Neutron stars as sources of continuous GWs

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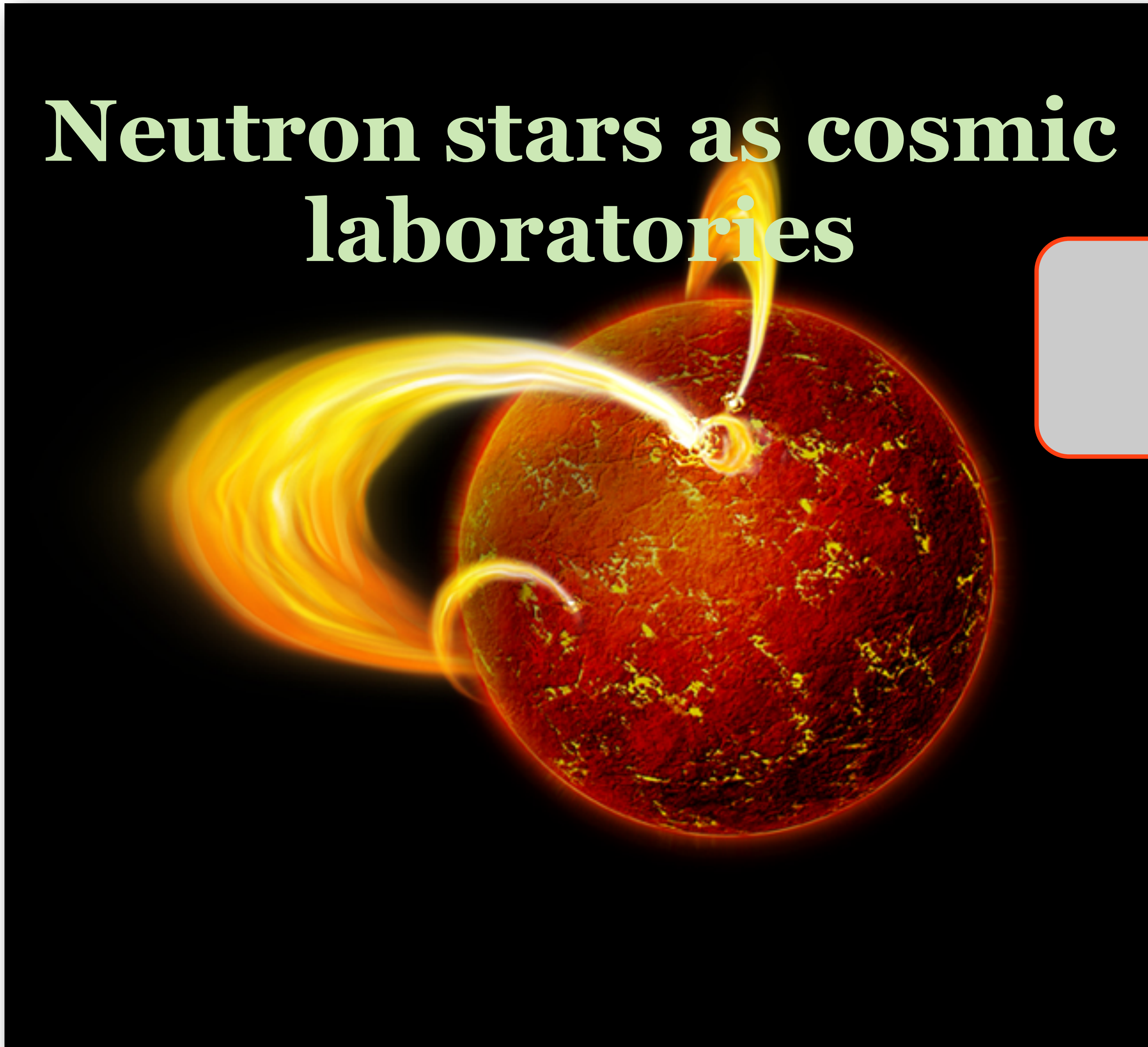


Properties of matter at supra-nuclear density

Infer the equation of state $p(\rho)$

Infer the composition (pairing etc.)

Infer the ground state of matter (hadrons vs hyperons & quarks)

GW emission

Relativistic Gravity

Use relativistic effects for measuring stellar parameters.

Test GR

On the verge of GW Astronomy

- The advanced detectors generation is at hand, data collection begins in late 2015.
- First direct detection of GWs is expected before the end of the decade!

www.advancedligo.mit.edu

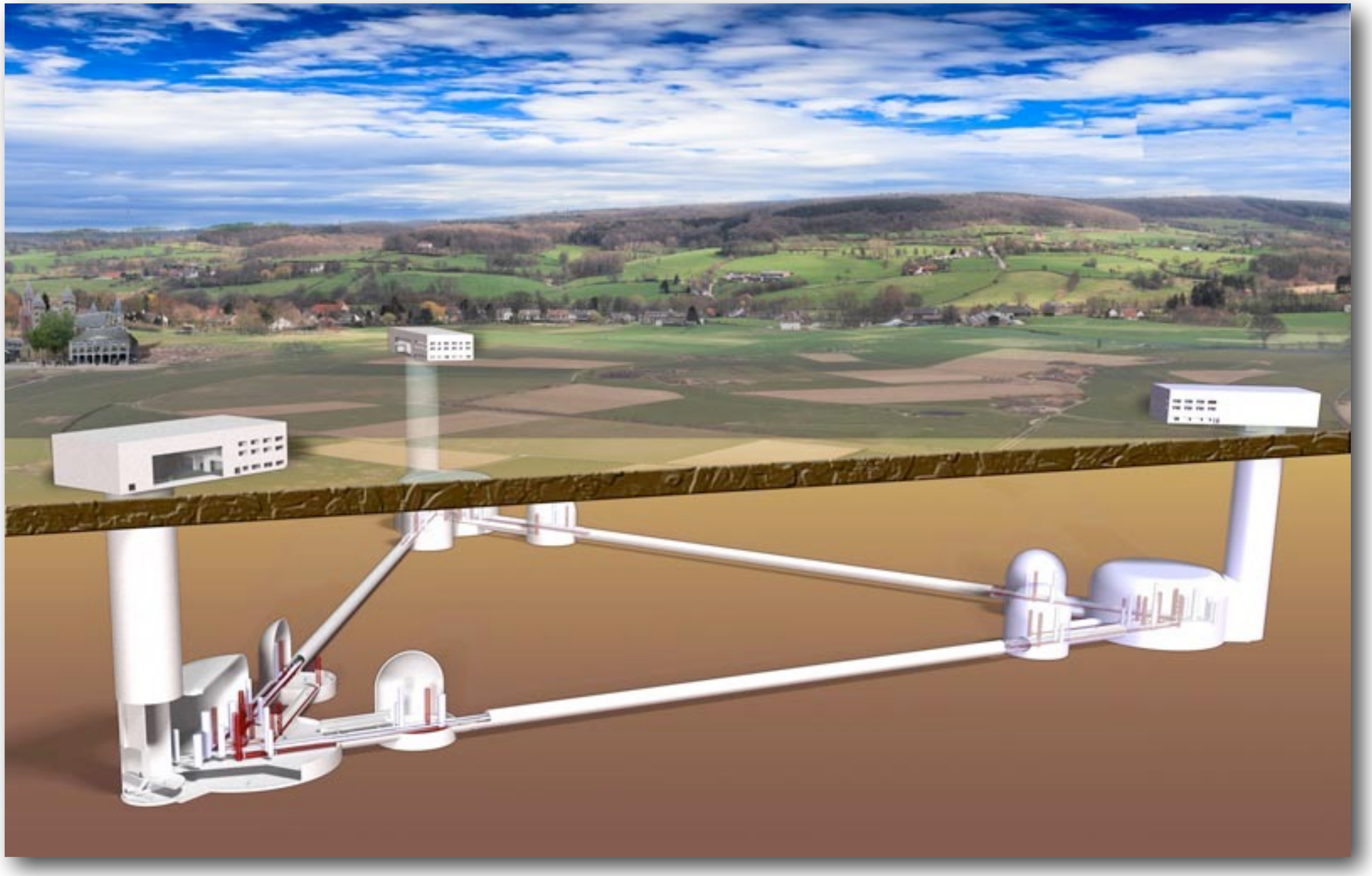
LIGO detector (Hanford site)

www.ego-gw.it

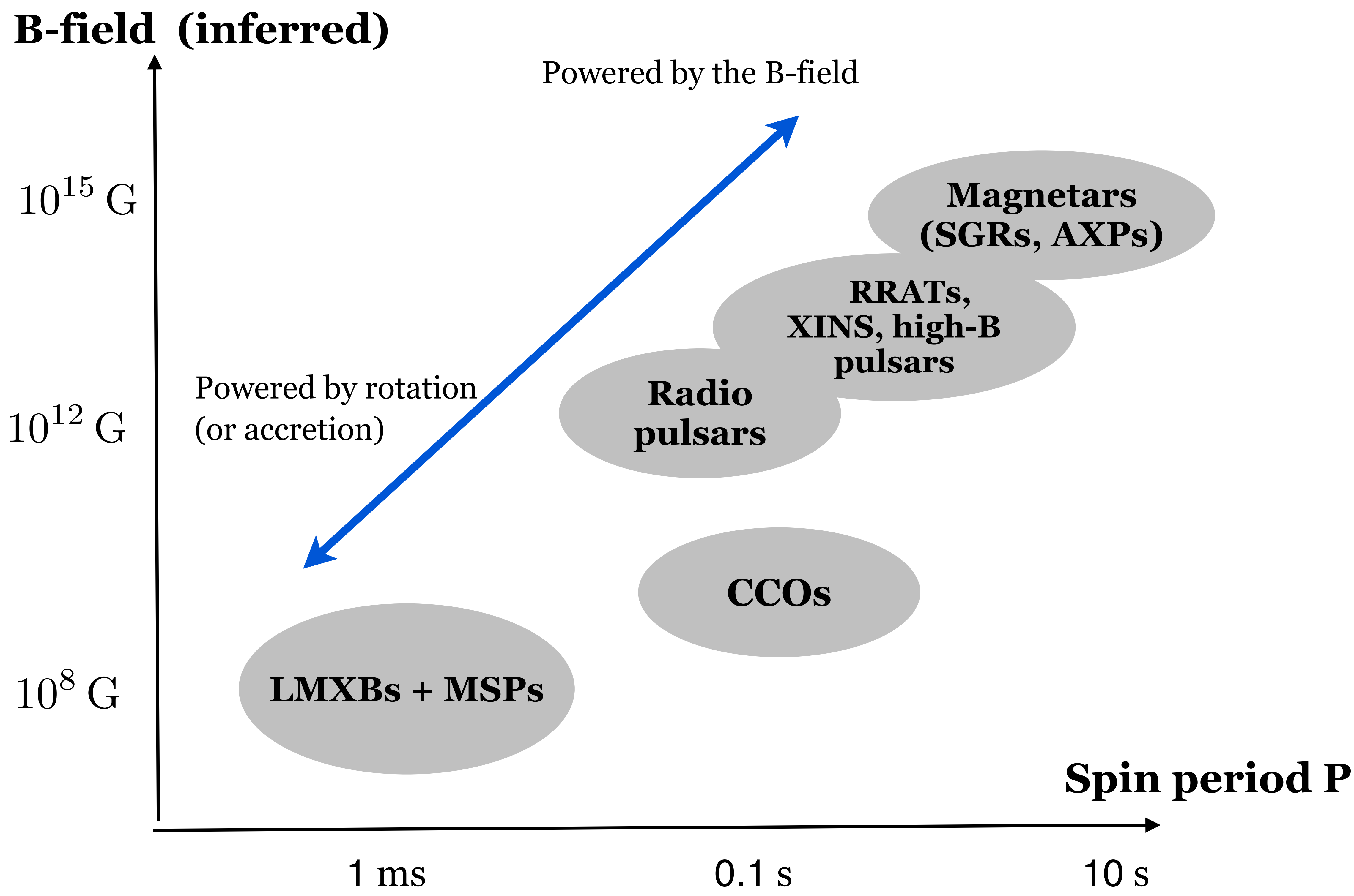
Virgo detector (near Pisa)

The 2020s

- The third generation of GW observatories: the Einstein Telescope
- Underground installation, about 10 times more sensitive than aLIGO/aVirgo.

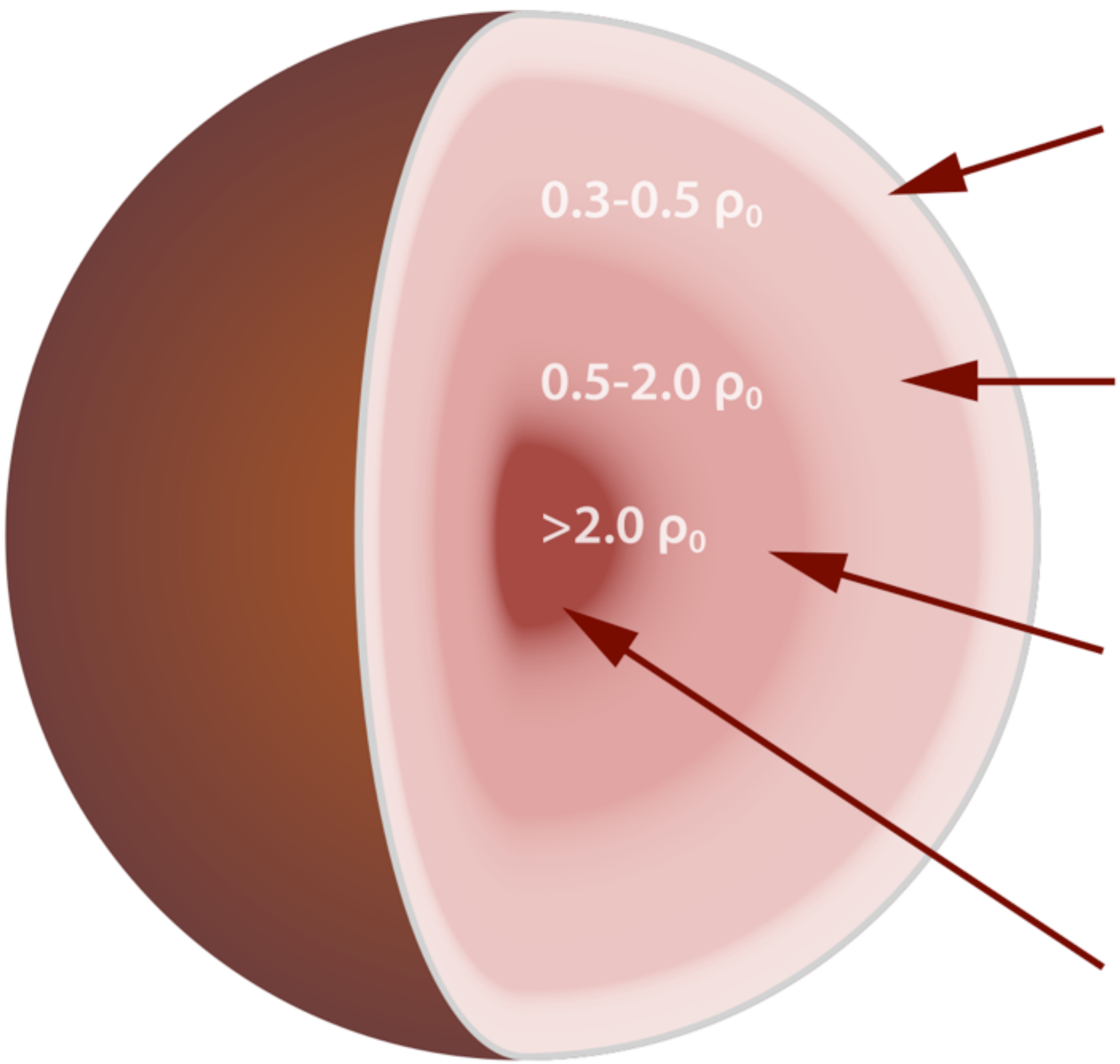


A neutron star zoo



Dissecting a neutron (or is it quark?) star

$\rho_0 = 2.8 \times 10^{14}\,\mathrm{gr/cm^3}$



outer crust 0.3-0.5 km
ions, electrons

inner crust 1-2 km
electrons, neutrons, nuclei
superfluidity, “pasta” phases

outer core ~ 9 km
neutron-proton Fermi liquid
few % electron Fermi gas
superfluidity, superconductivity

inner core 0-3 km
quark gluon plasma?
color superconductivity?

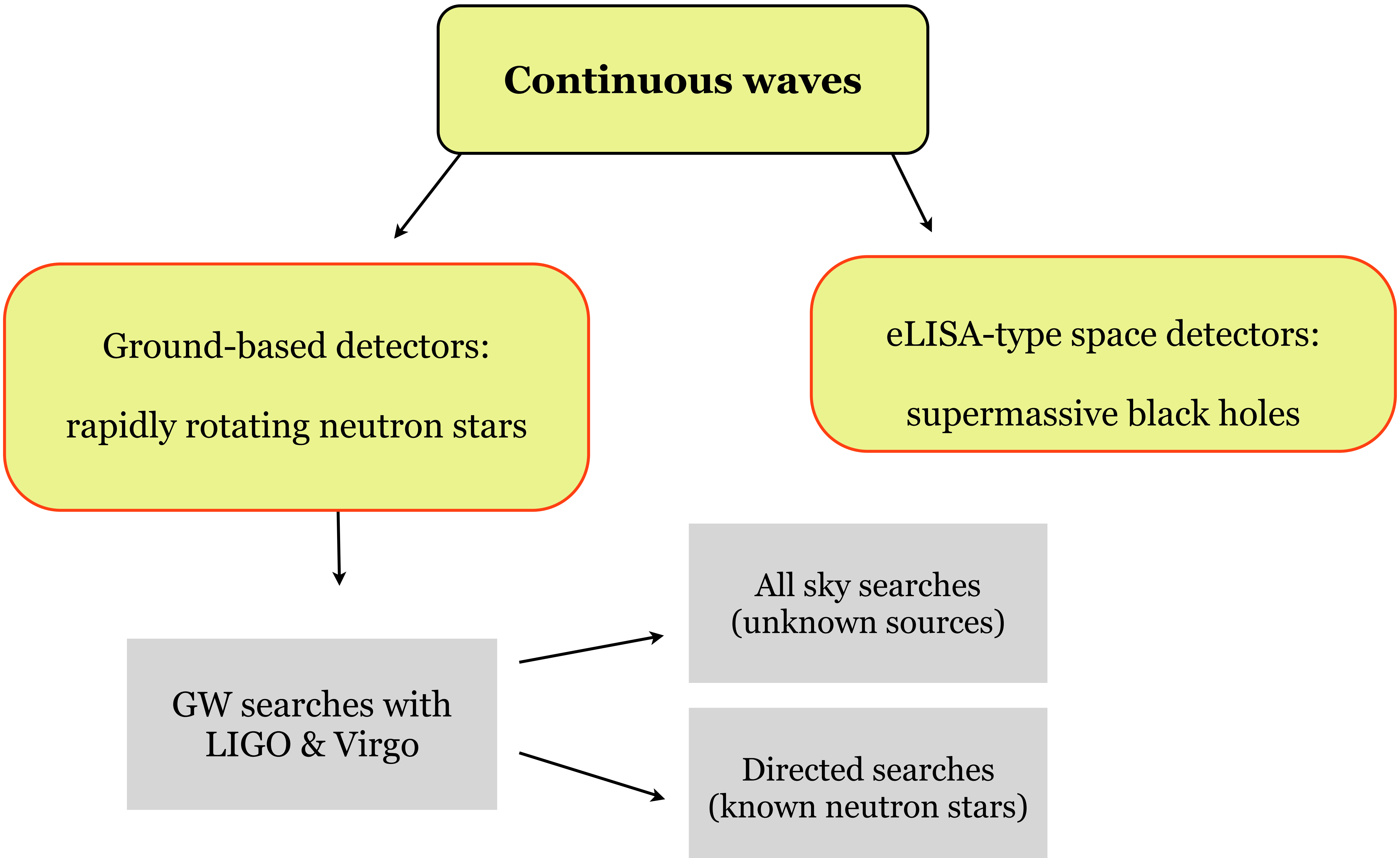
Neutron stars as GW sources

Burst emission
(not discussed here)

- Binary neutron star mergers (our safest bet for detection)
- Pulsar glitches
- Magnetar flares

Continuous emission
(next slide)

