Gravitational Wave Signature Of Non-Rotating Core-Collapse Supernovae with magnetic fields

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For videos of the simulations see the CAMAP web site:
http://www.uv.es/camap/camap_content_1.html

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- CCSN – standard scenario
- How to achieve explosions – **revival mechanisms**

- Alternatives – **Non-rotating CCSN**

  - Magnetic Field Amplification
  - Weak fields
  - Strong fields

✓ GW signal: identify the features
  - matter asymmetric motions
  - Asymmetric neutrino emission

✓ Detectability

Standard scenario – Onset of the collapse

- ***Progenitor*** -> Massive stars:

$$(9M_{sun} < M < 25M_{sun})$$

- Hydrostatic burning lead to ***onion shell structure*** -> nuclear fuel is exhausted. Achieve:

$$M_{ch} \approx 5.8(Y_e)^2 M_{sun}\ ,\ Y_e = 0.45 \rightarrow M_{ch} = 1.45\ M_{sun}$$

$$\boldsymbol{M \geq M_{ch},} \quad \boldsymbol{radial\ instability\ \rightarrow onset\ of\ the\ collapse}$$

How to get there?

✓ ***Reduction of Y_e ->*** $\boldsymbol{e^- + p \rightarrow n + \nu_e}$

- ***Collapse procceds (adiabatically) up to nuclear densities -> $\boldsymbol{\rho_{nuc} \sim 2 \times 10^{14} gcm^{-3}}$ -> proto – neutron star formation***

How ?

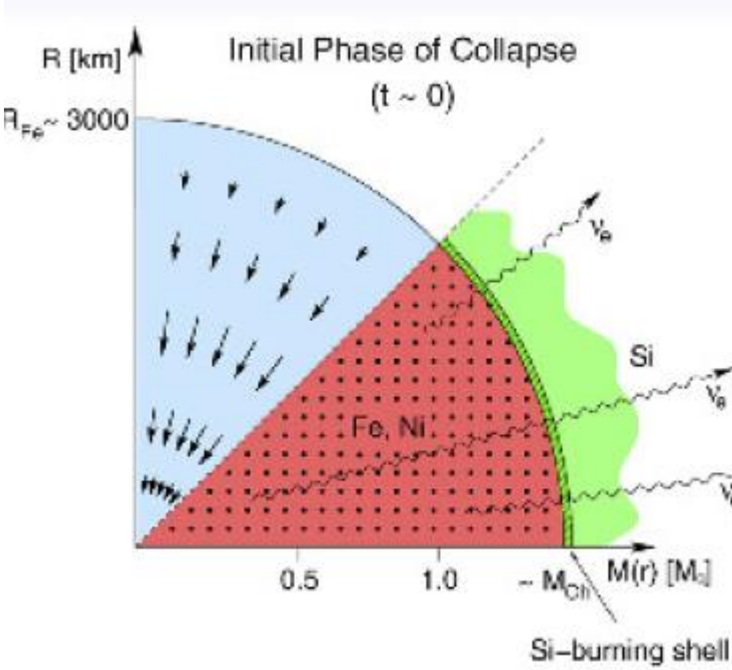
1. ***Photodissociation*** *of heavy nuclei*: $124.4\ MeV/reaction$

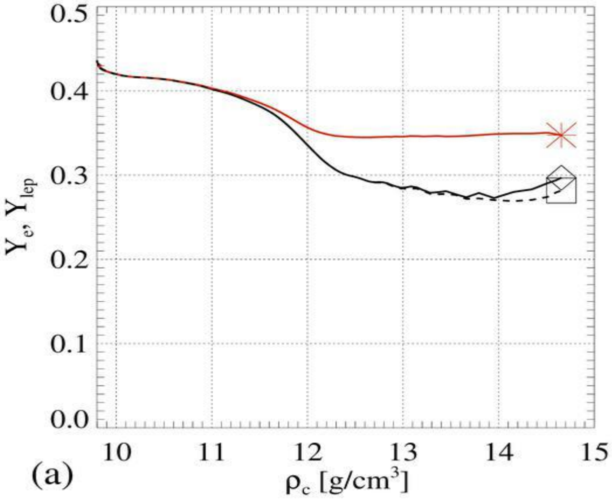
$$\gamma\ +\ Fe^{56} \rightarrow 13\alpha + n$$

2. ***Further Electron Capture*:**

$$e^- + (Z, A) \rightarrow (Z-1, A) + \nu_e$$
$$e^- + p\ \rightarrow n + \nu_e$$

- ***Neutrinos stream off*** ***freely*** **at densities below** $\boldsymbol{\rho \sim 10^{12}\ gcm^{-3}}$ **->** ***core deloptonization during collapse***





evolution of the electron and lepton fractions, *Ye* and *Ylep*, during collapse as a function of the central density of the core, c.

Standard scenario – **Neutrino trapping**

- ***Collapse phase***: Neutrino opacity is dominated by coherent neutrino heavy nuclei scattering:

$$\nu + (\mathrm{A,Z}) \leftrightarrow \nu + (A, Z)$$

- When $\rho > 3 \times 10^{12}\ gcm^{-3}$ -> $\tau_{diff} \gg \tau_{coll}$

-> ***Neutrinos trapping***

- **Deloptonization stops:**

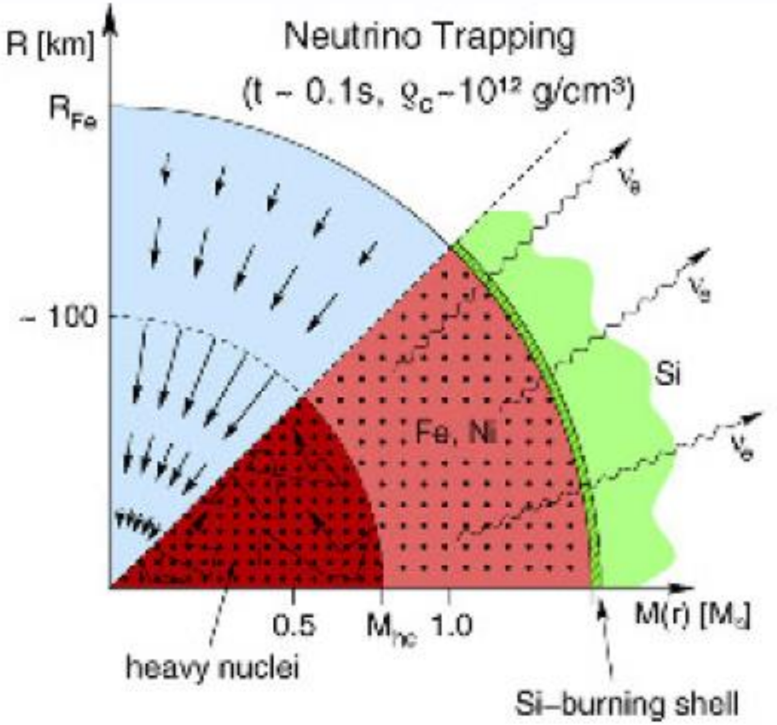
$$Y_{lep} = Y_e + Y_\nu \approx 0.32 = \text{constant}$$

- **Chemical equilibrium** -> electron capture rate is balanced by electron neutrino capture:

$$\boldsymbol{e^- + p \leftrightarrow n + \nu_e}$$

Trapped lepton fraction leads to -> Separation into ***homologously collapsing inner core*** and ***supersonically collapsing outer core.***

$$\boldsymbol{Collapse\ still\ proceeds}!$$



Standard scenario – **core's bounce + shock formation**

- Nuclear Physics:

$$R_{nuc} = A^{1/3} r_0 \ , r_0 = 1.25 \ fm$$

- Nuclear density:

$$\bar{\rho}_{nuc} = \frac{A m_b}{\frac{4}{3}\pi r^3} \approx 2 \times 10^{14} g cm^{-3}$$

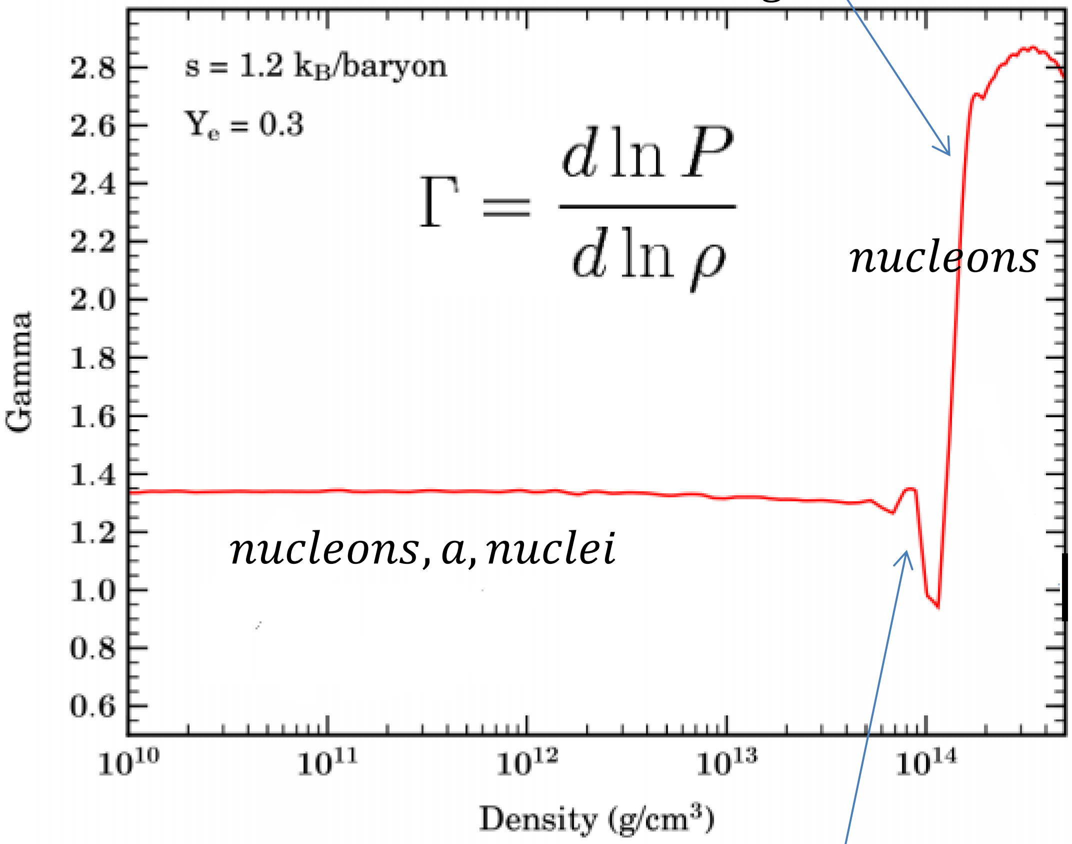
$$At \ \bar{\rho}_{nuc} > 2 \times 10^{14} g cm^{-3}$$

-> n and p are so close -> "repulsive core" due to the strong force kicks -> stiffening of the equation of state (EoS)

Inner core bounces into still infalling outer core!

Stiffening of the EoS -> HD shock formation!

Stiffening of EoS



$$\Gamma = \frac{d \ln P}{d \ln \rho}$$

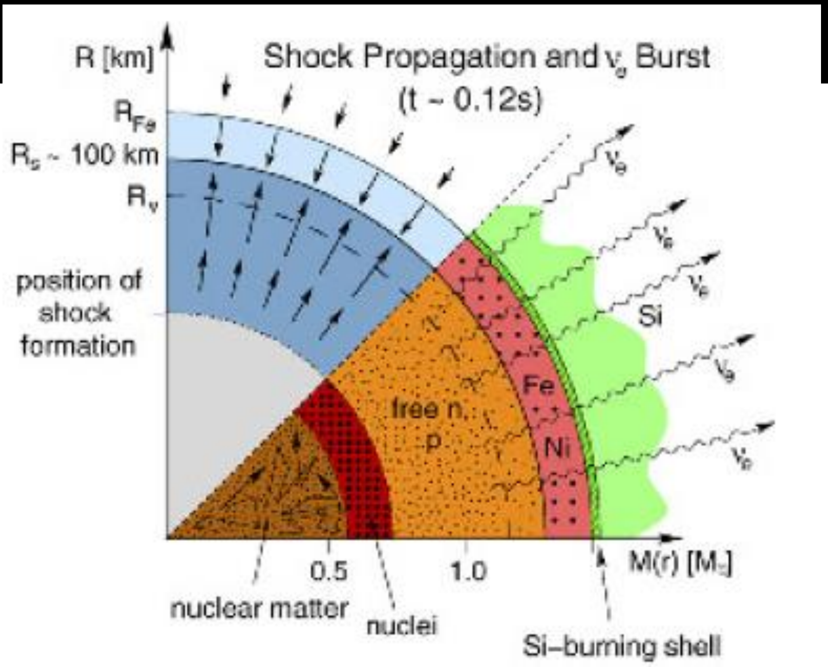
$nucleons$

$nucleons, a, nuclei$

Phase transition to
homogeneous nuclear matter ->

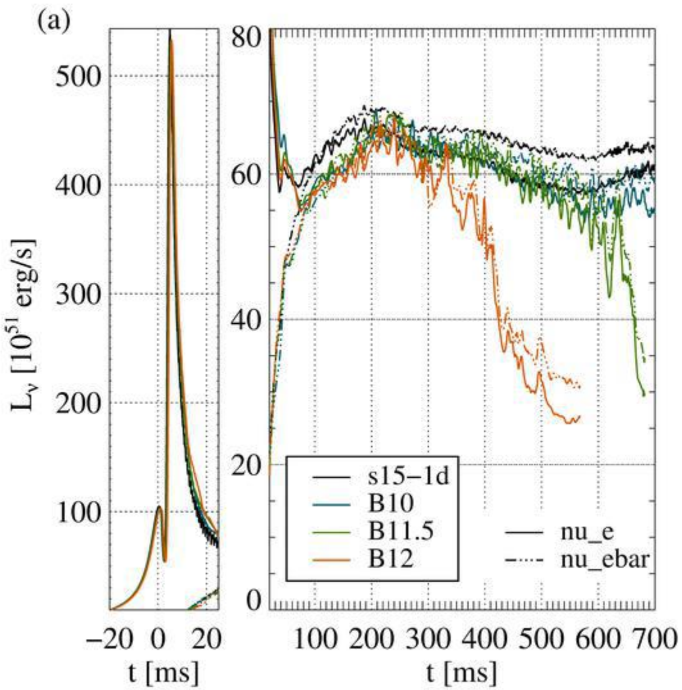
$$\boldsymbol{proto - neutron \ star \ formation \ (PNS)}$$

Standard scenario – shock's stagnation



- Shock loses energy:
  - Dissociation of infalling heavy nuclei
  - ***Neutrinos burst*** -> neutrinos stream away from behind the shock -> released by electron captures on free protons when the shock "breaks-out" of the $\nu_e - neutrinospheres$

Shock stalls -> no prompt explosion !



Collapse to a neutrons star -> $\sim 3 \times 10^{53}\ erg$

$300\ Bethe\ [B]\ gravitational\ energy$

99 % of the energy is radiated as neutrinos over hundreds of seconds as the PNS cools

Explosion mechanism must tap the gravitational energy fuel and convert the necessary fraction into energy of the explosion!

