Gravitational Wave as a Probe of Dark Matter

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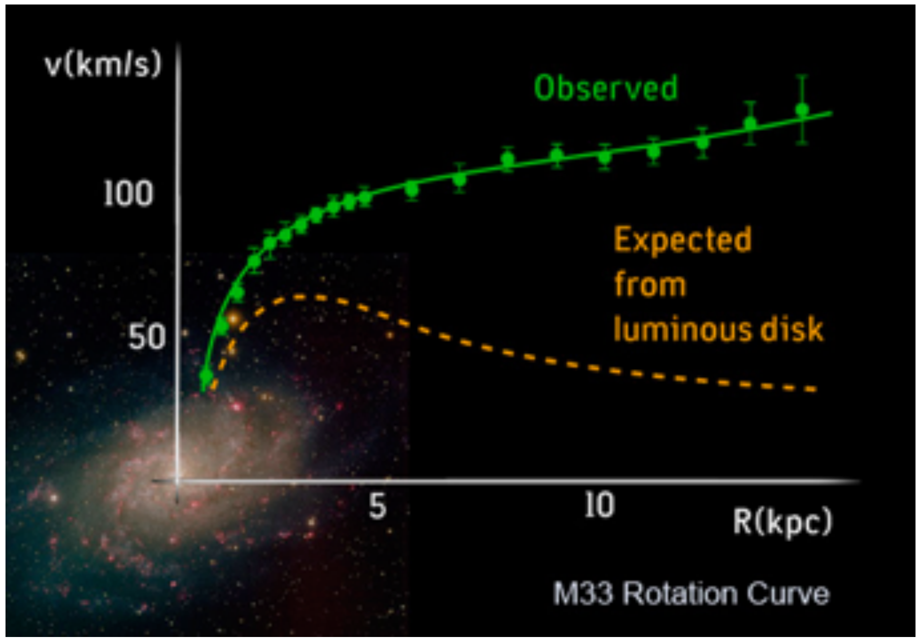
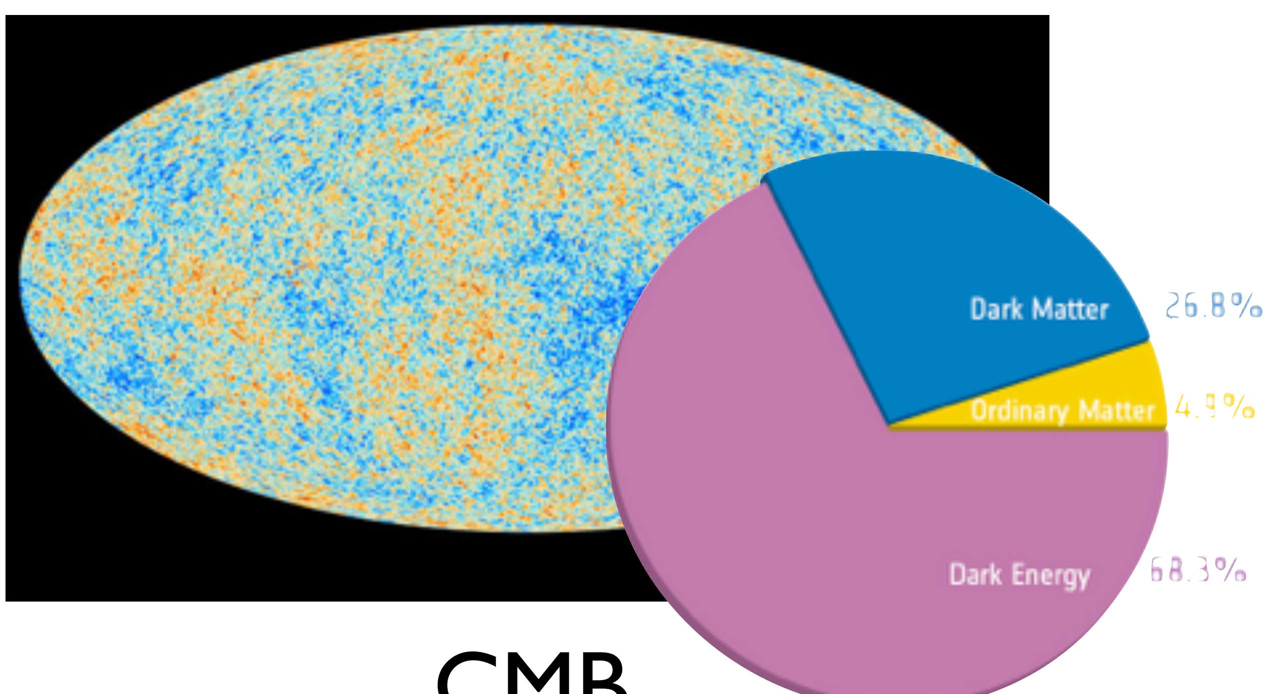
Reference: K. Eda et al. PRL 110, 221101 (2013)
PRD 91, 044045 (2015)

Dark matter - evidences

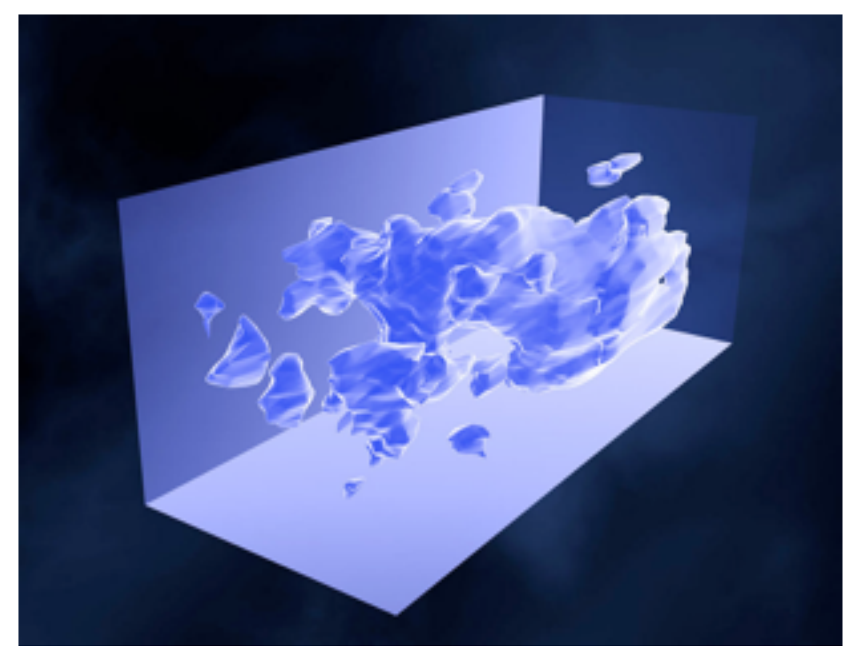


Galaxy rotation curve

The Bullet cluster



CMB



Lensing map

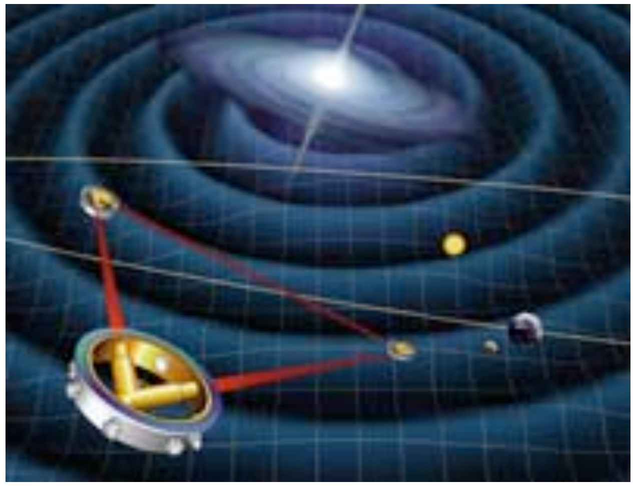
Unsolved problems

- **What is dark matter?**
  - Particles?
    - Weakly interacting? → Direct detection experiment
    - Annihilation? → Gamma-ray observation
    - Cold or Warm? → Galaxy survey
- **How did they evolve and how do they distribute in the Universe?**
  - Key to understanding the overall evolution and structure of the Universe

Unsolved problems

- **What is dark matter?**
  - Particles?
    - Weakly interacting? → Direct detection experiment
    - Annihilation? → Gamma-ray observation
    - Cold or Warm? → Galaxy survey
- **How did they evolve and how do they distribute in the Universe?**

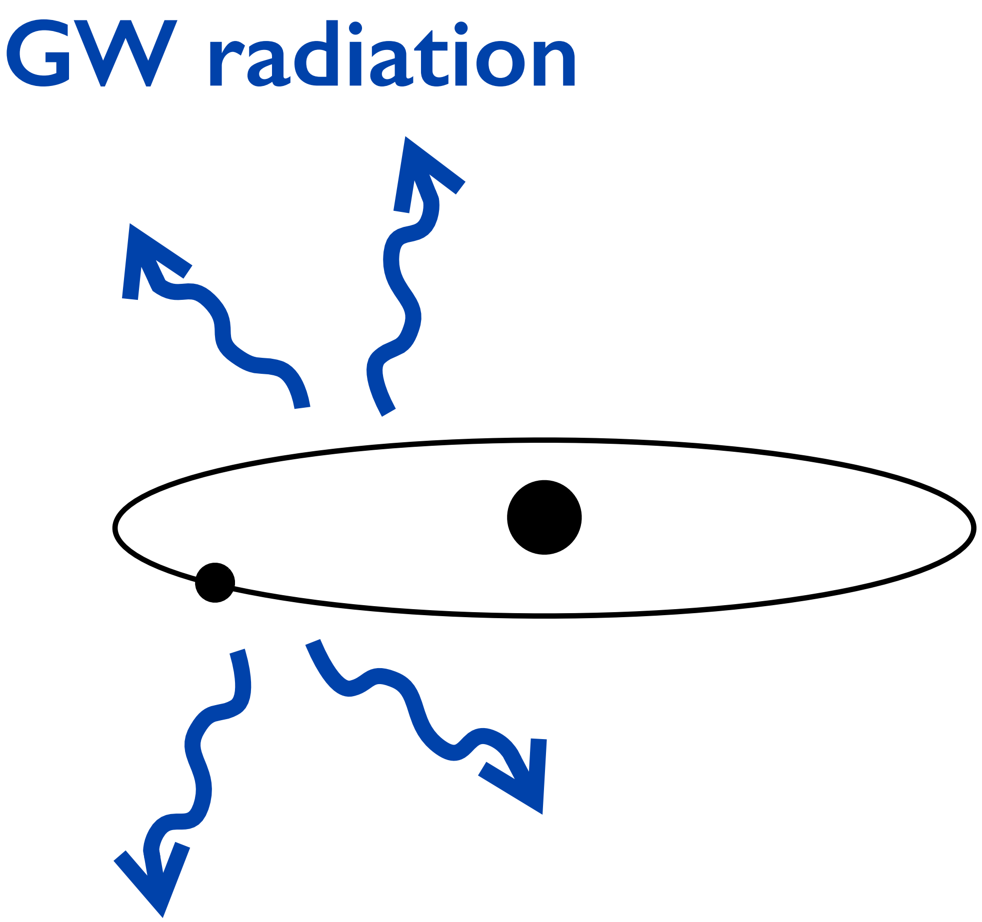
Gravitational wave experiments may help!



