# Gravitational waves and tests of general relativity

Mostly based on arXiv:1501.07274

Special thanks: E. Barausse, V. Cardoso, L. Gualtieri,
P. Pani, U. Sperhake, L. Stein, N. Wex, K. Yagi

See also: Gair+, 1212.5575; Yunes-Siemens, 1304.3473

Emanuele Berti, Mississippi/IST Lisbon/Caltech

8th Aegean Summer School, Rethymno, June 29 2015

INVESTIGADOR FCT
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## Not in this talk:

- Intro to GWs [van Holten]
- Binary pulsars [Kramer]
- GW sources [Glampedakis, Laguna, Shoemaker, Rezzolla, Stergioulas…]
- GW data analysis [Agathos, Shoemaker, Binetruy, Barsuglia…]
- Massive gravity [Bergshoeff, Babichev, Tsoukalas, Saridakis…]
- Primordial GWs/dark matter [Maggiore, Silk, Kuroyanagi…]

## Outline:

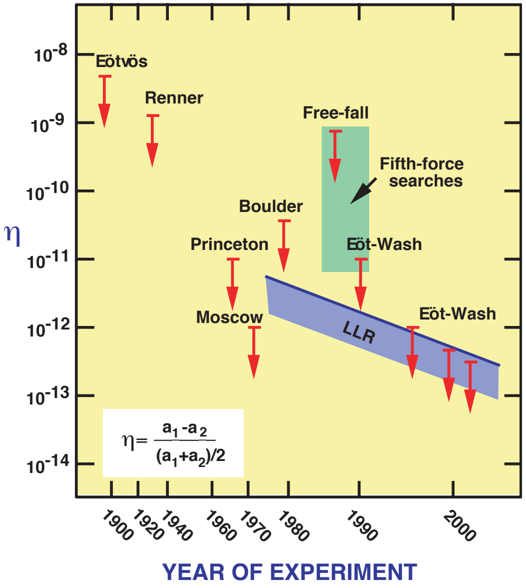
1) Weak- vs strong-field tests of GR
2) Black holes in GR and beyond
3) Compact stars in GR and beyond
4) Compact binaries in GR and beyond: signs of new gravitational physics?

# Weak-field tests
# vs
# strong-field tests

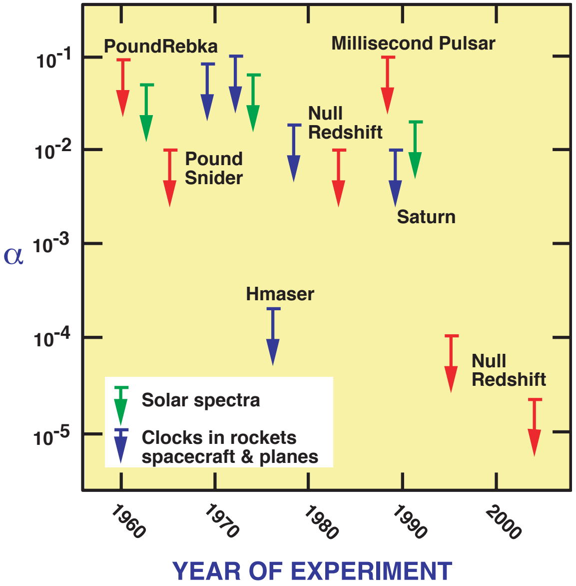
# The foundations of general relativity

[Will, 1403.7377]

TESTS OF THE
WEAK EQUIVALENCE PRINCIPLE

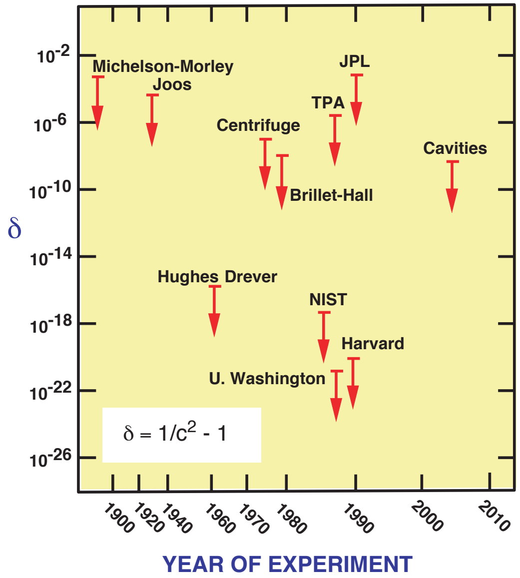


TESTS OF
LOCAL POSITION INVARIANCE



$$\Delta\nu/\nu = (1+\alpha)\Delta U/c^2$$

TESTS OF
LOCAL LORENTZ INVARIANCE

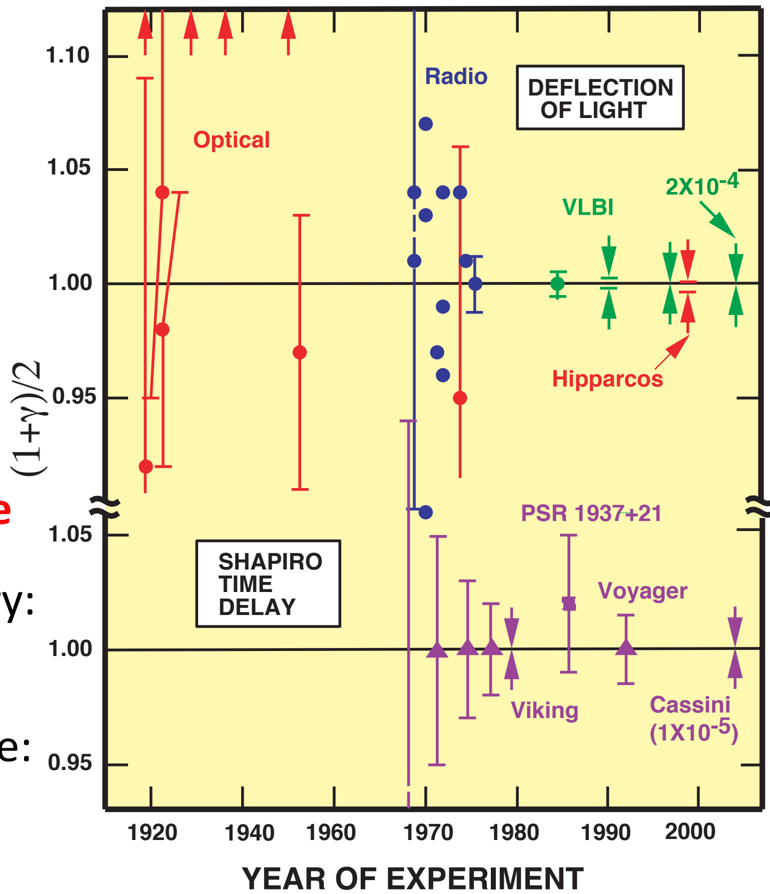


Weak Equivalence Principle+
Local Lorentz Invariance+
Local Position Invariance=
**Einstein's Equivalence Principle**

Implies gravity is a metric theory:
gravity is **spacetime curvature**

Best test of spacetime curvature:
**Cassini bound**

THE PARAMETER $(1+\gamma)/2$



# Modifications of GR: why bother?

(Circa 1919)
Journalist: *"Herr Einstein, what if the theory turned out to be wrong?"*
Einstein: *"I would feel sorry for the dear Lord. The theory is correct."*

(Circa 1970)
Chandrasekhar to his postdoc Clifford Will*:*
*"Why do you spend so much time testing GR? We know the theory is right."*

1) Theory: GR is not renormalizable
Becomes renormalizable by adding
higher-order curvature terms to the action

2) Experiments: dark matter, dark energy
Due to modified gravity?

Problem: GR is extremely well tested
"in between" these two regimes

"Short blanket problem!"

# Tests of general relativity – against what?

- Action principle
- Well-posed
- Testable predictions
- Cosmologically viable; allows for black holes, neutron stars

$$\begin{aligned}\mathcal{L} \quad &= f_0(\phi)R \\ &-\omega(\phi)\partial_a\phi\partial^a\phi - M(\phi) + \mathcal{L}_{\mathrm{mat}}\left[\Psi, A^2(\phi)g_{ab}\right] \\ &+f_1(\phi)(R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}) \\ &+f_2(\phi)R_{abcd}{}^{*}R^{abcd} \text{ + Lorentz violation, massive gravity...}\end{aligned}$$

Alternative theories usually:

Introduce more fields (scalars, vectors) or higher-curvature terms

**Need strong-field tests!** Challenge pillars of general relativity:

- Equivalence principle
- Lorentz invariance (Einstein-aether, TeVeS…)
- Parity conservation…

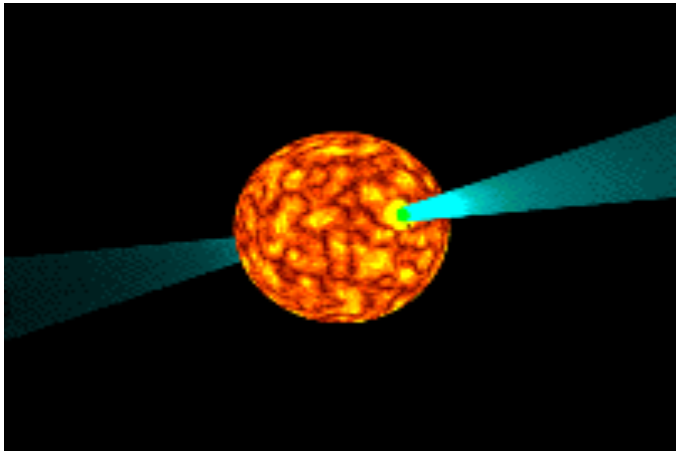
[Gair+,1212.5575; Clifton+, 1106.2476]

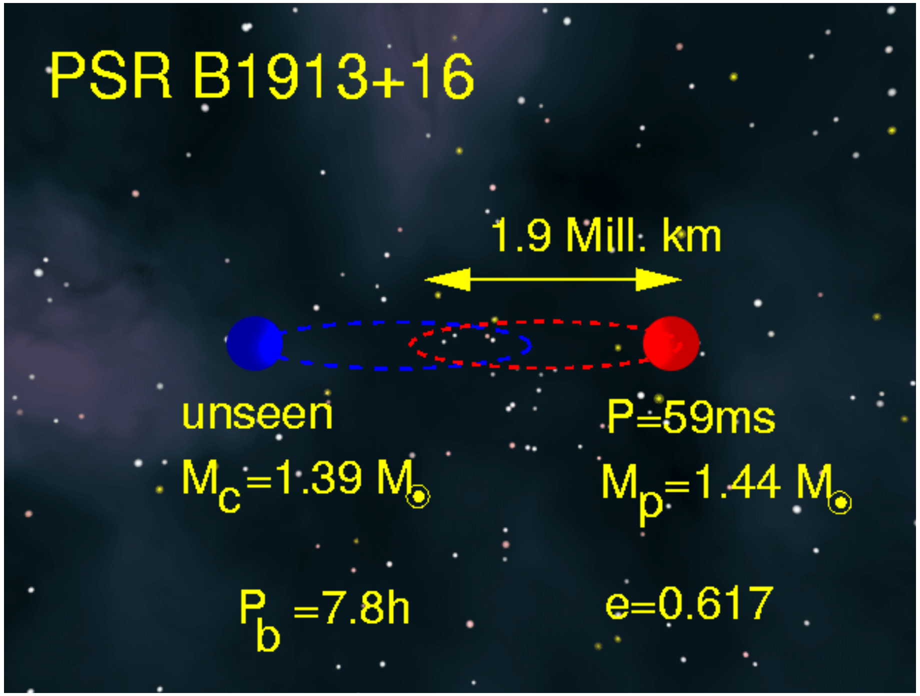
# What is “strong” gravity?

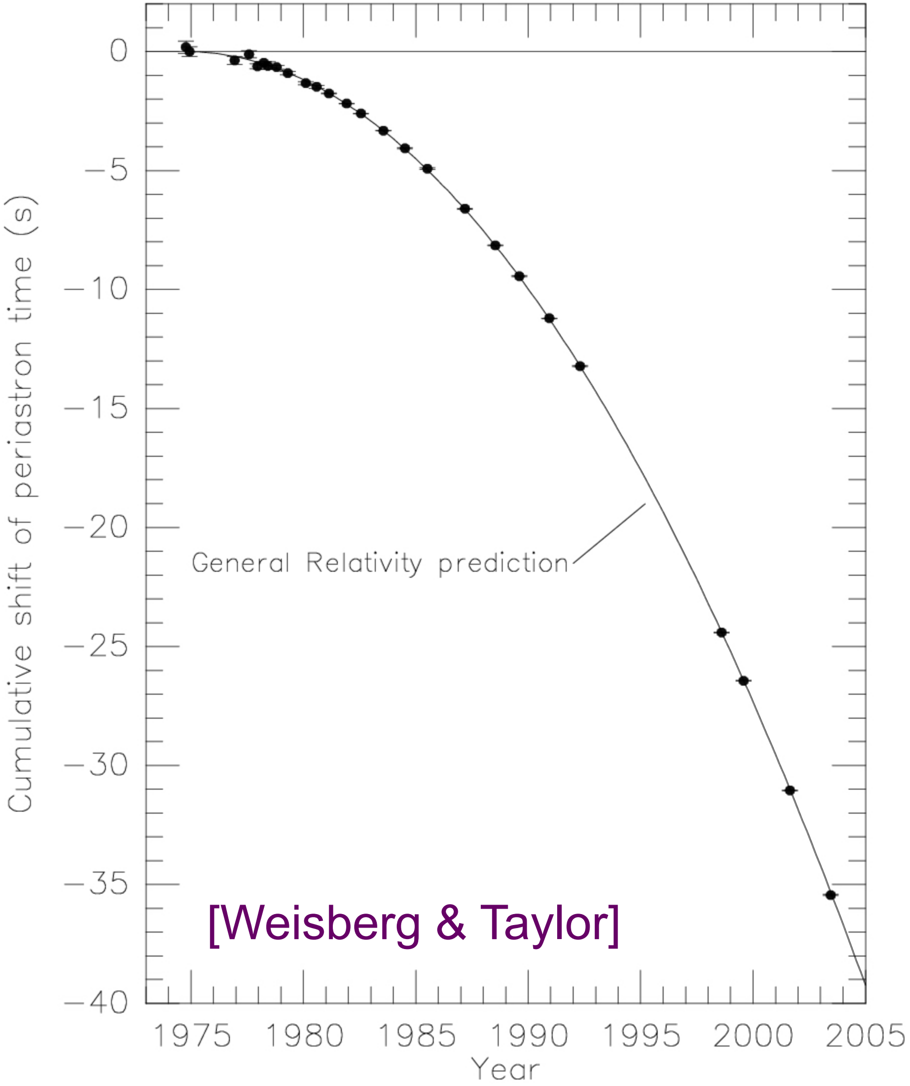
# Indirect detection of gravitational waves: binary pulsars

1993 Nobel Prize to Hulse and Taylor: discovery of the binary pulsar 1913+16

**Strongest test of GR**: PSR J0348+0432, P=2.46hr, **$v/c=2x10^{-3}$** [Antoniadis+, 1304.6875]

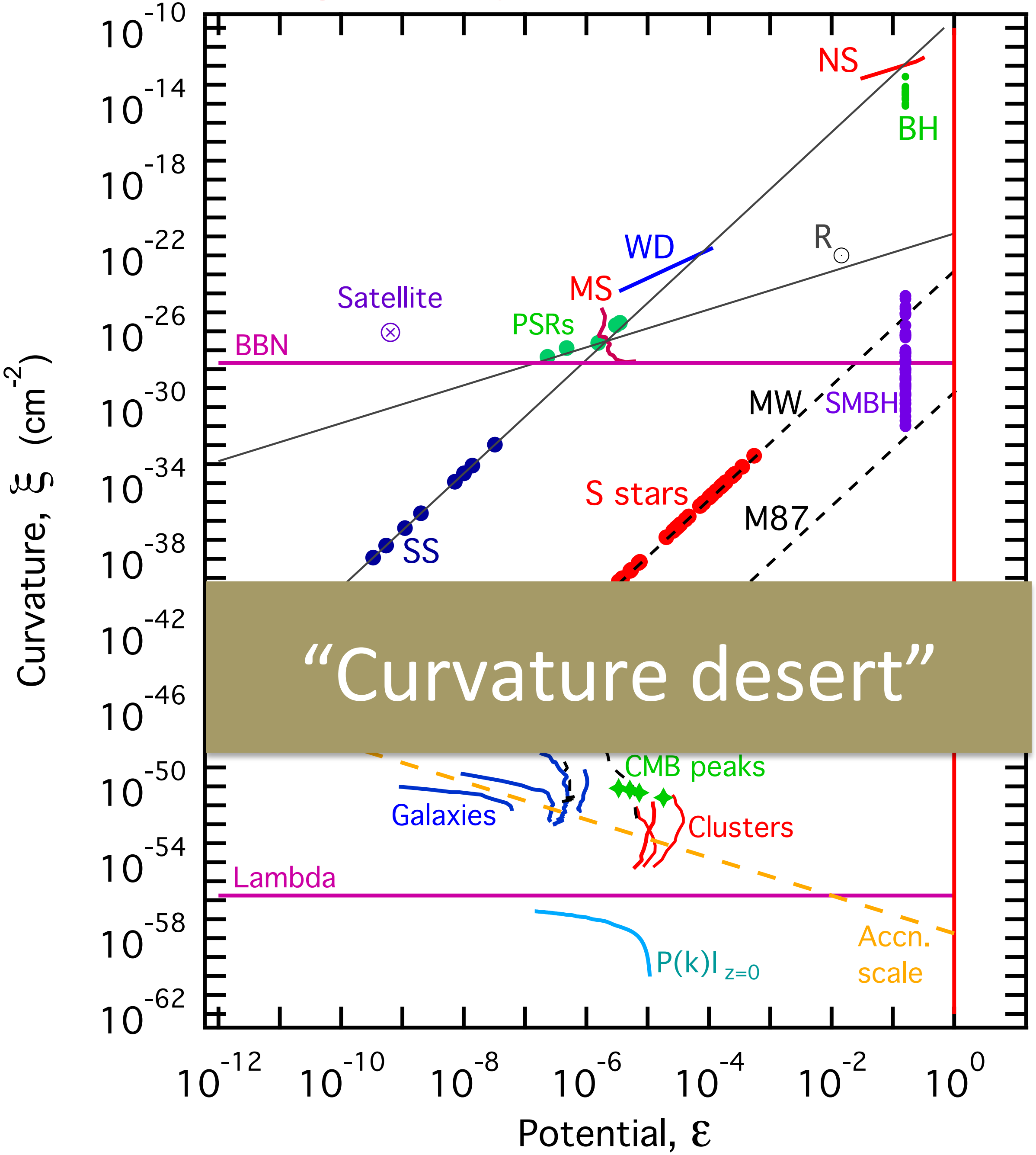






[Weisberg & Taylor]

