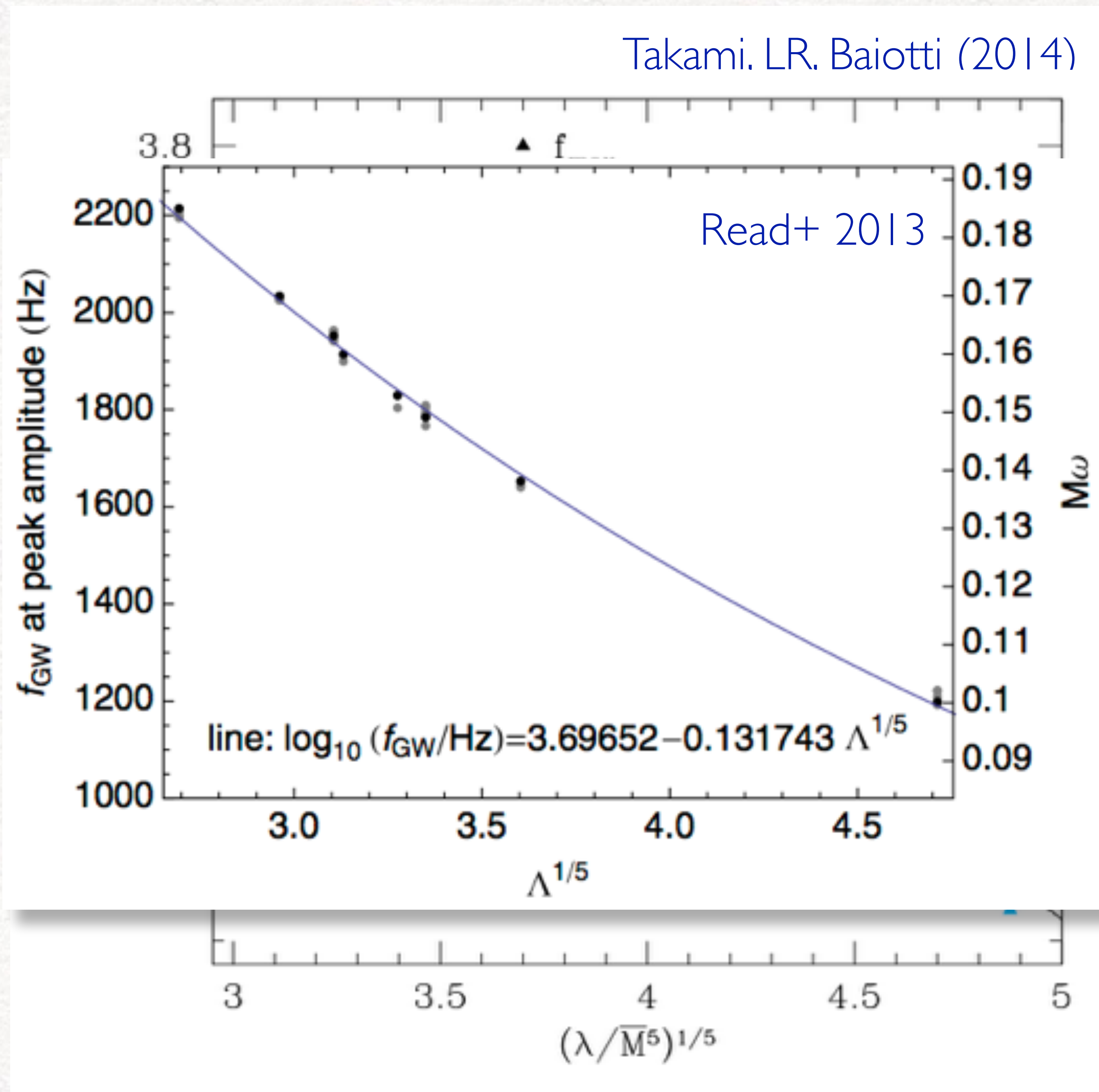


Anatomy of the GW signal

Inspiral



Hints of quasi-universality



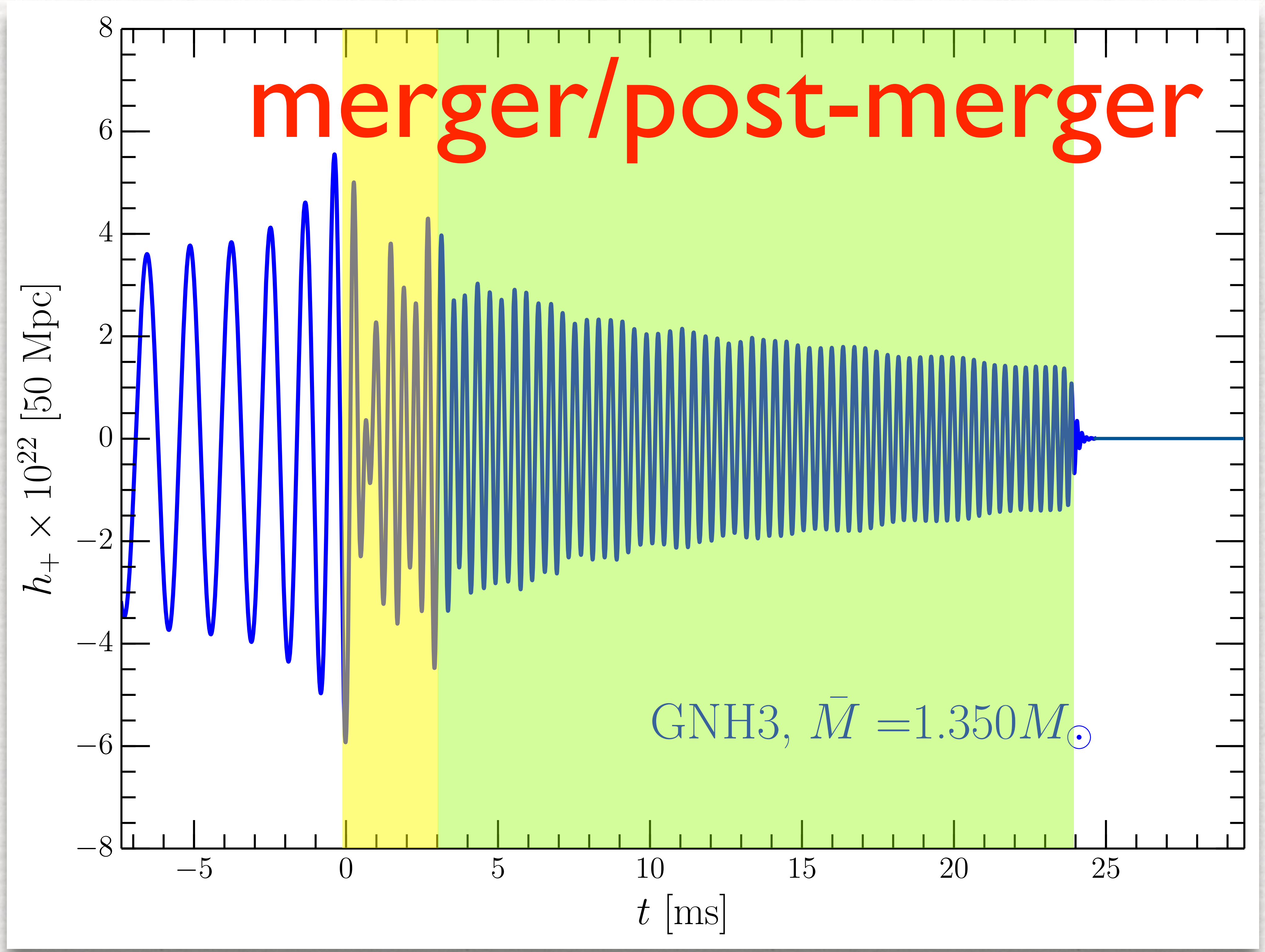
Read+, 2013, found "surprising" result: quasi-universal behaviour of GW frequency at amplitude peak

Bernuzzi+, 2014 and Takami+, 2015 confirmed with new simulations.

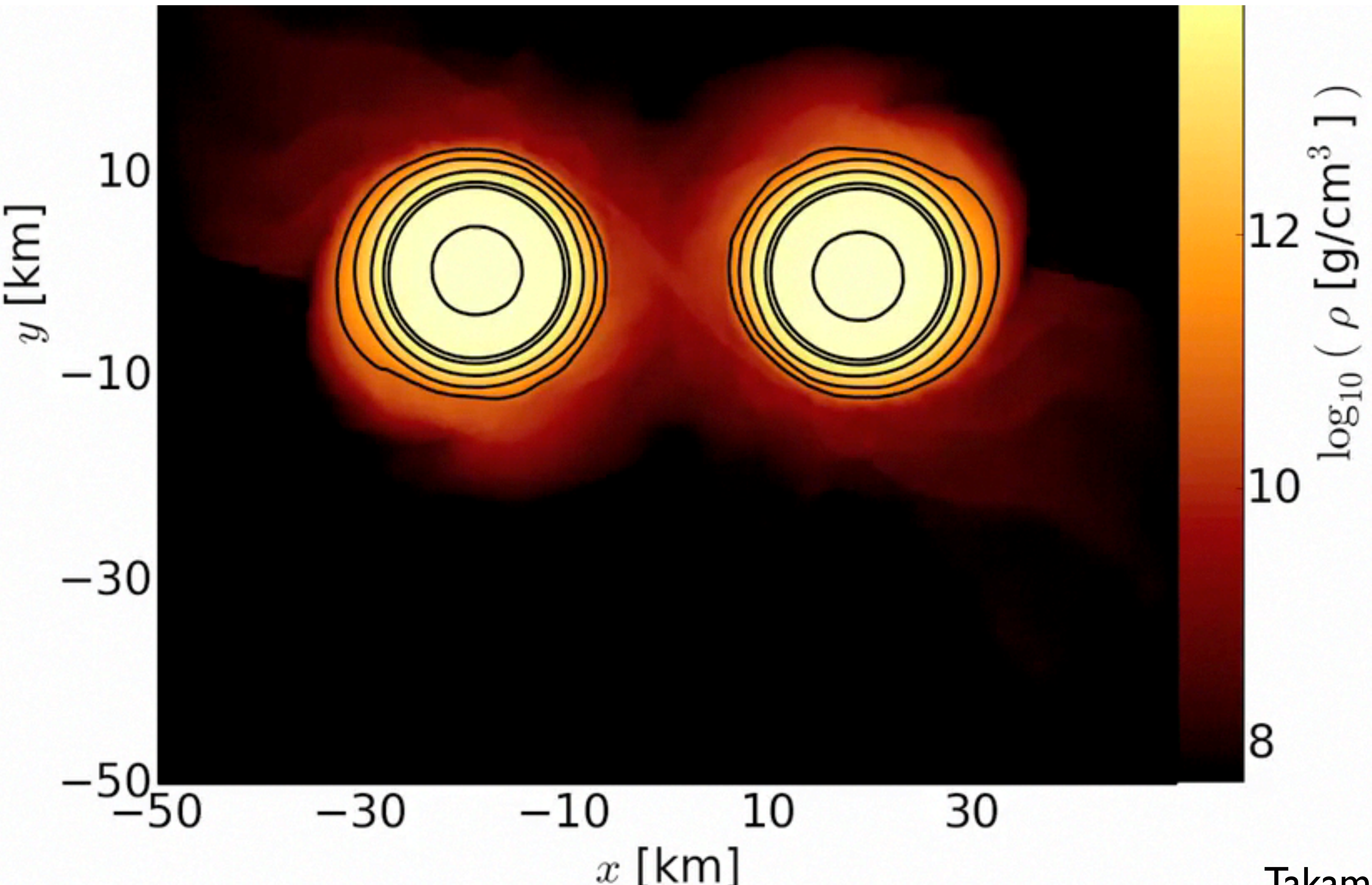
Quasi-universal properties exist in the **inspiral** of BNSs: once $\mathbf{f_{max}}$ is measured, so is tidal deformability.