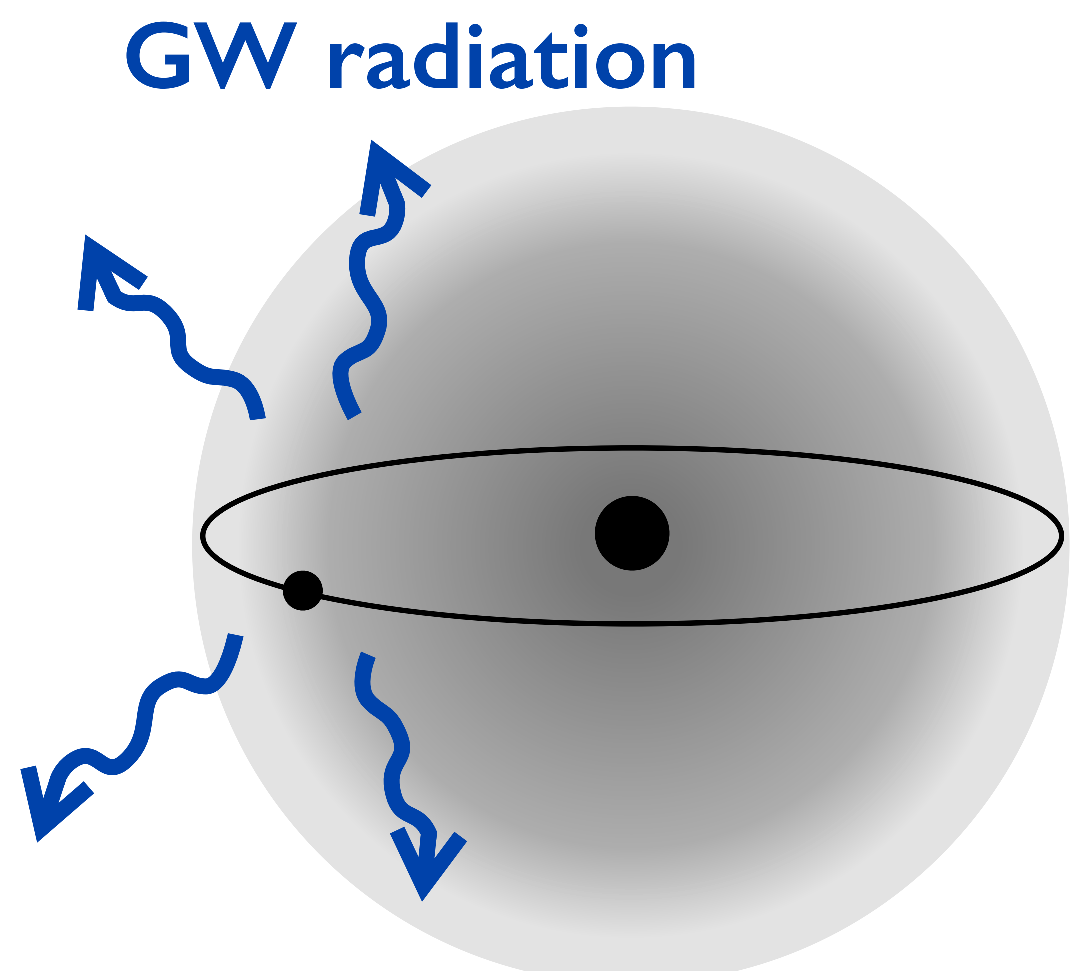
The idea

Gravitational wave inspiral signal

without DM halo



with DM halo

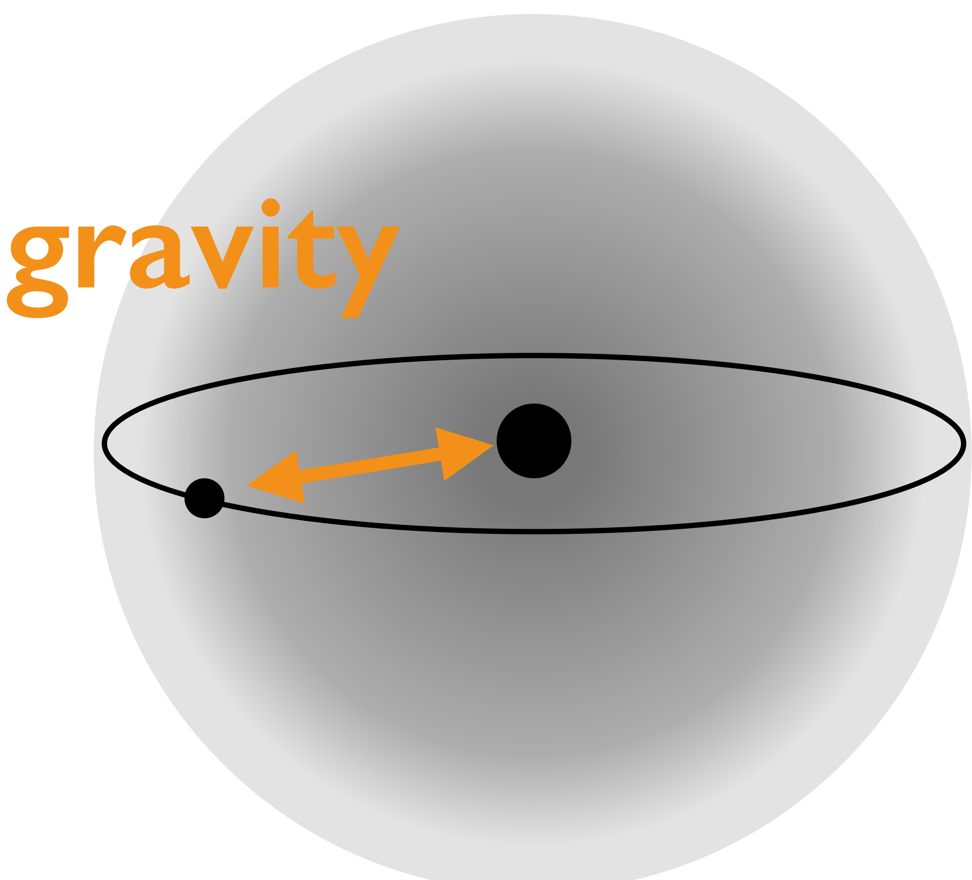


difference in GW signal?

DM effect 1: Gravitational potential

The star orbit is affected by
the gravitational potential of the DM halo

with DM halo



Equation of Motion
for the orbiting star

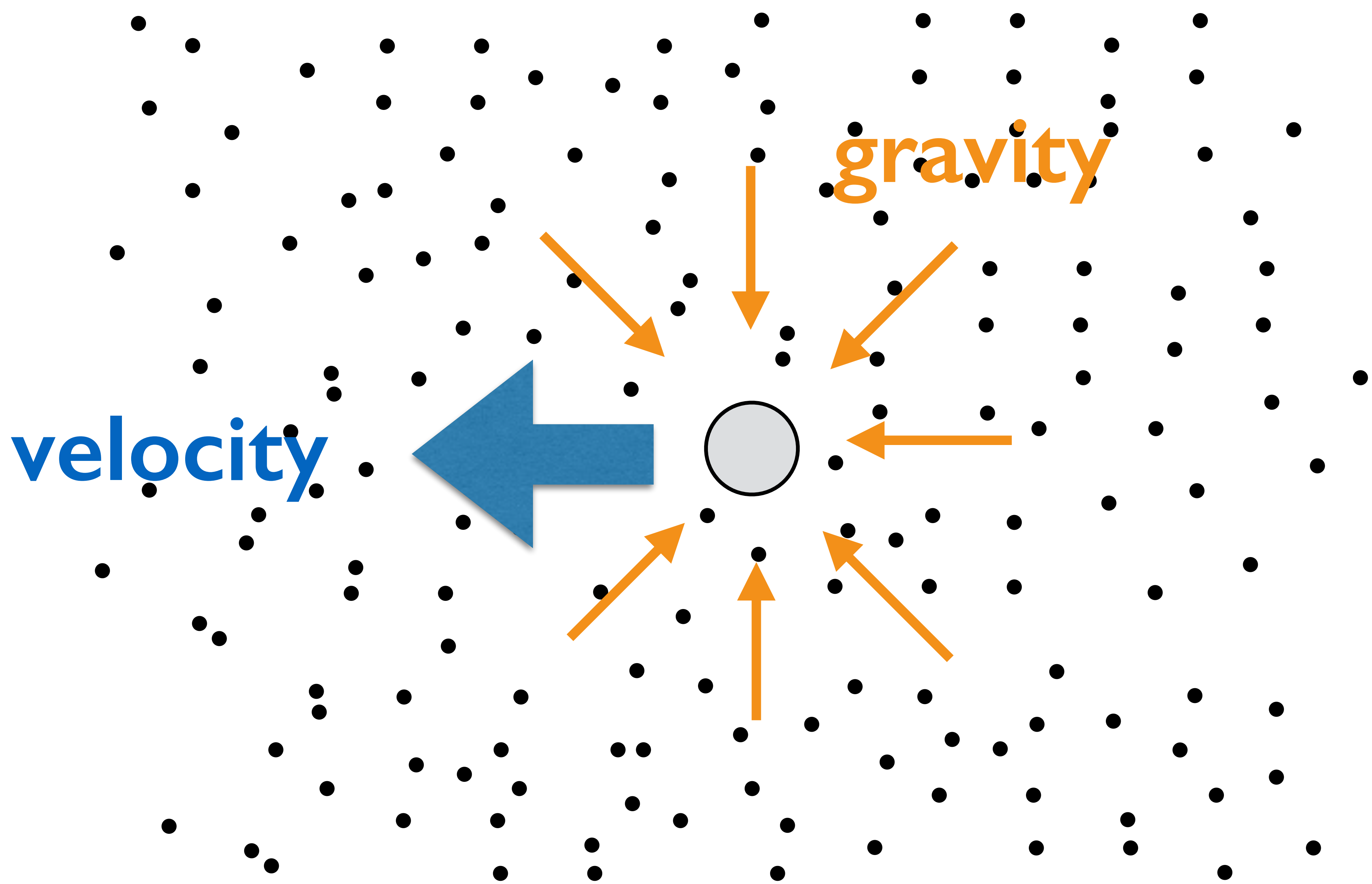
$$\boxed{\frac{d^2r}{dt^2} = -\frac{G(M_{\rm BH} + M_{\rm DM})}{r^2} + \frac{l^2}{r^3}}$$

centrifugal force

l: angular momentum

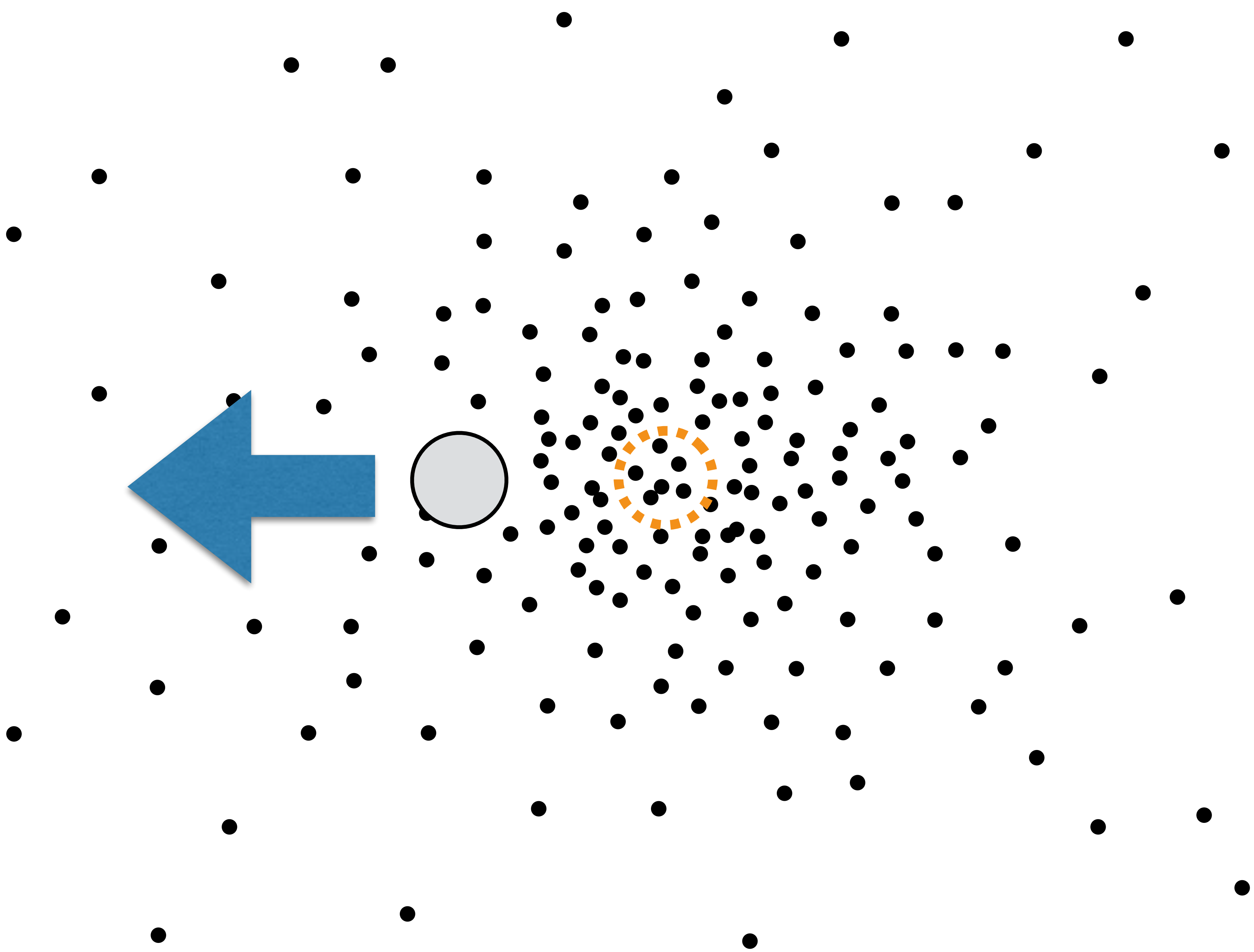
M_{DM}: DM mass inside the orbital radius

DM effect 2: Dynamical friction



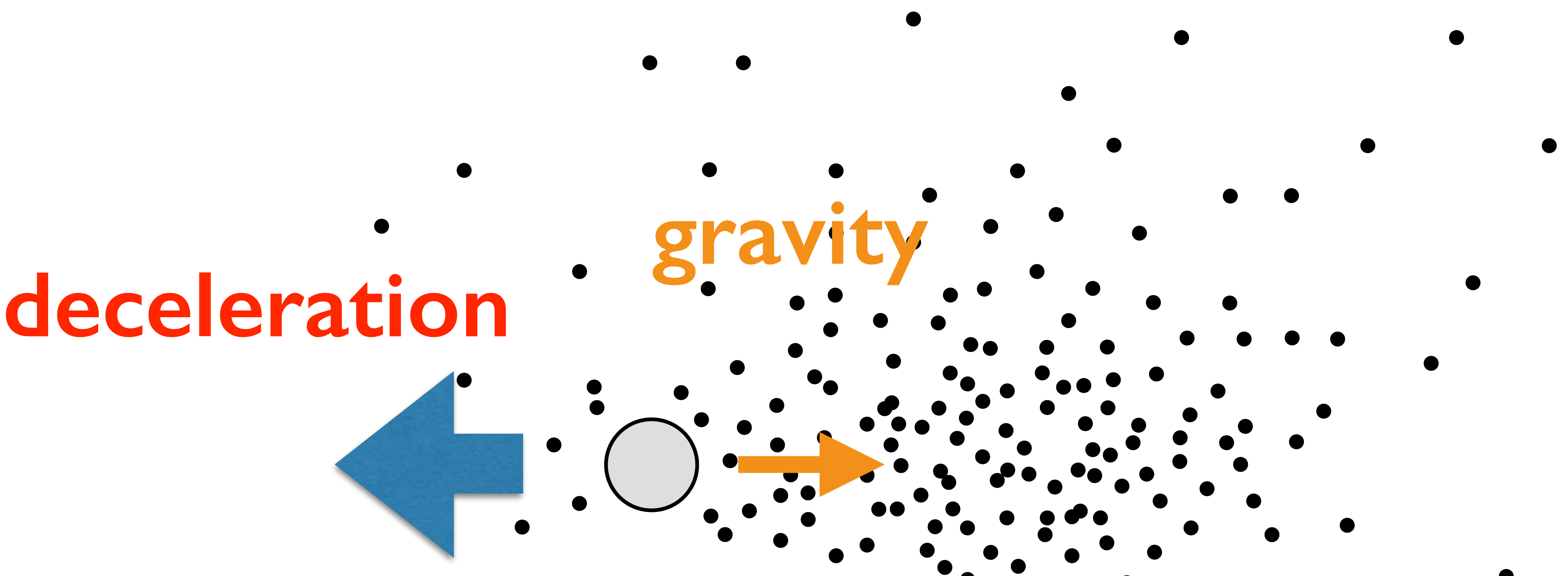
The star attracts
DM particles by gravity

DM effect 2: Dynamical friction



Dense region follows
after the star's trajectory

DM effect 2: Dynamical friction



$$\boxed{\frac{d\mathbf{v}}{dt} = -4\pi G^2 \mu \rho \ln \Lambda \frac{\mathbf{v}}{v^3}}$$

μ: mass of the star
ρ: DM density
v: velocity
lnΛ: Coulomb logarithm ~3

The star feels gravity from the dense DM cloud

Short summary

Star's orbital motion is affected
by existence of a DM halo around a BH through…

① Gravitational potential
② Dynamical friction

Equation of Motion
for the orbiting star

$$\boxed{\frac{d^2r}{dt^2} = -\frac{G(M_{\rm BH}+M_{\rm DM})}{r^2} + \frac{l^2}{r^3}}$$

→ modification due to energy losses
by GW emission and dynamical friction

① ① ②

$$\boxed{\frac{dE_{\rm orbit}}{dt} = -\frac{dE_{\rm GW}}{dt} - \frac{dE_{\rm DF}}{dt}}$$

Are there BHs with a dense DM halo?