# Strong-field probes: black holes and neutron stars



"Curvature desert"

**Gravitational field**

$$\epsilon \equiv \frac{GM}{rc^2}$$

**Curvature (Kretschmann scalar)**

$$\xi = \left(R^{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta}\right)^{1/2} = \sqrt{48}\,\frac{GM}{r^3c^2}$$

**$\ell_{\mathrm{Planck}}$=1.6x10$^{-33}$cm**

**$(\ell_{\mathrm{Planck}})^{-2}$= 10$^{66}$cm**

**Stellar mass black holes,**
**neutron stars:**

**$\ell_{\mathrm{BH}}$=10km=10$^{6}$cm**

**$(\ell_{\mathrm{Planck}}/\ell_{\mathrm{BH}})^{2}$=10$^{-78}$...**

**but:**

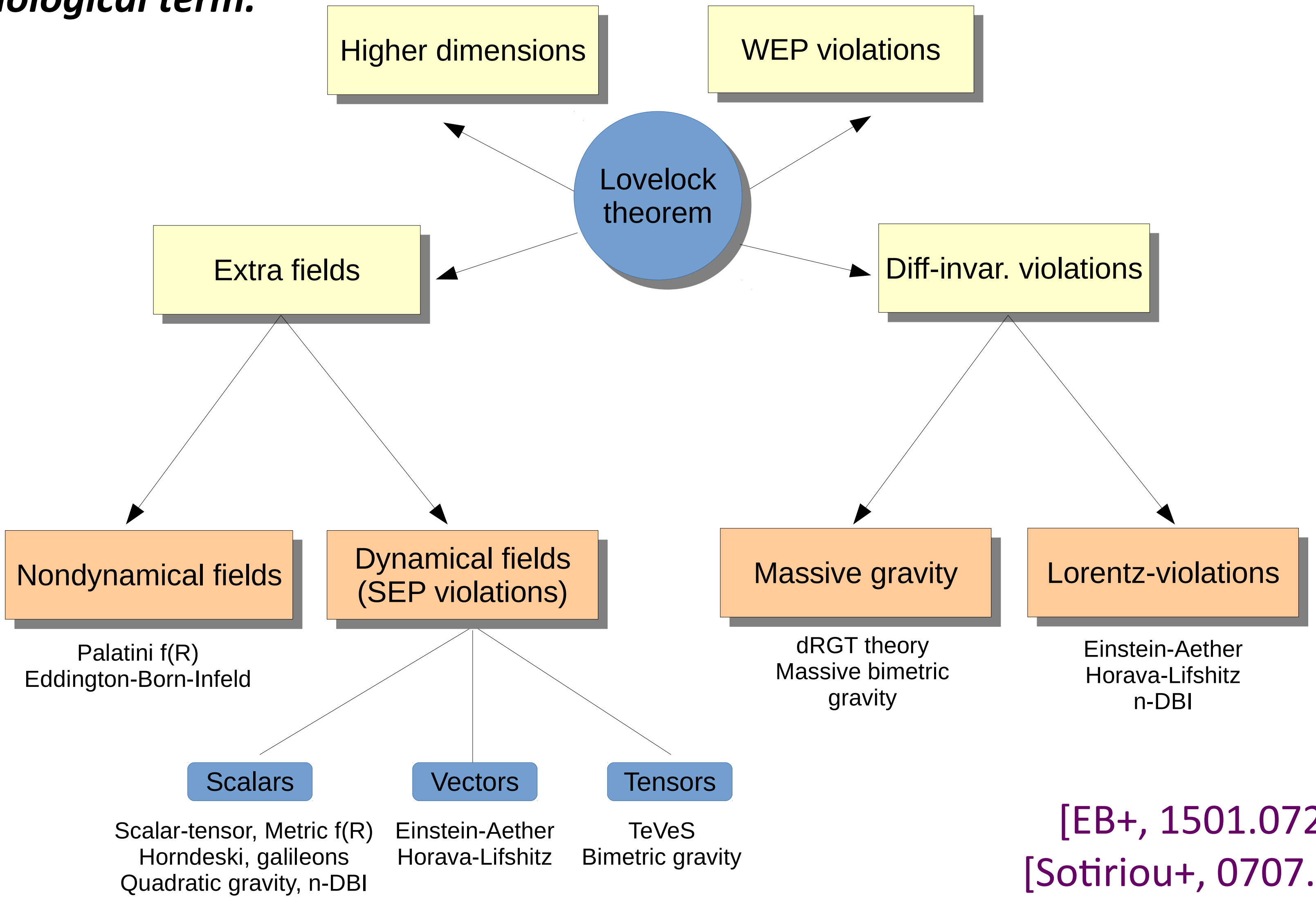
*** Modifications may not be Planck-suppressed;**

*** Untested extrapolations are always dangerous!**

Figure: [Baker+, 1412.3455]

# A guiding principle to modify GR: Lovelock's theorem

***"In four spacetime dimensions the only divergence-free symmetric rank-2 tensor constructed solely from the metric and its derivatives up to second differential order, and preserving diffeomorphism invariance, is the Einstein tensor plus a cosmological term."***



Lovelock theorem

Higher dimensions

WEP violations

Extra fields

Diff-invar. violations

Nondynamical fields

Palatini f(R)
Eddington-Born-Infeld

Dynamical fields (SEP violations)

Scalars

Scalar-tensor, Metric f(R)
Horndeski, galileons
Quadratic gravity, n-DBI

Vectors

Einstein-Aether
Horava-Lifshitz

Tensors

TeVeS
Bimetric gravity

Massive gravity

dRGT theory
Massive bimetric gravity

Lorentz-violations

Einstein-Aether
Horava-Lifshitz
n-DBI

[EB+, 1501.07274]
[Sotiriou+, 0707.2748]

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SKY & TELESCOPE

S&T TEST REPORT:
Vixen's Computerized
"Go To" Telescope Mount

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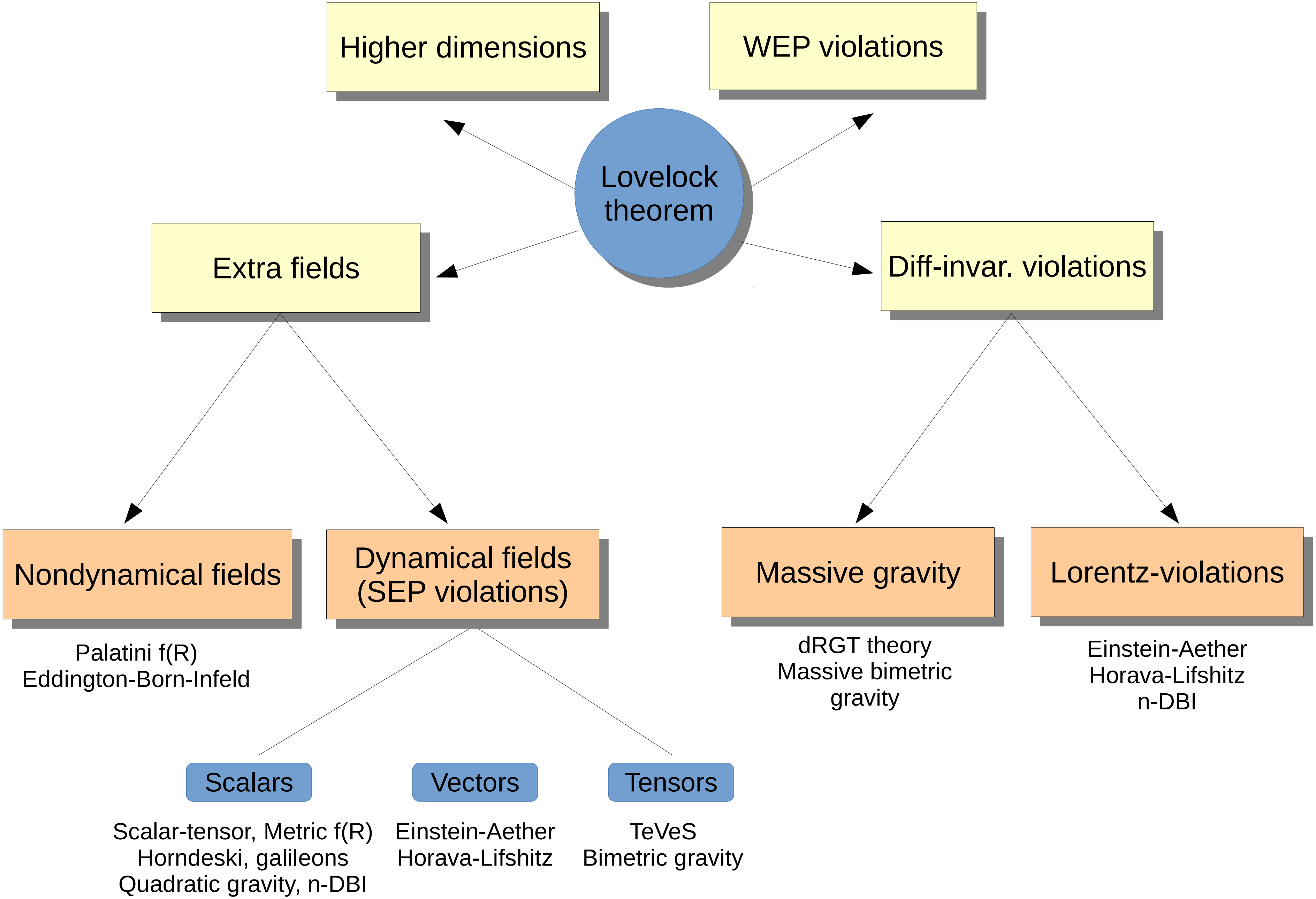
…and about what?

# Lovelock theorem as a map for the modified gravity zoo



Lovelock theorem

Higher dimensions

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TeVeS
Bimetric gravity

Massive gravity

dRGT theory
Massive bimetric gravity

Lorentz-violations

Einstein-Aether
Horava-Lifshitz
n-DBI

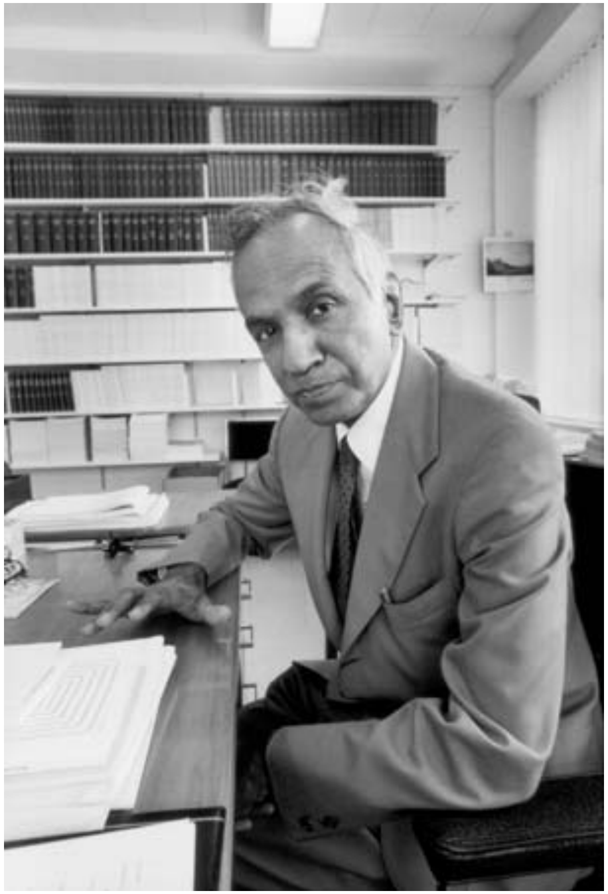
# Properties of (some) modified gravity theories

| Theory | Field content | Strong EP | Massless graviton | Lorentz symmetry | Linear $T_{\mu\nu}$ | Weak EP | Well-posed? | Weak-field constraints |
|---|---|---|---|---|---|---|---|---|
| Extra scalar field | | | | | | | | |
| Scalar-tensor | S | ✗ | ✓ | ✓ | ✓ | ✓ | ✓ [34] | [35–37] |
| Multiscalar | S | ✗ | ✓ | ✓ | ✓ | ✓ | ✓ [38] | [39] |
| Metric $f(R)$ | S | ✗ | ✓ | ✓ | ✓ | ✓ | ✓ [40, 41] | [42] |
| Quadratic gravity | | | | | | | | |
| Gauss-Bonnet | S | ✗ | ✓ | ✓ | ✓ | ✓ | ✓? | [43] |
| Chern-Simons | P | ✗ | ✓ | ✓ | ✓ | ✓ | ✗✓? [44] | [45] |
| Generic | S/P | ✗ | ✓ | ✓ | ✓ | ✓ | ? | |
| Horndeski | S | ✗ | ✓ | ✓ | ✓ | ✓ | ✓? | |
| Lorentz-violating | | | | | | | | |
| Æ-gravity | SV | ✗ | ✓ | ✗ | ✓ | ✓ | ✓? | [46–49] |
| Khronometric/ Hořava-Lifshitz | S | ✗ | ✓ | ✗ | ✓ | ✓ | ✓? | [48–51] |
| n-DBI | S | ✗ | ✓ | ✗ | ✓ | ✓ | ? | none ( [52]) |
| Massive gravity | | | | | | | | |
| dRGT/Bimetric | SVT | ✗ | ✗ | ✓ | ✓ | ✓ | ? | [17] |
| Galileon | S | ✗ | ✓ | ✓ | ✓ | ✓ | ✓? | [17, 53] |
| Nondynamical fields | | | | | | | | |
| Palatini $f(R)$ | – | ✓ | ✓ | ✓ | ✗ | ✓ | ✓ | none |
| Eddington-Born-Infeld | – | ✓ | ✓ | ✓ | ✗ | ✓ | ? | none |
| Others, not covered here | | | | | | | | |
| TeVeS | SVT | ✗ | ✓ | ✓ | ✓ | ✓ | ? | [37] |
| $f(R)\mathcal{L}_m$ | ? | ✗ | ✓ | ✓ | ✓ | ✗ | ? | |
| $f(T)$ | ? | ✗ | ✓ | ✗ | ✓ | ✓ | ? | [54] |

[EB+, arXiv:1501.07274]

# Black holes in GR and beyond

# No-hair theorems



**Black holes in GR are uniquely described by only two parameters – mass and spin**

[Carter, Israel, Hawking, Robinson, 1970s]

*"**In my entire scientific life**, extending over forty-five years, **the most shattering experience** has been the realization that an exact solution of Einstein's equations of general relativity, discovered by the New Zealand mathematician, Roy Kerr, provides the absolutely exact representation of untold numbers of massive black holes that populate the universe."*
(S. Chandrasekhar)

**Similar "no hair" theorems apply to modified gravity**

- ✓ Brans-Dicke [Hawking, Thorne & Dykla, Chase, Bekenstein]
- ✓ Multiple scalars [Heusler, gr-qc/9503053]
- ✓ Bergmann-Wagoner, f(R) [Sotiriou & Faraoni, 1109.6324]
- ✓ Higher-order curvature [Psaltis+, 0710.4564]

**…but beware:**
**same metric does not mean same dynamics!**

# Black holes in (some) modified gravity theories

| Theory | Solutions | Stability | Geodesics | Quadrupole |
|---|---|---|---|---|
| Extra scalar field | | | | |
| Scalar-tensor | $\equiv$GR [50–55] | [56–62] | – | – |
| Multiscalar/Complex scalar | $\supset$GR [51, 63, 64] | ? | ? | [63, 64] |
| Metric $f(R)$ | $\supset$GR [53, 54] | [65, 66] | ? | ? |
| Quadratic gravity | | | | |
| Gauss-Bonnet | NR [67–69]; SR [70, 71]; FR [72] | [73, 74] | SR [70, 75, 76]; FR [72] | [71, 77] |
| Chern-Simons | SR [78–80]; FR [81] | NR [82–85]; SR [74] | [69, 86] | [80] |
| Generic | SR [75] | ? | [75] | Eq. (3.12) |
| Horndeski | [87–89] | ? [90, 91] | ? | ? |
| Lorentz-violating | | | | |
| Æ-gravity | NR [92–94] | ? | [93, 94] | ? |
| Khronometric/ Hořava-Lifshitz | NR, SR [93–96] | ? [97] | [93, 94] | ? |
| n-DBI | NR [98, 99] | ? | ? | ? |
| Massive gravity | | | | |
| dRGT/Bimetric | $\supset$GR, NR [100–103] | [104–107] | ? | ? |
| Galileon | [108] | ? | ? | ? |
| Nondynamical fields | | | | |
| Palatini $f(R)$ | $\equiv$GR | – | – | – |
| Eddington-Born-Infeld | $\equiv$GR | – | – | – |

[EB+, arXiv:1501.07274]

# Black hole solutions

Most interesting targets for strong-gravity modifications? Maybe not!
Observations probe at most the horizon scale – curvatures not so large!