Searching for Continuous GWs

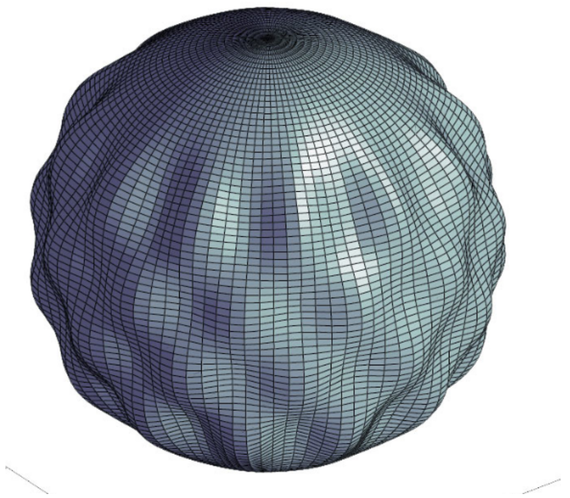


GWs from rotating neutron stars

Mechanisms for continuous emission

Non-axisymmetric mass quadrupole ("mountains")

Fluid part (oscillations)

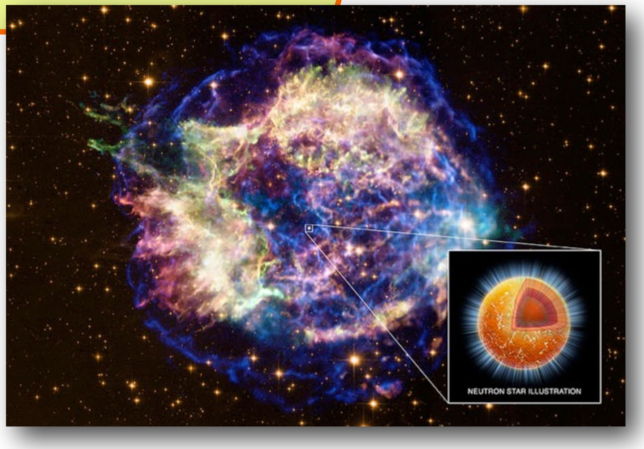


Targeting neutron stars

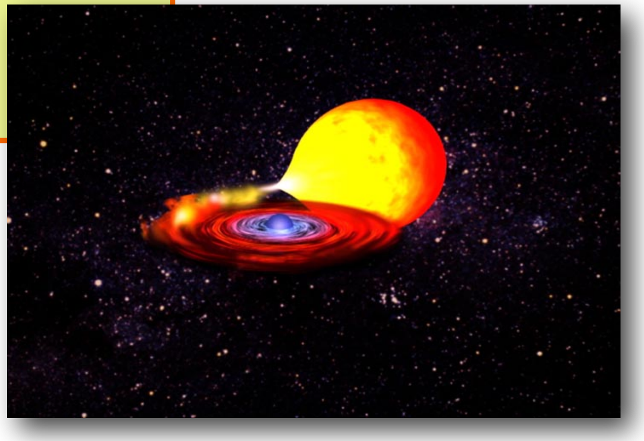
- **Four types** of continuous GW targets

Known pulsars (e.g. Crab)

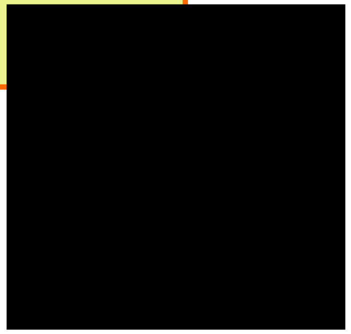
Non-pulsating, non-accreting systems (e.g. Cas A)



Accreting neutron stars (e.g. Sco X-1)



"Gravitars" (systems electromagnetically invisible!)

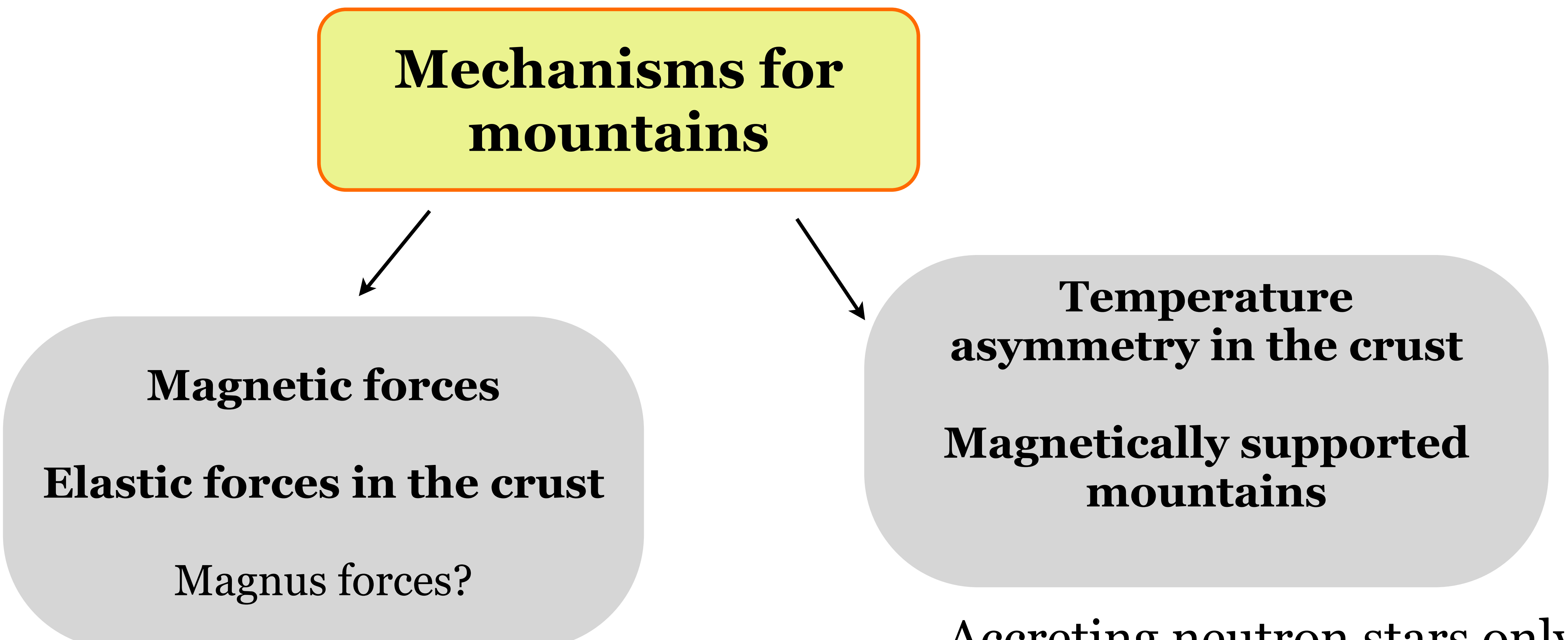


Neutron star mountains

“Mountains” in neutron stars

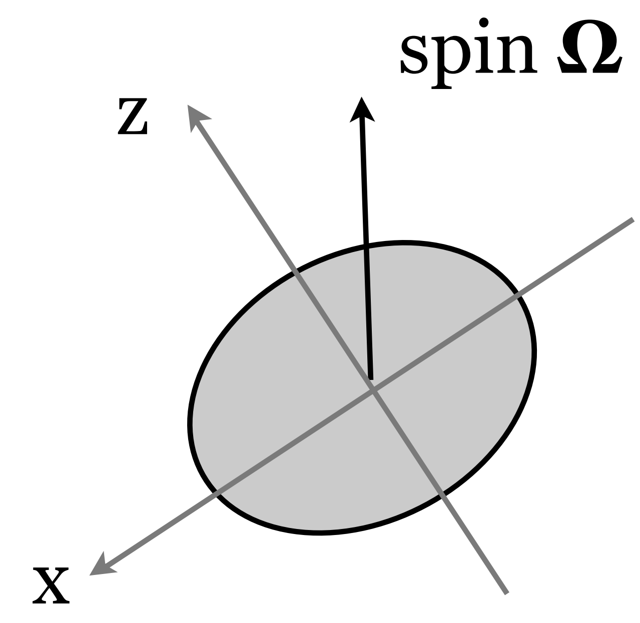
- Any mechanism leading to a ***non-axisymmetric mass quadrupole*** is interesting for GW emission!
  (note: in this regard the rotational deformation is irrelevant).

- The “mountain” may be “buried” in the stellar interior.



GWs from a rotating ellipsoid

- A textbook result: a rotating body with non-zero ellipticity (=quadrupole moment) is emits GWs if the symmetry axis is misaligned with the spin axis.



- **GW frequency:** $2f_{\rm spin}$
(under certain circumstances $f_{\rm spin}$ can also appear).

- **GW amplitude** (for a source at distance D):

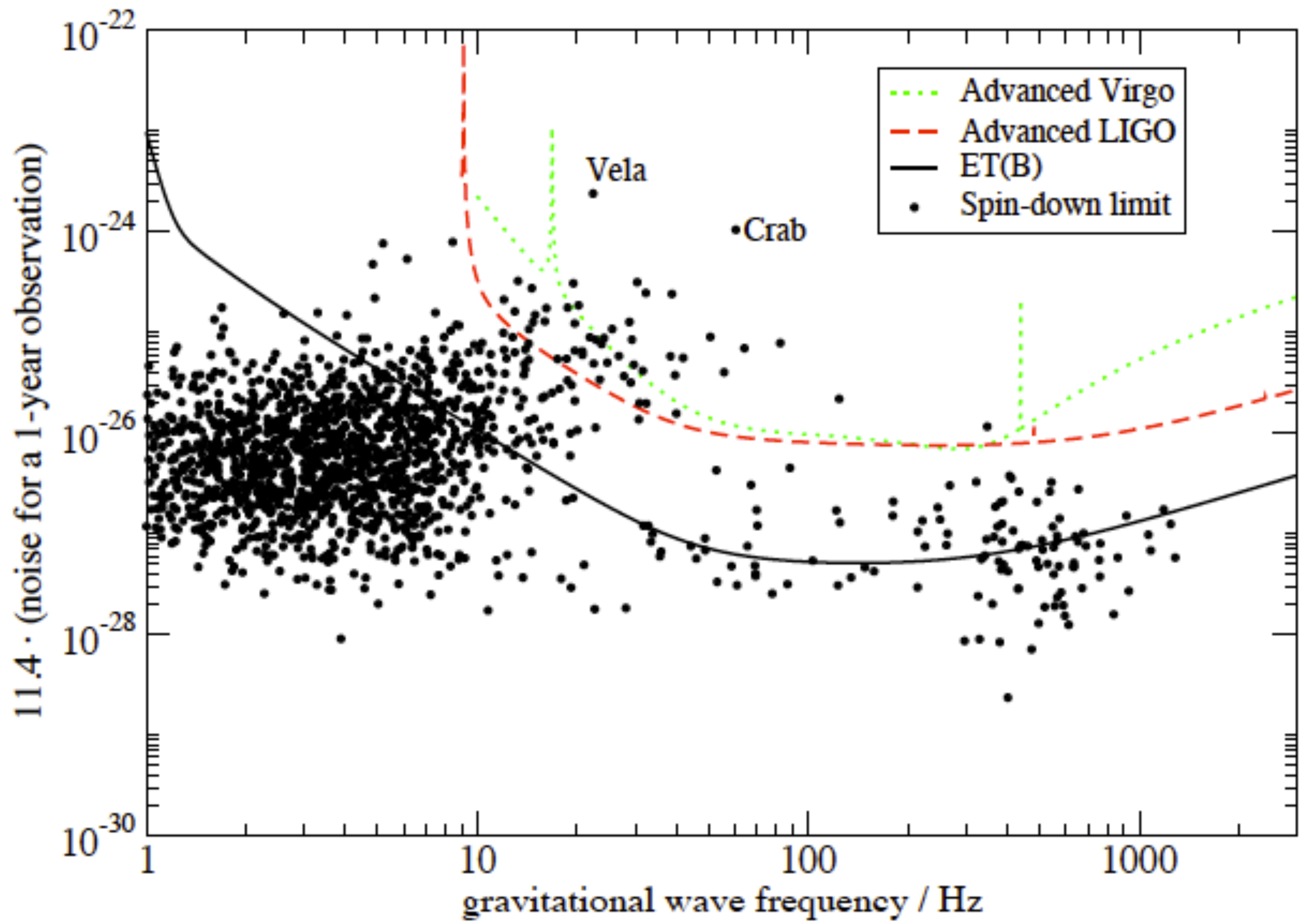
$$h_{\rm gw} \approx \frac{G}{c^4 D}\,\epsilon I_{\rm zz}\Omega^2 \approx 10^{-28}\left(\frac{1\,{\rm kpc}}{D}\right)\left(\frac{f_{\rm spin}}{10\,{\rm Hz}}\right)^2\left(\frac{\epsilon}{10^{-6}}\right)$$

stellar ellipticity: $\epsilon = (I_{\rm xx} - I_{\rm zz})/I_{\rm zz}$

Fast spinning systems strongly favored for detection!

Spin-down upper limits

- It is assumed a 100% conversion of the kinetic spin-down energy into GWs.
- The **no-detection of GWs** places an upper limit on the size of the ellipticity, and this becomes interesting if is comparable to the theoretical predictions.



Spin-down upper limits

- In fact, LIGO/Virgo no-detections have already "beaten" the spin-down limit for two pulsars [Aasi et al. 2014].

- **Crab pulsar**:

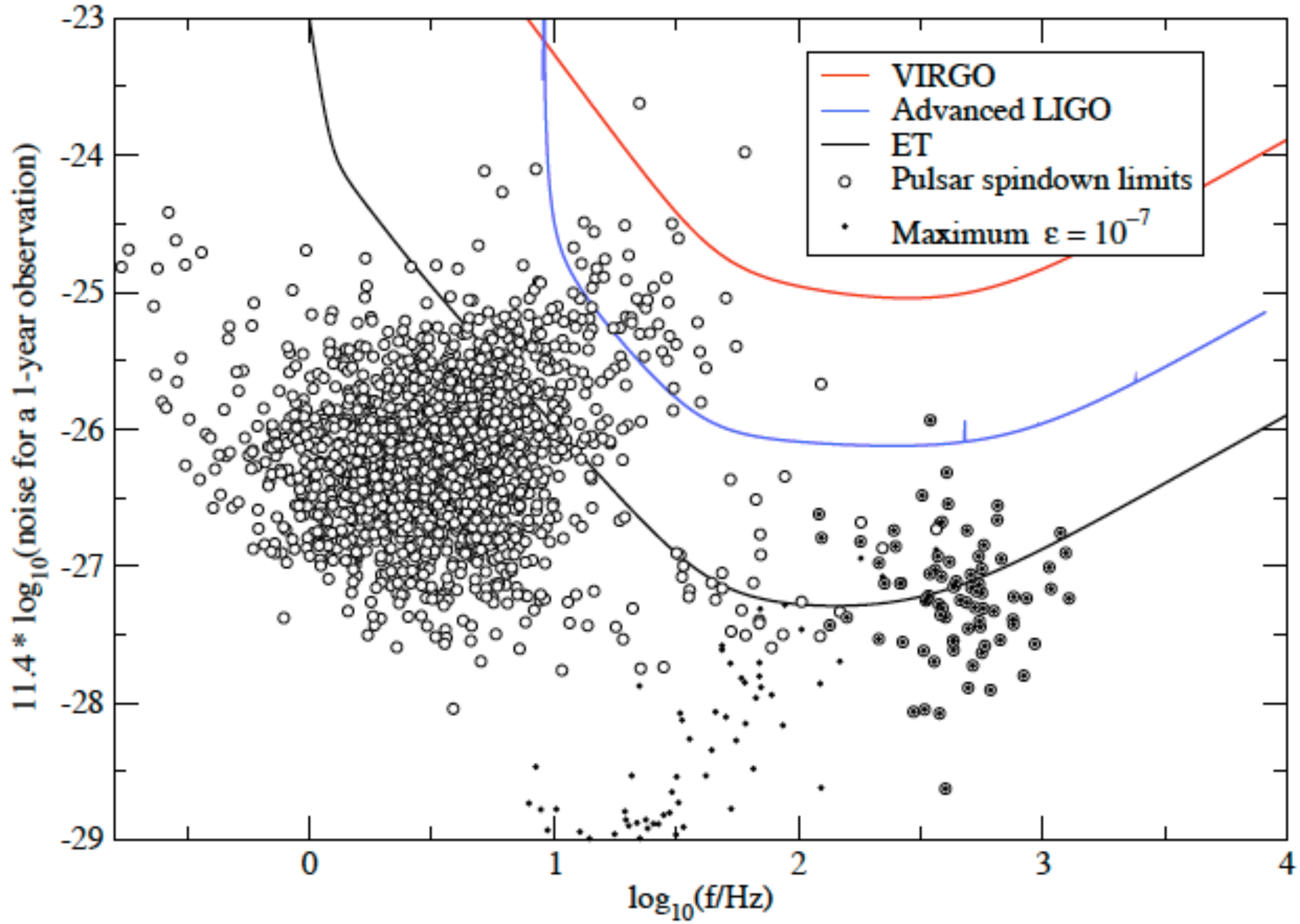
$$\frac{\text{energy in GWs}}{\text{spin-down energy}} \leq 1\% \quad \longrightarrow \quad \epsilon \lesssim 10^{-4}$$

- **Vela pulsar**:

$$\frac{\text{energy in GWs}}{\text{spin-down energy}} \leq 10\% \quad \longrightarrow \quad \epsilon \lesssim 6 \times 10^{-4}$$

- The data are already becoming theoretically interesting.

Mountains: GW detectability



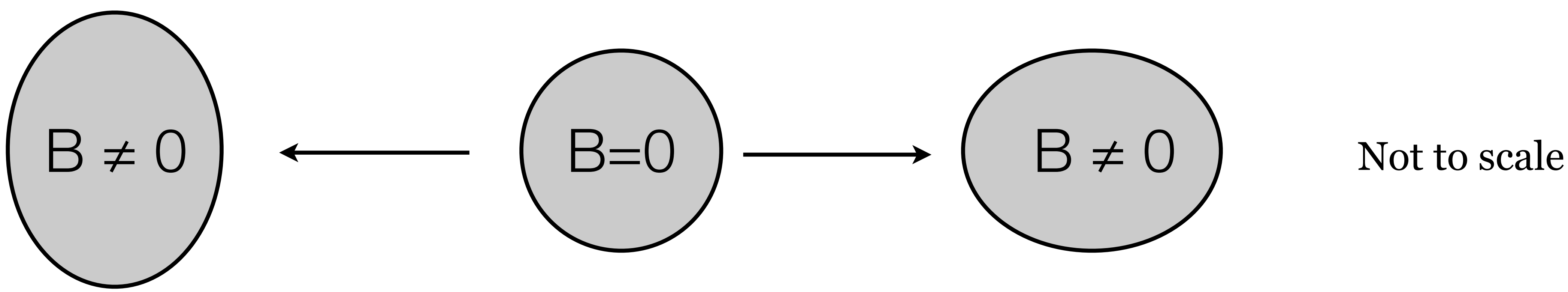
assumed ellipticity: $\epsilon = 10^{-7}$

[Andersson et al. 2011]

Magnetic mountains

- The **magnetic field** is the most "robust" way of deforming a star (first proposed in the 1950s by Chandrasekhar & Fermi).

- The deformation is generically non-axisymmetric and the shape can be oblate or prolate with respect to the magnetic axis.