$$\Lambda = \frac{\lambda}{\bar{M}^5} = \frac{16}{3}\kappa_2^T$$ tidal deformability or Love number

Anatomy of the GW signal



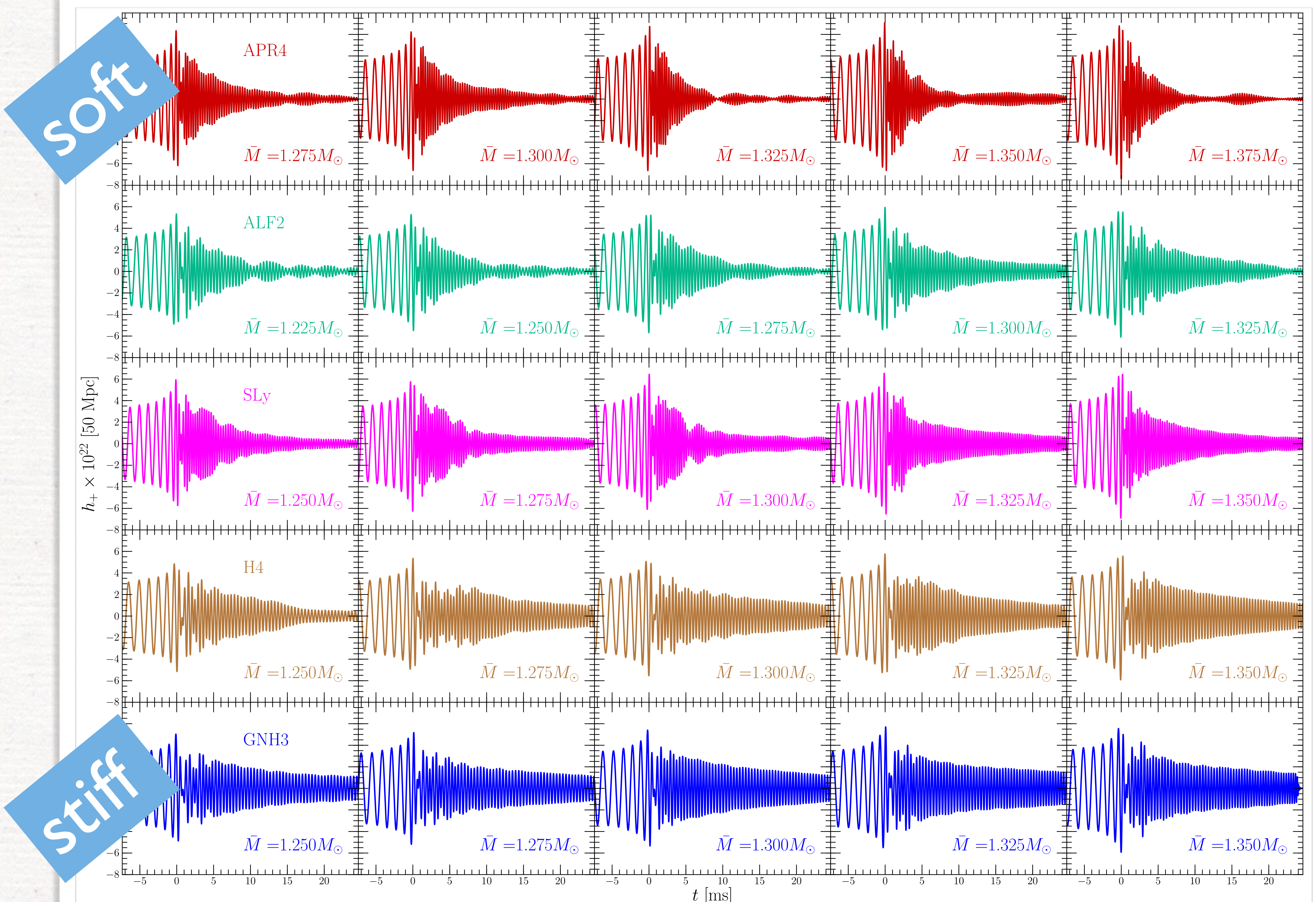
Prototypical simulation corotating frame: H4 EOS, M=1.30 $M_\odot$,