Scalar fields:

- Tensor-(multi)scalar and f(R) theories:
  black holes **same as in GR**…unless boundary conditions are nontrivial
- Quadratic gravity:
  Horndeski (nonrotating) BHs **same as in GR** [Hui-Nicolis, 1202.1296]
  Hairy solutions known numerically (EdGB), for slow rotation (dCS)
  Corrections suppressed by small coupling…but effective field theory?

Auxiliary fields:

- Black holes **same as in GR**

Lorentz-violating theories:

- "Universal horizons" with unclear stability properties…testable?

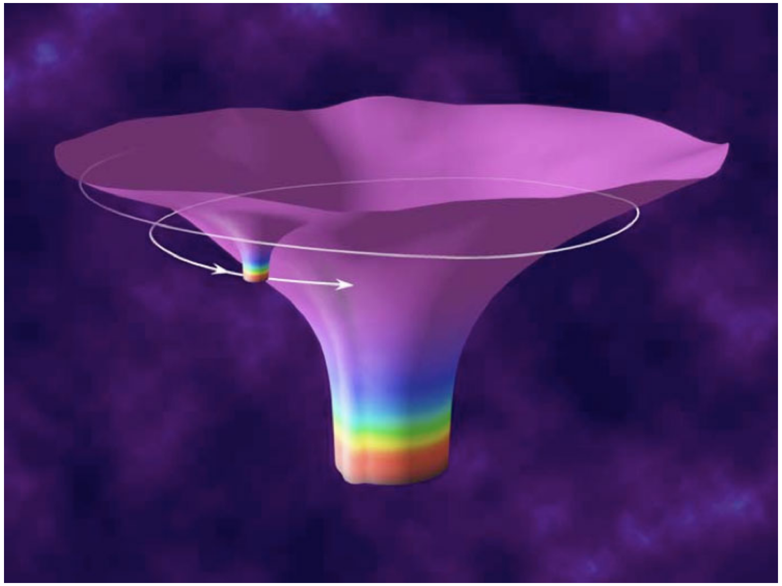
Massive gravity:

- Schwarzschild (and Kerr) solutions **unstable** for all (astrophysical) masses
  End-state of instability unclear…testable?

# Extreme mass-ratio inspirals

No-hair theorem: for black holes, (mass and current) multipoles depend only on mass and spin:

$$M_l+iS_l=(ia)^l M^{l+1}$$



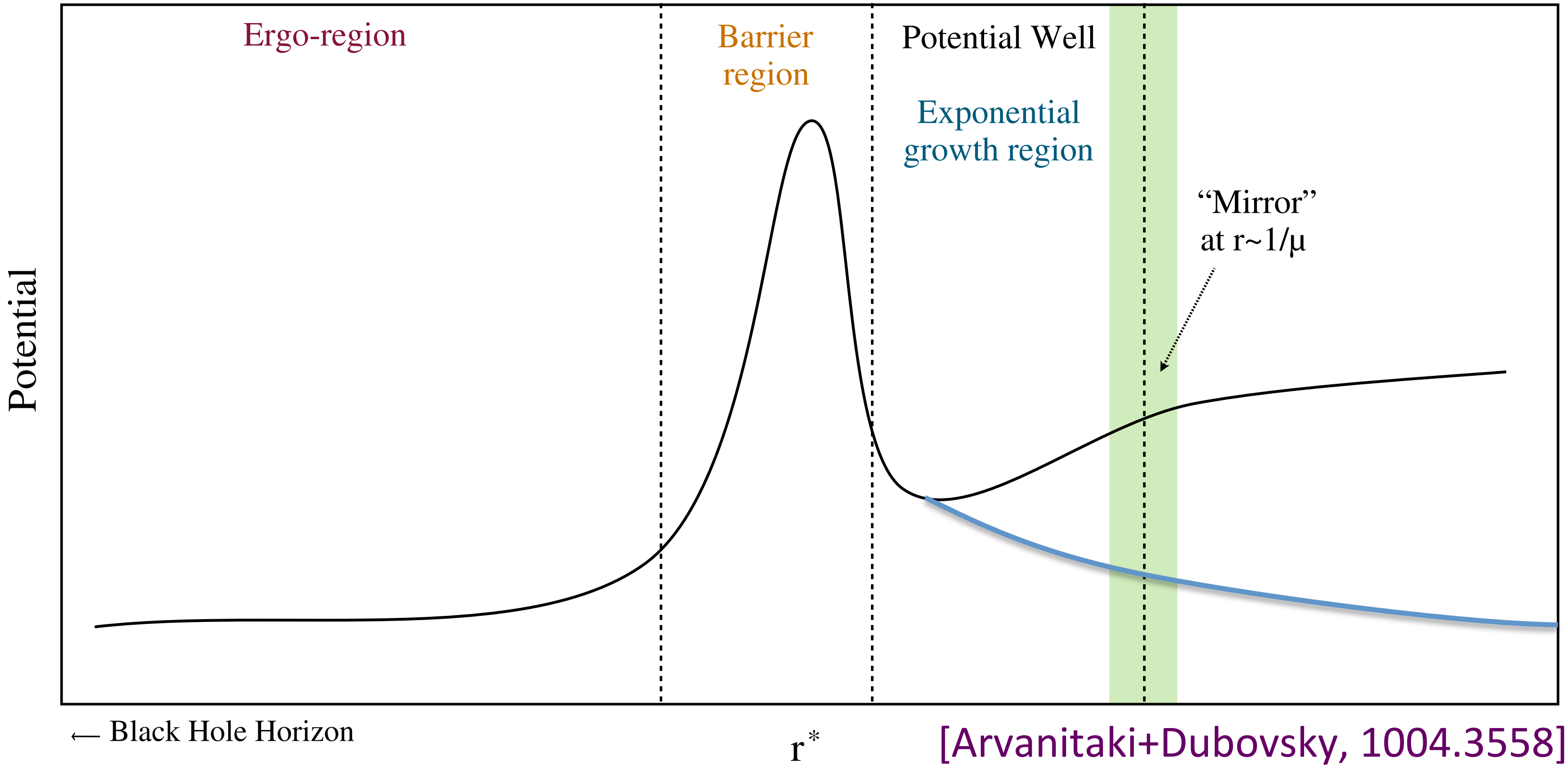
eLISA-like detectors may observe ~tens of 1-10$M_{sun}$ BHs (or NSs) spiralling into ~$10^6 M_{sun}$ BHs.

**~$10^4$-$10^5$ cycles, periapsis/orbital plane precession.** Payoff:

- ✓ map Kerr spacetime, probe nature of central object
- ✓ measure masses of stellar-mass BHs/SMBHs
- ✓ test GR (NS inspirals emit dipole radiation in scalar-tensor theories)
- ✓ smoking gun of alternative theories?

[Amaro-Seoane+, astro-ph/0703495]

# Wave scattering in rotating black holes



[Arvanitaki+Dubovsky, 1004.3558]

## Quasinormal modes:

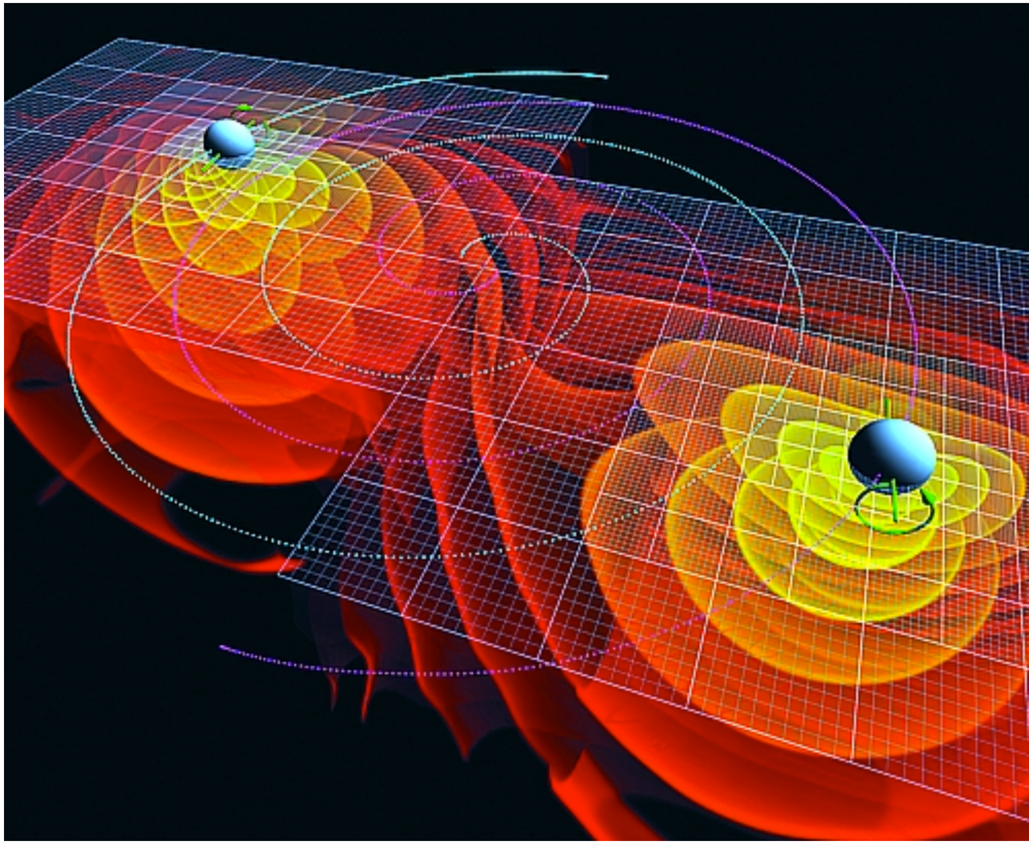
- ☐ Ingoing waves at the horizon, outgoing waves at infinity
- ☐ Discrete spectrum of damped exponentials ("ringdown") [EB++, 0905.2975]

## Massive scalar field:

- ☐ Superradiance: black hole bomb when $0 < \omega < m\Omega_H$
- ☐ Hydrogen-like, unstable bound states [Detweiler, Zouros+Eardley…]

# Massive black hole mergers: black hole spectroscopy

[Visualization: NASA Goddard]



- In GR, black holes oscillations are a set of complex-frequency modes determined only by mass and spin

- One mode: **(M,a)**
  Any other mode frequency:
  **No-hair theorem test**
  Relative mode amplitudes:
  pre-merger parameters
  [Kamaretsos+,Gossan+]

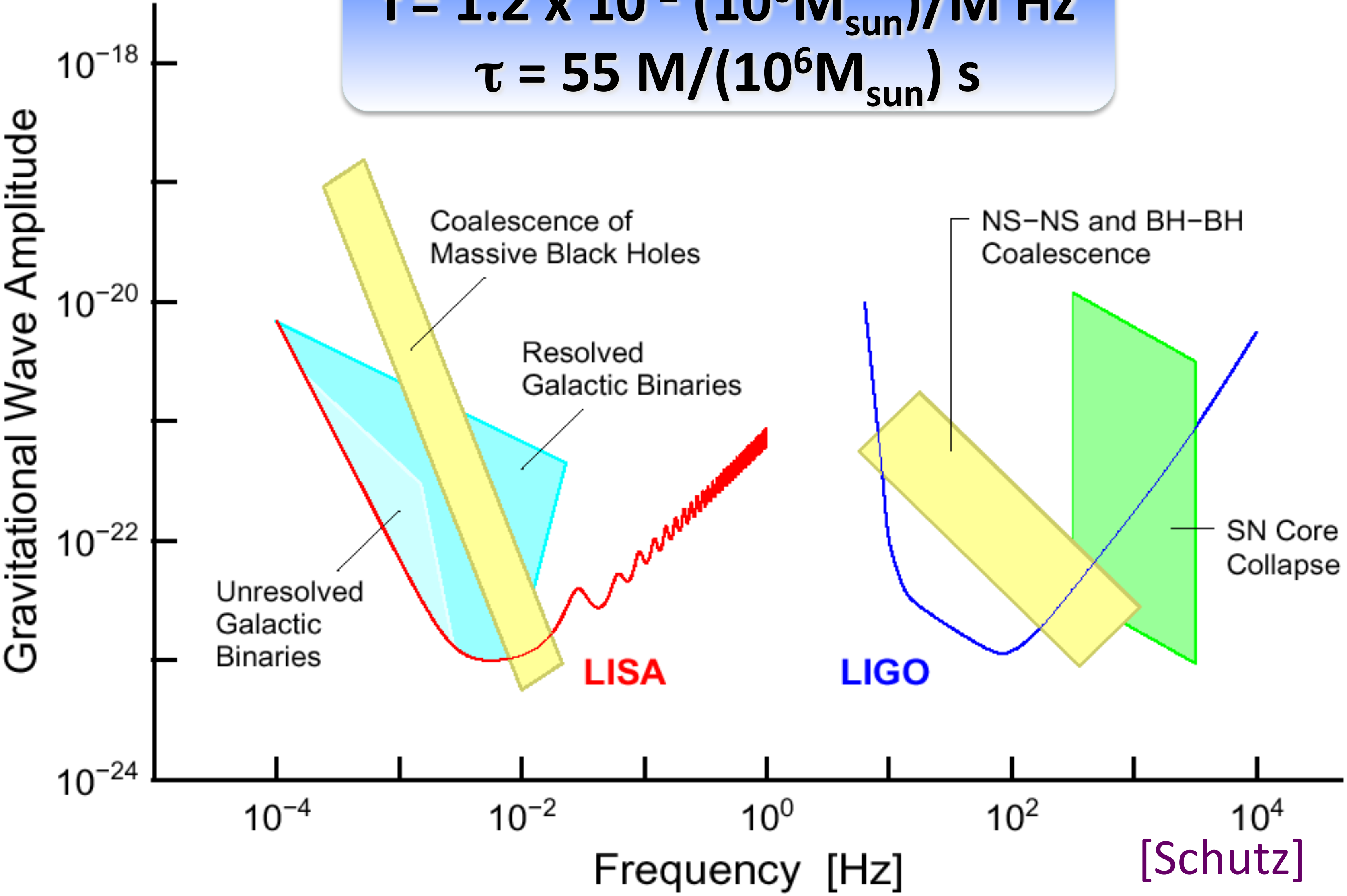
- Feasibility depends on SNR:
  **Need SNR>30** [EB+, 2005/07]
  1) Noise $S(f_{QNM})$
  2) Signal $h \sim E^{1/2}$, $\mathbf{E=\varepsilon_{rd} M}$
  $\varepsilon_{rd} \sim 0.01(4\eta)^2$ for comparable-mass mergers, $\eta = m_1 m_2/(m_1+m_2)^2$

$$f = 1.2 \times 10^{-2}\,(10^6 M_{sun})/M \text{ Hz}$$
$$\tau = 55\, M/(10^6 M_{sun}) \text{ s}$$

# (e)LISA vs. (Ad)LIGO

$f = 1.2 \times 10^{-2}\,(10^6 M_{sun})/M$ Hz

$\tau = 55\, M/(10^6 M_{sun})$ s



[Schutz]