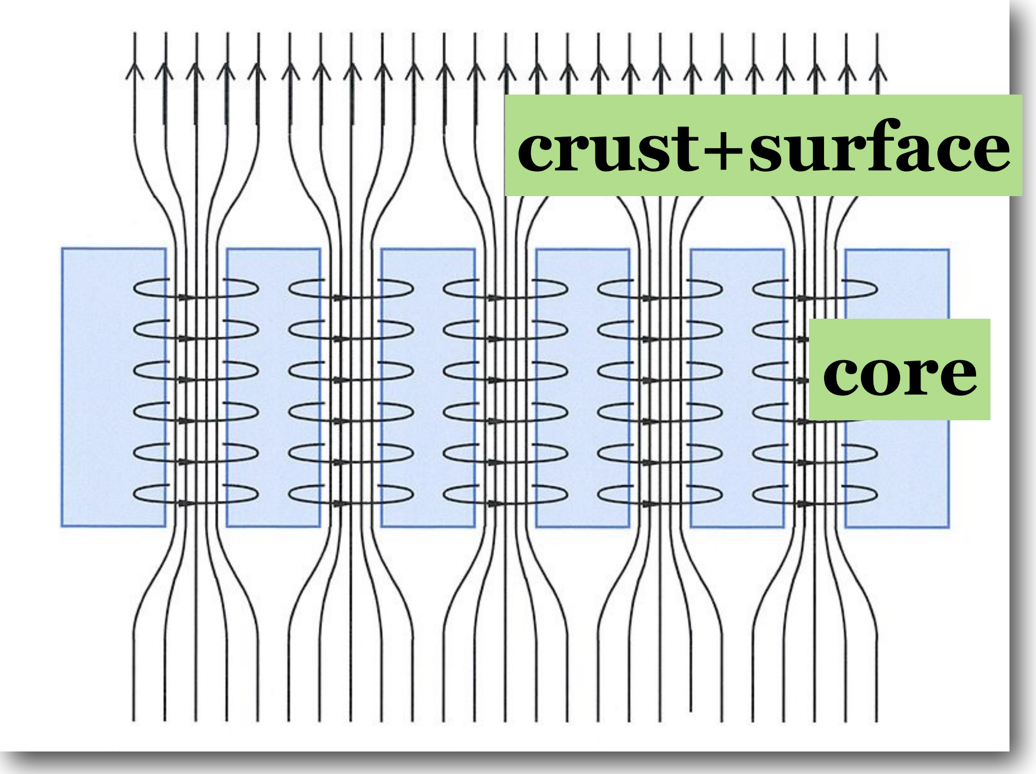
- The deformation (ellipticity) roughly equals the ratio of the volume-averaged magnetic energy to gravitational binding energy:

$$\epsilon \approx \frac{E_{\rm mag}}{E_{\rm grav}} \approx \frac{\int B^2 dV}{GM^2/R} \approx 10^{-12} \left(\frac{\bar{B}}{10^{12}\ {\rm G}} \right)^2$$

Adding proton superconductivity

- In fact, the magnetic mountain becomes bigger if we account for the likely presence of (type II) **proton superconductivity** in neutron star cores.

- Superconductivity amplifies the magnetic force by a factor ~ H_{c1}/B where the critical field is $H_{c1} \approx 10^{15}\,\mathrm{G}$.

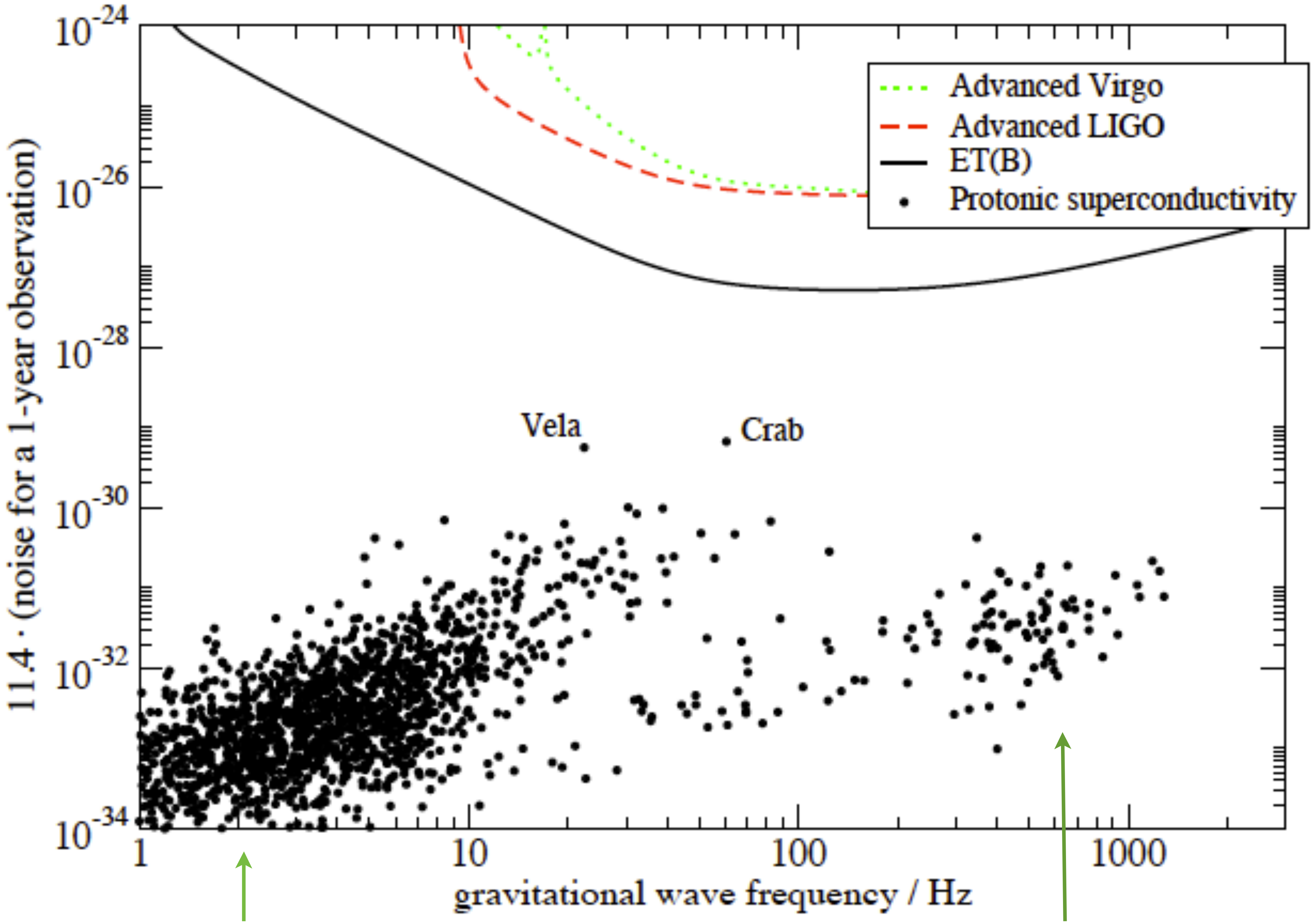


- The magnetic field threads the stellar core in the form of **quantised proton fluxtubes** and the tension of these fluxtubes gives rise to the magnetic force.

- The ellipticity is given by:

$$\epsilon \approx \frac{\text{averaged fluxtube tension}}{E_{\text{grav}}} \approx 10^{-9} \left(\frac{\bar{B}}{10^{12}\,\mathrm{G}} \right)$$

Magnetic mountains: detectability



High B, low Ω systems (like magnetars)

High Ω, low B systems (like LMXBs & MSPs)

Elastic stresses in the crust

- The **maximum** ellipticity supported by elastic forces in the solid crust:

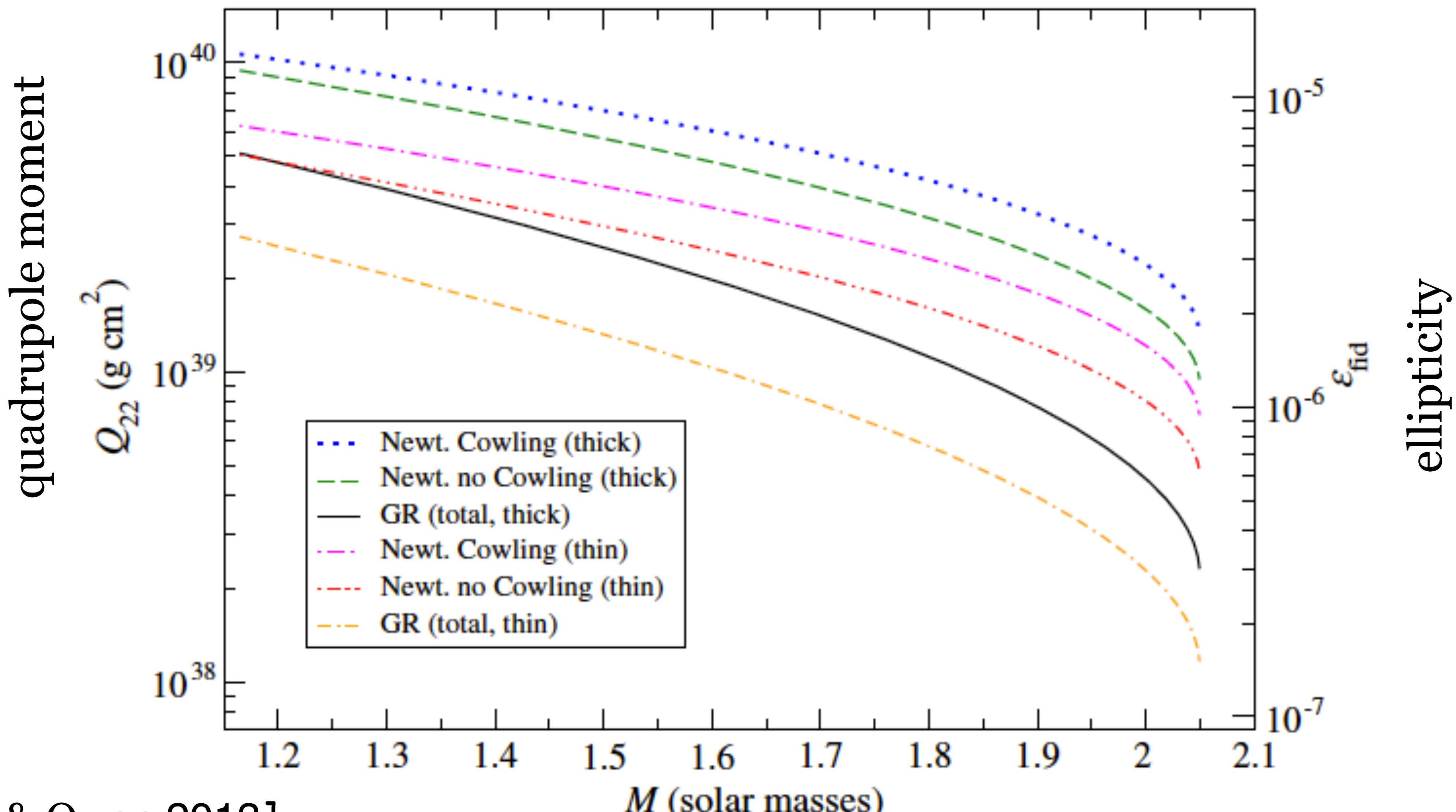
$$\epsilon \approx \frac{E_{\rm elastic}}{E_{\rm grav}} \approx \frac{\mu V_{\rm crust}\sigma_{\rm br}}{GM^2/R} \approx 10^{-7}\left(\frac{\sigma_{\rm br}}{10^{-2}}\right)$$

- The crustal "breaking strain" $\sigma_{\rm br}$ is the key unknown: state-of-art molecular simulations suggest $\sigma_{\rm br} \sim 0.01 - 0.1$ [Horowitz & Kadau 2009]

- Ushomirky et al. 2000: rigorous calculation, in good agreement with the above approximate estimate.

- **Need mechanism for deforming the crust in the first place!**

Maximum elastic deformation in GR



[Johnson-McDaniel & Owen 2013]

- GR **reduces** the maximum deformation by a factor ~ a few.
- Small mass systems are favoured.

Thermal mountains

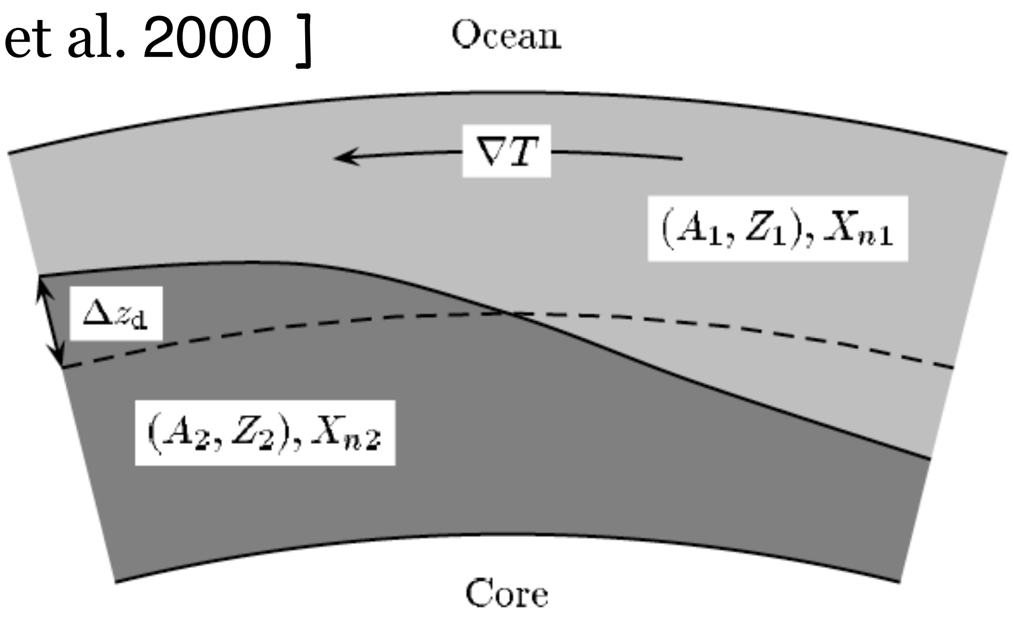
- This mechanism is based on having **asymmetric heating** of the crust in accreting neutron stars. This in turn leads to an asymmetry in the crust composition and the mass quadrupole [Bildsten 1998, Ushomirsky et al. 2000]

- The ellipticity is:

$$\epsilon \sim 10^{-9} \left(\frac{\delta T_{\ell=2}}{10^5\,\mathrm{K}} \right) \left(\frac{Q}{30\,\mathrm{MeV}} \right)^3$$

Q = reaction energy threshold

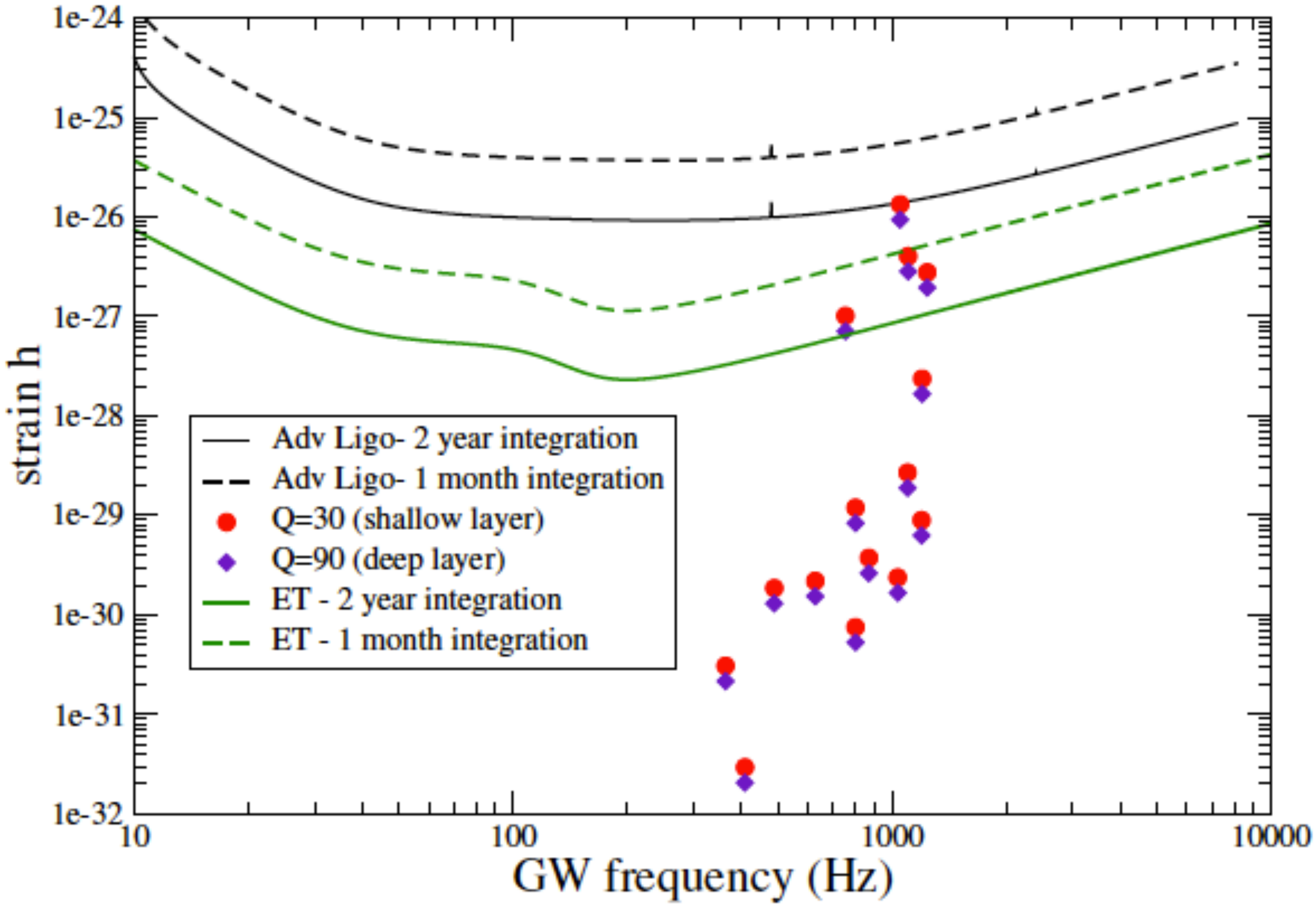
[Ushomirsky et al. 2000]



- Unclear if a significant quadrupolar T-gradient can be sustained (the following figures assume an optimistic fiducial value $\delta T_{\ell=2}/\delta T_{\ell=0} = 0.1$).

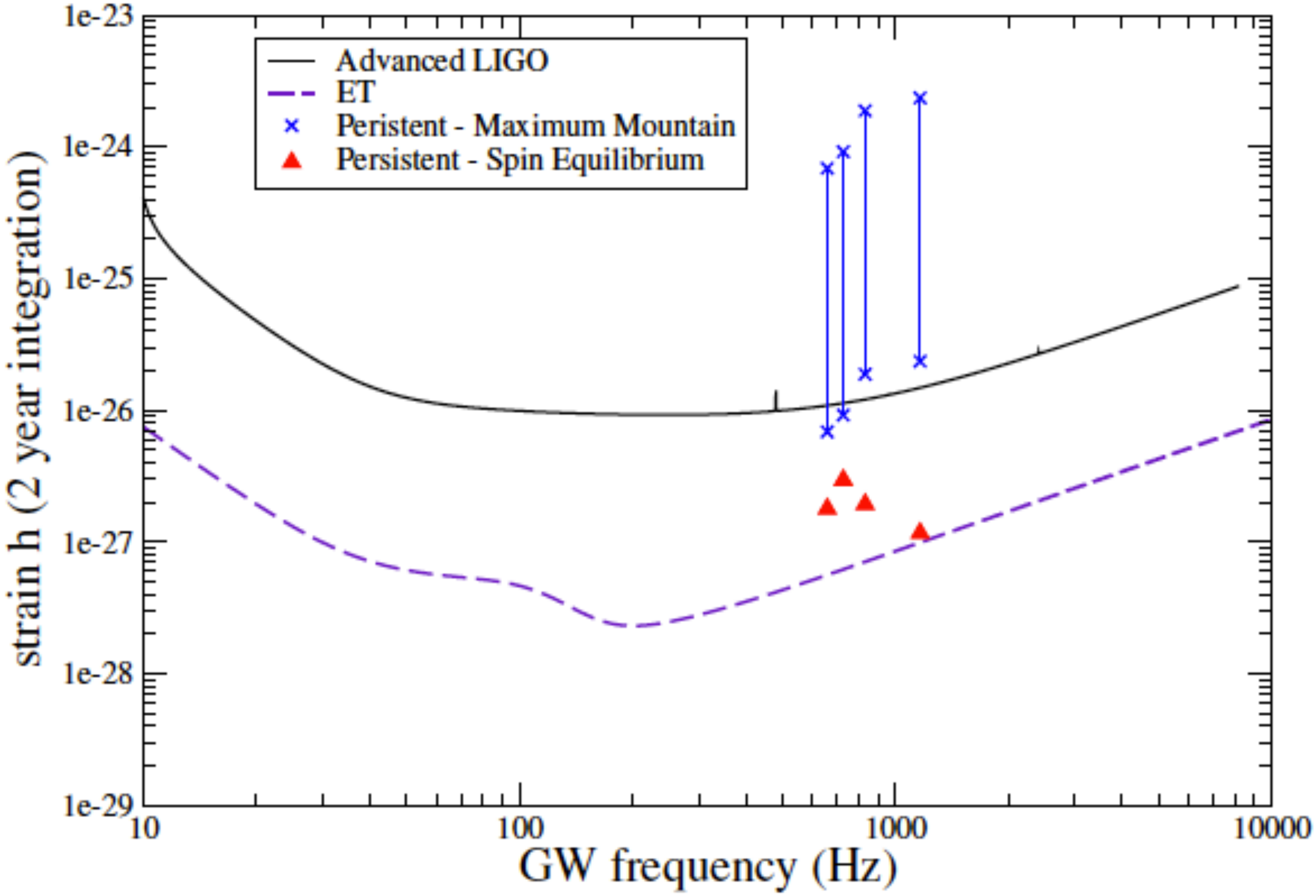
- Persistent accretion sources are favoured, otherwise the mountain could be "washed away" during accretion periods (thermal relaxation ~ few years).