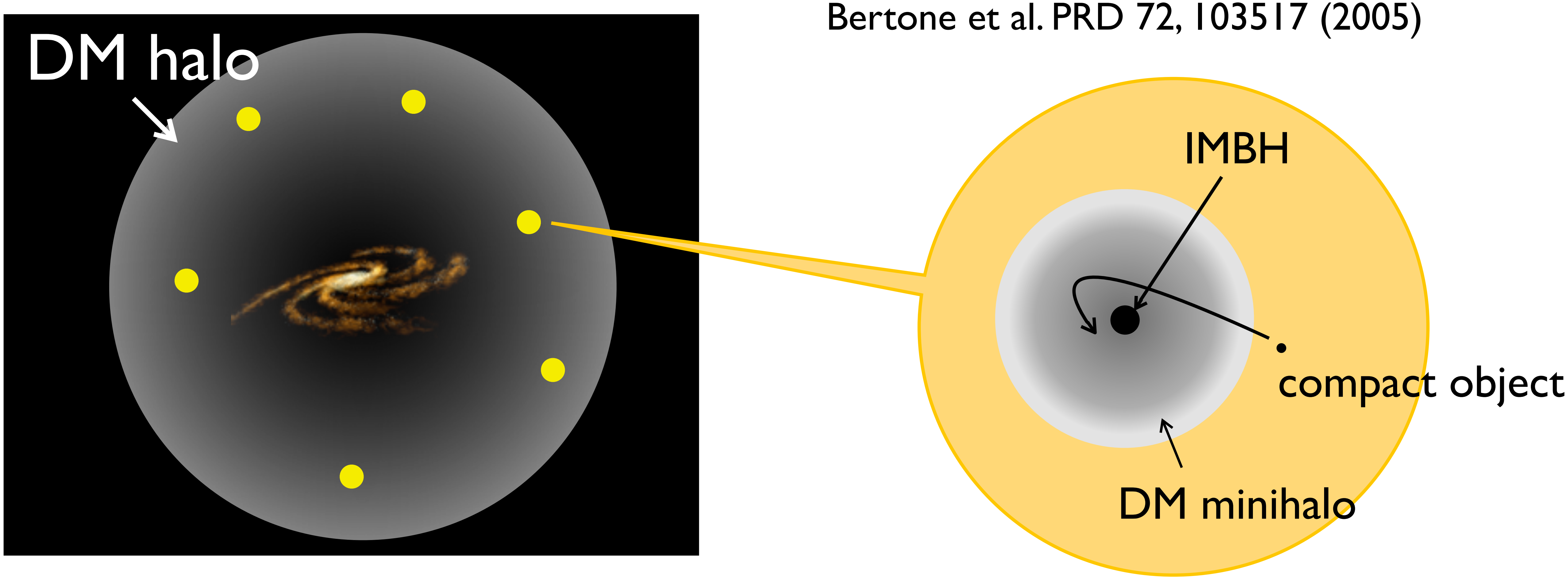
Gravity of BH attracts DM particles

BUT merger events destroy DM halo

→ Not likely for SMBHs in a galactic center

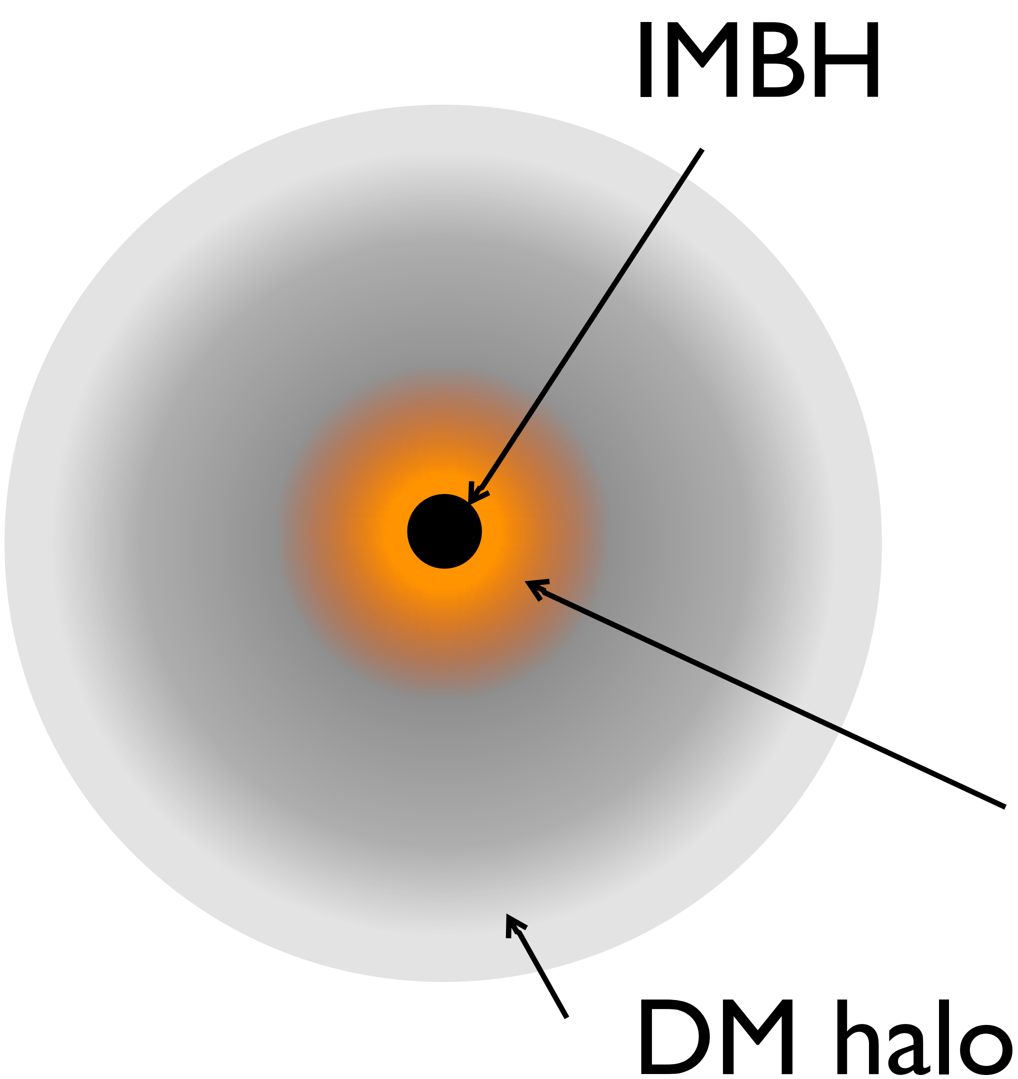
Candidate: isolated IMBHs around our galaxy

Bertone et al. PRD 72, 103517 (2005)



Dark matter mini spike

"Spikey" structure in a dark matter mini halo developed around an intermediate-mass black hole



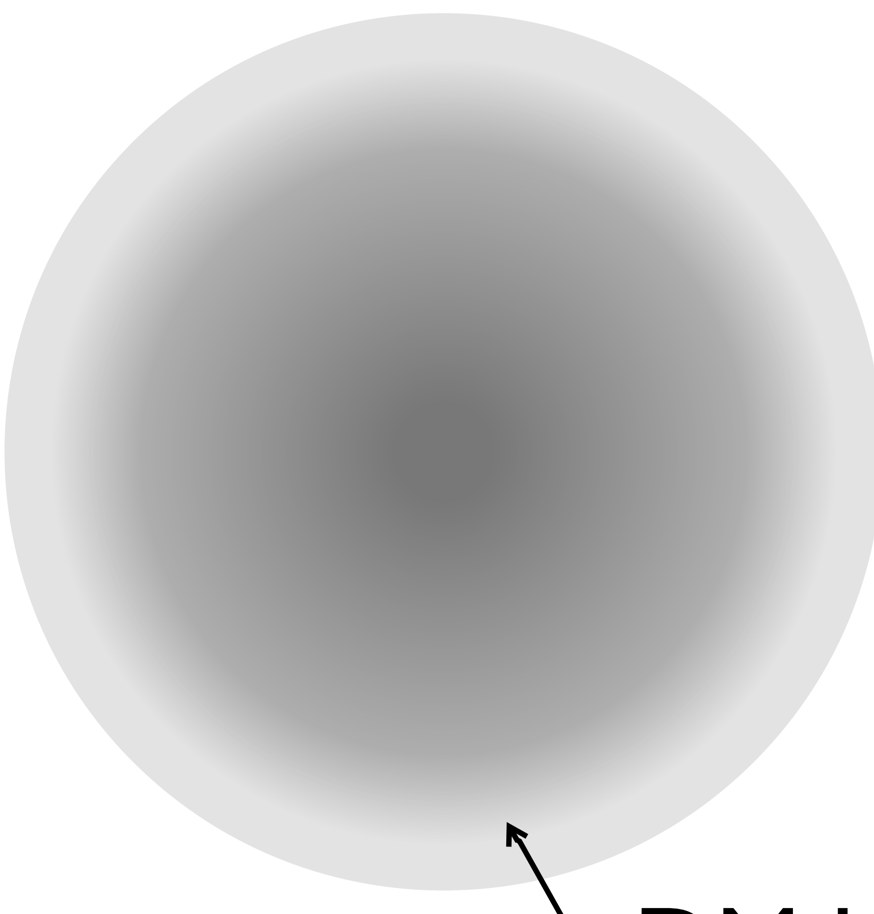
→ proposed in the context of gamma-ray observations as a mechanism to enhance the DM annihilation rate

P. Gondolo and J. Silk, PRL 83, 1719 (1999)

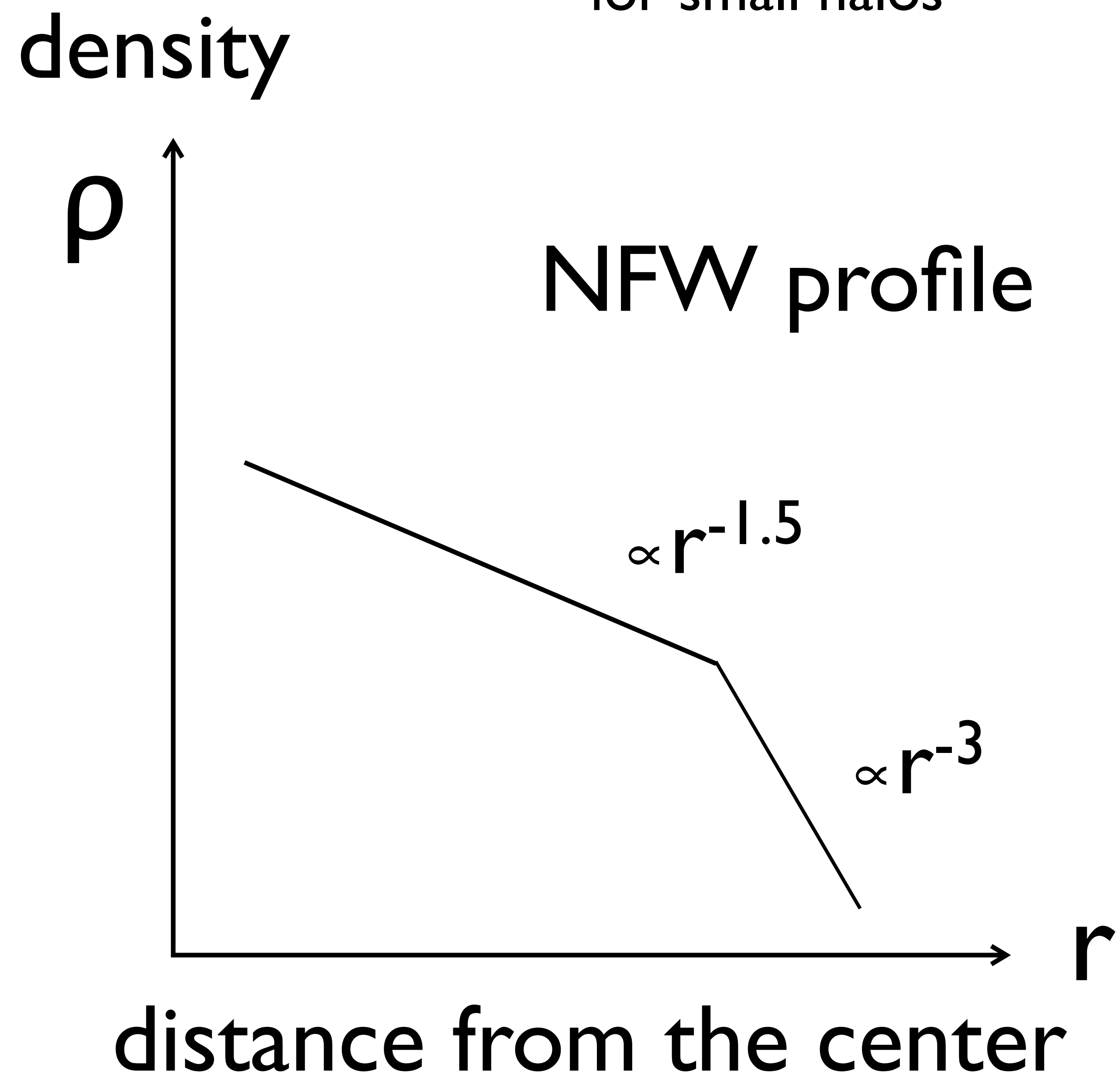
DM density profile

without BH

Note: NFW is based on simulations for **galaxy clusters** and may not true for small halos