SNR=h/S: S comparable, $h \sim \eta M^{1/2}$

# Compact stars in GR and beyond

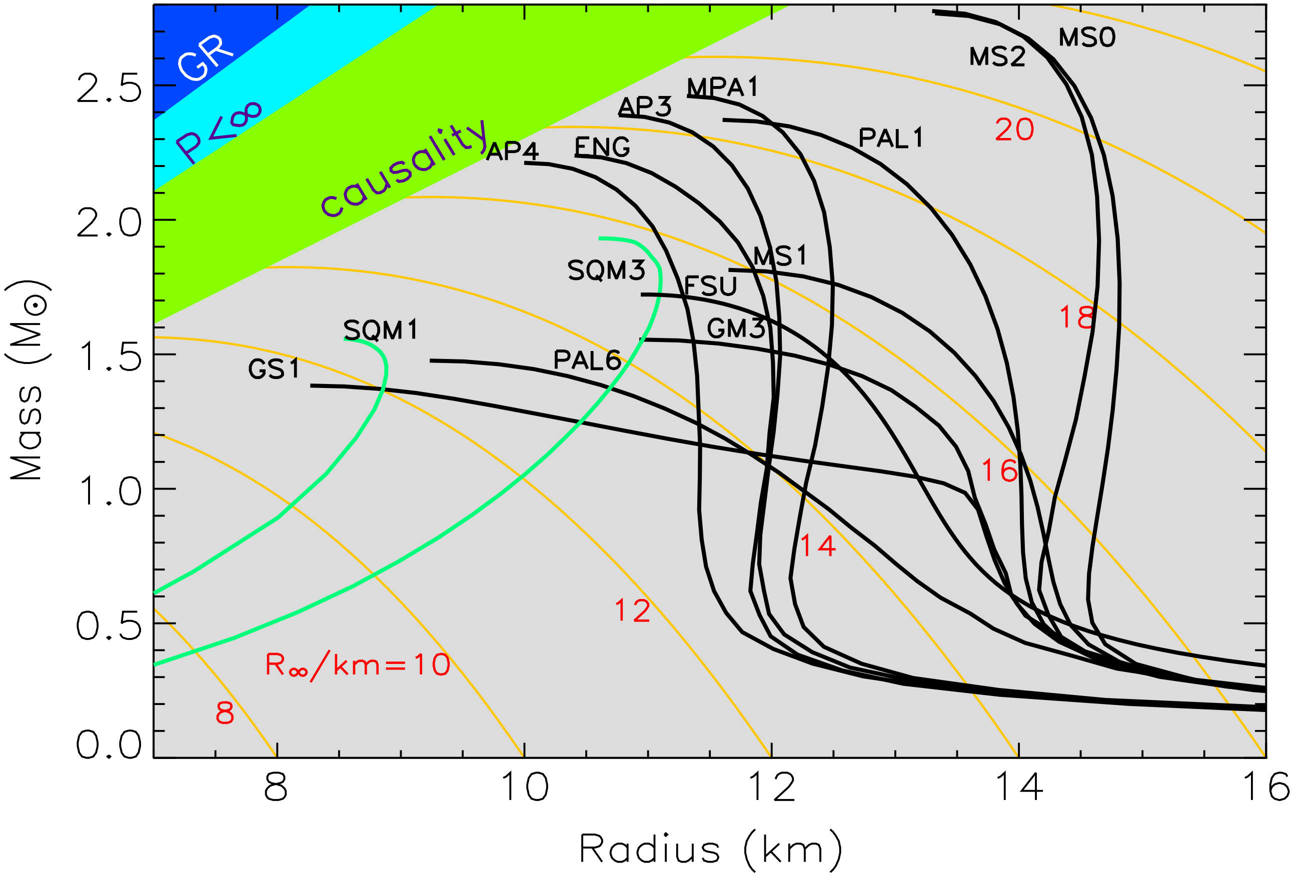
# “Internal” tests: neutron stars

Strong-field signatures:
high curvatures in interior, spontaneous scalarization…

**Observables?** Consider the Hartle-Thorne expansion in $W/(M/R^3)^{1/2}$

Zero order in rotation: **M(R)** - mass-radius relation

Radii hard to measure, both in binaries and in isolated systems



# Neutron stars as EOS probes

Strong-field signatures:
high curvatures in interior, spontaneous scalarization…

**Observables?** Consider the Hartle-Thorne expansion in $\Omega/(M/R^3)^{1/2}$

Zero order in rotation: **M(R)** - mass-radius relation
Radii hard to measure, both in binaries and in isolated systems

Corrections:

**Moment of inertia I** may be measurable in binary pulsars
[Lattimer-Schutz, Kramer, Wex…]

**Tidal "Love number"** may be measurable in binary inspirals
[Mora-Will, Berti-Iyer-Will, Read, Hinderer, Lang, Binnington, Poisson, Vines, Damour, Nagar, Bernuzzi, Villain, Favata, Yagi, Yunes…]

**Quadrupole Q or higher-order moments**: light curves or QPOs
[Laarakkers-Poisson, Berti-Stergioulas, BWMB, Baubock+, Pappas…]

**Stellar oscillations**

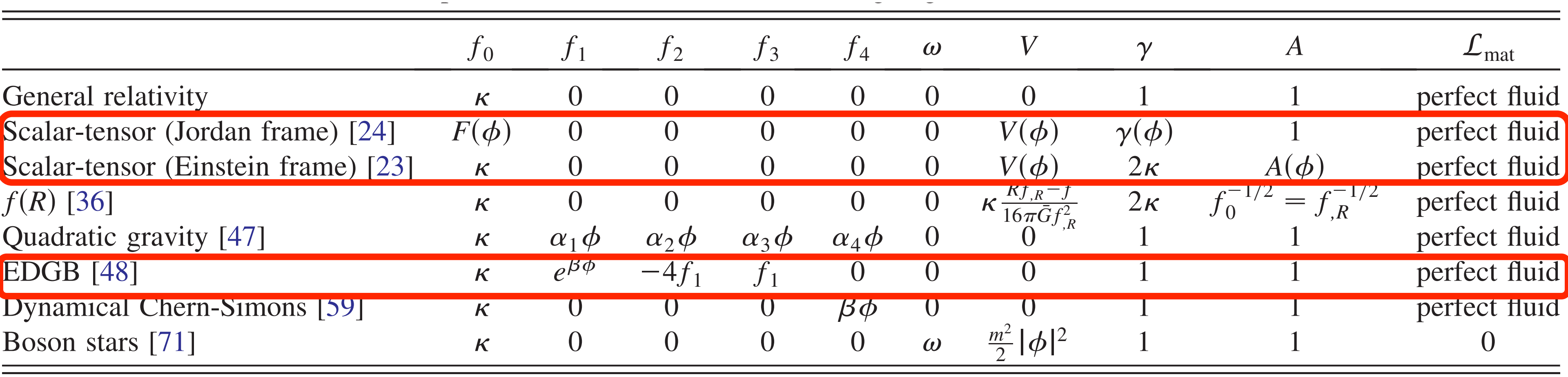
# Neutron stars in (some) modified gravity theories

| Theory | Structure |  |  | Collapse | Sensitivities | Stability | Geodesics |
|---|---|---|---|---|---|---|---|
|  | NR | SR | FR |  |  |  |  |
| Extra scalar field |  |  |  |  |  |  |  |
| Scalar-Tensor | [109–114] | [112, 115, 116] | [117–119] | [120–127] | [128] | [129–139] | [118, 140] |
| Multiscalar | ? | ? | ? | ? | ? | ? | ? |
| Metric $f(R)$ | [141–153] | [154] | [155] | [156, 157] | ? | [158, 159] | ? |
| Quadratic gravity |  |  |  |  |  |  |  |
| Gauss-Bonnet | [160] | [160] | [77] | ? | ? | ? | ? |
| Chern-Simons | $\equiv$ GR | [25, 40, 161–163] | ? | ? | [162] | ? | ? |
| Horndeski | ? | ? | ? | ? | ? | ? | ? |
| Lorentz-violating |  |  |  |  |  |  |  |
| Æ-gravity | [164, 165] | ? | ? | [166] | [43, 44] | [158] | ? |
| Khronometric/ Hořava-Lifshitz | [167] | ? | ? | ? | [43, 44] | ? | ? |
| n-DBI | ? | ? | ? | ? | ? | ? | ? |
| Massive gravity |  |  |  |  |  |  |  |
| dRGT/Bimetric | [168, 169] | ? | ? | ? | ? | ? | ? |
| Galileon | [170] | [170] | ? | [171, 172] | ? | ? | ? |
| Nondynamical fields |  |  |  |  |  |  |  |
| Palatini $f(R)$ | [173–177] | ? | ? | ? | – | ? | ? |
| Eddington-Born-Infeld | [178–184] | [178, 179] | ? | [179] | – | [185, 186] | ? |

[EB+, arXiv:1501.07274]

# A “theory of theories” of sufficient generality

$$\mathcal{L} = f_0(|\phi|)R - \gamma(|\phi|)\partial_a\phi^*\partial^a\phi - V(|\phi|) + f_1(|\phi|)R^2 + f_2(|\phi|)R_{ab}R^{ab} + f_3(|\phi|)R_{abcd}R^{abcd} + f_4(|\phi|)R_{abcd}{}^*R^{abcd} + \mathcal{L}_{\text{mat}}[\Psi, A^2(|\phi|)g_{ab}],$$



| | $f_0$ | $f_1$ | $f_2$ | $f_3$ | $f_4$ | $\omega$ | $V$ | $\gamma$ | $A$ | $\mathcal{L}_{\text{mat}}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| General relativity | $\kappa$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | perfect fluid |
| Scalar-tensor (Jordan frame) [24] | $F(\phi)$ | 0 | 0 | 0 | 0 | 0 | $V(\phi)$ | $\gamma(\phi)$ | 1 | perfect fluid |
| Scalar-tensor (Einstein frame) [23] | $\kappa$ | 0 | 0 | 0 | 0 | 0 | $V(\phi)$ | $2\kappa$ | $A(\phi)$ | perfect fluid |
| $f(R)$ [36] | $\kappa$ | 0 | 0 | 0 | 0 | 0 | $\kappa\frac{Rf_{,R}-f}{16\pi\bar{G}f_{,R}^2}$ | $2\kappa$ | $f_0{}^{-1/2} = f_{,R}{}^{-1/2}$ | perfect fluid |
| Quadratic gravity [47] | $\kappa$ | $\alpha_1\phi$ | $\alpha_2\phi$ | $\alpha_3\phi$ | $\alpha_4\phi$ | 0 | 0 | 1 | 1 | perfect fluid |
| EDGB [48] | $\kappa$ | $e^{\beta\phi}$ | $-4f_1$ | $f_1$ | 0 | 0 | 0 | 1 | 1 | perfect fluid |
| Dynamical Chern-Simons [59] | $\kappa$ | 0 | 0 | 0 | $\beta\phi$ | 0 | 0 | 1 | 1 | perfect fluid |
| Boson stars [71] | $\kappa$ | 0 | 0 | 0 | 0 | $\omega$ | $\frac{m^2}{2}\|\phi\|^2$ | 1 | 1 | 0 |

[Yunes & Stein, 1101.2921]
[Pani+, 1109.0928]

# Scalar-tensor theory and spontaneous scalarization

- Action (in the Einstein frame):

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g^\star}\left[R^\star - 2g^{\star\mu\nu}\left(\partial_\mu\varphi\right)\left(\partial_\nu\varphi\right) - V(\varphi)\right] + S_M[\Psi, A^2(\varphi)g^\star_{\mu\nu}]$$

- Gravity-matter coupling:

$$\alpha(\varphi) \equiv d(\ln A(\varphi))/d\varphi$$

$$\alpha(\varphi) = \alpha_0 + \beta_0(\varphi - \varphi_0) + \dots$$

- Field equations:

$$G^\star_{\mu\nu} = 2\left(\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g^\star_{\mu\nu}\partial_\sigma\varphi\partial^\sigma\varphi\right) - \frac{1}{2}g^\star_{\mu\nu}V(\varphi) + 8\pi T^\star_{\mu\nu}\,,$$

$$\Box_{g^\star}\varphi = -4\pi\alpha(\varphi)T^\star + \frac{1}{4}\frac{dV}{d\varphi}\,,$$

[Damour+Esposito-Farese, PRL 70, 2220 (1993); PRD 54, 1474 (1996)]

# Scalarization threshold: a back-of-the-envelope derivation

$$\Box_{g^*}\varphi = -4\pi\alpha(\varphi)T^*$$

$$\alpha(\varphi) = \beta_0\varphi$$

$$-T^* = A^4(\epsilon^* - 3p^*) \sim \frac{3}{4\pi R^2}\frac{m}{R} \quad \text{for} \quad r < R$$

$$\nabla^2\varphi = \text{sign}(\beta_0)\left[\frac{3|\beta_0|(m/R)}{R^2}\right]\varphi = \text{sign}(\beta_0)\kappa^2\varphi$$

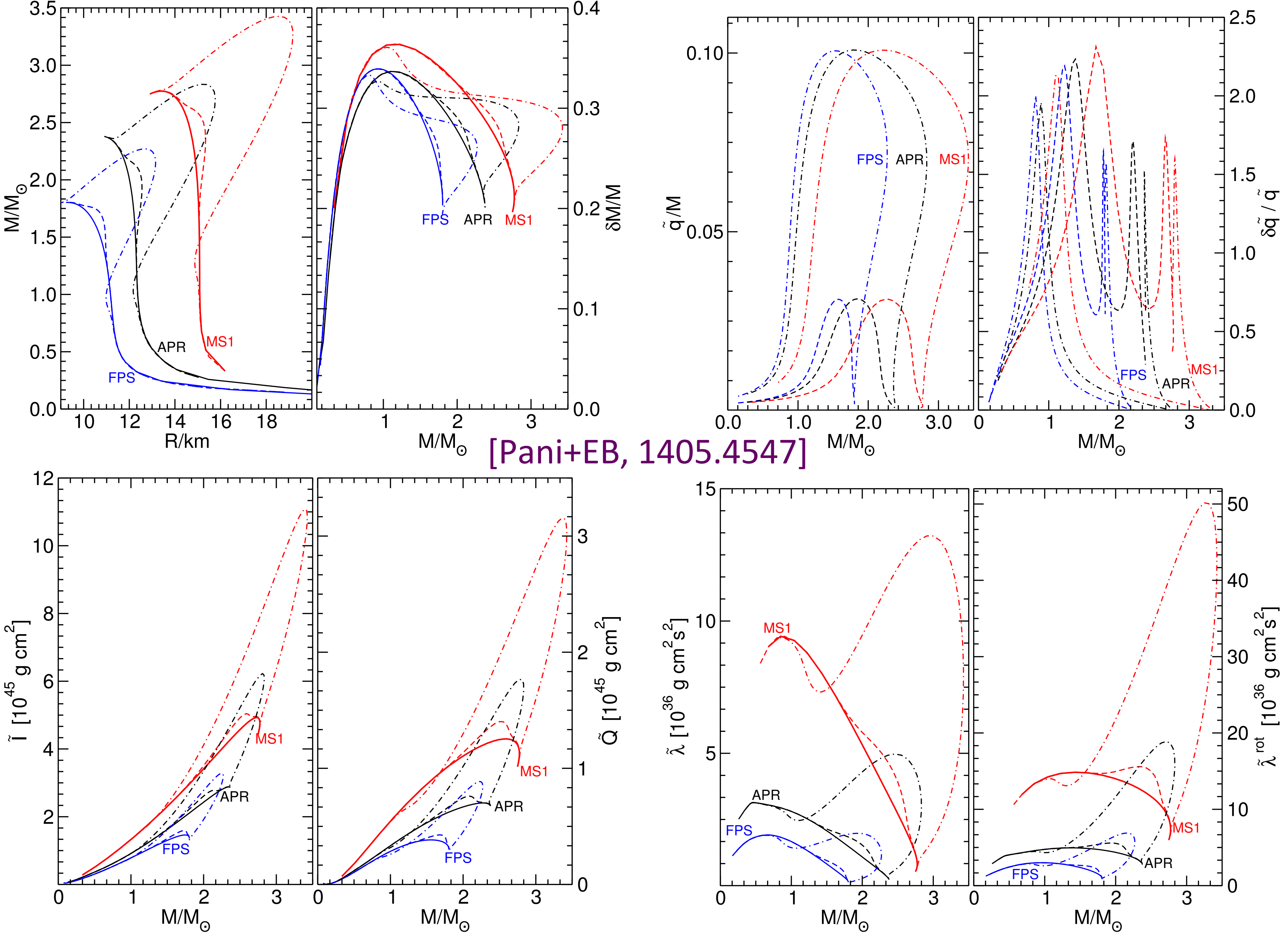
$$\beta_0 < 0 \Longrightarrow \varphi_{\text{inside}} = \varphi_{\text{c}}\frac{\sin(\kappa r)}{\kappa r}$$

$$\varphi_{\text{c}} = \frac{\varphi_0}{\cos(\kappa R)} \gg \varphi_0 \qquad \boxed{\kappa R \sim \pi/2}$$