CCSNe complexity

Multi-scale problem

- Progenitor
- Pre-collapse core $\sim(1-3)\times 10\ km$
- Stalled shock radius $\sim(1-1.5)\times 100\ km$
- Many dynamical scales (<1s for explosion)

Multi-D nature, multi physics

- MHD/plasma physics -> *dynamics of the stellar fluid*
- General relativity -> *Gravitation*
- Nuclear + neutrino physics -> *EoS, nuclear + neutrino interactions*
- Transport theory -> transition from optically thick region to trasparent -> *Neutrino transport*

Shock revival – Explosion

1. Neutrino heating in spherical symmetry

How is the failed explosion revived -> not a matter of energy, $e_{core} \gg e_{env}$, but energy transfer.

Stirling Colgate

Colgate & White 1966

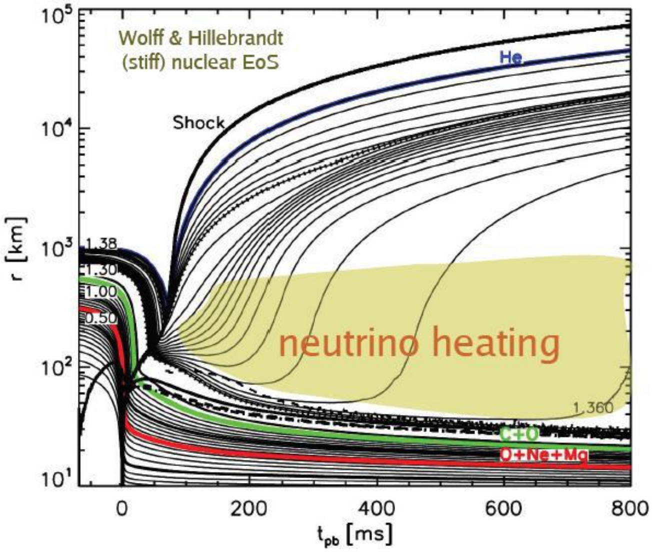
Dave Arnett

Arnett 1966

Jim Wilson

- Neutrinos diffusing out of the PNS
- Transfer thermal energy behind the shock
- Explosions in a limited mass range $\sim 8 - 10\ solar\ masses$

steep density profile outside the core -> easier to unbind the envelope



Kitaura, Janka, & Hillebrandt (2006)

Shock's revival – Assymmtric Explosion Predictions

2. **Neutrino heating + hydro-instabilities**

Explosion Criterion: Heating faster than advection of the infalling matter ->

$$\boldsymbol{\tau_{heat}} = \frac{\int_{r_{gain}}^{r_{sh}} dV\varepsilon}{\int_{r_{gain}}^{r_{sh}} dVq_{\nu}} < \boldsymbol{\tau_{adv}} = \int_{r_{gain}}^{r_{sh}} \frac{dV}{|V_r|}$$

-> not fullfilled in 2D simulations

i) Convection

neutrino heating creates a region of high entropy at the bottom of the gain layer

$$C_L \equiv \left(\frac{\partial\rho}{\partial s}\right)\bigg|_{Y,p}\frac{ds}{dr} + \left(\frac{\partial\rho}{\partial Y}\right)\bigg|_{s,p}\frac{dY}{dr}$$

Overturning convection and turbulence develops in the unstable region.

Effects

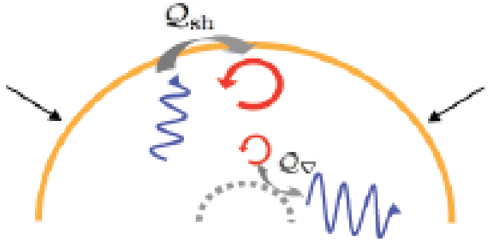
Dwell time of the material in the heating region is increased -> higher τ_{adv}

τ_{heat}/τ_{adv} *favorable*

ii) SASI

The shock wave sits at constant radius

In the subsonic post-shock region, sound waves can couple the PNS surface to the shock. Perturbations travel between the shock wave and the PNS and form an unstable feedback loop.



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ops in

eating

adv

PNS convection (beneath neutrinospheres) -> Boosts neutrino luminosities (asymmetric emission)

Neutrino driven (beneath stalled shock) -> Boosts shock radius

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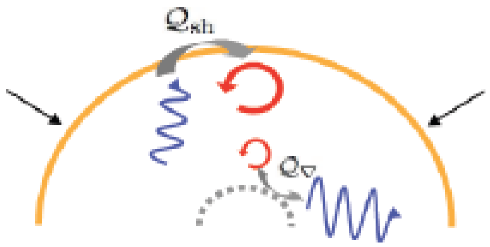

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In the subsonic post-shock region, sound waves can couple the PNS surface to the shock. Perturbations travel between the shock wave and the PNS and form an unstable feedback loop.

Similar role as convection, but acting on larger scales and producing strong shock deformations -> aspherical explosions

Key feature- asymmetric explosions!

Pulsar fields - Magnetars

Magnetic fields ?

Asymmetric explosions
–> may driven by large scale fields

Pulsar’s spin
–> rotation as an additional reservoir

Magnetically affected explosions very much dependent on the amount of the *field amplification* happening during and after collapse.

Heger et al. (2005)

Shock's revival – Asymmetric Explosions

3. MHD driven explosions – Magnetic fields amplification

a) with rotation

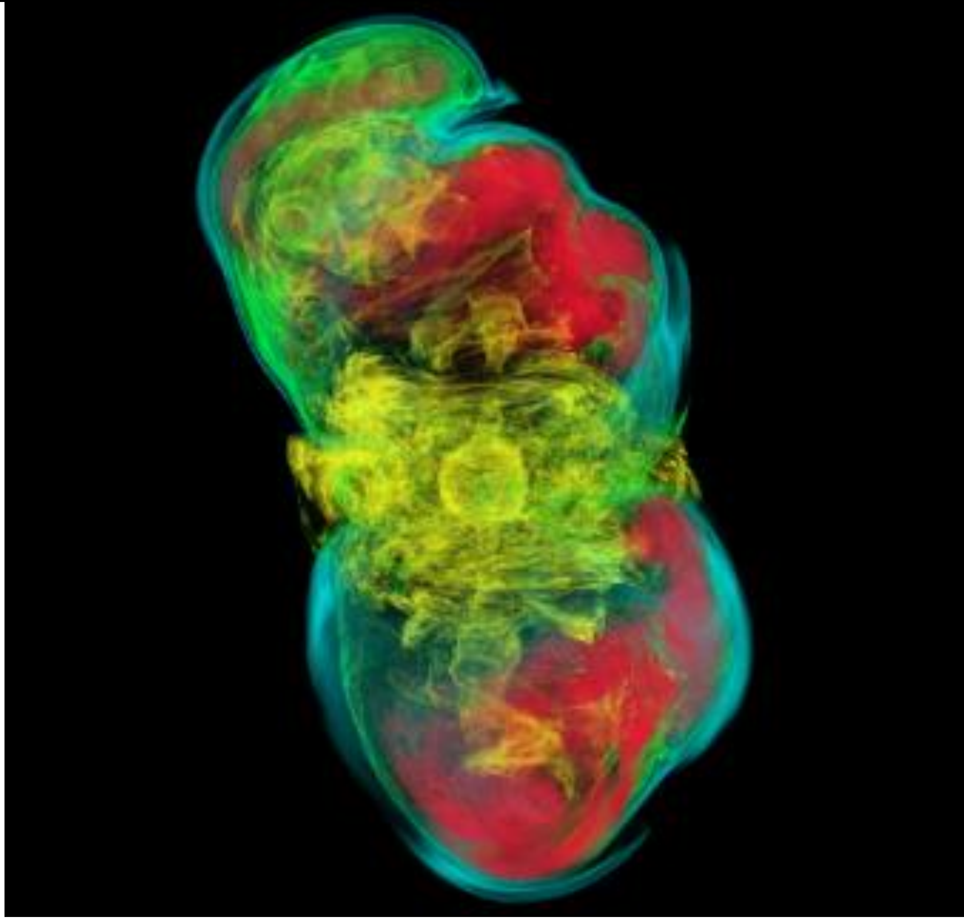
- Fraction of the gravitational BE -> stored in the free energy of differential rotation

Need for rapid rotation + stong magnetic fields

$$P_o < 4 - 6\ s \rightarrow \text{ miliecond PNS}$$

- PNS rotational energy of B-field up to equipartition, ***field amplification*** by:
  - Compression
  - Dynamos (α – Ω effect)
  - Magnetorotational instanility (MRI)

Rotational energy taped into, by magnetic fields may lead to Jet like explosions



Highly magnetized spiral plasma funnels expelled from the core push out the shock in polar regions
Mosta et al. (2014)

Limitations

- MHD mechanism insufficient for precollapse $P_0 > 4\ s$ but

-> stellar evolution and NS birth spin estimates $P_0 > 30\ s$

→ ***slowly rotating progenitors*** due to loss of angular momentum during evolution: stellar winds and magnetic braking (Heger et al. 2005; Maynet et al. 2011)

- ***Resolution*** does not allow to capture magnetorotational instability(MRI)

Alternatives!
Slowly Rotating or non-rotating cores

(Obergaulinger et al. 2014)

Shock's revival – Asymmetric Explosion

4. MHD driven explosions (WAVES) – Magnetic fields amplification

b) no rotation

- ***Compression*** **->** 3 orders of magnitude **->** no change of topology
- ***Small scale dynamo*** **->** only α-effect – driven by convection. But large scale field requires differential rotation (Ω-effect)

✓ ***non-radial fluid motions*** triggered by SASI and convection, lead to the magnetic fields perturbation and Alfven waves propagation

Shock's revival – Asymmetric Explosion

4. MHD driven explosions (WAVES) – Magnetic fields amplification

b) no rotation

Mass density of the ions -> perturbation
Magnetic fields -> restoring force

Alfven waves propagate along the field lines:

- Compete with the accretion flow -> stagnation point near the shock -> $C_A = V_{accretion}$
- Increase gas entropy -> energy dissipation behind the shock

Transmission of energy from the convective active PNS to the much less dense surrounding medium -> shock revival and explosion

Physical model

ideal MHD + neutrino transport

- ***Conservation Laws***

-> *mass*, electron number, *gas momentum*, total energy of matter, magnetic flux.

$$
\begin{aligned}
\partial_t \rho + \bar{\nabla}\cdot(\rho\bar{v}) &= 0 \\
\partial_t(\rho Y_e) + \bar{\nabla}\cdot(\rho Y_e \bar{v}) &= S_n^0 \\
\partial_t \rho v^i + \nabla_j (P_{tot}\delta^{ij} + \rho v^i v^j - b^i b^j) &= \rho\nabla^i\Phi + S^{1;j} \\
\partial e_{tot} + \bar{\nabla}\cdot((e_{tot} + P_{tot})\bar{v} - (\bar{v}\cdot\bar{b})\bar{b}) &= \rho\bar{v}\cdot\bar{\nabla}\Phi + S^0 + \bar{v}\bar{S}^1 \\
\partial_t \bar{b} - \bar{\nabla}\times(\bar{v}\times\bar{b}) &= 0
\end{aligned}
$$

- ***Source terms***

-> *pseudo - relativistic potential* -> account for the contributions of energy and pressure + metric terms of GR -> good approximation for moderately compact objects such as the PNS (Marek (2006))

-> *reactions with neutrinos* -> exchange of electron number , energy and momentum between the gas and the neutrinos

$$
\begin{gathered}
\boldsymbol{\rho} \\
\boldsymbol{Y_e} \\
\bar{\boldsymbol{v}} = v_r\hat{r} + v_\theta\hat{\theta} \\
\bar{\boldsymbol{b}} = b_r\hat{r} + b_\theta\hat{\theta} \\
\boldsymbol{e_{tot}} = \epsilon + \tfrac{1}{2}\rho\bar{v}^2 + \tfrac{1}{2}\bar{b}^2 \\
\boldsymbol{P_{tot}} = P + \tfrac{1}{2}\bar{b}^2 \\
\boldsymbol{\Phi}
\end{gathered}
$$

- ***Microphysics***

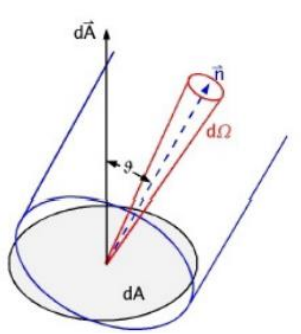
-> Neutrino reactions

-> Equation of state for the nuclear matter (Lattimer & Swesty, 1991), incompressibility K=220 MeV

$$
\begin{aligned}
& n + \nu_e \rightleftharpoons p^+ + e^- \\
& p^+ + \bar{\nu}_e \rightleftharpoons n + e^- \\
& (A,Z) + \nu_e \rightleftharpoons (A,Z+1) + e^- \\
& n/p + \nu_X \rightleftharpoons n/p + \nu_X \\
& (A,Z) + \nu_X \rightleftharpoons (A,Z) + \nu_X \\
& e + \nu_X \rightleftharpoons e + \nu_X
\end{aligned}
$$

Solve coupled equations of MHD + neutrinos transport

Neutrino Transport -> Boltzman equations



- ***Neutrinos Field -> $I(\vec{x}, \vec{n}, \epsilon, t)$***

-> energy carried by all ν of energy ϵ in direction $\vec{n} = \vec{p}/|\vec{p}|$ through a unit surface dA at position $\vec{x}$, time t

- ***Boltzmann equation***

advection, emission, absorption, and Doppler shift etc.

$$\begin{aligned} \partial_t I + \vec{n}\cdot\vec{\nabla} I &= \eta_0(\epsilon) - \chi_0(\epsilon) I \\ &\quad + \vec{n}\cdot\vec{v}\,(2\eta_0(\epsilon) - \epsilon\partial_\epsilon\eta_0 + [\chi_0(\epsilon) + \epsilon\partial_\epsilon\chi_0(\epsilon)]I) \end{aligned}$$

-> *expand the intensity in angular moments*

$$M^{i_1 i_2 \dots i_m} = \int d\vec{n}\, n^{i_1} n^{i_2} \dots n^{i_m} I$$

-> solve system of the first two moments (***energy density + energy flux***). Also, specify the second order moment (pressure tensor), by local algebraic Eddington factor closure.