Takami, LR

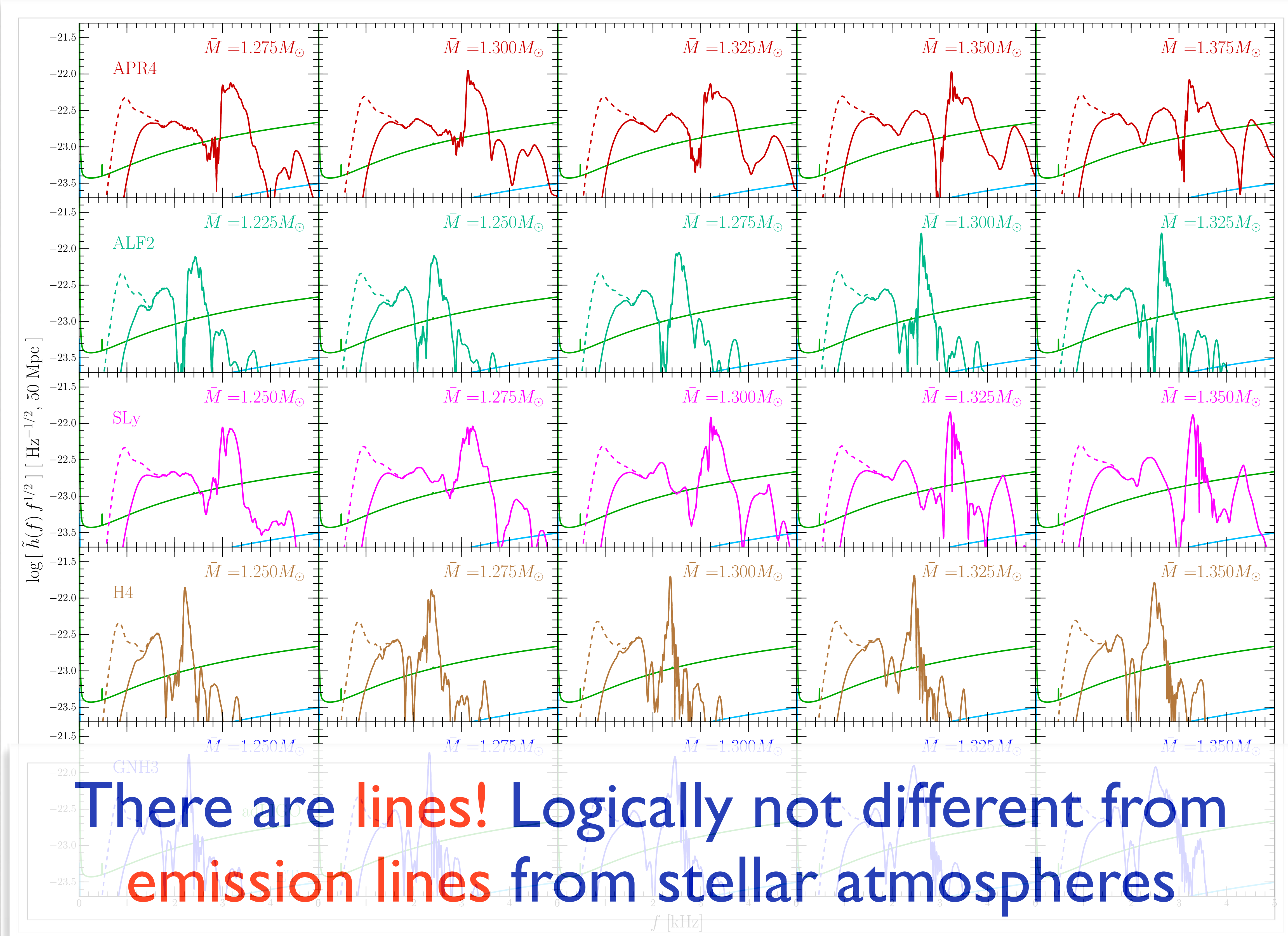
realistic EOSs

Takami, LR, Baiotti, 2014a, 2014b



extracting information from the EOS

Takami, LR, Baiotti, 2014

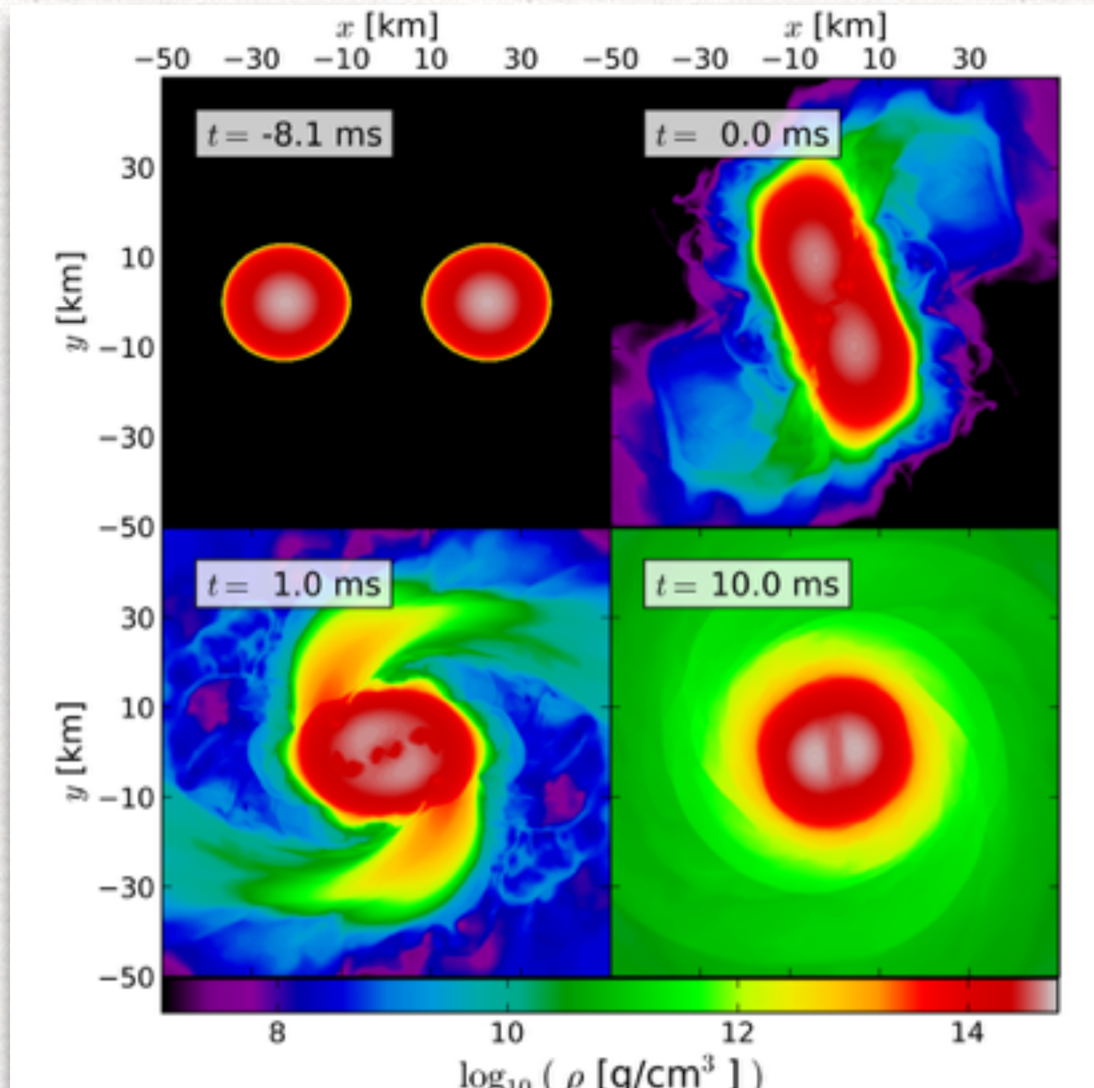


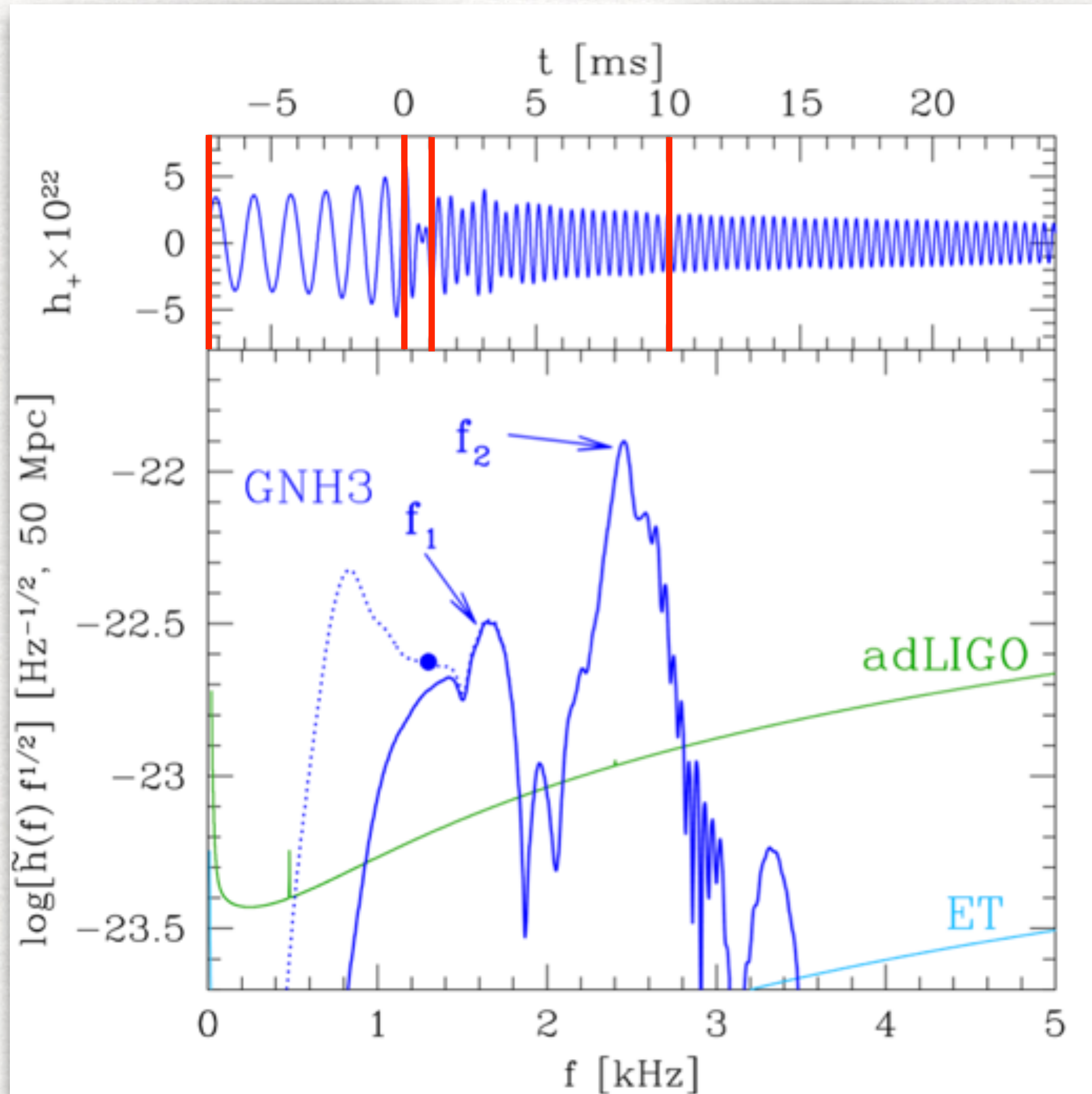
There are lines! Logically not different from emission lines from stellar atmospheres

A new approach to constrain the EOS

Takami, LR, Baiotti (2014)

We have carried out numerical-relativity simulations of NS binaries with nuclear EOS and thermal contribution via ideal-fluid contribution





A new approach to constrain the EOS

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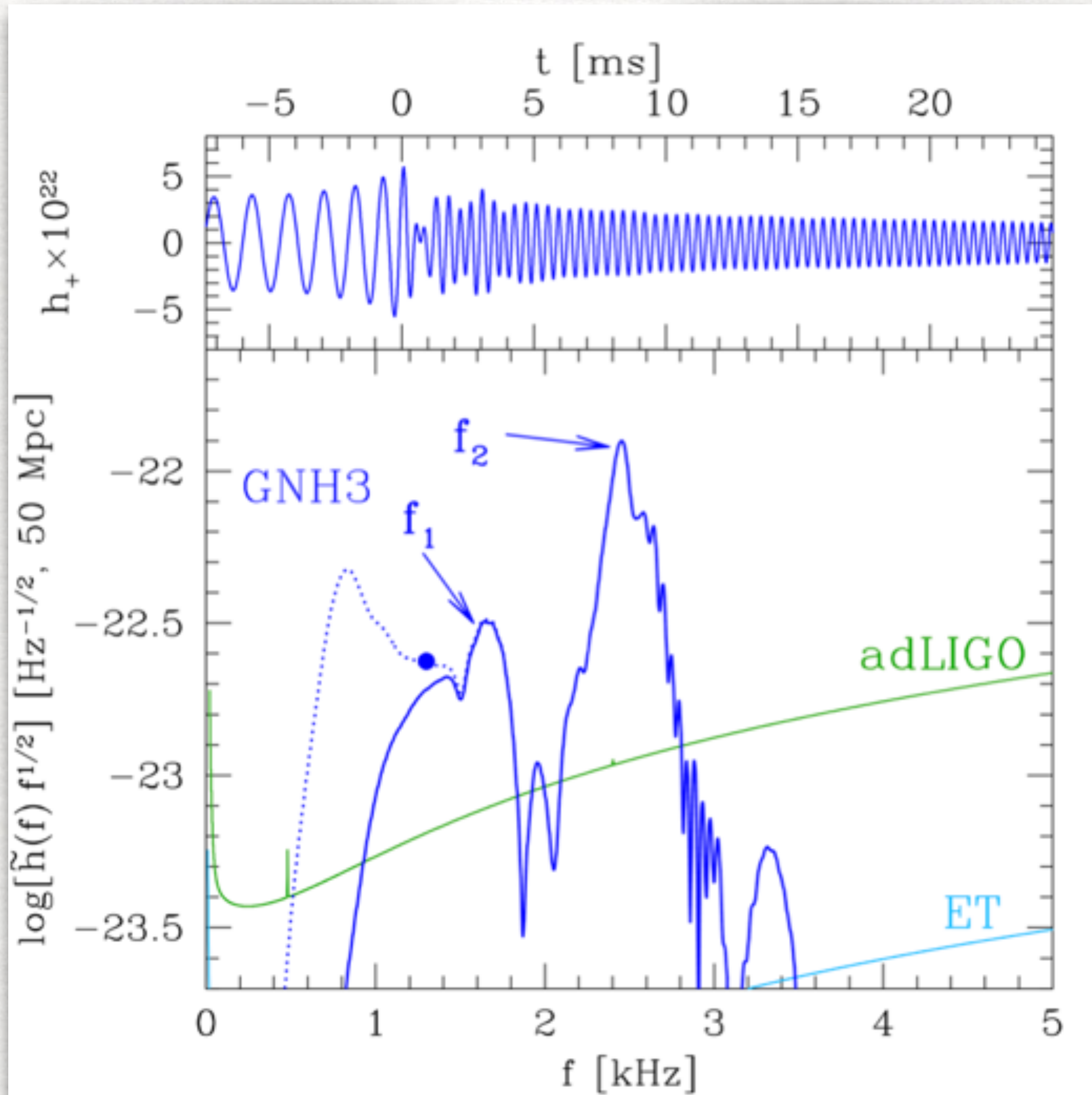
PSD of post-merger GW signal has number of peaks (Oechslin+2007, Baiotti+2008)

The high-freq. peak (f_2) been studied carefully and produced by HMNS (Bauswein+ 2011, 2012, Stergioulas+ 2011, Hotokezaka+ 2013)