Thermal mountains: transient sources



[Haskell et al. 2014]

Thermal mountains: persistent sources



[Haskell et al. 2014]

Magnetically supported mountains

- These require the combined action of accretion & the B-field [Melatos & Payne 2005]

- Matter from the accretion disk is funneled onto the magnetic poles. Once there, it spreads pushing the magnetic lines towards the equator.

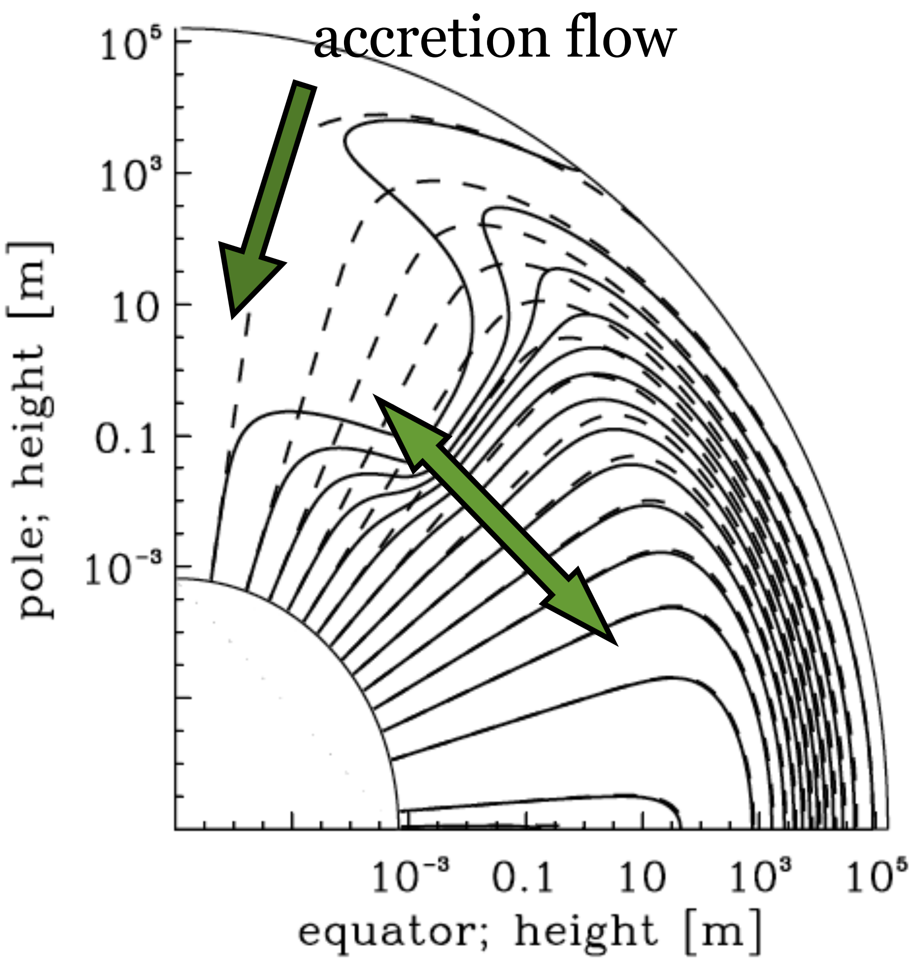
- A mass quadrupole can be sustained, misaligned with the spin axis --> GWs

- The estimated ellipticity is (for $M_{\rm acc} \lesssim M_{\rm crit}$):

$$\epsilon \sim \frac{M_{\rm acc}}{M_\odot}\left(1+\frac{M_{\rm acc}}{M_{\rm crit}}\right)^{-1}$$

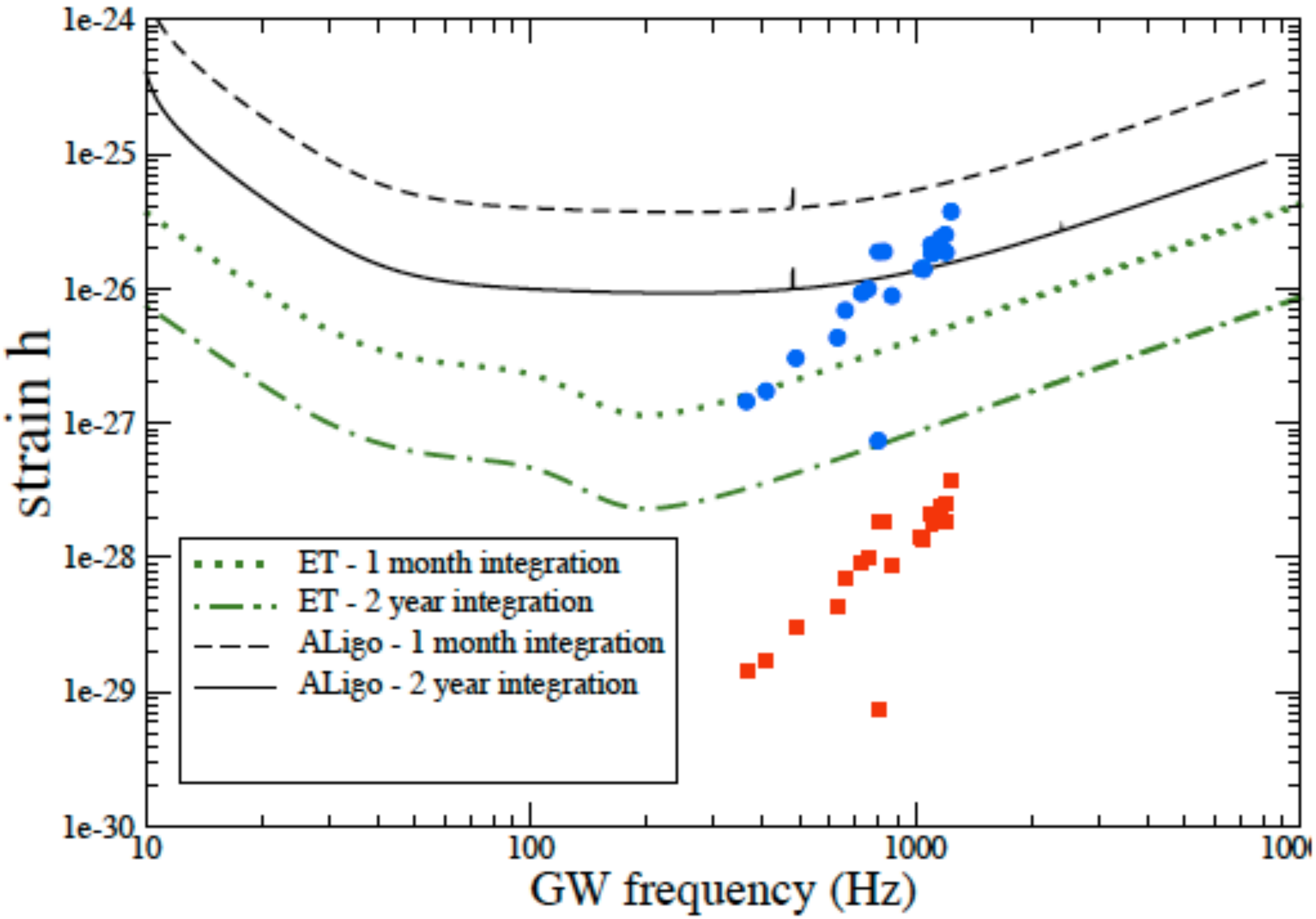
$$\frac{M_{\rm crit}}{M_\odot} \sim 10^{-7}\left(\frac{B_{\rm in}}{10^{12}\,{\rm G}}\right)^{4/3} \qquad \dot{M}_{\rm acc} \sim 10^{-9} M_\odot/{\rm yr}$$

- More work is required on the stability of these mountains.



[Melatos & Payne 2005]

B-field supported mountains: detectability



initial field:

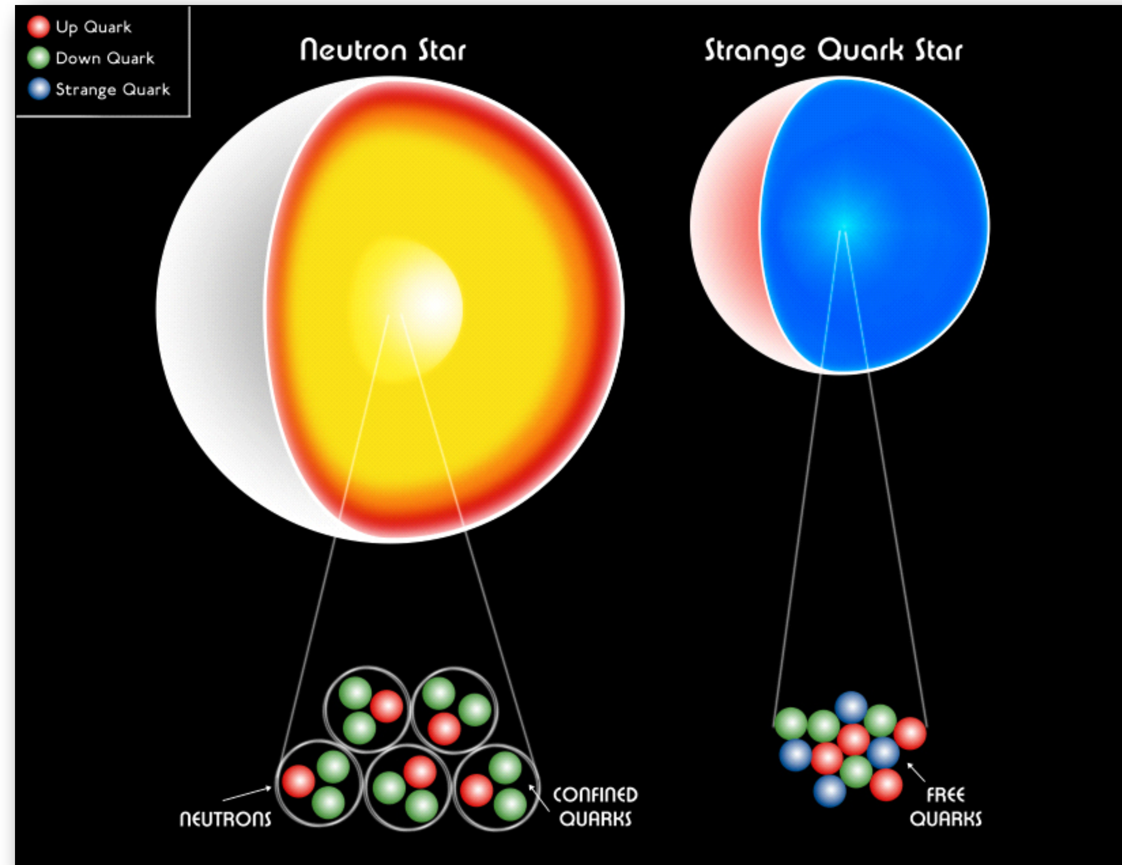
$B_{\rm in} = 10^{12}\,{\rm G}$

initial field:

$B_{\rm in} = 10^{10}\,{\rm G}$

[Haskell et al. 2014]

Mountains: more exotic scenarios



- These scenarios usually invoke the presence of some quark matter phase in the stellar core.

- Typically, quark matter can sustain **much higher** magnetic/elastic deformations. This implies that strange quark stars could be more "visible" sources of GWs.

- An exciting prospect: use GW observations to unveil the presence of stable quark matter inside neutron stars.

The quest for quark matter

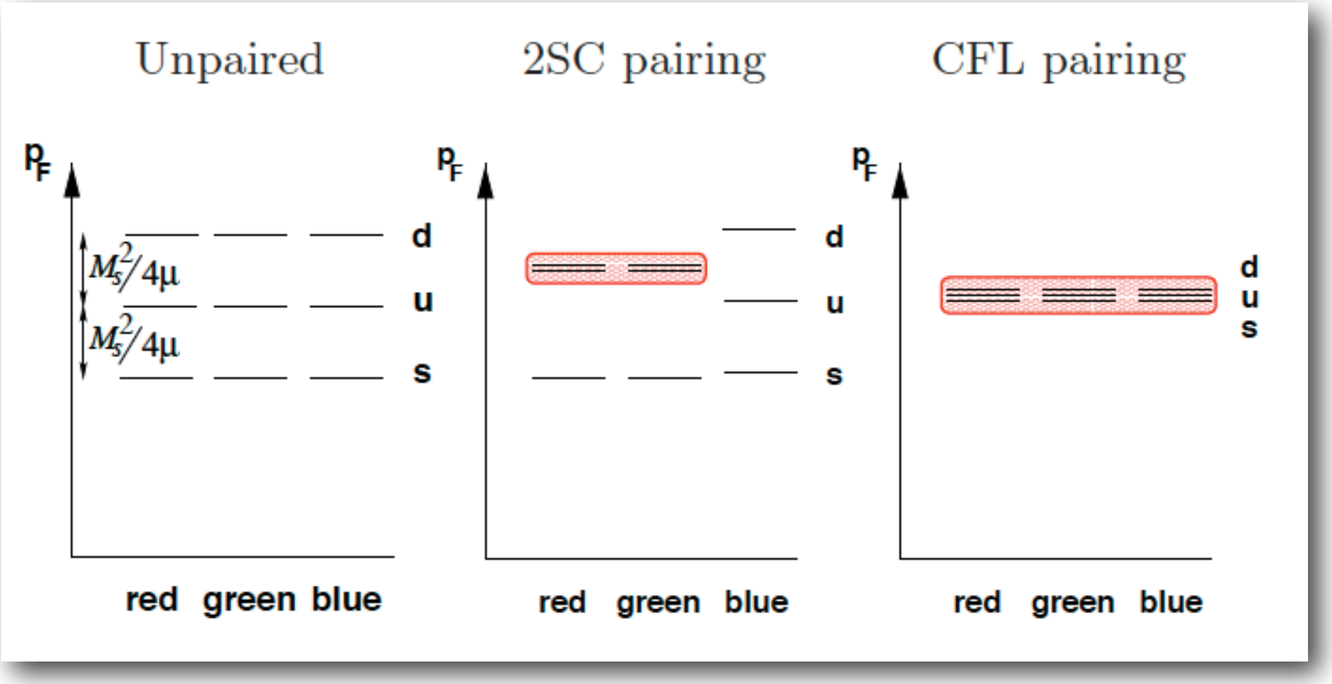
- **Are deconfined quarks the ground state of matter?**
  This is a fundamental question for neutron star astrophysics.

- In its stablest form, quark matter is a color superconductor, as a result of quark color/flavor pairing.

- "Popular" phases:

✓ "2SC" phase: up-down quark pairing

✓ "CFL" phase: all quarks pair.



[Alford et al. 2008]

- Real systems can be **hybrids** with quark inner core and normal hadronic outer mantle --> challenge for observations!

Color-magnetic mountains

- The B-field penetrates a 2SC/CFL superconductor by forming **color-magnetic fluxtubes** [Iida & Baym 2002, Alford & Sedrakian 2010].

- These are ~ 1000 more tensile than protonic fluxtubes in "normal" superconductivity, while their numbers are comparable => the magnetic force goes up by the same factor!

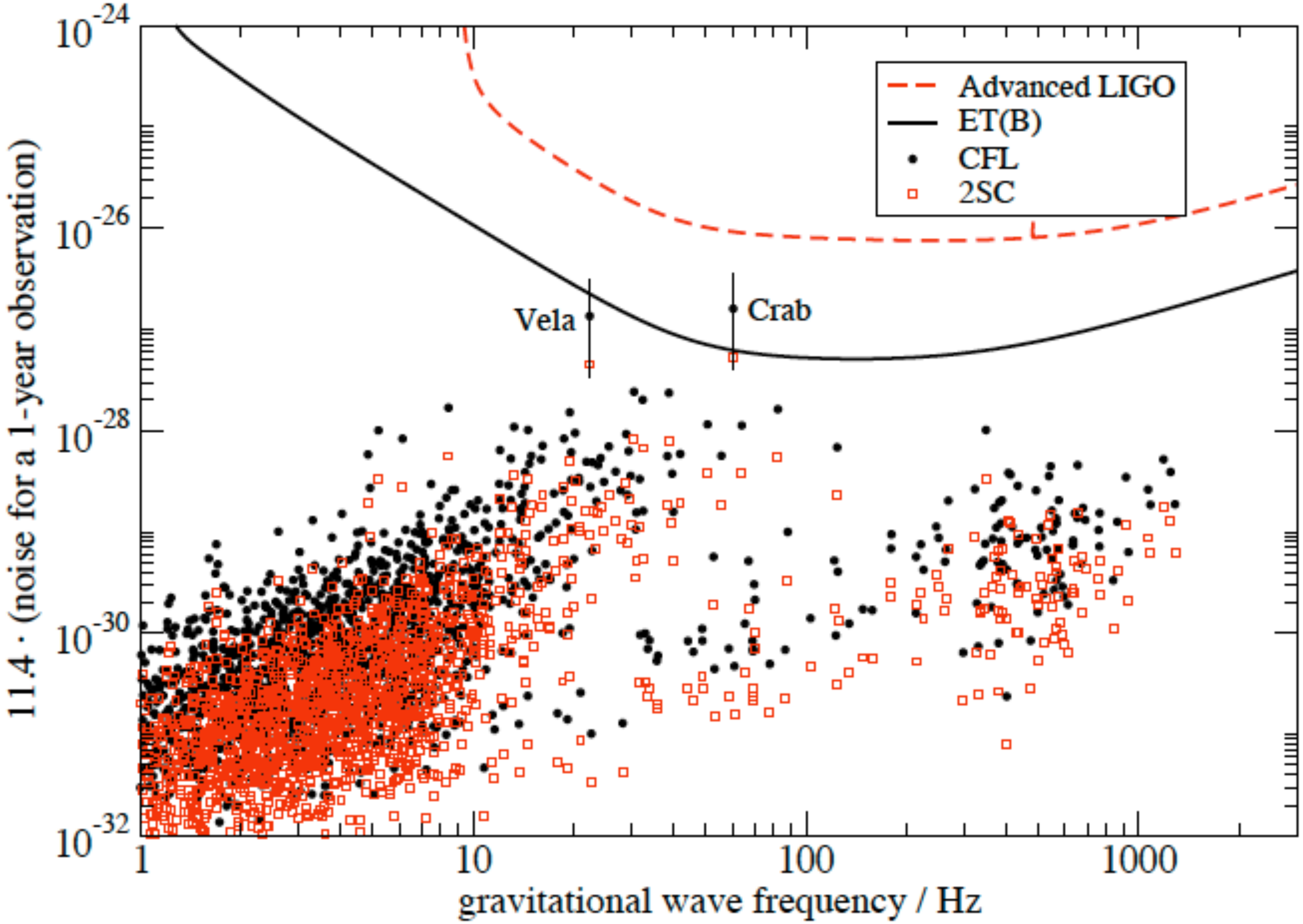
- The resulting "interior mountain" deformation is:

$$\epsilon_{\rm cfl} \approx 10^{-7} \left(\frac{\bar{B}}{10^{12}\,{\rm G}}\right) \left(\frac{V_q}{V_{\rm star}}\right) \left(\frac{\mu_q}{400\,{\rm MeV}}\right)^2$$