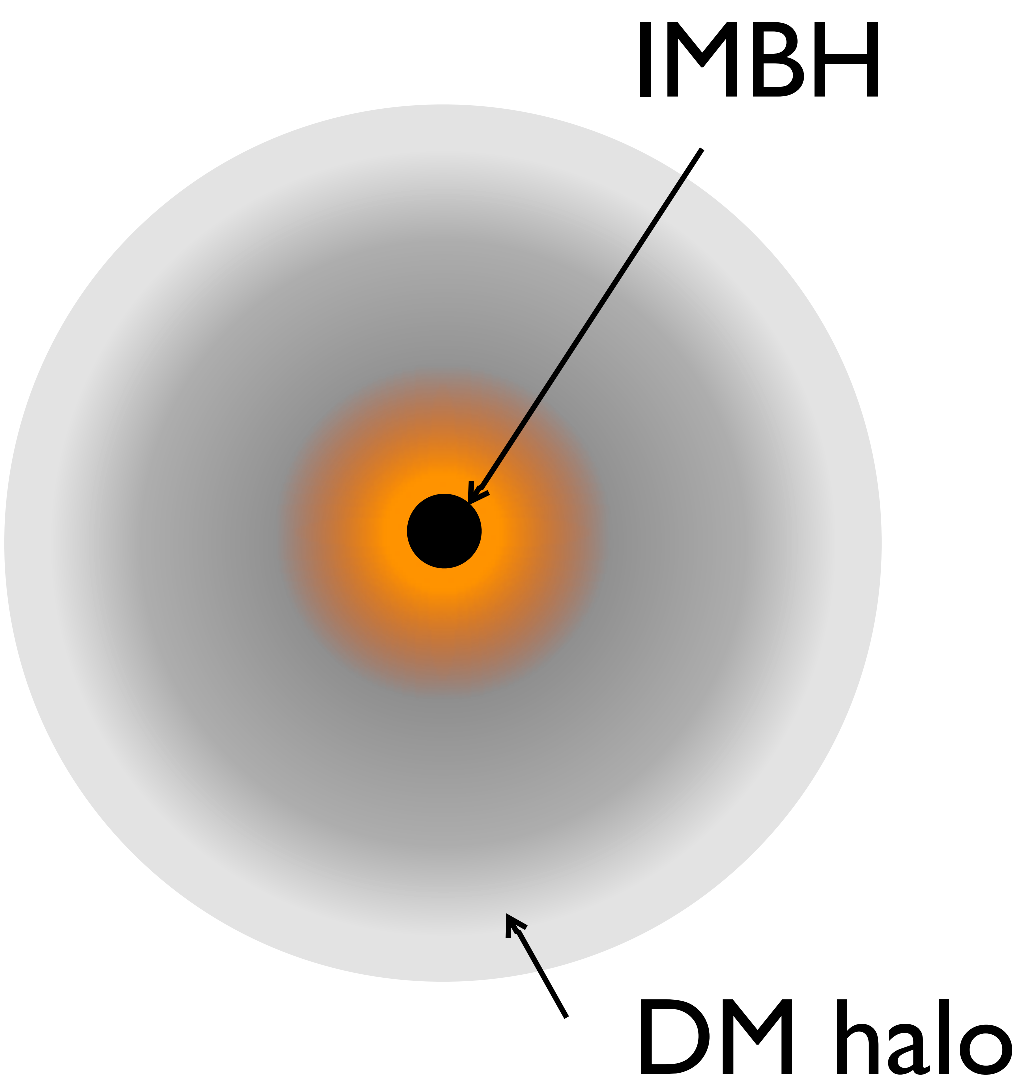


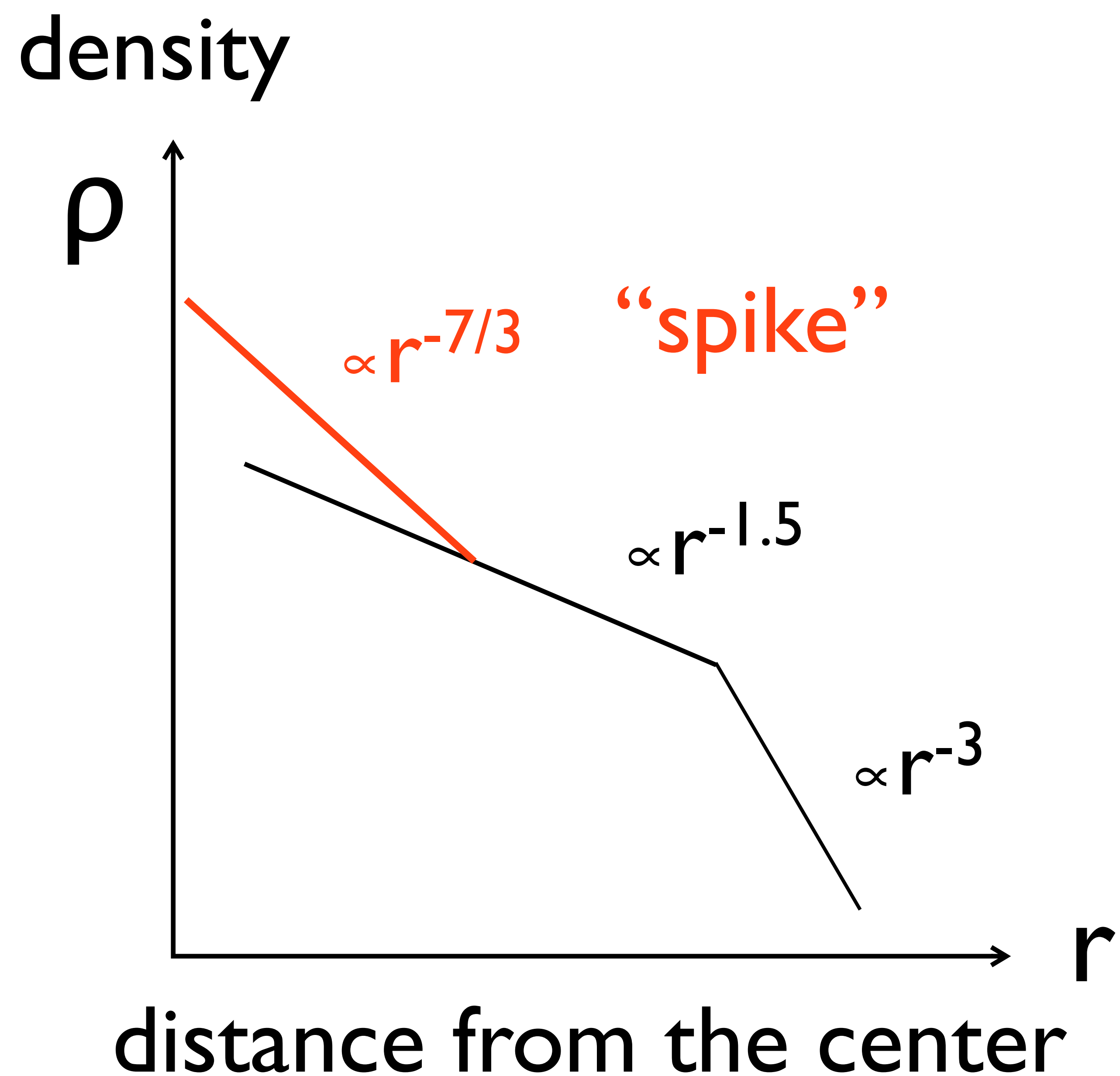
DM halo



DM density profile

with BH

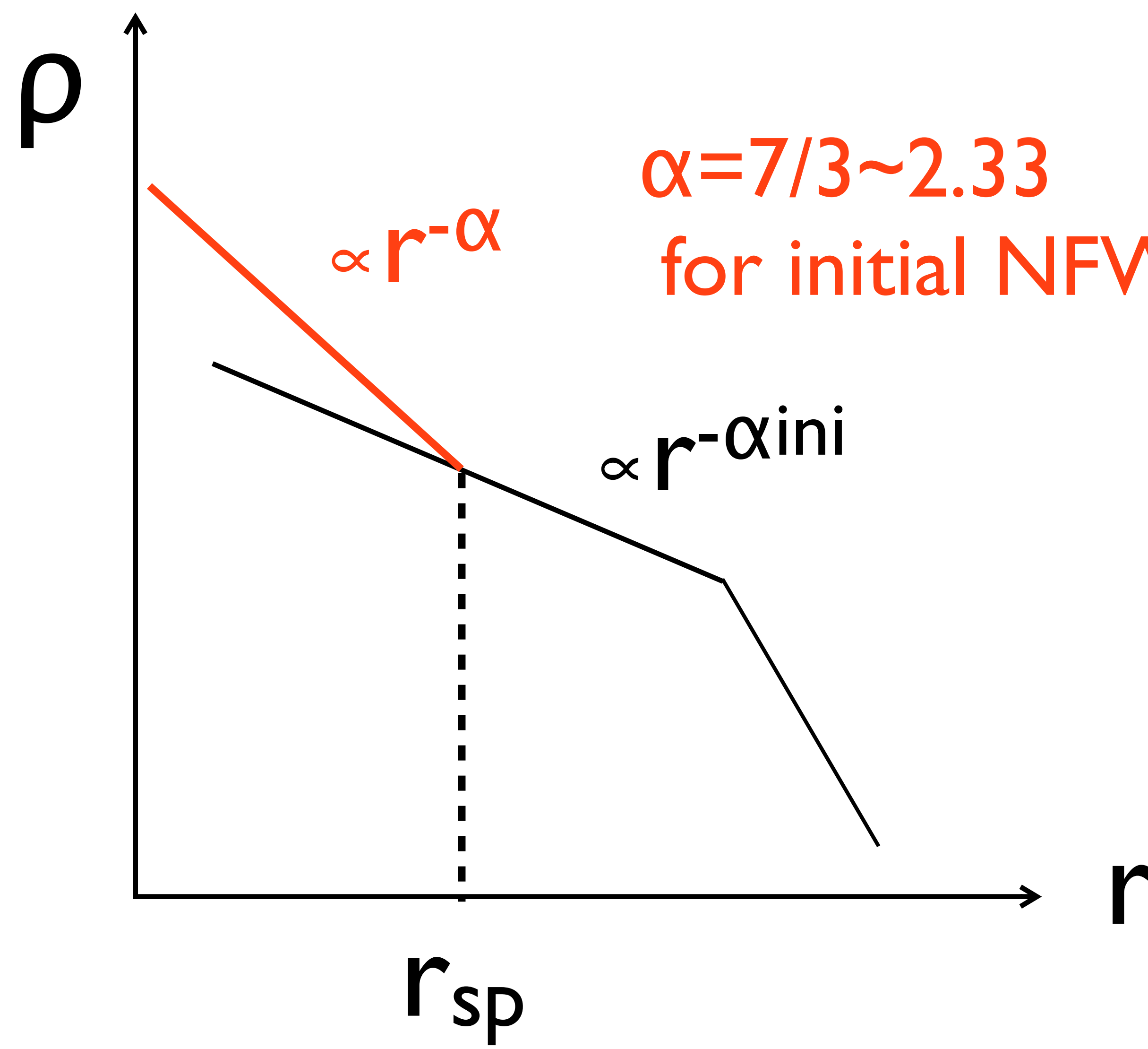




DM density profile

DM density of a spike: $\rho(r) = \rho_{\rm sp}\left(\frac{r_{\rm sp}}{r}\right)^{\alpha}$

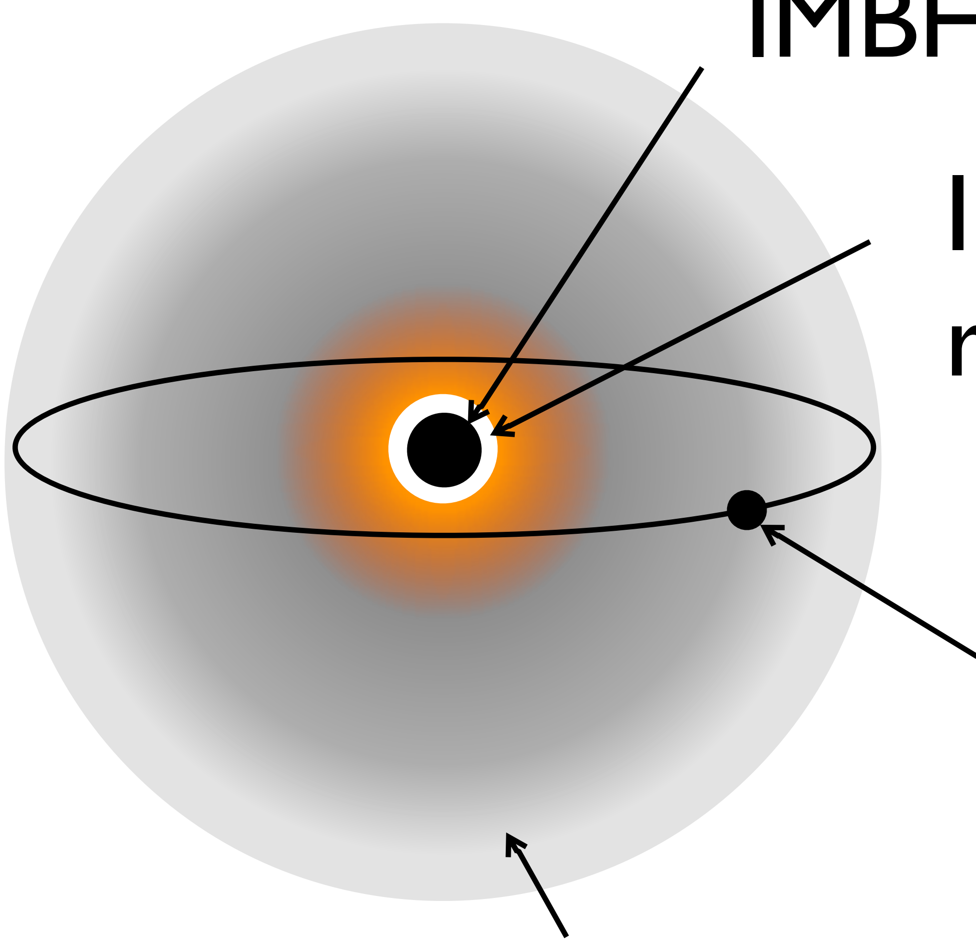
$$\alpha = (9 - 2\alpha_{\rm ini})/(4 - \alpha_{\rm ini})$$