The low-freq. peak (f_1) is related to the early post-merger phase



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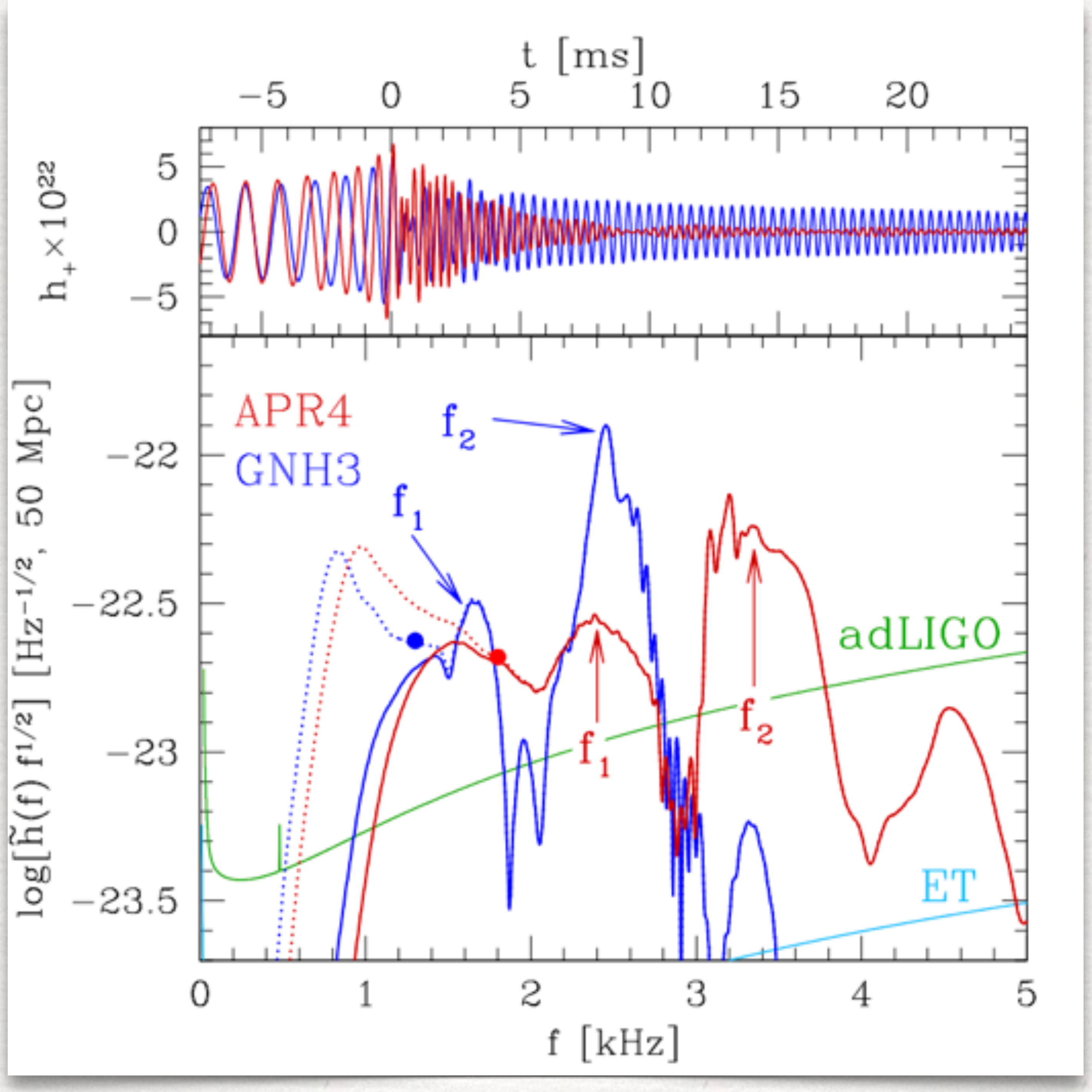
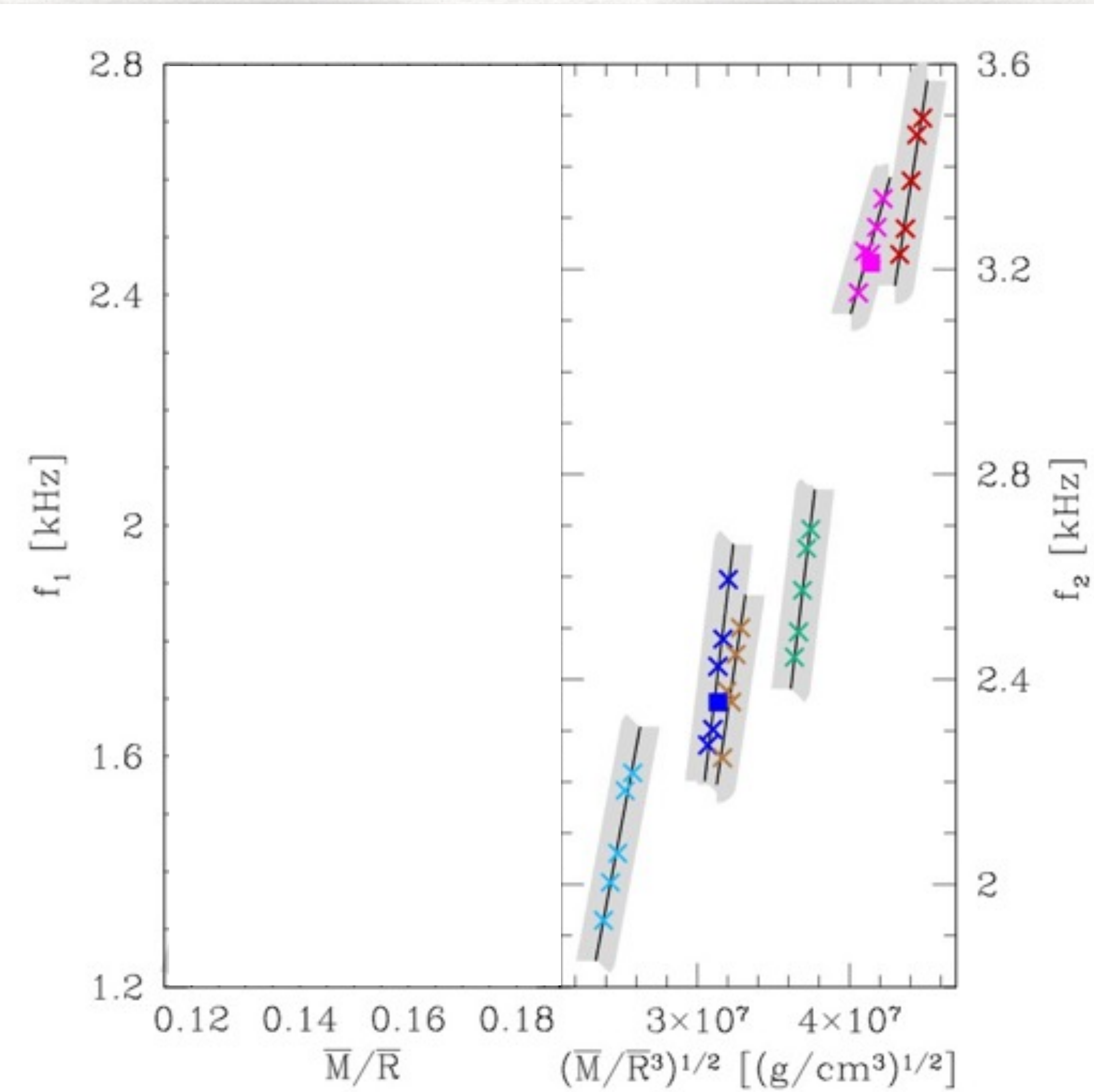


A new approach to constrain the EOS

It is possible to correlate the values of the peaks with the properties of the progenitor stars, i.e. M, R, and combinations thereof.



Each cross refers to a given mass and crosses of the same color refer to the same EOS

The high-freq. peak f_2 has been shown to correlate with stellar properties, e.g., R_{max}, $R_{1.6}$,. etc (Bauswein+ 2011, 2012, Hotokezaka+ 2013).

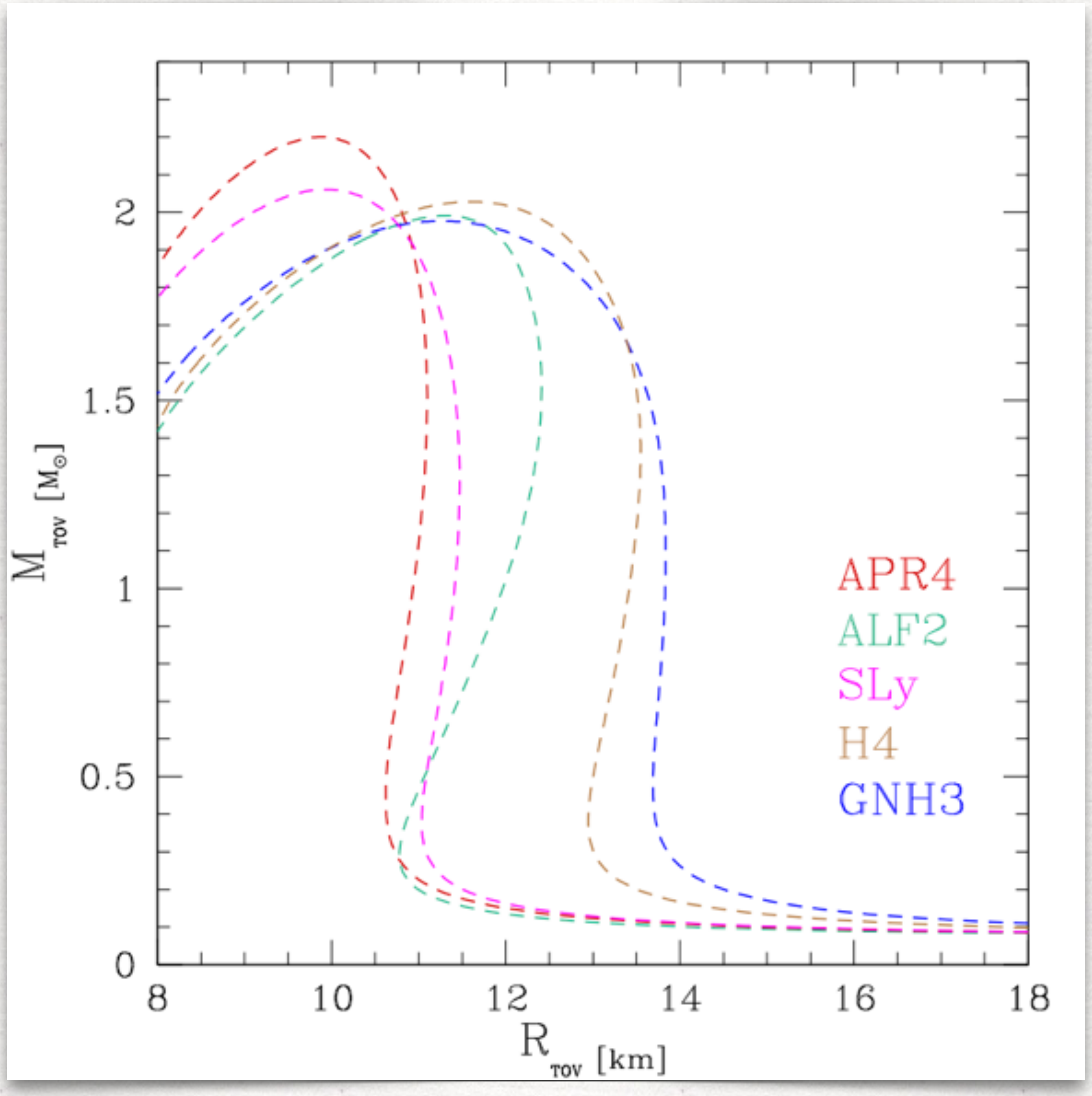
The correlation **depends** on mass

The low-freq. peak f_1 shows a much tighter correlation; most importantly, it **does not depend** on the EOS

An example: start from equilibria

Assume that the GW signal from a binary NS is detected and with a SNR high enough that the two peaks are clearly measurable.

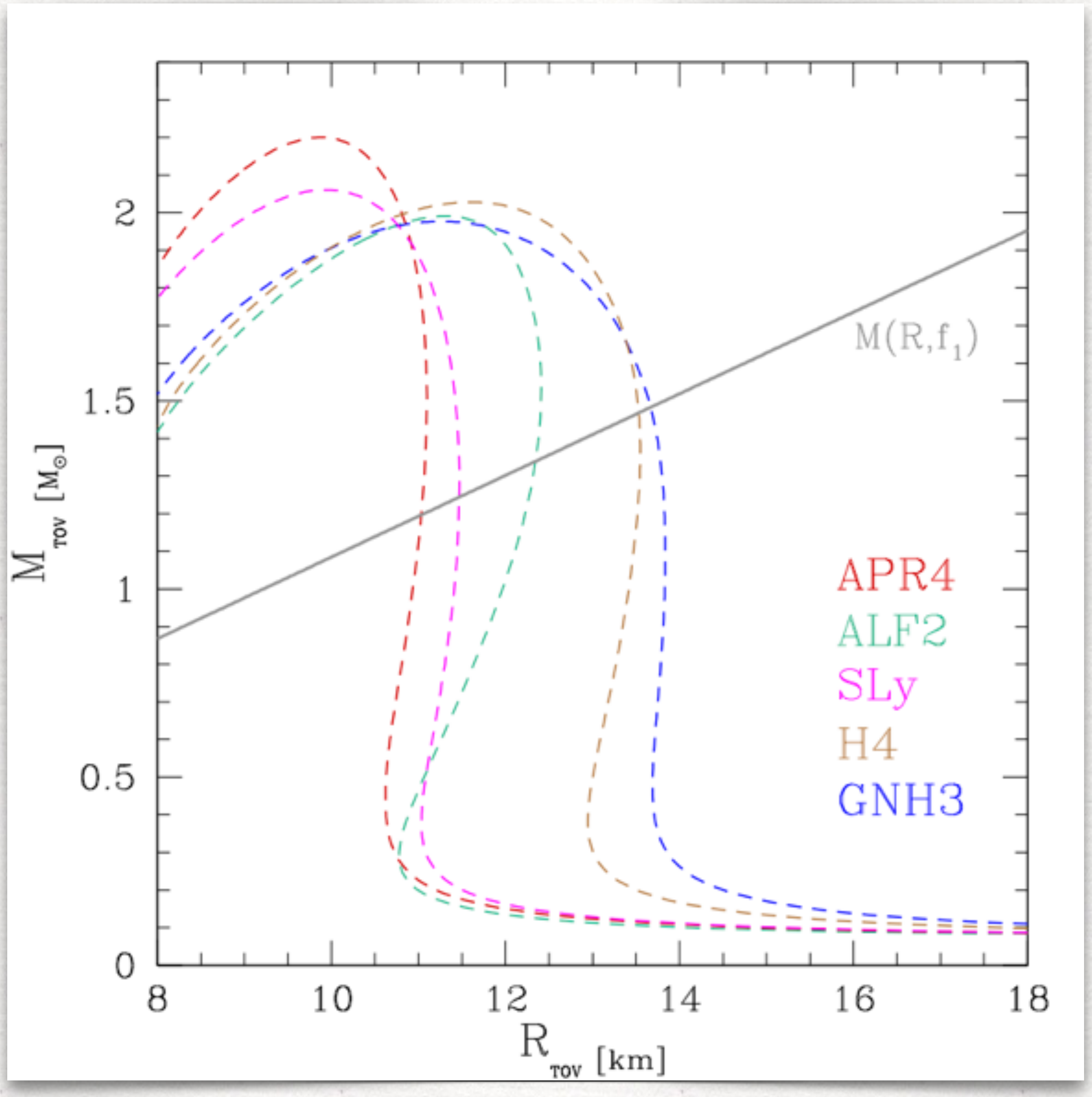
Consider your best choices as candidate EOSs