- Adjustable parameters: the quark core volume and the interior B-field

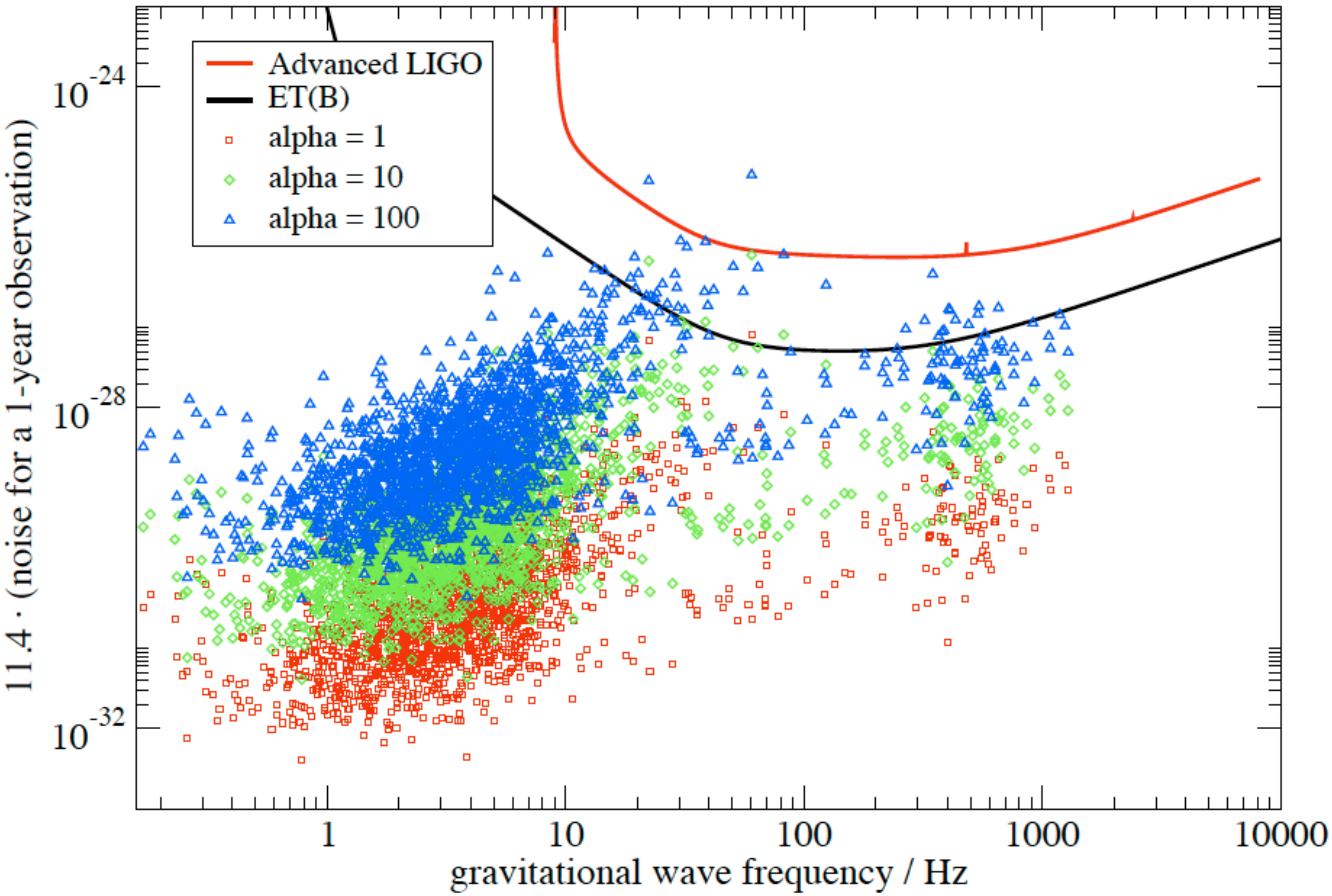
- We show results for the fiducial values: $V_q = 0.5 V_{\rm star} \quad \bar{B} = 2 B_{\rm surf}$

Color-magnetic mountains: detectability



[KG, Jones & Samuelsson 2012]

Color-magnetic mountains: detectability



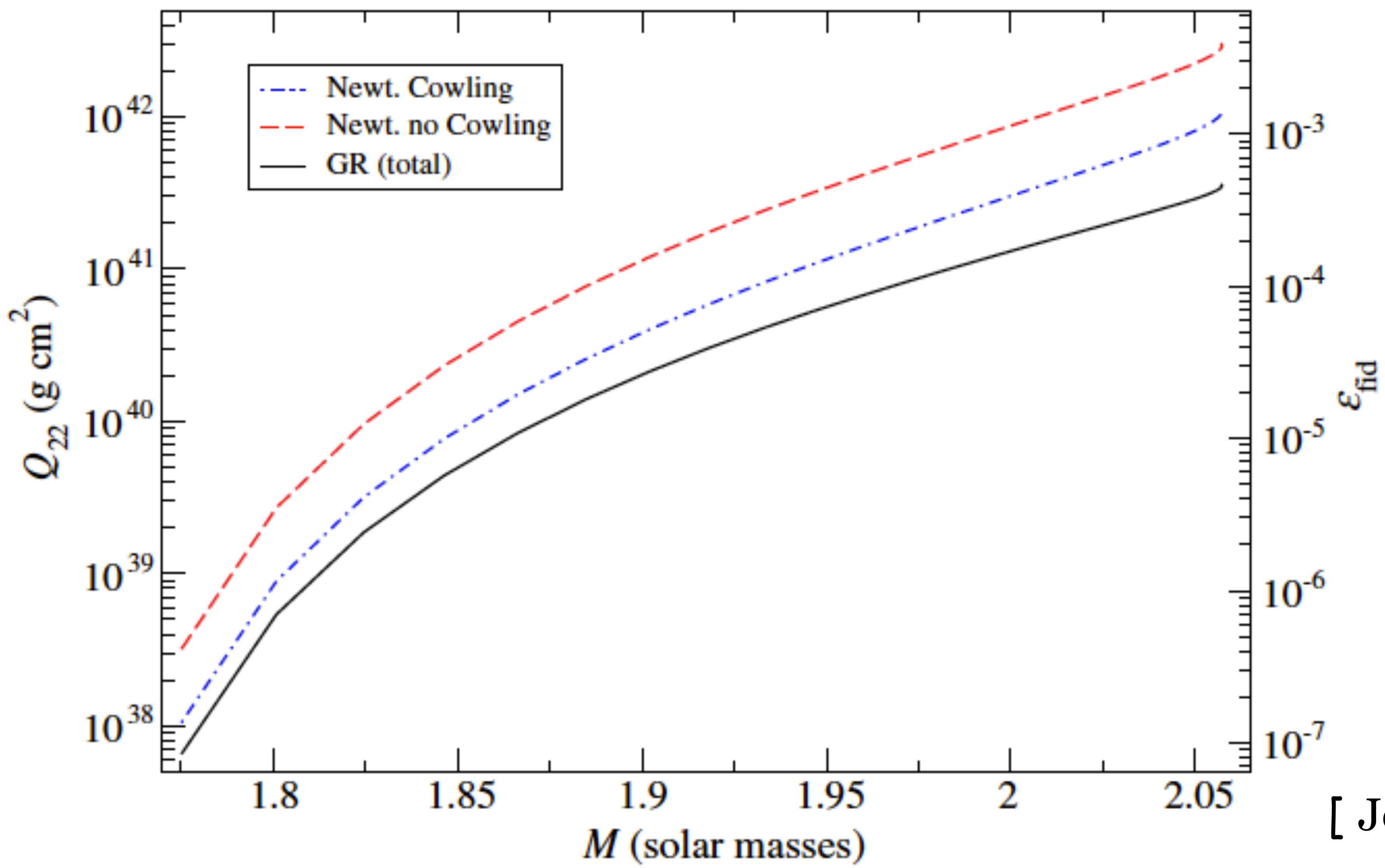
Re-adjusting the parameter:

$$\alpha = \frac{\bar{B}}{B_{\rm surf}}$$

[KG, Jones & Samuelsson 2012]

Elastic deformation in solid quark matter

- Some models predict solid color-superconducting quark phases, and these tend to have a much higher shear modulus than crustal matter.
- Detectability: solid quark cores favour high-mass systems

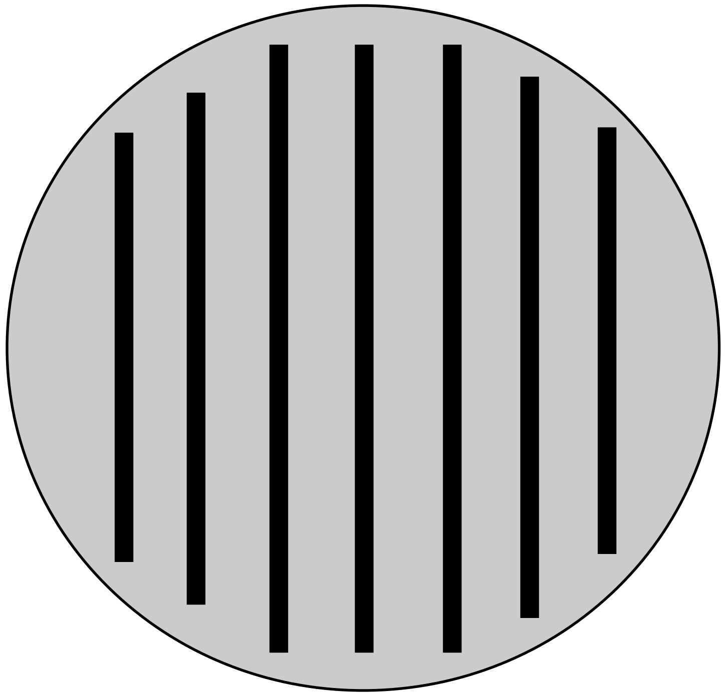


[Johnson-McDaniel & Owen 2013]

Magnus mountains ?

- This mechanism requires the presence of pinned superfluid vortices somwhere in the stellar interior (crustal lattice, proton fluxtubes).

- These pinned vortices experience a Magnus force. If pinning is non-axisymmetric, then a "Magnus mountain" should arise.



- So far, no detailed analysis exists; a simple calculation predicts [Jones 2002]:

$$\epsilon \approx 5 \times 10^{-9} \left(\frac{\Omega_{\rm pn}}{0.01\,{\rm Hz}} \right) \left(\frac{f_{\rm spin}}{100\,{\rm Hz}} \right) \left(\frac{\delta M_{\rm pin}}{0.1 M_\odot} \right)$$

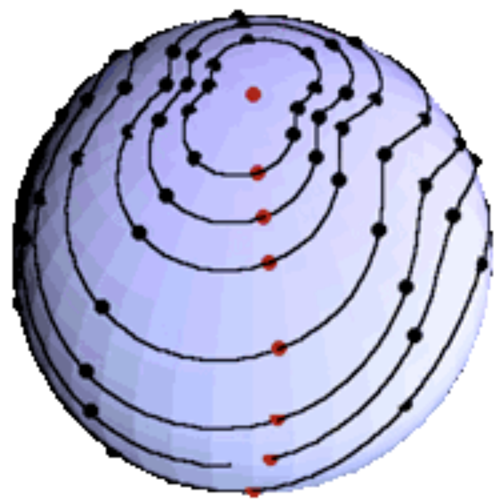
$\delta M_{\rm pin} =$ mass of pinned superfluid

- This is comparable to other mechanisms, therefore it warrants more study.

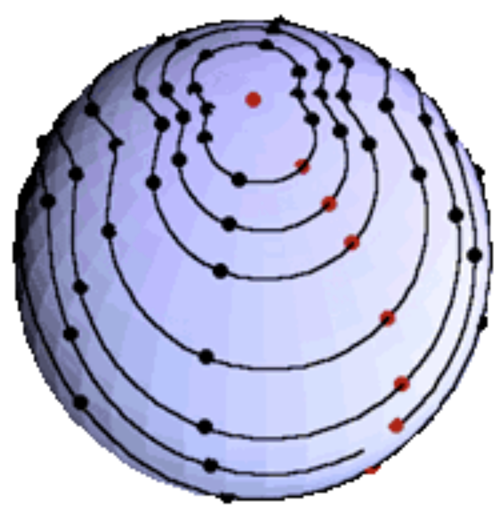
GWs from r-modes

The r-mode instability

- The r-modes is a special class of **inertial waves**, characterised by nearly horizontal fluid motion.
- r-modes may be driven unstable by the emission of GWs via the **CFS mechanism**: this involves the reverse-dragging of the mode by the rotating background.
- The r-mode GW radiation is special in the sense that it is dominated by the **current multipole.**
- The $\ell = m = 2$ r-mode is the most unstable one, with a growth timescale of ~ 1 min.
- **GW frequency:** $f_{\text{gw}} = f_{\text{mode}} \approx \frac{4}{3} f_{\text{spin}}$



corotating frame



inertial frame

Figure credit: Hanna & Owen

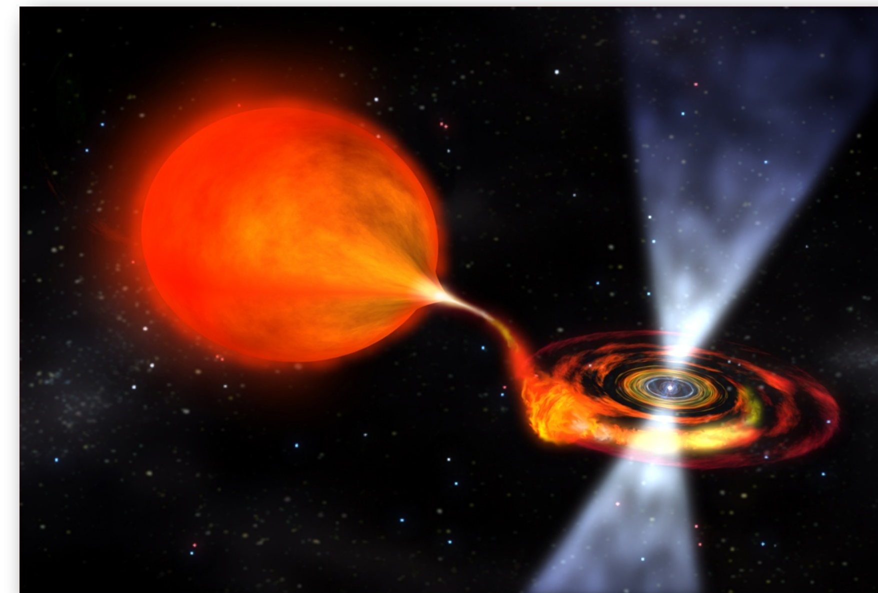
LMXBs: spin equilibrium

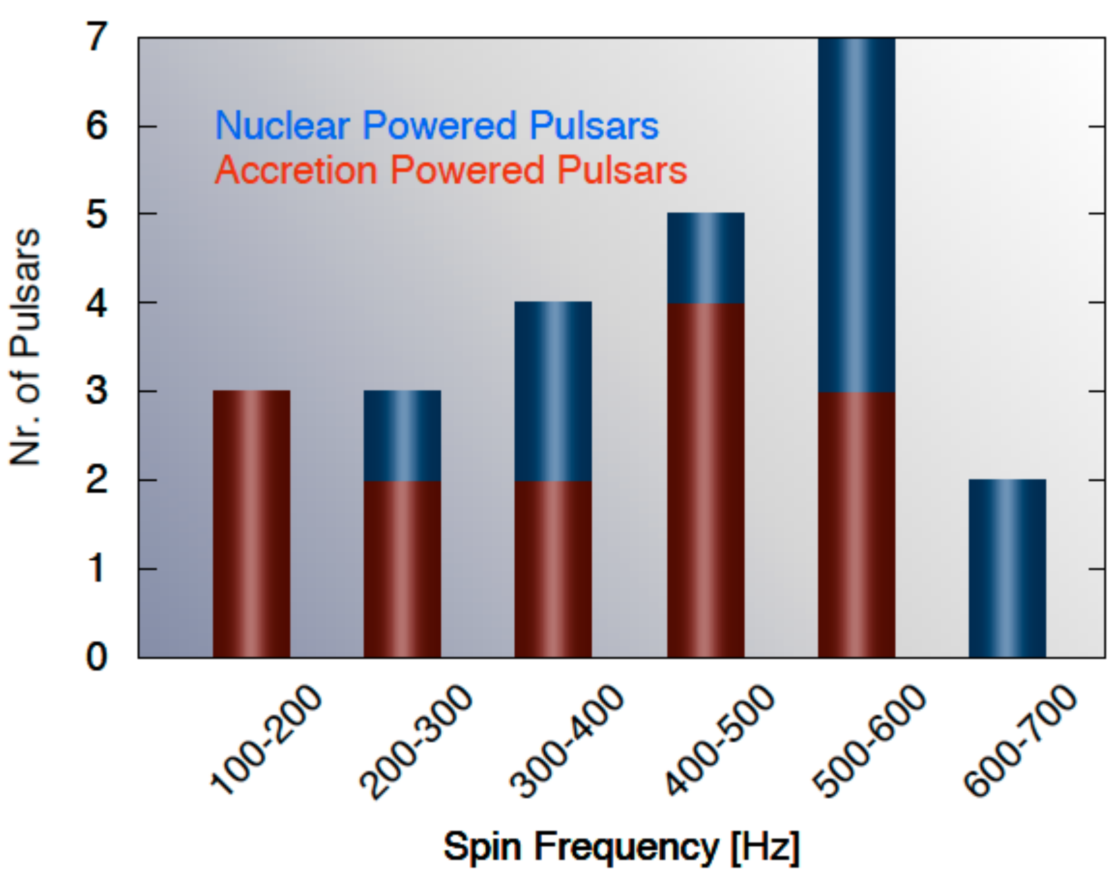
- LMXB spin distribution:

$$200\,\mathrm{Hz} \lesssim f_{\mathrm{spin}} \lesssim 600\,\mathrm{Hz}$$

- This is well below the mass-shedding limit:

$$f_{\mathrm{spin}} \ll f_{\mathrm{Kepler}} \sim 1.5\,\mathrm{kHz}$$





- Accretion lasts $\sim 10^7$ yr, Kepler limit should be reached.

- **Some process seems to halt the spin-up.**

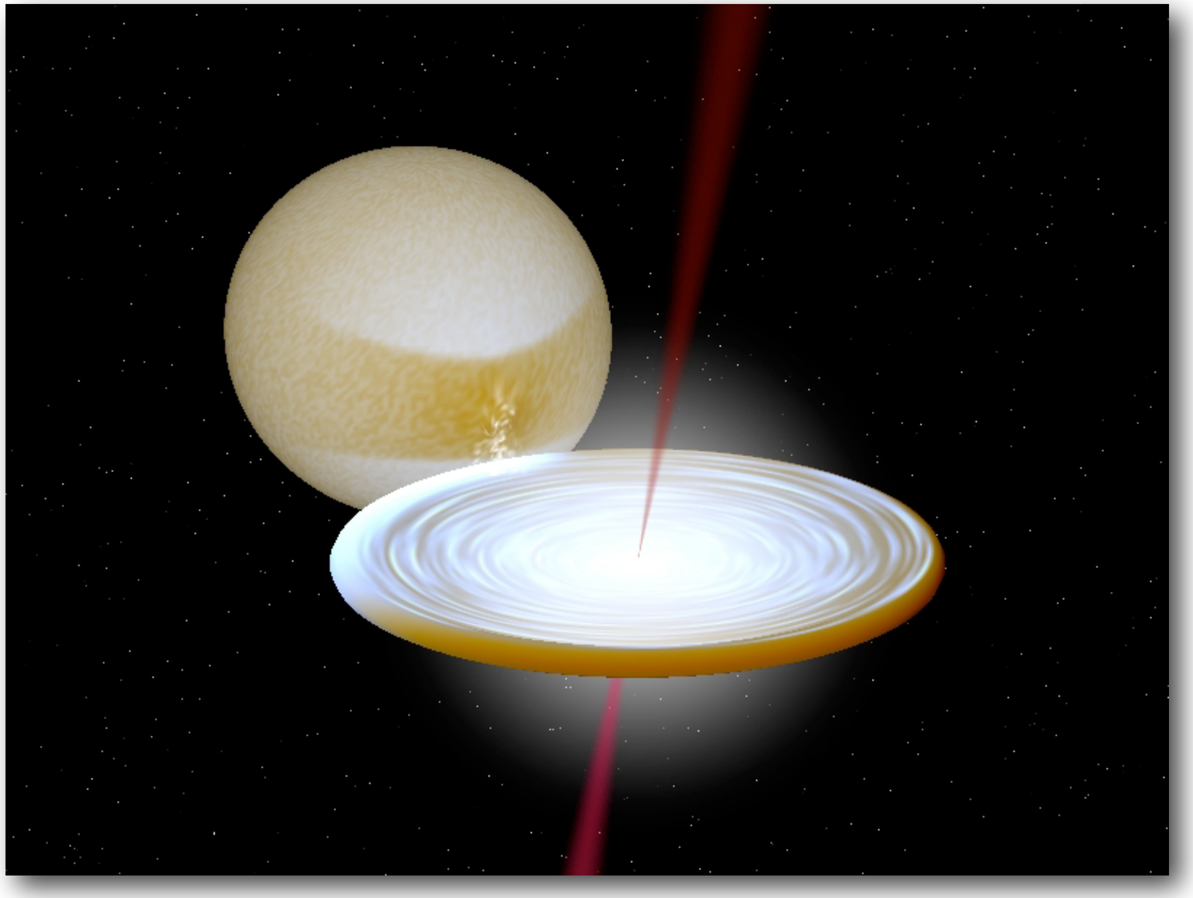
[Patruno & Watts 2012]

LMXBs: halting accretion

- Three mechanisms have been invoked to explain the data:

✓ Coupling between the stellar magnetic field and the accretion disc.