$$\boxed{m/R \sim 0.2 \Longrightarrow \beta \sim -4}$$

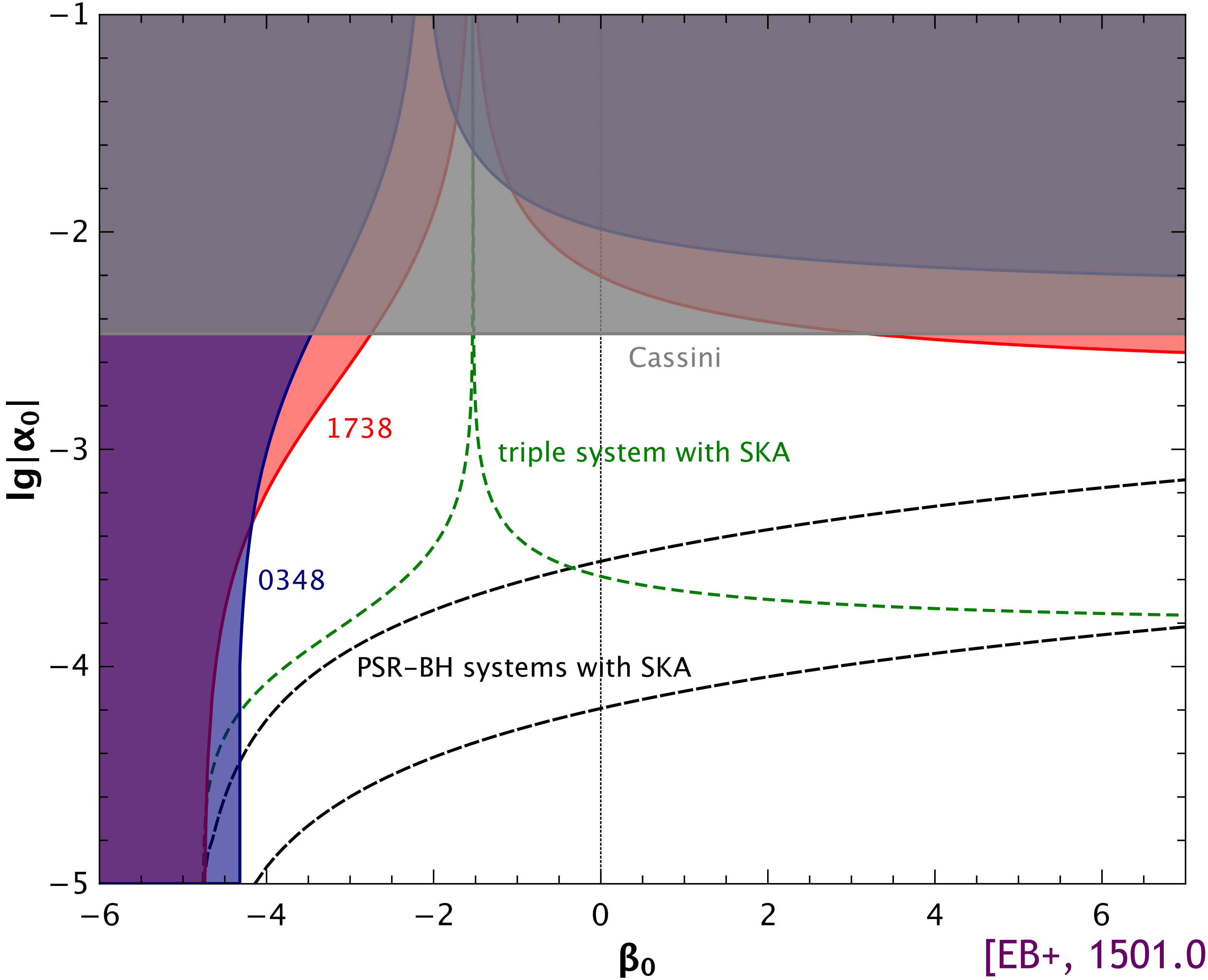
[Damour+Esposito-Farese, PRL 70, 2220 (1993)]

# Scalar-tensor theory: spontaneous scalarization



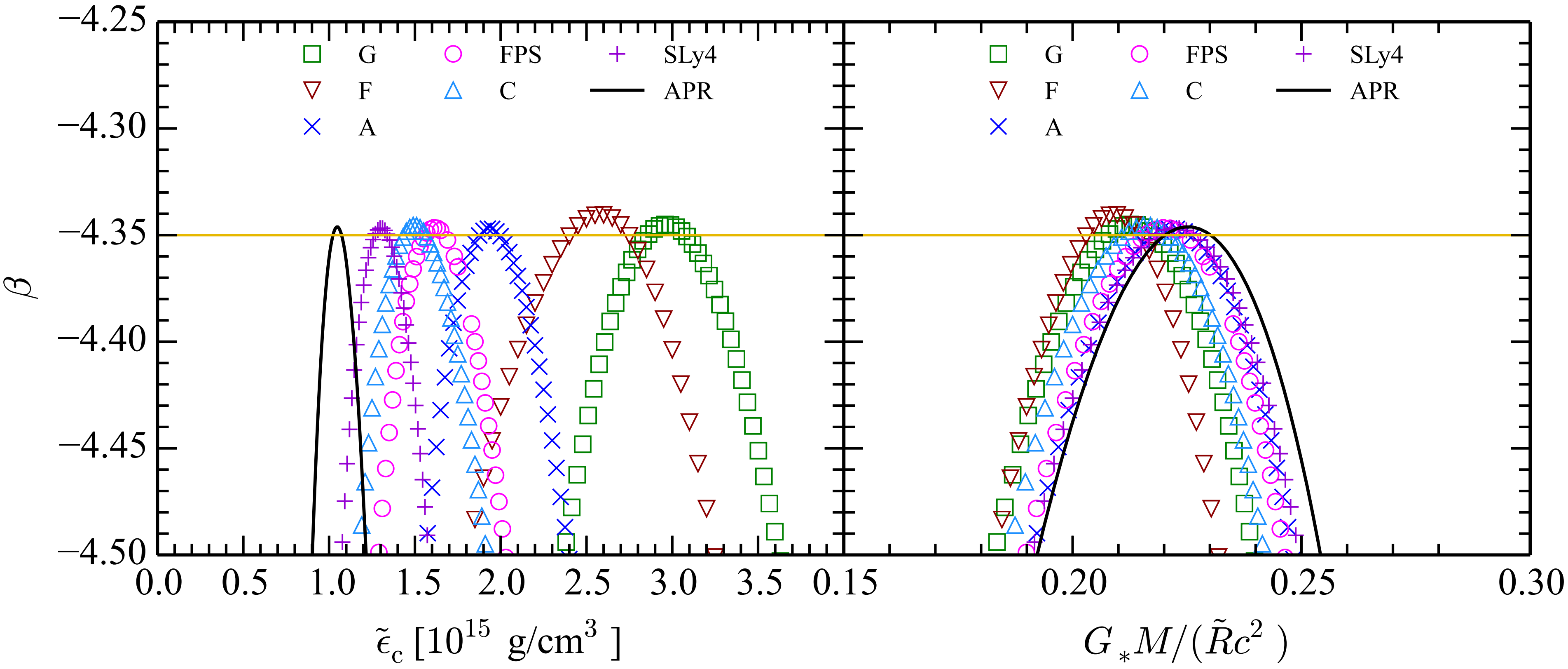
[Pani+EB, 1405.4547]

# Binary pulsar bounds on spontaneous scalarization



[EB+, 1501.07274]

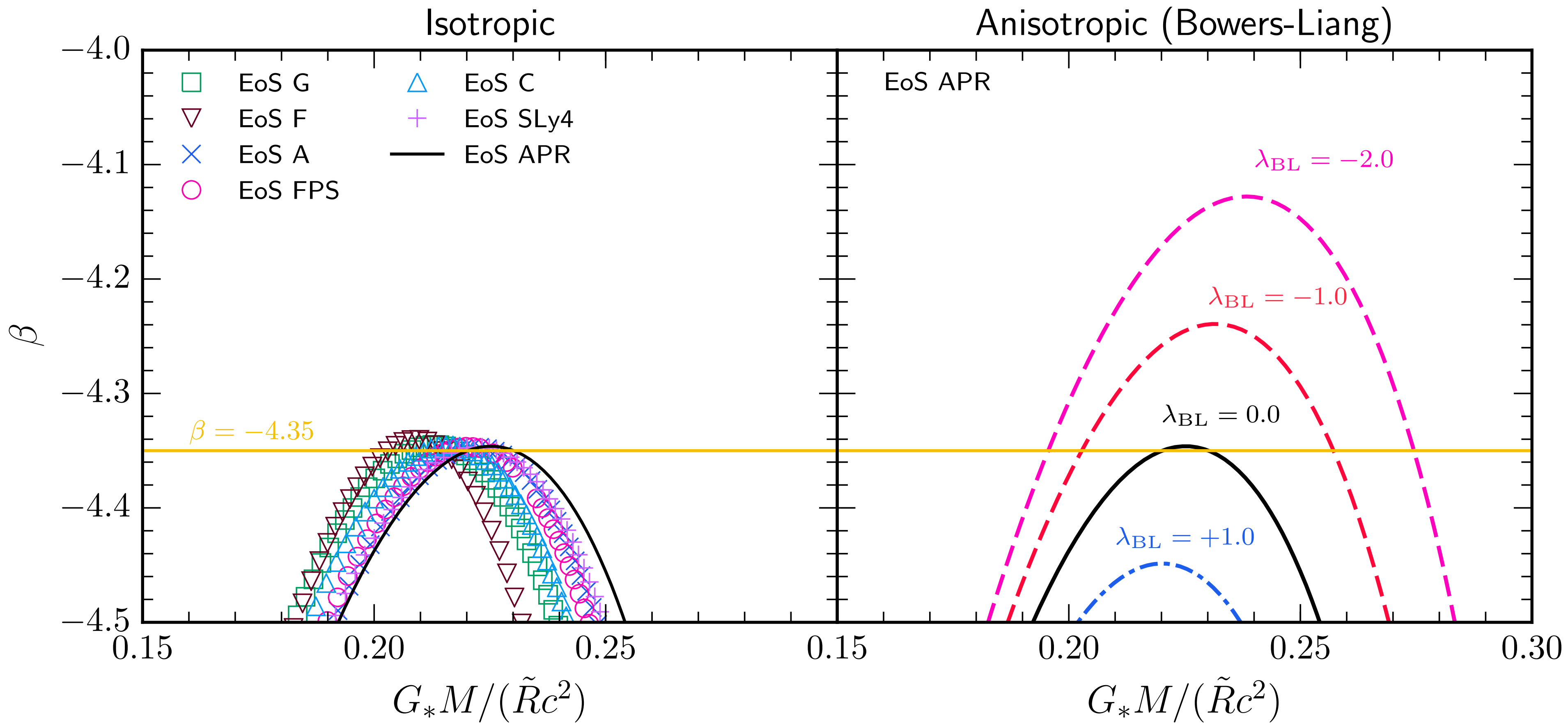
# EOS dependence of scalarization threshold



Dependence of β on EOS is too mild
for ordinary models of high-density nuclear matter

[Silva+, 1411.6286]

# Anisotropy dependence of scalarization threshold



λ = degree of anisotropy

Aside: in the limit λ=-2π the Bowers-Liang model for constant-density stars has R=2M – and the low-order multipole moments also tend to those of Kerr!

[Yagi-Yunes+, 1502.04131]

# Multiscalarization?

## Damour/Esposito-Farese, CQG 9, 2093 (1992)

$$S = \frac{1}{4\pi G_\star} \int d^4x \sqrt{-g} \left( \frac{R}{4} - \frac{1}{2} g^{\mu\nu} \gamma_{AB}(\phi) \partial_\mu \phi^A \, \partial_\nu \phi^B - B(\phi) \right)$$

$$+ S_{\rm m}[A^2(\phi) g_{\mu\nu}; \Psi] \,,$$

## Two-scalar model:

$$S = \frac{1}{4\pi G_\star} \int d^4x \sqrt{-g} \left( \frac{R}{4} - g^{\mu\nu} \gamma(\varphi, \bar\varphi) \nabla_\mu \bar\varphi \, \nabla_\nu \varphi - B(\varphi, \bar\varphi) \right)$$

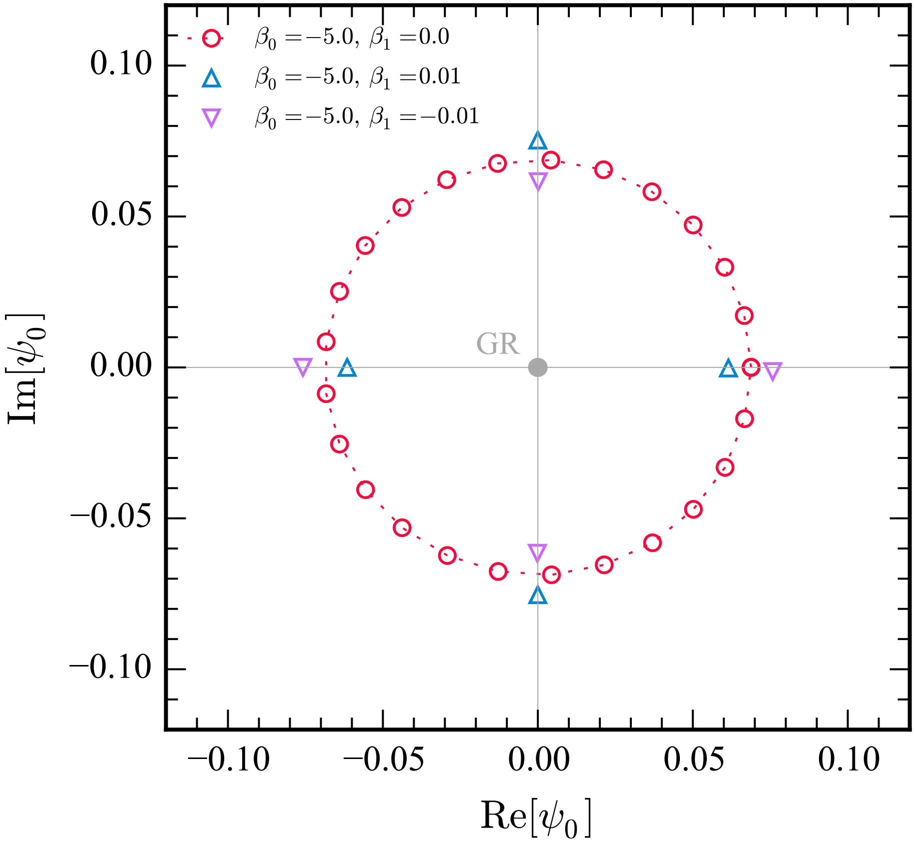
$$+ S_{\rm m}[A^2(\varphi, \bar\varphi) g_{\mu\nu}; \Psi] \,,$$

$$\gamma(\varphi, \bar\varphi) = \frac{1}{2} \left( 1 + \frac{\bar\varphi \varphi}{4\mathfrak{r}^2} \right)^{-2} \qquad \psi = \varphi e^{\mathrm{i}\theta_1 \breve{/} 2}$$

$$\log A(\psi, \bar\psi) = \alpha\psi + \bar\alpha\bar\psi + \frac{1}{2}\beta_0 \psi\bar\psi + \frac{1}{4}\beta_1 \psi^2 + \frac{1}{4}\beta_1 \bar\psi^2 + \ldots \,.$$

# α=0: symmetry breaking

$$\log A(\psi, \bar{\psi}) = \alpha\psi + \bar{\alpha}\bar{\psi} + \frac{1}{2}\beta_0\psi\bar{\psi} + \frac{1}{4}\beta_1\psi^2 + \frac{1}{4}\beta_1\bar{\psi}^2 + \ldots .$$

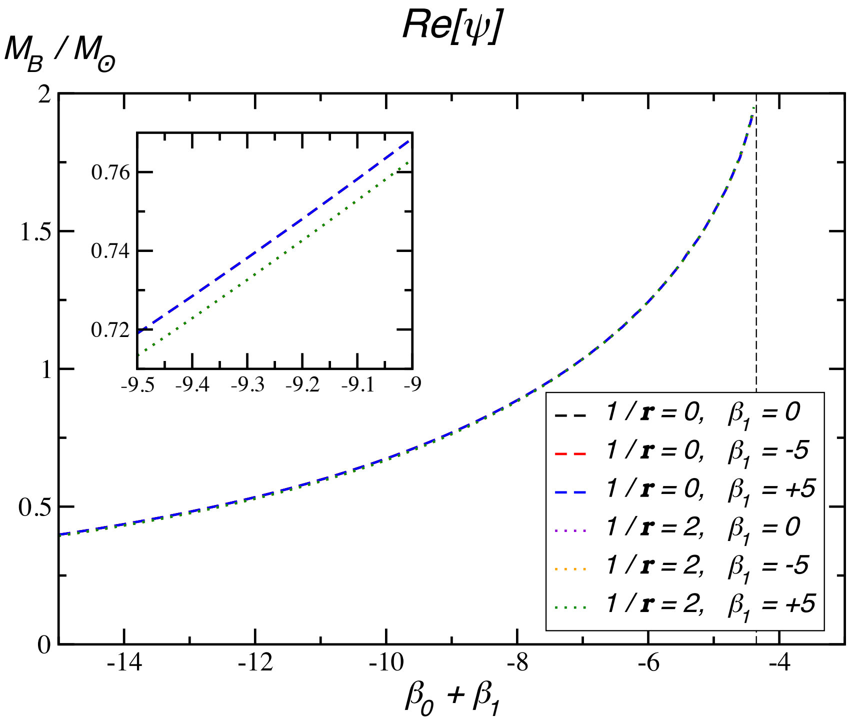


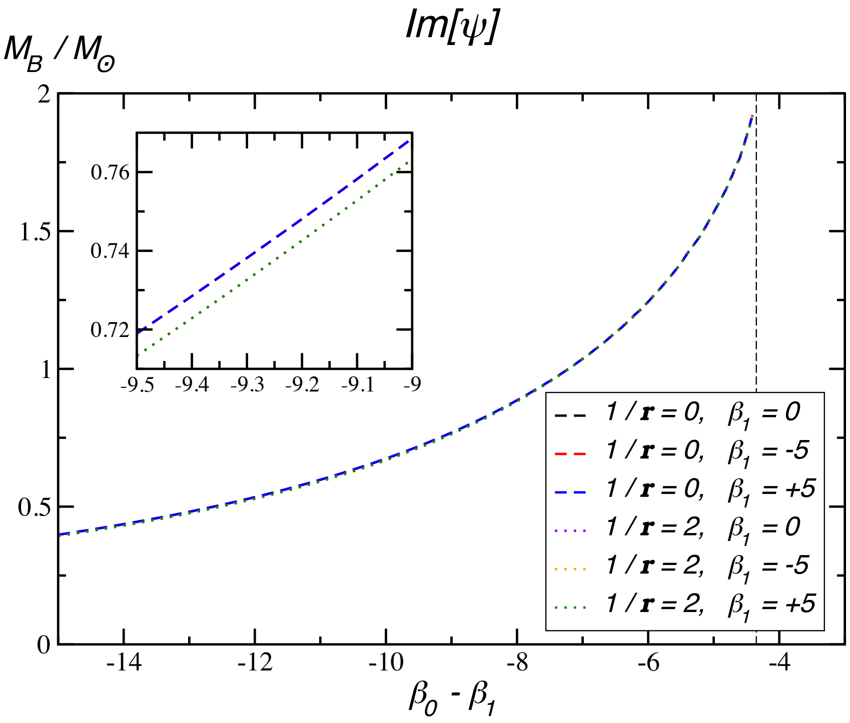
$$A(\psi, \bar{\psi}) = \exp\left(\frac{1}{2}\beta_0\psi\bar{\psi}\right)$$

$$\log A(\psi, \bar{\psi}) = \frac{1}{2}\left[(\beta_0 + \beta_1)\mathrm{Re}[\psi]^2 + (\beta_0 - \beta_1)\mathrm{Im}[\psi]^2\right]$$