(Cernohorsky & Bludman (1994))

$$\partial_t M^{i_1 \dots i_m} + \nabla_j M^{i_1 i_2 \dots i_{m+1}} + velocity\; terms = S_m^{i_1 \dots i_m}$$

$$E_\alpha = \int_{4\pi} d\Omega I \qquad F_\alpha = \int_{4\pi} d\Omega n^j I$$

- ***Source terms***

-> f (netrino energy, number of species)

-> flavors → $\nu_e - \bar{\nu}_e$

- ***Hyperbolic terms***

-> **radiative fluxes** + **advection and compression by the flow**

$$\begin{aligned} \partial_t E_\alpha(\omega) + \vec{\nabla}\cdot(E_\alpha(\omega)\vec{v}) + \vec{\nabla}\cdot\vec{F}_\alpha(\omega) - \omega\nabla_j v_k \partial_\omega P_\alpha^{jk}(\omega) &= S_\alpha^0(\omega) \\ \partial_t F_\alpha^i(\omega) + \nabla_j F_\alpha^i(\omega) v^j + \nabla_j P_\alpha^{ij}(\omega) + F_\alpha^i(\omega)\nabla_j v^j &= S_\alpha^{1;i}(\omega) \end{aligned}$$

- ***Velocity terms***

-> **spectral redistribution (Doppler shift) etc.**

Initial conditions

scope: identify the effects of variations of the magnetic fields

- **Progenitor ->** core of 15 Msun (Woosley et al. 2002)

- **2D - Grid** -> 360 logarithmically spaced zones radius, r and 144 zones in θ
- **Magnetic fields topology (complex)**

Initial field

modified dipole (Suwa et al (2007))

models -> Bb with b representing the various initial field strengths, $b = 0, 10^{8,10,11,11.5,12}\ G$

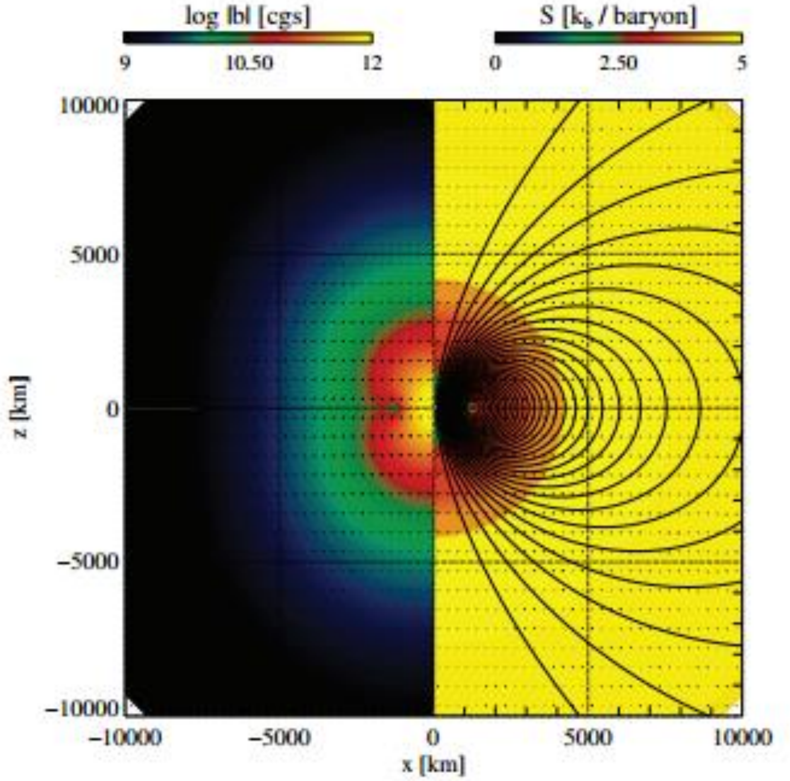
small scale – turbulent -> due to hydro-instabilities (SASI + convection)

Pre - collapse field

-> spherical collapse

-> amplification by compression in the infall

-> never strong enough to modify the collapse dynamics



Initial field geometry and strength

$$A^{\phi} = \frac{b_0}{2}\frac{r_0^3}{r^3 + r_0^3} r \sin\theta$$

Dynamical important regions – instabilities location

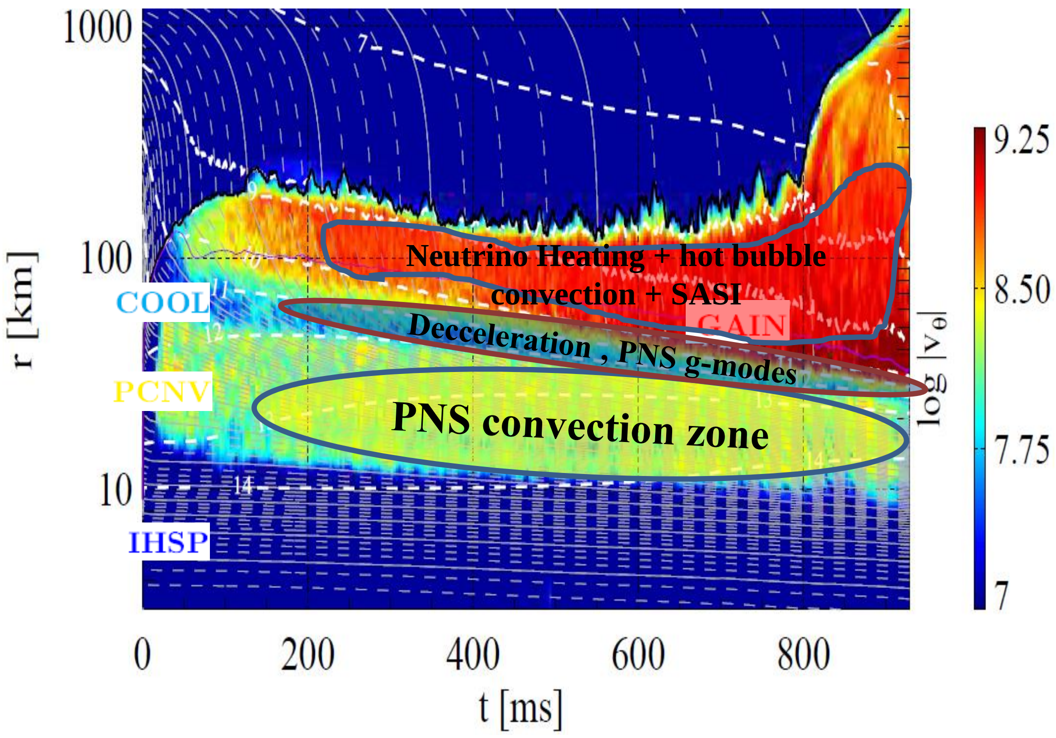
- **IHSP**: inner hydrodynamically ***stable PNS***

  $up\ to\ \rho \sim 10^{14} gcm^{-3}$

- **PCNV**: the *PNS convection zone*

  - Negative gradient of the electron fraction
  - Slightly negative entropy gradient. outer boundary -> location of the minimum of the Y_e profile

    $up\ to\ \rho \sim 10^{11} gcm^{-3}$



- **COOL**: outside the PNS convection zone

  - accreted matter suffers a net energy loss -> ***production of neutrinos***.
  - ***Deceleration of the accreted flow*** -> a stable layer on the "surface" of the PNS.

- **GAIN**: ***Hot-bubble convection***, ***SASI activity***, ***Neutrino heating***

  - Gain radius -> transition from neutrino cooling below to neutrino heating above
  - Neutrino heating -> ***negative entropy gradient*** and thus ***postshock convection***.
  - SASI activity -> larger volume -> encompassing parts of the neutrino-cooling -> ***SASI modes*** amplification between the shock and the deceleration zone. This happens typically between neutrinosphere and gain radius.

Field evolution

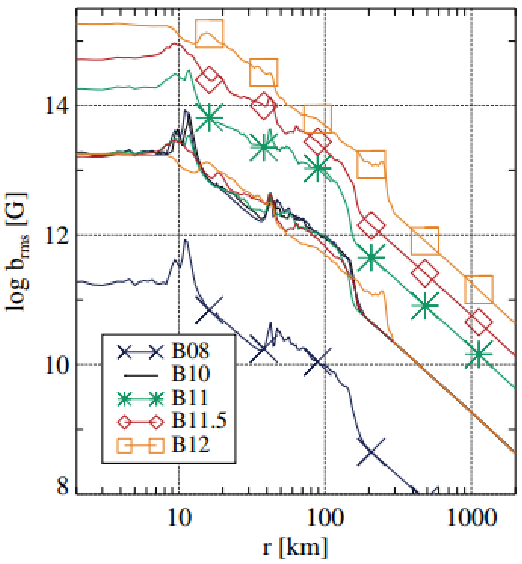
- Amplifications due to the ***overturning flow***
- loses due to ***expulsion of magnetic flux*** from the convection cells

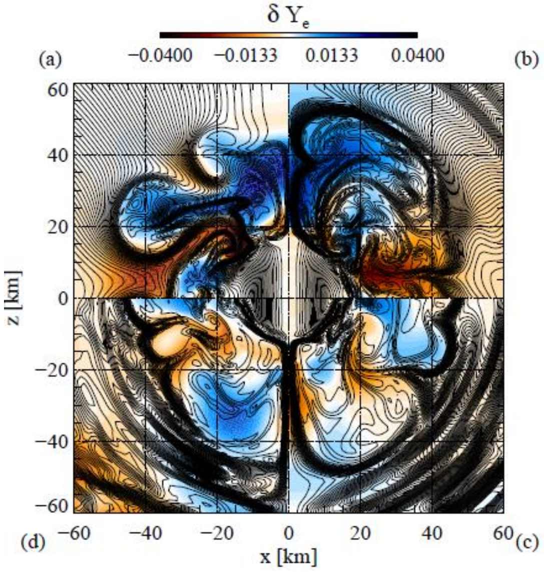
-> competition between the ***radial advection and the overturning flows***

-> PNS surrounded by strong field parallel to the surface (non-radial structures are advected towards the PNS convection zone

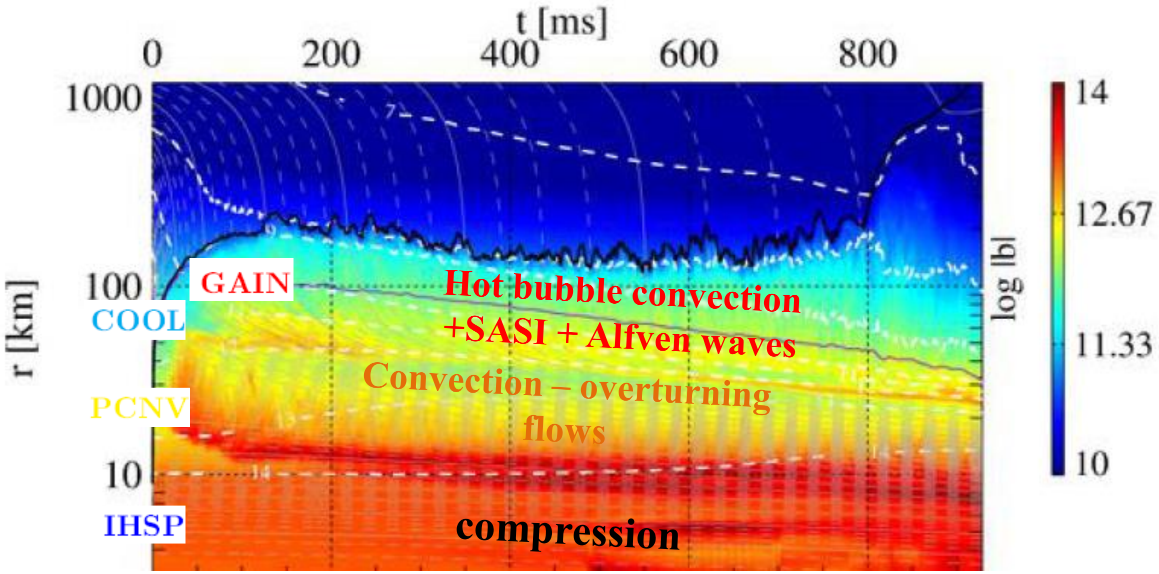
! ***shielded newly formed NS – unconventional field structure***

profiles of the magnetic field strength for different initial fields





Snapshots of the PNS convection zone at four times after bounce: deviation of Y_e from the angular average and field lines



Hot bubble convection +SASI + Alfven waves

Convection – overturning flows

compression

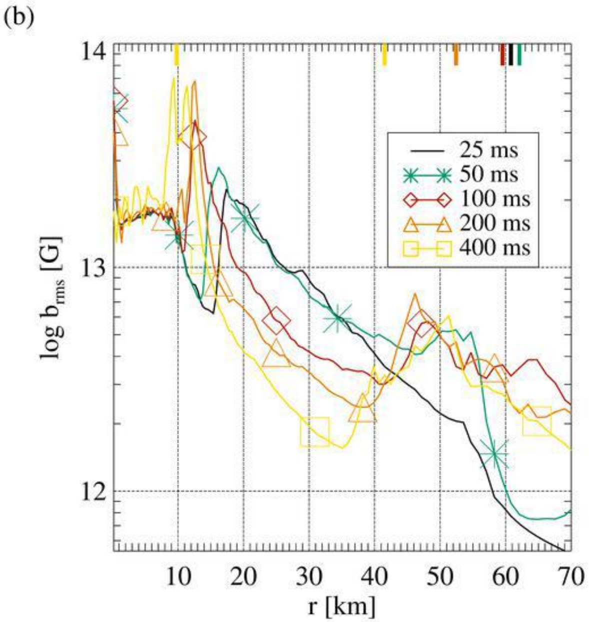
angularly averaged profiles of the strongest magnetised model as a function of time

Field amplification in the outer layers

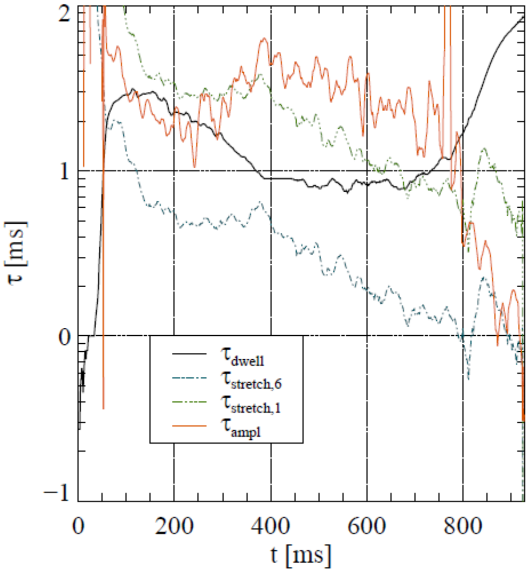
- moderate amplification at the deformed shock wave

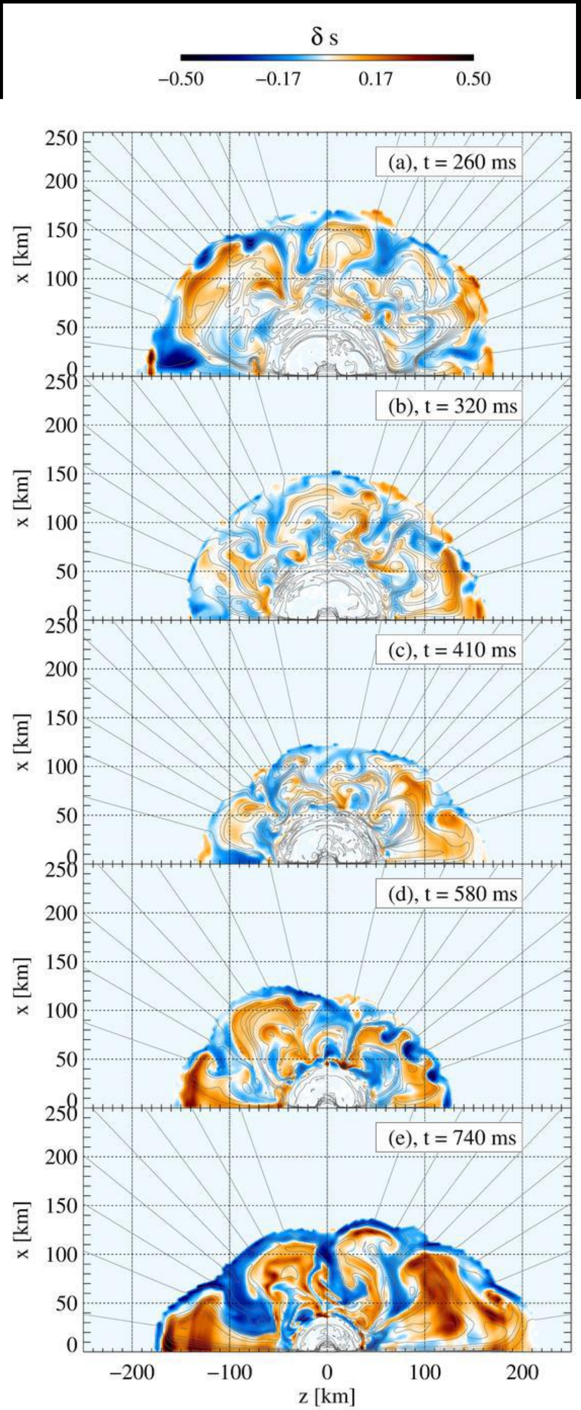
-> stretching and folding of the field lines