✓ GW torque by unstable r-modes or a "mountain".



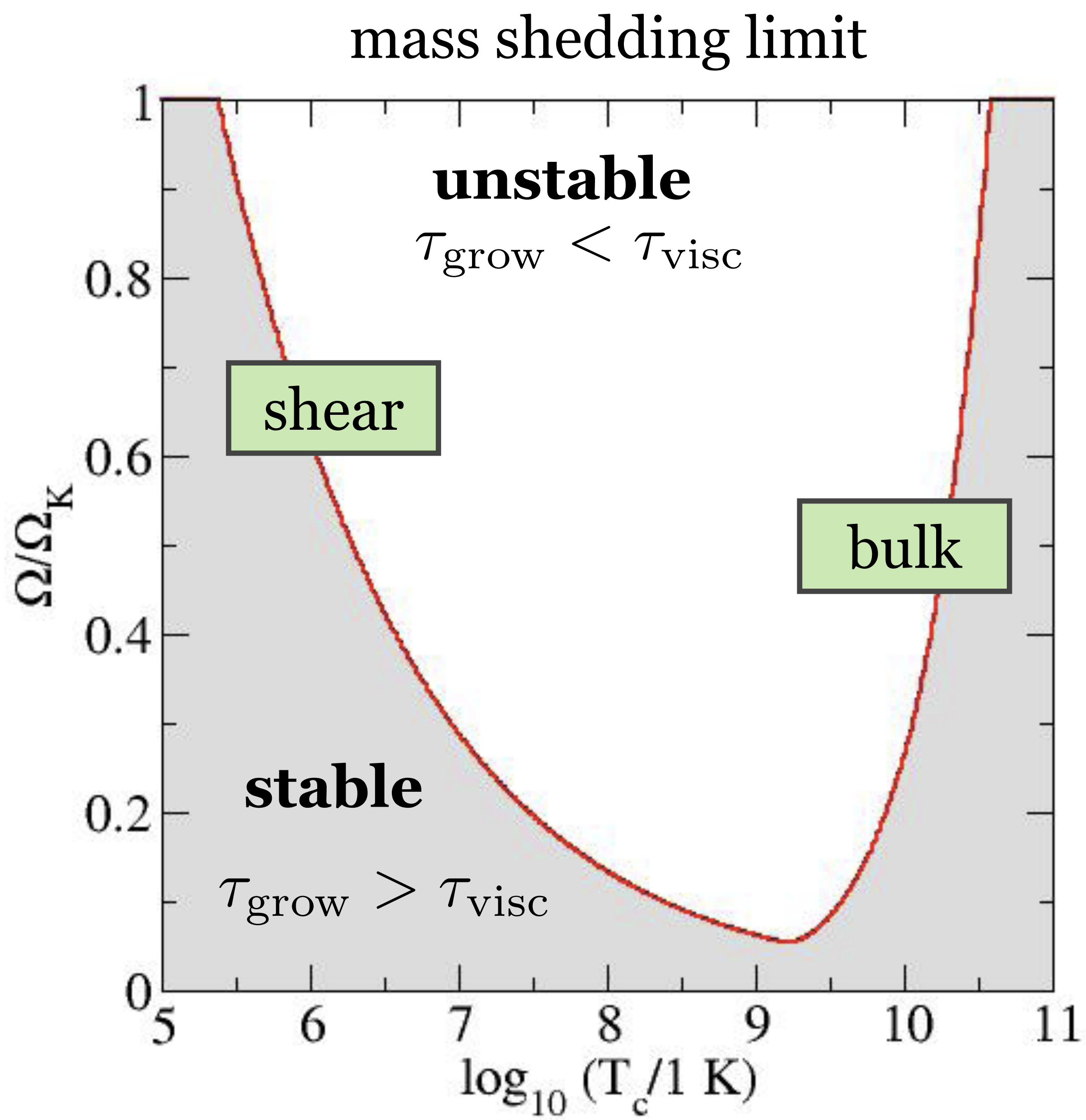
- Magnetic coupling with the disk can provide the necessary spin-down torque (although the underpinning accretion theory needs improvement)

- The measured long-term spin-down of two AMXPs (SAX J1808 & XTE J1814) are **consistent with dipole spin-down** by a $B \sim 10^8$ G "canonical" surface dipole field [Haskell & Patruno 2011].

The r-mode instability window

- The r-mode instability is active for any rotation but can be damped by viscous processes.
- The **spin-temperature instability window** is “large” but depends on uncertain core-physics.
- **A “minimal” model** accounts for damping due to shear (particle collisions, mostly e-e) and bulk viscosity (β-equilibrium reactions).
- Once active, the instability’s GW signal is largely determined by the mode’s **velocity amplitude α**:

$$\delta v_{\mathrm{mode}} \sim \alpha \left(\frac{r}{R}\right)^2 \Omega R$$



Spin-temperature evolution

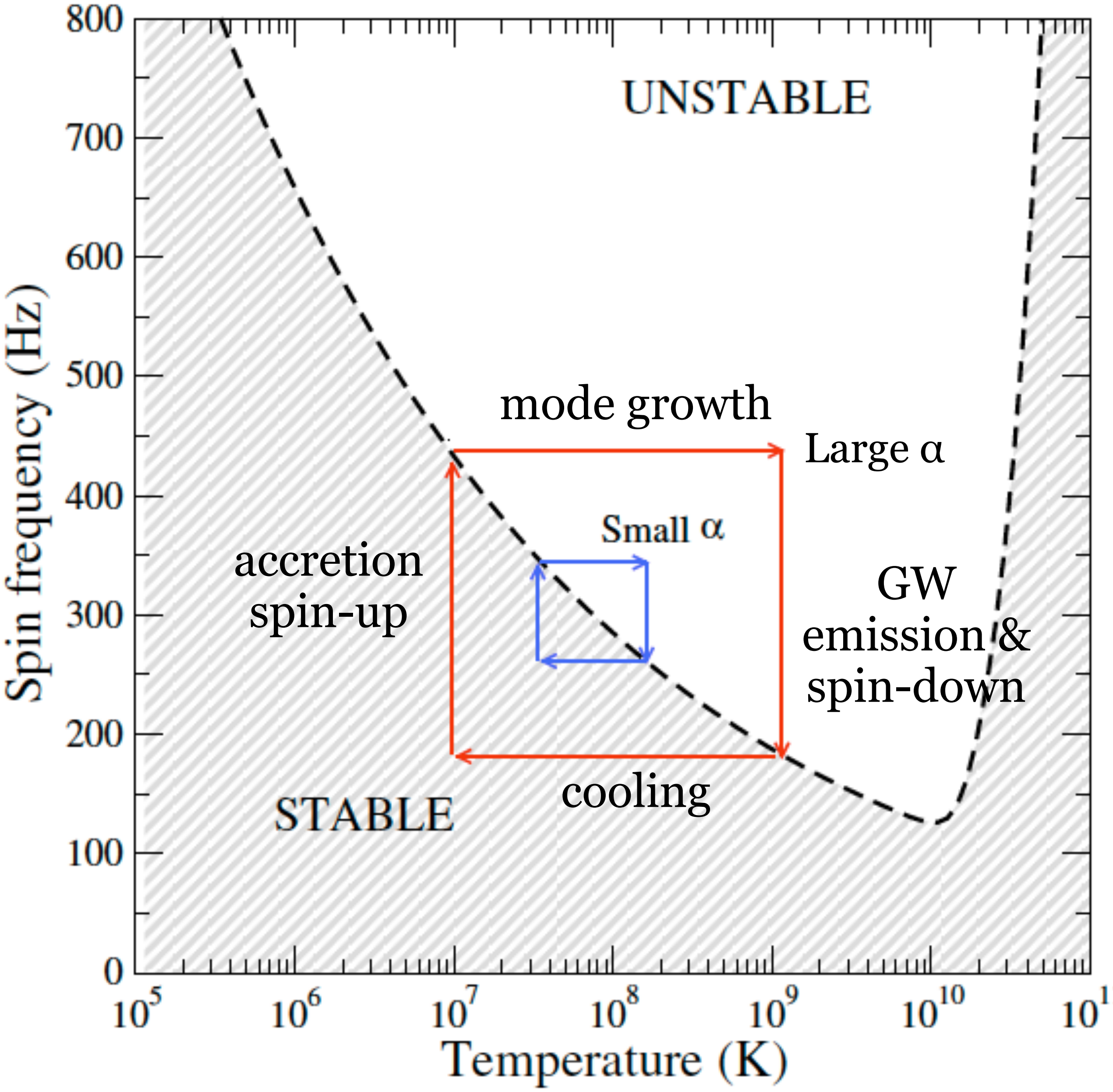
- The r-mode-driven evolution depends on two main factors:

✓ The T-slope of the curve at the point of entry.

✓ The mode's saturation amplitude.

- LMXBs are likely to become unstable in the negative T-slope portion of the instability curve.

- The figure shows the resulting **thermal "runaway"** evolution [Levin 1999].



[Haskell et al. 2014]

r-mode cycle: detectability

- The detectability of "r-mode cycling" LMXBs is a subtle issue.

- If α is too high, the duty cycle (=fraction spent in GW emission) is too low => no system would be observed during the unstable phase.

- The duty cycle is: $$D \approx \frac{t_{\rm cycle}}{10^7\,{\rm yr}} \approx \frac{10^{-11}}{\alpha^2}$$

- Combine this with the LMXB birth rate $10^{-5}/{\rm yr}/{\rm galaxy}$ and lifetime 10^7 yr and estimate the amplitude for which a system **is always "on"** in our galaxy:

$$D \lesssim 10^{-2} \rightarrow \alpha \lesssim 10^{-4}$$

- For the system to be detectable at (say) 10 kpc we need: $\alpha \gtrsim 10^{-6}$

- **Conclusion**: a small-ish amplitude is actually better for detecting LMXBs!

r-mode paradox?

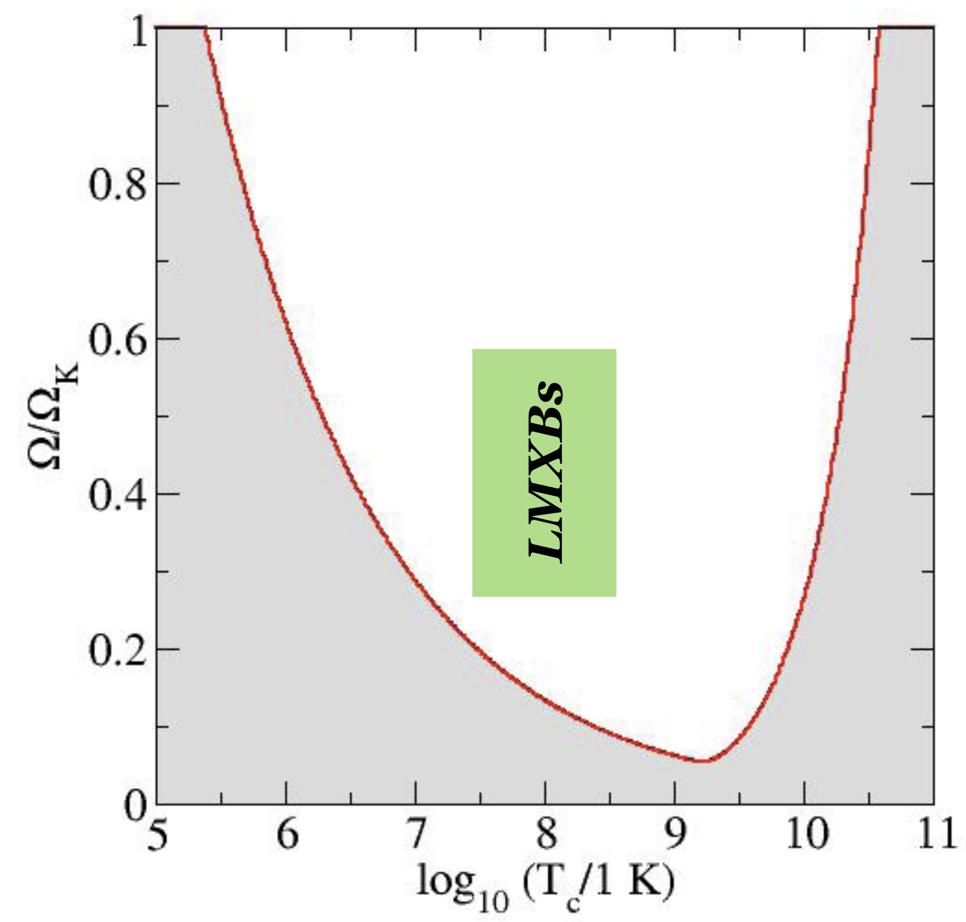
- Several LMXBs (and perhaps some MSPs) reside well inside the "minimal" instability window.

- These systems should experience r-mode-driven evolution and a GW spin-down torque.

- … but this is not what observations suggest. Possible resolutions:

✓ **Additional damping** (e.g. friction at the crust-core boundary, exotica in the core, …).

✓ r-mode amplitude **much smaller** than current theoretical predictions.



r-modes: extra damping

- Several other mechanisms could dampen the r-mode instability:

✓ **An Ekman-type boundary layer at the crust-core interface.**

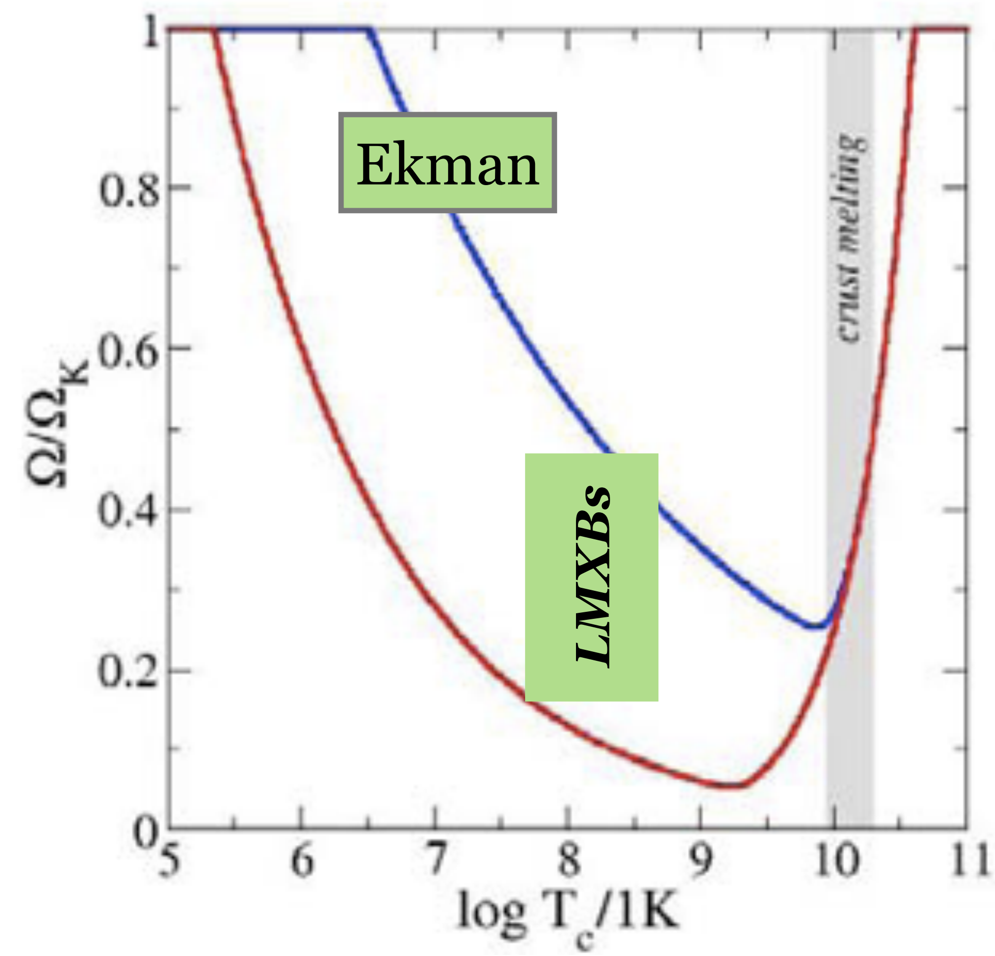
✓ Bulk viscosity due to exotica (hyperons/quark matter).

✓ Superfluid “mutual friction” due to vortex-fluxtube interactions.

✓ Coupling between the r-mode and superfluid modes.

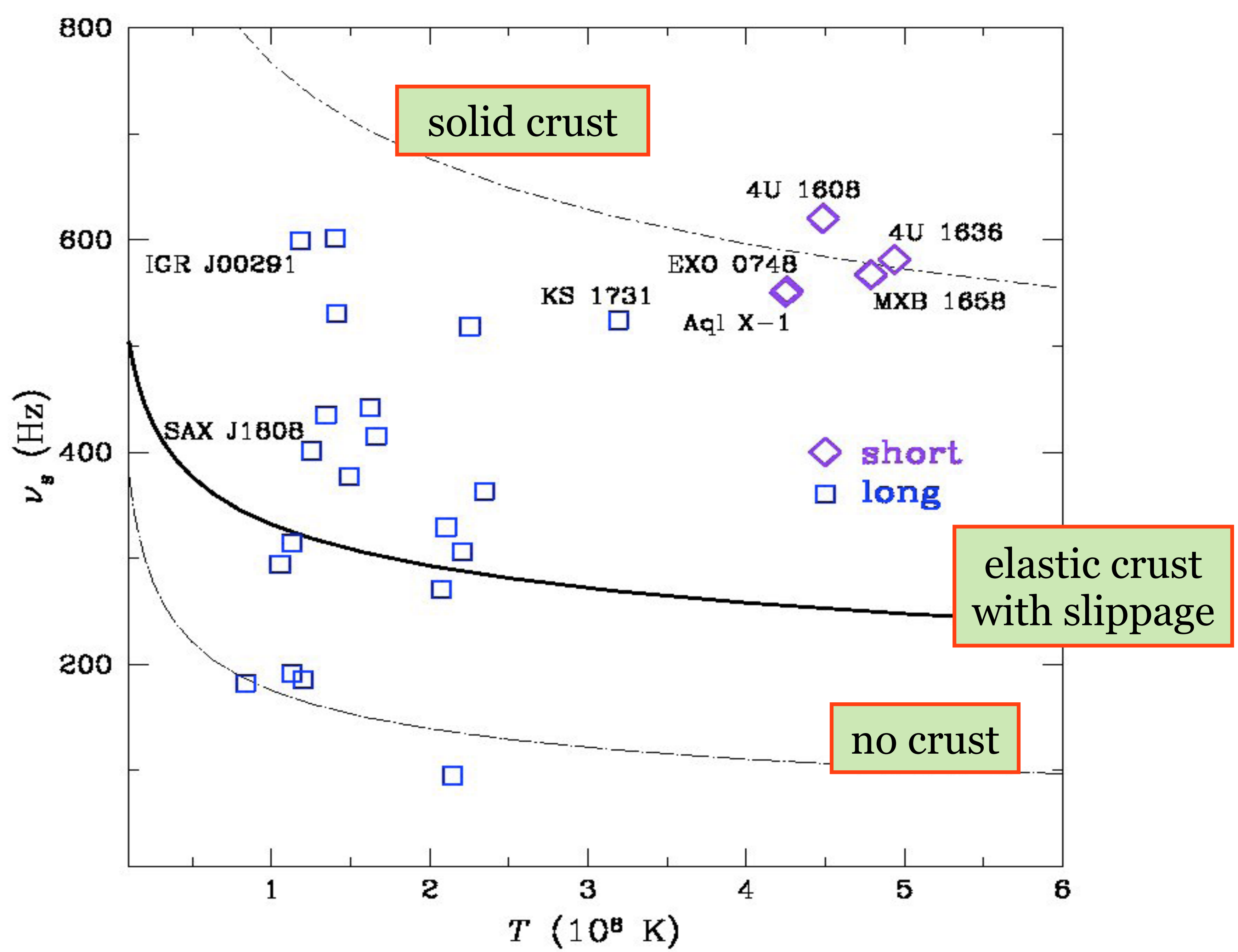
The role of the crust

- r-mode damping could be easily dominated by the viscous “rubbing” at the base of the crust [Bildsten & Ushomirsky 2000].
- The crust is more like a jelly than solid: the resulting crust-core “slippage” reduces damping [Levin & Ushomirsky 2001].
- Resonances between the r-mode and torsional crustal modes may also play a key role.
- Existing work assumes a “sharp” crust-core transition … but how safe is this assumption?