# “Independent” biscalarization

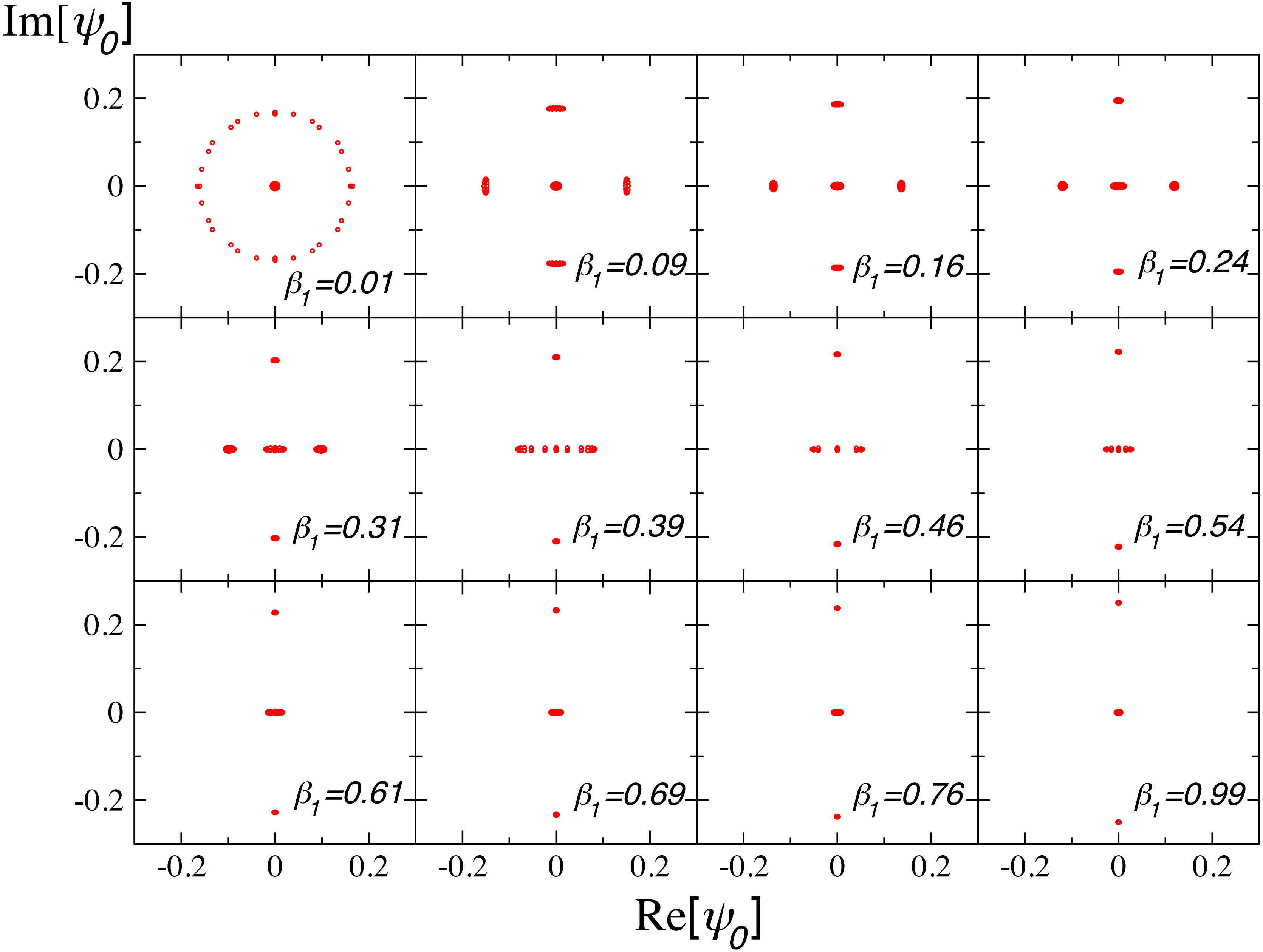
$$\log A(\psi,\bar{\psi}) = \frac{1}{2}\left[(\beta_0+\beta_1)\mathrm{Re}[\psi]^2 + (\beta_0-\beta_1)\mathrm{Im}[\psi]^2\right]$$


Re[ψ]


Im[ψ]

# “True” biscalarization

$|\alpha| = 0.001, \quad 1/\mathbf{r} = 0$



# More neutron star solutions in modified gravity

Main advantage over BHs: NSs probe how matter couples to gravity
Curvatures can be very large inside neutron stars

Scalar fields:

- Scalarization tightly constrained by binary pulsars
  - Torsional oscillations (flares) would not yield new constraints
  - …but anisotropic matter or multiscalarization
- f(R) theories: do compact stars even exist? Dynamical studies?

Quadratic gravity:
In EdGB, NS constraints already tighter than BH constraints!

Auxiliary fields:

- Surface singularities may rule out theory! degeneracy with EOS

Lorentz-violating theories:

- Only nonrotating stars

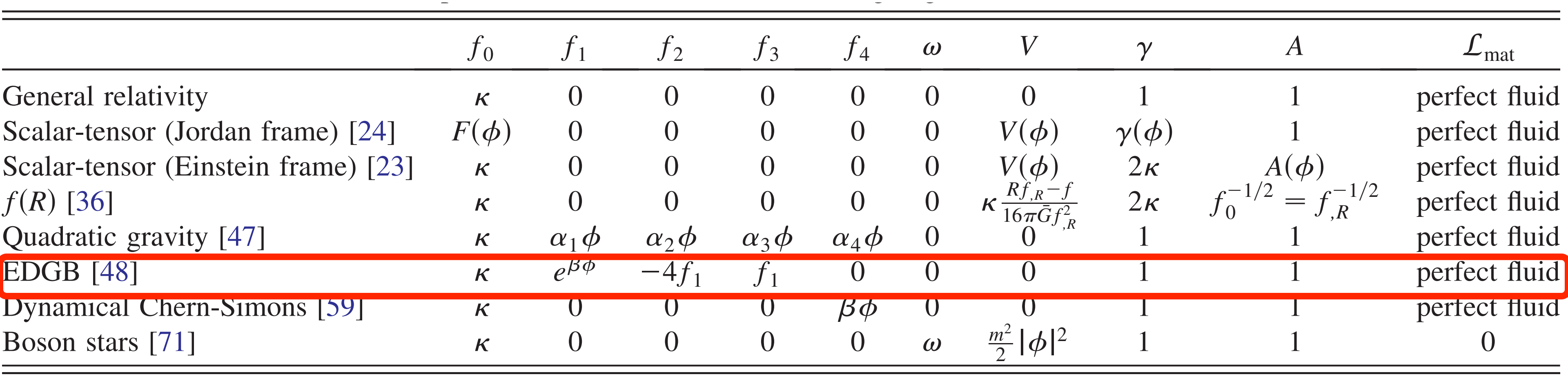
Massive gravity:

- Maybe no solutions? [Damour+, hep-th/0212155]

# A “theory of theories” of sufficient generality

$$\begin{aligned}\mathcal{L} = {}& f_0(|\phi|)R - \gamma(|\phi|)\partial_a\phi^*\partial^a\phi - V(|\phi|) + f_1(|\phi|)R^2 \\ &+ f_2(|\phi|)R_{ab}R^{ab} + f_3(|\phi|)R_{abcd}R^{abcd} \\ &+ f_4(|\phi|)R_{abcd}{}^*R^{abcd} + \mathcal{L}_{\rm mat}[\Psi, A^2(|\phi|)g_{ab}], \qquad (2)\end{aligned}$$

[Yunes & Stein, 1101.2921]
[Pani+, 1109.0928]



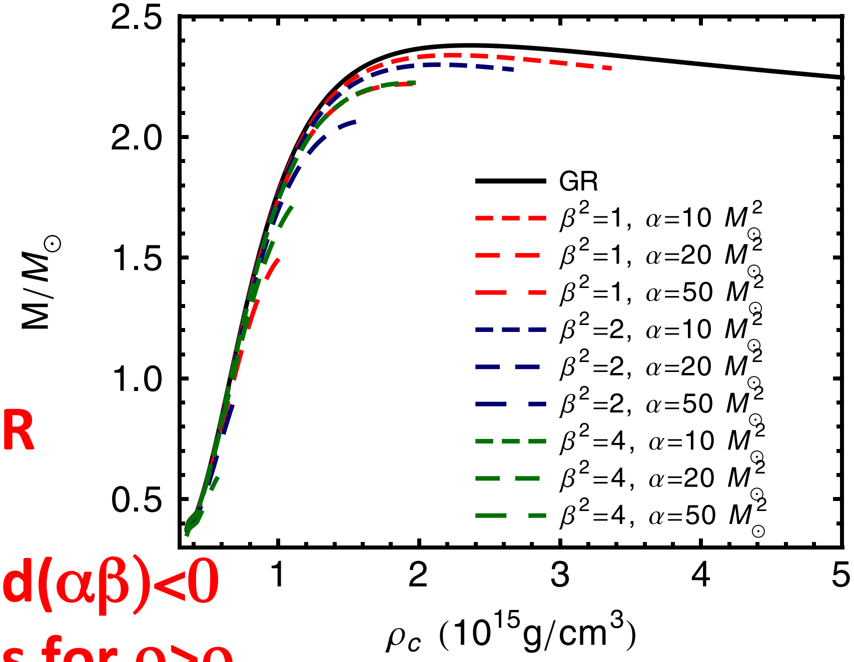
| | $f_0$ | $f_1$ | $f_2$ | $f_3$ | $f_4$ | $\omega$ | $V$ | $\gamma$ | $A$ | $\mathcal{L}_{\rm mat}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| General relativity | $\kappa$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | perfect fluid |
| Scalar-tensor (Jordan frame) [24] | $F(\phi)$ | 0 | 0 | 0 | 0 | 0 | $V(\phi)$ | $\gamma(\phi)$ | 1 | perfect fluid |
| Scalar-tensor (Einstein frame) [23] | $\kappa$ | 0 | 0 | 0 | 0 | 0 | $V(\phi)$ | $2\kappa$ | $A(\phi)$ | perfect fluid |
| $f(R)$ [36] | $\kappa$ | 0 | 0 | 0 | 0 | 0 | $\kappa\frac{Rf_{,R}-f}{16\pi\bar{G}f_{,R}^2}$ | $2\kappa$ | $f_0^{-1/2}=f_{,R}^{-1/2}$ | perfect fluid |
| Quadratic gravity [47] | $\kappa$ | $\alpha_1\phi$ | $\alpha_2\phi$ | $\alpha_3\phi$ | $\alpha_4\phi$ | 0 | 0 | 1 | 1 | perfect fluid |
| EDGB [48] | $\kappa$ | $e^{\beta\phi}$ | $-4f_1$ | $f_1$ | 0 | 0 | 0 | 1 | 1 | perfect fluid |
| Dynamical Chern-Simons [59] | $\kappa$ | 0 | 0 | 0 | $\beta\phi$ | 0 | 0 | 1 | 1 | perfect fluid |
| Boson stars [71] | $\kappa$ | 0 | 0 | 0 | 0 | $\omega$ | $\frac{m^2}{2}\|\phi\|^2$ | 1 | 1 | 0 |

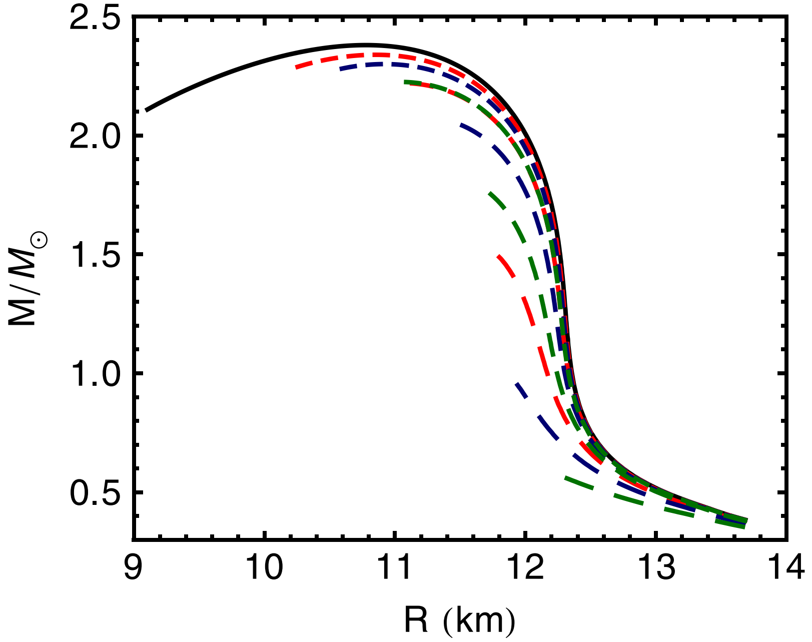
EDGB: $f_1 \equiv \dfrac{\alpha}{16\pi}e^{\beta\Phi}, \qquad 16\pi f_1(\Phi) \sim \alpha + \boxed{\alpha\beta}\Phi$

Set $\alpha > 0$ ; natural string theory choice is $\beta = \sqrt{2}$

[e.g. Kanti+, hep-th/9511071]