Setup

DM density of a spike: $\rho(r) = \rho_{\rm sp}\left(\frac{r_{\rm sp}}{r}\right)^{\alpha}$

$$\alpha = (9 - 2\alpha_{\rm ini})/(4 - \alpha_{\rm ini})$$



IMBH: $10^3 M_\odot$

Innermost Stable Circular Orbit

$r_{min} = r_{ISCO} = 6GM_{BH} \sim 3\times10^{-10}$pc

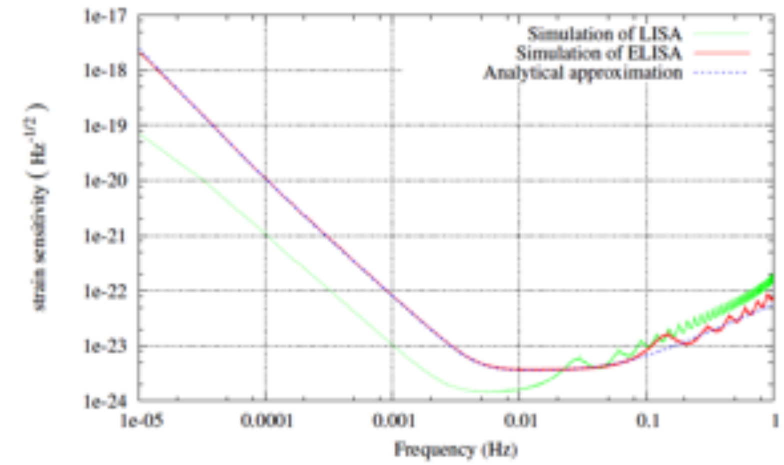
compact object: $1M_\odot$

DM halo: $10^6 M_\odot$

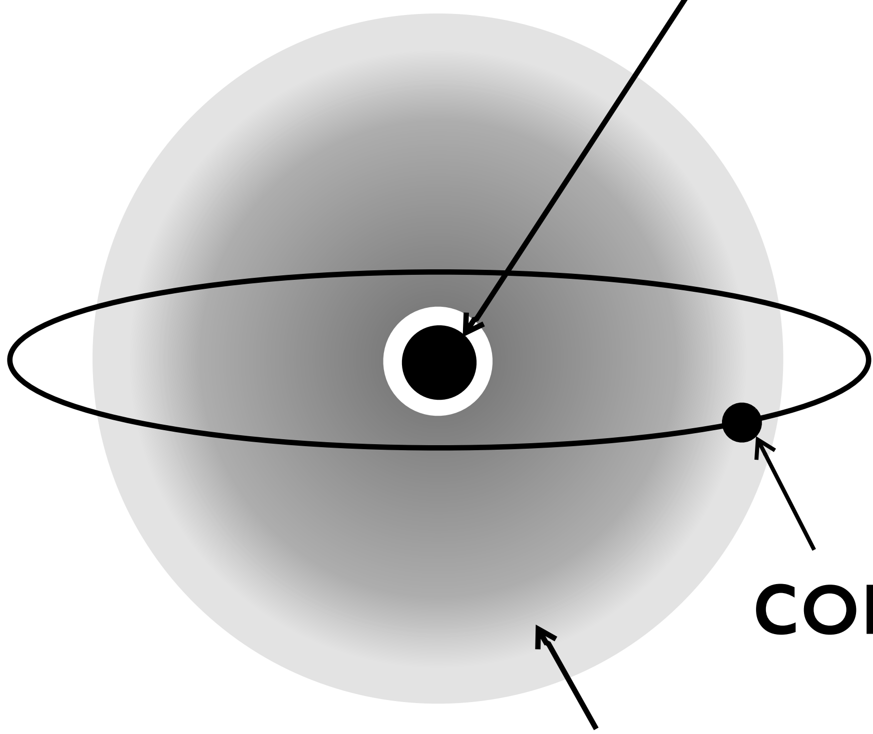
Assumption:

- circular star orbit
- DM is not perturbed by the object

Observation

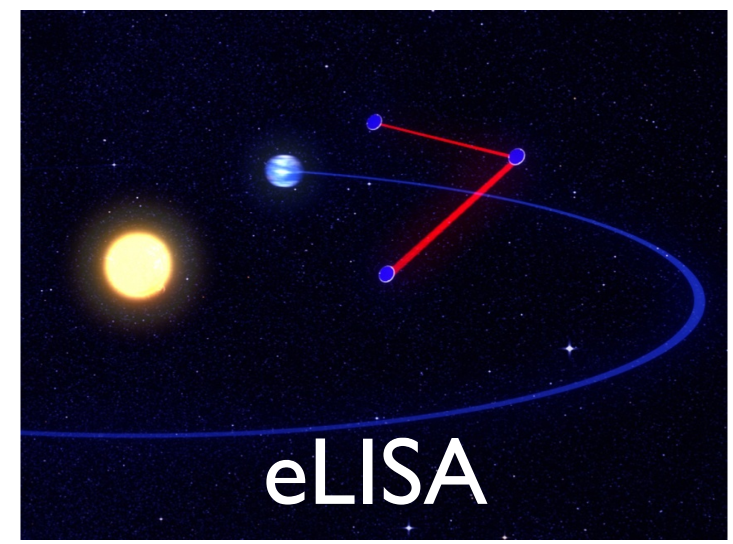


IMBH: $10^3 M_\odot$



GW

~0.01Hz



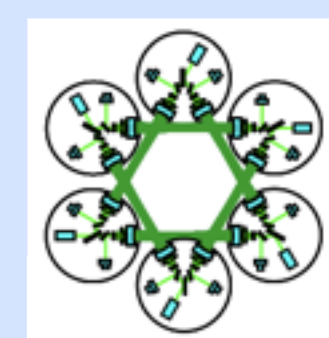
compact object

DM minihalo: $10^6 M_\odot$

Other possibilities

Ground-based
100Hz

DECIGO/BBO
0.1Hz



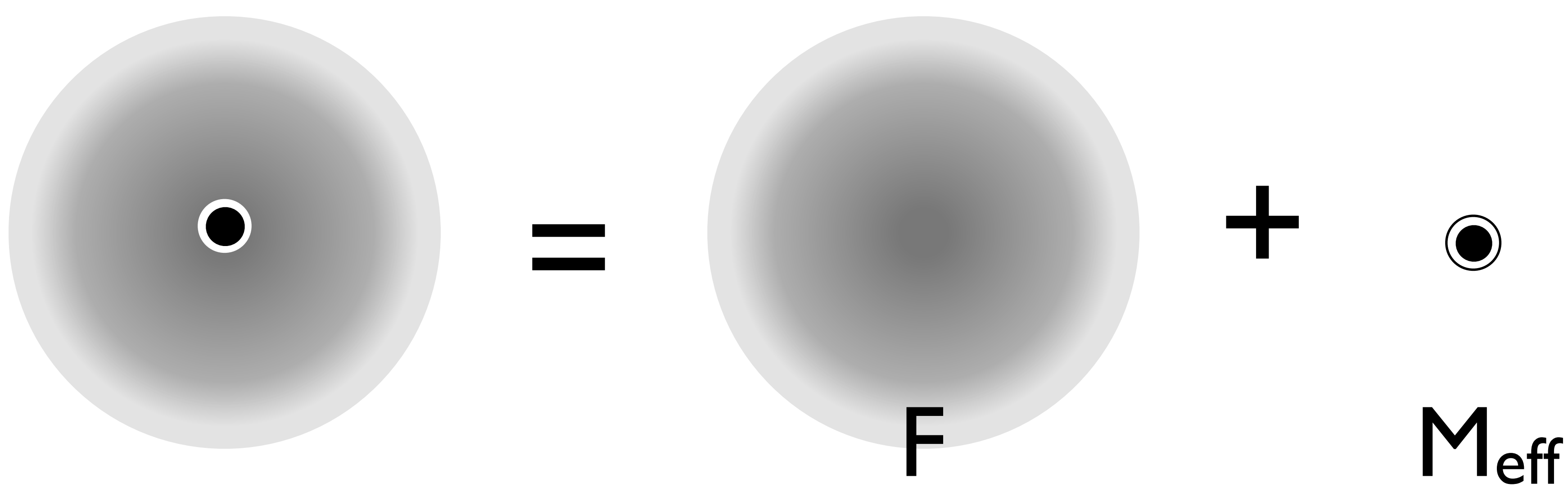
Equation of Mortion

$$\boxed{\frac{d^2r}{dt^2} = -\frac{GM_{\mathrm{eff}}}{r^2} - \frac{F}{r^{\alpha-1}} + \frac{l^2}{r^3}}$$

effect of DM

centrifugal force

$$M_{\mathrm{eff}} = M_{\mathrm{BH}} - \frac{4\pi r_{\mathrm{sp}}^{\alpha}\rho_{\mathrm{sp}}}{3-\alpha} r_{\mathrm{min}}^{3-\alpha} \qquad F = \frac{4\pi G r_{\mathrm{sp}}^{\alpha}\rho_{\mathrm{sp}}}{3-\alpha}$$



$$\varepsilon \equiv \frac{F}{GM_{\mathrm{eff}}} = \frac{\text{2nd term (effect of DM)}}{\text{1st term (gravity of BH)}}$$

GW waveform for circular orbit

quadrupole formula

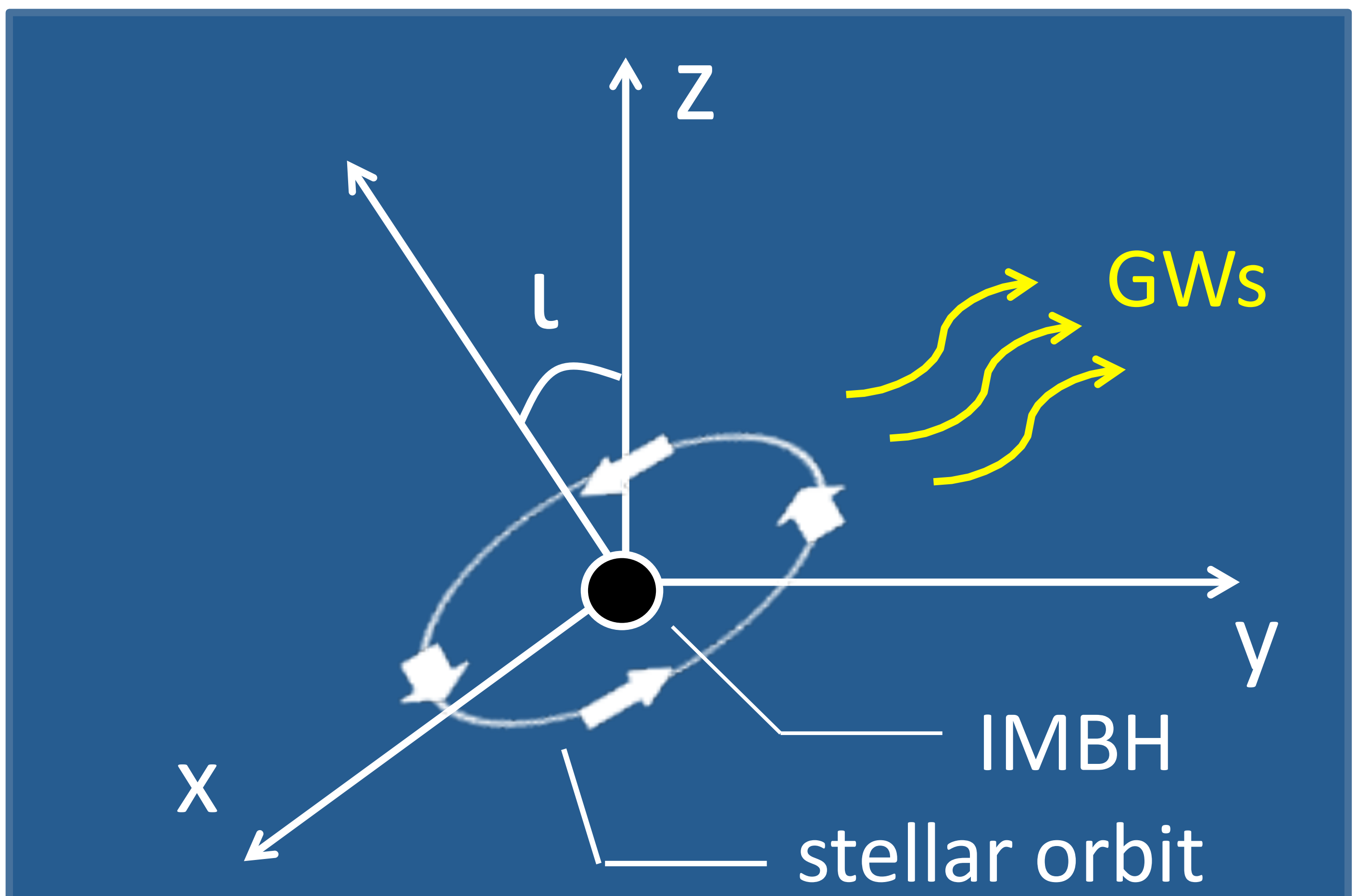
$$h^{jk}_{\mathrm{TT}} = \frac{2G}{c^4 R}\ddot{I}^{jk}_{\mathrm{TT}}$$

$$I^{jk} \equiv \int d^3x\ \rho x^j x^k$$

$$h_+ = \frac{1}{r}\frac{4G\mu\omega_s^2 R^2}{c^4}\frac{1+\cos^2\iota}{2}\cos(2\omega_s t)$$

$$h_\times = \frac{1}{r}\frac{4G\mu\omega_s^2 R^2}{c^4}\cos\iota\sin(2\omega_s t)$$

$$\omega_s^2 = \left(\frac{GM_{\mathrm{eff}}}{R^3} + \frac{F}{R^\alpha}\right)$$



orbital radius R

orbital frequency ωs

stellar mass μ

Inclination ι

distance from observer r

Modification by the energy losses

$$\boxed{\frac{dE_{\rm orbit}}{dt} = -\frac{dE_{\rm GW}}{dt} - \frac{dE_{\rm DF}}{dt}} \Rightarrow \frac{dR}{dt} = \cdots \Rightarrow \begin{array}{l} R \to R(t) \\ \omega_s \to \omega_s(t) \end{array}$$

① ① ②

energy loss by GW emission Dynamical friction

① Gravitational potential

$$\frac{dE_{\rm orbit}}{dt} = \left(\frac{GM_{\rm eff}}{2R^2} + \frac{4-\alpha}{2}\frac{F}{R^{\alpha-1}}\right)\mu\frac{dR}{dt}$$

→ gives R(t)

$$\frac{dE_{\rm GW}}{dt} = \frac{32}{5}\frac{G\mu^2}{c^5}R^4\omega_s^6 \qquad \omega_s = \left(\frac{GM_{\rm eff}}{R^3} + \frac{F}{R^\alpha}\right)^{1/2}$$

② Dynamical friction

$$\frac{dE_{\rm DF}}{dt} = vf_{\rm DF} = 4\pi G^2\frac{\mu^2\rho_{\rm DM}(r)}{v}\ln\Lambda$$