[KG & Andersson 2006]

r-mode window: “theory vs observations”



[Ho, Andersson & Haskell 2011]

Magnetic boundary layer

- The Ekman layer physics is significantly modified by the local magnetic field:

✓ Crust-core slippage is suppressed (=amplified damping)

✓ Above a threshold, the B-field enhances the damping rate [Mendell 2001].

✓ The layer's thickness grows with B, so the B-field shouldn't be too strong.

- In LMXBs (and MSPs) the magnetic field (B ~ 10^8 G) can indeed lead to enhanced damping, **provided the outer core is superconducting**:

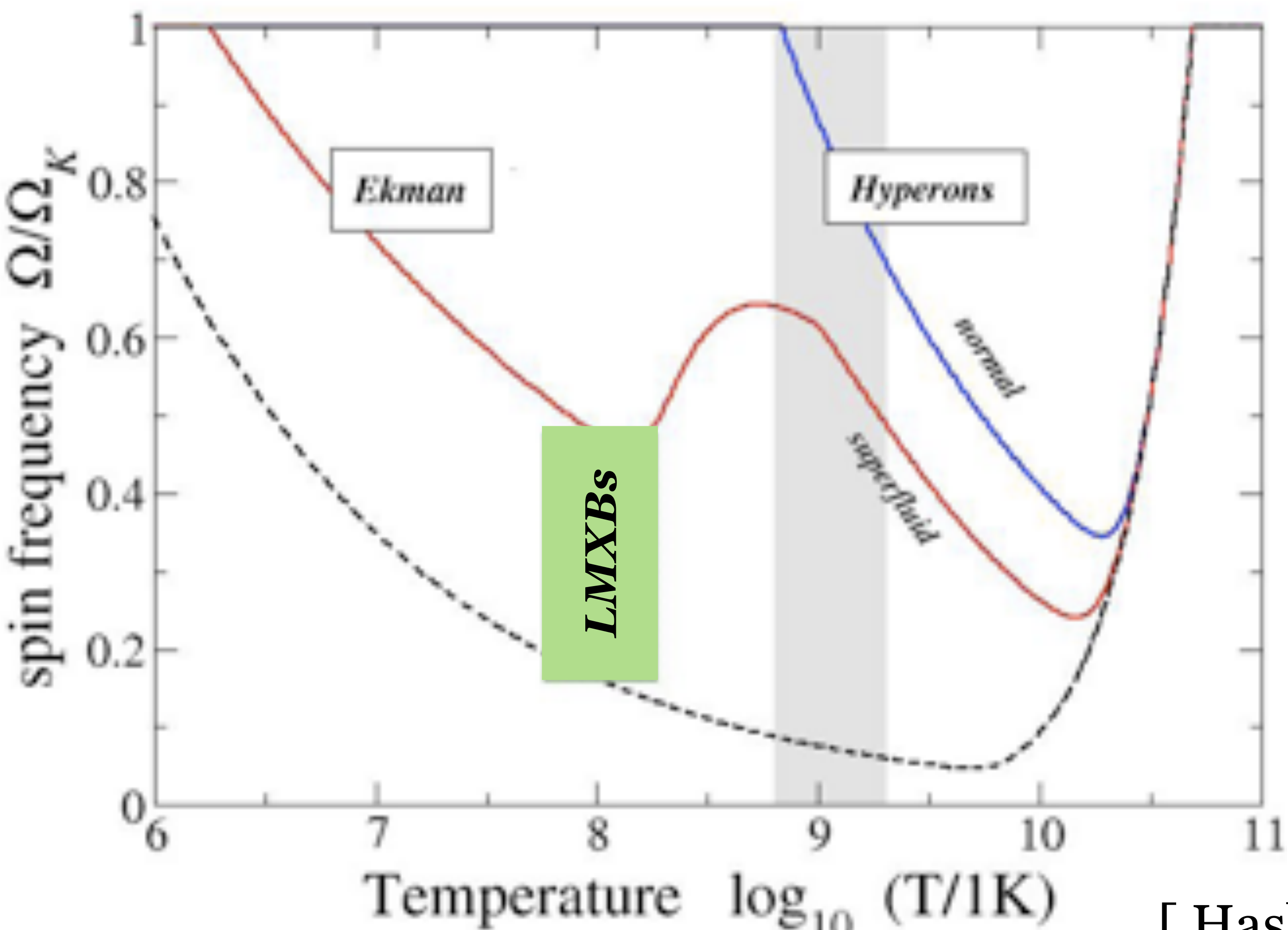
$$\frac{\dot{E}_B}{\dot{E}_{\rm visc}} \approx 30 \left(\frac{B}{10^8 \text{ G}} \right)^{1/2}$$

- This (approximate) revised damping would render these systems r**-mode stable**.

- But: we need more realistic crust-core interface modeling (with superfluidity, superconductivity, finite thickness transition etc.)

Exotic bulk viscosity

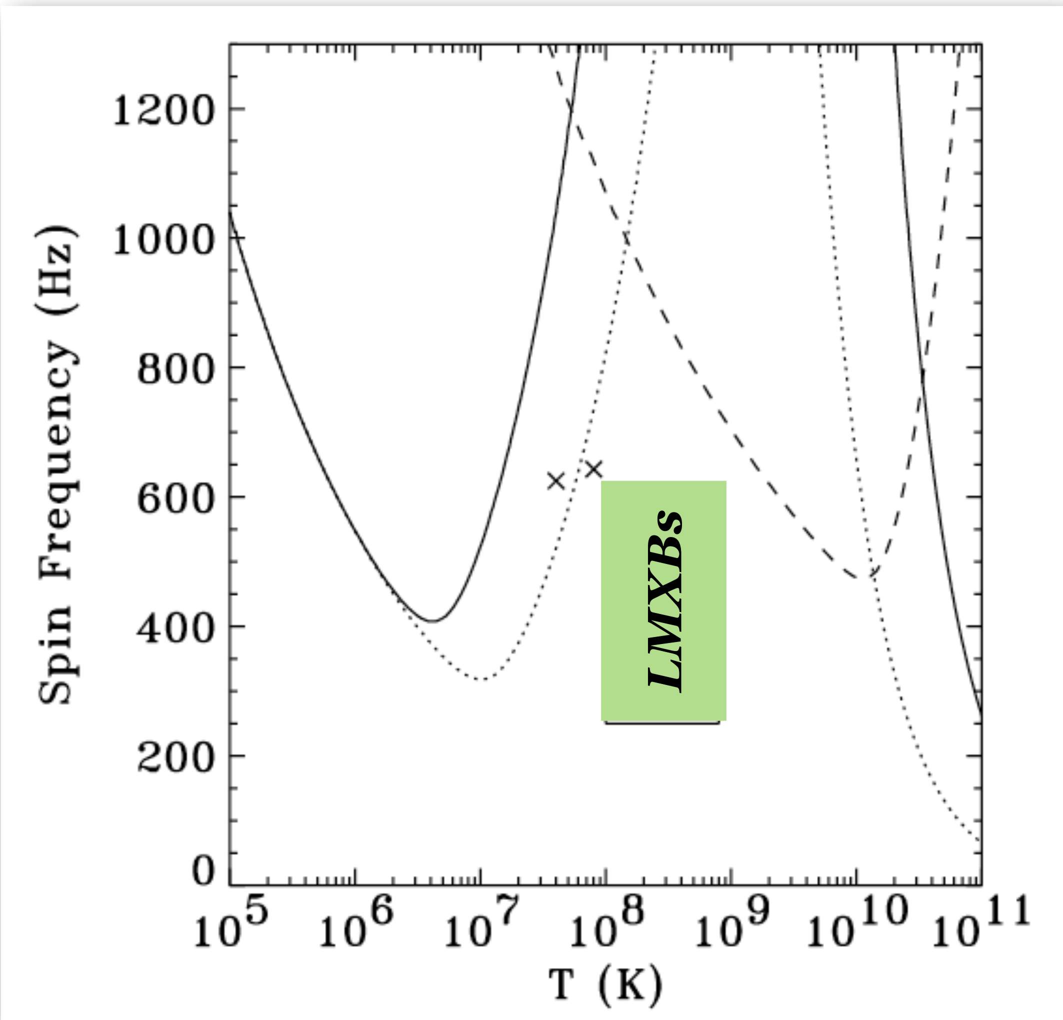
- A neutron star core populated by hyperons and/or quarks leads to strong bulk viscosity and a significantly modified r-mode instability window.
- Below we show examples of such windows but these can vary as a function of the poorly known properties of exotic matter.



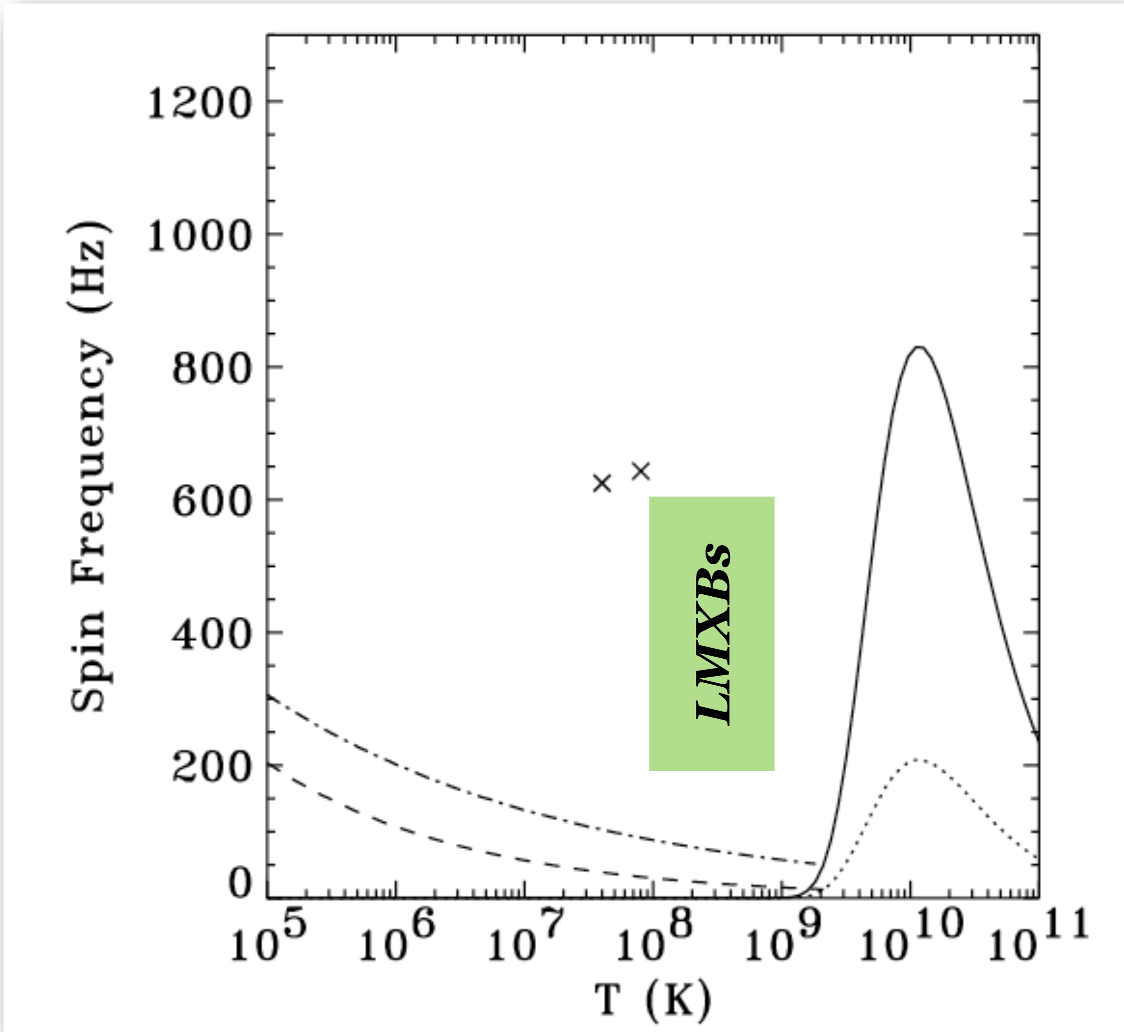
[Haskell & Andersson 2010]

Exotic bulk viscosity

Quarks (without pairing)



Quarks (with pairing)

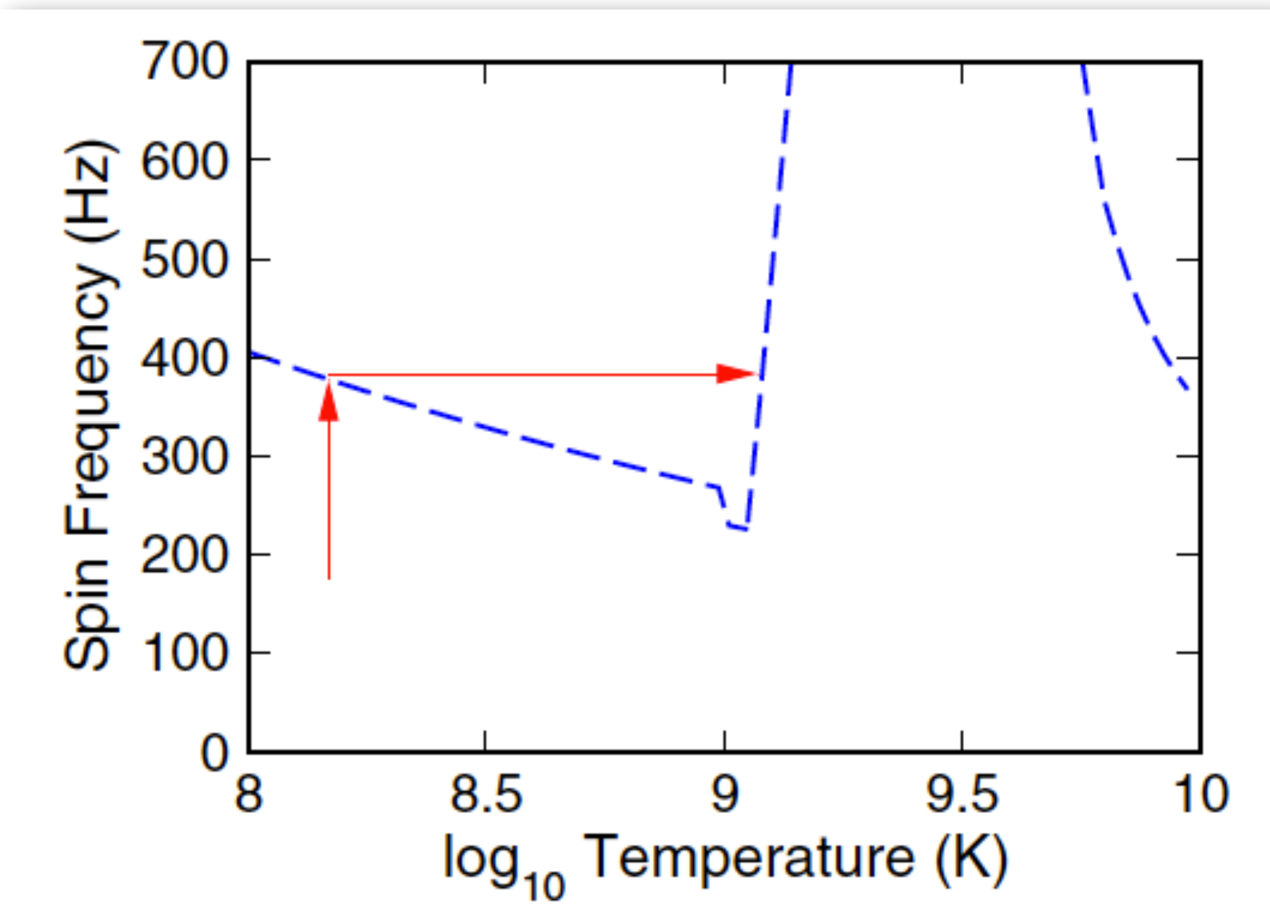


[Madsen 2000]

Persistent r-mode emission

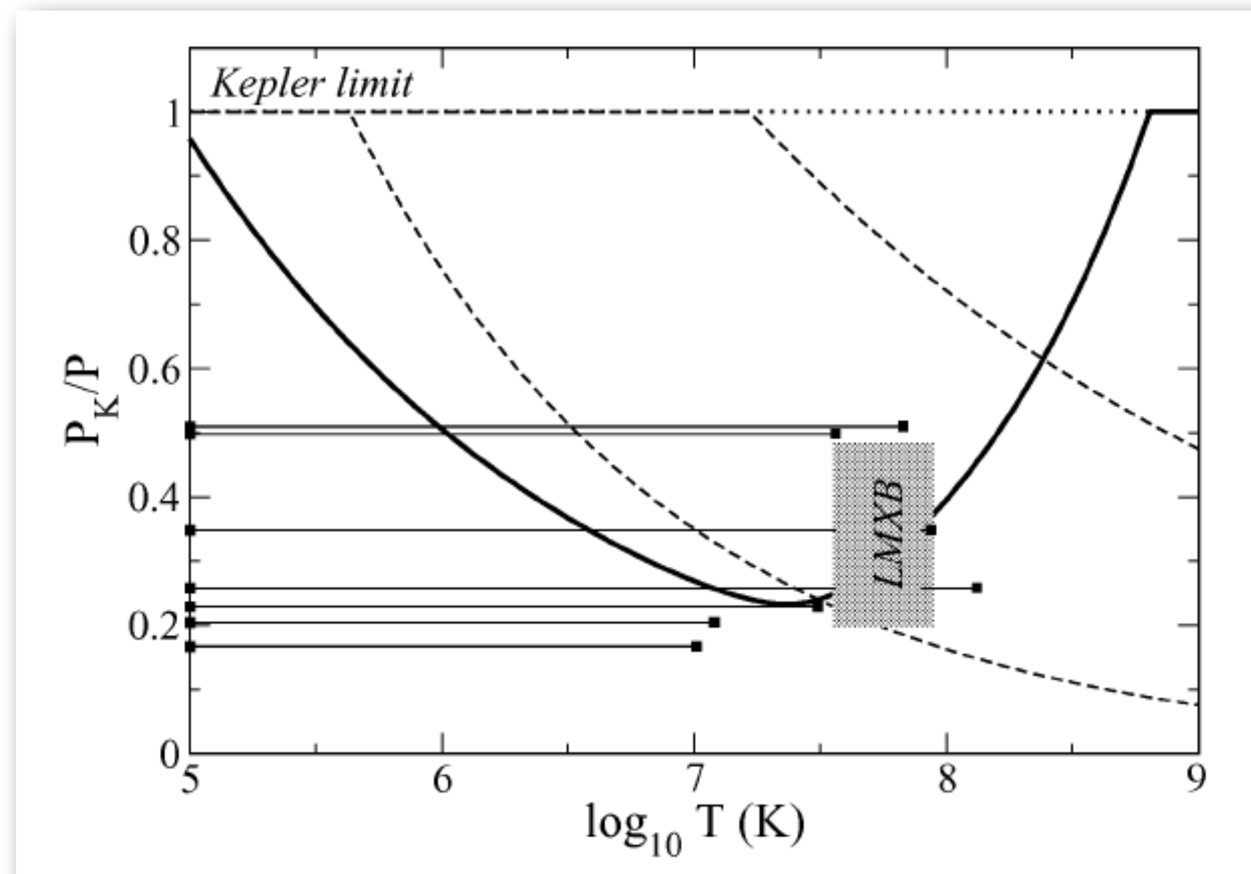
- The presence of exotica may drive LMXB-evolution near the positive T-slope instability curve, and the emission of persistent GWs. This could be potentially detectable by advanced detectors.