# Stellar structure in Einstein-dilaton-Gauss-Bonnet theory

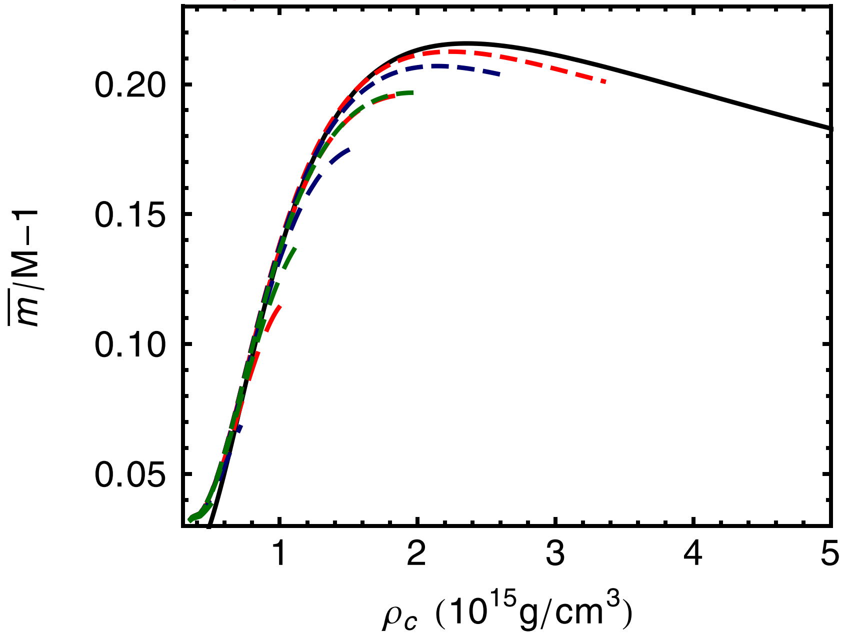


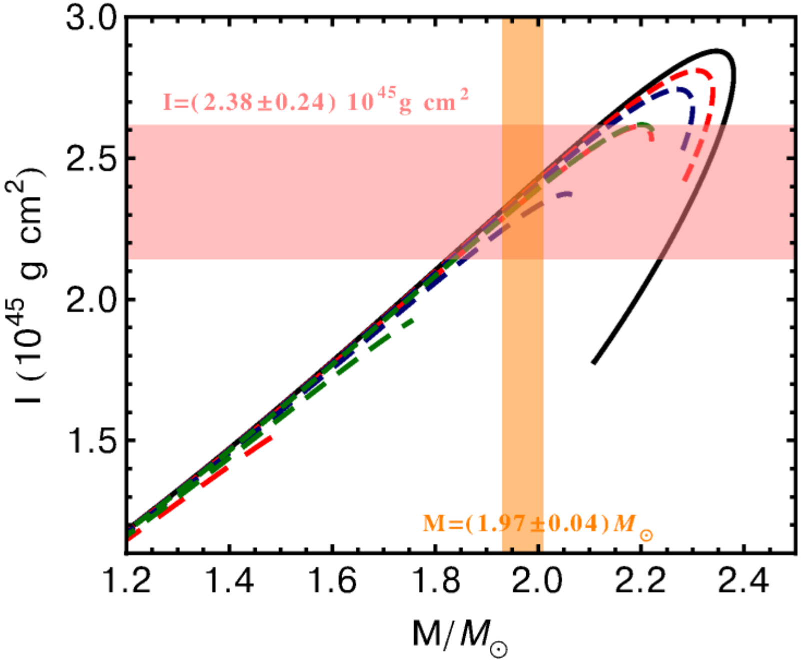


**EOS APR**

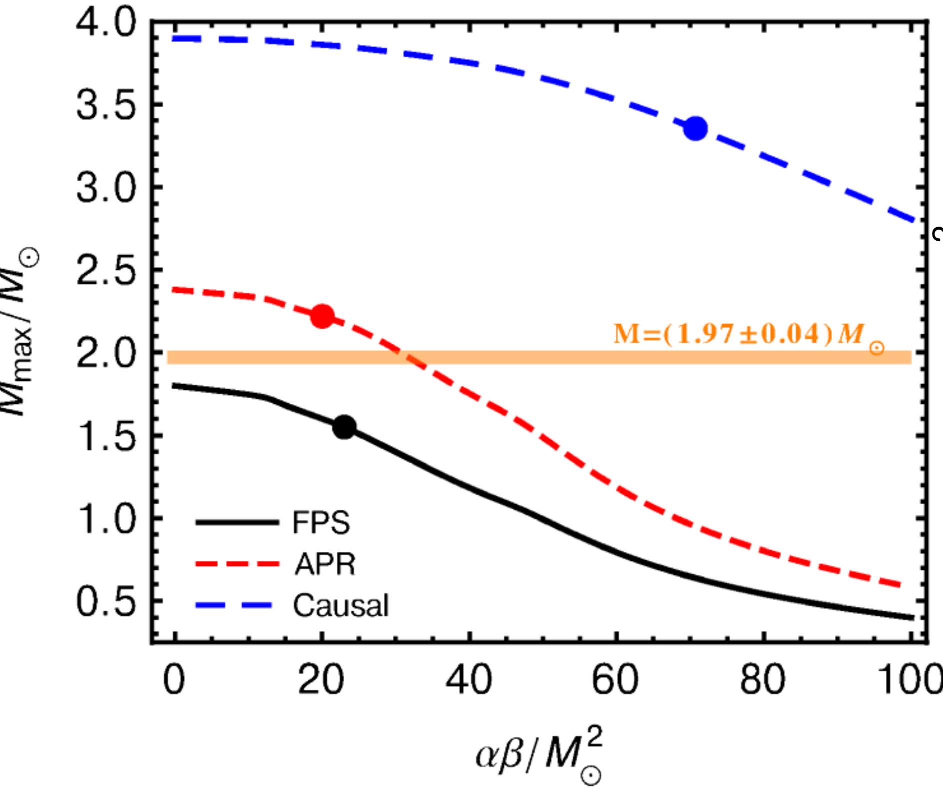
**$dM_{max}/d(\alpha\beta)<0$**

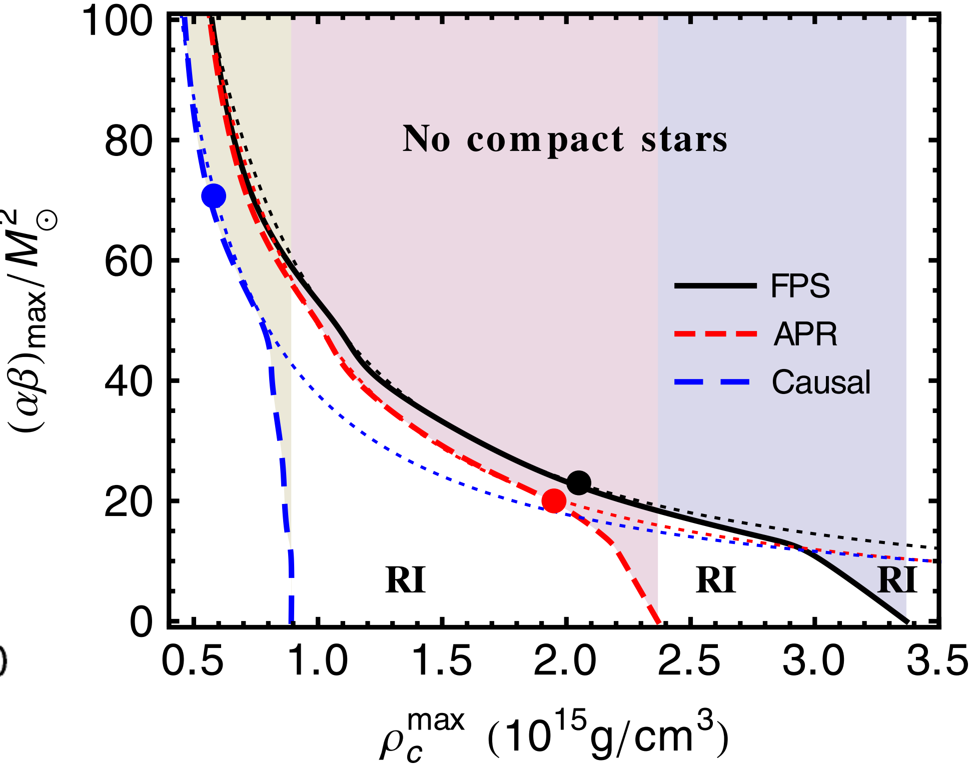
**No stars for $\rho>\rho_c$**





# Constraints on Einstein-dilaton-Gauss-Bonnet couplings





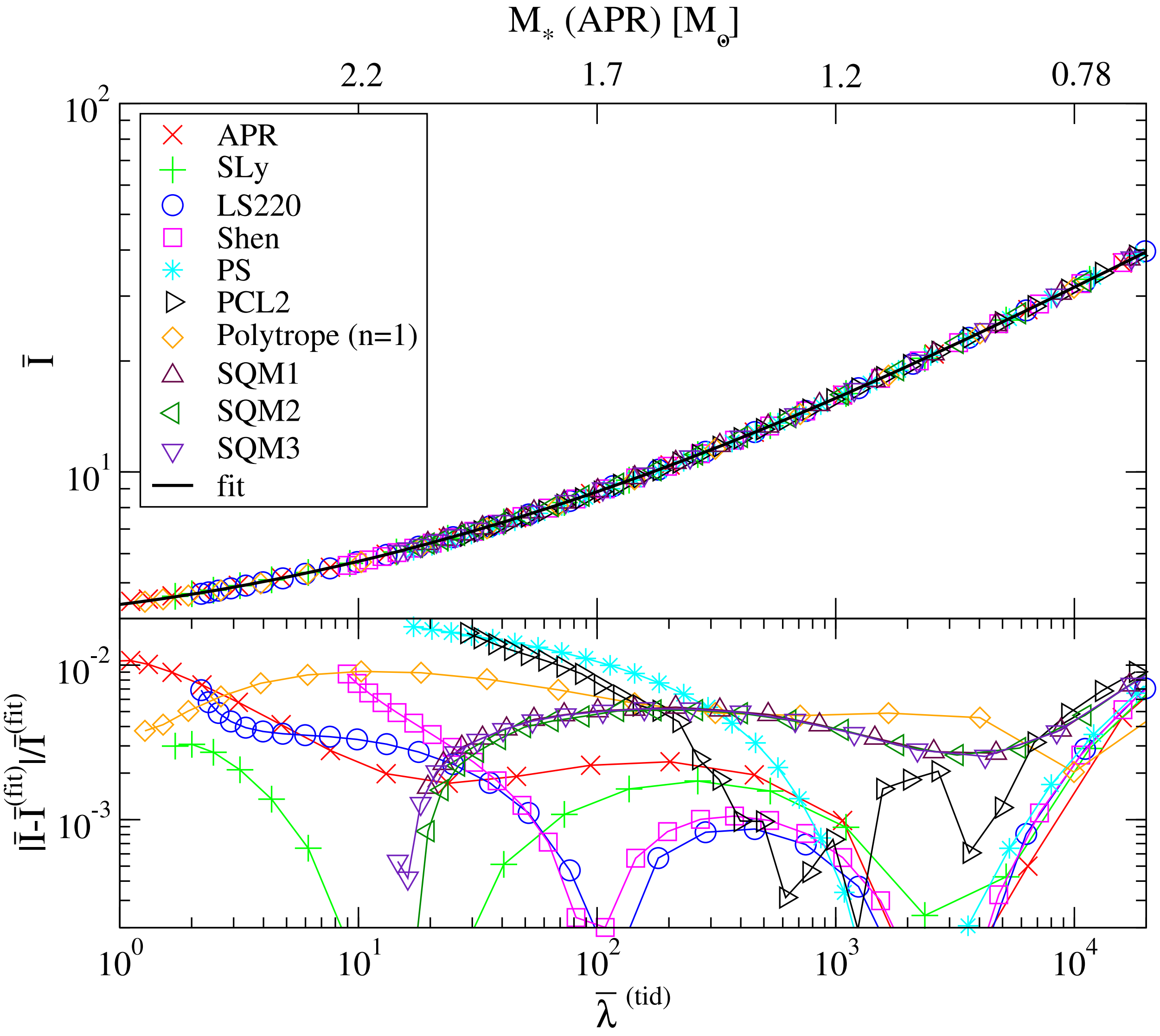
| EOS | $M_{\rm max} \gtrsim 1.4 M_\odot$ | $M_{\rm max} \gtrsim 1.7 M_\odot$ | $M_{\rm max} \gtrsim 1.93 M_\odot$ |
|---|---|---|---|
| FPS | $\alpha\beta \lesssim 30.1 M_\odot^2$ | $\alpha\beta \lesssim 13.9 M_\odot^2$ | no models |
| APR | $\alpha\beta \lesssim 50.3 M_\odot^2$ | $\alpha\beta \lesssim 41.9 M_\odot^2$ | $\alpha\beta \lesssim 33.6 M_\odot^2$ |

$$\beta = \sqrt{2} \qquad \alpha \lesssim 23.8 M_\odot^2$$

$$\frac{\alpha}{M_\odot^2} \lesssim 70\left[\frac{M_{\rm BH}}{10 M_\odot}\right]^2$$

Best bound on α already comes from NSs, not BHs!

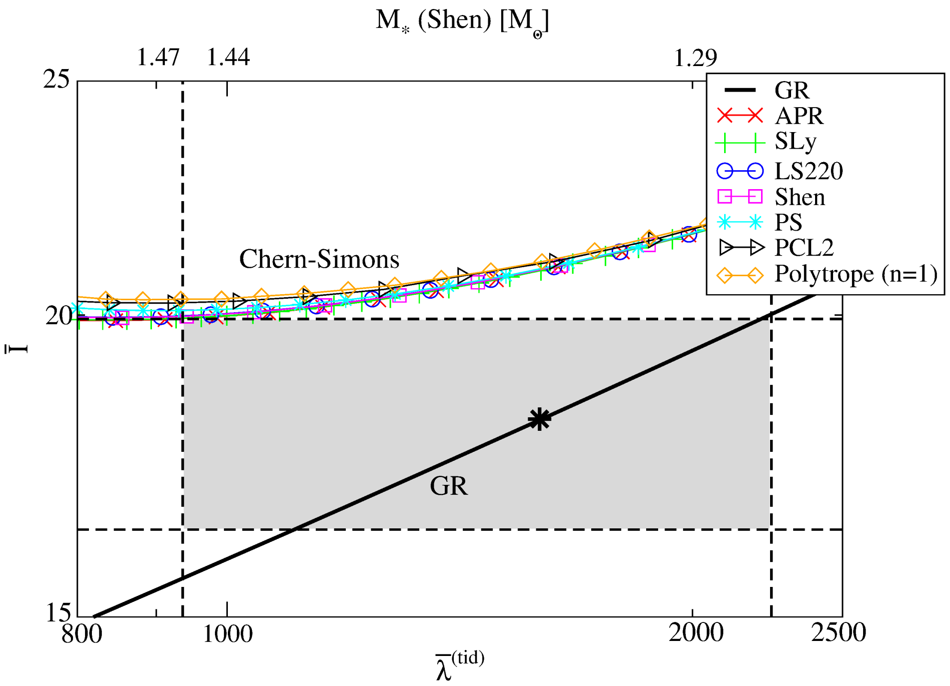
# Are we testing the EOS or gravity? Universal relations

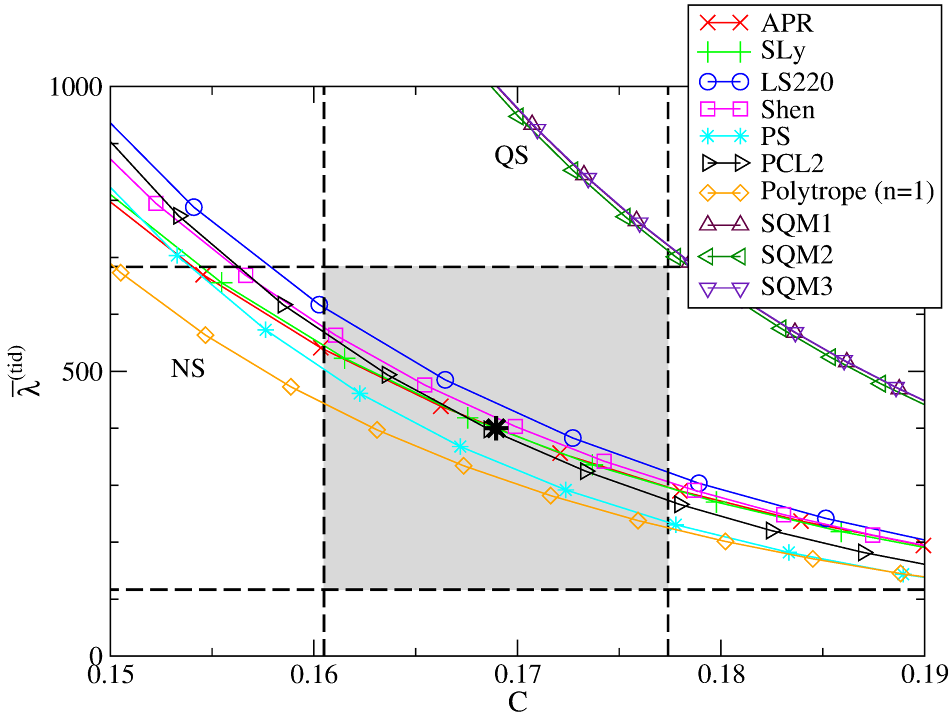


**I-Love-Q and three-hair relations could help tell theories apart**

[Yagi-Yunes,1302.4499]

# Are we testing the EOS or gravity? Universal relations





Issues:

**In most theories**

other than dynamical Chern-Simons (e.g. scalar-tensor, Eddington-inspired gravity)