Dimensionless radius: $x \equiv \varepsilon^{1/(3-\alpha)} R$

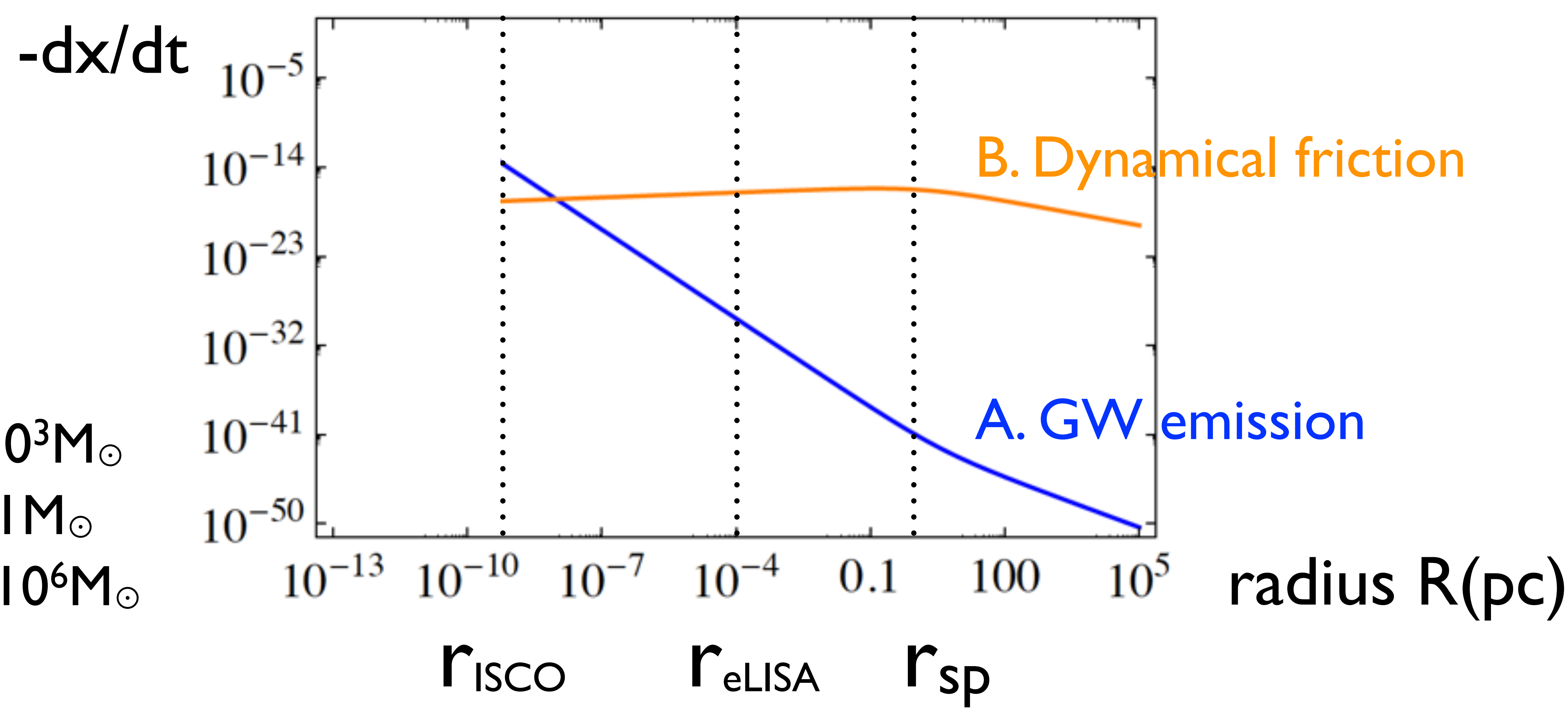
A. GW emission

B. Dynamical friction

$$\frac{dx}{dt} = -c_{\mathrm{GW}} \frac{(1+x^{3-\alpha})^3}{4x^3[1+(4-\alpha)x^{3-\alpha}]} - c_{\mathrm{DF}} \frac{1}{(1+x^{3-\alpha})^{1/2}[1+(4-\alpha)x^{3-\alpha}]x^{-5/2+\alpha}}$$

$$c_{\mathrm{GW}} \equiv \frac{256}{5}\left(\frac{G\mu}{c^3}\right)\left(\frac{GM_{\mathrm{eff}}}{c}\right)^2 \varepsilon^{4/(3-\alpha)}$$

$$c_{\mathrm{DF}} \equiv (8\pi G^2 \mu \rho_{\mathrm{sp}} r_{\mathrm{sp}}^{\alpha} \ln \Lambda)(GM_{\mathrm{eff}})^{-3/2} \varepsilon^{(2\alpha-3)/[2(3-\alpha)]}$$



$M_{\mathrm{BH}} = 10^3 M_\odot$

$M_{\mathrm{star}} = 1 M_\odot$

$M_{\mathrm{halo}} = 10^6 M_\odot$

$\alpha = 7/3$

Fourier transformed waveform (for + mode)

$$\boxed{\tilde{h}(f) = \underline{\mathcal{A}} f^{-7/6} e^{i\underline{\Psi(f)}}}$$

amplitude

$$\mathcal{A} = \left(\frac{5}{24}\right)^{1/2} \frac{1}{\pi^{2/3}} \frac{c}{D} \left(\frac{GM_c}{c^3}\right)^{5/6} \frac{1+\cos^2\iota}{2} \qquad M_c \equiv \mu^{3/5} M_{\rm eff}^{5/2}$$

phase

$$\Psi(f) = 2\pi f \tilde{t}_c - \Phi_c - \frac{\pi}{4} - \tilde{\Phi}(f) \qquad \tilde{t}_c \equiv t_c + D/c$$

$$\tilde{\Phi}(f) = \frac{10}{3}\left(\frac{8\pi G M_c}{c^3}\right)^{-5/3}\left[-f\int_{f_{\rm ISCO}}^{f} df' f'^{-11/3} L^{-1}(f') + \int_{f_{\rm ISCO}}^{f} df' f'^{-8/3} L^{-1}(f')\right]$$

$$L(f) = 1 + \underline{4 c_\varepsilon \tilde{\delta}^{(11-2\alpha)/[2(3-\alpha)]}}$$

$$\tilde{\delta} = \left(\frac{G}{\pi^2 f^2}\right)^{(3-\alpha)/3} \qquad c_\varepsilon = 5\pi c^5 G^{-5/2} M_{\rm eff}^{-(\alpha+5)/3} \rho_{\rm sp} r_{\rm sp}^{\alpha} \ln\Lambda$$

DM parameter: α, c_ε

Observation: Matched filtering

Signal to noise ratio

Signal

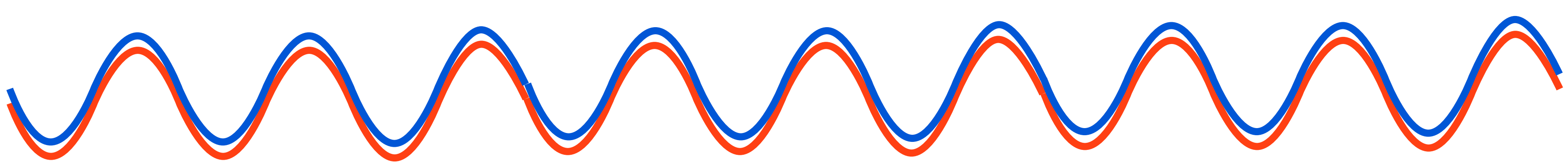
Template

= expected waveform

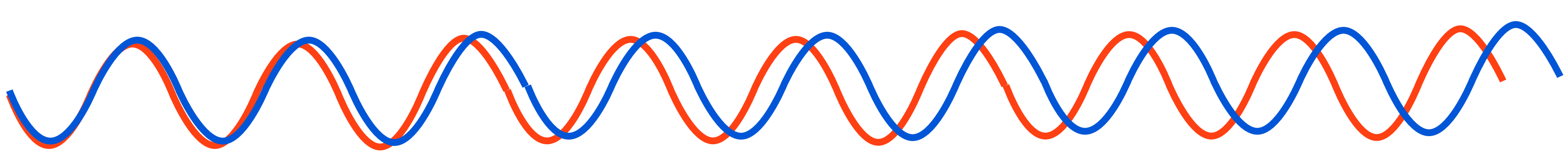
$$\left(\frac{S}{N}\right)^2 = \frac{\left[\int_{f_{\rm ini}}^{\infty} df \, \frac{\boxed{\tilde{h}(f)}\,\boxed{\tilde{h}_t^*(f)} + \boxed{\tilde{h}^*(f)}\,\boxed{\tilde{h}_t(f)}}{S(f)}\right]^2}{\int_{f_{\rm ini}}^{\infty} df \, \frac{|\tilde{h}_t(f)|^2}{S(f)}}$$

Noise

Normalization for template

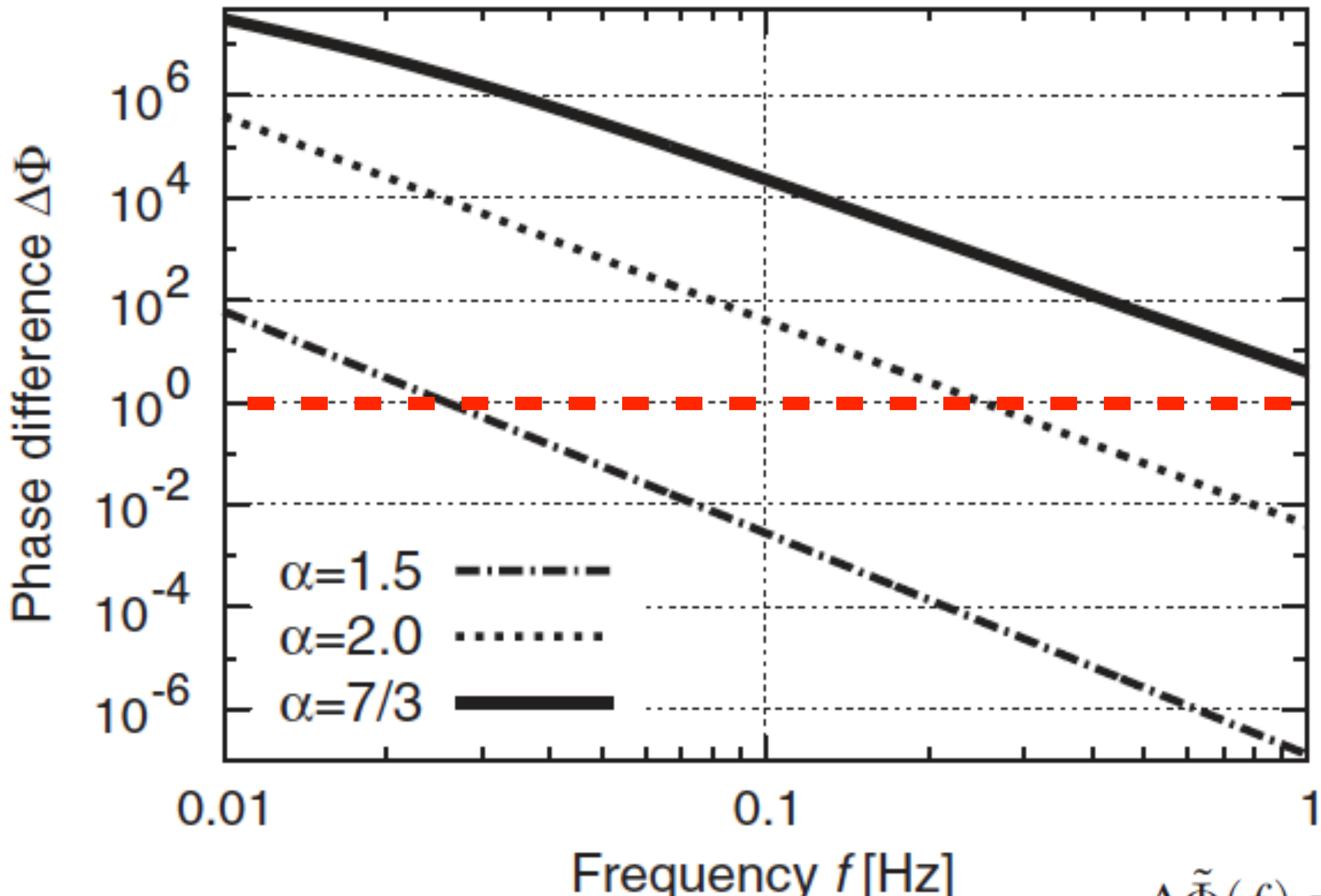


→matched: SN is maximized



→mismatch: SN is reduced

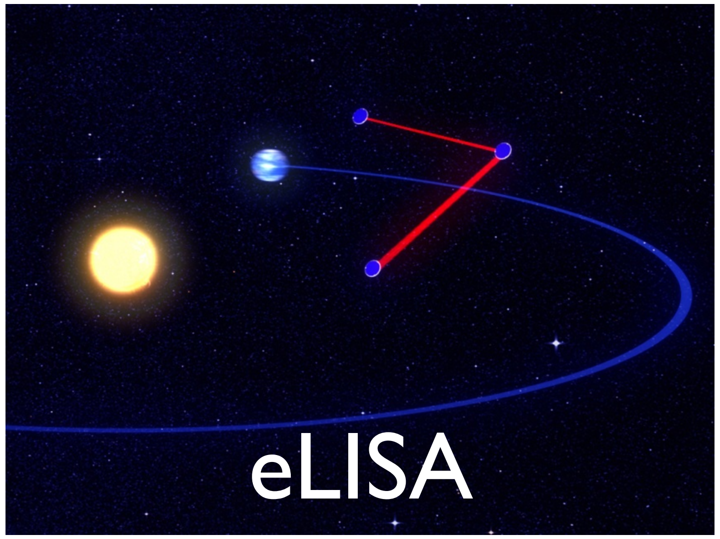
Phase difference is important!