- amplification factor in the gain layer results from a competition between advection and eddy turn-over time



Profiles of angularly averaged field strength for different times

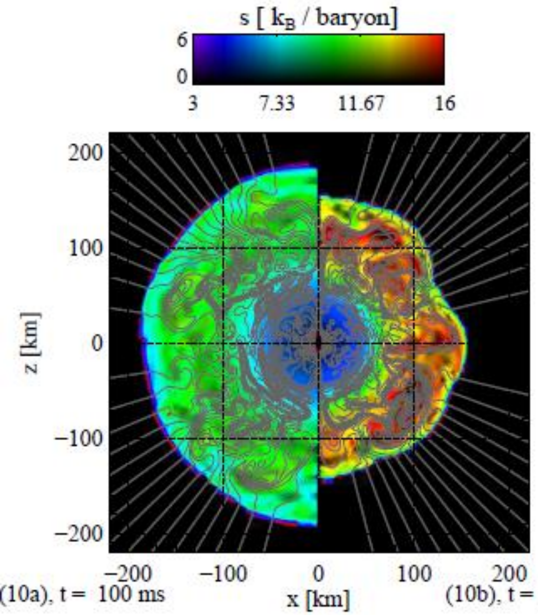




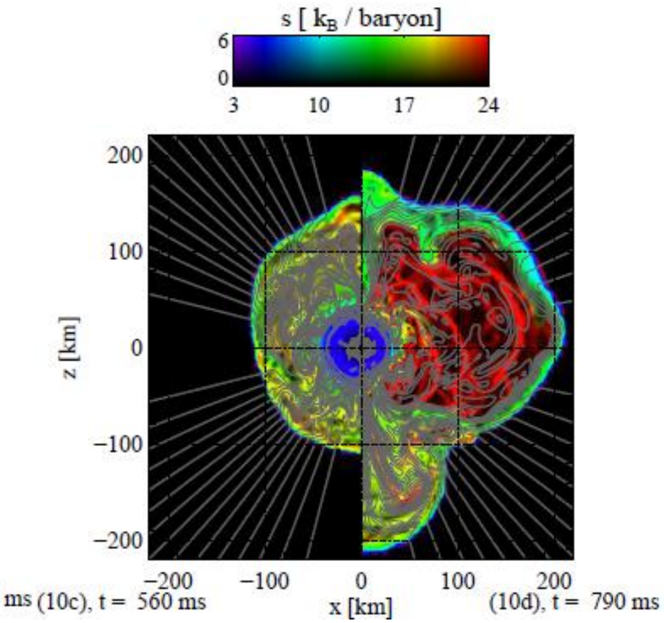
Explosion

a) Weak magnetic fields – pulsar final fields

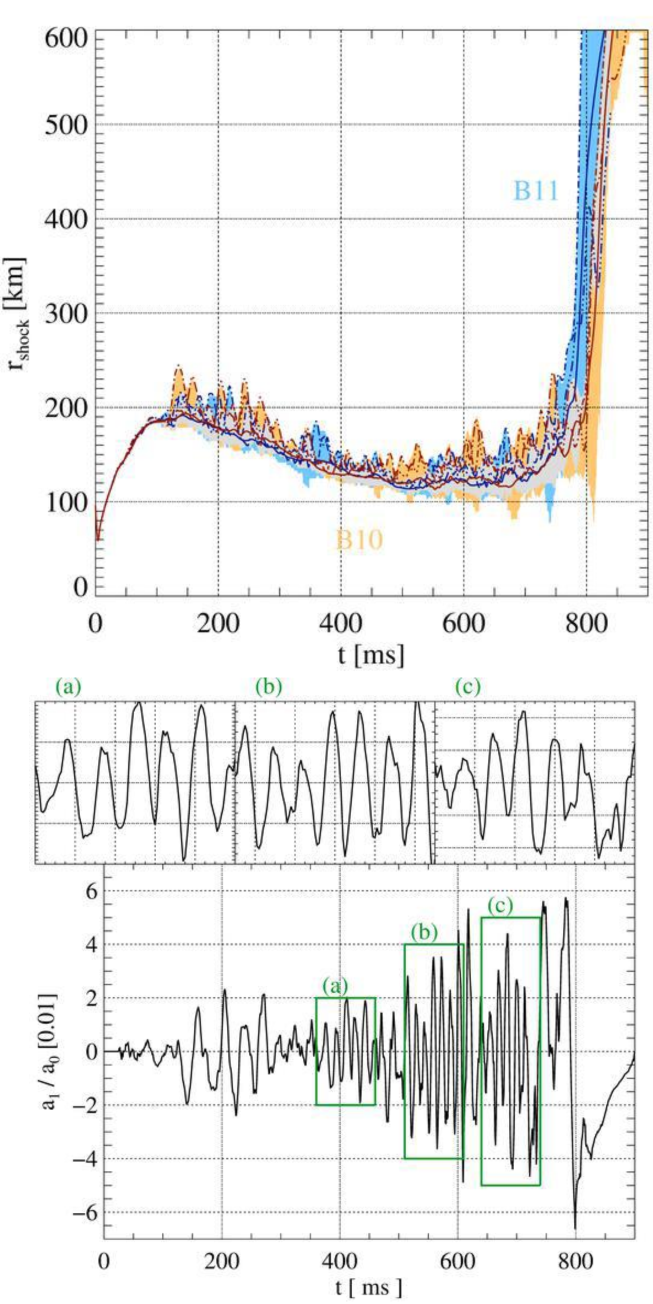
- long phase of shock contraction and more or less regular oscillations
- Slowly developing high-entropy bubbles of intermediate size appear and are quickly destroyed (formed by convective +SASI modes)
- explosion -> one large bubble of high entropy



(10a), t = 100 ms (10b), t = 330 ms



(10c), t = 560 ms (10d), t = 790 ms



Shock radii and the dipole mode

Explosion

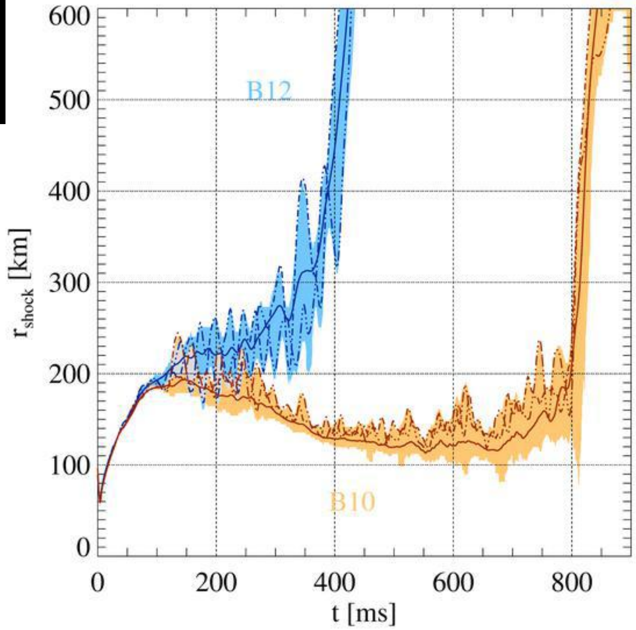
b) Strong magnetic fields – magnetar like

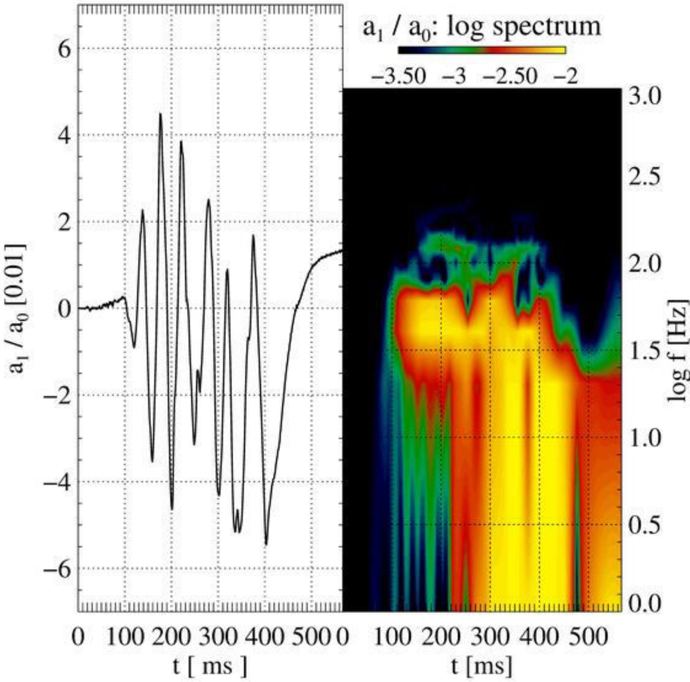
- the Alfven time scale is similar to the hydrodynamic time scale
- field resists bending, slows down motion across field lines

-> modifies the growth of SASI, convection

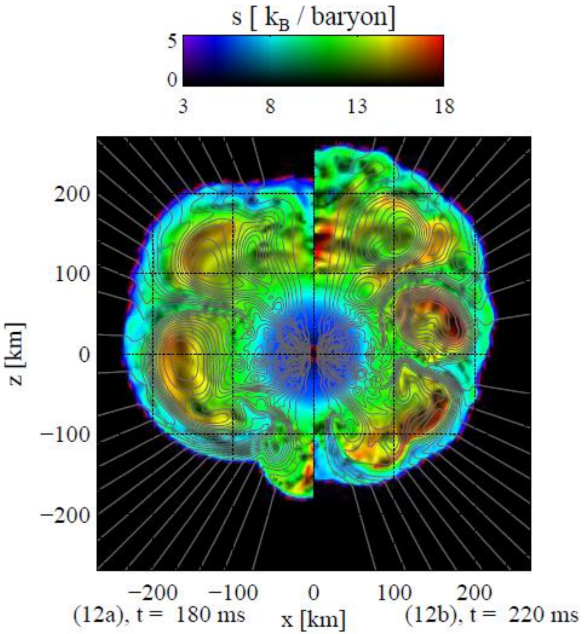
-> development of very persistent large-scale patterns of upflows and downflows, stronger shock expansion

-> much earlier explosions

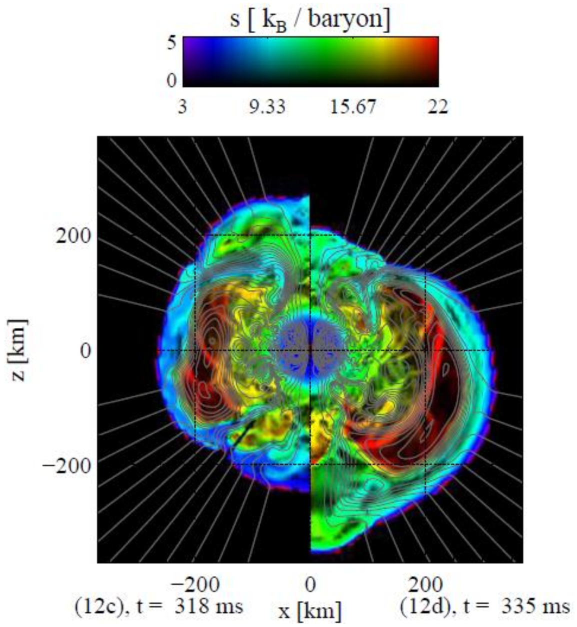




Shock radii and the dipole mode



(12a), t = 180 ms (12b), t = 220 ms



(12c), t = 318 ms (12d), t = 335 ms

GW in the Core-Collapse Supernova context:

- *Live* **dynamical information** from deep inside SN core.
- Diagnosis tool for multi-D dynamics and **explosion mechanism**.
- Can help to **infer macrophysical *and* microphysical details**.

Issue: ***Need detailed theoretical understanding*** of GW emission for GW data analysis and extraction of physics.

-> waveform predictions!

GW emission processes in CCSN

- Asymmetric collapse and core bounce
- Postbounce convection and SASI
- PNS core oscillations – g modes
- Anisotropic neutrino emission

GW generation – **the quadrupole formula** **a) matter asymmetric motions**

☐ $\bar{h}_{\alpha\beta} = -16\pi \cdot T_{\alpha\beta}$

- Solve Einstein equation coupled to matter

Solution

-> take the ***TT – part***

-> ***Slow motion*** approximation (SMA)

$$h_{ij}^{TT} = \left[4 \int \frac{T_{ij}(x', t' = t - |x - x'|)}{|x - x'|} d^3x'\right]^{TT}$$

$$h_+ = h_{\theta\theta}^{TT} = \frac{1}{r} \cdot \frac{d^2}{dt^2} I^{20}(t - r) T_{ij}^{E2,20}.$$

-$v \ll c = 1 \rightarrow \frac{L}{T} \sim \frac{L}{\lambda} \ll 1 \rightarrow \lambda \gg \mathrm{L}\ , |x - x'| = r$

-> ***Perfect fluid***

- Resolve -> ***Angular depedence*** -> expand in tensor spherical harmonics → axisymmetry → $(\mathrm{l} = 2, \mathrm{m} = 0)$, (Thorne (1980))

- ***Quadrupole moment*** - ***shape of the core***

-> 1st **derivative** - ***asphierisity of the mass flux + momentum distribution***

-> 2nd **derivative** – ***GW amplitude –forces acting on the fluid***)

-> remove time derivatives (due to *numerical noise*)

-> use Euler equations ***mass + momentum conservation***

$$I^{20} = \frac{32\pi^{3/2}}{\sqrt{15}} \int_0^1 d\mu \int_0^\infty d\left(\frac{r^3}{3}\right) \rho \cdot \left(\frac{3}{2}\mu^2 - \frac{1}{2}\right) d\left(\frac{r^3}{3}\right)$$

$$N_{20}^{E2} = \frac{d}{dt} I^{20} = \frac{32\pi^{3/2}}{\sqrt{15}} \int_0^1 d\mu \int_0^\infty d\left(\frac{r^3}{3}\right) (\partial_t \rho) \cdot \left(\frac{3}{2}\mu^2 - \frac{1}{2}\right)$$

$$A_{20}^{E2} = \frac{d^2}{dt^2} I^{20} = \frac{32\pi^{3/2}}{\sqrt{15}} \int_0^1 d\mu \int_0^\infty d\left(\frac{r^3}{3}\right) r \cdot \left(\partial_t(\rho v_r)(\frac{3}{2}\mu^2 - 1) - 3 \cdot \partial_t(\rho v_\theta)\mu\sqrt{1 - \mu^2}\right)$$

$$\partial_t \rho + \bar{\nabla} \cdot (\rho \bar{v}) = 0$$

$$\partial_t \rho v^i + \nabla_j (P_{tot} \delta^{ij} + \rho v^i v^j - b^i b^j) = \rho \nabla^i \Phi + S^{1;j}$$