Hyperon core



[Nayyar & Owen 2006]

Quark core



[Andersson, Jones & Kokkotas 2002]

r-mode saturation

- **Several mechanisms could limit the r-mode's maximum amplitude.**

✓ Non-linear coupling with short-wavelength modes [Arras et al. 2003, Bondarescu et al. 2007, 2009]:

$$\alpha_{\rm sat} \sim 10^{-4} - 10^{-3}$$

✓ Vortex-fluxtube cutting [Haskell, KG, Andersson 2014]:

$$\alpha_{\rm sat} \sim 10^{-6} - 10^{-5}$$

✓ Winding up of magnetic field lines by the r-mode's differential flow ? [Rezzolla, Lamb & Shapiro 2000]

- The r-mode amplitude required to balance the accretion torque:

$$\alpha_{\rm acc} \sim 10^{-6}$$

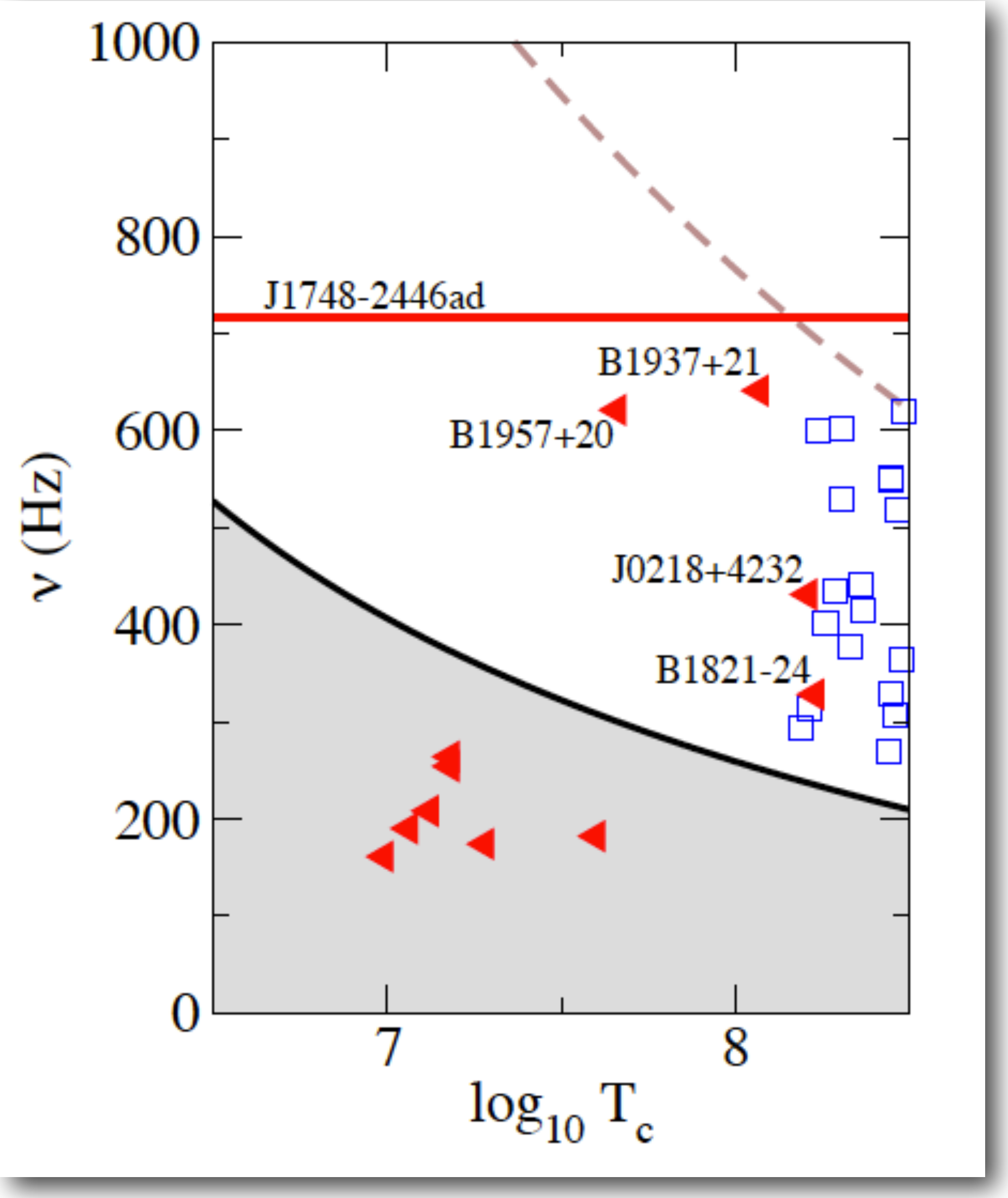
- Spin-down upper limits (if applicable) suggest:

$$\alpha_{\rm sat} < 10^{-7}$$

r-modes: spin-down upper limits

- Assume a “minimum-physics” instability window (no Ekman layer, no exotic matter, just standard shear and bulk viscosity).
- Then, several LMXBs and MSPs with measured $f_{\rm spin}, \dot{f}_{\rm spin}$ are potentially r-mode unstable.
- Obtain upper limits for the amplitude by assuming spin down only via r-mode GW radiation [Alford & Schwenzer 2015]

$$\alpha_{\rm sat} \lesssim 10^{-7}$$

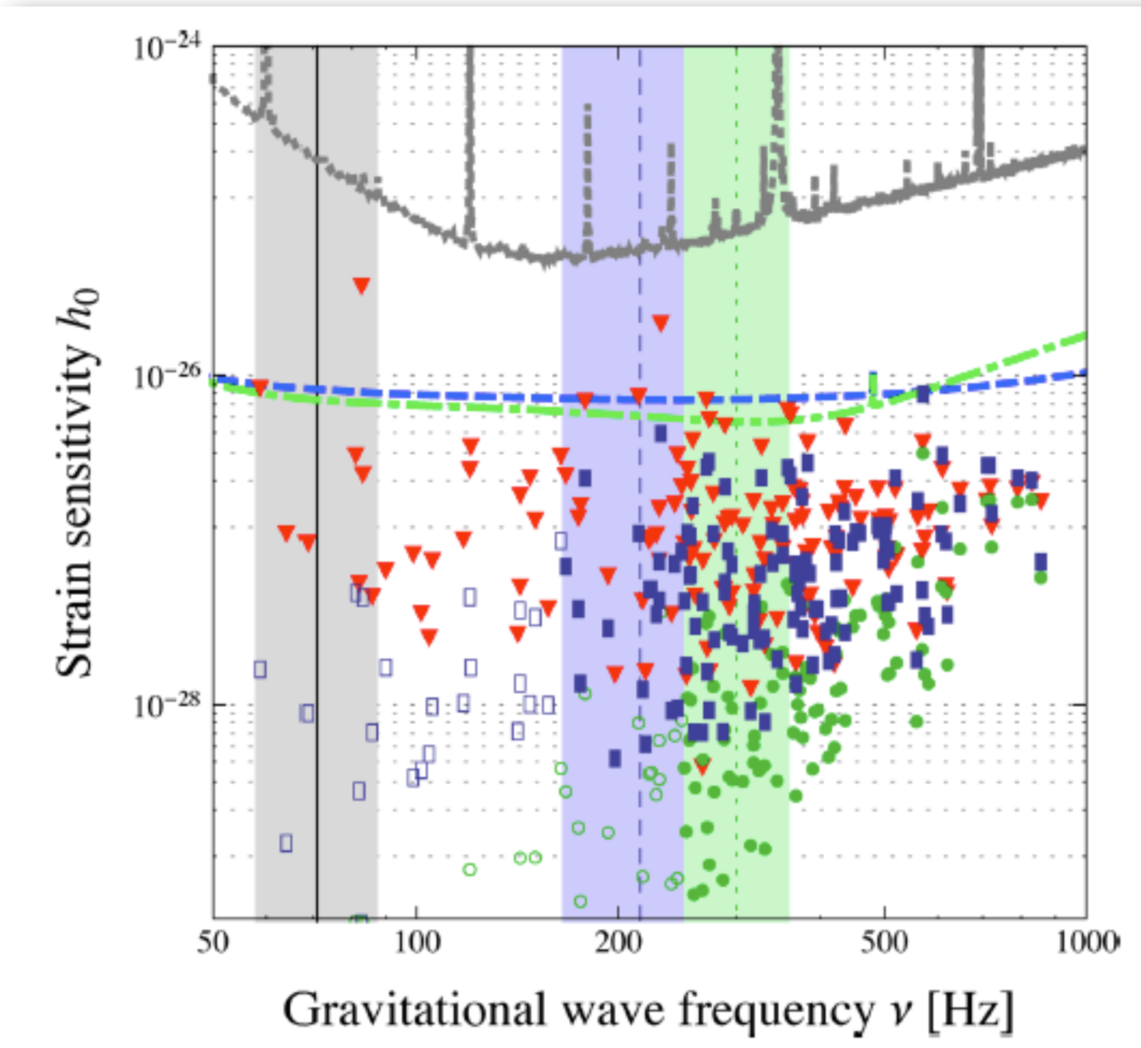


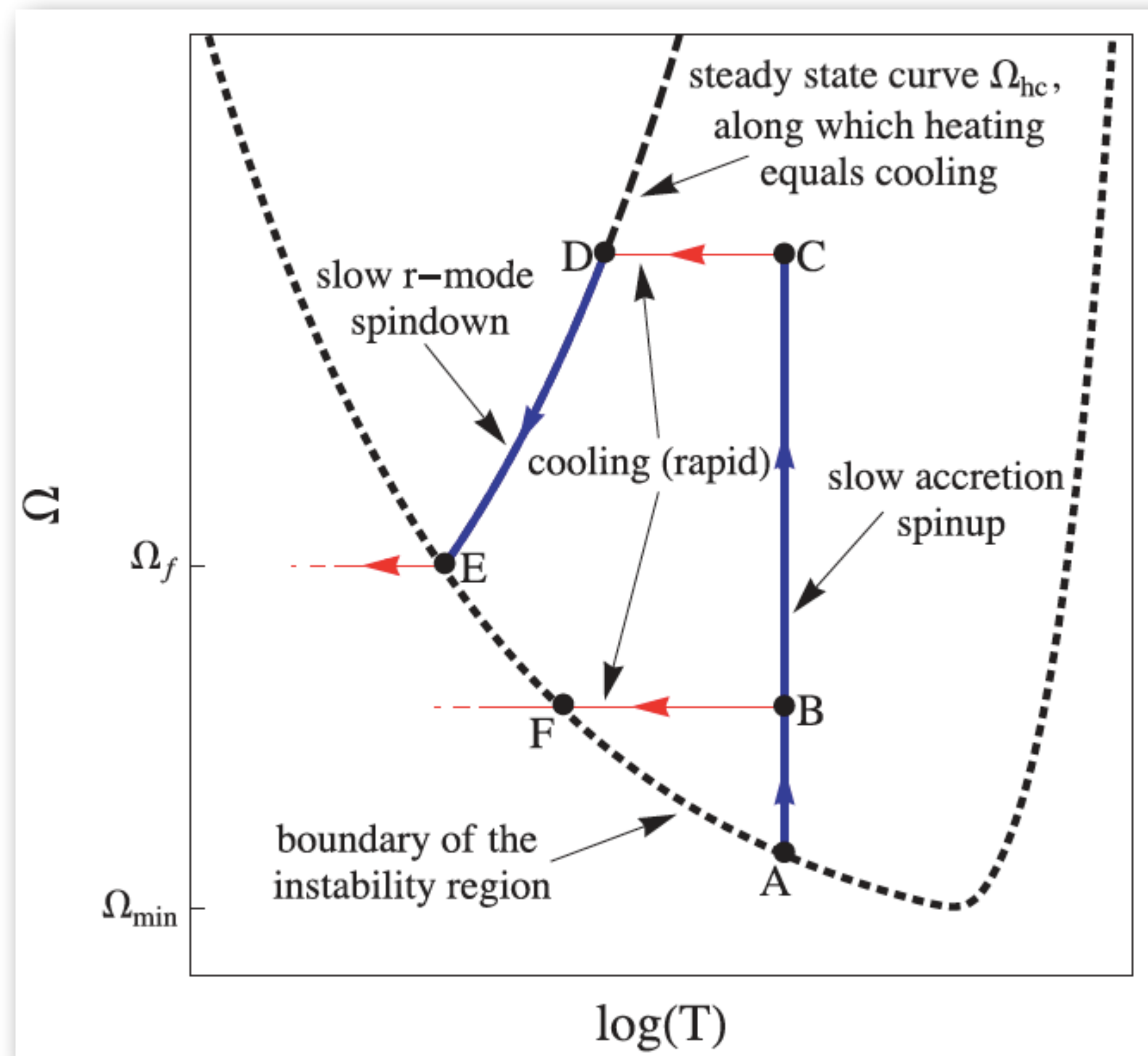
blue: LMXBs
red: MSPs (T data: upper limits)

[Figure credit: N. Andersson]

“Tiny” amplitude r-mode scenario

- Recent work has focused on unstable r-mode evolution/detectability using the spin-down inferred amplitude





[Alford & Schwenzer 2015]

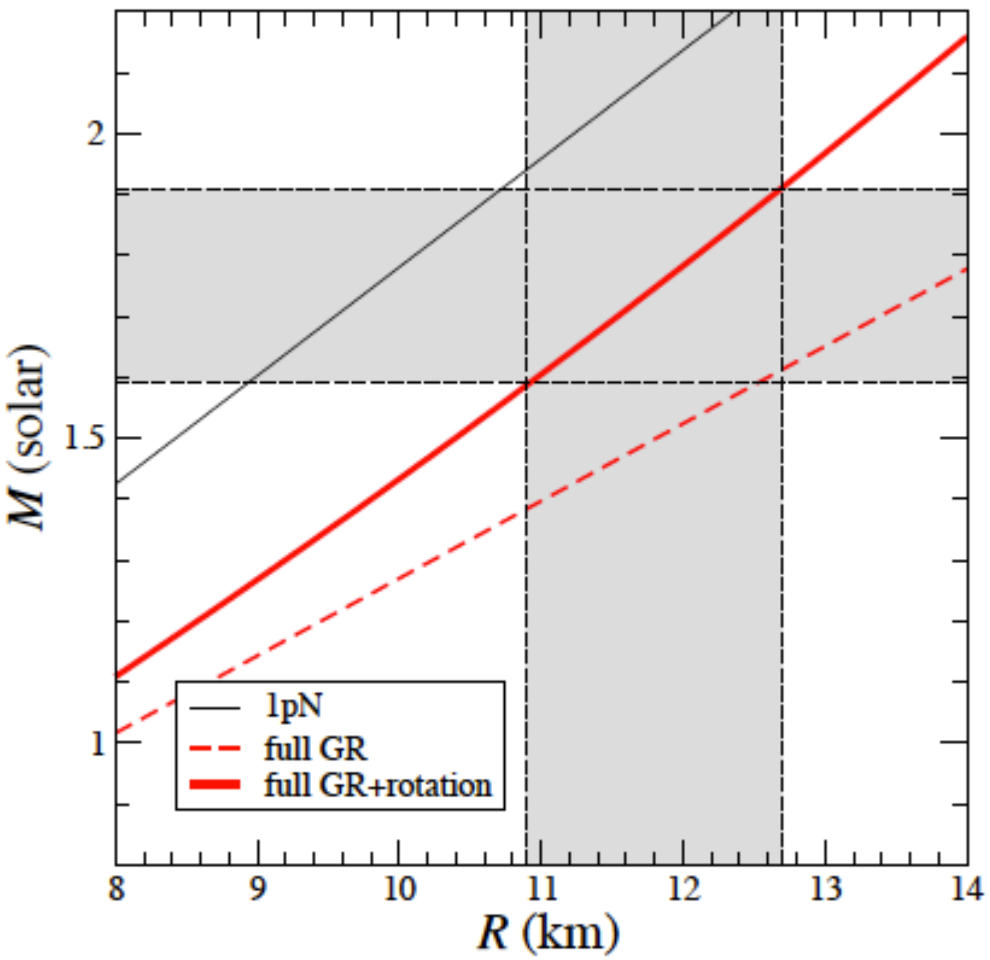
r-mode observed? XTE 1751

- XTE 1751-305 is an AMXP (accretion-powered X-ray pulsar)

- Recent result [Strohmayer & Mahmoodifar 2014]: coherent oscillation was discovered in the light curve of a 2002 burst:

$$f_{\rm osc} = 0.572 f_{\rm spin}$$

- Provided the light curve is modulated by a global mode, the observed signal could be an r-mode. The numbers can match provided we account for relativistic corrections in the mode frequency.



[Andersson, Jones & Ho 2014]

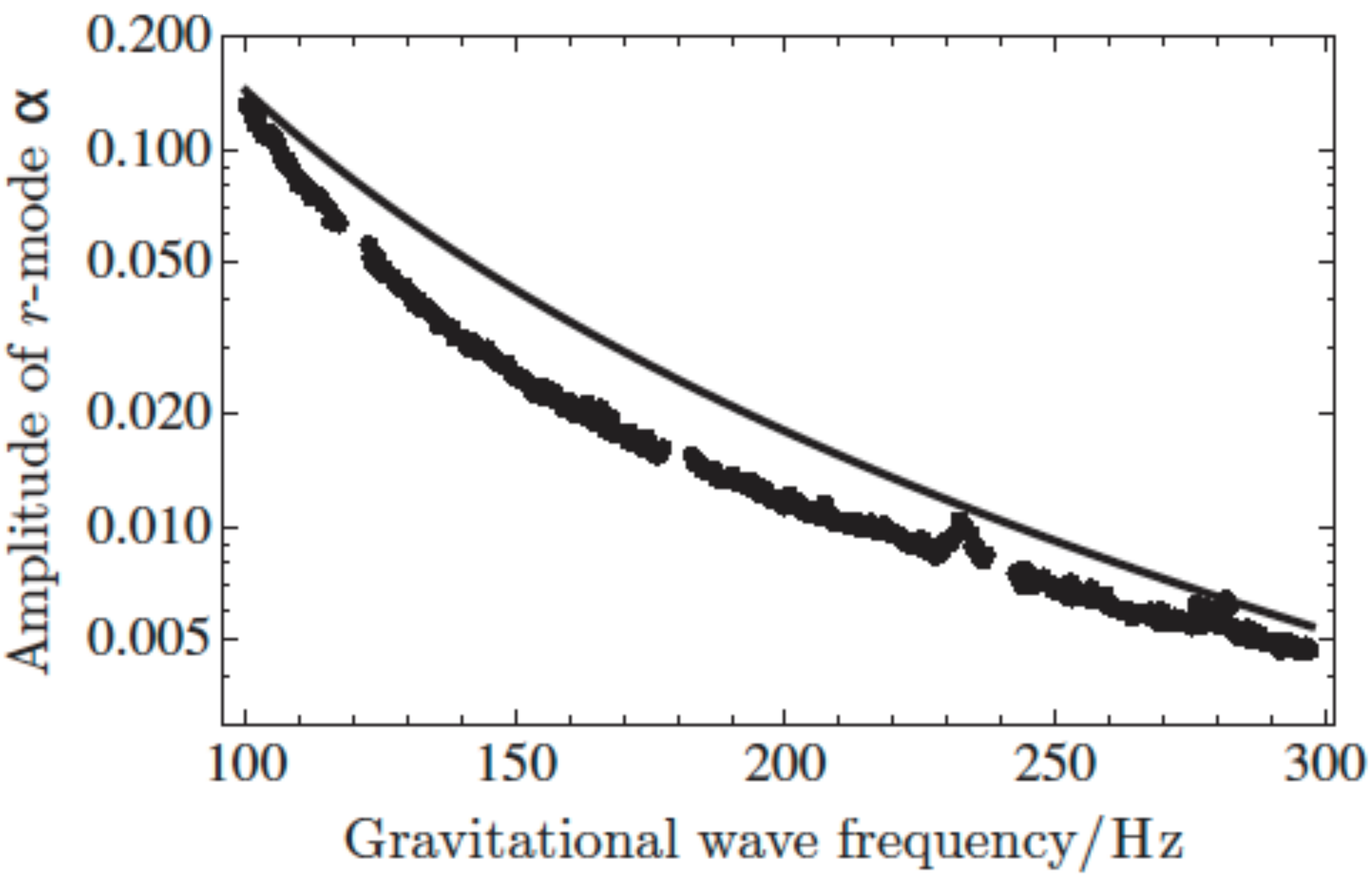
- But: inferred r-mode amplitude is too large to be reconciled with the system's spin evolution.

Direct upper limit: Cassiopeia A

- The Cas A supernova remnant hosts the youngest known neutron star (~300 years old). The system is "famous" for its thermal evolution: it is seen cooling in "real" time (~ 4% T-drop in a decade).

- LIGO has set direct upper limits on GW strain assuming r-mode emission

- The spin frequency is not know, but is likely to be too low for ground-based detectos (as in other CCOs).



Summary

- **Take home message:** prospects for detection of continuous waves from neutron stars are not too optimistic, but both mountains and r-modes remain viable GW sources for present and next generation GW observatories.

- More theoretical work is clearly required (r-mode instability window, accretion torque physics, mountains in quark stars etc.)