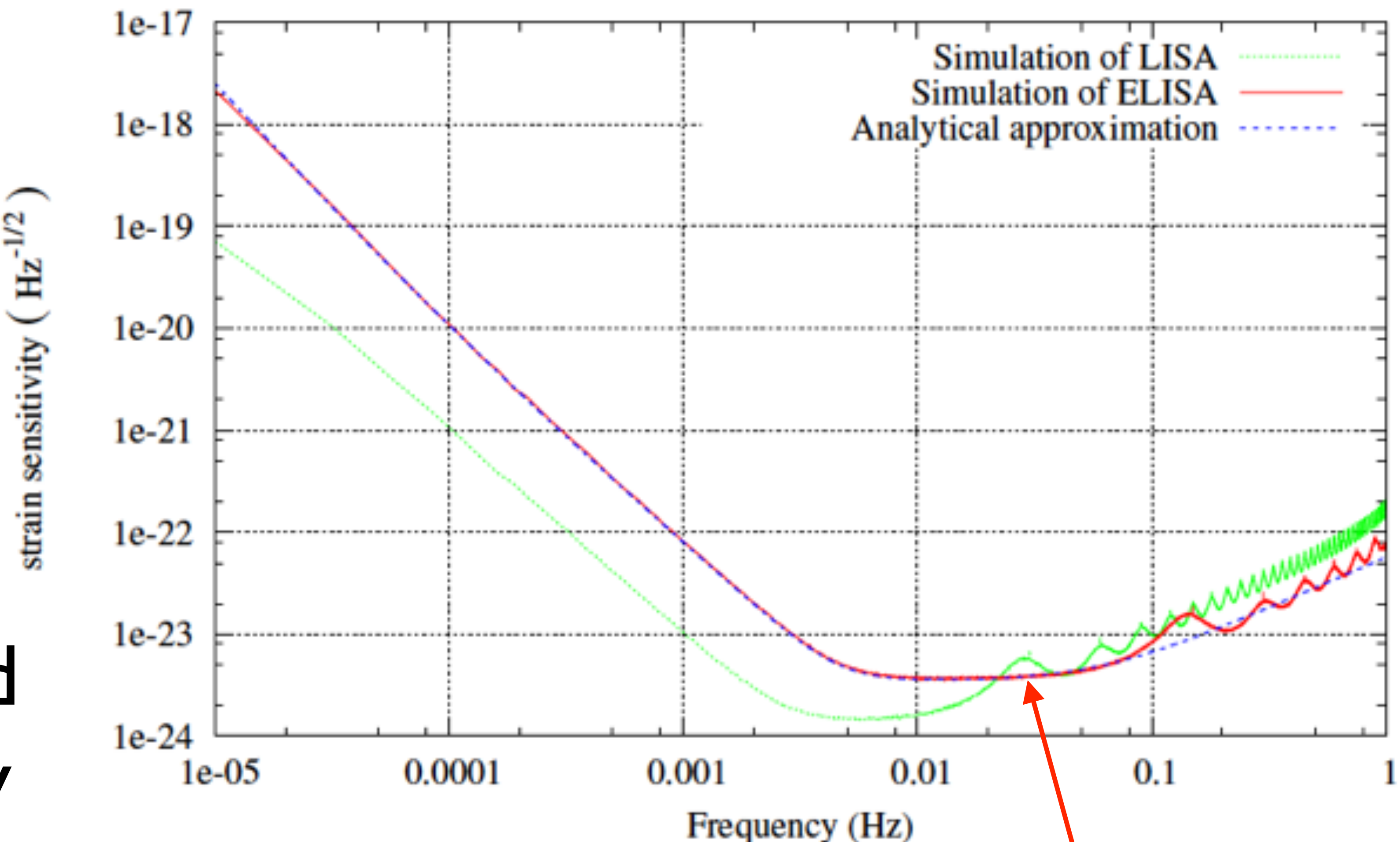


$$\Delta\tilde{\Phi}(f) \equiv \tilde{\Phi}(f) - \tilde{\Phi}_0(f)$$

$\Delta\Phi > 1$ would be necessary to distinguish DM effect

Prediction for eLISA





need to be integrated in terms of frequency

Fisher marix: $\Gamma_{ij} \equiv \left(\frac{\partial h}{\partial \theta^i} \middle| \frac{\partial h}{\partial \theta^j} \right)$ $\qquad (h_1|h_2) \equiv 4\mathrm{Re} \int_{f_{\mathrm{ini}}}^{f_{\mathrm{ISCO}}} \frac{\tilde{h}_1(f)\tilde{h}_2^*(f)}{S_n(f)} df$

$$\Delta\theta^i \equiv \sqrt{\langle (\Delta\theta^i)^2 \rangle} = \sqrt{(\Gamma^{-1})_{ii}}$$

→ gives expected measurement errors on parameter θ

Parameter estimation

GW waveform

$$\tilde{h}(f) = \underline{\mathcal{A}} f^{-7/6} e^{i\Psi(f)}$$

$$\Psi(f) = 2\pi f \underline{\tilde{t}_c} - \underline{\Phi_c} - \frac{\pi}{4} - \tilde{\Phi}(f)$$

$$\tilde{\Phi}(f) = \frac{10}{3}\left(\frac{8\pi G \underline{M_c}}{c^3}\right)^{-5/3}\left[-f\int_{f_{\rm ISCO}}^{f} df' f'^{-11/3} L^{-1}(f') + \int_{f_{\rm ISCO}}^{f} df' f'^{-8/3} L^{-1}(f')\right]$$

$$L(f) = 1 + \underline{4c_\varepsilon}\tilde{\delta}^{\underline{(11-2\alpha)/[2(3-\alpha)]}}$$

6 free parameters

A: overall amplitude

$\tilde{t}_c$: coalescence time

Φ_c: coalescence phase

M_c: chirp mass

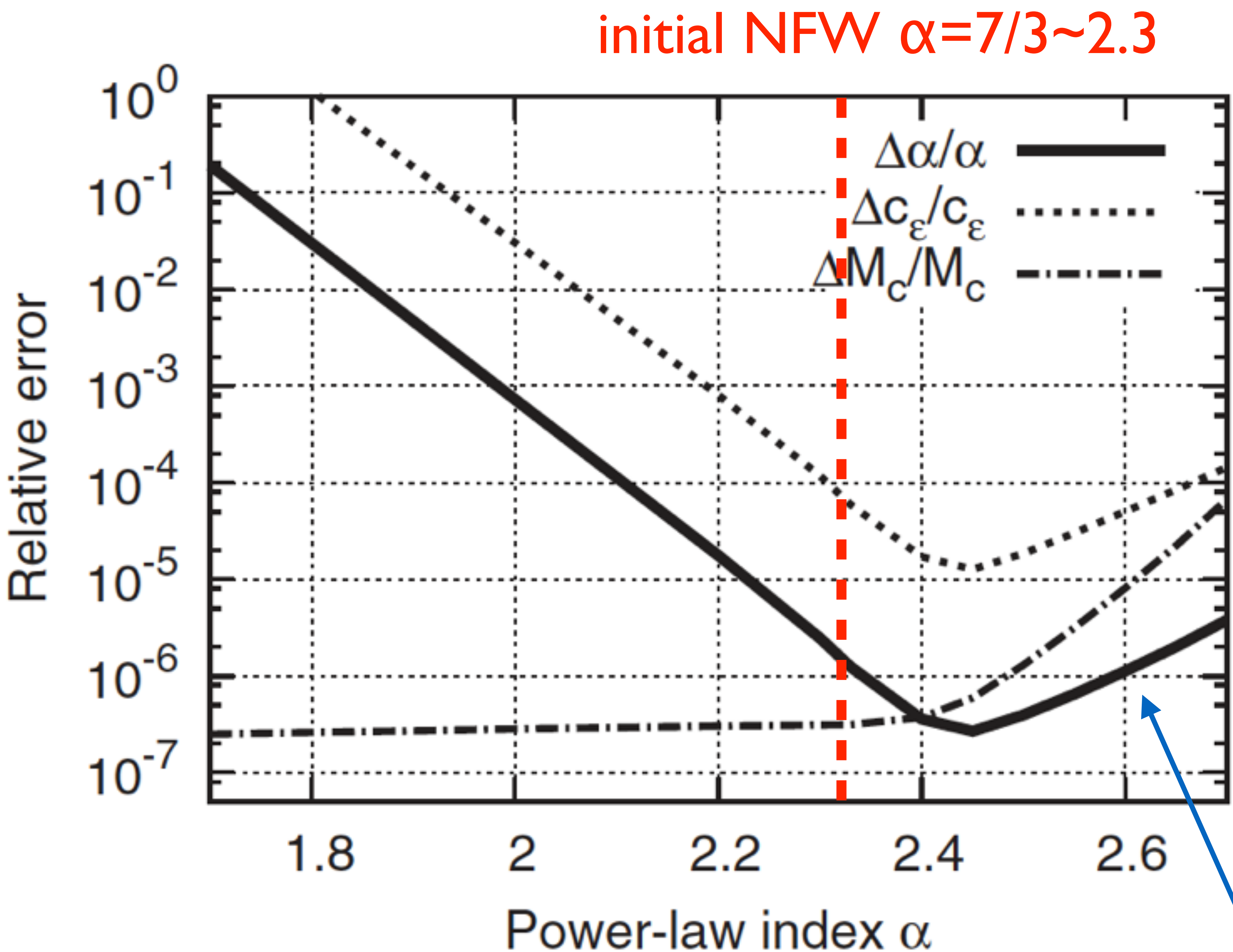
α: power low index of the DM spike

$$\rho(r) = \rho_{\rm sp}\left(\frac{r_{\rm sp}}{r}\right)^{\alpha}$$

c_ε: numerical coefficient of DM effect

$$c_\varepsilon = 5\pi c^5 G^{-5/2} M_{\rm eff}^{-(\alpha+5)/3} \rho_{\rm sp} r_{\rm sp}^{\alpha} \ln\Lambda$$

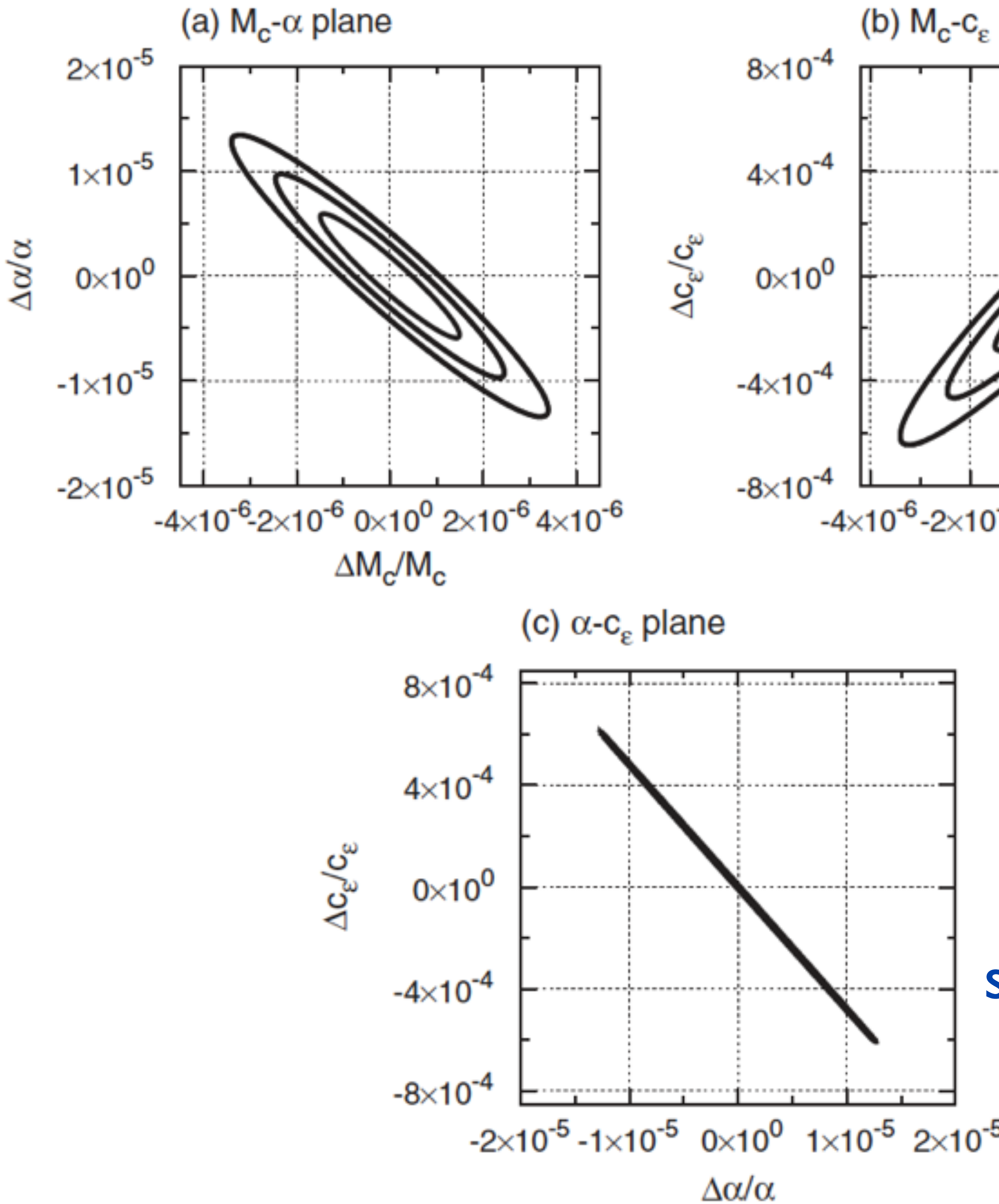
assumed 5-year observation by eLISA



DM effect is so strong that the frequency of GW goes out from eLISA sensitivity curve quickly