GW generation – **the quadrupole formula**
a) matter asymmetric motions

$$h_+ = \frac{A_{20}^{E2}}{r} \cdot \frac{1}{8}\sqrt{\frac{15}{\pi}} sin^2\theta$$

$$f_{ij} = \rho v_i v_j - b_i b_j$$

$$\begin{aligned} A_{20}^{E2} &= \frac{32\pi^{3/2}}{\sqrt{15}} \int_0^1 d\mu \int_0^\infty d\left(\frac{r^3}{3}\right) \cdot \\ &\cdot \left(f_{rr}(3\mu^2-1) + f_{\theta\theta}(2-3\mu^2) - 6f_{r\theta}\left(\mu\sqrt{1-\mu^2}\right)\right) \\ &\cdot \left(-r\rho\partial_r\Phi(3\mu^2-1) + 3\rho\partial_\theta\Phi\left(\mu\sqrt{1-\mu^2}\right)\right) \end{aligned}$$

- GW signal due to ***hydromagnetic stresses*** (the velocity terms)
- Magnetic contribution due to the ***Lorentz force action*** on the fluid
- Signal due to term of the ***gravitational potential***
- **No** momentum transfer due to the ***neutrino radiation pressure***

GW generation – the quadrupole formula

b) Neutrino asymmetric emission

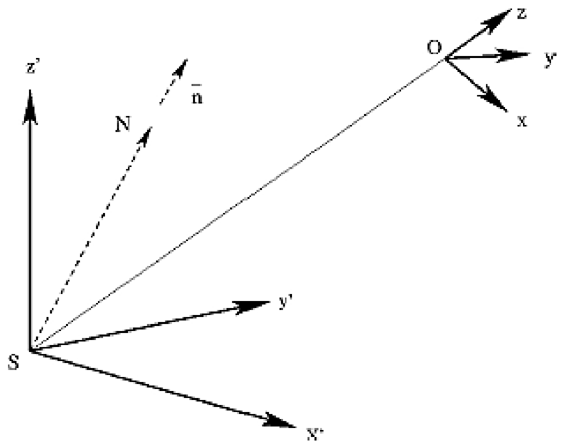
- any source with ***non-zero quadrupole*** or higher order moments result in gravitational wave emission
- GW memory

-> GW at t is caused by radiation at $t' = t - \frac{r}{c}$

-> $\Delta t = t - t' \cong \frac{r}{c} = cnst$ -> the GW leaves behind a constant “DC” offset after the asymmetric neutrino emission recedes

-> permanent displacements of the detector’s test masses when a GW “train” passes trough

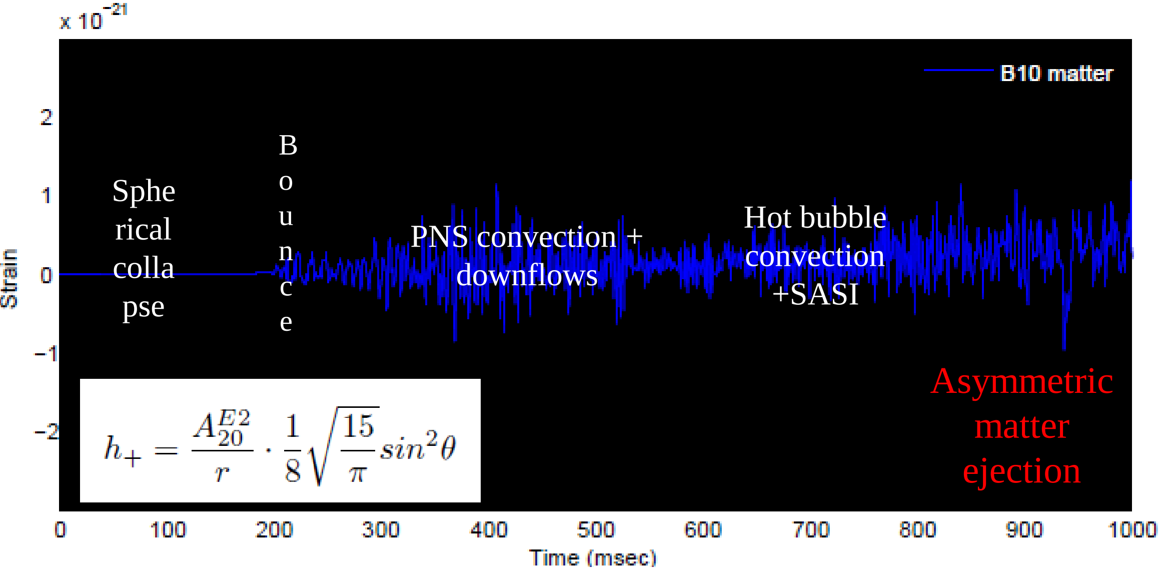
-> duration of the signal is critical for achieving a detection of the memory contributions



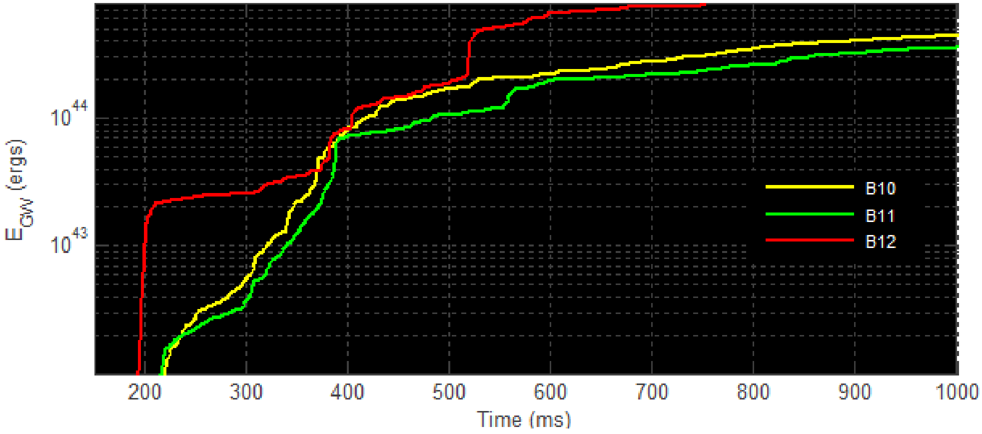
$$h_{ij}^{TT} = \frac{4G}{c^2 r}\int_{-\infty}^{t-\frac{r}{c}} dt' \int_{\Omega} d\Omega' \frac{(n_i n_j)^{TT}}{1 - cos\theta}\frac{dL}{d\Omega'}(\vartheta', \varphi', t')$$

- $\boldsymbol{n_i}$: indicates the direction of the neutrino emission
- $\boldsymbol{\theta}$: angle between n_i and the direction to the observer, ***O - frame***
- $(\boldsymbol{n_i n_j})^{\boldsymbol{TT}}$: projects out the TT-part with respect to the z-axis, ***O - frame***
- $\boldsymbol{\frac{dL}{d\Omega}}$: direction dependent neutrino luminosity where $\boldsymbol{\theta', \varphi'}$ define the beam direction in the ***S – frame***

GW matter signal– **weak magnetic fields – no equipartition**

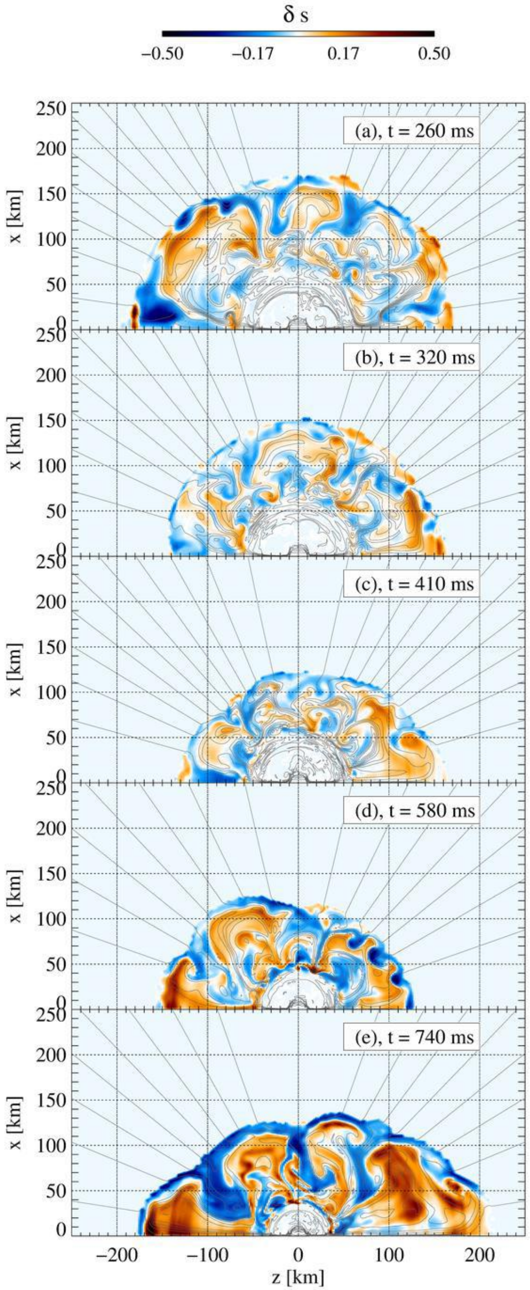


$$h_+ = \frac{A^{E2}_{20}}{r} \cdot \frac{1}{8}\sqrt{\frac{15}{\pi}} sin^2\theta$$



$$E^{GW} = R^2 \int d\Omega \int_{-\infty}^{+\infty} dt t^{GW}_{0z} = \frac{1}{32\pi}\frac{c^3}{G}r^2 \int d\Omega \int_{-\infty}^{+\infty} dt (2\pi f)^2 \left(\frac{d}{dt}h_+\right)^2$$

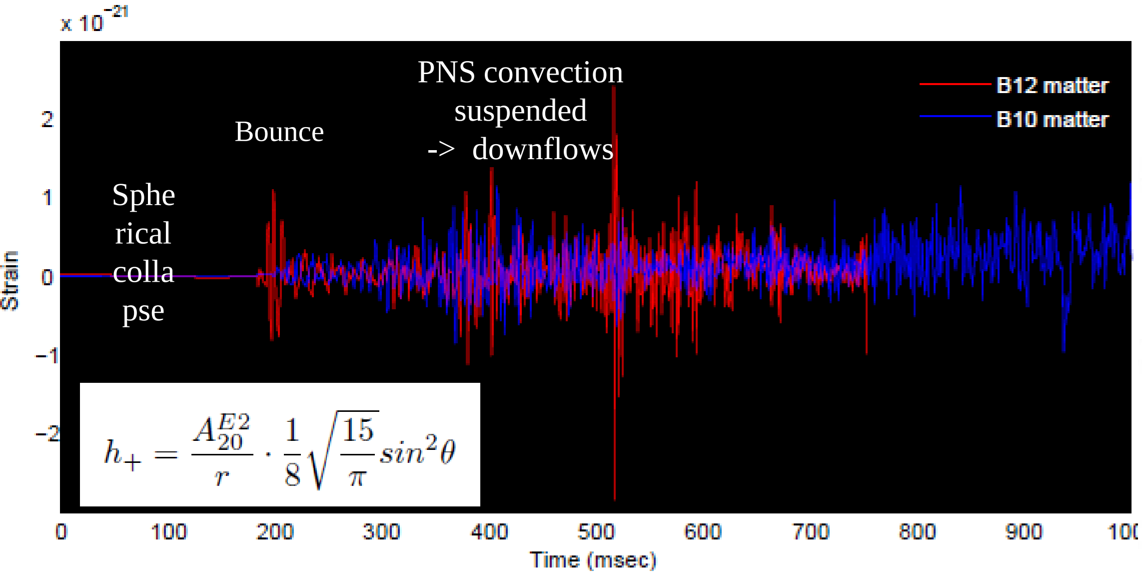
$$= \frac{1}{32\pi}\frac{c^3}{G}r^2 \int d\Omega \int_{-\infty}^{+\infty} dt (2\pi f)^2 \left(\frac{1}{r}\frac{dA^{E2}_{20}(t)}{dt}\right)^2$$

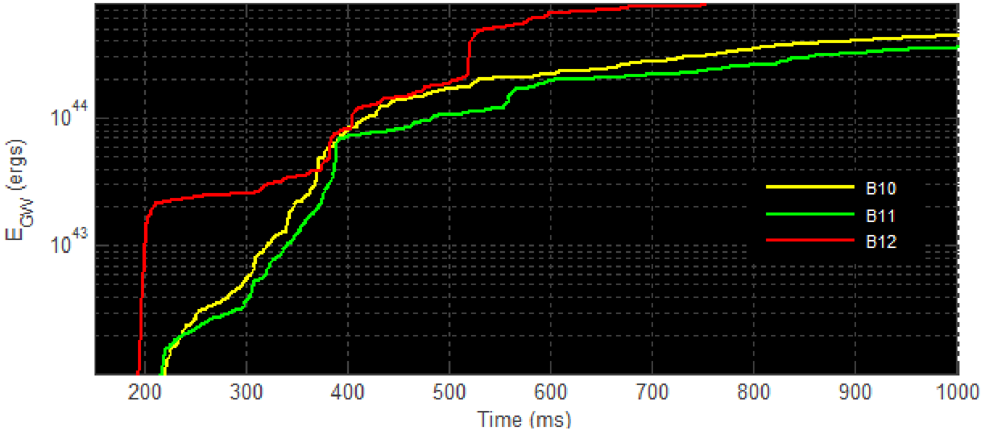


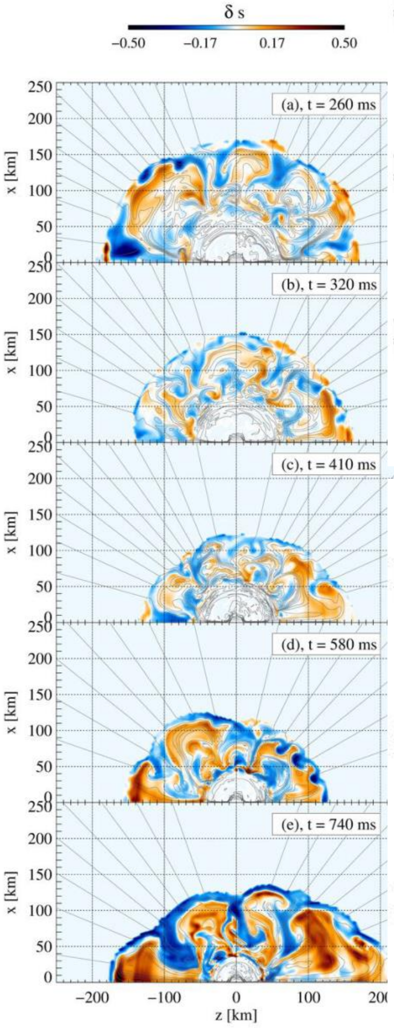
relative deviation of the entropy from its angular average and magnetic field lines