An example: use the $M(R,f_l)$ relation

The measure of the f_l peak will fix a $M(R,f_l)$ relation and hence a **single** line in the (M, R) plane.

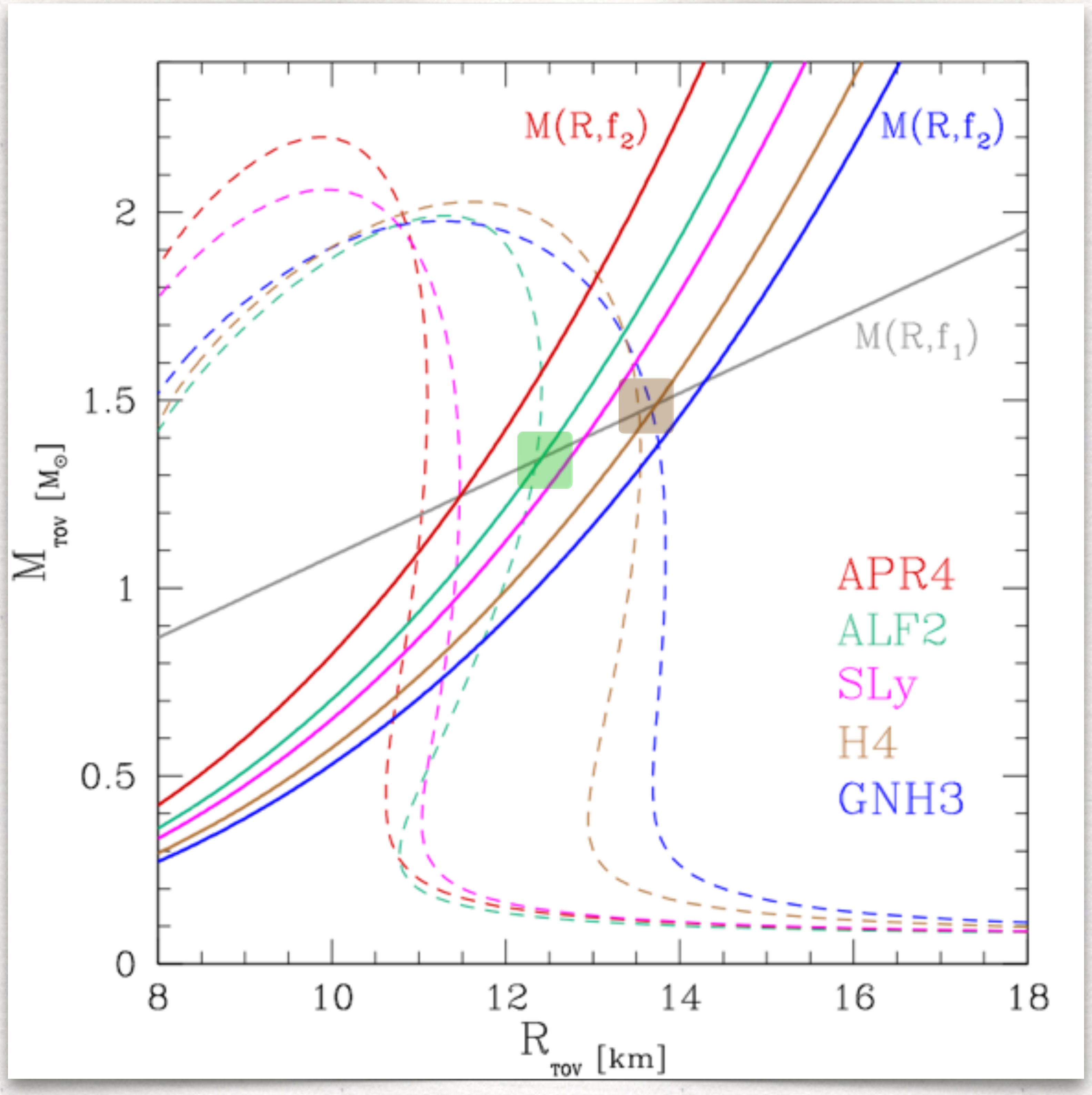
All EOSs will have **one** constraint (crossing)



An example: use the $M(R, f_2)$ relations

The measure of the f_2 peak will fix a relation $M(R, f_2, EOS)$ for each EOS and hence a **number** of lines in the (M, R) plane.

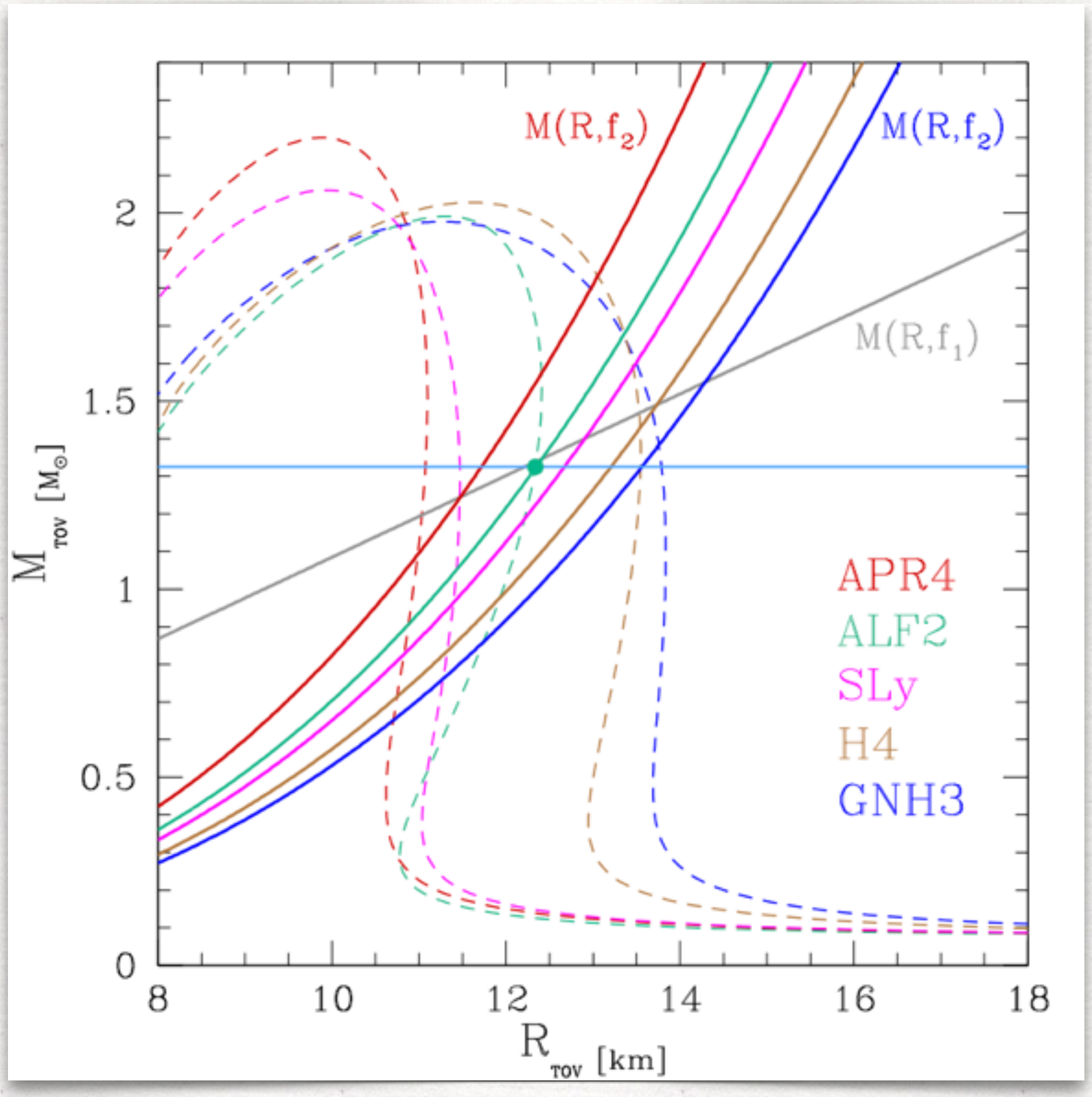
The right EOS will have **three** different constraints (APR, GNH3, SLy excluded)



An example: use measure of the mass

If the mass of the binary is measured from the inspiral, an additional constraint can be imposed.

The right EOS will have **four** different constraints. Ideally, a single detection would be sufficient.