**universal relations same as in GR**

$R^2$, Lorentz-violating theories: universal relations not studied

Massive gravity, general Horndeski: no studies of stellar structure

**All theories in one sweep? post-TOV formalism**

# Compact binaries in modified gravity

# Gravitational-wave tests

**Polarization:**

Up to **six polarization states**

**Propagation:**

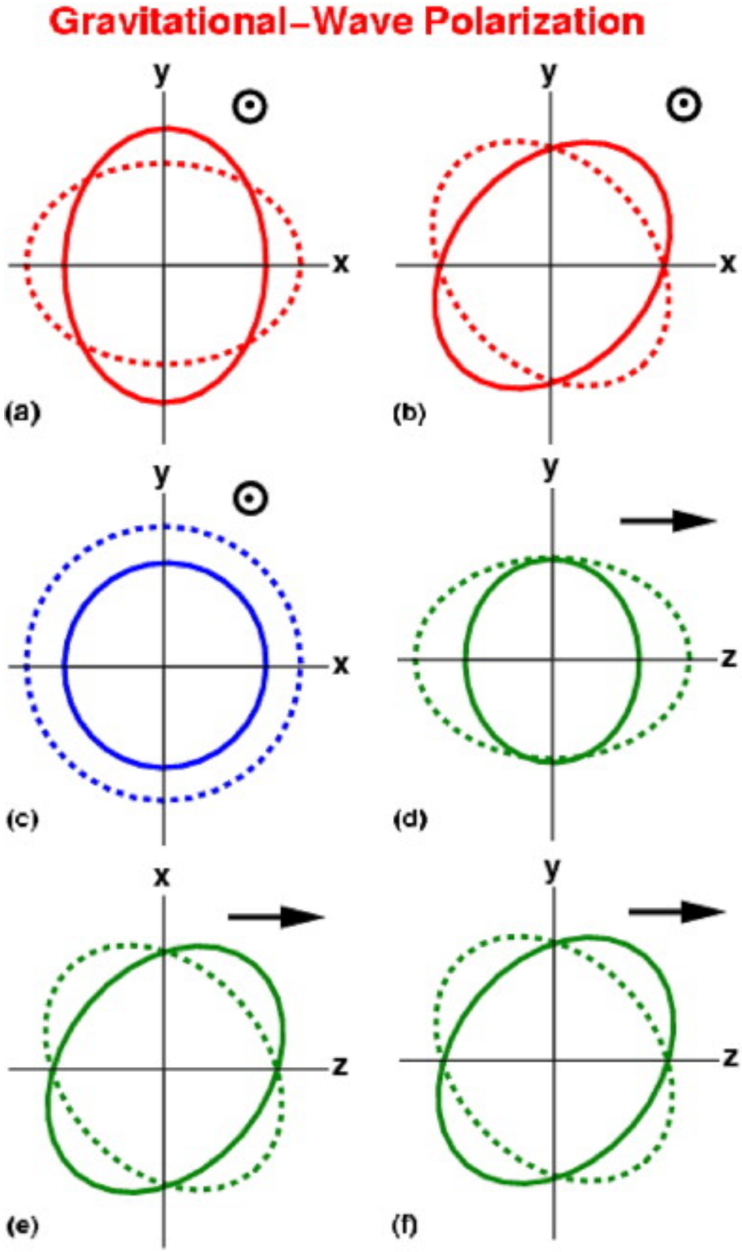
**$m_g \neq 0$** changes dispersion relation
GWs travel slower than EM waves

eLISA: black hole inspirals set **$m_g < 10^{-26}$eV**
($10^4$-$10^6$ better than Solar System)
[Will, gr-qc/9709011]
[EB+, gr-qc/0411129; 1107.3528]

AdLIGO: graviton oscillations in bigravity if
**$m_g > 10^{-22}$eV** [Narikawa+, 1412.8074]

**Energy flux:**

E.g. scalar-tensor theories predict
**dipole radiation** because
(inertial mass)≠(gravitational mass)



[e.g. Gair+,1212.5575]

# Potential: post-Newtonian effects with a mass term

$$S = \frac{1}{16\pi}\int\left[\phi R - \frac{\omega(\phi)}{\phi} g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} + M(\phi)\right](-g)^{1/2} d^4x$$

$$+ \int \mathcal{L}_{\mathrm{M}}(g^{\mu\nu}, \Psi) d^4x,$$

[Alsing+, 1112.4903]

✓ Shapiro time delay **(Cassini)**

✓ Nordtvedt effect **(Lunar Laser Ranging)**

✓ Orbital period derivative **(binary pulsars)**

$$\frac{\dot{P}}{P} = -\frac{8}{5}\frac{\mu m^2}{r^4}\kappa_1 - \frac{\mu m}{r^3}\kappa_D S^2$$

$$\kappa_1 = \mathcal{G}^2\left[12 - 6\xi + \xi\Gamma^2\left(\frac{4\omega^2 - m_s^2}{4\omega^2}\right)^2 \Theta(2\omega - m_s)\right],$$

$$\kappa_D = 2\mathcal{G}\xi\frac{\omega^2 - m_s^2}{\omega^2}\Theta(\omega - m_s),$$

$$\xi = \frac{1}{2 + \omega_{\mathrm{BD}}},$$

$$\mathcal{G} = 1 - \xi(s_1 + s_2 - 2s_1 s_2),$$

$$\Gamma = 1 - 2\frac{s_1 m_2 + m_1 s_2}{m}.$$

1) No dipole if **$S=s_1-s_2=0$** **(need NS-BH!)**

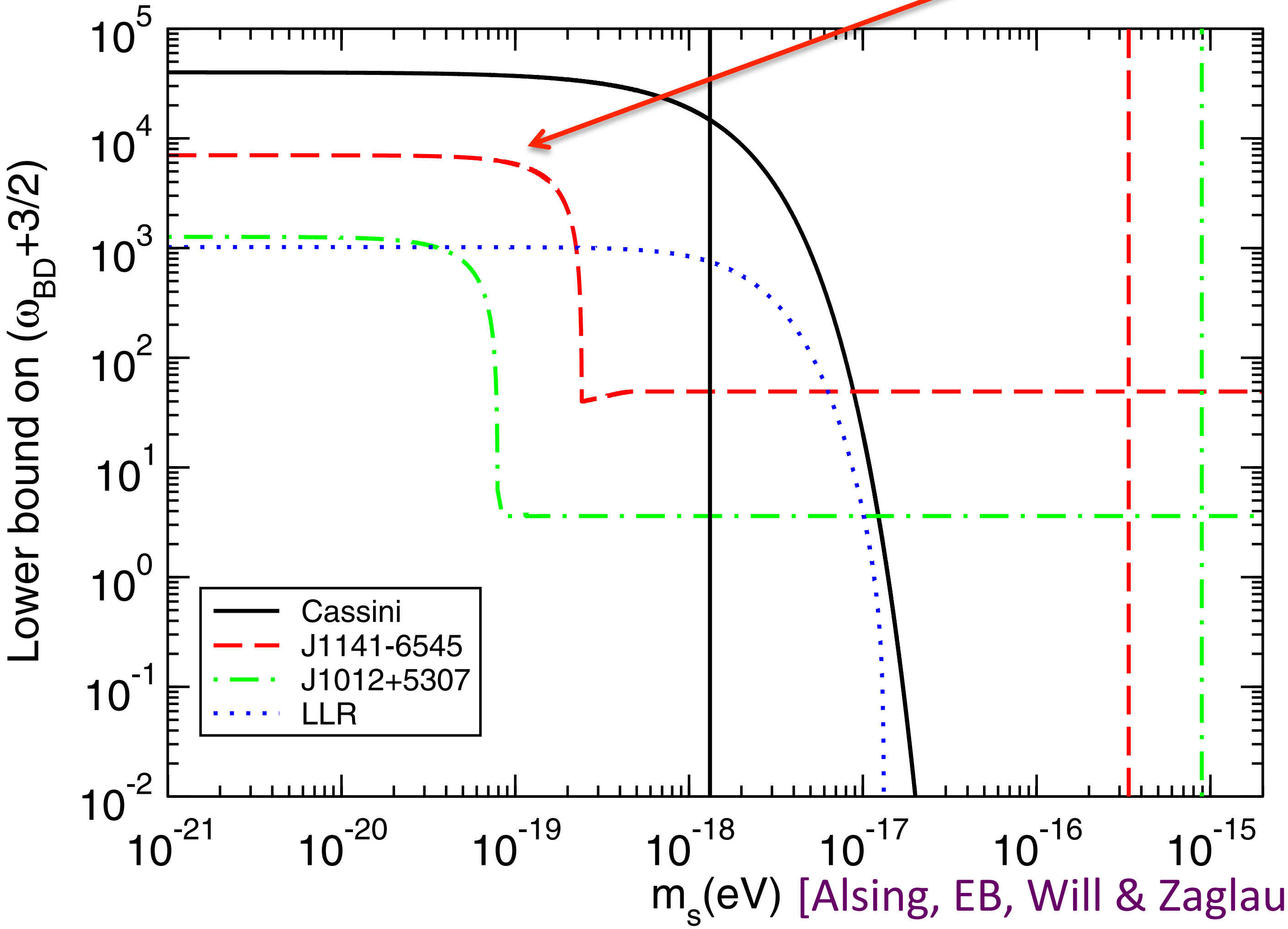
2) For binary black holes **$\Gamma$=0**: indistinguishable from GR?

# Are massive scalar fields viable?

Bounds from:

- ✓ Shapiro time delay: **$\omega_{BD}>40{,}000$** [Perivolaropoulos, 0911.3401]
- ✓ Lunar Laser Ranging
- ✓ Binary pulsars: **$\omega_{BD}>25{,}000$** [Freire++, 1205.1450]

WD-NS with e=0.172



[Alsing, EB, Will & Zaglauer, 1112.4903]

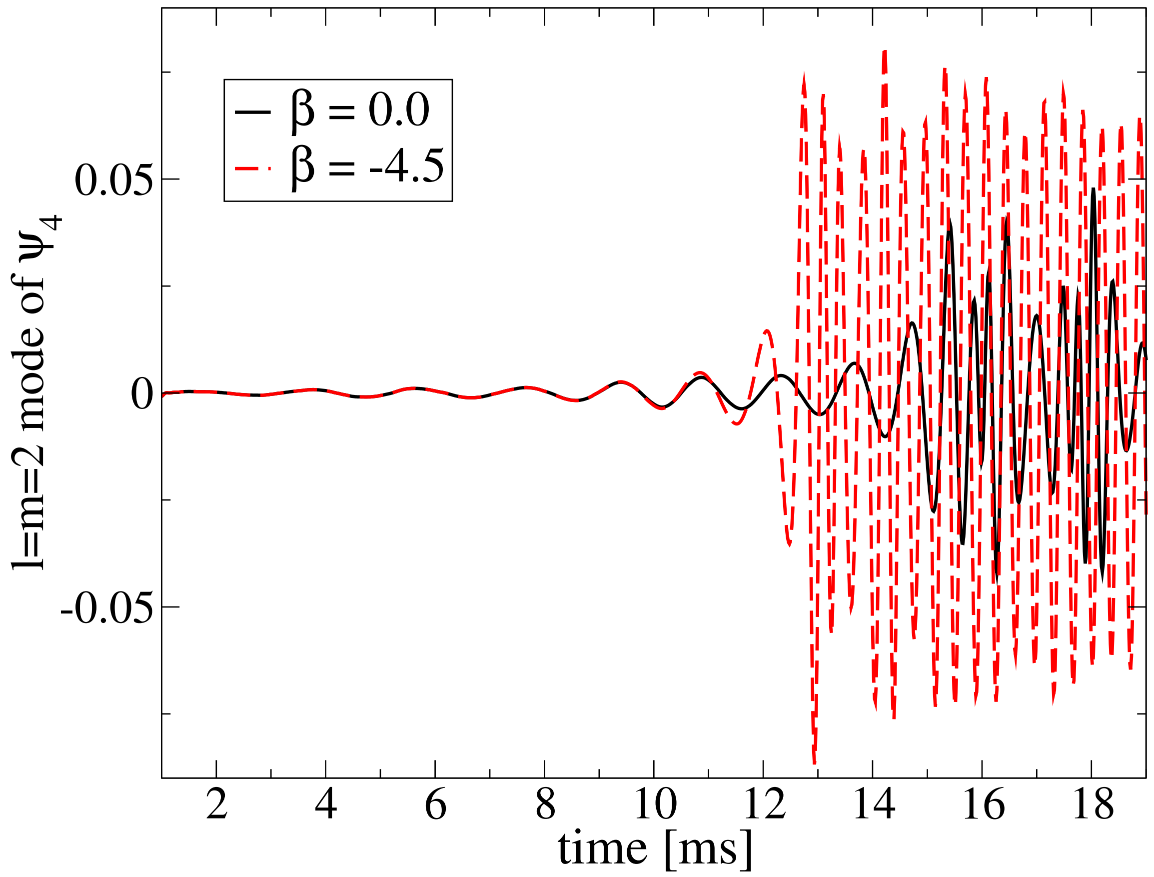
# Generalized no-hair theorems for binary black holes

- ✓ To leading order
  [Will & Zaglauer ‘89]
- ✓ Equations of motion up to 2.5PN
  [Mirshekari & Will, 1301.4680]
- ✓ To all orders in extreme mass ratio limit
  [Yunes+, 1112.3351]

**Key assumptions:**

1) No matter
   [Barausse+, 1212.5053]
2) Scalar field has zero potential (e.g. no mass term)
   [Healy+, 1112.3928]
3) Asymptotic flatness, scalar field asymptotically constant
   [Horbatsch-Burgess, 1111.4009; Berti+, 1304.2836]

# Matter: dynamical scalarization

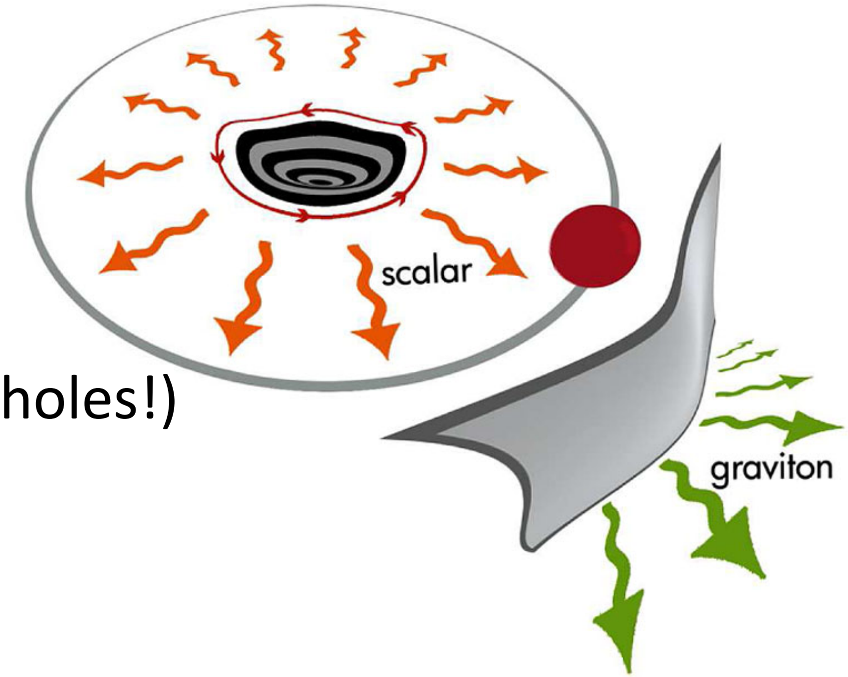


[Barausse-Palenzuela+, 1212.5053; 1310.4481; Taniguchi+, 1410.0738]

## Potentially detectable with Advanced LIGO?

[Sampson+, 1407.7038]

# Expect the unexpected: an example
# Massive scalars and superradiant instabilities



Superradiance when $\omega < m\Omega_H$

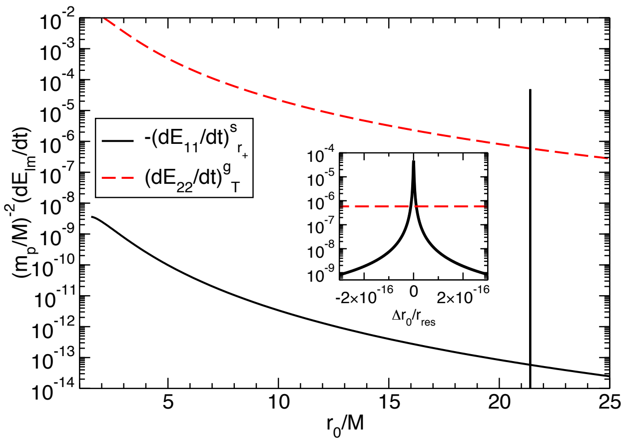
Strongest instability: $\mu_s M \sim 1$
[Dolan, 0705.2880]

For $\mu_s = 1\mathrm{eV}$, $M = M_{\mathrm{sun}}$ : $\mu_s M \sim 10^{10}$
Need light scalars (or primordial black holes!)

**Negative** scalar flux at the horizon
close to superradiant resonances at

$$\omega_{\mathrm{res}}^2 = \mu_s^2 - \mu_s^2 \left( \frac{\mu_s M}{l+1+n} \right)^2 , \quad n = 0, 1, ..$$

**“Floating orbits”** when $\boxed{\dot{E}_p + \dot{E}^g + \dot{E}^s = 0}$



**Compatible with current experiments!**

[Press+Teukolsky 1972, Detweiler 1980, Cardoso++ 1109.6021]

# What can GWs do for strong-gravity tests?

- ✓ Scalar fields:

  Small couplings → small deviations from GR in the dynamical regime?

  - \+ Scalar fields suggest the answer is no!

    Spontaneous/dynamical scalarization, floating orbits

    Strong constraints [talk by Kramer]…but tensor-multiscalar?
  - \- Is nature hiding deviations from us?

    Scalarization: not a cosmological attractor [Damour-Nordtvedt 1993]

    Floating orbits: fine tuning…

- ✓ Higher-order gravity

  Well posed? Astrophysical corrections Planck-scale suppressed?

  …not a problem? effective field theory

- ✓ Parametrized frameworks:

  "parametrized post-Einstein" [Yunes+] for binaries

  "bumpy Kerr" [Hughes, Glampedakis, Johannsen, Cardoso+…] for BHs

  "post-TOV" [Glampedakis+] for NSs

- ✓ Precision gravitational-wave astronomy: what control of systematics do we need to test strong-field gravity? [Agathos' talk]