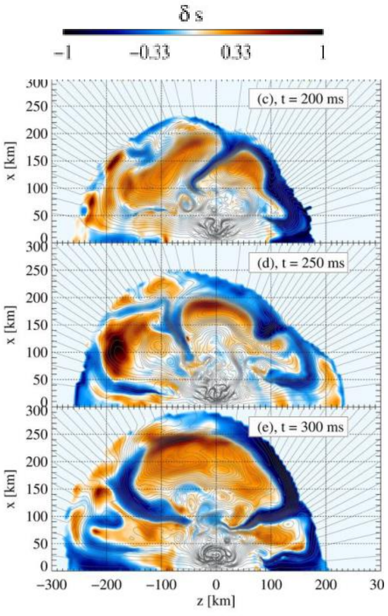
GW matter signal– **strong magnetic fields – equipartition**

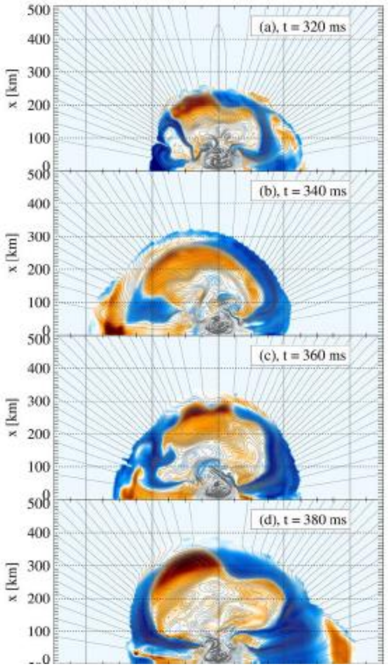
$$h_+ = \frac{A^{E2}_{20}}{r} \cdot \frac{1}{8}\sqrt{\frac{15}{\pi}} sin^2\theta$$







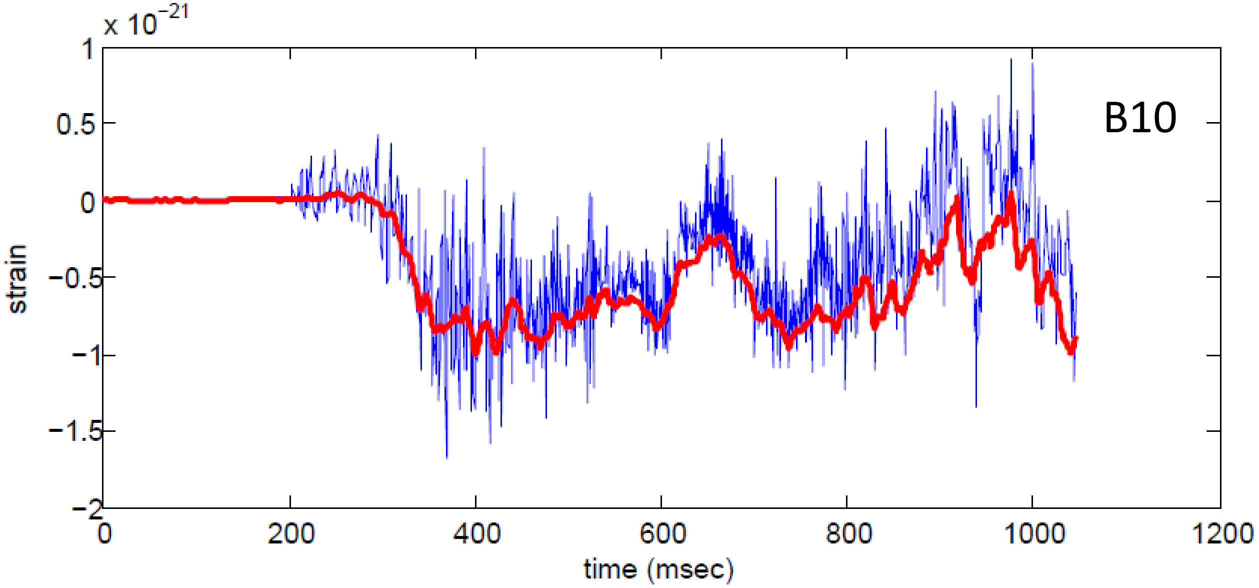


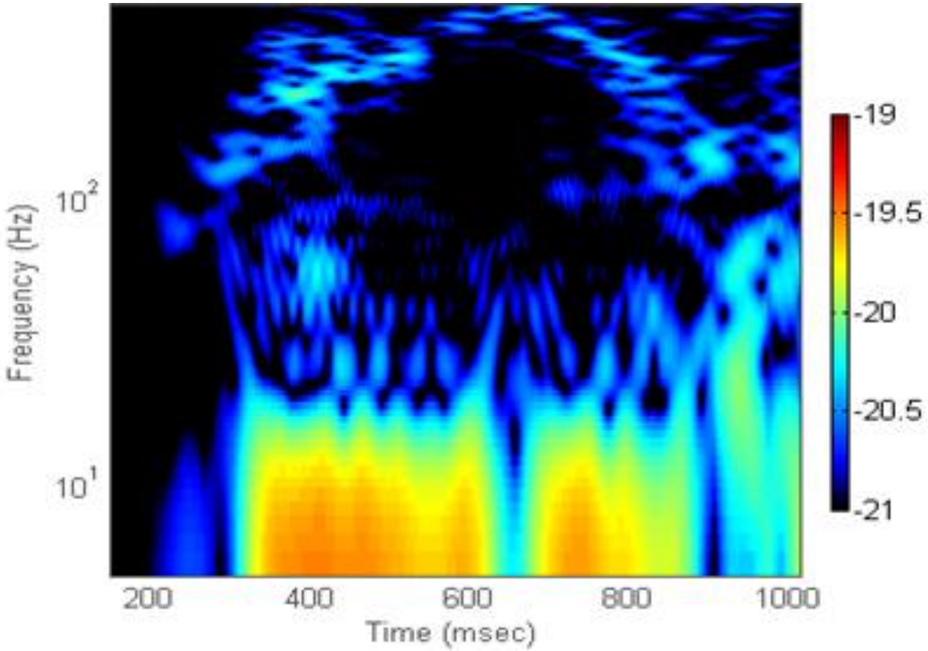


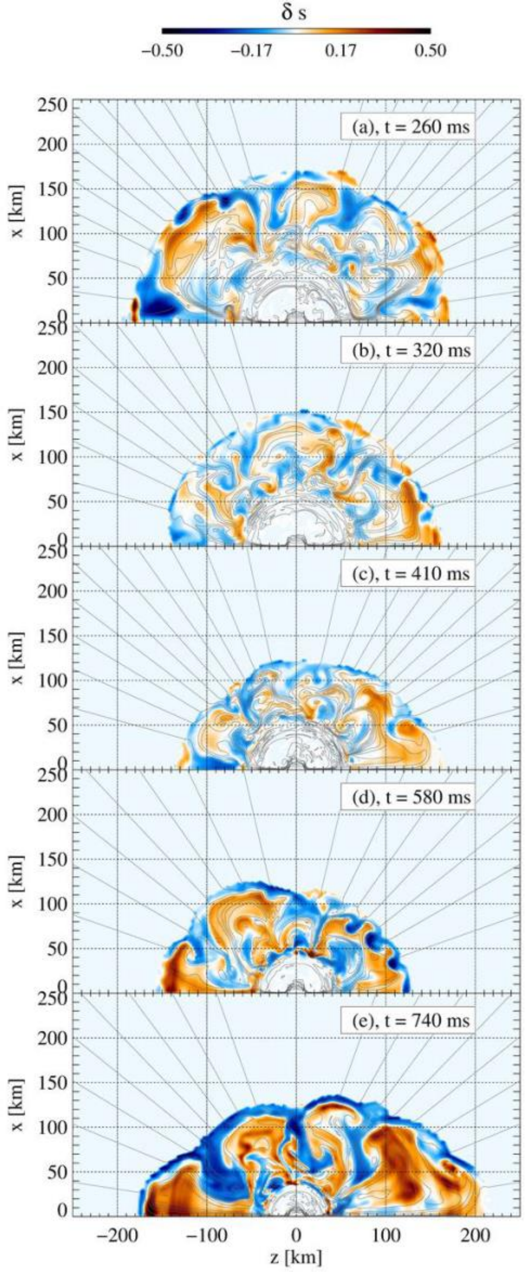
relative deviation of the entropy from its angular average and magnetic field lines

Total signal–
weak magnetic fields

$$h_+ = \frac{A^{E2}_{20}}{r} \cdot \frac{1}{8}\sqrt{\frac{15}{\pi}} si$$

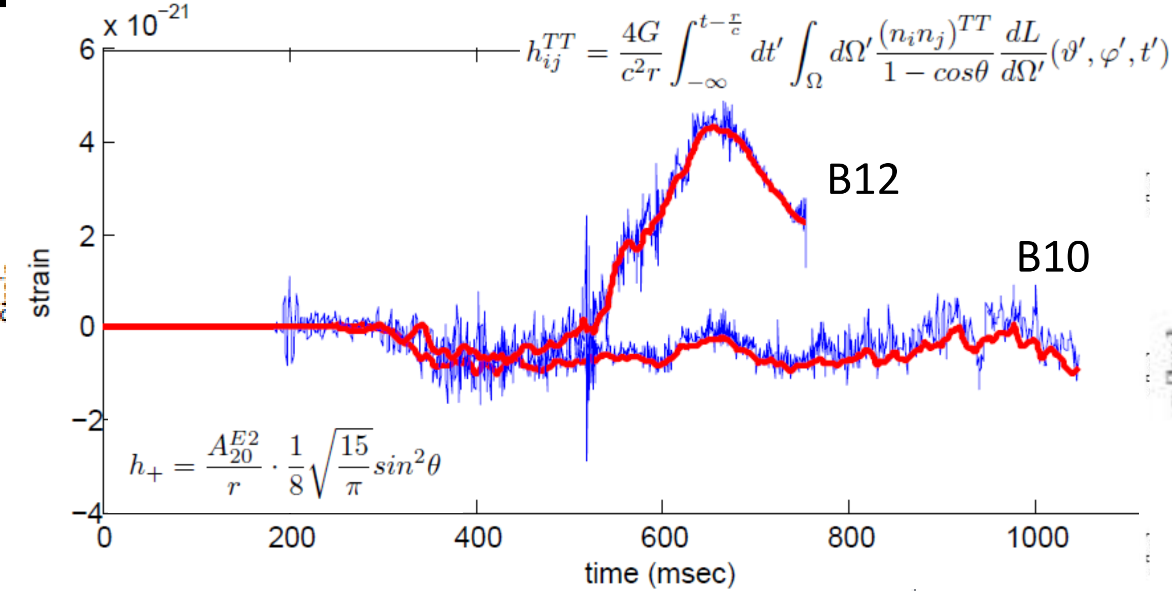






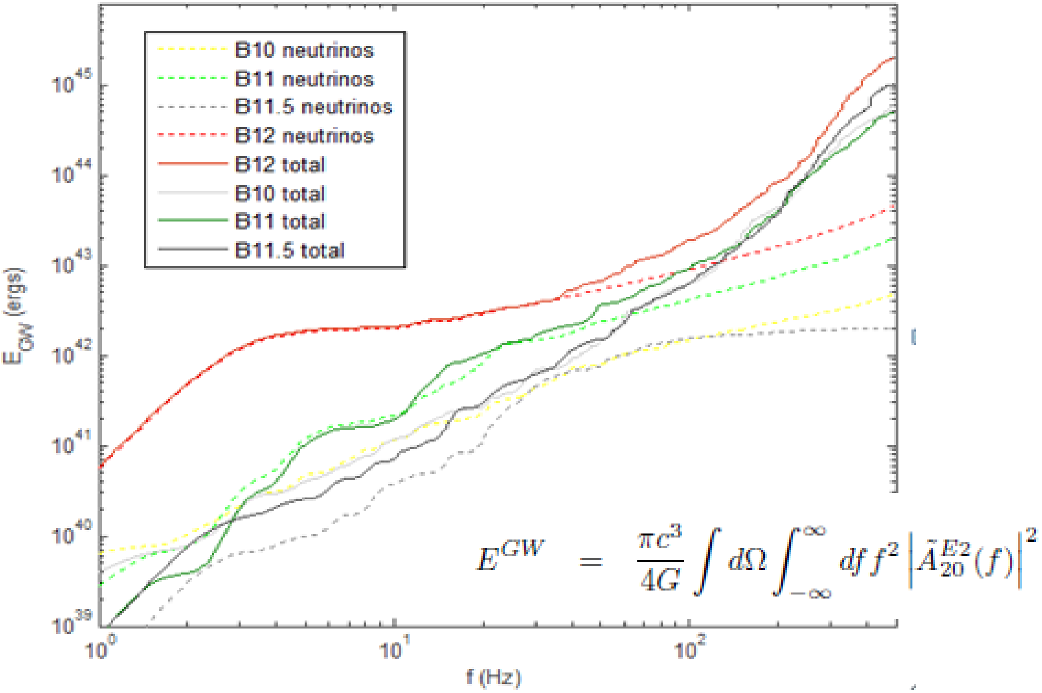
relative deviation of the entropy from its angular average and magnetic field lines