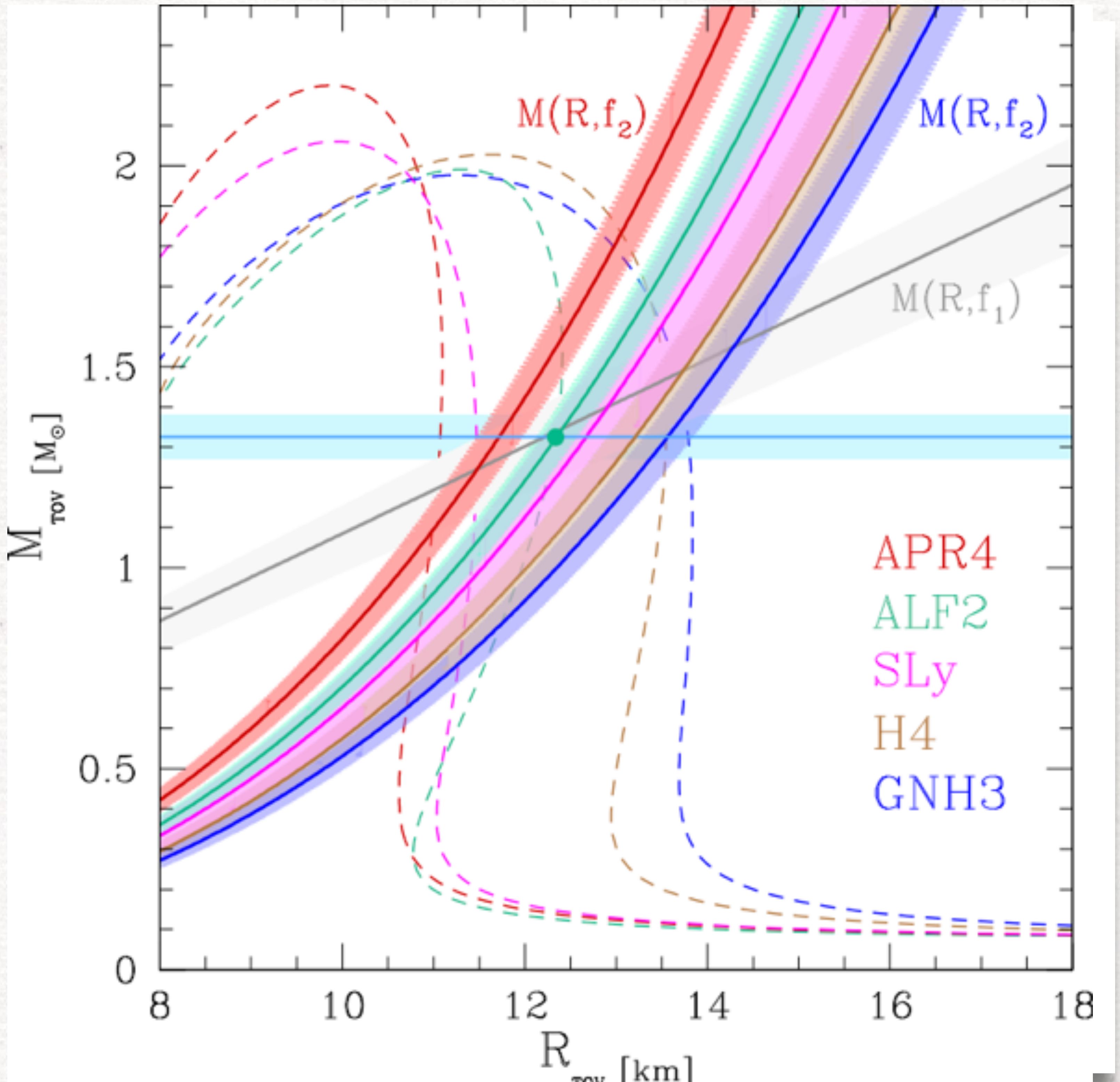


This works for all EOSs considered

In reality things will be more complicated. The **lines** will be **stripes;** Bayesian probability to get precision on *M, R*.

Some numbers:

- at 50 Mpc, freq. uncertainty from Fisher matrix is 100 Hz
- at SNR=2, the event rate is 0.2-2 yr^{-1} for different EOSs.

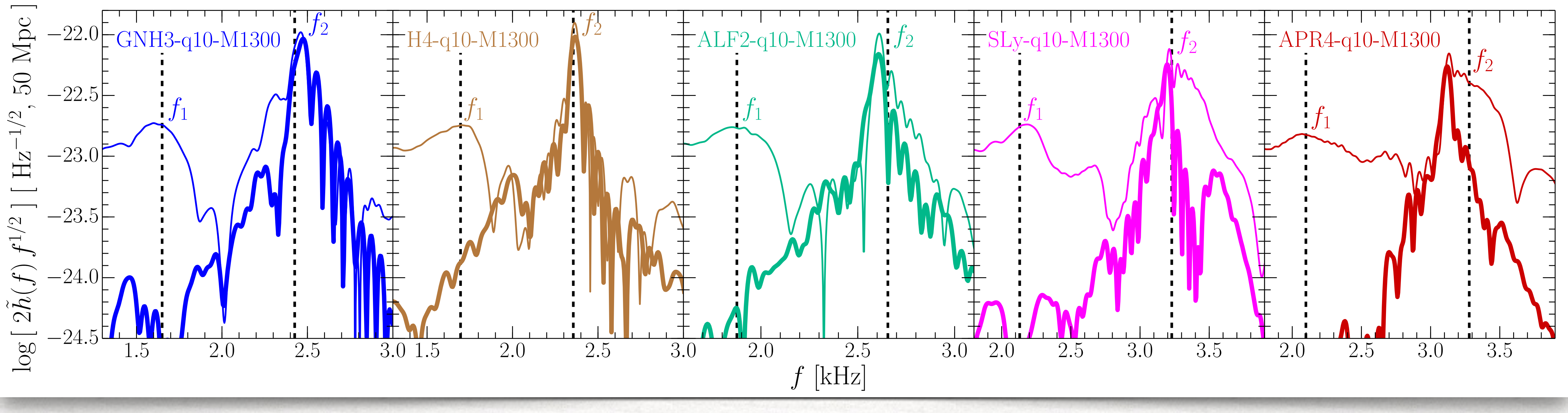


What produces the peaks?

- f_2 peak obviously related to long-term periodic rotation of bar-deformed HMNS (l=2=m fundamental oscillation Shibata 05, Baiotti+ 08, Bauswein+ 11, 12, Stergioulas+ 11, Hotokezaka+ 13).

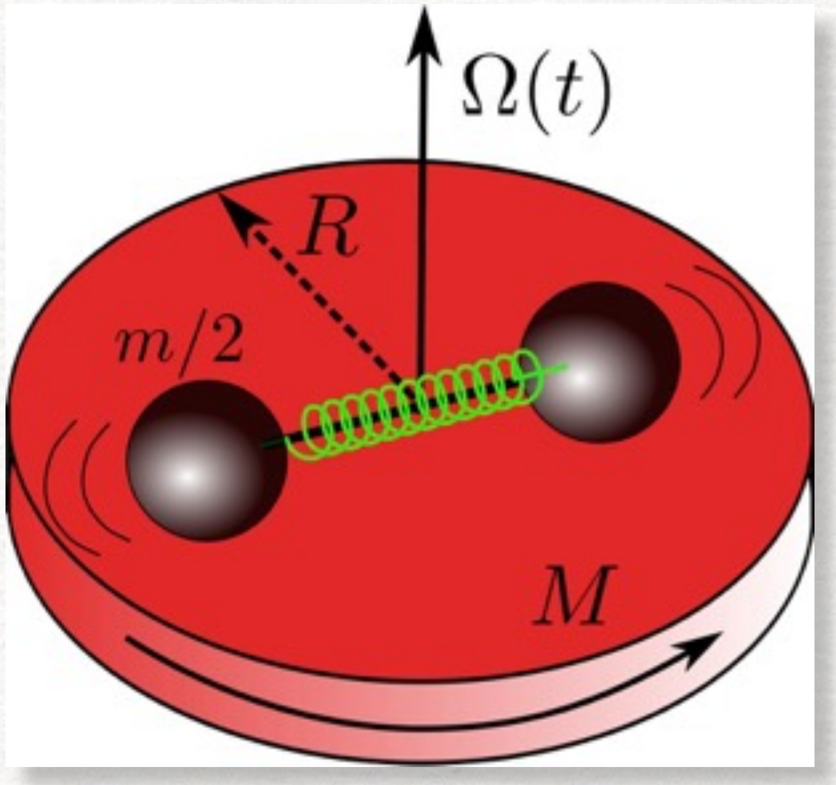
- f_1 peak is less obvious but there is a possible explanation.

- Thick lines are PSDs when first 3ms are removed.

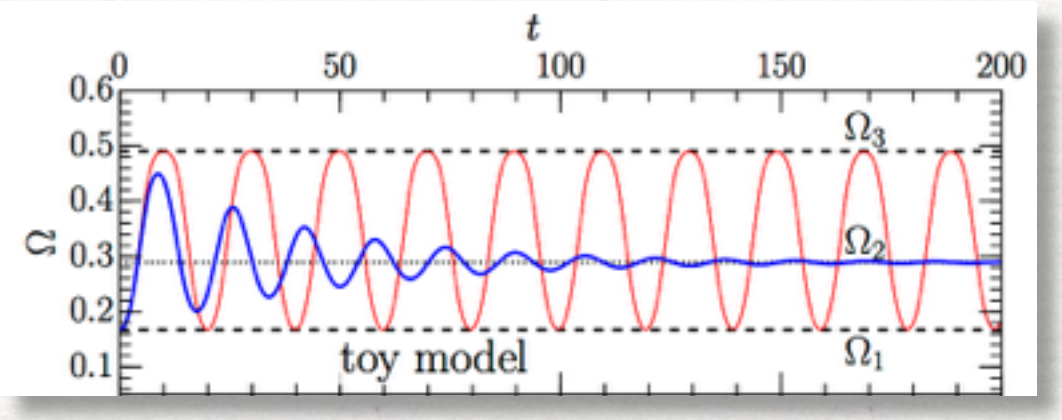


- **f_1 is formed only in short time window after merger!**

A mechanical toy model

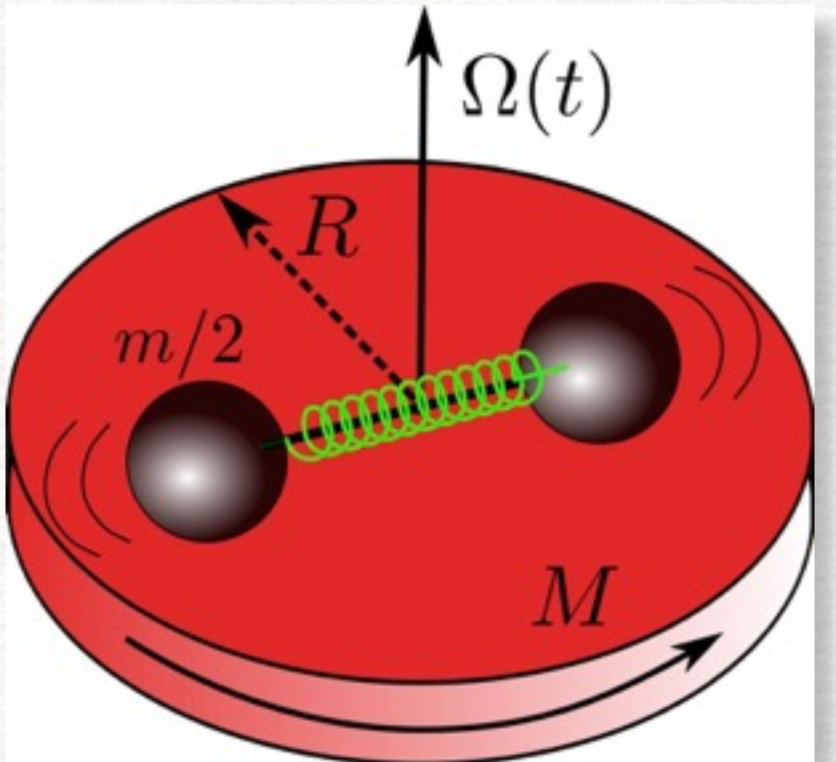


- Consider disk with 2 masses moving along a shaft and connected via a spring ~ HMNS with 2 stellar cores
- Let disk rotate and mass oscillate while conserving angular momentum



- If no friction is present, system will spin between two freqs: low (**f_1**) when masses are far apart, and high (**f_3**) when masses are close.
- If friction is present, system will tend asymptotically to spin at frequency **$f_2 \sim (f_1+f_3)/2$**.