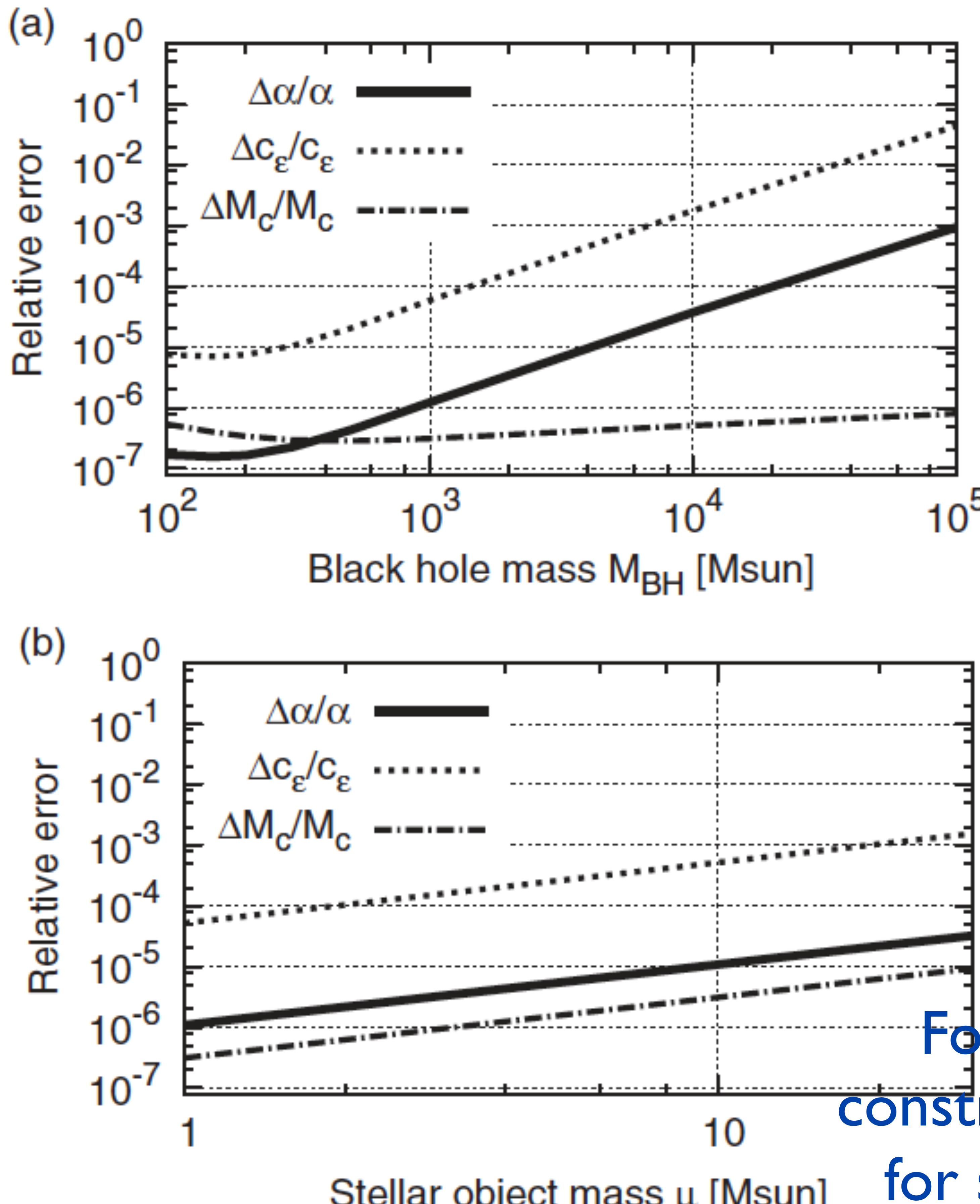
assumed 5-year observation by eLISA

$\alpha=7/3$



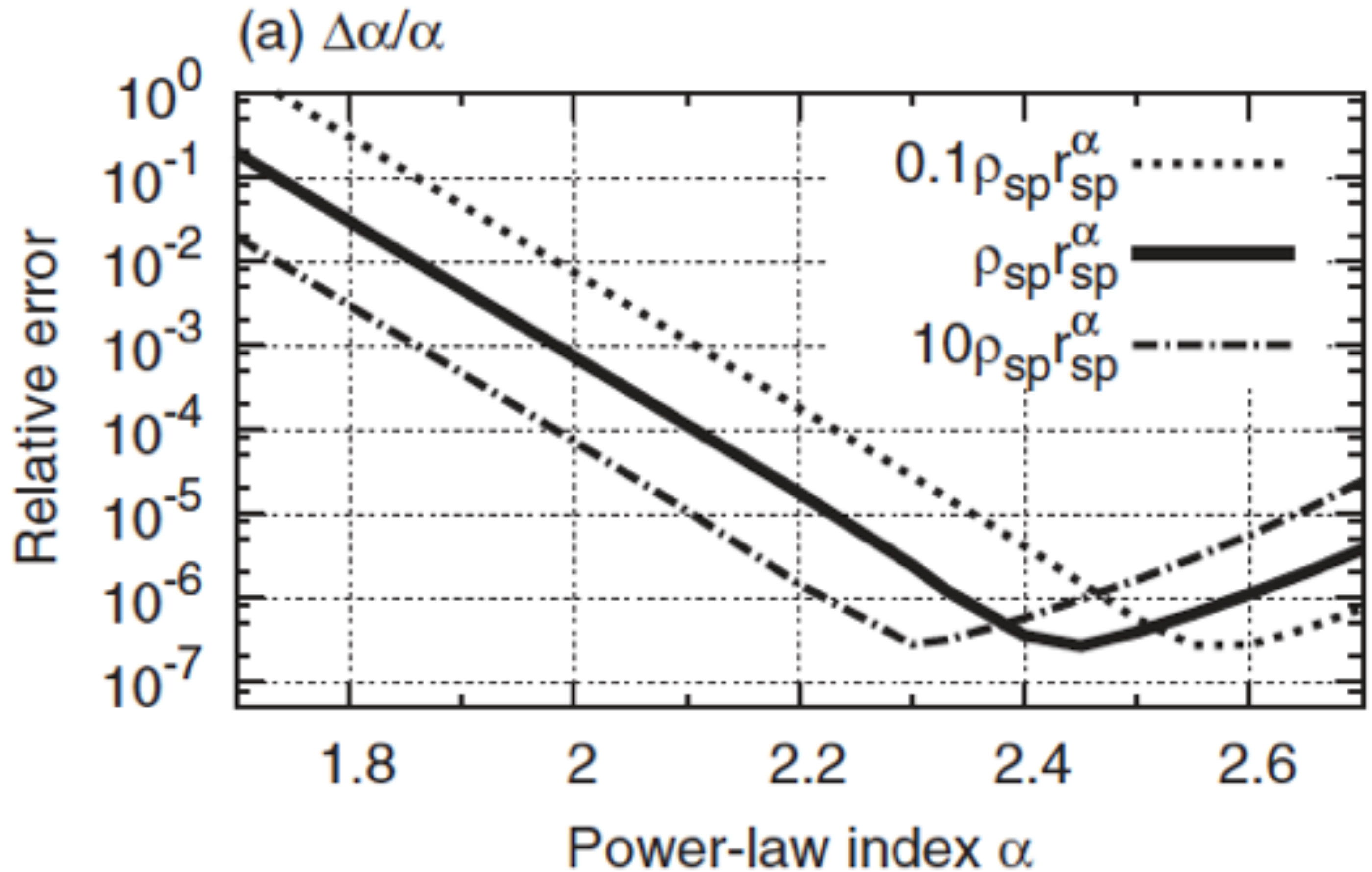
strong degeneracy between α and c_ε

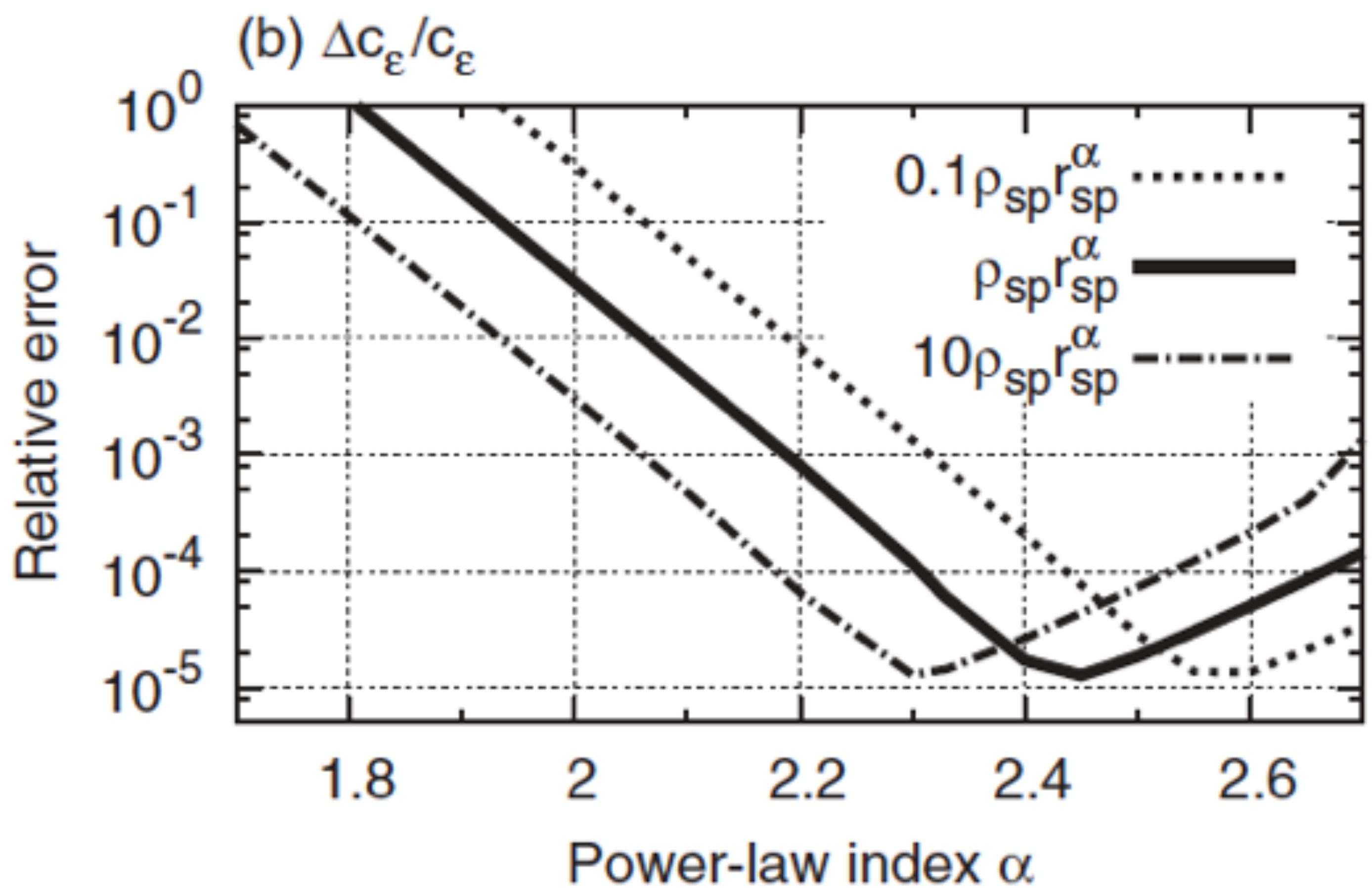
BH mass dependences



For fixed M_{DM}, the constraints get better for small M_{BH} and μ

DM density normalization dependences





Summary

Observation of GWs from IMBHs could be a new tool to probe the DM distribution around IMBH.

・Existence of the DM spike makes difference in the GW inspiral signal.

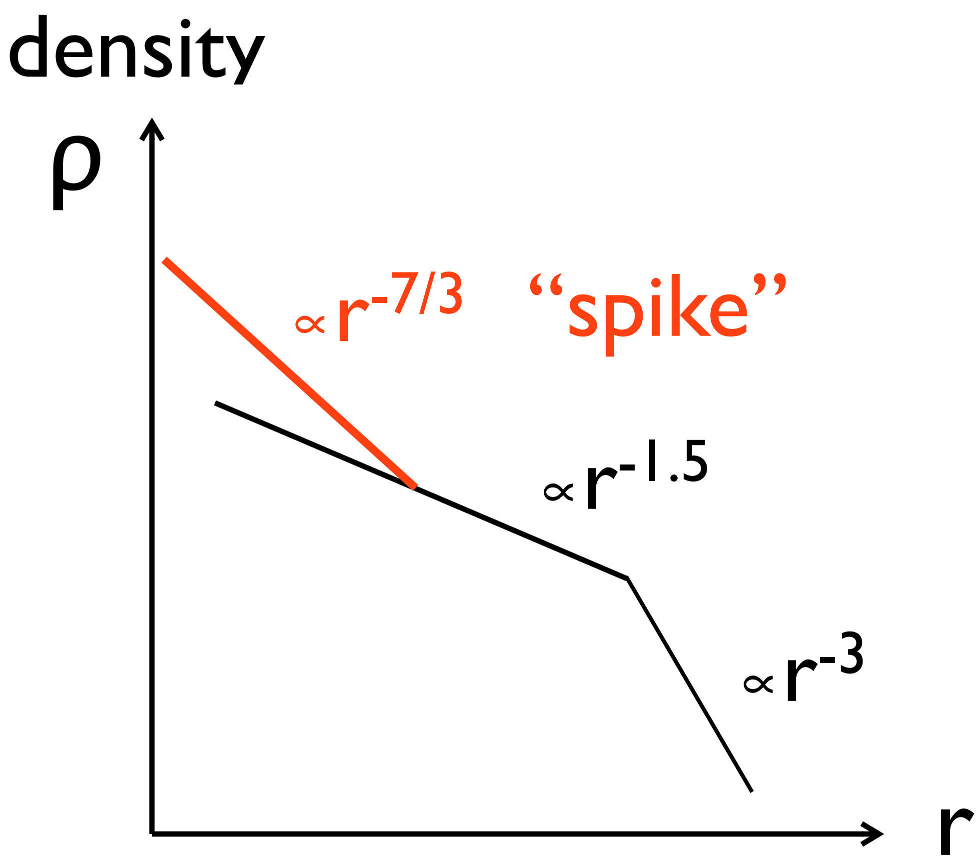
・If the density profile of the DM halo is steep enough, eLISA can measure the difference and even enable us to know the density profile of the DM halo with a good accuracy.

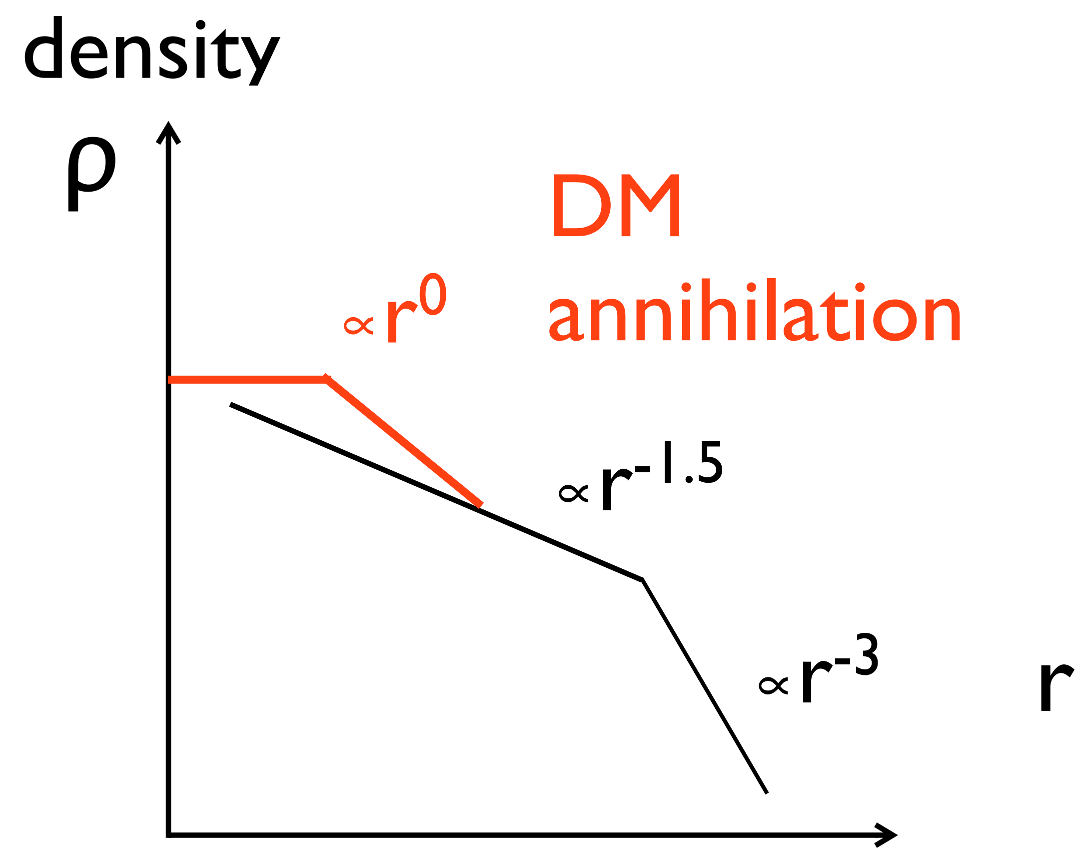
This may even offer hints on the BH formation history, since formation of DM spikes strongly depends on how BHs evolved.

Summary

- eLISA may determine the power index α

$$\rho(r) = \rho_{\rm sp}\left(\frac{r_{\rm sp}}{r}\right)^{\alpha}$$





→ GW observation

→ Gamma ray observation

complement each other