Total signal–
strong magnetic fields

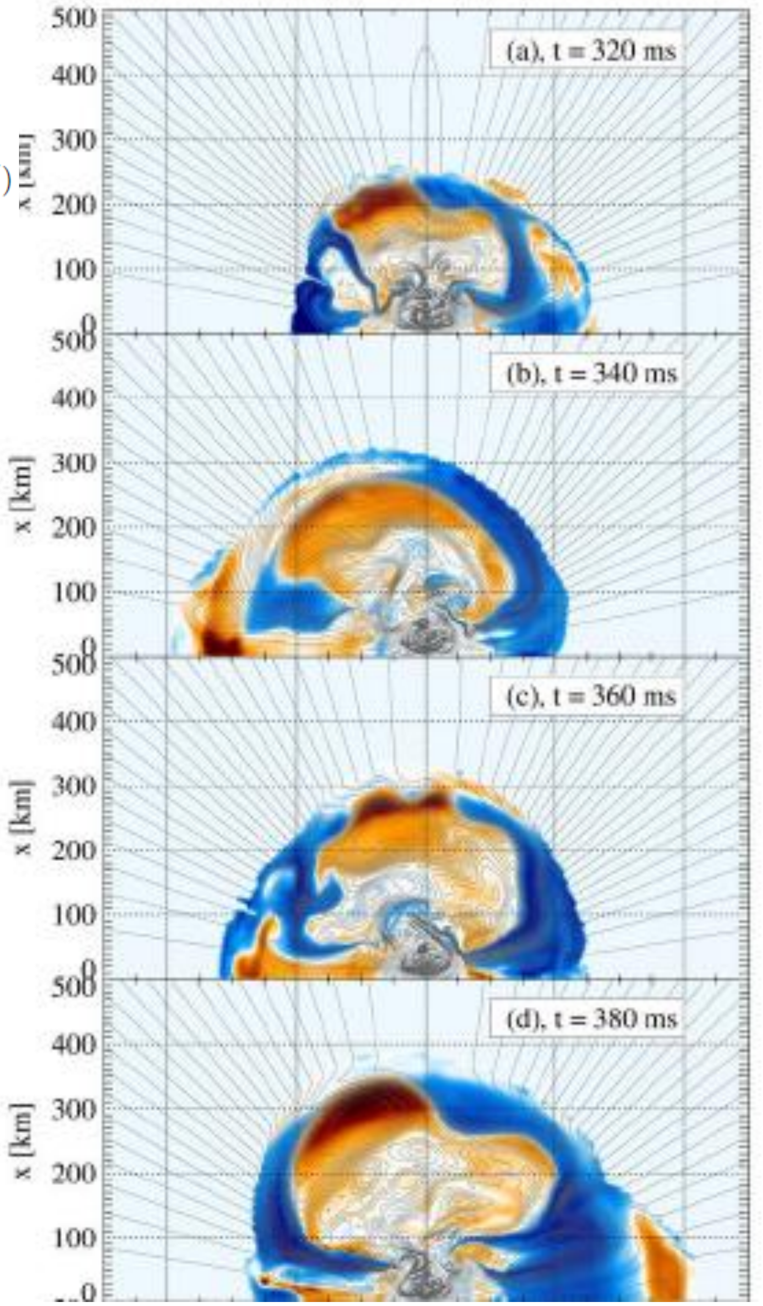


$$h_{ij}^{TT} = \frac{4G}{c^2 r}\int_{-\infty}^{t-\frac{r}{c}} dt' \int_{\Omega} d\Omega' \frac{(n_i n_j)^{TT}}{1-cos\theta}\frac{dL}{d\Omega'}(\vartheta',\varphi',t')$$

$$h_+ = \frac{A_{20}^{E2}}{r}\cdot\frac{1}{8}\sqrt{\frac{15}{\pi}}sin^2\theta$$



$$E^{GW} = \frac{\pi c^3}{4G}\int d\Omega \int_{-\infty}^{\infty} df\, f^2 \left|\tilde{A}_{20}^{E2}(f)\right|^2$$



relative deviation of the entropy from its angular average and magnetic field lines

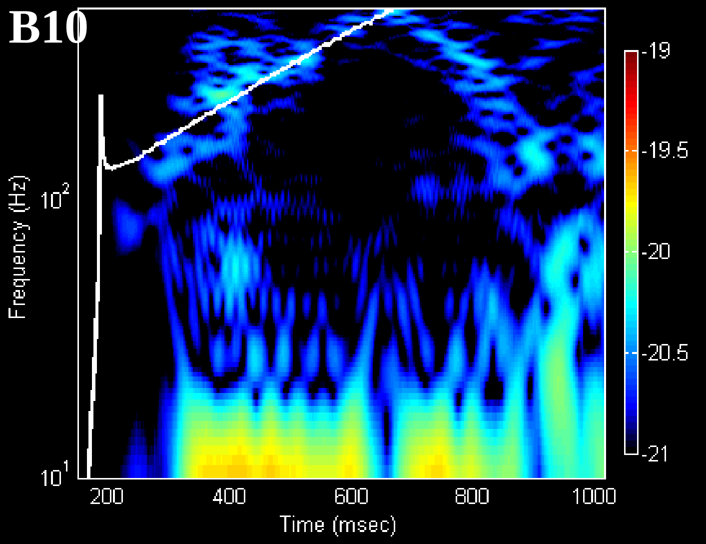
Characteristic frequencies - PNS properties

Buyancy frequency Brunt Vaisala

$$f_p = \frac{1}{2\pi}\frac{GM_{PNS}}{R^2_{PNS}}\sqrt{1.1\frac{m_n}{\langle E_{\nu_e}\rangle}}\left(1-\frac{GM_{PNS}}{R_{PNS}c^2}\right)^{3/2}$$

GW signal

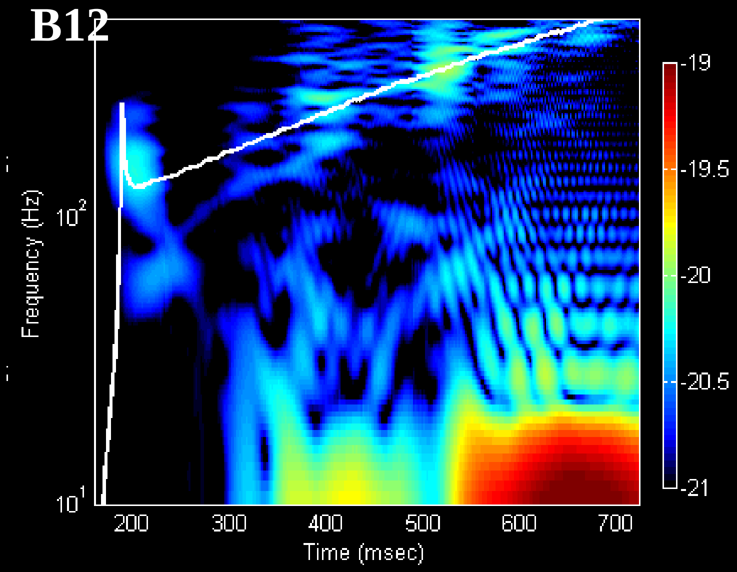
→ deceleration of **infalling convective plums** as a direct source for the gravitational wave signal **surface g-mode oscillations** excited by the downflows as a source. (Marek et al. (2009))



PNS structure → mass + radius ?

$f_{peak} = F(surface\ gravity,$ $surface\ temperature,$ $compactness\ parameter)$

-> time-variable, but stable g-mode frequency, but deceleration of plumes with random frequency and penetration depth is responsible for the more noisy part of the spectrum above this band



→ sensitive to factors that affect the ***contraction*** and ***thermal evolution*** of the proto-neutron star, such as the ***EoS*** and the ***neutrino treatment***

→ change the proto-neutron star surface temperature and the neutrino mean energies considerably.

GW matter signal
a) weak magnetic fields- no equipartition

Shock falls back- PNS contraction

-***—PNS surface and inside*** $\rho > 10^{11}\ gcm^{-3}$

-> **increasing mode** $\sim 100 - 500\ Hz$ -> eigenmodes of the COOL layer - the energy pumped into the oscillations is given by the downflows

—Hot bubble region, $\rho < 10^{11} gcm^{-3}$

-> **Medium frequency mode** ~ 60 Hz due to shock oscillations

Steady phase + Shock revival and Explosion

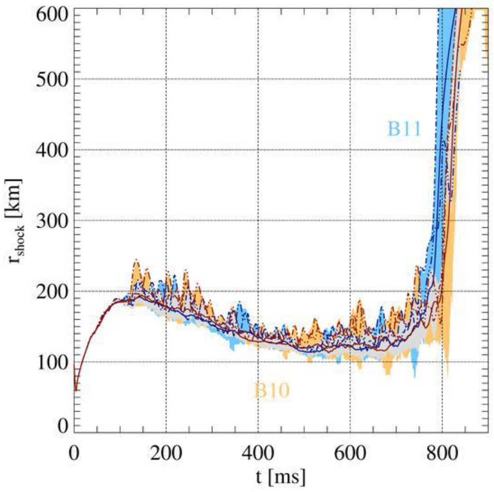
-*PNS surface,* $\rho > 10^{11}\ gcm^{-3}$

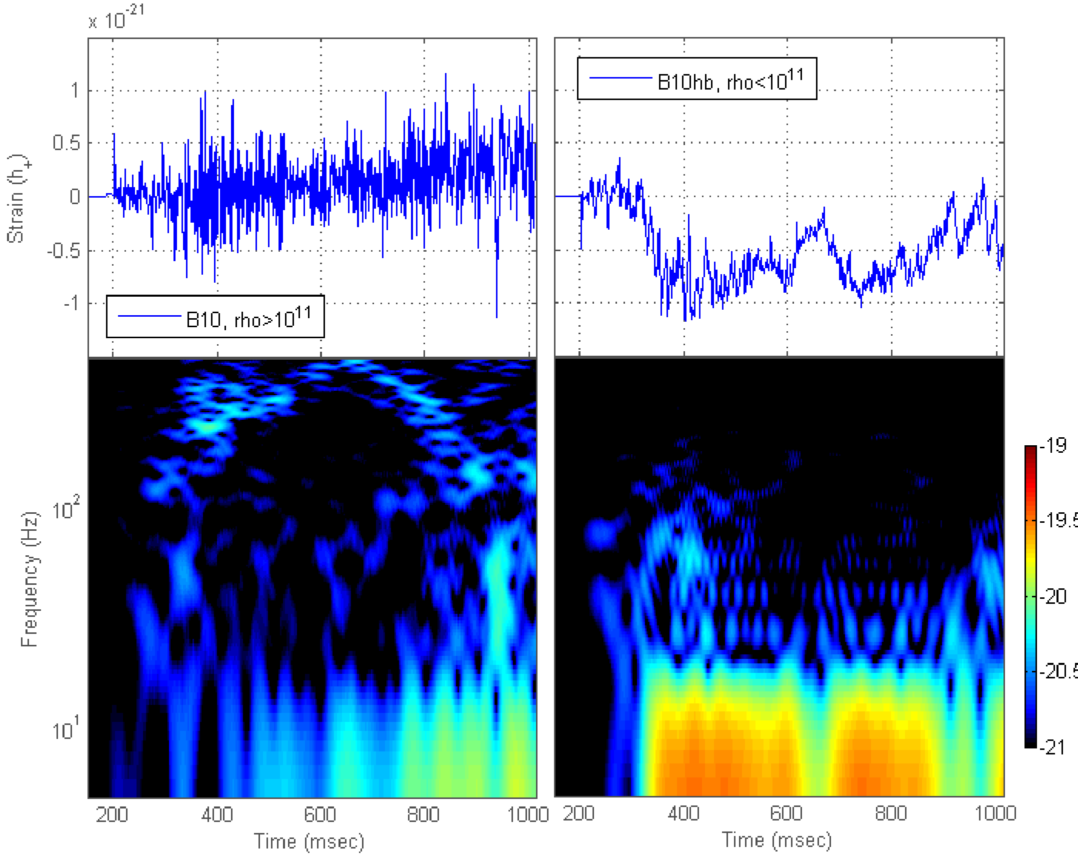
->**Decreasing frequency mode** from $\sim 400\ HZ\ to\ 100\ Hz$

-PNS surface + Hot bubble region

-> **Low freq. mode** ~ 10 Hz - related to the anisotropic expansion of the ejecta – large scale downflows

memory effect!





Detectability

$$\sqrt{S_h} = \frac{1}{\sqrt{f}} h_c(f) = 2\sqrt{f}\left|\tilde{h}(f)\right|$$

$$h_c = \frac{1}{r}\left(\frac{2G}{\pi^2 c^3}\frac{dE}{df}\right)^{1/2}$$

✓ **h_c properties:**

- Preliminary quantification of the *neutrinos + matter* memory effect (Christodoulou 1991)
- -> ***permanent displacements*** of the detector's test masses when a "GW train" passes through. (Thorne 1991)
- Area between the signal and the noise curves related to the signal to noise ratio (SNR)

▪ **$\sqrt{S_h}$ properties:**

- Quantifies directly the ***instantaneous displacements*** of the test masses when a GW passes through – direct observation of the source amplitudes
- But does not take into account the memory effect