A mechanical toy model



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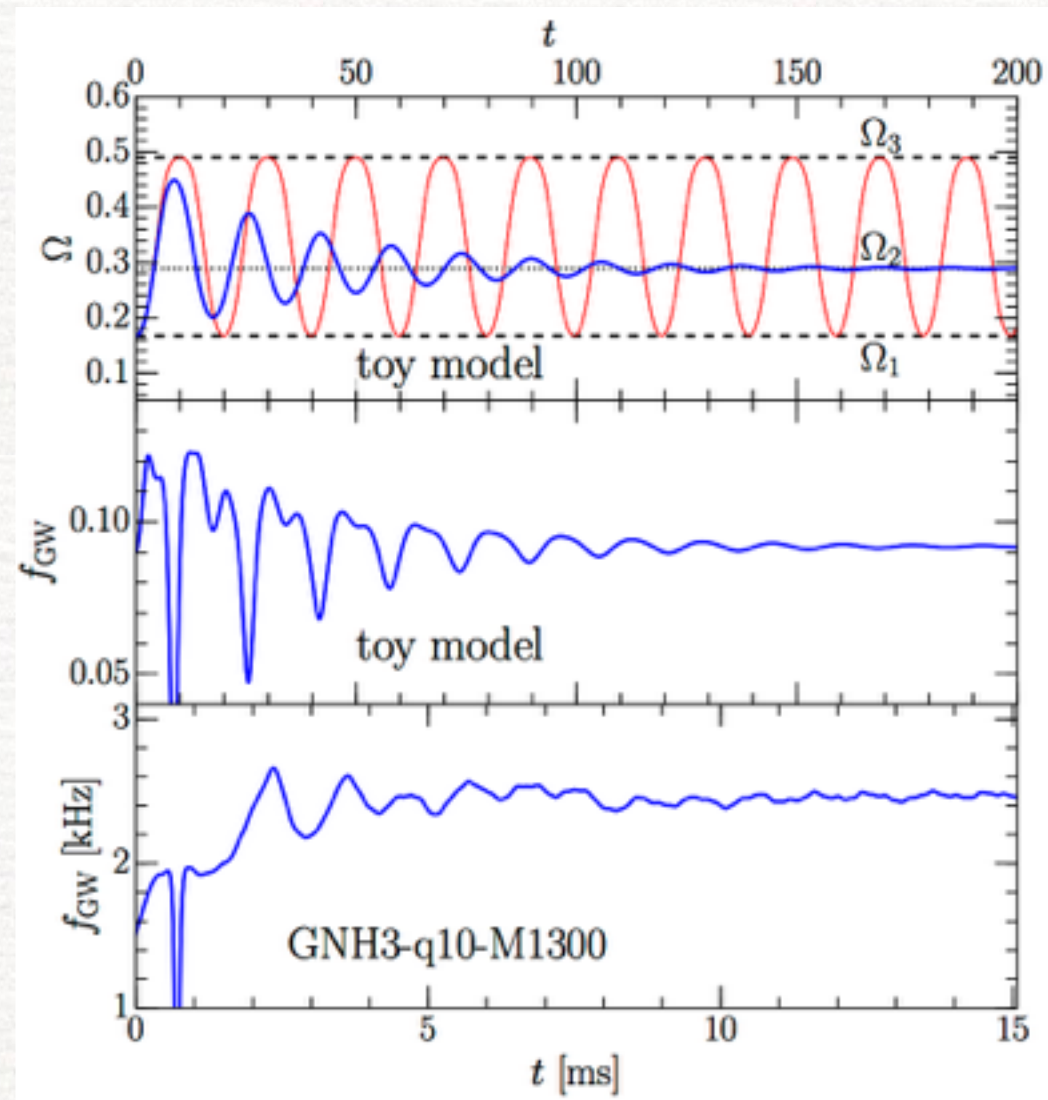

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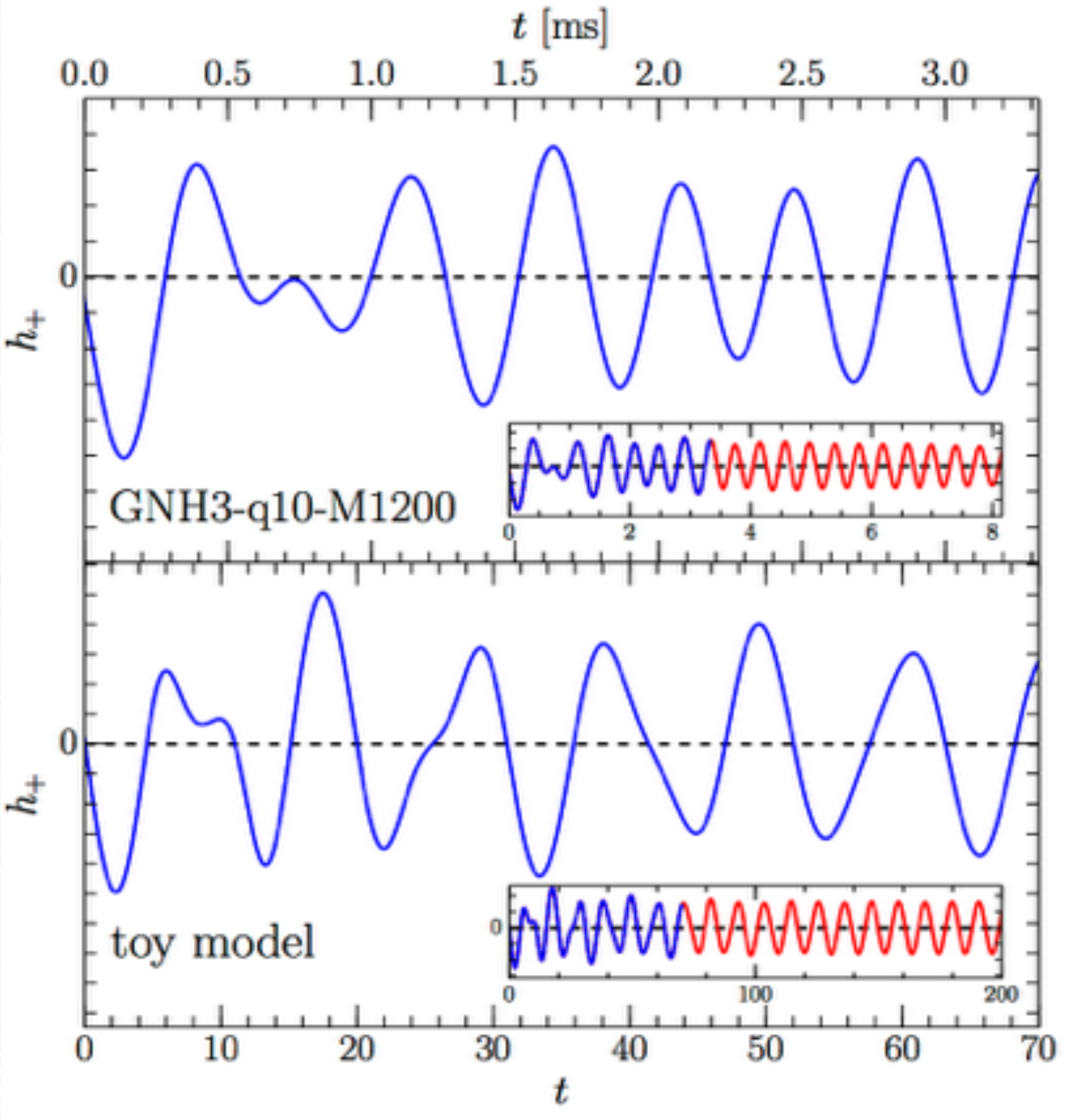
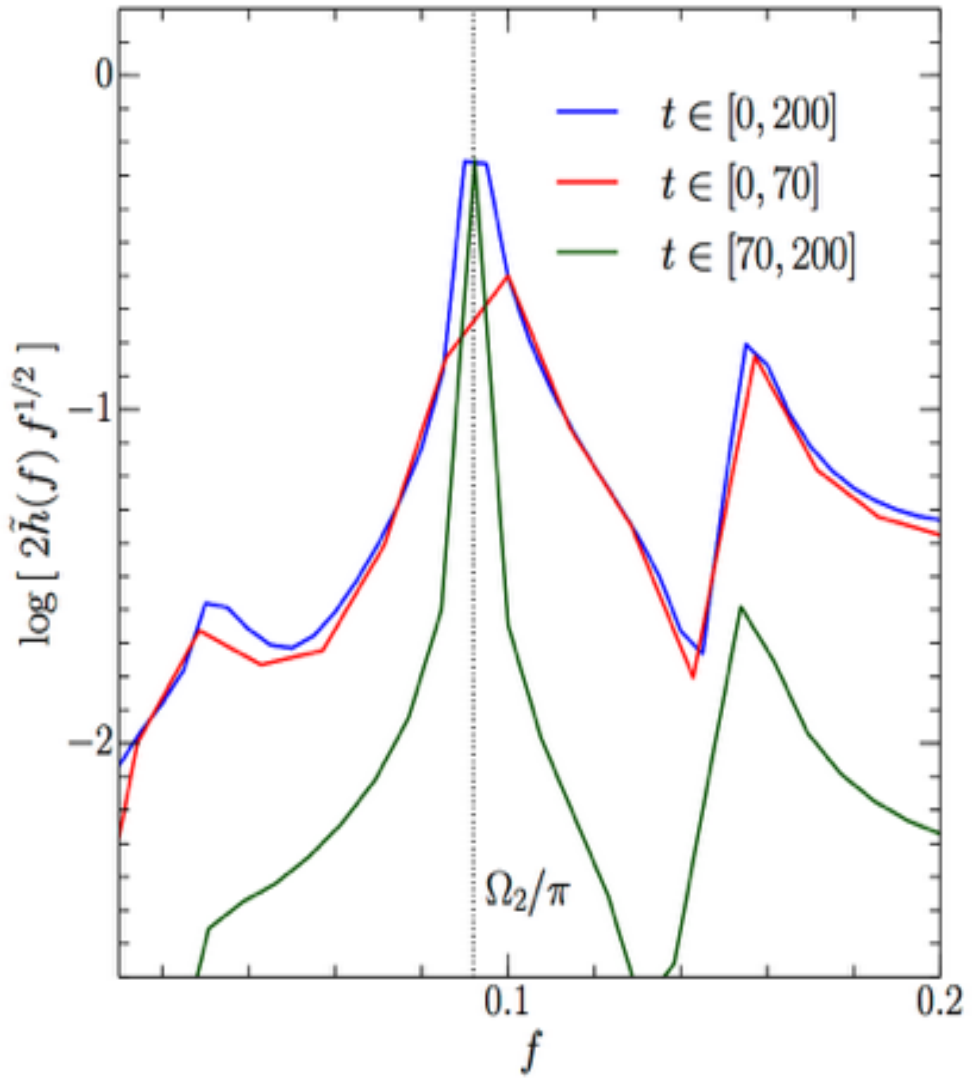
A mechanical toy model

The system emits GWs with features that can be computed via quadrupole formula

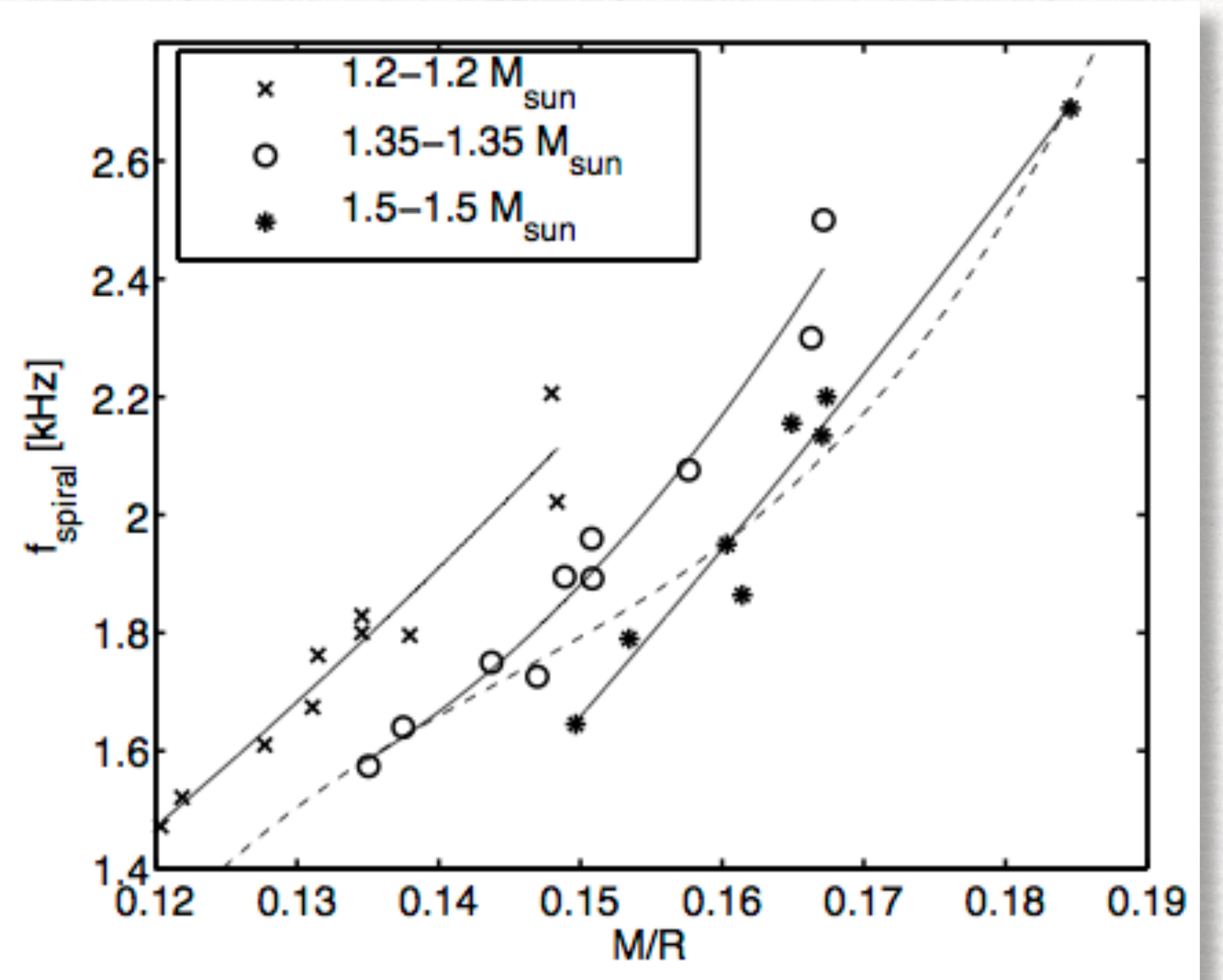




Also in this case: three peaks present and low-frequency peak disappears after transient.

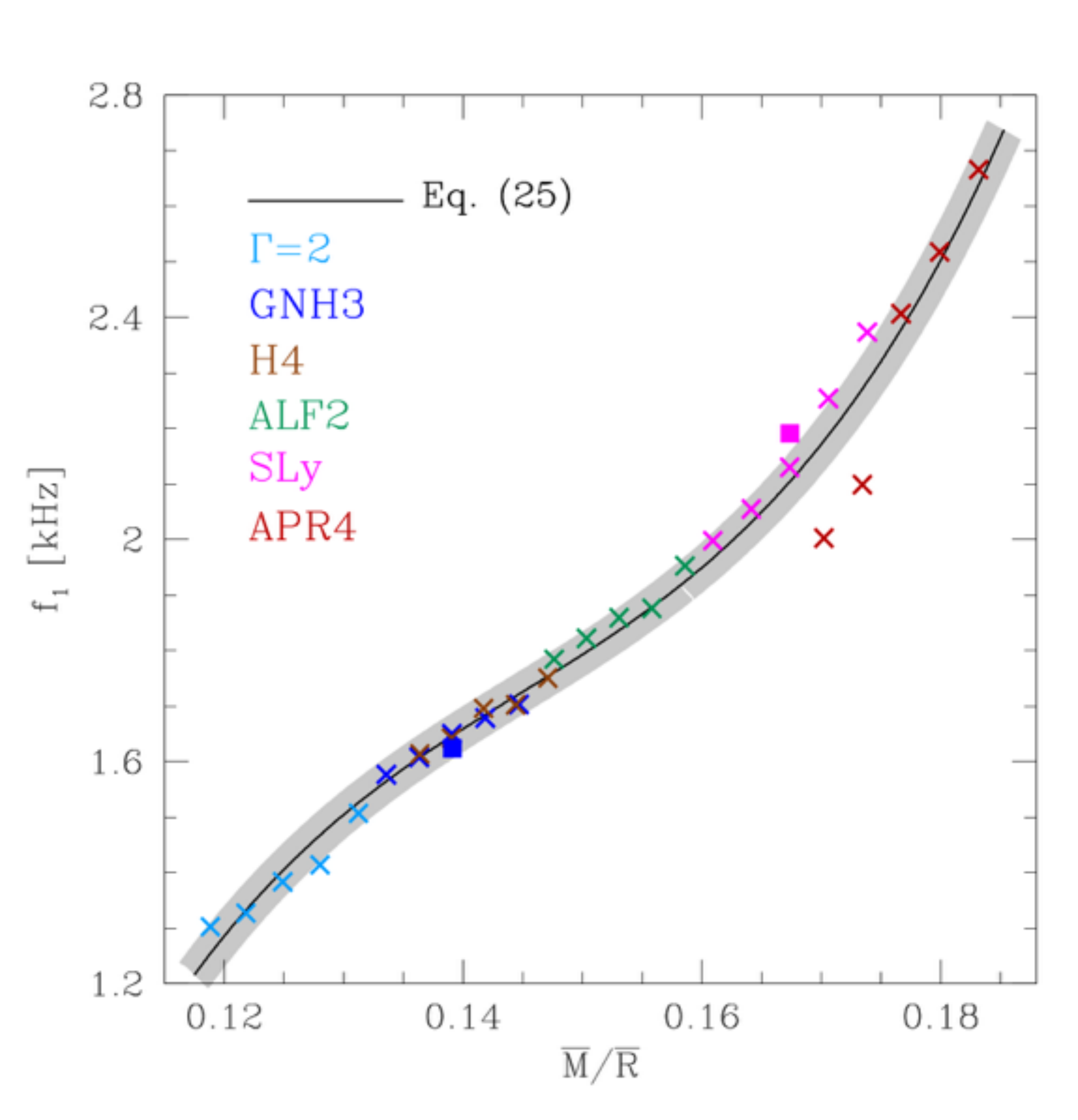
Quasi-universal or not?

- Consensus there is "quasi-universality" in **inspiral**.
- Recent calculations (Bauswein and Stergioulas 2015) suggest that there is no quasi-universal behaviour for f_1 (f_{spiral}).



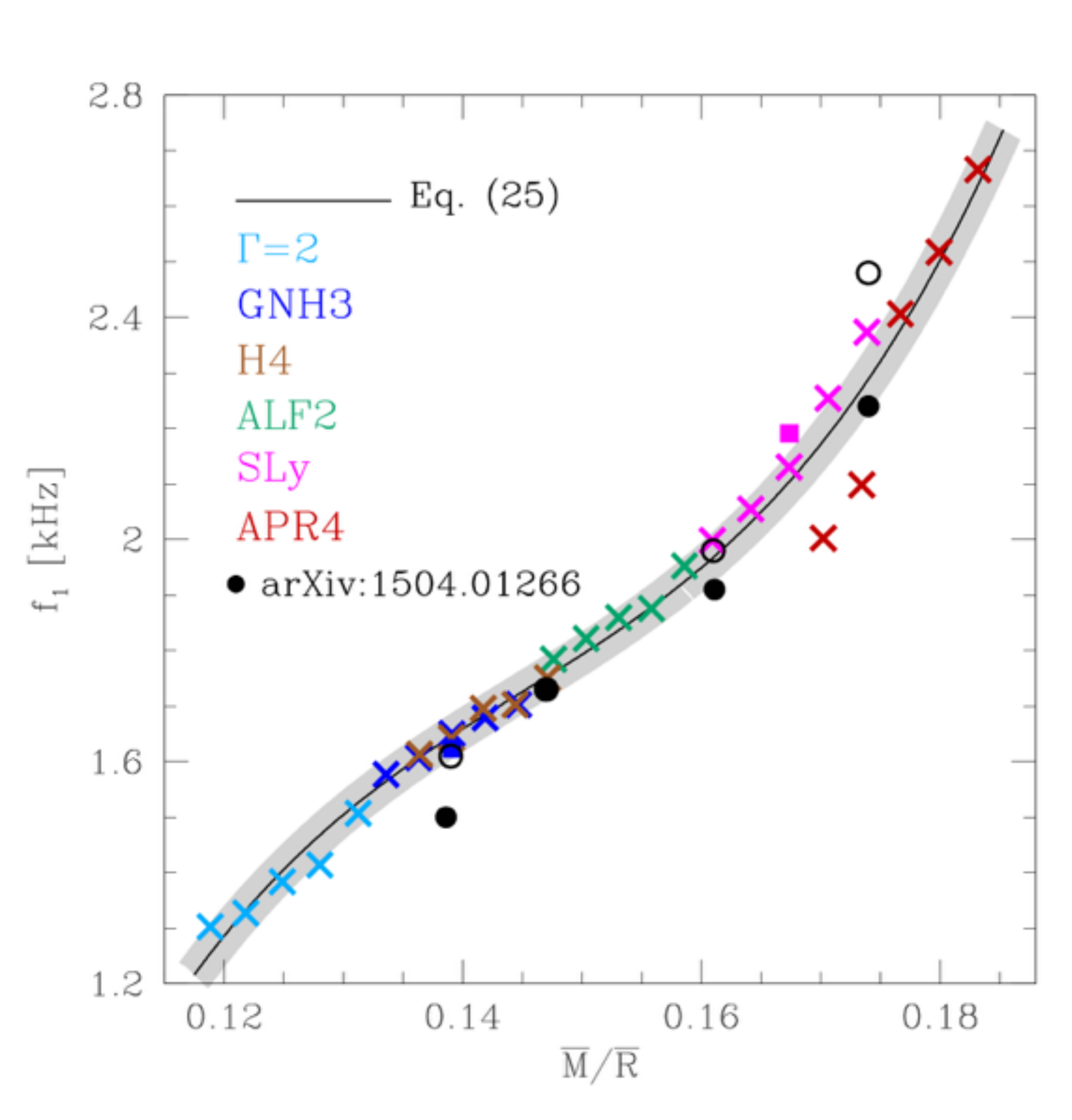
- Also interpreted peak in PSD as coupling between the quadrupolar mode f_2 (f_{peak}) and axisymmetric quasi-radial mode: $f_{2\text{-}0}$.
- Given the scatter, this may be a matter of definition.

Quasi-universal or not?



Identification of mode in PSD is clearly very delicate, especially for f_1 which is created in short time window.

Quasi-universal or not?



Identification of mode in PSD is delicate, especially for **f_I** which is created in short time window.

Recent calculations of Bernuzzi+ 1504.01266 seem to confirm the quasi-universality.

Is universality lost at very low masses?

Is SPH not accurate enough to measure **f_I?**

Conclusions

✶Modelling of binary NSs in full GR is **mature**: GWs from the inspiral can be computed with precision of binary BHs.

✶Spectra of post-merger shows clear peaks: cf lines for stellar atmospheres. Some peaks are **”universal”.**

✶If observed, post-merger signal will set tight constraints on EOS.

Binary neutron stars are a rich lab of physics and astrophysics. Numerical relativity is a perfect tool to explore it.