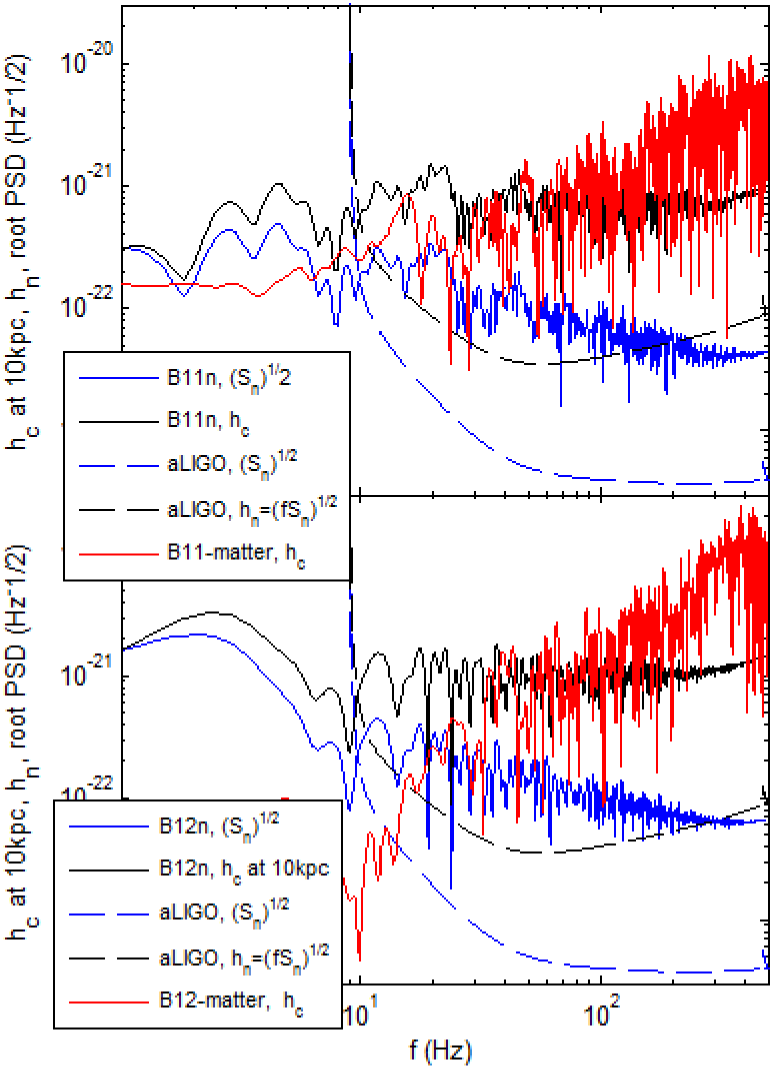
Predictions

- Low frequencies $\sim 10 - 20\ Hz$

-> *Neutrino driven GW signal* dominates – detectable

- Higher frequecies $\sim 40\ - 500\ Hz$

-> *Matter GW signal* more important – higher posibilities for detection

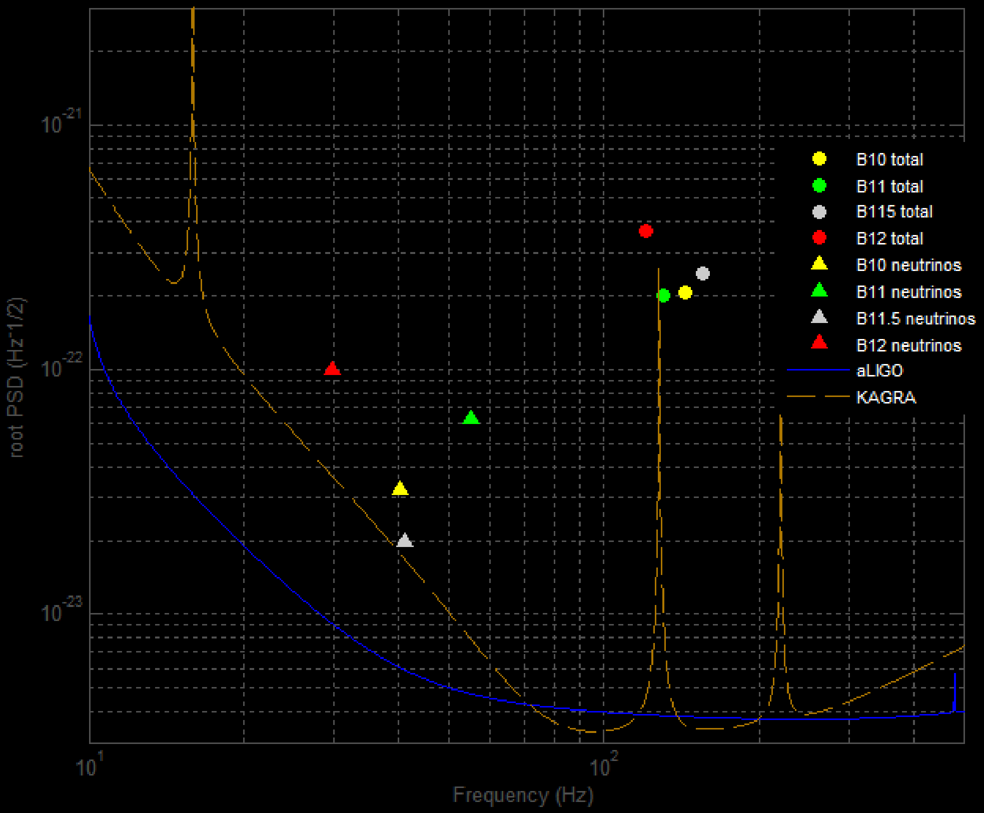


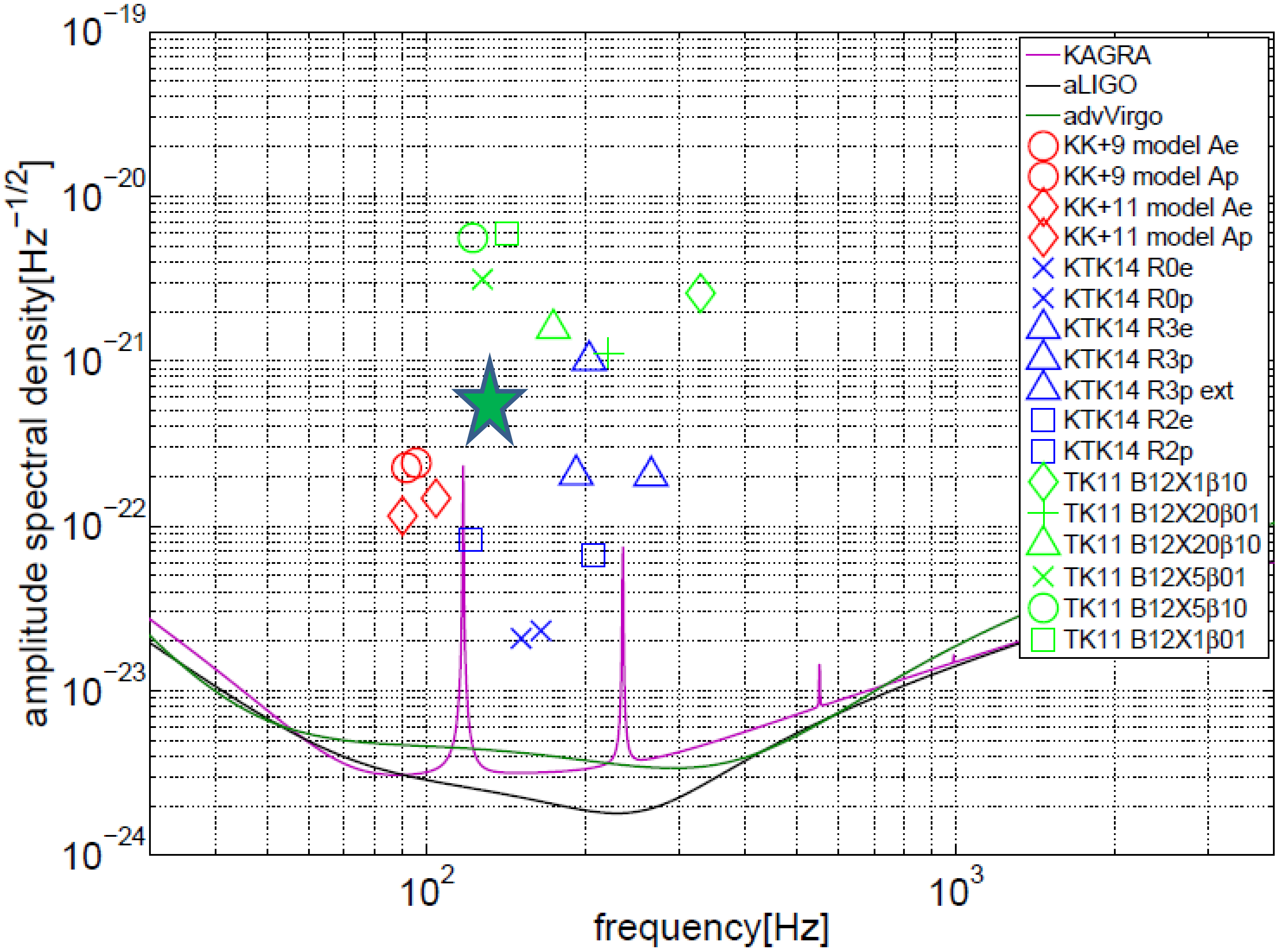
Detectability – **location of the signal predictions in the $f_c - \sqrt{S_h}$ plane**

- *GW emission of non-rotating magnetized models* -> mostly dominated by the matter asymmetric motions emits, in a frequency range of $100 - 200\ Hz$ in average

- **Neutrino contribution** is barely detectable despite its large amplitude

low frequency -> seismic noise

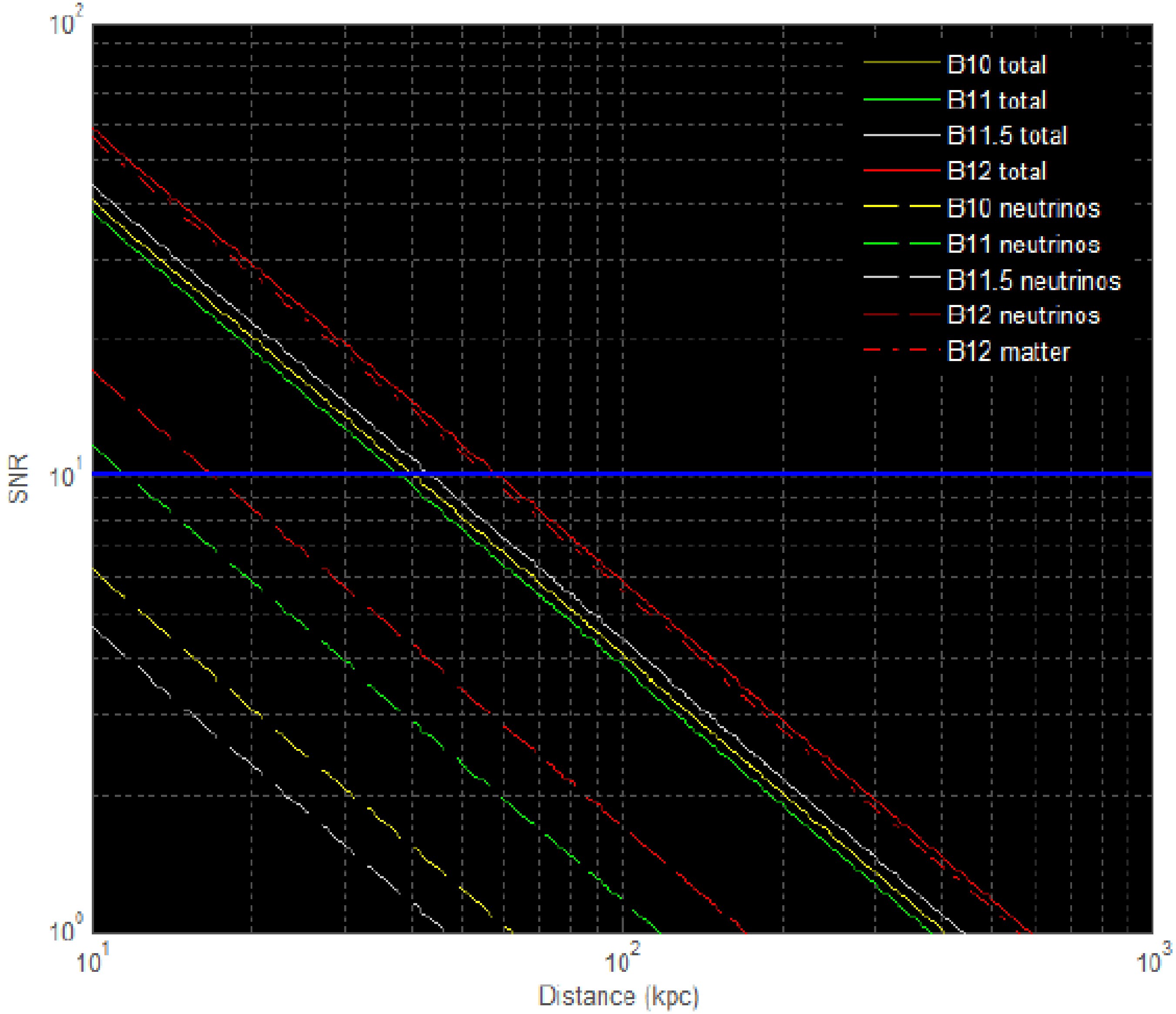




Kotake et al (2015)

Detectability- **signal to noise ratios (SNR)**

$$\rho^2 = \int_0^\infty d(\log f)\frac{|h_c(f)|^2}{f \cdot S_n(f)}$$



Conclusions

GW due to asymmetric mass motions

- ✓ Emitted mostly from PNS – PNS surface – cooling layer
- ✓ May provide information about the PNS properties
- ✓ GW amplitudes and frequencies -> level of activity of instabilities (convection + SASI)
- ✓ Signal endurance – absence of decreasing frequency mode + suspension of convection -> magnetic fields influence

GW due to neutrino asymmetric emission

- ✓ Hot bubble dynamics + structure
- ✓ Explosion morphology

- ✓ Detection limited for CCSN explosions in the Milky Way

-> the event rate up to 1-2 in 100 years

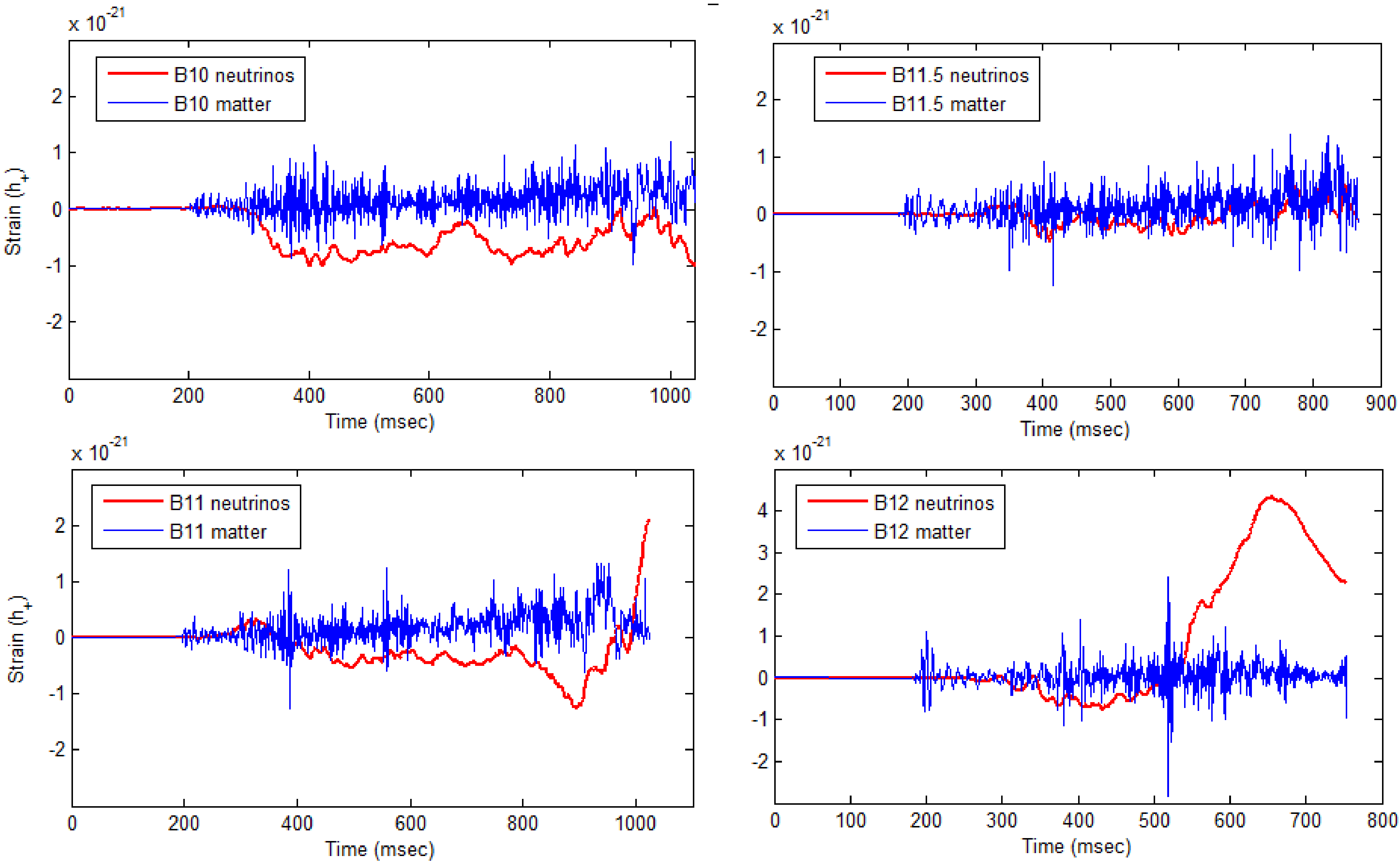
Not enough to get solid information about the explosion mechanism ->

-> wait for the third generation !

Supplemented material

Neutrino asymmetric emission

$$h_{ij}^{TT} = \frac{4G}{c^2 r}\int_{-\infty}^{t-\frac{r}{c}} dt' \int_{\Omega} d\Omega' \frac{(n_i n_j)^{TT}}{1 - cos\vartheta'}\frac{dL}{d\Omega'}(\vartheta', \varphi', t')$$



Suplemented material

Total signal + spectrograms

- $|h_+|_\nu > |h_+|_m$ but $f \sim 10 - 20\ Hz$
- Neutrino driven convection + SASI activity, in the hot bubble region may be modified by magnetic fields

-> Hot bubble characteristics (number , position, size, time evolution) = f (neutrino heating efficiency, SASI activity , magnetic fields)

- Cold matter accumulated to the core absorbs the neutrinos more efficiently than the hot matter in the polar regions

-> strong asymmetries to the ν's radiation field – GW emission -> may appear increasing positive tails (B11, B12) -> associate with large bubbles at late times

+ neutrino memory